Optimal Design of Composite Structures using the Particle Swarm Method and Hybridizations

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Master Dissertation

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Resumo

A otimização é uma área importante da Engenharia geralmente devido ao potencial de economizar custos e melhorar a segurança ao nível estrutural. As estruturas em engenharia são tipicamente complexas, e o Método dos Elementos Finitos é frequentemente necessário para avaliar tais estruturas. Encontrar metodologias de otimização para resolver tais problemas é desafiante, mas necessário, principalmente na otimização de compósitos estruturais. Este trabalho começa por apresentar uma revisão de literatura original dos mais recentes problemas de otimização estrutural e metodologias baseadas em Algoritmos Genéticos (GA) e Otimização por Enxame de Partículas (PSO). A novidade desta revisão de literatura está relacionada à categorização das publicações existentes, à extensão mais ampla dos problemas a todas as áreas da engenharia, com maior foco nas técnicas de otimização. Esta revisão visa apoiar futuros desenvolvimentos na otimização de estruturas. Um dos problemas de otimização mais comuns em estruturas compósitas é encontrar os parâmetros ótimos do material para minimizar o peso. Noutra perspetiva, a otimização baseada na robustez é uma abordagem que visa considerar a variabilidade da resposta do sistema devido à incerteza nas variáveis de projeto ou nas propriedades dos materiais. Nestas condições, para além do problema de otimalidade relacionado com a minimização do peso das estruturas, tem-se o problema adicional da maximização da robustez traduzido no problema de minimização da variabilidade da resposta estrutural. Anteriormente os GAs foram utilizados para encontrar um conjunto de ótimos de Pareto relativamente tanto ao peso mínimo quanto à robustez máxima da estrutura compósita. Este trabalho apresenta uma nova abordagem baseada no PSO, e outra baseada em uma hibridização do PSO com GA, para resolver o problema de otimização bi-objetivo. Os resultados mostram que ambas as abordagens baseadas no PSO levam a um conjunto de Pareto melhorado e com mais soluções para o mesmo número de avaliações, quando comparados aos resultados da literatura.

Optimal Design of Composite Structures using the Particle Swarm Method and Hybridizations

Abstract

Optimization is an important area of research in Engineering, usually due to the potentiality of saving costs and improving structural safety. Engineering structures are typically complex, and the Finite Element Method (FEM) is frequently required to evaluate such structures. Finding optimization methodologies to solve such problems is challenging but required, namely in composite design optimization. This work starts to present an original literature review of the most recent structural optimization problems and methodologies based on Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). The novelty of this review is related to the categorization of the existing publications, the broader extension of the problems to all the engineering fields and the clear focus on the optimization techniques themselves. This review is aimed to support future research and developments for the optimization of structures. One of the most common optimization problems in composite structures is finding the optimal parameters of the material to minimize the weight. From another perspective, Robustness Design Optimization is an approach that aims to consider the variability of the system response due to uncertainty in design variables or material properties. Under these conditions, in addition to the optimality problem related to minimizing the weight of the structures, the additional problem of maximizing robustness translated into the problem of minimizing the variability of the structural response. Previously, GAs were used to find a set of Pareto optima regarding the composite structure's minimum weight and maximum robustness. This work presents a new approach based on PSO, and another based on a hybridization of PSO with GA, to solve the bi-objective optimization problem. The results show that both PSO-based approaches lead to an improved Pareto set with more solutions for the same number of evaluations compared to the literature results.

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1 Introduction

Engineering problems have been growing in terms of complexity. Finding analytical solutions for the most common Engineering problems is improbable and impracticable. Therefore, numerical solutions have been preferable. Optimization is shown to be crucial for engineering problems and is demanded to reduce complexity and time costs (Schäfer 2006). "Optimization" can be defined as the process of obtaining the maxima or minima in a set of available alternatives (Ghosh et al. 2019). There is almost no Engineering field in which optimization is not involved. Due to its importance, optimization techniques have grown in recent years (Chinneck 2007). Over the last decades, and/or bio-inspired algorithms have been used more frequently, changing the optimization paradigm (Freitas et al. 2020). Three of these best-known techniques are GA, Ant Colony Optimization (ACO) and PSO. GA was first introduced in 1975 by John Holland (Holland 1984), and Darwin's principle of natural selection inspired it. Some years later, in 1991, Dorigo et al. proposed the ACO method, which initially aimed to solve the well-known NP-Hard Traveling Salesman Problem (Dorigo et al. 1999). Since then, the scientific community has improved both GA and ACO (Chelouah and Siarry 2000). ACO was used to solve non-combinatorial Optimization problems (Alharbi et al. 2020). In 1995, a novel population-based computational algorithm, PSO (Eberhart and Kennedy 1995; Kennedy and Eberhart 1995), was initially proposed and developed by the electrical engineer Russell Eberhart and the social psychologist James Kennedy. After these introductions, plenty of publications have been conducted on such novel techniques' improvements. For instance, continuous GAs have been designed to work in continuous search spaces instead of discretized traditional GAs (Chelouah and Siarry 2000). Another GA variant with an improved chromosome selection operator is developed to deal with constraint optimization problems (Yokota et al. 1996). In other variations of GA, considering different encoding methods is possible, such as real value and binary value representation and the requirement of specific crossover and mutation, which are easier in the case of binary, octal or hexadecimal representations. In PSO, Clerc and Kennedy (Clerc and Kennedy 2002) have developed an improved velocity update to avoid local optima. More recently, a self-adaptive swarm algorithm has been developed with the goal of turning algorithm constant parameters into variables (Salgotra et al. 2021). This is one of many other novel developments on algorithmic improvements currently held.

Structural optimization focuses on improving the structure's safe and efficient design (Kashani et al. 2021). Engineering projects can be more straightforward or more complex. More complex structures lead to high expensive operational costs. Structural engineers are then looking for a practical design that leads to lower costs without compromising the necessary conditions, such as assuring stress limits. Optimality criteria lead, in its turn, to additional complexities. Nonlinearities, non-convex problems, and discontinuous search spaces are common in structural engineering problems (He and Wang 2021; Pakalidou et al. 2020; Iancovici et al. 2022). Many works have been developing metaheuristic algorithms to optimize the design of structural engineering problems (Kashani et al. 2021). According to the authors, two main factors have given rise to optimization to be applicable in practical engineering: high-performance calculations at low cost, rapid advances in design optimization requiring hundreds of design variables and satisfying given constraints.

Optimization is even more important in some progressive fields of engineering, like aeronautics and automotive industries (Nikbakt et al. 2018). Composite materials are increasingly utilized in these industries due to their blend of high strength, toughness, stiffness, and low weight (Barman et al. 2021). However, there are problems associated with composite materials. Some of them are susceptibility to anomalies manifesting during manufacturing or the service period, fibre breakage, matrix cracking, delamination, and fibre buckling (Barman et al. 2021). From these, delamination damage is more critical. Due to the high number of design variables and objective functions associated with composite structures (Nikbakt et al. 2018), added to their susceptibility to damages of the referred kind, considerable work has been done in composite structure optimization (António and Hoffbauer 2009; António 2002; António and Hoffbauer 2008; António 2001, 2006; António and Hoffbauer 2010; António and Hoffbauer 2007; António et al. 1995, 1996; António and Hoffbauer 2013; António 2013; António et al. 2000; António 2009b; António 1999; António 2014; António 1995; das Neves Carneiro and António 2019b; das Neves Carneiro and Antonio 2018; Soeiro et al. 1994; das Neves Carneiro and António 2019a; António and Hoffbauer 2017; António and Hoffbauer 2016; Antonio 1993; das Neves Carneiro and António 2019; António and Hoffbauer 2017; das Neves Carneiro and António 2020; António 2011; António and Lhate 2003; das Neves Carneiro and António 2021).

Along with the development of methodologies to solve composite structure optimization problems, a robust design optimization approach is proposed by Antonio and Hoffbauer (António and Hoffbauer 2017) for the minimization of the structure weight and the variability of the structural response due to the uncertainty in design variables or material properties. The feasibility robustness is there defined through the determinant of the variance-covariance matrix of constraint functions. The bi-objective optimization problem is in their work solved using GA technique, where elitism is used to preserve the best individuals. Since they are preserved, a direct comparison between the new individuals and the best ones is possible. This comparison is useful so that a larger Pareto set can be achieved throught the dominance technique (António 2013).

The implementation of PSO to solve the multi-objective problem of the robust design optimization of the laminated shell structure would have the advantage of considering a continuous domain so that more numerical solutions can be found through exploitation. However, the difficulty of adapting the same methodology to PSO is related to that PSO does not preserve the best individuals in the local population.

The main scope of the present work is to perform a literature review and verify recent applications and methodologies performed on structural optimization, using GA, PSO and their hybridizations, to perform a PSO numerical demonstration so that beginner users can learn how to implement and perform the steps related to PSO technique and to use and adapt PSO and its hybridizations to the methodology used by Antonio and Hoffbauer (António and Hoffbauer 2017) to the shell structure.

This work is organized as follows: the next chapter is a literature review of the most recent applications and developments related to PSO, GA and their hybridizations on the structure optimization problems; chapter 3 is a numerical application of PSO in a benchmark function; chapter 4 aims to expose the methodology to solve robust design optimization problems and to present the proposed optimization methods based on PSO and its hybridizations; chapter 5 aims to present the application problem and the results related to the proposed multi-objective methods; chapter 6 concludes the present work and presents suggestions as future works.

2 State of the art

The main scope of the review that is done in this chapter is to collect and verify recent applications and methodologies performed on structural optimization, using GA, PSO and their hybridizations. Due to its novelty, composite structural optimization is also a review topic. The novelty of this review is related to the following factors: structural optimization is reviewed generically; composite structural optimization using non-traditional optimization is reviewed in more detail; a more detailed overview of the used optimization techniques and model simplifications is visually shown. This review is organized as follows: the next section, a background of Genetic Algorithms and Particle Swarm Optimization is given; Sections 2.2 and 2.3 focus on the literature review of a variety of works related to structural optimization and previous reviews on the topic; the last section remarks on the most critical findings.

2.1 Background of Concepts

Single and Multiple Objectives

Let N be the number of design variables $\mathbf{X} = \{x_1, \dots, x_N\}$ and $f(\mathbf{X})$ an objective function. The number of inequality constraints is given by n_g and the functional inequality constraints are $g_i(\mathbf{X}), i = 1, \dots, n_g$. Moreover, the number of equality constraints is given by n_h and $h_j(\mathbf{X}), j = 1, \dots, n_h$, are the functional equality constraints. $\mathbf{X} \in S^N$, where $S^N = [x_{1_L}, x_{1_U}] \times \dots \times [x_{N_L}, x_{N_U}]$ is the search space. Each interval in this Cartesian product is associated to the side constraints of the optimization problem formulation. The standard form of a single-objective, constrained optimization problem is given in (2.1a).

$$
\begin{aligned}
&\text{Minimize:} \quad f(\mathbf{X}) \\
\text{Subject to:} \quad & g_i(\mathbf{X}) \leq 0, i = 1, \dots, n_g \\
& h_j(\mathbf{X}) = 0, j = 1, \dots, n_k \\
& x_{k_L} \leq x_k \leq x_{k_U}, k = 1, \dots, N
\end{aligned}
\tag{2.1a}
$$

In a more general case, the objective and constraint functions can be explicit or implicit functions of the design variables. Implicit functions are used when numerical simulation (e.g., a finite element simulation) appears to evaluate a response function, such as stress and/or displacement values. Using numerical simulations for more complex problems can be time-consuming. Some alternative techniques, denoted by surrogate models, such as Artificial Neural Networks (Villarrubia et al. 2018), Response Surface Modelling (Roux et al. 1998) and the Kriging method (Kaymaz 2005), are being used to save significant computational time. Variables can be restricted to integer or discrete values (Fitas et al. 2022). In general, the local algorithms have difficulties solving problems with these variables. However, most global algorithms can be adapted to solve them. The side constraints in equation (2.1a) can be effectively and easily handled by a direct implementation. The remaining constraints in equation (2.1a) are included in the minimization procedure using several methodologies depending on the adopted optimization method. The procedure of finding the optimal solution $\mathbf{X}^*$ consists of finding the combination of values that may result in the best objective function value. This optimal solution must satisfy equality, inequality, and side constraints as well (Venter 2010).

When two or more objectives exist in an optimization problem, let there have n_f objectives, one may be referred to as multiobjective optimization problems, given in (2.1b).

$$\begin{aligned} \text{Minimize:} \quad & \mathbf{F}(\mathbf{X}) = (f_1, \ldots, f_{n_f}) \\ \text{Subject to:} \quad & g_i(\mathbf{X}) \le 0, i = 1, \ldots, n_g \\ & h_j(\mathbf{X}) = 0, j = 1, \ldots, n_k \\ & x_{k_L} \le x_k \le x_{k_U}, k = 1, \ldots, N \end{aligned} \tag{2.1b}$$

According to Gunantara (Gunantara 2018) and António (António 2020), two main methods divide the way one deals with multiobjective optimization: Pareto dominance and scalarization. In Pareto dominance, a set of solutions called the Pareto set is defined as solutions that dominate all the other solutions within the search space, i.e., the solutions that are not dominated. Therefore, no element of the Pareto set can dominate any of the others. Dominance can be defined by the mathematical statement in (2.2), where $\mathbf{X_1}, \mathbf{X_2} \in S^N$ (Sun et al. 2019).

$$\mathbf{X_1} \prec \mathbf{X_2}: \forall i = 1, \ldots, n_f: f_i(\mathbf{X_1}) \le f_i(\mathbf{X_2}) \wedge \exists i \in \{1, \ldots, n_f\}: f_i(\mathbf{X_1}) < f_i(\mathbf{X_2}) \tag{2.2}$$

The Pareto set is a set with multiple valid solutions. An adequate solution is left for the user to choose. In scalarization, the various objectives are combined in a single fitness function to transform the problem into a single optimization problem (António 2020).

Classification on Structural Optimization

Structural optimization is a specific optimization topic. It can be divided into three categories (Mei and Wang 2021; Xiao et al. 2013): size optimization, shape optimization and topology optimization. In the context of structural design with FEM size optimization, cross-sectional areas, thicknesses, and other geometric variables are usually considered design variables. Here, the members of the structure have the same position and their nodal coordinates the same reference coordinates. In shape optimization, the coordinates of the nodes of the structure are variable, being then considered as defining the shape of the boundaries. According to Rajan (D. 1995b), shape optimization achieves more promising results when compared to size optimization. Topology optimization is associated to optimal material distribution concerning a predefined domain. Topology optimization is the most efficient type compared to the other two (Ruiyi et al. 2009) but is more challenging (Bendsoe and Sigmund 2003; Kaveh and Laknejadi 2013), leading to non-convex and discontinuous design space (Hajela and Lee 1995, 1997). In Figure 1, the three structural optimization types are illustrated with an exemplary structure. A fourth group is added to the previous ones: the multi-objective optimization group (Mei and Wang 2021; Xiao et al. 2013). Here, the three types above can be combined. Although shape and topology variables are usually combined using multilevel optimization (Azid et al. 2002b, 2002a; D. 1995a), researchers (Souza et al. 2016; Tejani et al. 2018) also combine all types of variables in order to perform a more complex structural design optimization.

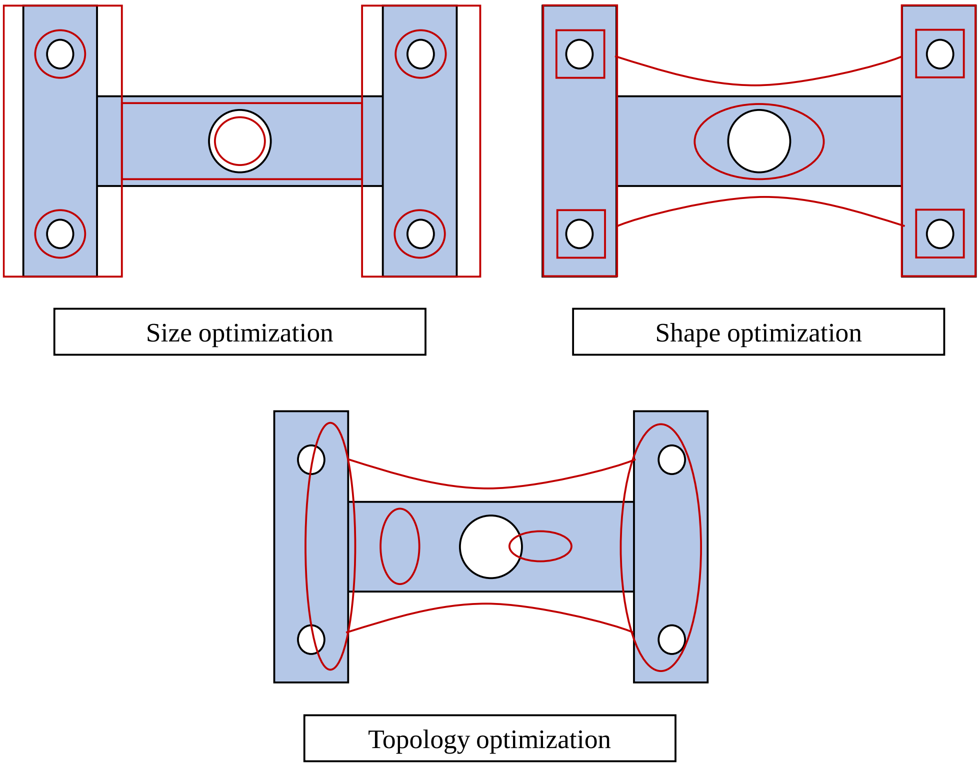

Figure 1. Illustration on size, shape and topology optimization. Adapted from (Gao et al. 2020)

Genetic Algorithms (GAs)

GAs are bio-inspired population-based algorithms emulating the principle of Darwin's theory of evolution (Darwin 1859). GAs combine several operators such as parent selection, crossover, mutation and survivor selection (António 2020). Typically, binary string format is the most usual format for representing the chromosomes. These chromosomes are denoted by genotype and are used for the representation of the values of the design variables (phenotype), within the search space. According to a certain fitness function, a fitness value is assigned to the chromosomes (Michalewicz and Schoenauer 1996).

Along with the generations, the population of chromosomes is changed chiefly depending on the fitness evaluation. Overall, the evolutionary process manipulates the population, aiming to improve the average fitness of the chromosomes over time (Sampson 1976).

1. Parent Selection

Parent selection is a stochastic mechanism that randomly selects individuals with a bias towards the best individuals of a population, according to two different methodologies: fitness and ranking selection. In the last one, despite this ranking being created based on fitness values, individuals are selected based on their hierarchy within the population. In general, various operators are developed in the literature to improve the efficiency of GA. Some of the most known are Roulette Wheel selection, rank selection, tournament selection, Boltzmann selection or stochastic universal sampling (António 2020). For instance, Roulette Wheel selection is based on allocating fitness-based probabilities for the correspondent individuals and randomly selecting them to form the next generation of individuals (Jebari and Madiafi 2013; Katoch et al. 2021). As referred to in the literature (Katoch et al. 2021), they all have advantages and disadvantages. For instance, Roulette Wheel selection is easy to implement, but premature convergence is a risk to the operator; in turn, the Boltzmann selection operator is less prone to premature convergence, but it is computationally expensive.

2. Crossover

Crossover is the most critical operator regarding the time convergence of the evolutionary process (Murata and Ishibuchi 1996). From the values of the parent chromosomes of the previous generation, the crossover is intended to combine and mix the information of both to create new chromosomes. The aim is to inherit the information of the previous generation at the same time new strings are being created. According to Herrera et al. (Herrera et al. 2003), different operators can be grouped into Discrete, Aggregation-Based and Neighbourhood-Based Crossover Operators. In binary encoding, single-point, multipoint, uniform and segmented crossover are the most known operators (Pachuau et al. 2021; Eshelman et al. 1989). For example, in the single-point crossover, parent strings are divided into two parts of different sizes. One for both parts is directly passed to the offspring, whereas in the other part each bit is exchanged. In multipoint crossover, the process is the same as that occurred in single-point optimization, but the parent strings are divided into more than two parts. In real encoding, mean-centric (Ono 1997) and parent-centric (Deb et al. 2002) recombination operators are used (Deb and Jain 2011). In recent years, some studies have reported synergies are found when multiple crossover operators are considered, see e.g. (António 2009a; Yoon and Moon 2002).

3. Mutation

Mutation is an operator that involves a small percentage of bits in relation to the entire population. With a given probability, bits are converted from 0 to 1 and 1 to 0. In other codification types, the process must be different. For instance, in real encoding, the input values are added to a small number around 0 following a uniform or normal distribution. On the limit, the mutation operator assures the convergence in GA's probabilities or the population diversity increasing depending on the mutated gene (Rudolph 1998; António 2001, 2002; das Neves Carneiro 2020).

4. The survivor selection and the elitist strategy

Survivor selection is a deterministic mechanism that aims to select individuals who survive in the next generation. Despite the operator's similarity concerning parents' selection, survivor selection is not stochastic and can be based on the corresponding age or fitness. Elitist strategy is added into account. It aims to directly select the best individuals to ensure that the best, along with the generations, co-exist in the current generation (António 2001, 2002; das Neves Carneiro 2020).

The basic steps of a GA are illustrated in the flowchart represented in Figure 2. Firstly, a random population of a fixed number of elements is created. After evaluating the fitness values, if the stopping criteria are met, the algorithm stops. Otherwise, the population is subjected to the selection, crossover and mutation operators, characteristic of GA. A new generation of children is created. Fitness function and stopping criteria are, once again, evaluated.

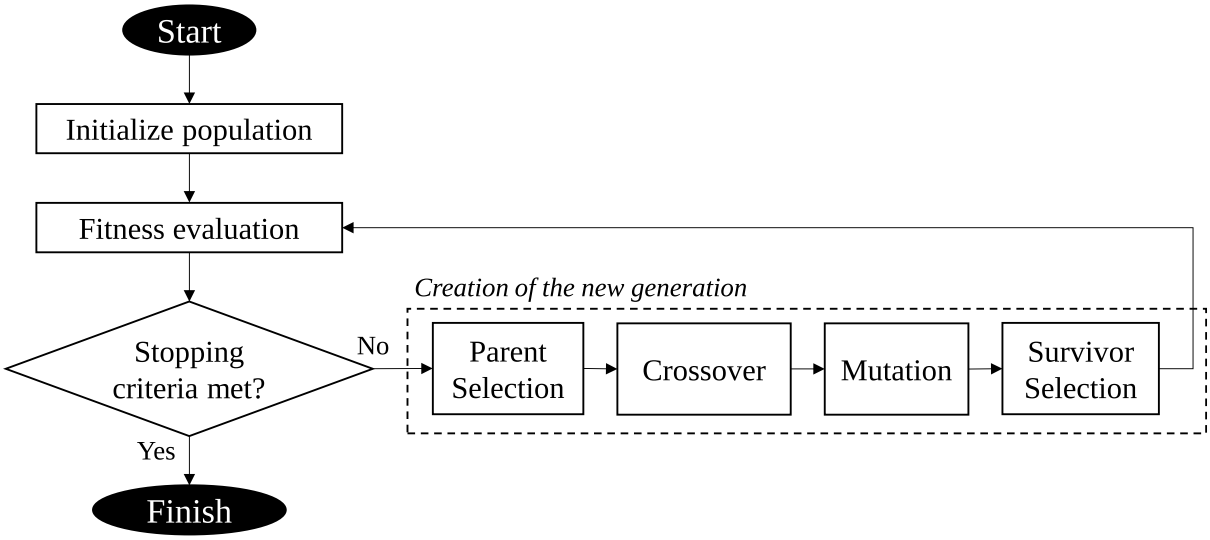

Figure 2. Generic flowchart of GA

Some years later, GA started to be hybridized with other optimization methods. According to El-Mihoub et al. (El-Mihoub et al. 2006), better solution quality, better efficiency, the guarantee of feasible solutions, and optimized control parameters can be achieved using hybridized methods. Also, population size can significantly influence the sampling capability of GA (Piszcz and Soule 2006).

Particle Swarm Optimization

PSO is a technique that was inspired by social behaviours like bird flocking, fish schooling and particularly in swarming theory (Eberhart and Kennedy 1995; ping Tian 2013). PSO is based on updating velocities and positions of each population particle i, at each generation t, and for each dimension d, as it is shown in equations (2.3) and (2.4).

$$v^i_{d,t+1} = \omega v^i_{d,t} + \phi_1 R^i_{1\,d,t}\left(p^i_{d,t} - x^i_{d,t}\right) + \phi_2\, R^i_{2\,d,t}\left(g^i_{d,t} - x^i_{d,t}\right) \tag{2.3}$$

$$x^i_{d,t+1} = x^i_{d,t} + v^i_{d,t+1} \tag{2.4}$$

In the equations above, v is the velocity of a particle, x is the position of a particle, ω is the inertia weight, which has then been introduced in later publications (Shi and Eberhart 1998), ϕ_1 and ϕ_2 are cognitive and social coefficients, respectively, p is the best position of each particle among all the generations between the first and t, g is the best position among all particles and generations between first generation and generation t, and R_1 and R_2 are values sampled at random.

PSO has received more attention since its first implementation in 1995, leading the scientific community to verify some systematic issues resulting in the most known premature convergence and consequently developing variations of PSO. In 1999, Maurice Clerc introduced a constriction factor K (Clerc 1999) related to ϕ_1 and ϕ_2. Clerc and Kennedy (Clerc and Kennedy 2002) generalized discrete-time PSO to a continuous-time PSO from an analytical point of view. K is determined by controlling the exploration and exploitation, and it is multiplied by the right-hand side of (2.3), turning the resulting equation a variation of the original one. Random PSO (RPSO) (van den Bergh 2001) is another PSO variation that is aimed to find a global minimum. For that, a particle is chosen and is sampled at random. In 2006, Van den Bergh (van den Bergh 2001) had demonstrated that PSO is a local but not a global search algorithm. However, RPSO has been proven to be a global search method. Since RPSO introduces such an operator, in which particles are randomly sampled into the search space during the iterative process, each iteration is achieved without depending on the starting population.

Apart from the variations, these PSO single objective variant methods generally have a generalised flowchart represented in Figure 3. It is possible to observe that the flowchart is like GA, except for the core part of creating the new population, where particles' velocity and positions are updated based on mathematical expressions. Also, the values of the design variables assigned to the population of particles are usually real values.

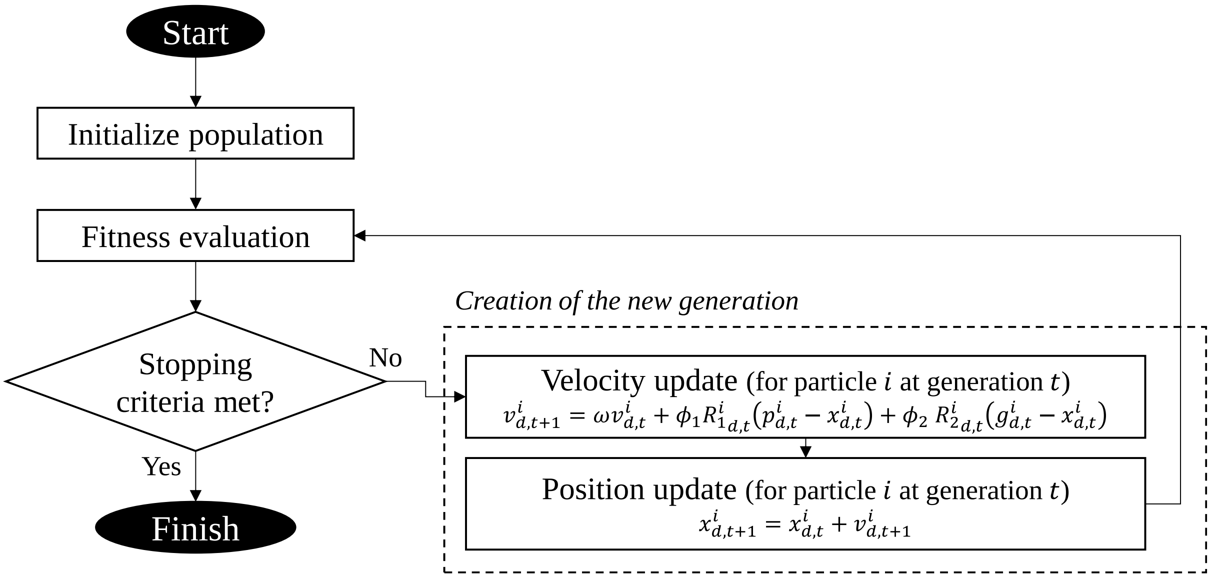

Figure 3. Flowchart for the Classic PSO Algorithm

2.2 Literature Review

Structural optimization is a topic of review, in which include a large number of papers. These are separated according to Figure 4: type of publication; number of objectives; local or global search; metaheuristic technique used; type of structural optimization; constraints. When compared to all the types and materials in structural optimization, the hypothesis that the literature has a low percentage of studies addressing composites is assumed. Due to its importance, specific keywords are used to enlarge the literature review on composite structures. Therefore, the type of material is not considered in the list of sub-section. However, instead, an extra sub-section addressing composite structures is added.

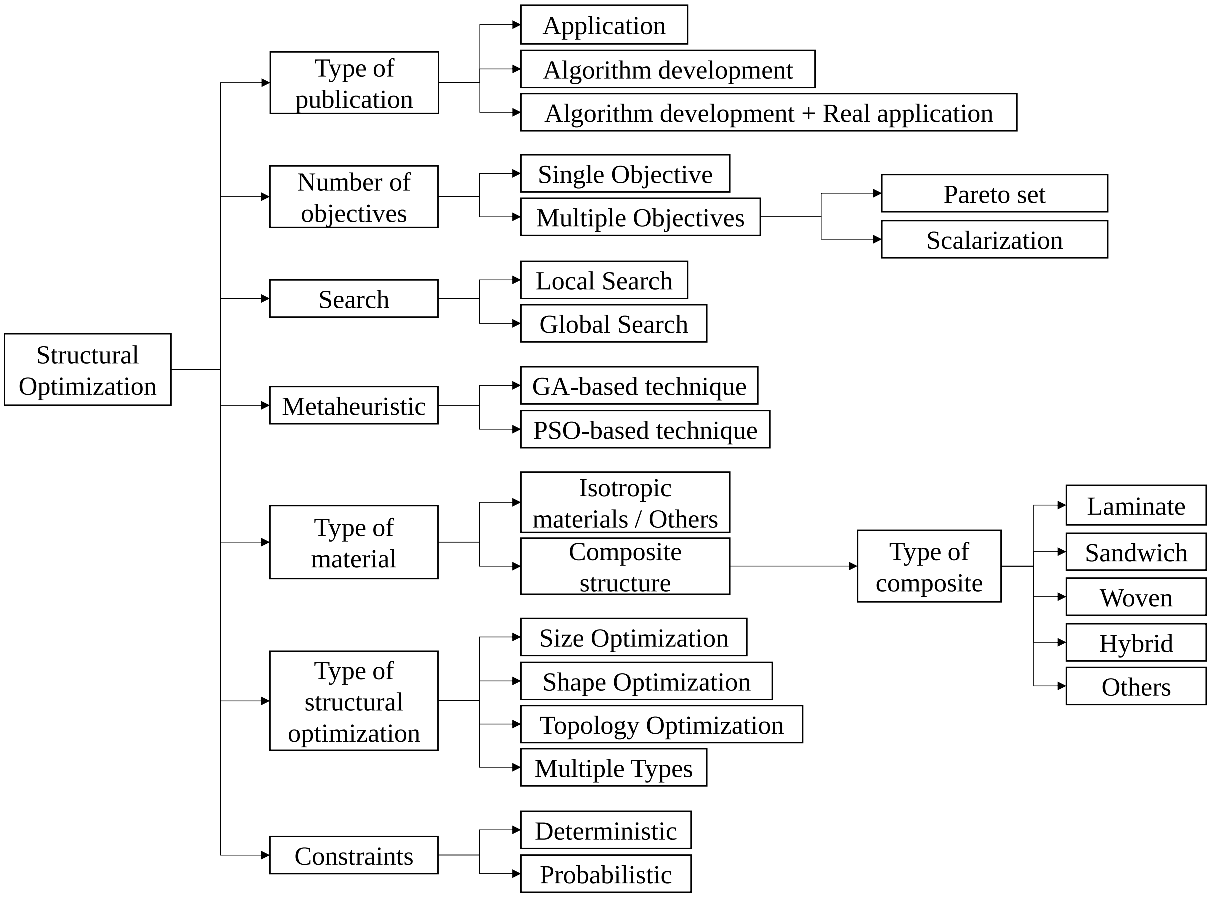

Figure 4. Flowchart representative of the current literature review

Search Method

For the elaboration of this review, Scopus is used as the platform for the literature search. It is divided into two main parts: structural optimization and composite structural optimization. A different string of keywords is used for each case. Due to many works published in structural optimization, the keywords are filtered by PSO and GA algorithms. Therefore, the keywords "Particle Swarm Optimi*" (catching both British and American versions of the name) and "Genetic Algorithm*" are used. Figure 5 and Table 1 present the resulting output from Scopus. From the figure, it is possible to verify that structural optimization is a clear trending topic for the future ($p<0.05$, using a Mann Kendall t-test (Yue and Pilon 2004)). Also, thousands of references are obtained. In order to get more filtered results, the keywords listed in Table 1 are applied for the article title solely. This has ensured that only works within the referred scope are considered. However, potential indexed papers within the scope could belong to the selected list. Considering that keywords are only applicable to the article title was not done for the achievement of Figure 5 since no trending significance would be obtained.

Table 1. Number of publications according to the keywords used in the search

Abbreviation	Keywords	Total No. Publications (abstract, keywords and title)	Total No. Publications (only title)
Keywords 1 (K1)	(("Particle Swarm Optimi* ") OR ("Genetic Algorithm*")) AND "structural optimi*")	4255	159
Keywords 2 (K2)	(composite* AND (woven* OR laminate* OR sandwich* OR hybrid*) AND ("genetic algorithm*" OR "particle swarm optimi*"))	1697	183

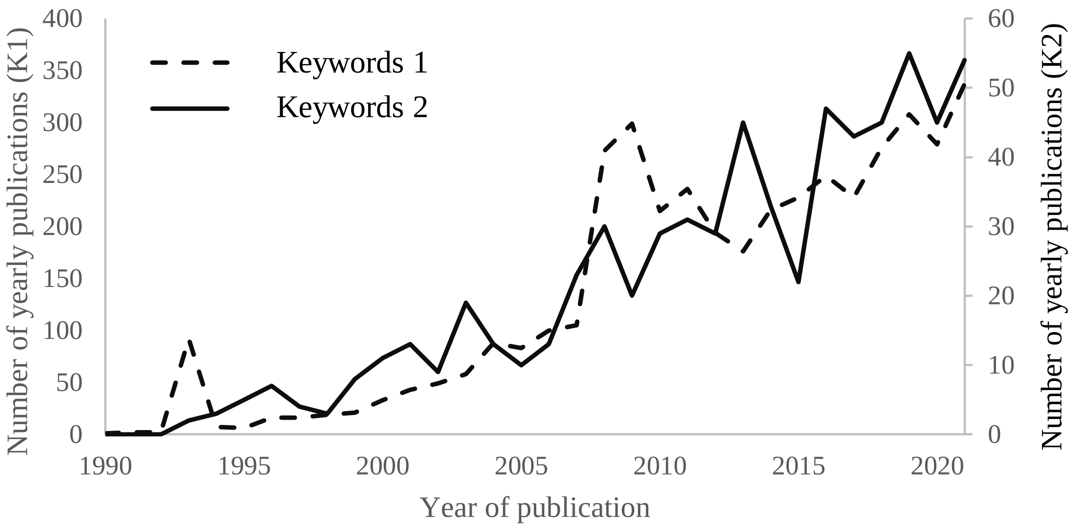

Figure 5. Number of yearly publications by year of publication according to the keywords used in the search. Source: Scopus

Related Reviews

In the literature, reviews on structural optimization have been already developed. For instance, Ghadge et al. (Ghadge et al. 2022) performed a review on multi-disciplinary composite structures. Multi-disciplinary composites involve the presence of several industries in which composite structures are used. The review addresses the challenge of finding the optimal sets, which is usually done using trial and error approaches, and that may depend on the user and longer time intervals to have a properly optimized design. The review does not present the state-of-the-art overview explicitly on the used techniques and genetic algorithm variations. However, it was concluded that optimization models had been implemented in this field.

Fernandes et al. (Fernandes et al. 2021) performed a literature review on the design and optimization of self-deployable damage tolerance of composite structures. Firstly, the authors have referred to the importance of a deeper investigation into such a topic, supported by the European Space Agency identification of large demand design requirements associated with elastic structures with high natural frequencies. Damage tolerance design is seen as a potential solution. The optimization strategies are similar when comparing the specific application of deployable composite structures with other composite structures. Still, only one study has been done (Ferraro and Pellegrino 2019) regarding topology optimization of deployable composite structures, which has been unexpected due to the highest interest and demand for damage predictive models by several industries (Alderliesten 2015; McGugan et al. 2015).

Wu et al. (Wu et al. 2021) addressed the topic of topology optimization of multi-scale structures for review. The authors have separated the existing approaches into the full-scale process, where structural analysis on fine-scale is done under pattern repetition and local volume constraints, and the multi-scale approach, where analytical or numerical homogenization is used to reduce computational time. Some warnings are given. There is a lack of real significance of homogenization-based solutions, the lack of comparison of homogenization-based approaches with standard mono-scale methods and the lack of validation and verification solutions under explicit statements of superiority on multi-scale structures.

Also, Mei and Wang (Mei and Wang 2021) reviewed structural optimization studies in civil engineering. Here, the problems are divided into discrete optimization and continuous optimization. The design variables of the studies are, in general, cross-sectional areas, nodal coordinates and the connectivity of structural elements. In general, the objectives are minimizing the total weight of the structure, compliance with the structure and total strain energy. Safety and serviceability are requirements of the structure, and, in consequence, they are accounted for as constraints of the problems. Metaheuristic-based algorithms are primarily used when compared to conventional algorithms. The authors have identified some gaps: the lack of comprehensive explanation and criterion for the adoption of the weights when performing scalarization of the multiobjective functions; the lack of standardization among algorithmic optimization approaches; and the lack of categorization of the structural optimization algorithms regarding the optimization problem.

Afzal et al. (Afzal et al. 2020) have reviewed previous literature on reinforced concrete structural design and optimization techniques used in that regard. They are based on four different topics: material efficiency; material and cost efficiency; material and environmental efficiency; and sustainable design efficiency. The authors have identified some gaps: the lack of consideration of the performed-based design approach and the whole frame structure; the lack of clash detection and resolution at the structure joints. Regarding the optimization methodologies, no significant gaps are reported.

Another review (Raut et al. 2021) has been performed on damage detection of composite structures. This time, the authors have concluded on the general topic instead of the specific topic of deployable structures addressed by (Fernandes et al. 2021). They concluded that GA and PSO are two of the most used techniques for the optimization problems of classification and prediction of damage. ACO and fuzzy logic are used in hybridization with GA and PSO and not alone as optimizers. GA and PSO have been recognized for their efficiency, compliance, autonomy, complication and optimization capacity characteristics.

Pan et al. (Pan et al. 2020) addresses methods for designing and optimising uniform and non-uniform lattice structures. Significant properties of the lattice structures are found to be closer to the topology structures of unit cells. However, unique lattice structures with more complex properties, such as non-uniform Poisson's ratio, different materials in the same structure, and different lattice scales, have been turning more into a more familiar setting. In that regard, a broader extension of information is expected to be collected so that the optimal design of such structures can accompany the advances in lattice novel structures and settings.

The proposed literature review on structural optimization differs from the works identified in this subsection. The most important ones are the following:

- In this review, structural optimization has a broader extension abroad in all the engineering fields, not carrying out a specific area or component, except for the composite structures that are held in the final phase of the review;
- The base optimization techniques are only GA and PSO;
- The optimization techniques are more deeply detailed.

2.3 Structural Optimization

A total of 81 of the 159 works have been reviewed using the search methodology explored in the previous section. The following subsections are divided into the categories in Figure 4. Moreover, the literature review on composite structural optimization is based on 60 papers of 183.

Application-based and Development-based works

The literature review is here proposed to be divided into three major types of publications: papers on the application of already-developed optimizers in concrete engineering structures (application-type publications); papers on the development of optimizers (development-type papers); papers on the development of optimizers followed by a concrete engineering application (D&A-type papers). Development papers aim to improve or develop new operators compared with previously used methods using benchmark structural optimization problems. Usually, 10-bar or other number truss structures are used for such a comparison. Application papers use previous methods and tested operators for the application in a real engineering problem. The aim is to solve an optimization engineering problem that has been not solved. Development and application papers are a combination of both. Figure 6 shows the proportion of papers existing in the literature for each of the different types. D&A type is found to be less common in the literature. Application-type papers are the most common in this review.

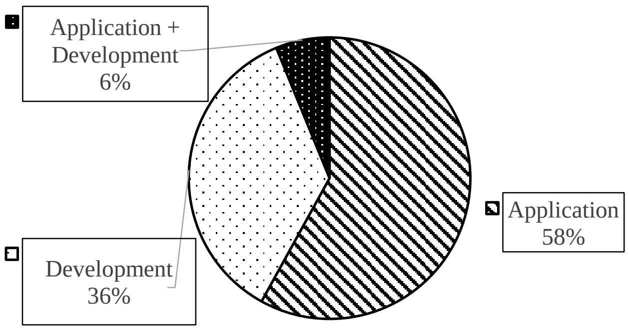

Figure 6. Proportion of the three types of publications in the literature

Examples of development works are listed in Table 2 and detailed in the following paragraphs.

Table 2. Examples of works for each type of publication

Type of publication	Examples of references
Development	(Ren et al. 2020), (Ren and Liu 2017), (Carvalho et al. 2017), (Im and Park 2013), (Guo et al. 2011), (S. Ai and Wang 2011), (Zuo et al. 2011), (Seyedpoor et al. 2010), (Bai et al. 2010), (Pengfei et al. 2009), (Bekiroğlu et al. 2009), (Froltsov and Reuter 2009), (Wang and Tai 2007), (Kaveh and Shahrouzi 2007), (Potgieter and Engelbrecht 2007), (Ward and Mccarthy 2006), (Park et al. 2006), (Lemonge and Barbosa 2004), (Takahama and Sakai 2002), (Hasançebi and Erbatur 2000), (Botello et al. 1999), (Annicchiarico and Cerrolaza 1999), (Kasabov and Watts 1997), (Yang and Soh 1997), (Gholizadeh et al. 2008), (Adeli and Cheng 1994)
Application	(Mohan and Maiti 2013), (Jang et al. 2011), (Boudjemai et al. 2007), (Pugnale and Sassone 2007), (Gallet et al. 2005)

D&A	(Niu et al. 2021b), (Niu et al. 2021a), (Shao et al. 2015), (Guo and Li 2009), (Sakamoto et al. 2001)

An improvement of GA is made by Ren et al. (Ren et al. 2020) to better deal with discrete and continuous variables to solve the complex design of frame structures that involve beam elements. For that, a two-level multipoint approximation strategy is adopted. In the first-level approximate problem, objectives and constraints are mixed in a single expression, used for discrete values. Continuous variables are optimized using dual methods, using Taylor first-order approximation in the second-level approximate problem.

A modified particle swarm optimization algorithm has been developed by Ren and Liu (Ren and Liu 2017). First, the chaotic mapping generates the initial population, which can guarantee diversity and improve the quality of the initial solution. Constraints are added to the objective function in different common penalty-based fitness functions. The authors have used the root mean square of both normalized objective function and constraint in the current work. Then, the result value is summed with the ratio between the number of violated constraints and the total number of constraints. Also, when the proportion of feasible solutions in the population is small, exploitation operations are adopted and executed. Oppositely, feasible particles enhance the diversity of the population. Also, adaptive parameters (constants in the PSO form) are based on sine functions. At last, to increase the population diversity, this paper introduces the particle reset strategy. When the particle is trapped in the optimal local solution, it is given a random position at a certain probability.

The investigation carried out by Carvalho et al. (Carvalho et al. 2017) aims to modify the PSO algorithm for an alternative method based on a new velocity and an operator called "craziness velocity". This velocity is based on giving high values for the velocity of certain particles at very low probabilities. It is aimed to solve the convergence problem of PSO.

A novel methodology for structural optimization using PSO, surrogate models and Bayesian statistics is developed by Im and Park (Im and Park 2013). In the method, surrogate models replace the response of the search space. Sequential methods are used when surrogate models are used in optimization. The concept of sequential approximation is then considered because it is difficult to express the real phenomenon of response, such as the objective function and constraint functions using one approximation model for the whole design space. Surrogate models are updated for each iteration within the trust region in a sequential approach. Bayesian statistics are used to reduce the errors' effects and suggest the reliable value of optimum that is safer.

Guo et al. (Guo et al. 2011) have developed a novel optimization algorithm for structural applications. The traditional GA operators are recovered, but a quasi-full stress design is introduced, so the search space is restricted to the boundaries of the problem, where those boundaries are assumed to have optimal points, and, as a consequence of the constraint, computational time and exploitation are improved. Two examples are given to test the algorithm: the weight of the 72-bar truss structure shown to be minimized with stress and displacement constraints; in-plane trussed structure with two load cases.

Ai and Wang (S. Ai and Wang 2011) proposed two novel variations of GA to optimise structures. Those new variations are aimed at solving the premature convergence of GA. The first one combines GA and the downhill simplex method, whereas the second combines GA and the conjugate gradient method. According to the authors, the initial set of points is almost independent of the resulting outcomes of the algorithms. An example of 10 bar truss problem is given to compare these two variations with traditional GA. These new approaches are said to improve the corresponding accuracy, efficiency, and reliability results.

Seyedpoor et al. (Seyedpoor et al. 2010) proposed a combination of PSO with simultaneous perturbation stochastic approximation (SPSA), which uses gradient information. In the first phase, an initial optimization is carried out using only SPSA. Then, PSO is used in its more traditional way, with the difference that the initial population is gotten utilising the solution of the optimization performed previously by SPSA. Structural optimization is done using benchmark truss structures to compare PSO, SPSA and SPSA-PSO.

Bai et al. (Bai et al. 2010) improved traditional GA, adding a niche for the selection operation. According to the authors, hybrid GA with niche technology is especially suitable for complex multi-modal function optimization problems. Also, Pengfei et al. (Pengfei et al. 2009) developed an Imitative Full-stress Design algorithm based on GA, similarly to the work developed by Guo et al. (Guo et al. 2011).

Bekiroglu et al. (Bekiroğlu et al. 2009) added an adaptive mutation operator to the traditional GA algorithm. In a variety of steel space truss structures, binary encoding, quaternary encoding, value encoding and real encoding are compared for different population sizes and measurements (iteration number, weight and convergence).

An improved GA for the structural optimization of the atomic cluster was developed by Froltsov and Reuter (Froltsov and Reuter 2009). Suitably, an existing cluster geometry is cut into two halves along an arbitrarily oriented plane and then spliced two halves together. This is done by either recombining the two halves of the same cluster after rotation by a random angle (Wolf and Landman 1998) or by recombining the halves of different configurations in the current population, the mating or crossover operator. For each cluster size and population size, and concerning the number of trials for reaching a global solution, full mating, hybrid mating and mutation, and full mutation are compared. Potential energy is also being compared for mutation when compared to mating.

Wang and Tai (Wang and Tai 2007) have integrated a constrained multiobjective evolutionary algorithm (RAY et al. 2001) in a simple genetic local search algorithm. The population of the proposed method is divided into three, based on Pareto ranking: 1. elite individuals (do not suffer any change); 2. good individuals (mutate, Local search); 3. apply crossover. The weights of the linear combination for the fitness function are fixed. A penalty function is added. Target Matching Problem example is given for testing.

Kaveh and Shahrouzi (Kaveh and Shahrouzi 2007) use hybrid GA and ACO to optimize 10-bar planar and space truss structures. The problem of minimization of the mass is constrained. Therefore, the authors have used a linear combination of penalty functions to transform the constrained problem into an unconstraint - the size of the population changes over the generations. The results show that the size of the population increases along with them.

Potgieter and Engelbrecht (Potgieter and Engelbrecht 2007) developed a novel hybrid GA algorithm. Its pipeline is built based on the following strategy: a fast, rough k-means clustering algorithm, a genetic algorithm, and the ranking operation where the best and the worst particles are determined, the “hall-of-fame”. This method deals with one of the primary problems with most computational intelligence paradigms: the need to iterate over each training pattern to calculate an error metric or to calculate the fitness function. The proposed method uses clustering to try to break the restriction above.

Ward and Mccarthy (Ward and Mccarthy 2006) improved fitness evaluation using Neural Networks. The authors have reported two significant GA optimization problems: expensive fitness evaluation and high epistasis. To overcome them, the authors have used back-propagation neural networks. However, it does not solve the problem correctly due to training expensive costs. Experiments have shown that only a subset of the population is needed to train the neural network to classify fitness. It led to cost-saving for costly and time-consuming tasks of performing fitness evaluations.

Since traditional GA methods are computationally expensive for practical use in structural optimization, particularly for large-scale problems, the successful implementation of GA-based optimization algorithms by Park et al. (Park et al. 2006) involves using trial-and-error for tuning GA parameters. A high-performance GA is developed in this paper in the form of a distributed hybrid genetic algorithm for structural optimization, implemented on a cluster of personal computers to overcome the difficulties mentioned above. The distributed hybrid genetic algorithm proposed in this paper consists of a microGA running on a master computer and multiple simple GAs running on slave computers. The algorithm is applied to the minimum weight design of steel structures.

Lemonge and Barbosa (Lemonge and Barbosa 2004) proposed a penalty methodology for constraint optimization problems based on Deb's proposal. The maximum evaluation is replaced by the average of all evaluations of the current population. Also, a coefficient is added to the constraints. The main idea is to penalize those more complicated constraints highly.

Takahama and Sakai (Takahama and Sakai 2002) aimed to optimise the neural network architecture by using the number of neurons and layers parameters. Due to the difficulty of selecting a proper network structure and the difficulty of interpreting the hidden units, it had been important to perform this optimization task. The proposed method uses GA with a damaged gene operator. It is an operation like mutation, but the state of the mutated gene is different from the normal. The main goal is to avoid local minima, creating more exploration.

A modified version of Joines and Houck's penalty function method (Joines and Houck 1994) is introduced in GA and studied by Hasançebi and Erbatur (Hasançebi and Erbatur 2000). Then, a new methodology for the penalty is proposed. Two different parameters are here introduced. The parameters are introduced to control the average generation value throughout the generations and avoid the increase in the penalty, which may force the search and optimization process to locate only feasible points.

Botello et al. (Botello et al. 1999) have presented a family of parallel stochastic search algorithms that include several popular schemes, such as GA and its hybridization with Simulated Annealing (SA), abbreviated ESSA, with multiple starting points. It also consists of a hybrid algorithm that combines parallel SA with selection. Trussed structures are examples of applications for the evaluations of the proposed algorithms for comparison.

Annicchiarico and Cerrolaza (Annicchiarico and Cerrolaza 1999) have developed a software tool for optimization, GENOSOFT. It is divided into modules: GENOSOFT, the main module to drive the use of the four program modules; GENOPRE, an interactive pre-processor to generate and edit the analytical and optimization model data; GENOPT, a genetic optimization module that carries out all the optimization process tasks; GENOPRO, an analysis module. The considered applications are 2D and 3D truss structures, frame structures, and 2D FEM.

Kasabov and Watts (Kasabov and Watts 1997) investigated the use of genetic algorithms as learning and adaptation strategies, called Fuzzy Neural Networks (FuNN). FuNN uses a multi-layer perceptron (MLP) network and a modified backpropagation training algorithm. The general FuNN architecture consists of 5 layers. It is an adaptable FNN where the membership functions of the fuzzy predicates and the fuzzy rules inserted before training or adaptation may adapt and change according to new data.

Yang and Soh (Yang and Soh 1997) presented tournament selection and compared it to the already-existing roulette selection.

Gholizadeh et al. (Gholizadeh et al. 2008) combined GA with a wavelet neural network in such a way that activation functions are wavelet functions. GA is used to optimise the weight of truss structures, with the introduction of normalized natural frequencies as a constraint. For the fitness function, the penalty function is added.

Adeli and Cheng (Adeli and Cheng 1994) introduced Augmented Lagrange method in GA to deal with constraints. It requires numerous numerical experiments and experience to choose suitable values for the penalty function coefficient. If a small value is used as the starting value for the penalty function coefficient, the solution usually drops rapidly to the infeasible region. The weight of the objective function is much greater than that of the penalty function, thus resulting in a negligible penalty and an infeasible solution (design). On the other hand, a large starting value for the penalty function coefficient causes ill-conditioning in the optimization solution, slow convergence, or numerical oscillation.

As previously referred to, application-type papers are the most common in the current review. In the next subsections, they are deeply explored. Some examples are given in Table 2 and detailed in the following paragraphs.

Mohan and Maiti (Mohan and Maiti 2013) performed the structural optimization of a rotating disk using GA. Since optimization involves several function evaluations, performing Finite Element Analysis (FEA) for a disk model for each evaluation increases computational cost. The response surface design model has been developed to reduce computational cost as an accurate alternative strategy significantly. It is aimed to minimize the cross-sectional disk area that leads to the reduction of weight. The design parameters are geometrical dimensions of the cross-sectional plane, and allowed stress is considered a constraint of the problem. A penalty factor transforms the problem into an unconstraint for the fitness evaluation.

Jang et al. (Jang et al. 2011) performed the structural optimization of a long span mobile bridge of 60 meters with six members and five nodes using GA. According to the authors, these kinds of bridges over 50 m in length are difficult to design because military equipment is usually required to meet rigid safety criteria and design requirements. Thicknesses of sections are the design variables, but those thicknesses may be only discrete values. Other constraints are the maximum weight and the maximum stress.

Boudjemai et al. (Boudjemai et al. 2007) optimized a small satellite based on the minimization of the satellite's mass. Three main applications are carried out: an isogrid structure, a sandwich structure and a satellite. According to the authors, modelling the honeycomb structure is difficult since it is costly in terms of computing time, memory capacity and scale problem plane /thickness. Choosing the function fitness is a difficult task since, between the physical problem to optimize and the genetic algorithm, many possibilities are available, but potentially for very different results.

Pugnale and Sassone (Pugnale and Sassone 2007) performed the shape optimization of free form shells using GA. The shape of the shell has been modelled using NURBS representation. The design variables are the vertical positions of control points, mapped on a net of 10x10. The minimization of the maximum vertical displacement of the structure under self-weight is aimed. Control points may pass through constraint points of the shell. Otherwise, a penalty value is increased.

Moreover, Gallet et al. (Gallet et al. 2005) aimed to minimize the weight of components in the aerospace field. Firstly, the main fuselage component is optimized and then extended to the fuselage section. The design variables are stiffener pitch, frame pitch, skin thickness, outer flange width, outer flange thickness, outer flange thickness, inner flange thickness, web width, and web thickness. Materials are added to the list later on. GA is compared to the Gradient-based method with the sequential quadratic method.

A deeper focus on D&A-type publications is now given. Their references are summarized in Table 2 as well.

A sandwich composite construction T-joints bounded with adhesive are analysed by Niu et al. (Niu et al. 2021b, 2021a). One may pretend to minimize the structure's weight using their variables, using the Multi-Island Genetic Algorithm. This algorithm expected a more global and efficient optimization since multiple GA separated populations work separately, but they operate together to reach a global solution. Single and multiple modes are considered for constraint definition, including Tsai-Wu failure criteria for single mode. The geometry of the structure and the number of plies have been considered as variables. FEM is considered for the model of the structure. The results show that the weight is reduced by 34% compared with the original structure weight.

Guo and Li (Guo and Li 2009) performed the structural optimization of a steel tower for a 1000 kV Nanyang-Jinmen line using adaptive GA. Presenting topology optimization theory, adaptive crossover and adaptive mutation, and Kuhn-Tucker theory for shape combination optimization, this novel variation aims to combine different variables to solve a more complex structure. Section sizes, shapes, and topology variables are considered. The minimization of the weight is the objective of the problem. Equivalent stress constraint, rod slenderness rate, and angle between two connected members are constraints. Topology combination optimization based on adaptive GA is shown to perform better in comparison to size and shape combination optimizations.

Sakamoto et al. (Sakamoto et al. 2001) aimed to use a fully stressed design method based on GA to perform the structural optimization of a CRT's 3D shell. Since maximum stress is added into account, a penalty is considered. The resulting optimum model has achieved a 25% lighter weight than the reference model. It is also confirmed that the computational time is lower than simple GA and others, and it highly assures the stability of the solution.

Figure 7 shows the evolution of the percentage of the application-type papers over the total number of papers published in the same year, where error bars are due to the confidence interval of the sample. Despite the increasing number of application works from 2009, using a Mann Kendall t-test, it is not possible to conclude about such increasing.

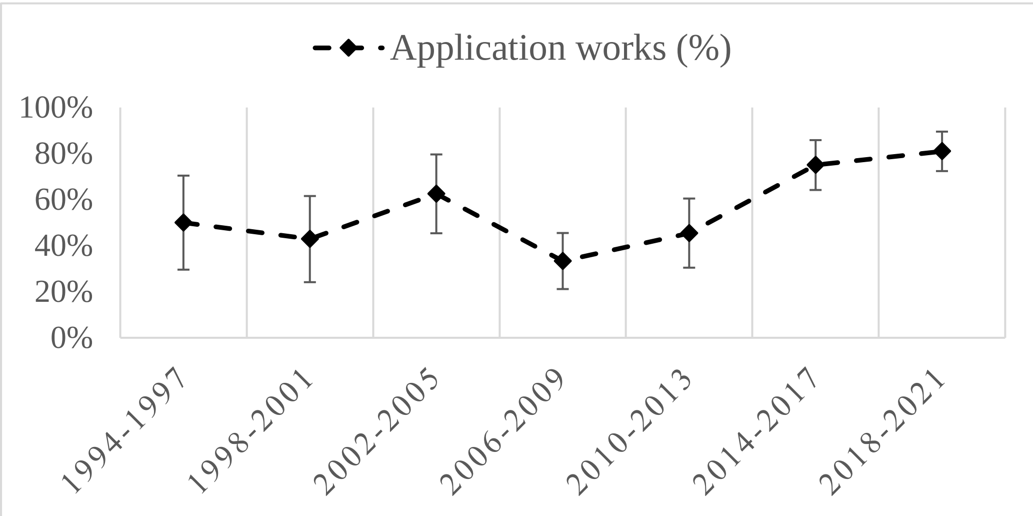

Figure 7. Percentage of application-type papers over the years

Objectives

Reviewed papers can also be divided according to the number of considered objectives: single and multi-objective optimization. Within multi-objective, Pareto set and scalarization methods can be used. The theoretical background is detailed in Section 2. Figure 8 represents the proportion between both single and multiple objectives and the ratio of the last between Pareto and scalarization. Single optimization is dominant in the literature review concerning multi-objective optimization. In multi-objective, the scalarization method is the most common in this literature review compared to the Pareto-based method.

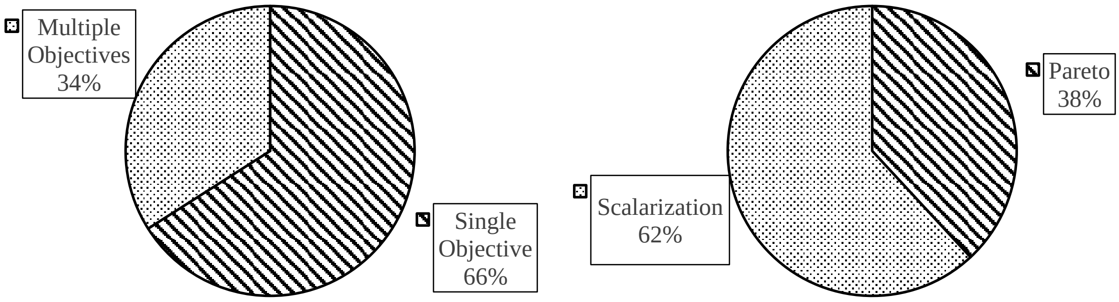

Figure 8. Proportion of single-optimization and multi objective-optimization works

Table 3 lists some references for each considered category in the current subsection.

Table 3. Examples of works by number of objectives

Category		Examples of references
Single objective		(Luo et al. 2018), (Gentils et al. 2017), (Pagnotta 2003), (Iwamatsu 2003)
Multiple objective	Pareto set	(Fadlallah et al. 2021), (Zheng et al. 2020), (Wang and Xie 2018), (Escusa et al. 2017), (Yazdi 2016), (Zhu et al. 2014)
	Scalarization	(Feng et al. 2021), (Lopes et al. 2019), (Takashi Yasui et al. 2017), (Yasui et al. 2021), (Yi et al. 2014)

A brief summary and the main results for the examples given for single-objective optimization are reported as follows.

Luo et al. (Luo et al. 2018) have investigated the structural, topological optimization of cable-net structures using GA. According to them, the literature had been lacking on spatial structures, with most of the studies being carried out on truss structures, which motivated the development of the work. The z-coordinates of the cable nodes and support nodes, initial pretension level and cross-sectional areas are considered in their work as design variables. Minimizing the total weight is the objective function. For the fitness function, a penalty is multiplied by the objective. This penalty is a function of constraints related to the maximum displacement of the net and the maximum stress that cables can support to assure the structure's safety, according to the architecture requirements. For this work, the adjacent exchange method is used to perform the information exchange in the parallel strategy that has been adopted in the optimization strategy. This adjacent exchange method has been developed by Lin et al (Lin et al. 2011) and Hummel (Hummel 2015). The improved strategies carried out in work decrease the calculation period of operations, and consequently, GA is more efficient.

Gentils et al. (Gentils et al. 2017), the optimization of offshore wind turbine support structures is performed using GA, where minimum mass is aimed to be achieved. A total of 13 design geometrical variables are considered for a more complex structure design. The design variables are put in a Finite Element model, and the resulting mass is given. Constraints such as vibration, stress, deformation, buckling, fatigue and geometry are considered. The proposed model reduces the support structure by almost 20% of the original design. According to the authors, fatigue and natural frequency are the main design drivers, in agreement with the recommendations from design standards.

Pagnotta (Pagnotta 2003) performed automotive dashboard support optimization using GA, considering the position and orientation of the ribs of the automotive dashboard support as design variables and natural frequency maximization as the objective.

Examples of multi-objective optimization works are now detailed. Fadlallah et al. (Fadlallah et al. 2021) have optimized a lightweight Sandwich composite heliostat, whose objective function is to minimize the weight, the displacement of the panel and the stress on the aluminium sheet using GA. A similar study had been already carried out before, but the variables used in this study had not been considered in the study, which are the core thickness, the cell wall angle, length, and thickness. The behavioural approach was modelled with an ANN. Hyperparameters of ANN (such as the training algorithm and the number of neurons in the hidden layer on the activation function) and the swarm size of PSO were manually tuned.

Zheng et al. (Zheng et al. 2020) tried to perfom the optimization of a wind turbine tower based on the tower's height, radius, and thickness of the external surface. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is used for the optimization. The minimization of the maximum deformation and total mass are aimed as objectives. After finding the Pareto set, three different solutions are considered for the final approach of the study. The final results indicated that the optimized tower is 1.5% lighter compared to the original design and that maximum deformation has been reduced by 16.5%. The mathematical relationships between functions and variables are done using Response Surface Methodology (RSM). The verification of the model is done with FEM. The error of the model is less than 2.5%.

Wang and Xie (Wang and Xie 2018) carried out the structural optimization of a magnetic shock absorber using PSO. The effective length of the piston, the inner diameter of the working cylinder, the radial height of the damping gap and the coil number are considered the design variables. Two performance indexes are considered objective functions. They are considered separately. The constraints are geometrical but also dynamic. The excitation current is constrained as well.

Escusa et al. (Escusa et al. 2017) carried out the optimization of hybrid sandwich panels. The geometry of the panels, Young's Modulus of the material, its density, and laminate stack architecture of the bottom layers and ribs are set as design variables. These properties result in consideration of chromosomes, assembling 22 design variables. Self-weight, price and environmental foot are objectives to be minimized. Since this is a multiobjective problem, the self-weight can achieve forbidden values so that it has been limited to a maximum of 75 kg/m^2.

Yazdi (Yazdi 2016) optimised a frame structure based on the improvement of weight and stiffness of it. Since two objectives are considered, the problem is transformed into a multiobjective optimization problem with multiple Pareto solutions. The aim of the study is to develop an interface where it is given the information from the user about how much important the weight of the structure is to minimize in relation to the stiffness. Since the interface has been designed for the user to select an option instead of the input of the parametric scalars of the linear combination, a Fuzzy logic approach is done to carry out the decision of the multi objective problem.

Zhu et al. (Zhu et al. 2014) aimed to perform the weight and cost optimization of a 1.5 MW commercial HAWT blade. Some of the design parameters used for the study are the number and the location of layers in the spar cap, the position of the shear webs and the width of the spar cap, in a total of twenty variables. Strain constraint, tip deflection, vibration constraint, buckling constraint and fatigue lifetime constraint are constraints used in the problem. The results have shown that the weights of three selected designs from the Pareto set decreased by 6.4%, 16.9% and 24.8%, respectively, while the values of cost change by 4.2%, −29.4% and −76.8%, respectively, in regards initial design.

Scalarization is applied, for instance, by Feng et al. (Feng et al. 2021). The structural optimization of the Calatrava Bridge is carried out in this study. Thicknesses of several parts of the structure are the design variables of fitness formed through the combination of various objective functions: the self-weight of the bridge, secondary dead load and maximum pedestrian load. Beyond this, the penalty factor considers constraints in the fitness function. Those constraints are mainly the maximum stress and deflection. Due to manufacturing issues, the thicknesses are assumed to vary as integers. ANSYS optimization toolbox results are compared to the GA algorithm. GA algorithm led to higher reductions for all the objectives.

The foundation designed to support a high-capacity motor-driven compressor is aimed to be optimized by Lopes et al. (Lopes et al. 2019). The minimum weight is set to be the objective function. The heights of blocks used to simplify the foundation model are used as design variables. Despite the objective function being linear, the fitness is calculated using non-linear constraints: natural frequencies, allowed displacement and acceleration. These constraints are considered using the Augmented Lagrangean Method. The results show that almost half of the weight has been reduced.

Yasui et al. (Yasui et al. 2017; Yasui et al. 2021) carried out the optimization of a 4x4 multimode interference coupler using the parallelization of genetic algorithms. Geometrical dimensions are used as variables. Both imbalance and excess loss, defined by the authors and dependent on the wavelength, are considered to calculate the fitness, summing up all those values for a set of wavelengths. The results are comparable with a coupler similar to the one obtained through optimization.

Yi et al. (Yi et al. 2014) optimized a planet carrier in a 1.5 MW wind turbine gearbox using GA. According to the authors, the improper structural design of planet carriers can cause a power split, uneven dynamic load, an increase in the vibration of the gearbox and its noise, and, in consequence, a significant reduction of service life. Therefore, analysing the dynamic characteristics of the planet carrier and optimising its geometric parameters is crucial when designing the wind turbine transmission system. A total of ten critical dimensions of the planet carrier are design variables. The objective functions are maximum deformation, maximum stress and mass. These are linearly combined using constant weights chosen in 5 different ways. Compared with the original design, the optimised design's mass and stress are reduced by 9.3% and 40%. Consequently, the cost of planet carriers is reduced, and their stability is also improved.

When looking for the objectives themselves, it is possible to verify, from Figure 9, that the minimum weight (see, e.g. (Li et al. 2020; Abd Elrehim et al. 2019; Tsiptsis et al. 2019)) is the most common objective in the literature review. Other relevant objectives in the literature are potential energy (see e.g. (Shao et al. 2018; Liu et al. 2016; Shao et al. 2015)), minimum displacement (see e.g. (Fadlallah et al. 2021; Zheng et al. 2020; Pugnale and Sassone 2007)) and maximum stress (see e.g. (Dong et al. 2022; Barbosa et al. 2008; Guo et al. 2021)) .

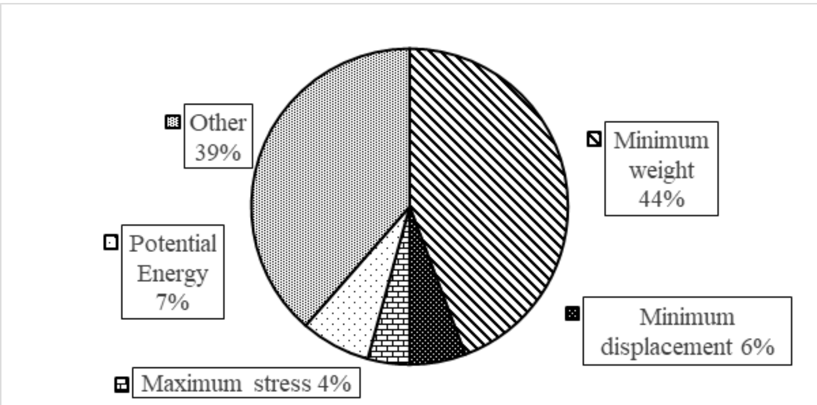

Figure 9. Literature review by objectives

Local and Global search

Some examples of local and global search implementations are given in Table 4.

Table 4. Examples of works for each type of search

Type of search	Examples of references
Local search	(Krishnapillai and Jones 2009; Primorac et al. 2016)
Global search	(Zayed et al. 2017; Gallet et al. 2005)

Some techniques can behave as global and local search (Cai et al. 2012). Here, PSO is applied using an inertia weight so that global search and local search are balanced. In other studies, local search techniques are combined with global search techniques, e.g. (Ruzbehi and Hahn 2019). Here, GA is combined with Greedy Search for the structural optimization of an electromagnetic actuator. The proposed algorithm is aimed to implement Greedy Search as a deterministic local search at the end of the GA optimizer. Although the combination of both algorithms has shown promising results, Greedy Search itself seemed to be the more efficient optimizer for the application problem.

Metaheuristic: GA and PSO

As referred in the introductory section, only GA and PSO are considered for this review. GA and PSO have been introduced in Section 2. Some examples of GA and PSO-based techniques implemented among structural engineering problems are listed in Table 5.

Table 5. Examples of works for each metaheuristic

Category	Examples of references
GA	(Dong et al. 2022; Khodzhaiev and Reuter 2021; Ni and Ge 2019)
PSO	(Shao et al. 2017, 2015; A. Wang et al. 2011; Tsiptsis et al. 2019)

Khodzhaiev and Reuter (Khodzhaiev and Reuter 2021) have studied the optimization of a transmission tower, using a new GA approach where a variable-length genome based on a two-stage mutation turns GA into a different algorithm. This concept of variable length genome had been already present in previous studies, but both did not apply for structural optimization of trusses. The design variables are sectional and material properties of the tower members and the number and heights of panels in tower segments. The "Death-Penalty" methodology for the fitness has been implemented for the constraints according to European Building codes EN 50341-1:2012 (EN 50341-1:2012) and EN 1993-3-1:2006 (EN 1993-3-1:2006). The result of the executed case study is a reduction of 10% of the weight. FEM solver is introduced for the model, but the authors stated the importance of ANN implementation to achieve lower computational costs.

PSO has been applied by Tsiptsis et al. (Tsiptsis et al. 2019), where it is combined with Non-Uniform Rational B-Splines (NURBS) for both shape and topology of structural optimization of frame structures, namely for towers. The objective function is the minimum mass. NURBS describes the geometry of the truss chords, so the number of parameters is significantly decreased. For the experiments, control points and bracing position are shape parameters. The results have shown that 16% of the mass has been reduced. This was possible since the number of iterations is increased with the decrease of the design variables due to NURBS for the same computational costs. FEM is also used to access the stresses and displacements. The choice of the PSO method as the optimizer used was based on the following factors: is easy to understand and implement due to the low number of setting parameters involved; existence of open-source codes that allow to use for algorithmic extension and validation; for tower optimization, PSO implementation previously done gives a good reference point and a way to compare the results.

PSO and GA are not combined in any of the considered works. PSO-GA combinations can be verified in the literature (Barroso et al. 2017; Moradi et al. 2021).

Figure 10 compares the proportion of GA and PSO within this review. It is possible to verify that GA applications are significantly greater when compared to PSO.

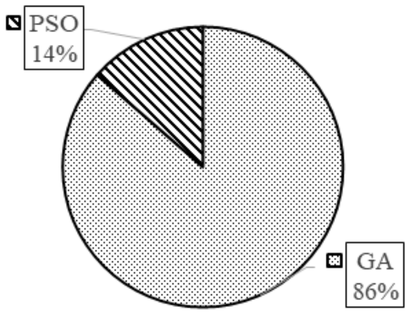

Figure 10. Literature review by metaheuristic approach: GA vs PSO

Constraints

Constraint problems can be divided into two categories: deterministic and probabilistic. Deterministic constraints may exist significantly in the literature review compared to probabilistic approaches. Only in the works of Niu et al. (Niu et al. 2021b, 2021a) probabilistic-based criteria (Tsai-Wu) have been used over the present literature review. In general, different deterministic constraints have been considered. The most important are drawn in Figure 11. Maximum stress and maximum deformation are the most used constraints within the literature review. Some examples for each of the main constraints (maximum stress, maximum deformation and admissible buckling) are given in Table 6.

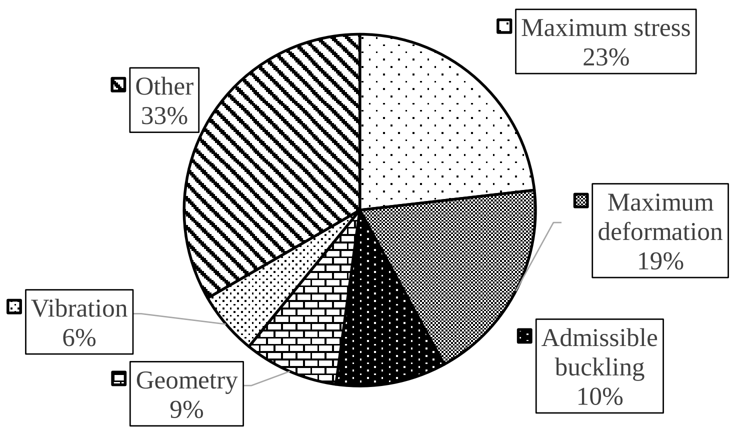

Figure 11. Proportions of the most considered constraints in the present literature review

Table 6. Examples of works for each constraint

Constraint	Examples of references
Maximum stress	(Fadlallah et al. 2021; Khodzhaiev and Reuter 2021; Ding et al. 2021; Abd Elrehim et al. 2019; Koumar et al. 2017; Wang et al. 2016; Lu and Xie 2014)
Maximum deformation	(Fadlallah et al. 2021; Ding et al. 2021; Tsiptsis et al. 2019; Lopes et al. 2019)
Admissible buckling	(Koumar et al. 2017; Gentils et al. 2017; Wang et al. 2016; Zhu et al. 2014; XU et al. 2005)

A flood wall structure is optimized by Ding et al. (Ding et al. 2021). The minimization of its costs is done with the linear combination of independent lengths of the structure, where scalars function as the price of the material used for the substructure. Maximum stress and deflection are taken into account. It is verified that the maximum displacement and stresses are much lower in the diagonal bracing type than without diagonal bracing.

Wang et al. (Wang et al. 2016) carried out the structural optimization of wind turbine composite blades using GA. For this, twenty-three design variables are set: 16 numbers of unidirectional plies for different regions, three thicknesses for three other regions, and four normalized locations. The weight of the blade is minimized to reduce the cost of the construction and to reduce both centrifugal and gravity loads on the blade. Allowed stress, deformation, vibration and buckling of the structure are constraints of the problem. Since the blade is intended to be made of laminated composite, manufacturing requirements and the laminates' continuity are also considered. The authors have reached an optimized blade of 228 kg, 17.4% lower than the initial design. Also, the maximum compressive stress is very close to the allowable values. At the same time, other constraint parameters maintain a large margin from the permissible values, which is indicative that the compressive stress may be the dominant constraint of the problem.

Lu and Xie (Lu and Xie 2014) performed the structural optimization of a hub unit bearing using GA. Minimizing the weight of the structure and the moment rigidity is aimed, being the maximum stress a constraint of the problem. The response surface method is used to simplify the model of the maximum stress and the moment rigidity. Three design parameters related to the geometry of the structure are set. The results obtained show that the weight reduction is 5.8%, while the inclination used to evaluate the moment rigidity and the maximum equivalent stress increase by 0.66% and 2.68%, respectively.

Moreover, the composite structural optimization of a hat stiffened laminated composite panel of a typical passenger bay of a blended wing body type transport aeroplane (Vitali 2000) is carried out by Xu et al. (XU et al. 2005), using Neural networks, response surface modelling and GA. The considered design variables are the distance from the panel end to the thickness discontinuity, the skin thickness near the panel end, the skin thickness in the interior of the panel, crown thickness near the panel end and crown thickness in the interior of the panel. Because the skin and all the components of the hat stiffener are made of graphite epoxy, the effects of strength and buckling in design are considered. According to the Hecht Nielsens Theorem (Hecht-Nielsen 1987), a three-layer backward propagation neural network is sufficient to achieve a global mapping of the structural response, and it is considered in the current work.

Although design variables are directly related to the structure, thermodynamic and heat transfer-based constraints can be applied. Wang et al. (Wang et al. 2022) used GA to optimize a double-layered capillary wick in a cryogenic loop heat pipe system. It is pretended to maximise the heat load for a specific evaporation temperature that cannot be higher than a certain given threshold. Dryness is also considered in this study as a constraint. For the constraints handling, if minimum values are achieved for a set of variables that violates the constraints, this set and its corresponding objective values are discarded.

Type of structural optimization

The literature review is now divided into size optimization, shape optimization, topology optimization, and multiple types, in line with the division considered in section 3. Figure 12 represents the distribution resulting from the present literature review regarding such a division. It is possible to verify that studies are carried out with multiple types in the literature review more often than in any of the others. Among the first three mentioned structural optimization types, size optimization can be considered the most frequent. Authors have combined size and topology optimization more often among all the possible combinations from the multiple types.

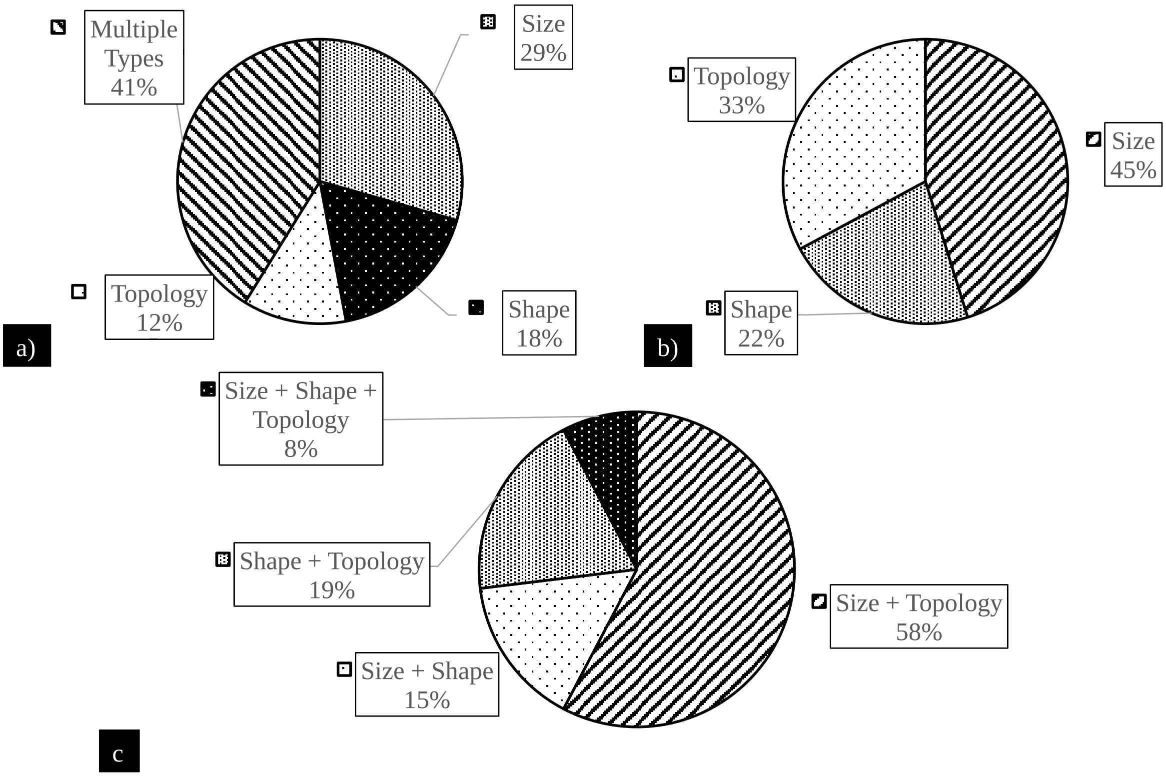

Figure 12. Proportions of the most considered types of structural optimization in the present literature review. a) according to the drawn division; b) according to the basic three types; c) distribution of the possible combinations of multiple types

Table 7 lists examples of references for each of the types.

Table 7. Examples of works for the type of structural optimization

Category	Examples of references
Size optimization	(Guo et al. 2021; Ni and Ge 2019; Barbosa et al. 2008; Adeli and Kumar 1995)
Shape optimization	(Shao et al. 2017; Roberts et al. 2000; Daven et al. 1996; Davies et al. 2007)
Topology optimization	(Shao et al. 2018; Nakanishi 2000)

Multiple types	(Dong et al. 2022; Wang et al. 2022; Li et al. 2020; Liu et al. 2016; Wang et al. 2011; Sekulski 2008; Palko 1996)

Dong et al. (Dong et al. 2022) have used GA to determine an ANN architecture that would model a Finite Element Analysis to replicate the anisotropic behaviour of Carbon Fibre Reinforced Polymers (CFRP) fabricated with 3D printing to optimize a structure. This structure would be optimized regarding the position of control points, sizes such as heigh of the layers, thickness of the entire specimen and radii of FDM bead and carbon fibre reinforcement. The objective function is to normalise four selected criteria: Poisson's ratio, maximum von Mises stress under 20% tensile strain; volume-specific energy absorption; buckling resistance under compressive load. All four objectives are normalized and sum to create a final fitness function.

Robotics research leads to new requirements like low speed and heavy loads. These requirements are achieved with the correct design of gear reducers, in which their optimization is proposed by Guo et al. (Guo et al. 2021). The radius of the lever and two-rod lengths of the structure are selected as design variables. The fitness value is achieved with a linear combination of various objectives: mass, stress, torque and deflection. In this study, only the lever mechanism in the reducer is optimized.

Li et al. (Li et al. 2020) did inlet structural optimization of an aircraft. For the modelling of the structure, Backpropagation neural networks are used. The project variables considered are the throat aspect ratio, slope inclination of the air inlet structure and the opening length. The optimization algorithm is more efficient, with a lower prediction error, considering the mass flow rate and the fuel penalty.

Abd Elrehim et al. (Abd Elrehim et al. 2019) aimed to investigate the structural optimization of arch bridges using GA. Despite the total number of joints of the structure, only 18 are considered due to the computational cost. For each joint, 16 positions are possible. The objective function is the arch weight and the constraints, which are included in the fitness function as a penalty factor, are the maximum stress and deflection, defined by the Egyptian Code of Practice for the design and construction of concrete structures. Compared to traditional designs, a 30% to 35% of weight reduction has been seen as a result of the optimization process. The developed Finite Element Analysis has been used to check the structural safety issue.

Ni and Ge (Ni and Ge 2019) optimized a jacket platform structure. The jacket's outer cross-sectional diameter and wall thickness are considered design variables. The mass of the structure is the objective function. Allowed stress and displacement are constraints of the problem and are included in the fitness function as penalty functions. The overall volume of the jacket platform has been reduced by 38% from the original design.

Koumar et al. (Koumar et al. 2017) have performed the structural optimization of the barrel vault Scissor structure using GA. The number of units of the barrel vault structure (topological optimization) and the height, width, and thickness of the rectangular tube cross-section for both the polar and the translational units (size optimization) are set as design variables. The minimum weight and compactness are aimed to be achieved. Fourteen constraints are considered, grouped into three main categories: maximum displacement, maximum stress and allowed buckling. The study concludes that stress is more dominant than global and local buckling and deformation. In consequency, buckling analysis is removed from the optimization and only verified at last for the optimal scissor structures. It has meant a significant reduction in computational time of 30%.

Using PSO, Cai et al. (Cai et al. 2012) have optimized Horizontal-Axis Wind Turbine Blades. The weight is aimed to be optimized, and the design variables are the number of layers in the spar cap, the locations of layers in the spar cap, and the position of the left shear web and right shear web. Some constraints are considered: the strain generated by the loads cannot exceed the failure limit; the natural frequency of the blade should be separated from the harmonic vibration associated with rotor rotation to prevent the occurrence of resonance, which under high amplitude of vibration could lead to the destruction of the structure; deflection is less than a specified value.

Wang et al. (Wang et al. 2011) optimized a permanent magnet drive. The objectives are to minimize volume and torque. Several material parameters have been assumed to be constant: parameters of the magnet material, copper conductivity and resistivity, steel relative permeability and operating temperature. The design variables are copper axial and magnet thicknesses, magnet radial depth, magnet width and the number of magnets. Artificial Neural Networks are used in this work to replace the Finite Element Method, which is more costly. The results show that the new design has improved the original design by 20% in the magnet material but with no loss of output torque.

Barbosa et al. (Barbosa et al. 2008) developed an encoding strategy in GA to lead with constraints, namely with cardinality constraints. Choosing the set of areas that minimize the volume of the structure is aimed. The set of areas to use is limited to the size of the set and the number of constraints. The adaptive penalty method is proposed by Lemonge and Barbosa (Lemonge and Barbosa 2004). Also, rank-based selection and a two-point crossover operator are used.

The work developed by Sekulski (Sekulski 2008) aims to optimize a high-speed craft. Due to the complexity of the optimization problem related to ship structures, only partial optimization tasks had been formulated in each area independently, and no attempt to unify them had been made. The minimization of multiple regions of the total structure uses a linear combination. Due to standardization and manufacturing reasons, constraints are introduced, like relationships between the plate thickness and the web frame thickness. Thirty-seven design variables are considered, for instance, the number of transversal frames in the considered section and the number of longitudinal stiffeners in the regions.

Palko (Palko 1996) have done the structural optimization of an induction motor using GA and FEM. Shape parameters are considered design variables. The objectives are to minimize the total electromagnetic losses, maximize the torque at a constant or variable slip, and minimize the error due to constraint violations. The breakdown torque, low losses, power factor, current ripple and torque ripple are constraints of the problem. GA is parallelized.

Adeli and Kumar (Adeli and Kumar 1995) have done size structural optimization using parallel GA with Dynamic Load Balancing. Cross-sectional areas are design parameters; the weight is aimed to be minimized, taking maximum allowed stress and displacements into account. The constrained optimization is converted into unconstrained using the quadratic penalty function and the Augmented Lagrangian method.

Davies et al. (Davies et al. 2007) have done the minimization of the energy of configurations of tubular nanotubes, using the positions of the atoms as design variables. Constraints of one-dimensionality (through periodic boundary conditions) and of radial confinement (via barrier potentials) are imposed.

Composite Design Optimization

Until now, the generic structural optimization review has been done. The corresponding studies have used both isotropic and composite materials (and potentially others). In this subsection, a set of 60 papers from Scopus only relative to composite structures are reviewed. Five types of composites are considered: woven, sandwich, laminated, hybrid composites and others. Figure 13 presents a distribution among these three types resulting from the literature review. Hybrid and laminated composites are more often used in the literature review than the other types.

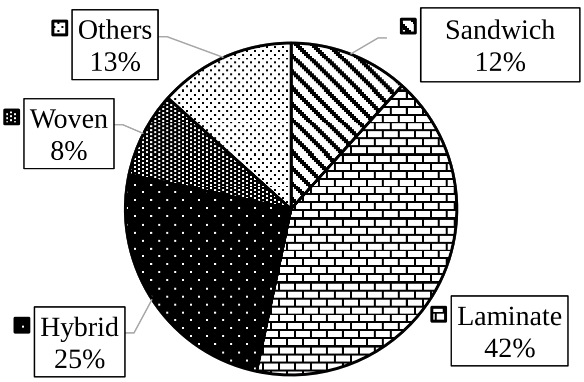

Figure 13. Proportions of the most used types of composites in the present literature review

Table 8 suggests a list of references for each of the types.

Table 8. Examples of works for each type of composites

Category	Examples of references
Laminate	(Pal et al. 2022; Li et al. 2021; Yang et al. 2021; Innami et al. 2020; Wei et al. 2019; Kanjirath and Thirupalli 2019; San et al. 2019; Vosoughi et al. 2017; Talebitooti et al. 2017; Ehsani and Rezaeepazhand 2016; Cho and Rowlands 2015; Cherniaev 2014; Bhise et al. 2014; Hwang et al. 2014; Le-Manh and Lee 2014; Hajmohammad et al. 2013; Barman et al. 2021; Zadeh et al. 2018; Liu et al. 2018; Khatir et al. 2017; Rocha et al. 2014)
Sandwich	(Namvar and Vosoughi 2020; Jiao et al. 2021; Kheirikhah 2020; Arikoglu 2017)
Woven	(Fu et al. 2017, 2015; Tao et al. 2017)
Hybrid	(Kayaroganam et al. 2021; Kumar et al. 2021; Srinivasan et al. 2021)
Others	(Xie et al. 2018; Niu and Feng 2020)

Several works have been done concerning composite structures. For instance, Barman et al. (Barman et al. 2021) have used vibration-based damage detection to detect delamination damages in composite beams and plates. According to the authors, this is a global damage detection method. Compared to other non-destructive methods such as ultrasonic techniques, eddy-current technology, radiography, x-ray, infrared thermography, and acoustic emission, it is not a costly method. A mixed unified PSO is proposed to combine the conventional continuous unified PSO and the binary version of the unified PSO. Size-type optimization is here applied. The fitness function is calculated based on the linear combination of the natural frequencies and the mode shapes. Only single interface delamination is considered.

Zadeh et al. (Zadeh et al. 2018) aimed to achieve the optimal sequence for symmetric composite structures. Two levels of optimization are considered. The first one is done to minimize the weight of the structure, and the second one is done to reduce the load-bearing capacity concerning buckling. The design variables are lamination parameters and the number of plies of specific angles. The total number of plies and the admissible buckling are constraints of the problem.

António (António 2002) used GA in its work to optimise composite structures. Cloning and niching are two explored operators to exchange genetic information. Tests for the influence of the genetic operators are reported. Similar study (António 2006) is used by the author a structure under a non-linear behaviour. The evolutionary method used in such a paper is based on dividing the original population into subpopulations. Members of each of them migrate so that an hierarchical relationship occurs between subpopulations.

António et al. (António et al. 1995) have optimized composite structures to minimize their weight. The efficiency of the material is firstly maximized using a bi-level strategy and considering only the ply angles as variables. Then, weight minimization is carried out using the thicknesses of the layers as variables. The same is done in ref. (António et al. 2000) for elastoplastic material behaviour optimization on composite structures of thermoplastic resins and in ref. (António 1999) for a composite beam with non-linear geometric behaviour.

Tao et al. (Tao et al. 2017) propose a multi-scale optimization scheme for the lightweight design of 3D woven composite automobile fenders. The minimum weight is achieved using mesoscale parameters such as distances between layers and angles of the layers for the main body and attachments as design variables. The stiffness of the fender tip must be larger than 100 N/mm, a constraint that is considered in the fitness function as a penalty. The Kriging modelling technique is used for the computational time reduction of the model. The component has achieved minus 20.65% of the weight compared to the initial design.

Lui et al. (Liu et al. 2018) tried to solve the CFRP battery box lightweight design problem using a modified PSO. Modified PSO is based on the classical PSO, using a velocity of reset that diminishes along the iterations to guarantee convergence and samples the particles at random to guarantee global optimization over the domain. The design parameters are meso- and macro-scale parameters such as yarn width, yarn distances between layers, and layer thicknesses. The reliability is aimed to be improved, and the minimum weight is sought to be achieved. Three core parts divide the methodology: uncertainty quantification & propagation, used to predict the elastic and strength properties of the studied composites; finite element analysis (stiffness and strength analysis); optimization (using particle swarm optimization (modified) & surrogate models). This approach has led to a 22.14% of weight reduction.

Using PSO, Khatir et al. (Khatir et al. 2017) tried to solve the problem of damage detection in composite beam-like X. Modal assurance criterion and natural frequencies are used as objective functions, and GA is used for comparison. The results show that PSO is close to the real damage in terms of computational cost and damage measurement accuracy compared to GA.

Namvar and Vosoughi (Namvar and Vosoughi 2020) have investigated the optimum design of a symmetric rectangular hexagonal honeycomb sandwich plate with a uniformly distributed load by introducing a new multiobjective optimization technique using a hybrid PSO-GA approach. Design variables are core height, face thicknesses, cell wall thickness, vertical and inclined cell wall length, and the angle between the inclined and horizontal lines. Weight reduction and the increase of the deflection of the plate are aimed as objectives. TOPSIS is used for the final design from the Pareto set.

Jiao et al. (Jiao et al. 2021) coupled Fourier series expansion, particle swarm optimization, and genetic algorithm methods in order to optimize a sandwich nanoplate. The maximum phase velocity of a nanoplate is aimed to be achieved. 27% more phase velocity is acquired using the adopted methodology.

In the work carried out by Pal et al. (Pal et al. 2022), the fundamental frequencies of composite shells aim to be maximised to avoid resonance. The design variables are the discrete ply angles. FEM uses a nine-node isoparametric element and first-order shear deformation theory, and it is used to model the natural frequencies. A variety of numerical studies is done to obtain robustness and reliability. Moreover, rectangular and cylindrical geometries are tested in the simulations.

Also, to increase the performance of a composite material subjected to the curing process, the study performed by Li et al. (Li et al. 2021) aims to minimize the residual stress that appears during such a process. Two stages of the GA technique are adopted: the first one promotes the exploration of the search space, and the second promotes the exploitation. Moulding parameters are two heating rates, two dwell times, two holding temperatures and a colling rate. The improved GA has been able to reduce 2% of the residual stress compared to the traditional GA.

The study carried out by Yang et al. (Yang et al. 2021) considers a fibre-reinforced resin matrix composite laminate to maximize the loss kinetic energy of the impact body, using ply angles as design variables. These ply angles have the following possible values: -45, 0, 45 and 90 degrees. Hashin criterion is used as a constraint. ABAQUS FEM software was used to model the system.

Kheirikhah (Kheirikhah 2020) considered an optimization problem for sandwich composite. The results show an increase of 8 to 9% in weight. Also, a decrease of 50% in deflection and an increase of 72% in the buckling load are obtained.

Innami et al. (Innami et al. 2020) maximized fundamental frequencies of laminated composites. Discrete orientation angles are set as design variables. For the calculation of the eigenvalues of composite fibre reinforced plastic rectangular plates with between 8 to 16 layers, the Ritz method is used. The fundamental frequency is improved up to 24%.

Wei et al. (Wei et al. 2019) aim to maximize the buckling load of laminated composite shells, optimizing their stacking sequence and using GA and FEM. The authors have proposed a stiffness coefficient-based method (SCBM) where extensional stiffness and bending stiffness coefficient ratios remain close to the optimum values. This methodology aims to replace FEM calculations to lead to a more efficient and accurate solution. Results have shown that the discrepancy between both methods is not significant and that SCBM can be even applied to more complex structures.

Kanjirath and Thirupalli (Kanjirath and Thirupalli 2019) aimed to use a variation of GA to optimize the stacking sequence and orientation angles used for a composite laminated structure. The considered design variables are encoded in discrete numbers. The objective functions considered are the weight, cost and thickness combined in a single fitness function, strength, fatigue and buckling in the other three fitness functions, one for each of the last three objectives. All fitness functions are ranked. Such rankings are linearly combined. In GA, a scout operator is added. This operator is based on the divided and conquers strategy. Also, the crossover rate grows, and the mutation rate decays over time.

San et al. (San et al. 2019) aimed to maximize the fundamental frequencies of a laminated composite structure using shape and topology optimization. The global vertical coordinates of the nodes of the control points of the NURBS shape are considered for shape optimization. Orientation angles are also aimed to be optimized. These orientation angles are discrete values. The authors have reached the conclusion that adding more control points at the edges of the structure is more efficient.

Vosoughi et al. (Vosoughi et al. 2017) tried to maximize the buckling load of a laminated composite structure. PSO is used as an operator of GA. FEM is used for obtaining shear deformation based on the fundamental plate equations. Different geometries have been used: box, T’s and rectangular shapes. Also, the number of layers and boundary conditions are varied.

Talebitooti et al. (Talebitooti et al. 2017) considered the maximization of the sound transmission loss and the minimization of the weight of a laminated cylindrical composite structure as objectives. Material, porous types and ply orientation angles are selected as design variables, and NSGA-II is applied. Two different configurations are taken from the resulting Pareto set: one with maximum STL and the other with the minimum weight.

Arikoglu (Arikoglu 2017) optimized hybrid viscoelastic/composite-sandwich beams. Material, ply orientation and thickness are design variables. Natural frequency and weight are objectives of the problem. NSGA-II is here implemented.

Moreover, Fu et al. (Fu et al. 2017) considered the weight of a stiffened panel made of 3D woven composite material for minimization. The problem corresponds to a size optimization problem since all the six design variables are around the dimensions, including the thickness of the yarns of the panel.

The maximum buckling load of laminated composite grid plates is obtained by Ehsani and Rezaeepazhand (Ehsani and Rezaeepazhand 2016). The stacking sequence and pattern composition of the grids are considered design variables. Weight and grid thickness are constraints of the problem.

Fu et al. (Fu et al. 2015) combined ANN and GA to optimize 3D woven composite stiffened panels, considering the minimum weight. Critical load, pre-buckling stiffness and post-buckling stiffness are constrained to given threshold values. A penalty function is evaluated to transform the constrained problem into an unconstrained problem. A multi-scale modelling approach is also considered.

The maximization of the buckling load of a generic laminated composite plate is aimed by Cho and Rowlands (Cho and Rowlands 2015). Fibre orientation angle is considered for each of the 40 elements.

The previous studies detailed here are examples of the total number of studies considered. Figure 14 represents two distributions of this collection concerning objective functions and structural optimization type. Minimum weight remains the most popular objective. Among the three basic types of structural optimization, topology optimization, especially at the level of material design, appears more often than size optimization.

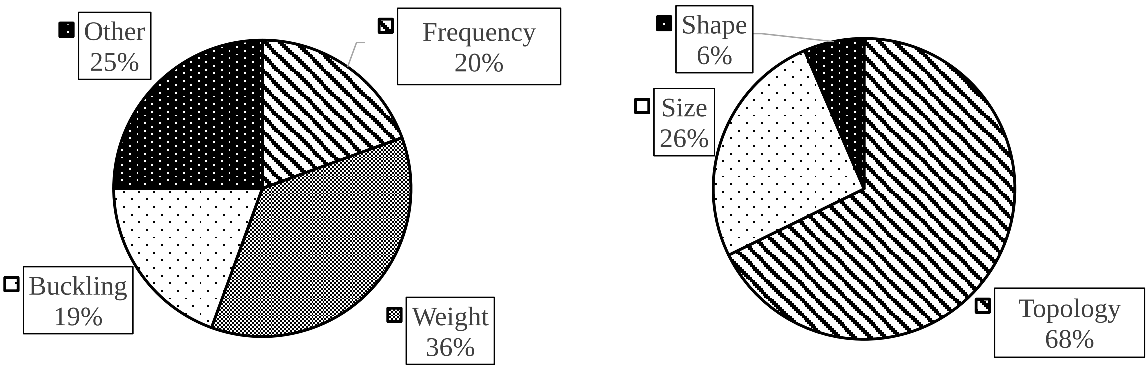

Figure 14. Left: proportion of objectives considered in composite structures. Right: proportion for each type of structural optimization in composite structures

2.4 Reliability-based and Robust Design Optimization

Reliability-based design optimization and robust design optimization are also addressed in the literature related to the design optimization of composite structures.

According to António and Hoffbauer (António and Hoffbauer 2009), reliability-based design is defined in the paper as the optimization problem where probabilistic constraints are held due to uncertainty effects. There are some works carried out in the literature related to the reliability-based design optimization (H Agarwal 2004; Harish Agarwal and Renaud 2004; Gunawan and Papalambros 2006).

In its turn, Robustness-based Design Optimization (RDO) is the optimization problem where those uncertainty effects are minimized (Taguchi 1987). The literature review carried out by Beyer and Sendhoff (H. G. Beyer and Sendhoff 2007) focuses on the different methodologies of doing RDO. Taguchi (Taguchi 1987) uses signal-to-noise measures based on the mean square deviation. Also, the concept of robust regularization is considered in ref. (Lewis 2002), where the maximum value of the function is taken when considering the evaluation of the neighbourhood, representing the worst-case scenario. Other approach, considered in ref. (H.-G. Beyer et al., n.d.) consists on using the expectancy and the variance of the variables in order to formulate a multi-objective optimization problem. Evidence-based design optimization, see e.g. (Mourelatos and Zhou 2005) is also a different robust-based design-optimization methodology.

Both concepts are considered in the methodology used by the authors (António and Hoffbauer 2009), which is divided into two steps: calculation of the maximum allowed load based on the solution of the inverse reliability-based design optimization problem; maximum robustness considering the previously calculated maximum permitted load.

Reliability-based design optimization of composite structures is also analyzed by António (António 2001). The reliability is based on the Reliability Index Approach using, for its evaluation, a Lind-Hasofer approximation combined with the Newton-Raphson iterative method and the arc-length method. GA is used to minimize the weight of the structure under failure probability constraints, using the mechanical properties of the ply laminates as random parameters, and ply angles and ply thicknesses as design variables.

A variation of the Performance Measure Approach is considered in the reference (das Neves Carneiro and Antonio 2018) in a different approach. In the said approach, the uncertainty space is defined in directional coordinates, being then reduced to a surface. The results are successful since they allowed an increase in the efficiency of the reliability-based design optimization approach.

Das Neves Carneiro and António (das Neves Carneiro and António 2019b) carried out the minimization problem off the two objectives, the weight of the cylindrical shell structure and the determinant of the variance-covariance matrix, as a quantification of the robustness, under deterministic and probabilistic constraints. The design-optimization methodology of such structures is reliability-based. The method is then based on the reliability index approach, and an elitist strategy is adopted.

In ref. (das Neves Carneiro and António 2019), the reliability-based design optimization of the composite structure used in the previous works is now used once again to evaluate the effects of different sources of uncertainty: random design variables and random parameters. The authors have divided them into four groups: mechanical properties, ply-angle of the laminates, laminate thicknesses and point loads. It has been possible to conclude that mechanical properties have a reduced influence. Since the reliability assessment is shown to significantly harm the efficiency of the optimization methods based on evolutionary algorithms, particularly in composite laminate structures, the dimensionality reduction of the original model is carried out in a different study (das Neves Carneiro and António 2021), considering the mechanical properties to be frozen as previously demonstrated by the authors (das Neves Carneiro and António 2019).

Antonio and Hoffbauer (António and Hoffbauer 2008) aimed to analyse the influence of the input variables over the output variables of composite structures, mainly concerning their mechanical properties, ply angles and thicknesses, and applied loads. Global sensitivity analysis, variance-based method, first-order local method and extension to global variance are considered for such an analysis. Other studies, see e.g. (Peng et al. 2021) or (António and Hoffbauer 2007), have also been conducted.

3 PSO: A Numerical Example

This chapter presents an example to let readers know how to apply PSO and understand it. The example here given is original and the figures are created in MATLAB. Classic PSO is here used with a constriction factor and linear variation of inertia weight. Let us consider the 2D Ackley function which is presented in (3.1).

$$f(x_1, x_2) = a \cdot \exp\left(-b \cdot \sqrt{\frac{x_1^2 + x_2^2}{2}}\right) - \exp\left(\frac{1}{2}\sum_{i=1}^{2} cos(c \cdot x_i)^2\right) - a + \exp\ 1 \qquad (3.1)$$

For this example, a = -20, b = 0.2 and c = 4π and $(x_1, x_2) \in [-5{,}5] \times [-5{,}5]$. The surface for the 2D Ackley function, with referred parameters, is presented in Figure 15. As well as most benchmark functions in the literature, the optimum solution is located at the origin of the coordinate system.

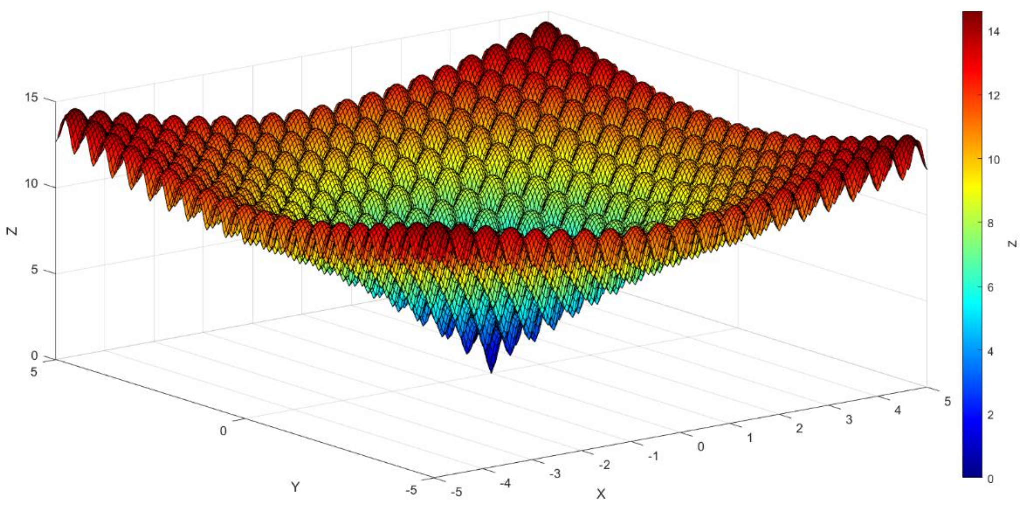

Figure 15. Surface plot of 2D Ackley function

In the next figures, the PSO flowchart of Figure 3 is followed step-by-step, including plots for every iteration. In those plots, the optimum solution, despite being known, will be displayed by an asterisk (*). A total of 50 generations and a swarm of P=10 particles are used.

- Step 1: Initialization

Typically, a value for the population size larger than ten is used for the swarm size; however, for this example, P=10. In the equations (2.3) and (2.4) of the previous chapter are modified so that inertia weight and constriction factor are added to create better convergence conditions. Inertia value is limited to $\omega = [0.4, 0.9]$. Two variables are set: $\omega_{min} = 0.4$ and $\omega_{max} = 0.9$ and ω is calculated as in (3.2), where *maxiter* = 50 is the maximum number of generations and t is the current generation index. Despite considering 50 generations of the process, only the first two generations are shown. Also, the constriction factor is calculated as in (3.3).

$$\omega_t = \omega_{max} + (\omega_{min} - \omega_{max}) \cdot \frac{t}{maxiter} \tag{3.2}$$

$$K = \frac{2}{|2 - c_1 - c_2 - \sqrt{(c_1 + c_2)^2 - 4 \cdot (c_1 + c_2)}|} \tag{3.3}$$

Values of c_1 and c_2 are both equal to 2.05. Also, t = 0 at the beginning. $\mathbf{X}_0$ and $\mathbf{V}_0$ are randomly set:

$$\mathbf{X}_0 = \begin{bmatrix} -2.0320 & 2.7283 \\ -4.7516 & 1.9328 \\ 4.1784 & 1.1967 \\ 1.8603 & 2.6230 \\ -2.6572 & 0.1148 \\ -0.0265 & -1.7059 \\ 2.9058 & -4.9866 \\ -1.1977 & 0.8064 \\ -2.8142 & -3.2816 \\ 2.2267 & -3.0509 \end{bmatrix}, \mathbf{V}_0 = \begin{bmatrix} -3.4968 & -2.6716 \\ 0.0542 & -2.5395 \\ -4.8261 & -1.4807 \\ -0.2879 & -0.9092 \\ 4.5392 & -3.8576 \\ -0.5773 & 0.9278 \\ -3.3644 & 1.7593 \\ 0.7210 & 0.4302 \\ 3.1832 & -1.4287 \\ -4.1466 & 1.6159 \end{bmatrix}$$

Figure 16 also shows this initial swarm scattered in the plot.

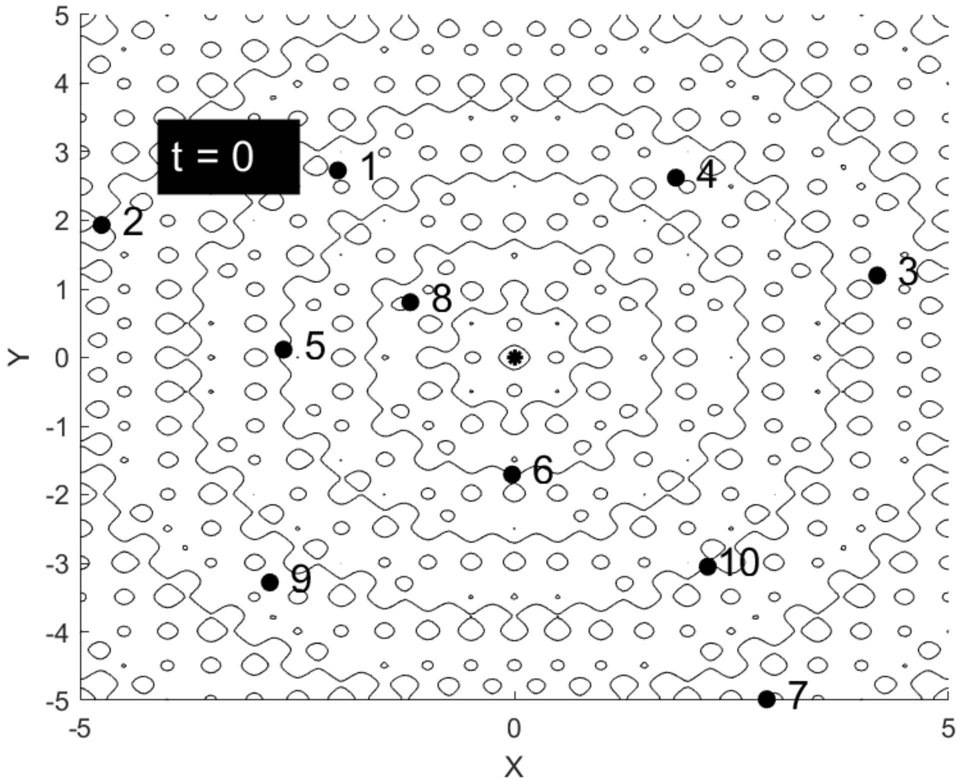

Figure 16. Swarm at generation t = 0

- Step 2: Fitness evaluation

In this step, (3.1) is evaluated. The first column of $\mathbf{X}_0$ is equivalent to the first coordinate for all particles, and the second column is for the second coordinate. Table 9 shows the results of step 2.

- Step 3: Update the personal best and the global best

For this first generation, $\mathbf{P}_0^{\mathrm{i}} = \mathbf{X}_0^{\mathrm{i}}$ holds. The best personal fitness score for the first generation and for the i-th particle is called $pbest_{\mathbf{0}}^{\boldsymbol{i}} = f\left(p_{1_0}^i, p_{2_0}^i\right)$, that is also the equivalent to say, in this generation, $pbest_0^i = f\left(x_{1_0}^i, x_{2_0}^i\right)$**.** For the next generations, this situation is not so likely to happen. The best fitness score is called $gbest_0$:

$$f\left(x_{1_0}^8, x_{2_0}^8\right) = f(-1.1977, 0.8064) = 5.9516 = \ gbest_0 = \ f\left(g_{1_0}, g_{2_0}\right)$$

Table 9. Fitness Values and personal best values, generation t = 1

Particle	$\boldsymbol{x_{1_0}}$	$\boldsymbol{x_{2_0}}$	$\mathbf{F}\left(\boldsymbol{x_{1_0}}, \boldsymbol{x_{2_0}}\right)$	$\boldsymbol{pbest_0}$	$\boldsymbol{p_{1_0}}$	$\boldsymbol{p_{2_0}}$
1	-2.0320	2.7283	9.3776	9.3776	-2.0320	2.7283
2	-4.7516	1.9328	12.1900	12.1900	-4.7516	1.9328
3	4.1784	1.1967	11.4070	11.4070	4.1784	1.1967
4	1.8603	2.6230	9.1028	9.1028	1.8603	2.6230
5	-2.6572	0.1148	8.1127	8.1127	-2.6572	0.1148
6	-0.0265	-1.7059	5.9575	5.9575	-0.0265	-1.7059
7	2.9058	-4.9866	11.8990	11.8990	2.9058	-4.9866
8	-1.1977	0.8064	5.9516	5.9516	-1.1977	0.8064
9	-2.8142	-3.2816	11.4200	11.4200	-2.8142	-3.2816
10	2.2267	-3.0509	10.0700	10.0700	2.2267	-3.0509

- Step 4: Update velocity and particle positions

The PSO equations being used are presented in (3.4) and (3.5):

$$\mathbf{V}_{t+1}^i = K \cdot [\omega \cdot \mathbf{V}_t^i + \varphi_1 \mathbf{R_1}_t^i \circ \left(\mathbf{P}_t^i - \mathbf{X}_t^i\right) + \varphi_2 \mathbf{R_2}_t^i \circ \left(\mathbf{g}_t - \mathbf{X}_t^i\right)] \tag{3.4}$$

$$\mathbf{X}_{t+1}^i = \mathbf{X}_t^i + \mathbf{V}_{t+1}^i \tag{3.5}$$

Generally, and for other applications, (3.5) is never changed. In PSO variations, only velocity is susceptible to changes. For this application, all components are met and are as follows:

- K = 0.7298, using (3.3). Note that $\varphi_1 = \varphi_2 = 2.05 = \text{const.}$, K will never change;
- $\omega = \ 0.89$ (using (3.2), for t = 1);
- $\mathbf{g}_1 = \left(g_{1_1}, g_{2_1}\right)^{\mathrm{T}} = (-1.1977, 0.8064)^{\mathrm{T}}$;
- $\mathbf{X}_0^{\mathrm{i}}$ values are displayed in Table 9;
- $\mathbf{R_1}_{\mathrm{t}}^{\mathrm{i}}$ and $\mathbf{R_2}_{\mathrm{t}}^{\mathrm{i}}$ are random values that can vary from 0 to 1. Those values given in this example for each particle i are displayed in Table 10.

With all components defined, (3.4) is used with the previous values. For the first particle, the new velocity is calculated as follows:

$$\mathbf{V}_1^1 = K \cdot [\omega \cdot \mathbf{V}_0^1 + \varphi_1 \mathbf{R_1}_0^1 \circ (\mathbf{P}_0^1 - \mathbf{X}_0^1) + \varphi_2 \mathbf{R_2}_0^1 \circ (\mathbf{g}_1 - \mathbf{X}_0^1)] =$$
$$= 0.7298 \cdot \left[0.89 \cdot \begin{Bmatrix} -3.4968 \\ -2.6716 \end{Bmatrix} + 2.05 \cdot \begin{Bmatrix} 0.6787 \cdot (-2.0320 + 2.0320) \\ 0.5707 \cdot (2.7283 - 2.7283) \end{Bmatrix} + 2.05 \cdot \begin{Bmatrix} 0.1730 \cdot (-1.1977 + 2.0320) \\ 0.1131 \cdot (0.8064 - 2.7283) \end{Bmatrix}\right] = \begin{Bmatrix} -2.0554 \\ -2.0607 \end{Bmatrix}$$

Also, the same particle position is calculated as follows:

$$\mathbf{X}_1^1 = \mathbf{X}_0^1 + \mathbf{V}_1^1 = \begin{Bmatrix} -2.0320 \\ 2.7283 \end{Bmatrix} + \begin{Bmatrix} -2.0554 \\ -2.0607 \end{Bmatrix} = \begin{Bmatrix} -4.0874 \\ 0.6676 \end{Bmatrix}$$

After this operation, the side constraints must be met so that the particle cannot be outside the domain. For this application, $\mathbf{X}_1^1$ is inside the possible domain because $-5 \leq -4.0874 \leq 5$ and $-5 \leq 0.6676 \leq 5$. In other applications, each component of $\mathbf{X}_1^1$ must be transformed according to (3.6).

$$x_{d_1}^i \leftarrow \begin{cases} x_{d_1}^i & \text{, if } x_{d_{min}} \leq x_{d_1}^i \leq x_{d_{max}} \\ x_{d_{max}} & \text{, if } x_{d_{max}} < x_{d_1}^i \\ x_{d_{min}} & \text{, if } x_{d_{min}} > x_{d_1}^i \end{cases} \tag{3.6}$$

For the first particle, $x_{1_1}^1 \leftarrow x_{1_1}^1$ and $x_{2_1}^1 \leftarrow x_{2_1}^1$. However, $x_{1_1}^3 < -5$, so $x_{1_1}^3 \leftarrow -5$

The rest of the particles are as Table 10 shows.

Table 10. Random Values for velocity and position update, generation t = 1

Particle	$r_{1_{1_0}}$	$r_{1_{2_0}}$	$r_{2_{1_0}}$	$r_{2_{2_0}}$	v_{1_1}	v_{2_1}	x_{1_1}	x_{2_1}
1	0.6787	0.5707	0.1730	0.1131	-2.0554	-2.0607	-4.0874	0.6676
2	0.1316	0.6348	0.6944	0.1079	3.7274	-1.8314	-1.0241	0.1014
3	0.4396	0.3989	0.9249	0.4457	-10.5750	-1.2221	-5.0000	-0.0253
4	0.9372	0.2489	0.3706	0.9389	-1.8826	-3.1423	-0.0223	-0.5193
5	0.5702	0.9353	0.6388	0.6338	-1.5536	-1.8498	-4.2108	-1.7350
6	0.5776	0.4489	0.8897	0.9297	-1.9340	2.8919	-1.9605	1.1860
7	0.7646	0.5318	0.4829	0.0629	-5.1504	1.6877	-2.2446	-3.2988
8	0.2470	0.6348	0.6298	0.3069	0.4683	0.2795	-0.7293	1.0859
9	0.6939	0.6681	0.4650	0.9498	3.1924	4.8812	0.3782	1.5996
10	0.2949	0.4390	0.1092	0.0104	-3.2528	1.1095	-1.0261	-1.9415

After this step, the generation number is set to 1. As *t* = 1 < 100 (maxiter), the algorithm returns to step 2. Now, the first generation is finished.

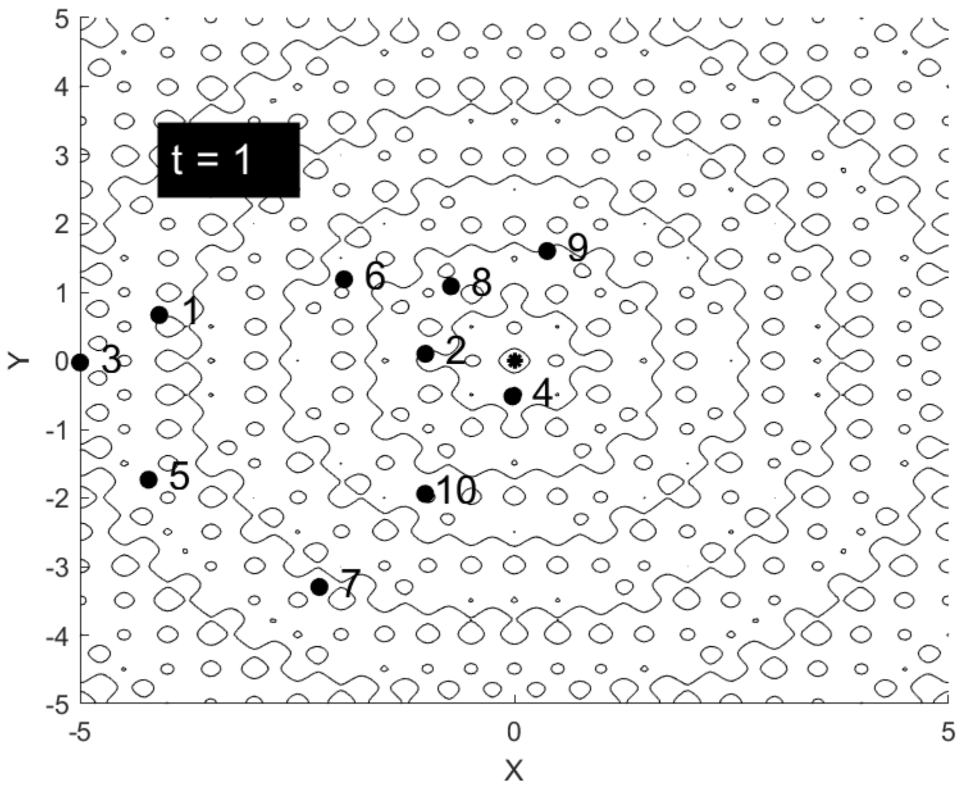

Figure 17. Swarm at generation t = 1

- Step 2 (2nd generation): Fitness evaluation

The equation (3.1) is re-evaluated. Table 11 shows the results of this step 2.

- Step 3 (2nd generation): Update pbest and gbest

In Table 11, it is possible to evaluate all pbest and gbest. Now, for the first particle, $pbest_1^1$ is given by the minimum between the current fitness value and the previous one,

$$pbest_1^1 = \min_{\tau}\{\, \mathrm{y}_0^1,\ \mathrm{y}_1^1\} = \min_{\tau}\{9.3776, 10.6120\} = \ 9.3776$$

And the vector of the best position is given by the position of the first particle at the best generation, called "tb",

$$\mathbf{P}_1^1 = \ \mathbf{X}_1^{1,\mathrm{tb}} = \ \mathbf{X}_0^1 = \begin{Bmatrix} -2.0320 \\ 2.7283 \end{Bmatrix}$$

For the calculation of $gbest_1$, only the $pbest_1$ is considered, but $\mathbf{g}_1$ is verified according to all occurrences in current and previous generations. The value of $y_1^{ib,1}$, where ib is the best particle for the generation t = 1, is given by:

$$y_1^{ib,1} = \min_i\{\, y_1^1, \dots,\ y_1^P\} = \ y_1^2 = \ f\left(x_{1_1}^2, x_{2_1}^2\right) = f(2.498, 1.488) = 0.682$$

It was verified in generation t=0 that $y_0^{ib,1} = \ y_0^8$. Therefore, gbest_1 is:

$$y_1^{ib,tb} = \ gbest_1 = \ \min_{\tau}\left\{\, y_0^{ib,0},\ y_1^{ib,1}\right\} = \min_{\tau}\{\, y_0^8,\ y_1^4\} = \min_{\tau}\{5.9516, 1.5086\} = 1.5086$$

It means that $y_1^{ib,tb} = \ y_1^{ib,1} = \ y_1^4$ and so

$$\mathbf{g}_1 = \ \mathbf{X}_1^{\mathrm{ib,tb}} = \ \mathbf{X}_1^4 = \begin{Bmatrix} -0.0223 \\ -0.5193 \end{Bmatrix}$$

Table 11. Fitness Values and personal best values, generation t = 2

Particle	x_{1_1}	x_{2_1}	$\mathrm{F}(x_{1_1}, x_{2_1})$	y_1^i < $pbest_0^i$? *(Yes / No)*	$pbest_1$	p_{1_1}	p_{2_1}
1	-4.0874	0.6676	10.6120	No	9.3776	-2.0320	2.7283
2	-1.0241	0.1014	3.5623	Yes	3.5623	-1.0241	0.1014
3	-5.0000	-0.0253	10.2060	Yes	10.2060	-5.0000	-0.0253
4	-0.0223	-0.5193	1.5086	Yes	1.5086	-0.0223	-0.5193
5	-4.2108	-1.7350	11.8210	No	8.1127	-2.6572	0.1148
6	-1.9605	1.1860	7.1564	No	5.9575	-0.0265	-1.7059
7	-2.2446	-3.2988	10.9390	Yes	10.9390	-2.2446	-3.2988
8	-0.7293	1.0859	5.3152	Yes	5.3152	-0.7293	1.0859
9	0.3782	1.5996	5.6726	Yes	5.6726	0.3782	1.5996
10	-1.0261	-1.9415	5.7318	Yes	5.7318	-1.0261	-1.9415

- Step 4: Update velocity and particle positions

For this generation, the methodology is the same, i.e., all the components are met:

- K = 0.7298. Note that $\varphi_1 = \varphi_2 = 2.05 = \text{const.}$
- $\omega = 0.88$ (using (3.4), for t = 1).
- $\mathbf{g}_t = (g_{1_1}, g_{2_1})^T = (1.954, 1.015)^{\mathrm{T}}$
- $\mathbf{V}_1^{\mathrm{i}}$ values are displayed in Table 12.
- $\mathbf{x}_1^{\mathrm{i}}$ values are displayed in Table 11.
- ${\mathbf{R}_1}_1^{\mathrm{i}}$ and ${\mathbf{R}_2}_1^{\mathrm{i}}$ are random values that can vary from 0 to 1. Those values given in this example for each particle i are displayed in Table 12.

With all components defined, application of (3.4) is done. For example, for the first particle, the new velocity can be calculated as follows:

$$\mathbf{V}_2^1 = K \cdot [\omega \cdot \mathbf{V}_1^1 + \varphi_1 {\mathbf{R}_1}_1^1 \circ (\mathbf{P}_1^1 - X_1^1) + \varphi_2 {\mathbf{R}_2}_1^1 \circ (\mathbf{g}_1 - \mathbf{X}_1^1)] =$$

$$= 0.7298 \cdot \left[0.88 \cdot \begin{Bmatrix} -2.0554 \\ -2.0607 \end{Bmatrix} + 2.05 \cdot \begin{Bmatrix} 0.1678 \cdot (-2.0320 + 4.0874) \\ 0.9641 \cdot (2.7283 - 0.6676) \end{Bmatrix} + 2.05 \cdot \begin{Bmatrix} 0.3315 \cdot (-0.0223 + 4.0874) \\ 0.9426 \cdot (-0.5193 - 0.6676) \end{Bmatrix}\right] = \begin{Bmatrix} 1.2125 \\ -0.0250 \end{Bmatrix}$$

Also, the same particle position can be calculated as follows:

$$\mathbf{X}_2^1 = \mathbf{X}_1^1 + \mathbf{V}_2^1 = \begin{Bmatrix} -4.0874 \\ 0.6676 \end{Bmatrix} + \begin{Bmatrix} 1.2125 \\ -0.0250 \end{Bmatrix} \approx \begin{Bmatrix} -2.8750 \\ 0.6427 \end{Bmatrix}$$

As in generation 1, the particle cannot be outside the domain. For this application, $\mathbf{X}_2^1$ is inside the possible domain because $-5 \leq -2.8750 \leq 5$ and $-5 \leq 0.6427 \leq 5$. In other applications, each component of $\mathbf{X}_2^1$ must be transformed as follows:

$$x_{d_1}^1 \leftarrow \begin{cases} x_{d_1}^1 & \text{, if } x_{d_{min}} \leq x_{d_1}^1 \leq x_{d_{max}} \\ x_{d_{max}} & \text{, if } x_{d_{max}} \leq x_{d_1}^1 \\ x_{d_{min}} & \text{, if } x_{d_{min}} \geq x_{d_1}^1 \end{cases} \tag{3.7}$$

According to (3.7), $x_{1_2}^1 \leftarrow x_{1_2}^1$ and $x_{2_2}^1 \leftarrow x_{2_2}^1$.

The rest of the particles are evaluated and the numerical results are displayed as Table 12 shows.

Table 12. Random Values for velocity and position update, generation t = 2

Particle	$r_{1_{1_1}}$	$r_{1_{2_1}}$	$r_{2_{1_1}}$	$r_{2_{2_1}}$	v_{1_2}	v_{2_2}	x_{1_2}	x_{2_2}
1	0.1678	0.9641	0.3315	0.9426	1.2125	-0.0250	-2.8750	0.6427
2	0.7553	0.9819	0.5782	0.6416	3.2607	-1.7720	2.2365	-1.6706
3	0.8338	0.9186	0.8545	0.8291	-0.4278	-1.3976	-5.0000	-1.4229
4	0.6059	0.2835	0.6424	0.3888	-1.2092	-2.0182	-1.2315	-2.5374
5	0.5179	0.0586	0.4201	0.1684	2.8387	-0.7195	-1.3721	-2.4545
6	0.9429	0.8210	0.5459	0.7401	3.0692	-3.5832	1.1087	-2.3972
7	0.7412	0.1795	0.1823	0.8728	-2.7018	4.7137	-4.9464	1.4149
8	0.7011	0.0349	0.2363	0.3555	0.5507	-0.6743	-0.1786	0.4116
9	0.7447	0.7713	0.2054	0.1788	1.9272	2.5681	2.3054	4.1676
10	0.8738	0.5405	0.2597	0.9964	-1.6991	2.8328	-2.7253	0.8914

After this step, generation number is set to 2. As $t = 2 < 50$ (maxiter), the algorithm returns to step 2. Now, the second generation is finished, and the algorithm would continue until $t = 50$. The current swarm in show in Figure 18.

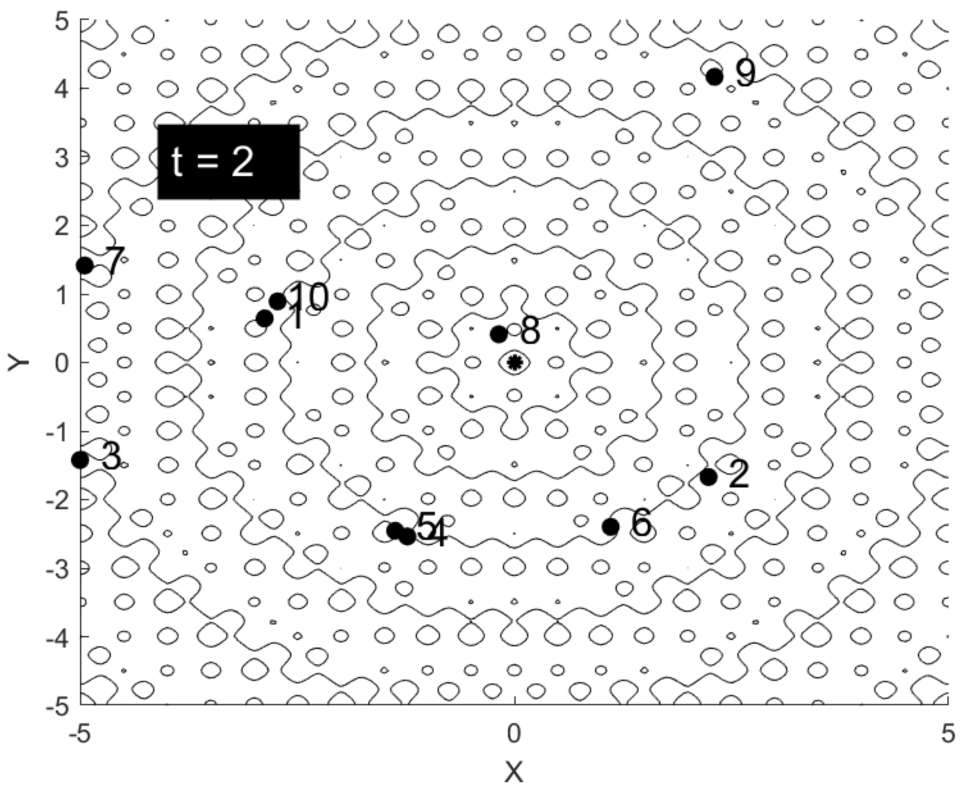

Figure 18. Swarm at generation t = 2

The process is the same for further generations. In Figure 19, it is possible to verify the swarm for t = 50. At this step, the particles are shown to converge to almost a unique point, the global optimum.

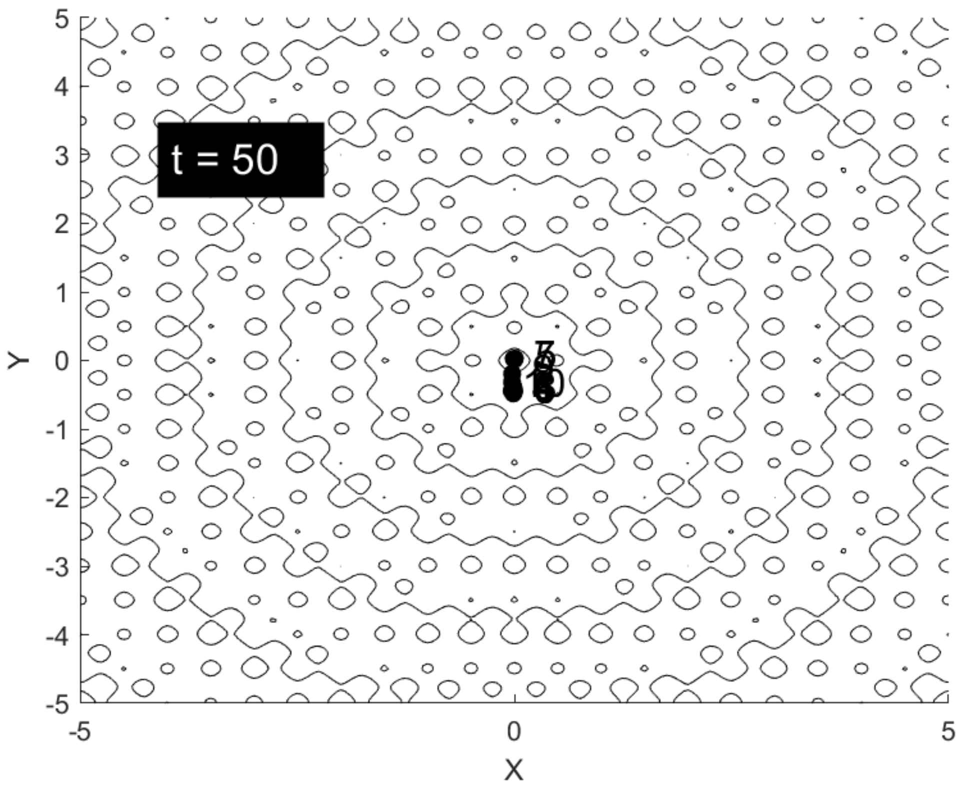

Figure 19. Swarm at generation t = 50

4 Proposed Approach

4.1 Structure response

The present work uses the finite shell element developed in (Ahmad 1969). Based on the Mindlin shell theory, this element is 3D, isoperimetric, with eight nodes and five degrees of freedom for each node. These degrees of freedom are three independent translations and two independent rotations. The shell consists of many perfect bonded plies. Each individual ply is assumed homogeneous and anisotropic.

The shape functions $\mathbf{N}_k(\xi,\eta)$, where ξ and η are local coordinates defined in the element's middle plan, are given. For each element i and each node k, the displacement vector $\bar{\mathbf{u}}_{ik}$, the thickness h_k and the cosines of the nodal coordinate system $\bar{\mathbf{v}}_{jk}, j = 1,2$, are also given and are referent to the shell's middle surface. The displacement field of the element i can be computed based on (4.1). In (4.1), ζ corresponds to the coordinate in the orthogonal direction of the middle plan.

$$u_i = \sum_{k=1}^{n} \mathbf{N}_k(\xi,\eta)\left(\mathbf{u}_{ik}^{mid} + \frac{1}{2}\zeta h_k[\bar{\mathbf{v}}_{1k}\bar{\mathbf{v}}_{2k}](\beta_{1k},\beta_{2k})^T\right) \tag{4.1}$$

The strain matrix **B** performs the relationship between the displacement vector and the strain vector $\boldsymbol{\epsilon}'$. Such a strain-displacement relationship is considered in (4.2).

$$\boldsymbol{\epsilon}' = \mathbf{Bu} \tag{4.2}$$

The stress-strain constitutive relation is written in (4.3).

$$\boldsymbol{\sigma}' = \mathbf{T}^{\mathrm{T}}\mathbf{DT}\boldsymbol{\epsilon}' \tag{4.3}$$

Where $\boldsymbol{\sigma}'$ is the stress vector, **T** is the transformation matrix of the system, and **D** is the matrix with the elastic constants. The above equation can also be represented by (4.4).

$$\begin{Bmatrix} s_{x'} \\ s_{y'} \\ s_{x'y'} \\ s_{x'z'} \\ s_{y'z'} \end{Bmatrix} = \mathbf{T}^{\mathrm{T}} \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & k_1G_{13} & 0 \\ 0 & 0 & 0 & 0 & k_2G_{23} \end{bmatrix} \mathbf{T} \begin{Bmatrix} \frac{\partial \mathrm{u}}{\partial \mathrm{x}'} \\ \frac{\partial \mathrm{v}}{\partial \mathrm{x}'} \\ \frac{\partial \mathrm{v}}{\partial \mathrm{x}'} + \frac{\partial \mathrm{u}}{\partial \mathrm{y}'} \\ \frac{\partial \mathrm{w}}{\partial \mathrm{x}'} + \frac{\partial \mathrm{u}}{\partial \mathrm{z}'} \\ \frac{\partial \mathrm{w}}{\partial \mathrm{y}'} + \frac{\partial \mathrm{v}}{\partial \mathrm{z}'} \end{Bmatrix} \tag{4.4}$$

Where $(\mathrm{x}',\mathrm{y}',\mathrm{z}')$ is the local coordinate system, (u,v,w) is the displacement system, and the elastic constants of the orthotropic ply are the longitudinal elastic modulus E_1, the transversal elastic modulus E_2, the in-plane shear modulus G_{12}, the out-of-plane modulus G_{13} and G_{23}, the in-plane Poisson's ratio ν_{12} and the constants k_1, k_2 that are shear correction factors. Moreover, $s_{x'}$, $s_{y'}$, $s_{x'y'}$, $s_{x'z'}$ and $s_{y'z'}$ are stresses related to the stress vector $\boldsymbol{\sigma}'$.

Considering the linear elastic behavior of composite structures, the equilibrium equation is set in (4.5).

$$\mathbf{K}\,\mathbf{u} = \mathbf{F} \tag{4.5}$$

Where **F** is the vector of the applied external loads and **K** is the stiffness matrix of the system. The global stiffness matrix is the result of the assembly of the stiffness matrix of each element e, $\mathbf{K}^e$, that is defined in (4.6).

$$\mathbf{K}^{\mathrm{e}} = \int_{V^e} \mathbf{B}^{\mathbf{T}}\mathbf{D}'\mathbf{B}\, dV^e \tag{4.6}$$

The system response is given by the function $\boldsymbol{\varphi}(\mathbf{x})$, with $\mathbf{x} = (x_1, \ldots, x_n)$. In the present study, two output parameters are considered. The first one concerning the maximum displacement, $u_{max} = \max\{u_i, i = 1, \ldots, N_{dis}\}$, being N_{dis} the number of displacements, and the second one concerning the most critical Tsai number, $R_{min} = \min\{R_i, i = 1, \ldots, N_{str}\}$, being N_{str} the number of points where the stress vector is evaluated. The Tsai number R_i is a function of the stresses calculated using the quadratic failure criterion of Tsai-Wu (Tsai 1987), as in (4.7).

$$\left(F_{jk}s_js_k\right)R_i^2 + \left(F_js_j\right)R_i = 1, \quad j,k = 1,2,6 \tag{4.7}$$

In (4.7), s_j is the j-th component of the stress vector, F_{jk} and F_j are strength parameters associated with the unidirectional laminated defined in the macro-mechanical perspective (António and Hoffbauer 2017).

Therefore, the system response is given in (4.8) for the current application.

$$\boldsymbol{\varphi}(\mathbf{x}) = \begin{Bmatrix} u_{max} \\ R_{min} \end{Bmatrix} \tag{4.8}$$

4.2 Uncertainty assessment based on sensitivity analysis

Uncertainty assessment is part of the Robust Design Optimization (RDO) of composite structures. Aleatory uncertainty is common to use in the literature. It arises due to the randomness in the behaviour of composites, such as the physical or geometric properties and loads of the model. The methodology is based on Taylor's series expansion. Its first order is written as in (4.9).

$$\boldsymbol{\varphi}(\mathbf{x}^0 + \delta\mathbf{x}) = \boldsymbol{\varphi}(\mathbf{x}^0) + \delta\boldsymbol{\varphi} \tag{4.9}$$

S is a matrix storing the sensitivity of the response φ_i to the system design variable x_j, represented by the first-order derivative as given in (4.10).

$$S_{ij} \cong \frac{\partial \varphi_i}{\partial x_j} \tag{4.10}$$

The response function $\boldsymbol{\varphi}$ can be then written in terms of the sensitivity matrix **S**, as in (4.11).

$$\boldsymbol{\varphi}(\mathbf{x}^0 + \delta\mathbf{x}) \cong \boldsymbol{\varphi}(\mathbf{x}^0) + \mathbf{S}\delta\mathbf{x} \tag{4.11}$$

As explained in (António and Hoffbauer 2013; António and Hoffbauer 2017), including formulation for higher-order become unnecessary since it is complex and is then avoided in practice. The expectation and covariance in the response function are obtained as in (4.12) and in (4.13), respectively.

$$\mathbf{E}(\boldsymbol{\varphi}) = \boldsymbol{\varphi}^0 \tag{4.12}$$

$$\mathbf{C}_{\boldsymbol{\varphi}} = \mathbf{E}(\mathbf{S}\delta\mathbf{x}(\mathbf{S}\delta\mathbf{x})^{\mathrm{T}}) = \mathbf{S}\mathbf{E}(\delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}})\mathbf{S}^{\mathrm{T}} = \mathbf{S}\mathbf{C}_{\mathbf{x}}\mathbf{S}^{\mathrm{T}} \tag{4.13}$$

The joint effects of the propagation of uncertainties concerning the response are essential to the structural reliability analysis. Each component of the matrix $\mathbf{C}_{\mathbf{x}}$ is defined as in (4.14).

$$\mathbf{C}_{\mathbf{x}_{\mathrm{ij}}} = \begin{bmatrix} \sigma_1^2 & \cdots & \rho_{1j}\sigma_1\sigma_j & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{i1}\sigma_i\sigma_1 & \cdots & \rho_{ij}\sigma_i\sigma_j & \cdots & \rho_{in}\sigma_i\sigma_n \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \cdots & \rho_{nj}\sigma_n\sigma_j & \cdots & \sigma_n^2 \end{bmatrix} \tag{4.14}$$

In (4.14), ρ_{ij} are correlation coefficients.

In regard to the vector $\boldsymbol{\varphi}(\mathbf{x})$ previously defined in (4.8), the matrix $\mathbf{C}_{\mathbf{x}}$ is applied to the current application as in (4.15).

$$\mathbf{C}_{\boldsymbol{\varphi}} = \begin{bmatrix} \mathrm{var}(u_{max}) & \mathrm{cov}(u_{max}, R_{min}) \\ \mathrm{cov}(R_{min}, u_{max}) & \mathrm{var}(R_{min}) \end{bmatrix} \tag{4.15}$$

In the RDO problem, the feasibility of the solutions is verified through the analysis of the design constraints. The design constraints define the design space to be considered over the optimization iterative process. The uncertainty propagation from the design variables leads to the design constraints not being deterministic. Therefore, the variability of the design constraints is associated with the feasibility robustness (António and Hoffbauer 2013; António and Hoffbauer 2017). This feasibility is then evaluated through the determinant of the variance-covariance matrix (António and Hoffbauer 2013; António and Hoffbauer 2017). In this way, the joint effects of the referred uncertainty are introduced.

4.3 The adjoint variable method

The adjoint variable method was developed for the structural analysis of composite structures (António 1995; António and Hoffbauer 2013). In this method, an augmented Lagrangian is defined based on the terms of the adjoint variable fields to eliminate implicit derivatives. Based on the response equilibrium equation (4.5), the Augmented functional is written as in (4.16).

$$\mathbf{L}(\mathbf{u}, \mathbf{x}, \boldsymbol{\phi}) = \boldsymbol{\varphi}(\mathbf{u}, \mathbf{x}) - \boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{\Psi}(\mathbf{u}, \mathbf{x}) \tag{4.16}$$

In (4.16), $\boldsymbol{\phi}$ is a vector of Lagrange multipliers and, based on the equilibrium equation above defined, $\boldsymbol{\Psi} = \boldsymbol{\Psi}(\mathbf{u}, \mathbf{x})$ is defined as in (4.17).

$$\boldsymbol{\Psi}(\mathbf{u}, \mathbf{x}) = \mathbf{K}\,\mathbf{u} - \mathbf{F} \tag{4.17}$$

The Lagrange multipliers are here selected so that the functional $\mathbf{L}$ is stationary about the displacement vector $\mathbf{u}$. It is formulated as in (4.18).

$$\frac{\partial \mathbf{L}}{\partial \mathbf{u}} = \frac{\partial \boldsymbol{\varphi}(\mathbf{u}, \mathbf{x})}{\partial \mathbf{u}} - \boldsymbol{\phi}^{\mathrm{T}} \frac{\partial \boldsymbol{\Psi}(\mathbf{u}, \mathbf{x})}{\partial \mathbf{u}} = \mathbf{0} \tag{4.18}$$

Considering $\mathbf{F}$ and $\mathbf{u}$ independent of each other, (4.19) can be obtained.

$$\mathbf{K}(\mathbf{x})\boldsymbol{\phi} = \frac{\partial \boldsymbol{\varphi}(\mathbf{u}, \mathbf{x})}{\partial \mathbf{u}} \tag{4.19}$$

Moreover, the tangent stiffness matrix is defined for the equilibrium equation, $\boldsymbol{\Psi}(\mathbf{u}, \mathbf{x}) = \mathbf{0}$.

Considering that (4.16) is stationary, for an equilibrium scenario, (4.20) is proven.

$$\frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}\mathbf{x}} = \frac{\partial \mathbf{L}(\mathbf{u}, \mathbf{x}, \boldsymbol{\phi})}{\partial \mathbf{x}} \tag{4.20}$$

The derivative of $\mathbf{L}$ is based on the equation obtained in (4.18) and is solved for $\mathbf{x}$, corresponding to the equation in (4.21).

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}\mathbf{x}} = \frac{\partial \boldsymbol{\varphi}(\mathbf{u},\mathbf{x})}{\partial \mathbf{x}} + \frac{\partial \boldsymbol{\varphi}(\mathbf{u},\mathbf{x})}{\partial \mathbf{u}}\frac{\partial \mathbf{u}}{\partial \mathbf{x}} - \boldsymbol{\phi}^{\mathrm{T}}\left[\frac{\partial \boldsymbol{\Psi}(\mathbf{u},\mathbf{x})}{\partial \mathbf{x}} + \frac{\partial \boldsymbol{\Psi}(\mathbf{u},\mathbf{x})}{\partial \mathbf{x}}\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right] \tag{4.21}$$

Considering the stationarity condition in (4.18), the definition in (4.16) and the independence of $\mathbf{F}$ and $\mathbf{x}$, (4.21) can be simplified, resulting in (4.22).

$$\frac{\mathrm{d}\boldsymbol{\varphi}}{\mathrm{d}\mathbf{x}} = \frac{\partial \boldsymbol{\varphi}(\mathbf{u},\mathbf{x})}{\partial \mathbf{x}} - \boldsymbol{\phi}^{\mathrm{T}}\frac{\partial \mathbf{K}(\mathbf{x})}{\partial \mathbf{x}}\mathbf{u} \tag{4.22}$$

The sensitivity analysis can be performed based on the following steps:

1. Solving the adjoint equations in (4.19);
2. Getting the sensitivities from (4.22).

From the obtained sensitivities, the components of $\mathbf{S}$ are determined and the variance-covariance matrix $\mathbf{C}_{\boldsymbol{\varphi}}$ is obtained.

4.4 The Robust Design Optimization Problem in Composite Shell Structures

Robust Design Optimization (RDO) aims to improve structural performance and minimise the propagation effects of uncertainties. The RDO formulations in the literature are based on the robustness of a performance associated with the dispersion around its mean (António and Hoffbauer 2017). In this work, the uncertainty is evaluated through the determinant of the variance-covariance matrix $\mathbf{C}_{\boldsymbol{\varphi}}$ from (4.15). The uncertainty analysis in RDO approach, aimed to be developed, is integrated so that the present bi-objective optimization formulation is based on the consideration of the following objective functions:

1. A function that describes the performance/cost of the structural composite structure;
2. A function that describes the constraints' feasibility robustness due to the structure response's uncertainty, measured by its variability.

The classes of variables and parameters considered for the formulation of the optimization problem are as follows: $\mathbf{d} \in \mathbb{R}^{\mathrm{k}}$ is the vector of deterministic design variables; $\mathbf{z} \in \mathbb{R}^{\mathrm{m}}$ is the vector of random design variables; $\boldsymbol{\pi} \in \mathbb{R}^{\mathrm{p}}$ is the vector of random parameters. The expected values are defined as follows: $\boldsymbol{\mu}_{\mathbf{z}} = \mathbf{E}(\mathbf{z})$ and $\boldsymbol{\mu}_{\boldsymbol{\pi}} = \mathbf{E}(\boldsymbol{\pi})$. The corresponding uncertainties are given by their standard deviations or coefficients of variations. The vector of deterministic design variables $\mathbf{d}$, the vector of random design variables $\mathbf{z}$ and its expected values $\boldsymbol{\mu}_{\mathbf{z}}$ intervene in the optimization process.

The weight of the structure depends on the vector of deterministic design variables and on the expected values of the random design variables, and it is represented by $W = W(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{z}})$. The weight is here used as the performance or cost of the structure. In consequence, it is used as the first objective. Moreover, the feasibility robustness associated to the variability of the constraints is given by a functional that depends on the vector of deterministic design variables, on the expected values of the random design variables and on the elements of the variance-covariance matrix $\mathbf{C}_{\boldsymbol{\varphi}}$. Considering $\mathrm{cov}(R_{min}, u_{max}) = \mathrm{cov}(u_{max}, R_{min})$, the referred functional in defined as $V = V(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{z}}, \mathrm{var}(u_{max}), \mathrm{var}(R_{min}), \mathrm{cov}(u_{max}, R_{min}))$. By minimizing it, the function assures that the constraints are satisfied under uncertainty. In consequence, it is used for the second objective.

Given the objective functions defined in (4.23),

$$\begin{gathered} f_1 = W(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{z}}) \\ f_2 = V\big(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{z}}, \mathrm{var}(u_{max}), \mathrm{var}(R_{min}), \mathrm{cov}(u_{max}, R_{min})\big) = \det \mathbf{C}_{\boldsymbol{\varphi}} \end{gathered} \tag{4.23}$$

and the functional constraints in (4.24),

$$g_1(\mathbf{d},\boldsymbol{\mu}_\mathbf{z}) = \frac{u_{max}(\mathbf{d},\boldsymbol{\mu}_\mathbf{z})}{u_a} - 1 \qquad (4.24)$$
$$g_2(\mathbf{d},\boldsymbol{\mu}_\mathbf{z}) = 1 - \frac{R_{min}(\mathbf{d},\boldsymbol{\mu}_\mathbf{z})}{R_a}$$

the bi-objective optimization problem can be then established as in (4.25).

$$\begin{aligned} \text{Minimize:} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x})) \\ \text{Subject to:} \quad & g_1(\mathbf{d},\boldsymbol{\mu}_\mathbf{z}) \le 0 \\ & g_2(\mathbf{d},\boldsymbol{\mu}_\mathbf{z}) \le 0 \\ & d_{j_L} \le d_j \le d_{j_U}, j = 1, \dots, k \\ & \mu_{z\,j_L} \le \mu_{z\,j} \le \mu_{z\,j_U}, j = 1, \dots, m \end{aligned} \qquad (4.25)$$

In equation (4.23), $\det \mathbf{C}_\boldsymbol{\varphi}$ is the determinant of the variance-covariance matrix. Also, in (4.24) and (4.25), u_a and R_a are the maximum allowed displacement and Tsai number, respectively, g_1 and g_2 are constraints of the problem concerning the displacement and Tsai number, respectively, d_{j_L} and d_{j_U} are the lower and upper values of the design variables and $\mu_{z\,j_L}$ and $\mu_{z\,j_U}$ are the lower and upper values of the expected values of the random variables, respectively. The last two of the four constraints are the side constraints of the problem.

4.5 The fitness assignment procedure

The fitness assignment procedure that is exposed in this section has been initially by (Deb 2001) and adopted by Antonio (António 2013). The goal of the fitness assignment is to properly rank and sort the population according to local non-constrain dominance. According to Deb (Deb 2001) and António (António and Hoffbauer 2013), an individual $\mathbf{u}_j$, where $\mathbf{u} = [\mathbf{x}\,\boldsymbol{\pi}]$, is considered constrained-dominated by $\mathbf{u}_i$ if at least one of the following conditions is met:

1. the individuals $\mathbf{u}_i$ and $\mathbf{u}_j$ are feasible, but $\mathbf{u}_i$ if not worse than $\mathbf{u}_j$ for every objective and $\mathbf{u}_i$ is strickly better than $\mathbf{u}_j$ at least for one of the objectives;
2. $\mathbf{u}_i$ is feasible, but $\mathbf{u}_j$ is not;
3. $\mathbf{u}_i$ and $\mathbf{u}_j$ are not feasible, but $\mathbf{u}_i$ has a smaller value for the constraint violation.

The concept of constraint violation is based on the mathematical combination of the values of all the objectives. In this work, it is given as in (4.26).

$$\xi(\mathbf{u}) = \sum_{i=1}^{2} \max\big(0, g_\mathrm{i}(\mathbf{x},\boldsymbol{\pi})\big) \qquad (4.26)$$

In the method of fitness assigned, the rank given to a certain individual corresponds to the number of particles of the population that dominates such an individual. Considering an individual dominated by $p_i < P$ individuals, where P is the population size, its score is given by (4.27).

$$r_i = p_i + 1 \qquad (4.27)$$

Therefore, it means that if an individual is non-dominated, it ranks 1. Moreover, disregarding all the individuals ranked 1, all the other individuals that are non-dominated are ranked 2. This process continues until no individuals are left to be ranked. It means that: 1. At least one individual must be ranked 1 and; 2. No individual has a rank greater than the size of the population (António and Hoffbauer 2013; António and Hoffbauer 2017).

After all the individuals have an associated ranking, a temporary fitness score is assigned to each individual based on their rank. This fitness score is aimed to be a larger value for the best individuals and 1 for the worst individual. In case multiple individuals have the same ranking, all those individuals are scored with the same value, corresponding to the average of their positions in the sorted array. Considering the function $\eta(r_i) = \#\mathbf{C}(r_i)$, where $\mathbf{C}(r_i) = \{j = 1, \dots, P: r_j = r_i\}$, the referred temporary fitness score is given by (4.28).

$$F_i^{aver} = \begin{cases} N - 0.5(\eta(1) - 1), & if\ r_i = 1 \\ N - \sum_{k=1}^{r_i - 1} \eta(k) - 0.5(\eta(r_i) - 1), & if\ r_i \neq 1 \end{cases} \tag{4.28}$$

After the average fitness score is calculated, the concept of niching among solutions of each rank is now adopted and based on the proposed methodology from Fonseca et al. (Fonseca et al. 1993). The proposed approach gives a solution located in a less-crowded region a better-shared fitness. This is aimed to ensure that the merit is given to the particles that situate in a less explored region of the Pareto front as a tie-breaking criterion. The shared fitness F_i^{shar} of a solution $\mathbf{u}_i$ is obtained if one divides the average fitness calculated previously by the niche count that depends on the individual $\mathbf{u}_i$ itself. F_i^{shar} is given by (4.29).

$$F_i^{shar} = \frac{F_i^{aver}}{nc(\mathbf{u}_i)} \tag{4.29}$$

The niche count is based on using a sharing function that is calculated based on the distance between a solution with index i, for which the niche count is being calculated, and a solution with index j, that shares with $\boldsymbol{u}_i$ the same ranking r_i. Then, the values resulting from the sharing functions with respect to all those solutions are summed, as in (4.30).

$$nc(\mathbf{u}_i) = \sum_{j=1}^{\eta(r_i)} Shar(\delta_{ij}) \tag{4.30}$$

The sharing function $\mathrm{Shar}(\delta_{ij})$ is defined in (4.31).

$$Shar(\delta_{ij}) = \begin{cases} 1 - \left(\dfrac{\delta_{ij}}{\sigma_{share}}\right)^{\alpha}, & \text{if } \delta_{ij} \leq \sigma_{share} \\ 0, & \text{otherwise} \end{cases} \tag{4.31}$$

Where $\alpha = 0.5$ is a shape parameter, σ_{share} is a distance of reference and δ_{ij} is the normalized distance between two solutions $\mathbf{u}_i$ and $\mathbf{u}_j$. Mathematically, considering $\mathbf{Y}_{i,k}$ as the result of the function evaluation, where $\mathbf{Y} \in \mathbb{R}^{P \times M}$, as the k-th objective value resulting from the function evaluation of the i-th individual, $f_k^{max} = \max_i \mathbf{Y}_{i,k}$, $f_k^{min} = \min_i \mathbf{Y}_{i,k}$, δ_{ij} is defined in (4.32).

$$\delta_{ij} = \sqrt{\sum_{k=1}^{M} \left(\frac{f_k^i - f_k^j}{f_k^{max} - f_k^{min}}\right)^2} \tag{4.32}$$

Moreover, a dynamic value is considered in (4.33) for the calculation of the reference value σ_{share}.

$$\sigma_{share} = \frac{\sum_{k=1}^{M} (f_k^{max} - f_k^{min})}{P - 1} \tag{4.33}$$

Finally, a scaling operation is considered. The resulting fitness assignment value is given in (4.34).

$$\bar{F}_i^{shar} = \frac{F_i^{aver}\eta(r_i)}{\sum_{k=1}^{\eta(r_i)} F_k^{shar}} F_i^{shar} \tag{4.34}$$

The fitness assignment is then given in (4.35).

$$\mathbf{M}(\mathbf{X}) = \{\bar{F}_i^{shar}, i = 1, \ldots, P\} \tag{4.35}$$

4.6 Multiple-objective PSO (MOPSO)

This work uses a dominance-based sorting methodology on the population to evaluate the particles. Initially, PSO needs merit to be global so that a particle's current performance can be compared to its best performance throughout past generations. Also, its usual configuration is illustrated in Figure 20. PSO population is represented by a matrix containing the values of the design parameters $\mathbf{X}_{\mathrm{t}} = (\mathbf{x}_{1_t}, \ldots, \mathbf{x}_{\mathrm{i}_t}, \ldots, \mathbf{x}_{\mathrm{P}_t})$, where $\mathbf{x}_{\mathrm{i}_t} = (x_{i_t}^1, \ldots, x_{i_t}^j, \ldots, x_{i_t}^k)$, i is the index of the particle, j is the index of the design variables, P is the population size, k is the total number of dimensions and t is the index of the generation. A velocity is associated with the position of each particle, and a matrix of size $P \times k$ also represents it. Positions and velocities are calculated using equations (2.3) and (2.4). The constriction factor is introduced in the velocity equation and the inertia weight varies along with the generations. Moreover, the best position $\bar{\mathbf{x}}_{\mathrm{i}_t}$ is also saved in a matrix with the same dimensions as previously referred.

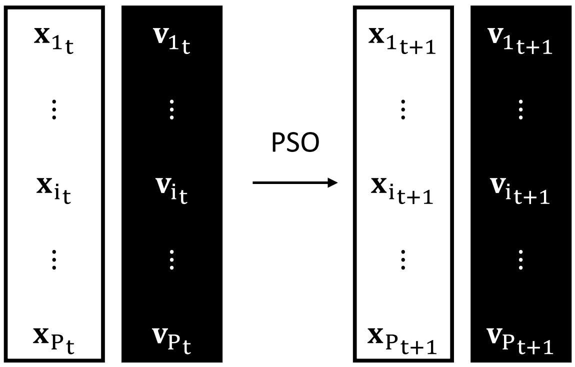

Figure 20. Configuration of the population during the optimization process, using PSO

However, the values resulting of the fitness assignment are only valid for the comparison of the performance of the particles within a generation, being not valid for the comparison to other generations. Therefore, an original methodology is here used to adapt the PSO methodology to the use of the relative score. This methodology is inspired by the preservation of the best solution in GA, and it is schematically represented in Figure 21.

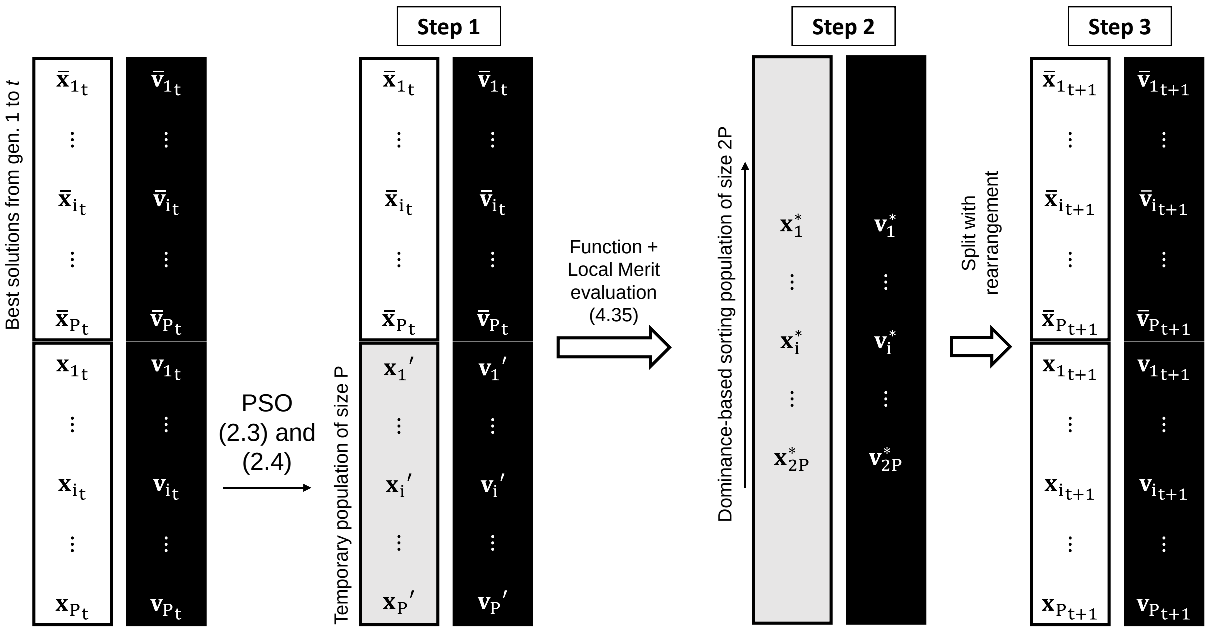

Figure 21. Configuration of the population during the optimization process, using MOPSO

The **first step** is as follows: considering the generation t, the set of particles previously referred to performs PSO equations described in the literature review. However, the matrix storing the personal best particles is not updated here. It becomes not possible because, as referred previously, the same particle cannot be compared between any two generations. The personal best storage is then updated in further steps.

The **second step** is aimed at evaluating the new particles that PSO has generated, and the resulting is given by $\mathbf{y}_{\mathrm{i}}' = \left(f_1(\mathbf{x}_{\mathrm{i}}'), f_2(\mathbf{x}_{\mathrm{i}}')\right), i = 1, \dots, P$. Being $\mathbf{Y} \in \mathbb{R}^{2\mathrm{P}\times 2}$, $\mathbf{Y}_{ij} = f_j(\bar{\mathbf{x}}_{\mathrm{i}}), i = 1, \dots, P \wedge j = 1,2$ and $\mathbf{Y}_{P+i,j} = \mathbf{y}_{\mathrm{i}}', i = 1, \dots, P \wedge j = 1,2$. Considering $\mathbf{X}'' = \{\bar{\mathbf{x}}_1, \dots, \bar{\mathbf{x}}_{\mathrm{P_t}}, \mathbf{x}_1', \dots, \mathbf{x}_{\mathrm{P_t}}'\}$, the resulting scores from the fitness assignment function $\mathbf{M}(\mathbf{X}'')$ allow the sorting of a new population of size 2P, which considers both populations containing the best particles and the local population. This new population is $\mathbf{X}^* = (\mathbf{x}_1^*, \dots, \mathbf{x}_i^*, \dots, \mathbf{x}_{2P}^*)$, and the respective velocities are $\mathbf{V}^* = (\mathbf{v}_1^*, \dots, \mathbf{v}_i^*, \dots, \mathbf{v}_{2P}^*)$. Furthermore, there is a relationship between the indexes of the set of particles before sorting the particles, performed according to their merit, and after it. Therefore, the relationship $j = s(i)$ holds, being i the position of a certain particle before the sorting phase and j is the position of the same particle after the sorting phase. In consequence, $i = s^{-1}(j)$ is referred to as the inverse function. The velocities are always associated with the same particles, i.e., the velocities are also sorted by the relative performance of the corresponding particle, following then the relation $j = s(i)$ as well.

The **third step** consists of dividing the temporary population of size 2P into two sub-populations: the first one is the new population of best particles, defined as $\bar{\mathbf{X}}_{\mathrm{t+1}} = \left(\bar{\mathbf{x}}_{1_{\mathrm{t+1}}}, \dots, \bar{\mathbf{x}}_{\mathrm{i_{t+1}}}, \dots, \bar{\mathbf{x}}_{\mathrm{P_{t+1}}}\right) = (\mathbf{x}_1^*, \dots, \mathbf{x}_{\mathrm{i}}^*, \dots, \mathbf{x}_{\mathrm{P}}^*)$ and the second one is the local population, $\mathbf{X}_{\mathrm{t+1}} = \left(\mathbf{x}_{1_{\mathrm{t+1}}}, \dots, \mathbf{x}_{\mathrm{i_{t+1}}}, \dots, \mathbf{x}_{\mathrm{P_{t+1}}}\right) = (\mathbf{x}_{\mathrm{P+1}}^*, \dots, \mathbf{x}_{\mathrm{P+i}}^*, \dots, \mathbf{x}_{2\mathrm{P}}^*)$. The current method for the rearrangement and splitting of the populations is carried out as shown in the flowchart in Figure 22. The methodology considers the following requirements:

1. At least half of the first-half population, with size P, shall have previously been scored in the first-half population. Mathematically: $\#\mathbf{A} \geq P/2$, where $\mathbf{A} = \{i = 1, \dots, P : \bar{\mathbf{x}}_{\mathrm{i_{t+1}}} \in \mathbf{X}^{**}\}$ and $\mathbf{X}^{**} = (\mathbf{x}_1^*, \dots, \mathbf{x}_i^*, \dots, \mathbf{x}_P^*)$;
2. The personal best position of a particle and its current position shall share the same relative position regarding the subpopulation they belong to. Generally, if the personal

best position of a particle has previously been $\bar{\mathbf{x}}_{i_t}$ and, consequently, $\mathbf{x}_i'$ has been the temporary position at generation t, and assuming $\bar{\mathbf{x}}_{j_{t+1}} \in \{\bar{\mathbf{x}}_{i_t}, \mathbf{x}_i'\}$, then $\mathbf{x}_{j_{t+1}} \in \{\bar{\mathbf{x}}_{i_t}, \mathbf{x}_i'\}$.

Moreover, the flowchart is adapted to work with two different methodologies. Such methodologies are chosen using the condition $V_2 = 1$ or $V_2 \neq 1$. In both situations, if the current position of a particular particle is different from its previous best position, the current best position of the particle is replaced by its current position, the same as what occurs in classic PSO. However, the difference between the situations $V_2 = 1$ and $V_2 \neq 1$ is as follows. When $V_2 = 1$, the particle's current position is replaced by the previous best position of such a particle. When $V_2 \neq 1$, the particle's current position is equal to its current best position. A disadvantage of using the latest methodology concerns the cloning of positions, leading to probable stagnation of the particles, whereas the first one enriches exploration.

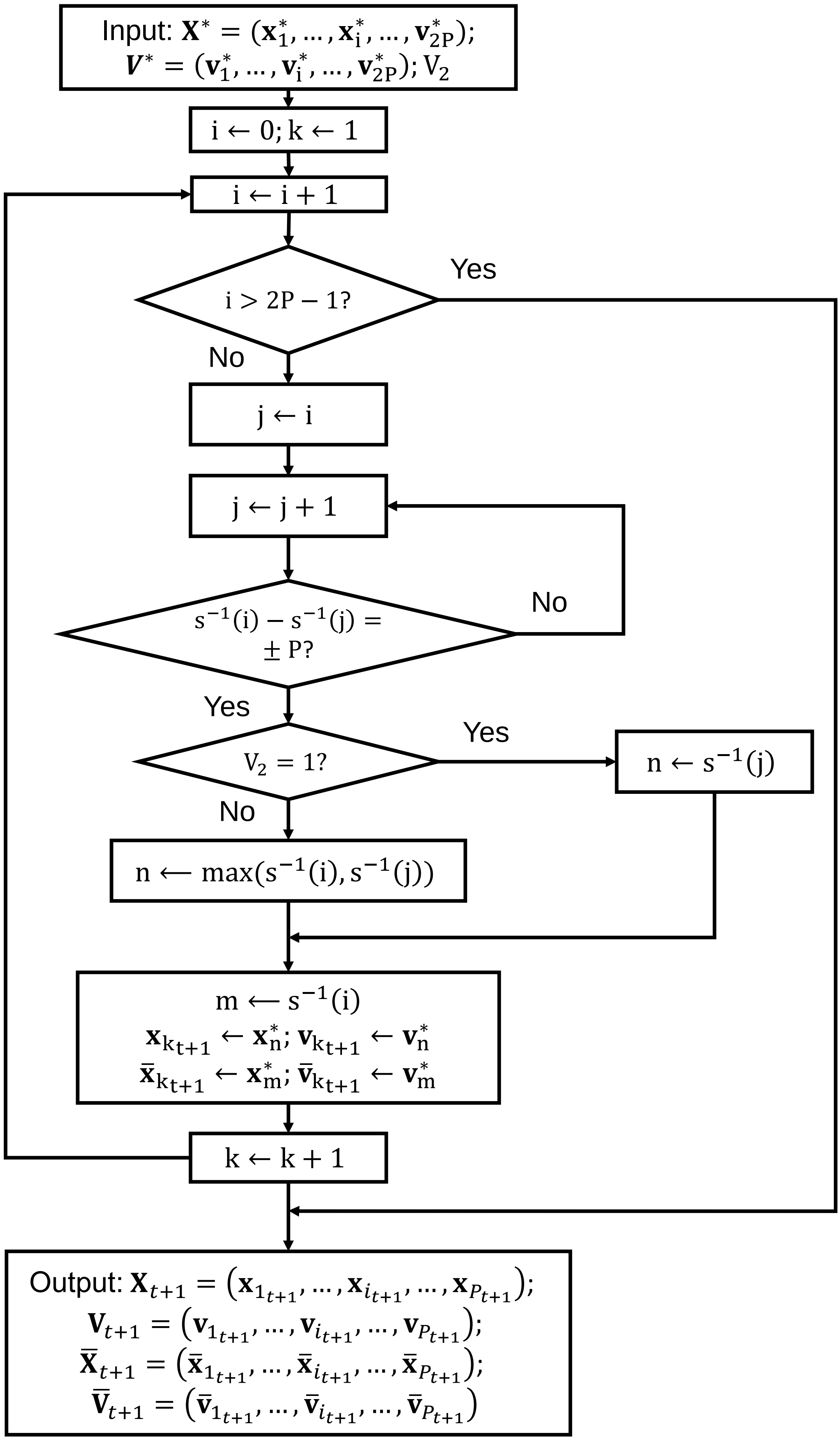

Figure 22. Flowchart for the rearrangement methodology in MOPSO between Step 2 and Step 3

The position of the best particles is then updated for each generation, and it is indeed based on a comparison after fitness assignment as established in equation (4.35), evaluated in the same generation. The connection between the top and the bottom population is then achieved, through the rearrangement procedure shown in flowchart of Figure 22.

This method is then intended to be a variation of the PSO adapted to the current assignment methodology. Simultaneously, the particle positions generated for each iteration are stored in an enlarged population aiming to define the Pareto front (António 2013).

Three different variations of MOPSO are listed in Table 13 and are tested. Results of such tests are shown in the results chapter.

Table 13. Configuration of different MOPSO versions being proposed

Method	Is the global best particle based on the enlarged population?	$V_2 = 1$?
$MOPSO_1$	YES	YES
$MOPSO_2$	YES	NO
$MOPSO_3$	NO	YES

4.7 Multiple-objective PSO hybridized with GA (MOPSOGA)

The same problem and methodology for merit evaluation are used. Other methods based on PSO are then proposed and presented here to deal with the same multi-objective optimization problem. In this section, two hybridization approaches are presented. Both use both PSO and the GA technique proposed by (António and Hoffbauer 2017). The difference between the proposed methods consists of the following: the first method consists of splitting the local population into two parts, from which PSO and GA are separately executed for each of sub populations. Both subpopulations are not required to have the same size. This approach is labelled $\mathrm{MOPSOGA}_1$. The second approach consists of executing PSO in the first step and using the GA technique (António and Hoffbauer 2017) in the second step. This approach is labelled $\mathrm{MOPSOGA}_2$.

The hybridization methods are more complex than MOPSO, even because GA and PSO use two different representations for the population.

The following sub-sections are aimed at presenting each of the methods.

$MOPSOGA_1$

Figure 23 represents the working population in positions and velocities, similarly to Figure 21 for MOPSO.

The local population is generally divided into two sub-populations: the sub-population of particles where positions and velocities are updated using PSO and another sub-population where positions are updated using the crossover proposed by (António and Hoffbauer 2017). The first one is highlighted in Figure 23 using a dashed and dotted border, whereas the second one is highlighted using a solid border.

In **step one**, the sub-population in which PSO is applied is of size $\mathrm{P_{PSO}}$ and the sub-population in which GA is applied is of size $\mathrm{P_{GA}}$. The total number of particles is $P_{PSO} + P_{GA} = P$.

GA technique is adopted for the calculation of positions and velocities using the following steps:

1. The position vector of a certain particle is converted to a binary string using an approximation procedure. It is carried out using the inverse procedure of the usual decoding for converting from binary strings to decimal values.
2. Crossover operation is used, considering the detailed methodology exposed by Antonio and Hoffbauer (António and Hoffbauer 2017).
3. Decoding from the resulting binary string to decimal is applied.
4. If the similarity between values is observed, the mutation is applied.
5. The velocity in the current position is calculated inversely to the velocity calculation procedure in PSO, i.e., velocity is here defined as the difference between the current and the previous position.

The following **steps two and three**, shown in Figure 23, are based on the methodology being used for MOPSO calculation. Therefore, the definition of the resulting position matrix and velocity matrix from the sorting operation due to merit evaluation, are once again given as $\mathbf{X}^* = (\mathbf{x}_1^*, \dots, \mathbf{x}_i^*, \dots, \mathbf{x}_{2P}^*)$ and $\mathbf{V}^* = (\mathbf{v}_1^*, \dots, \mathbf{v}_i^*, \dots, \mathbf{v}_{2P}^*)$, respectively.

In step four of Figure 23, one can verify that another split operation must be done to get the initial configuration of separated PSO and GA populations.

In MOPSO, the proposed rearrangement procedure puts the particles with a better merit score at the top of each sub-population, wherever it is the sub-population of the best positions or the local sub-population. If such methodology is fully adopted, the next generation of PSO particles would have a better merit score than GA particles since PSO is at the top position of the local population by default. Consequently, the solutions obtained by each technique would be biased, and information sharing between them is not as achievable as desired in a hybridization approach.

The proposed approach is based on tracking the particles that are initially assigned to each technique so that the positions and velocities of a particle are calculated with the same technique over the generations. Information sharing occurs as follows: if GA particles achieve a better global score, PSO particles have a better global best so that the swarm moves towards the best position achieved by GA; a similar situation occurs in a vice-versa situation.

Therefore, a different flowchart is proposed and illustrated in Figure 24. In this new flowchart, a new condition is added to check if the particles have been previously assigned to PSO or GA. This condition evaluates if the particle belongs to the PSO population, and it is defined as $C\colon (s^{-1}(i) \leq P \wedge s^{-1} \leq P_{PSO}) \vee (s^{-1}(i) > P \wedge s^{-1}(i) \leq P + P_{PSO})$. If so, the particle is assigned to the new PSO population; otherwise, it is given to the latest GA population.

In the **fourth step**, the population splits to have GA and PSO populations separated. This split operation is done without further rearrangement since PSO and GA particles have already been assigned to the respective populations. The result is four sub-populations, each with the particles with better scores at the top of them and the worse at the bottom.

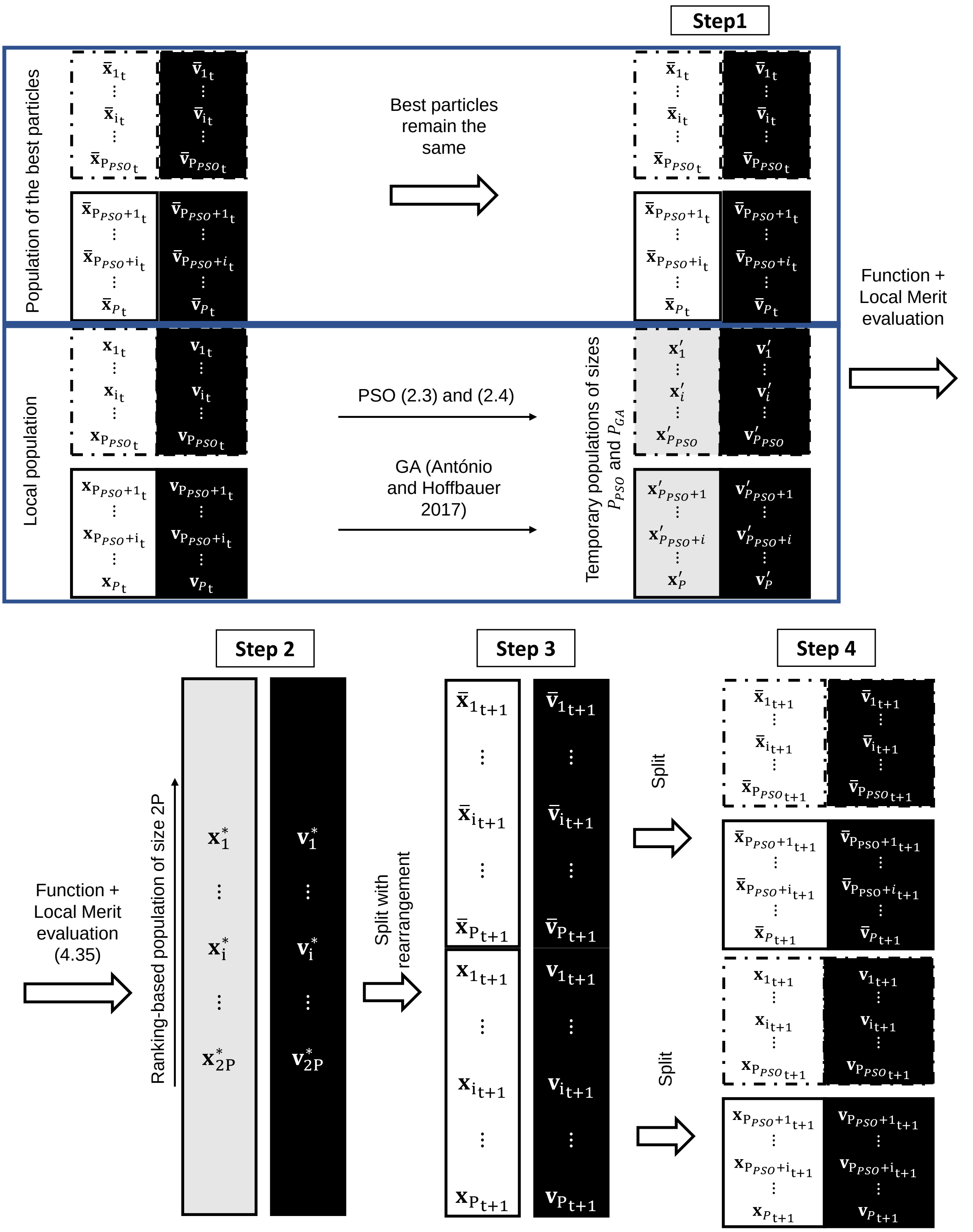

Figure 23. Configuration of the local population during the optimization process, using MOPSOGA$_1$

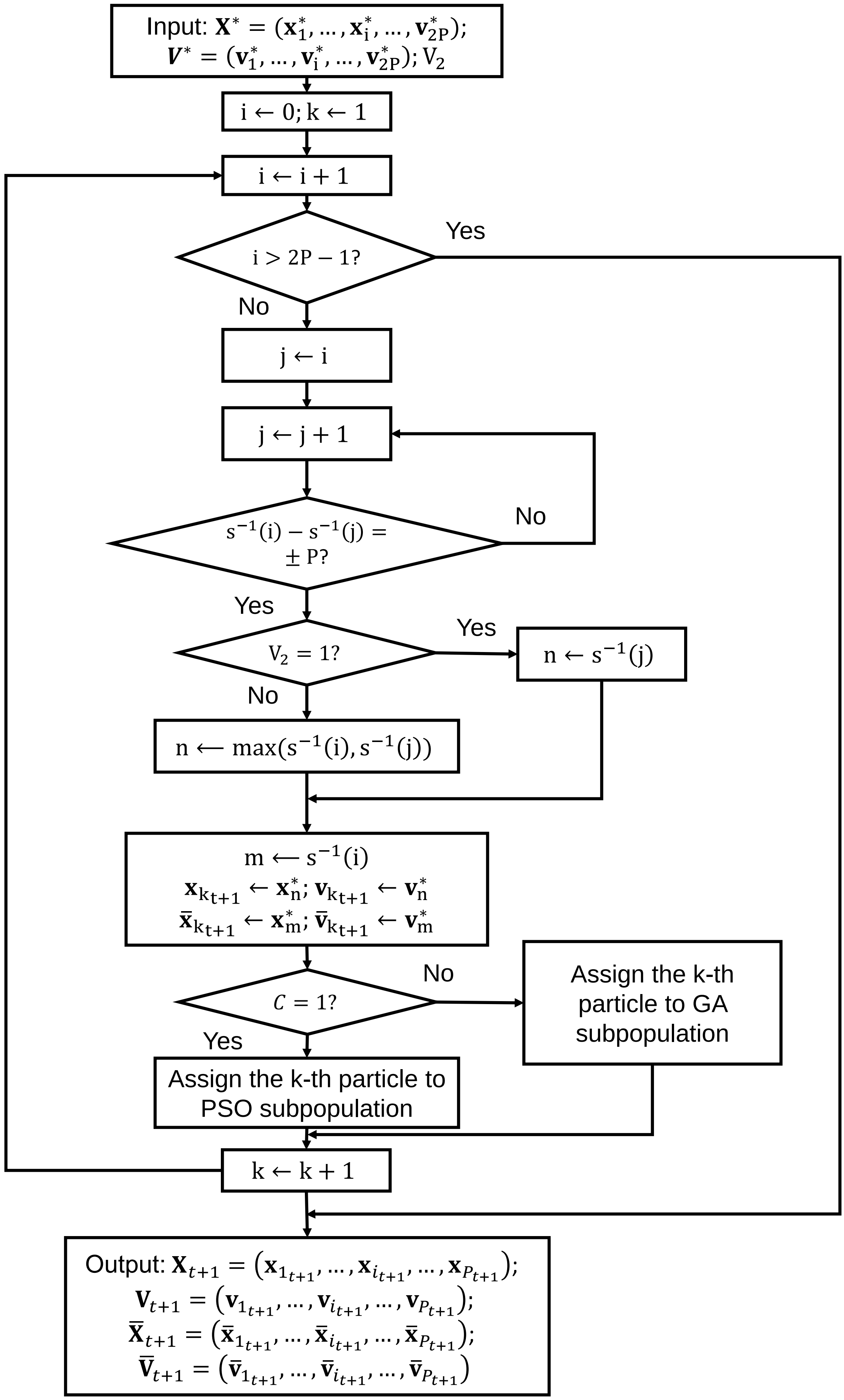

Figure 24. Flowchart for the rearrangement methodology in $MOPSOGA_1$

MOPSOGA₂

Figure 25 represents the working population in positions and velocities, similarly to Figure 21 for MOPSO and Figure 23 for $MOPSOGA_1$. In opposite to $MOPSOGA_1$, the approach being proposed in this section requires that $P_{PSO} = P_{GA} = P/2$ since it is based on cloning the PSO population to perform GA crossover operation right after. Cases where $P_{PSO} \neq P_{GA}$ are not performed in the thesis.

In the **first step**, PSO is performed for the first half of the population. After this, the cloning operation for the resulting sub-population is performed. GA crossover operation is applied to the cloned sub-population. In the end, the population of size P has half of its particles updated using the PSO technique, and the other half have been used PSO and GA.

Steps two and three are the same as that performed in MOPSO. At the end of the last step, two PSO and two GA populations exist. In opposition to the previous method, all the particles from the PSO population have the best from the fitness assignment; in the GA population, particles have the worst merit score and, therefore, are disregarded in the next generation. It implies that the new PSO population does not consist necessarily of particles from the previous PSO population. The particles from the last population GA necessarily have a corresponding velocity.

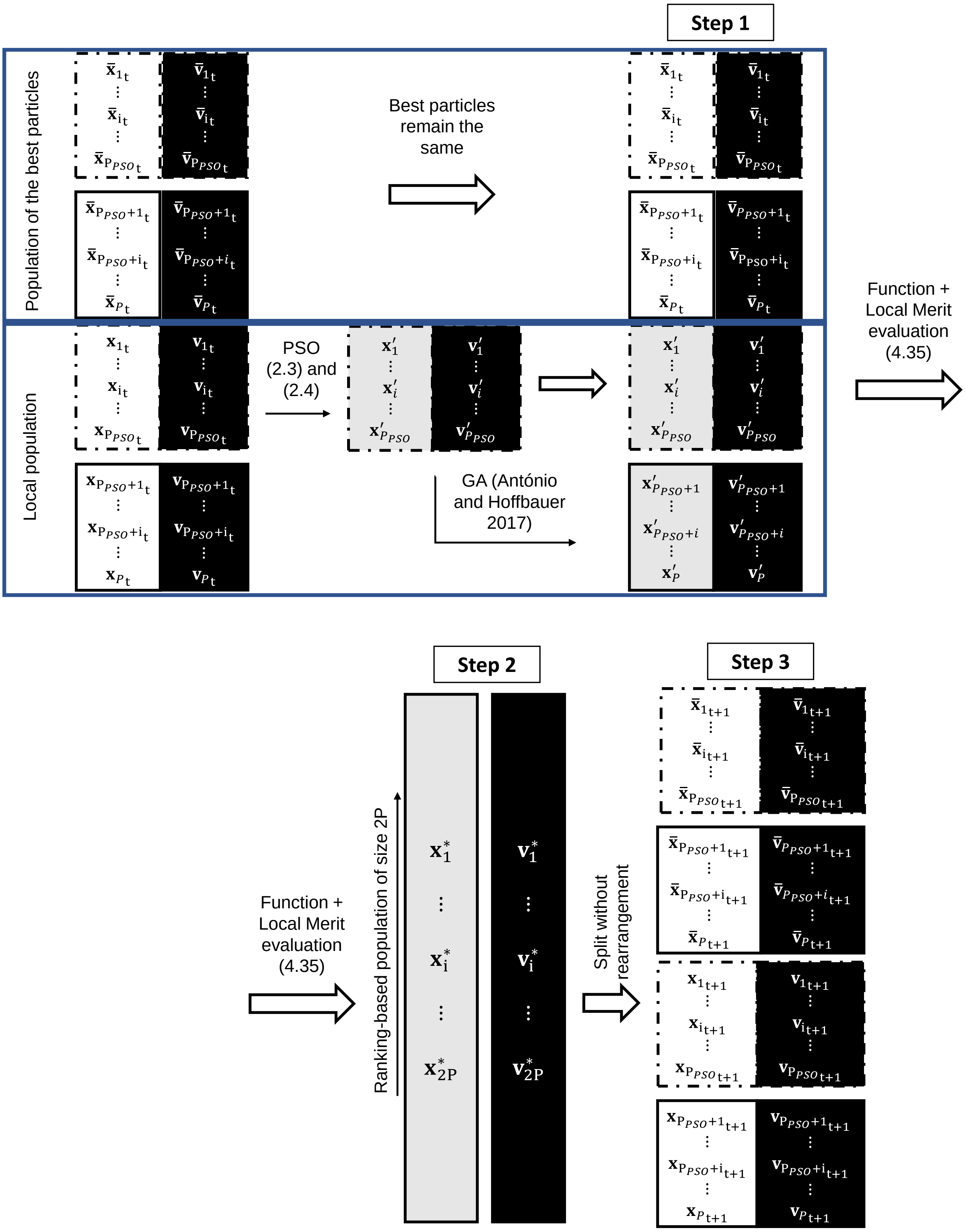

Figure 25. Configuration of the local population during the optimization process, using MOPSOGA$_2$

4.8 The proposed framework

The flowchart presented in Figure 26 summarizes the optimization process, when combining the optimization algorithm, the function evaluation and the merit evaluation (fitness assignment) referred to in the previous sections.

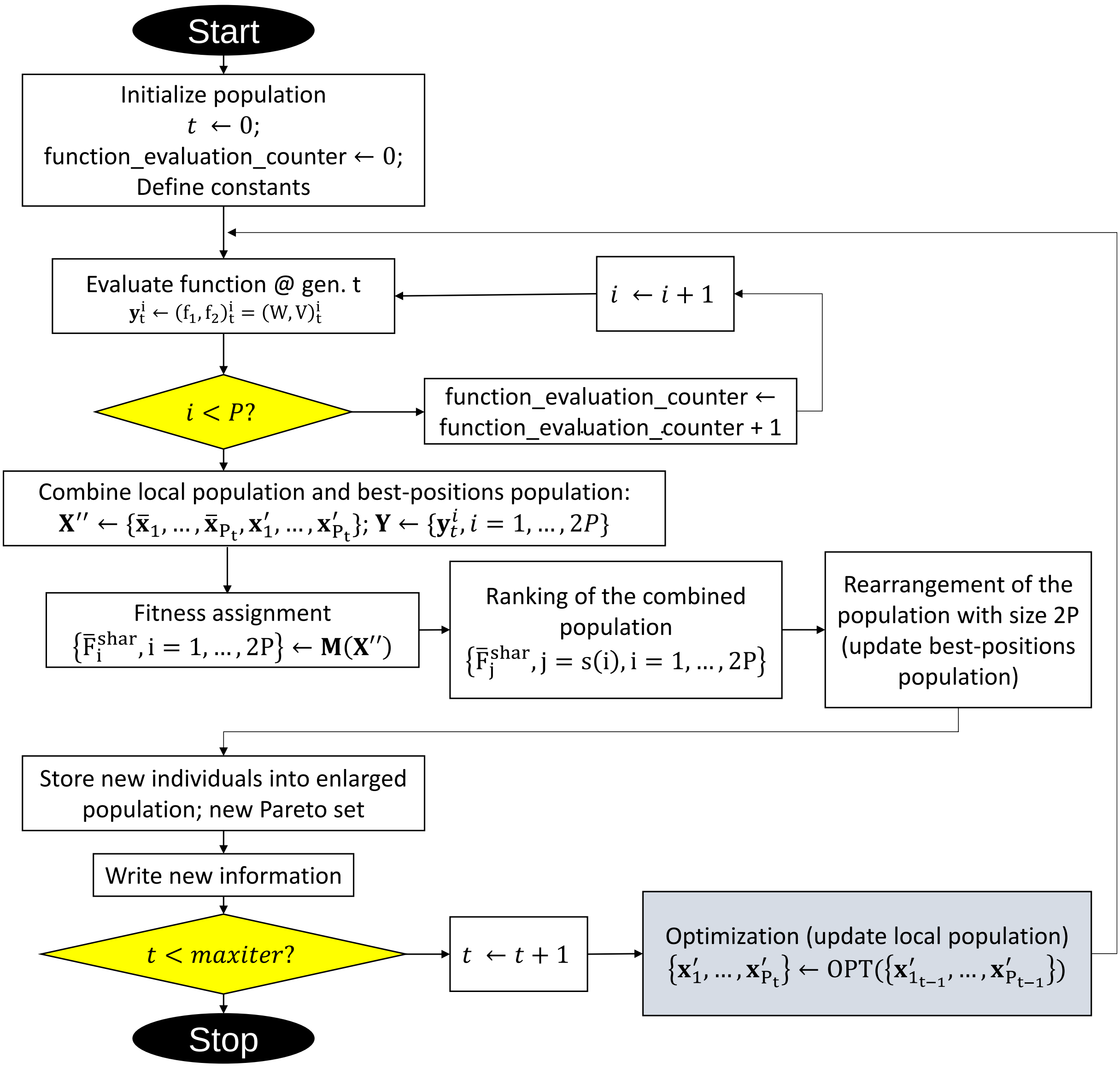

Figure 26. Flowchart of the global optimization procedure

5 Application in Composite Structures

5.1 Problem Definition

The outcomes of the proposed methodologies are tested using a given optimization problem of a composite structure. In Figure 27, a clamped cylindrical shell laminated structure is represented. Nine vertical loads of value P = 7kN are numerically applied, representing a distributed load along the free side of the structure AB. The structure is divided into four macro-elements, each one having one laminate. The distribution is also shown in the Figure. Moreover, the stacking sequence $[+\alpha, +\alpha, -\alpha, -\alpha]_s$, where s represents the symmetry, and the balanced angle-ply laminates with eight layers, are considered in the composite construction. Ply angle α is a design variable common to all the laminates. It is defined as illustrated in the figure. The design variables $h_i, i = 1, \dots, 4$ represent the thickness values of the shell, one for each laminate. A smoothing procedure at the boundaries detailed in section 3.3. of the reference (António 1995) is considered to guarantee the continuity of the structure between macro-elements / laminates.

A carbon/epoxy-based composite material, named T300/N5208 (Tsai 1987), is considered in the present analysis. This material is constituted by long fibres of carbon aggregated in an epoxy matrix. The values of the mechanical properties of the material are presented in Table 14. The ply strength properties are the longitudinal strength in tensile, represented by *X*, and in compression, represented by *X'*, the transversal strength in tensile and in compression, represented by *Y* and *Y'*, respectively, and the shear strength S.

Moreover, the allowable values in the constraints on displacement and Tsai number are $u_a = 8.0 \times 10^{-2}$ meters and $R_a = 1$, respectively. The side constraints are the following:

$$0 < \alpha < 90 \ \ [°]$$
$$0.005 < h_i < 0.040 \ [m], \qquad i = 1, \dots, 4$$

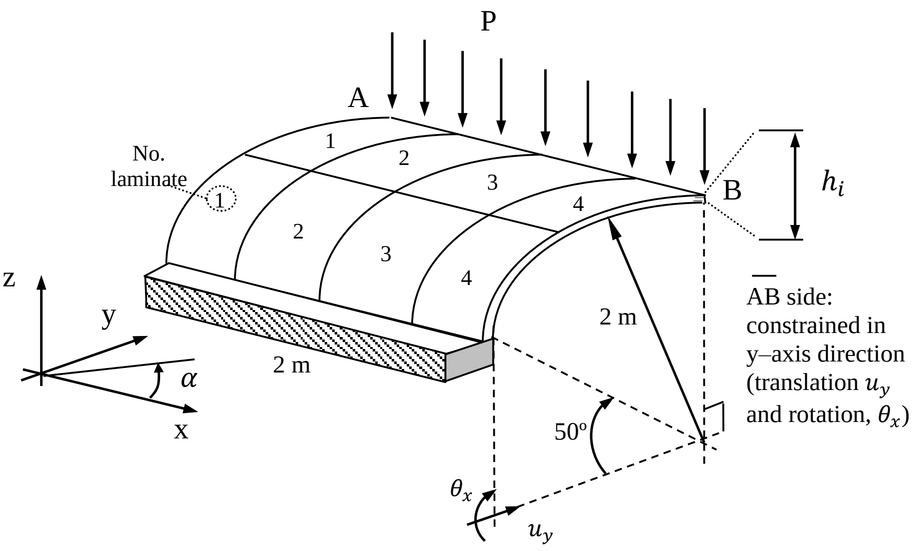

Figure 27. Cylindrical shell and ply angle representation

Table 14. Material properties being used in the proposed application

Material	E_1 **/ GPa**	E_2 **/ GPa**	G_{12} **/ GPa**	ν_{12} **/ GPa**
T300/N5208	181.00	13.30	7.17	0.28
Material	***X; X'*** **/ MPa**	***Y; Y'*** **/ MPa**	***S*** **/ MPa**	ρ **/ kg m^{-3}**
T300/N5208	1500; 1500	40; 246	68	1600

5.2 Results and Discussion

The results of the methods proposed are presented and discussed in this chapter. In the chapter, different methodologies with different parameters are tested. MOGA, from (António and Hoffbauer 2017), is here used for comparison. Pareto front concerning both objectives weight, in Newton, and the determinant of the variance-covariance matrix, $\det \mathbf{C}_\varphi$, is used to visualize the performance of the different methods better. Since $\det \mathbf{C}_\varphi$ values vary from 10^{-10} and 10^{-5}, a logarithmic scale is used, whereas a linear scale is used for the weight. The weight is represented in terms of mass multiplied by the gravitational constant g. After visualizing Pareto fronts, a few points are taken to visualize the exact value for the weight while visualizing the values for the robustness $\det \mathbf{C}_\varphi$, also abbreviated as “Rob.” throughout the figures. Such points are cases enumerated as C1, C2, and so on – depending on the number of points retrieved.

Figure 28 shows the resulting Pareto fronts from applying MOGA and MOPSO_1 for three cases. In MOGA, a population size of 30 and 300 generations has been set for the experiments. Moreover, ten individuals are the maximum size of the elitist population, and 20% of the total population is conducted to mutation operations during the iterative process. The values result in an estimate of 6000 evaluations. In MOPSO_1, a local population size of 30 and 200 generations has been set. All the particles of the local population are evaluated, so 6000 evaluations are carried out. The importance of having the same number of evaluations is to assess the methods better, fixing the computational time that is almost entirely spent in evaluating the set of design values.

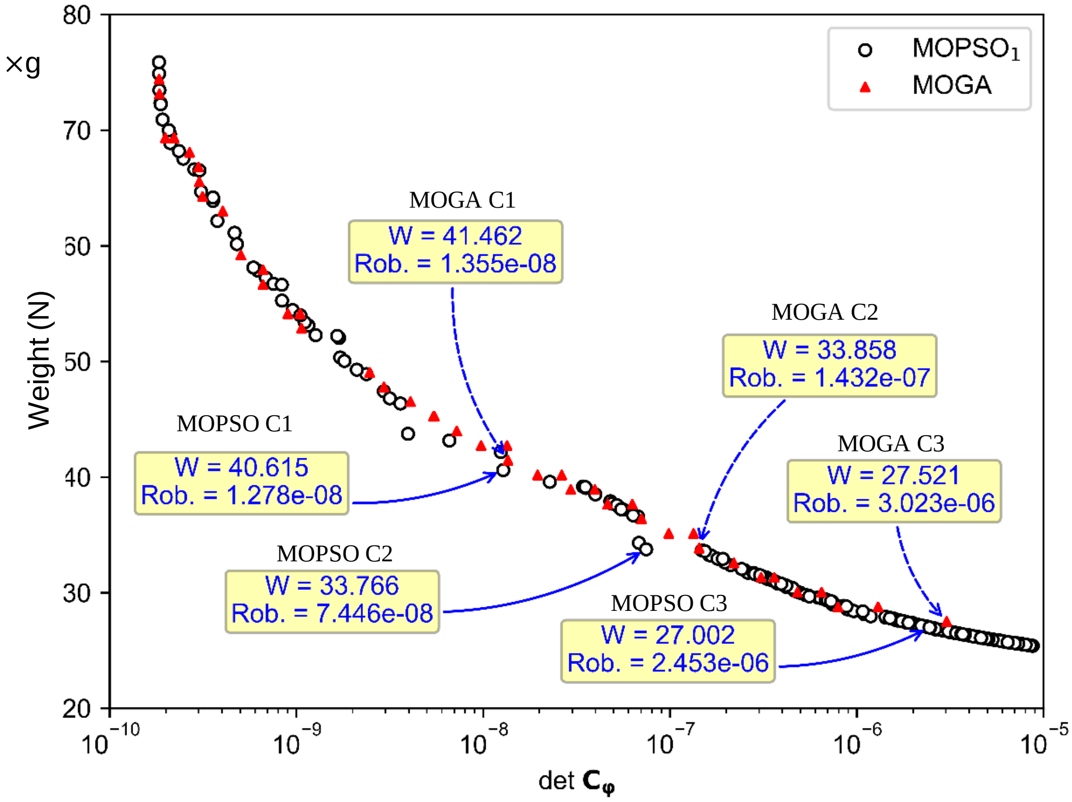

Figure 28. Pareto front resulted from the application of MOGA and MOPSO$_1$

It is possible to verify that MOPSO$_1$ compares well to MOGA. One of the first conclusions that can be taken from observing the plot is that both Pareto fronts from MOPSO$_1$ and MOGA are almost overlapping. Moreover, MOPSO$_1$ conducts many more solutions on the Pareto front. It may be considered an advantage due to the following reasons:

1. Pareto front is more evident represented;
2. More solutions can be retrieved for a wide variety of applications, depending on the importance that is given to both objectives;
3. Numerically, a more robust solution and a lighter solution are achieved;

Generally, this can be justified by the variables that GA and PSO use. In GA, the domain is theoretically discretized, whereas real variables are considered in PSO. Numerically, variables are discrete in both techniques, but the considered domain in PSO is much larger than the domain in GA. Another reason is related to the velocity scale that is used along with the generations in PSO. If small increments are added at a final stage of PSO, more solutions are expected to be achieved between previously achieved solutions.

The Pareto front can be divided into three parts in terms of the number of solutions that have been achieved. The first one is the set of lighter than 34g N solutions. A vast number of acquired solutions are verified. The region is dense in solutions compared to the rest of the plot. The second region has solutions with weights that are between 34g N and 44g N. Here, it seems that not so many solutions are represented here, and many discontinuities are observed. The last region is the set of solutions heavier than 44g N. Despite no significant discontinuities being not observed in this region, it is not so dense as in the first region. Generally, this division is related to the considered domain and its relationship with the objectives and the constraints. A deeper study of the resulting discontinuities and denser regions should be considered in future works.

Moreover, it is important to highlight that, in the second zone, PSO can achieve better solutions in terms of both weight and robustness.

Three different cases are taken from Pareto fronts. The design values and objectives corresponding to the retrieved points of the plot are listed in Table 15.

Table 15. Values of the objectives and design variables for different cases from the Pareto set resulted from MOGA and the proposed MOPSO

Variable	MOGA C1	MOGA C2	MOGA C3	MOPSO C1	MOPSO C2	MOPSO C3
Weight/N (×g)	41.462	33.858	27.521	40.615	33.766	27.002
$\det \mathbf{C}_{\varphi}$	1.355e-8	1.432e-7	3.023e-6	1.278e-8	7.446e-8	2.453e-6
α / °	90	90	90	89.50	89.48	89.50
h_1 / m	6.129e-3	6.129e-3	6.129e-3	5.254e-3	5.126e-3	5.330e-3
h_2 /m	7.258e-3	5.000e-3	5.000e-3	5.254e-3	5.126e-3	5.000e-3
h_3 / m	5.000e-3	5.000e-3	5.000e-3	5.080e-3	5.046e-3	5.000e-3
h_4 / m	1.855e-2	1.403e-2	8.387e-3	1.816e-2	1.399e-2	8.724e-3

It is possible to verify that a ply angle of almost 90° is recommended for laminated composites for one laminate per macro element. Using the PSO technique allows finding that a value to the ply angle near 89.5° would be better in some instances where better objective values are achieved. The results suggest using lower thickness values, especially for the third macro element. For lighter structures, the thickness values are closer to the lower side constraint.

Figure 29 represents the Pareto front of both methods, MOGA and $MOPSO_1$, but at the end of 2000 function evaluations. This figure has been intended to verify the earlier convergence of $MOPSO_1$ at a lower stage of the iteration process. It seems that using PSO leads to a good convergence at the level of GA. Although some PSO solutions dominate GA solutions, there are still more GA solutions than PSO solutions. Moreover, there are already a lot of solutions lighter than 34g N at such a stage.

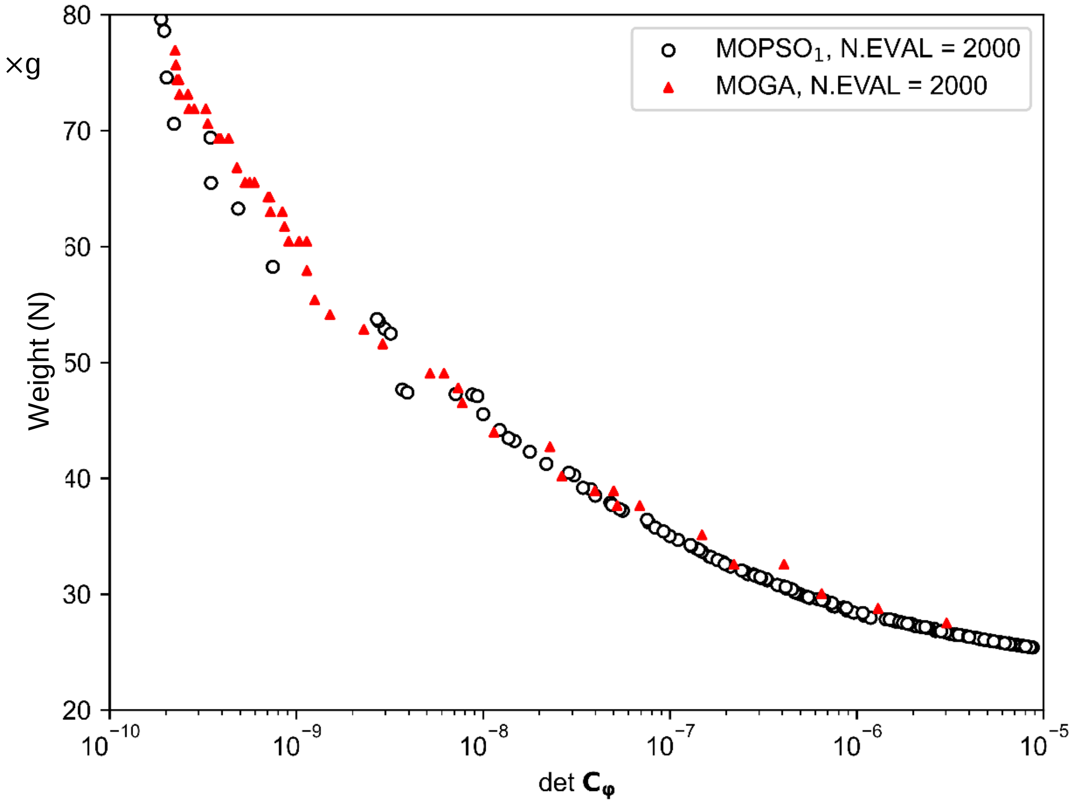

Figure 29. Pareto front resulted from the application of MOGA and MOPSO$_1$, at the end of 2000 function evaluations

Figure 30 compares the Pareto front of MOPSO$_1$ at the end of 600 evaluations and 6000 evaluations. It is possible to verify the significant difference between both stages. At the end of 600 evaluations, the Pareto front is not visible. However, at the end of 6000 evaluations, the Pareto front is visible. It is expected that the previously observed discontinuities can be avoided with more function evaluations.

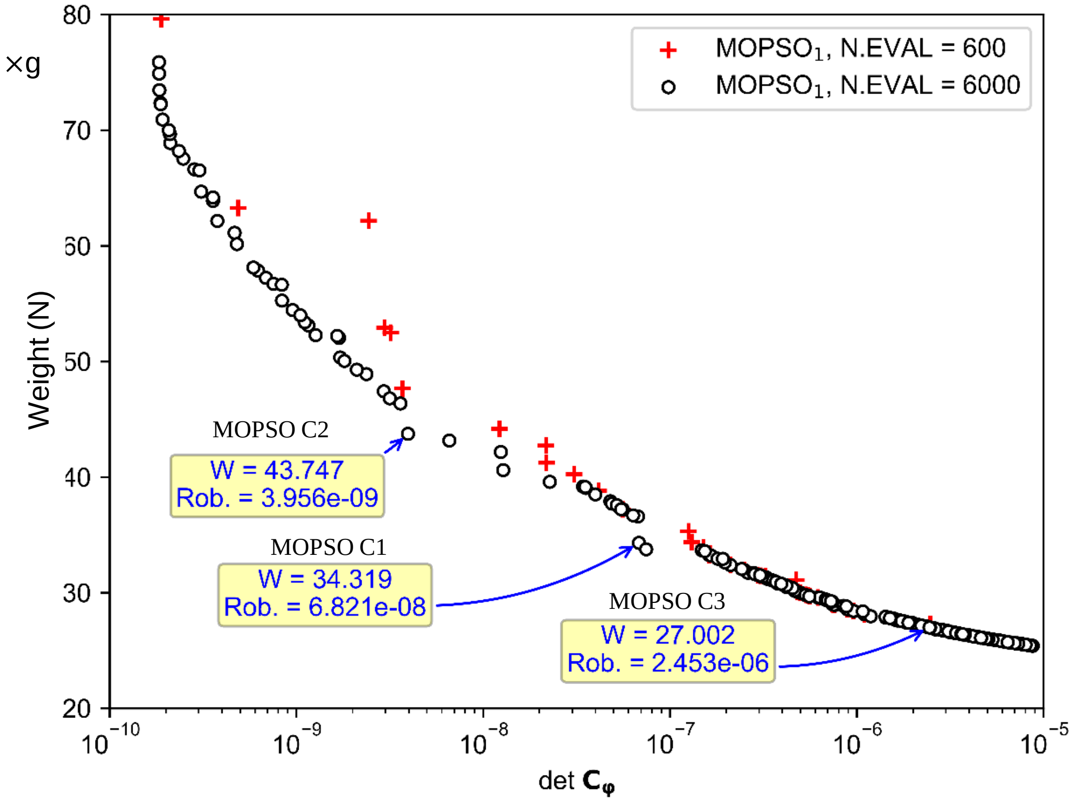

Figure 30. Pareto front resulted from the application of $MOPSO_1$, at different stages of the optimization process

Moreover, Table 16 summarizes three different cases on the second Pareto front.

Table 16. Values of the objectives and design variables for different cases from the Pareto set resulted from the proposed MOPSO1

Variable	MOPSO C1	MOPSO C2	MOPSO C3
Weight/N (×g)	34.319	43.747	27.002
det $\mathbf{C}_{\varphi}$	6.821e-8	3.956e-9	2.453e-6
α / °	89.47	89.48	89.50
h_1 / m	5.315e-3	5.290e-3	5.330e-3
h_2 /m	7.429e-3	6.042e-3	5.000e-3
h_3 / m	5.000e-3	5.000e-3	5.000e-3
h_4 / m	2.123e-2	1.424e-2	8.724e-3

Observing the previous figures, using PSO leads to a higher number of Pareto solutions than those when using GA. However, based on the performance of the techniques in finding improved solutions, it is unclear when the iteration process can be stopped. For instance, one may already know that stopping at the 600th function evaluation would not be a good idea since the Pareto front is significantly improved at the 6000th function evaluation. Figure 31 shows the evolution of the number of Pareto solutions along with the generations. One may verify that PSO always finds new solutions, whereas, in GA, the number of Pareto solutions is almost the same. In the plot corresponding to PSO, verifying an almost stable number of Pareto solutions from the 3000th to the 5000th function evaluation is possible. Still, the number grows again in a significant way. It means that one may not know when it will happen after the 7500th function evaluation, but it is more likely to happen compared to MOGA.

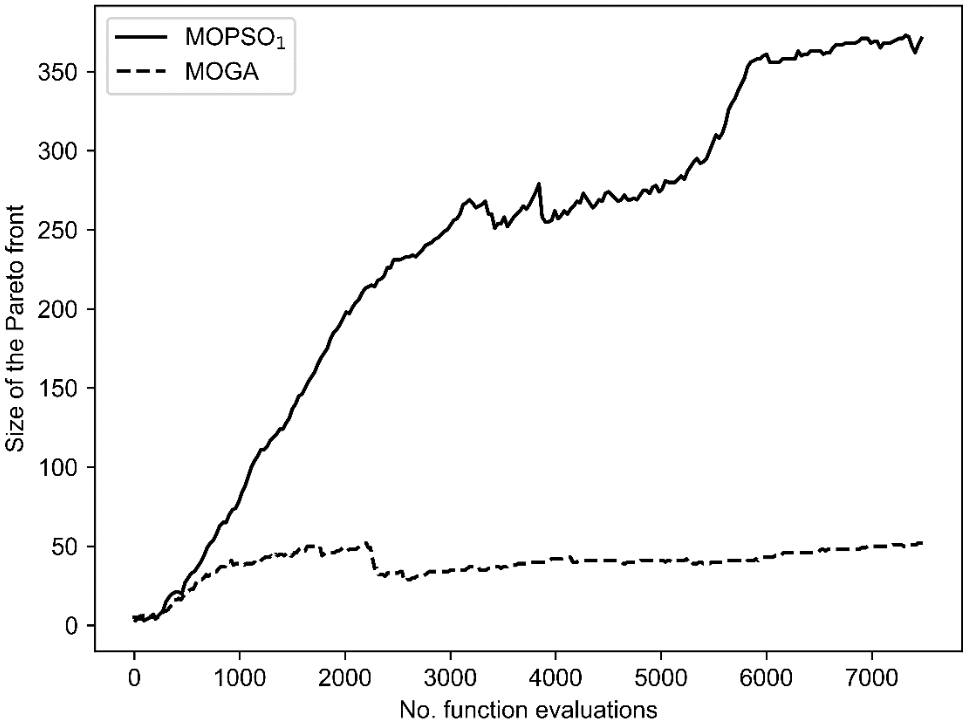

Figure 31. Variation in the size of the Pareto front resulted from the application of $MOPSO_1$ and MOGA at different stages of the optimization process

The solutions associated with other ranks are also registered. In Figure 32, the solutions of ranks 2 and 5 are compared to those of rank 1. It is possible to verify that no significant differences exist between the solutions of ranks 1 and 2 since weight and robustness values are very close. Even after zooming in the region below 34g N, it is challenging to find significant differences. However, between 34g N and 44g N, the differences are visible. Moreover, solutions of rank 5 are different from those of ranks 1 and 2.

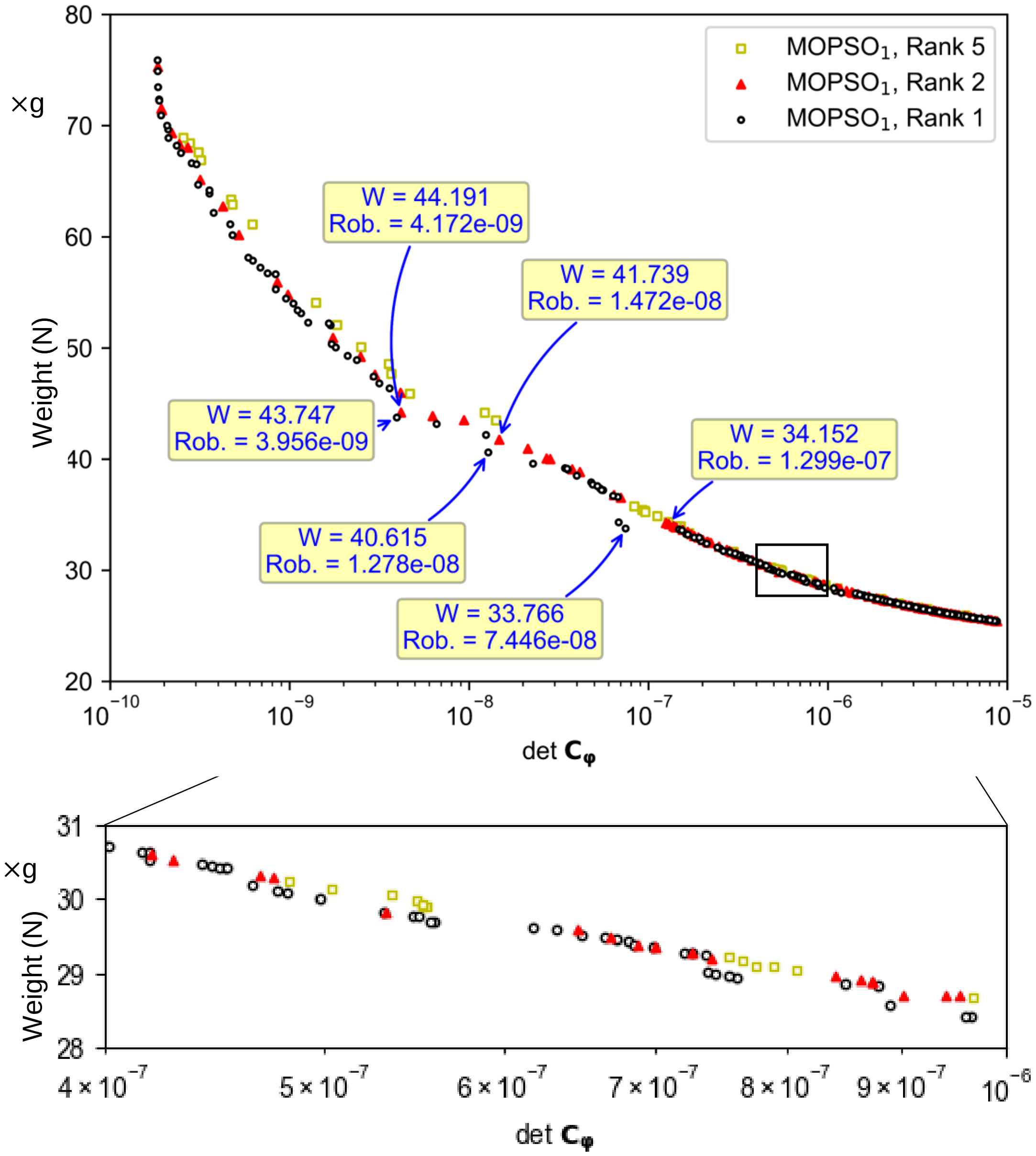

Figure 32. Pareto front resulted from the application of MOPSO$_1$, compared to other solutions of lower-ranking values, at the end of 6000 function evaluations

The other variations of MOPSO that have been presented in Table 13 of the previous chapter, MOPSO$_2$ and MOPSO$_3$, are also tested and compared to MOPSO$_1$. Pareto fronts for the three methods are shown in Figure 33. At first glance, it is possible to verify that MOPSO$_3$ performs better than the other two and MOPSO$_2$ performs worse than the others. This conclusion is clearer seen in the middle zone of the plots. On the right side of the Pareto front, the differences do not exist in practice.

As the third variation of MOPSO is the best, the cloning operation after shorting the population by their rank does not seem to be the best solution. One reason for obtaining such a result is that the operation may lead to an early stagnation of the iteration process. Also, it seems like using the best solution for the enlarged population as a matter of comparison is not the best solution. It can be related to the entropy that is created when increasing the probability of the global solution being significantly different between generations, which may increase the velocity of the particles and, in consequence, no stable solution. However, these hypotheses should be carefully investigated in future works.

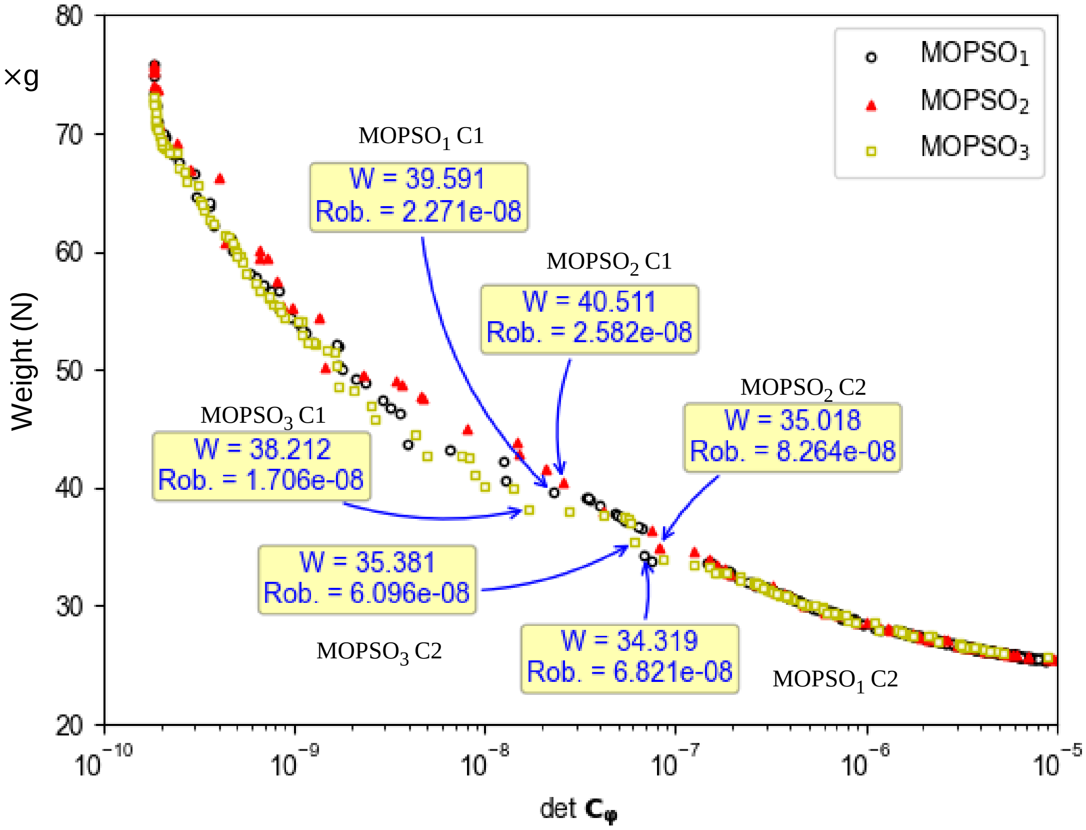

Figure 33. Pareto front resulted from the application of $MOPSO_1$, $MOPSO_2$ and $MOPSO_3$

Table 17 shows six solutions, two from each of Pareto fronts. When comparing the values of this Table with the values of the Tables previously shown, one may conclude that the ply angle has changed to values between 88° and 89.5°. The thicknesses have smaller values for the macro element 3 and much larger values for the macro element 4.

Table 17. Values of the objectives and design variables for different cases from the Pareto set resulted from the proposed $MOPSO_1$, $MOPSO_2$ and $MOPSO_3$

Variable	**$MOPSO_1$ C1**	**$MOPSO_1$ C2**	**$MOPSO_2$ C1**	**$MOPSO_2$ C2**	**$MOPSO_3$ C1**	**$MOPSO_3$ C2**
Weight/N (×g)	39.591	34.319	40.511	35.018	38.212	35.381
det $\mathbf{C}_{\varphi}$	2.271e-8	6.821e-8	2.582e-8	8.264e-8	1.706e-8	6.096e-8
α / °	89.67	89.48	88.14	84.89	87.99	89.46
h_1 / m	5.388e-3	5.126e-3	6.693e-3	6.545e-3	5.225e-3	5.287e-3
h_2 /m	9.010e-3	5.916e-3	5.000e-3	5.000e-3	6.579e-3	5.914e-3
h_3 / m	5.000e-3	5.046e-3	5.000e-3	5.000e-3	5.618e-3	5.000e-3
h_4 / m	1.587e-2	1.399e-2	1.939e-2	1.372e-2	1.814e-2	1.340e-2

The population size parameter is also interesting in the current study. The population size is changed with $MOPSO_3$ to the following values: 10, 20, 30 and 100. The maximum number of evaluations of 6000 is always considered for the tests. The results for such a variation are displayed in Figure 34. It is possible to verify that using very small population size, like 10 particles, does not conduct a good solution, whereas 20 and 100 are good options to consider. The variation between the results is much more evident for structure weights above 40g N. for lighter structures, the differences concerning Pareto fronts continue to be not relevant.

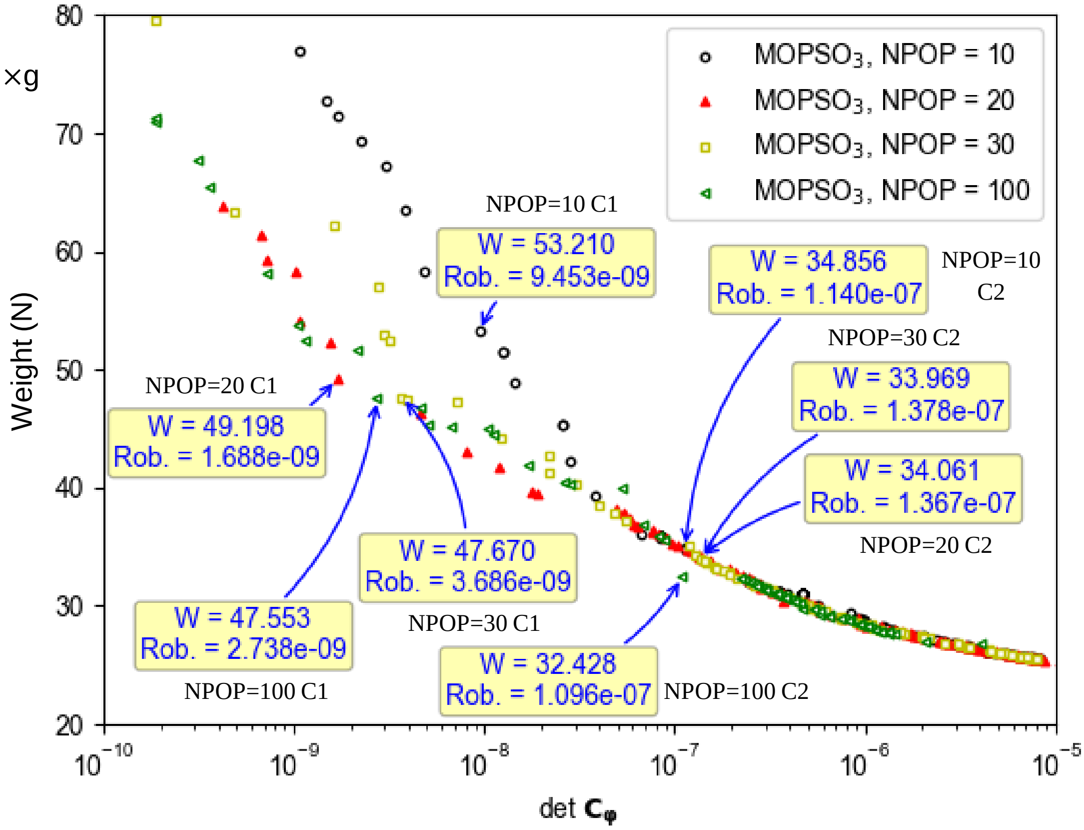

Figure 34. Pareto front resulted from the application of $MOPSO_3$ for different population sizes

Tables 18 and 19 list the results of two particular cases. The ply angle and thickness results are similar to those previously obtained. The ply angle is 90°, and the minimum thickness of 5 mm is obtained for the macro element 3. The second table obtains the minimum thickness for three out of four macro elements.

Table 18. Values of the objectives and design variables for different cases from the Pareto set resulted from the proposed $MOPSO_3$ with different population sizes: Case 1

Variable	$MOPSO_3$ #POP=10	$MOPSO_3$ #POP=20	$MOPSO_3$ #POP=30	$MOPSO_3$ #POP=100
Weight/N (×g)	53.210	49.198	47.670	47.553
det $\mathbf{C}_\varphi$	9.453e-9	1.688e-9	3.686e-9	2.738e-9
α / °	90.00	90.00	90.00	90.00
h_1 / m	5.387e-3	5.339e-3	5.000e-3	5.000e-3
h_2 /m	5.000e-3	7.562e-3	1.140e-2	5.000e-3
h_3 / m	5.000e-3	5.432e-3	5.000e-3	5.000e-3
h_4 / m	8.688e-3	2.549e-2	2.106e-2	1.236e-2

Table 19. Values of the objectives and design variables for different cases from the Pareto set resulted from the proposed $MOPSO_3$ with different population sizes: Case 2

Variable	$MOPSO_3$ #POP=10	$MOPSO_3$ #POP=20	$MOPSO_3$ #POP=30	$MOPSO_3$ #POP=100
Weight/N (×g)	34.856	34.061	33.969	32.428
$\det \mathbf{C}_{\varphi}$	1.140e-7	1.367e-7	1.378e-7	1.096e-7
α / °	90.00	90.00	85.93	90.00
h_1 / m	5.297e-3	5.000e-3	5.262e-3	5.000e-3
h_2 /m	5.000e-3	5.000e-3	5.000e-3	5.000e-3
h_3 / m	5.000e-3	5.000e-3	5.000e-3	5.000e-3
h_4 / m	8.688e-3	1.310e-2	1.300e-2	1.349e-2

The hybridization methods are also tested. Figure 35 shows the resulting Pareto fronts from applying MOGA and $MOPSOGA_1$ for two distinct cases. In MOGA, the same population size of 30 and 300 generations are used, the same as in previous experiments. In $MOPSOGA_1$, two different population sizes are used. The experiment associated with the circular mark uses a total population size of 60 and a PSO population size of 30, i.e., half of the total population. The experiment corresponding to the triangular mark uses a total population size of 40, but the PSO population size continues to be half the total population size. All the particles of the local population are evaluated the corresponding number of times, i.e., the maximum number of generations so that 6000 evaluations are carried out.

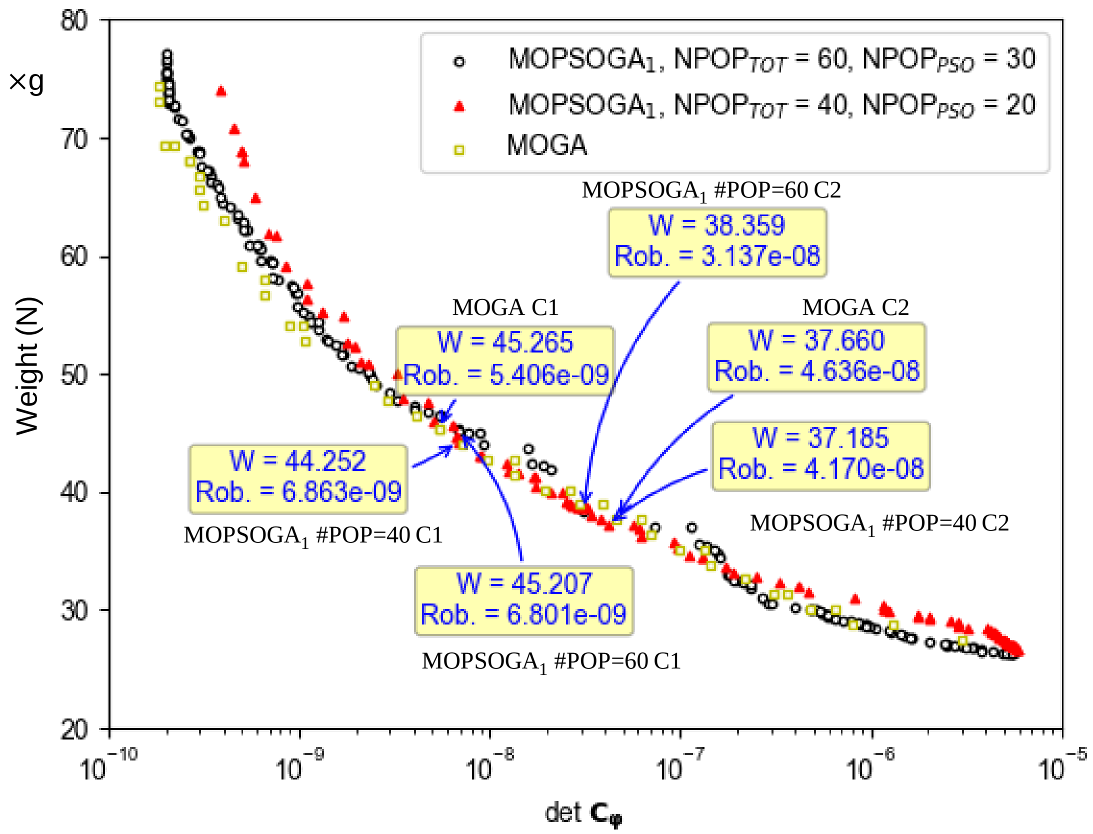

Figure 35. Pareto front resulted from the application of MOGA and $MOPSOGA_1$

At first glance, not many differences are seen between MOGA and $MOPSOGA_1$. When dividing the Pareto sets into the different regions that have been, it is possible to verify that:

1. Using $MOPSOGA_1$ with 60 particles is the best choice when choosing between lighter solutions (on the right side of the Pareto fronts);
2. Using $MOPSOGA_1$ with 40 particles is the best choice when choosing between the solutions in the middle region of the Pareto fronts;
3. Using MOGA is the best choice when choosing solutions with lower variability (solutions from the left side of the Pareto fronts).

Another observation about $MOPSOGA_1$ is related to the number of solutions found when compared to MOPSO methods. In fact, on the right side of the plot, it is impossible to verify the denser scattering as in MOPSO. It may be because GA is used in the MOPSOGA method, reducing the probability of exploitation between existing Pareto solutions.

Moreover, Table 20 presents the results of each of the methods for two different cases. As the previous results show, the third macro element has a thickness near the minimum possible value, and the suggested ply angle is near or equal to 90°.

Table 20. Values of the objectives and design variables for different cases from the Pareto set resulted from MOGA and the proposed $MOPSOGA_1$

Variable	$MOPSOGA_1$ #POP=60 C1	$MOPSOGA_1$ #POP=40 C1	MOGA C1	$MOPSOGA_1$ #POP=60 C2	$MOPSOGA_1$ #POP=40 C2	MOGA C2
Weight/N (×g)	45.207	44.252	45.265	38.359	37.185	37.660
$\det \mathbf{C}_{\varphi}$	6.801e-9	6.863e-9	5.406e-9	3.137e-8	4.170e-8	4.636e-8
α / °	90.00	90.00	84.00	87.32	90.00	90.00
h_1 / m	5.000e-3	5.000e-3	5.000e-3	5.071e-3	6.129e-3	6.129e-3
h_2 /m	1.243e-2	5.646e-3	6.129e-3	5.400e-3	7.258e-3	5.000e-3
h_3 / m	5.069e-3	5.987e-3	5.000e-3	5.000e-3	5.000e-3	5.000e-3
h_4 / m	3.567e-2	1.411e-2	1.968e-2	1.488e-2	1.855e-2	1.516e-2

Pareto fronts resulting from $MOPSOGA_1$ and $MOPSOGA_2$ are also compared and are displayed in Figure 36. Both hybridization methods are those presented in the previous chapter. The results show that $MOPSOGA_2$ performance is better than $MOPSOGA_1$. The difference between both methods is that $MOPSOGA_2$ uses the best half of the particles to perform PSO and GA techniques, combined or only using PSO, whereas $MOPSOGA_1$ performs PSO and GA separately.

Moreover, it is possible to verify that the Pareto front's middle zone has several discontinuities already reported in MOPSO methods. In this zone, it turns clear the advantage of using $MOPSOGA_2$ since all the points in the middle zone are dominated by the points resulting from it.

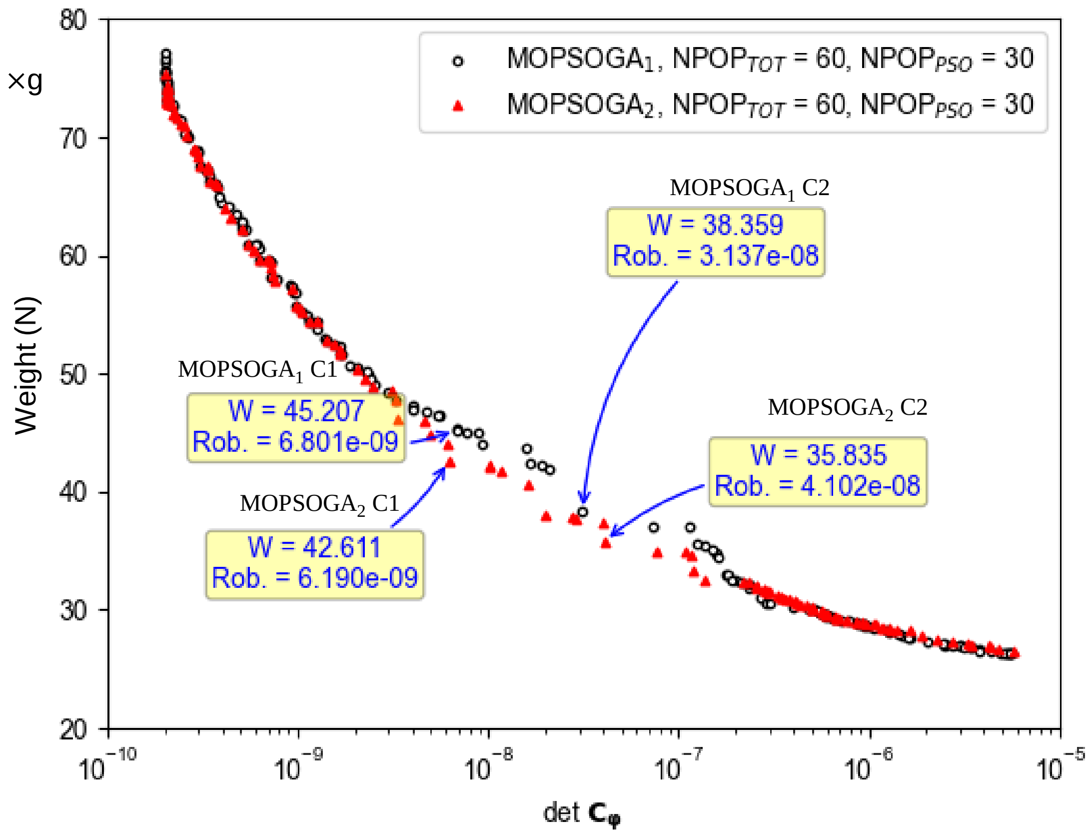

Figure 36. Pareto front resulted from the application of MOPSOGA$_1$ and MOPSOGA$_2$

Table 21 shows the results for two different cases and both methods. The results are shown to be similar to those previously obtained.

Table 21. Values of the objectives and design variables for different cases from the Pareto set resulted from MOGA and the proposed MOPSOGA$_1$ and MOPSOGA$_2$

Variable	MOPSOGA$_1$ C1	MOPSOGA$_1$ C2	MOPSOGA$_2$ C1	MOPSOGA$_2$ C2
Weight/N (×g)	45.21	38.36	42.61	35.83
det $\mathbf{C}_\varphi$	6.801e-9	3.137e-8	6.190e-9	4.102e-8
α / °	90.00	90.00	90.00	90.00
h_1 / m	5.000e-3	5.000e-3	5.000e-3	5.234e-3
h_2 /m	1.243e-2	5.646e-3	5.000e-3	7.859e-3
h_3 / m	5.069e-3	5.987e-3	5.000e-3	5.000e-3
h_4 / m	3.567e-2	1.411e-2	1.097e-2	1.723e-2

Figure 37 compares the Pareto front of MOPSOGA$_1$ at the end of 600 and 6000 evaluations. It is possible to verify the significant difference between both stages. At the end of 600 evaluations, the Pareto front is not visible. However, at the end of 6000 evaluations, the Pareto front is clearly visible. This observation had been already registered for the case of MOPSO. It is expected that the previously observed discontinuities can be avoided with more function evaluations.

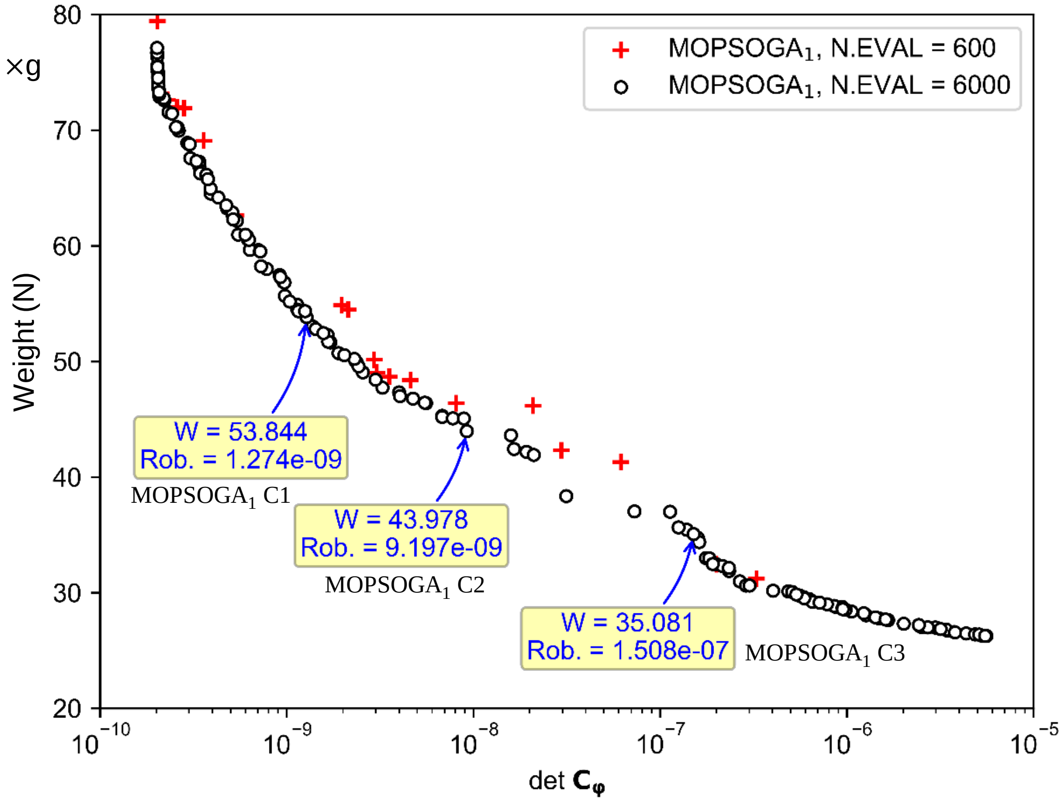

Figure 37. Pareto front resulted from the application of MOPSOGA$_1$, at different stages of the optimization process

Moreover, Table 22 summarizes three different cases on the second Pareto front.

Table 22. Values of the objectives and design variables for different cases from the Pareto set resulted from the proposed MOPSOGA$_1$

Variable	**MOPSOGA$_1$ C1**	**MOPSOGA$_1$ C2**	**MOPSOGA$_1$ C3**
Weight/N (×g)	53.844	43.978	35.081
det $\mathbf{C}_\varphi$	1.274e-9	9.197e-9	1.508e-7
α / °	90.00	90.00	90.00
h_1 / m	5.000e-3	5.000e-3	5.000e-3
h_2 /m	1.155e-2	5.625e-3	5.577e-3
h_3 / m	5.542e-3	5.000e-3	5.000e-3
h_4 / m	3.265e-2	7.513e-3	8.024e-3

Observing the previous figures, using the hybridization of PSO and GA also leads to a higher number of Pareto solutions than those using only GA. However, based on the performance of the techniques in finding improved solutions, it is unclear when the iteration process can be stopped. Figure 38 shows the evolution of the number of Pareto solutions along with the generations for MOPSOGA$_1$, MOPSOGA$_2$ and MOGA. One may verify that MOPSOGA methods always find new solutions, whereas, in MOGA, the number of Pareto solutions is more stable along with the generations. In the plots corresponding to MOPSOGA, it is possible to verify that, after 7500 function evaluations, the continuous growth of the size of the Pareto set is very likely. However, MOPSO leads always to a more extensive Pareto set. This situation may be caused due to the use of GA, which implies more exploration than PSO.

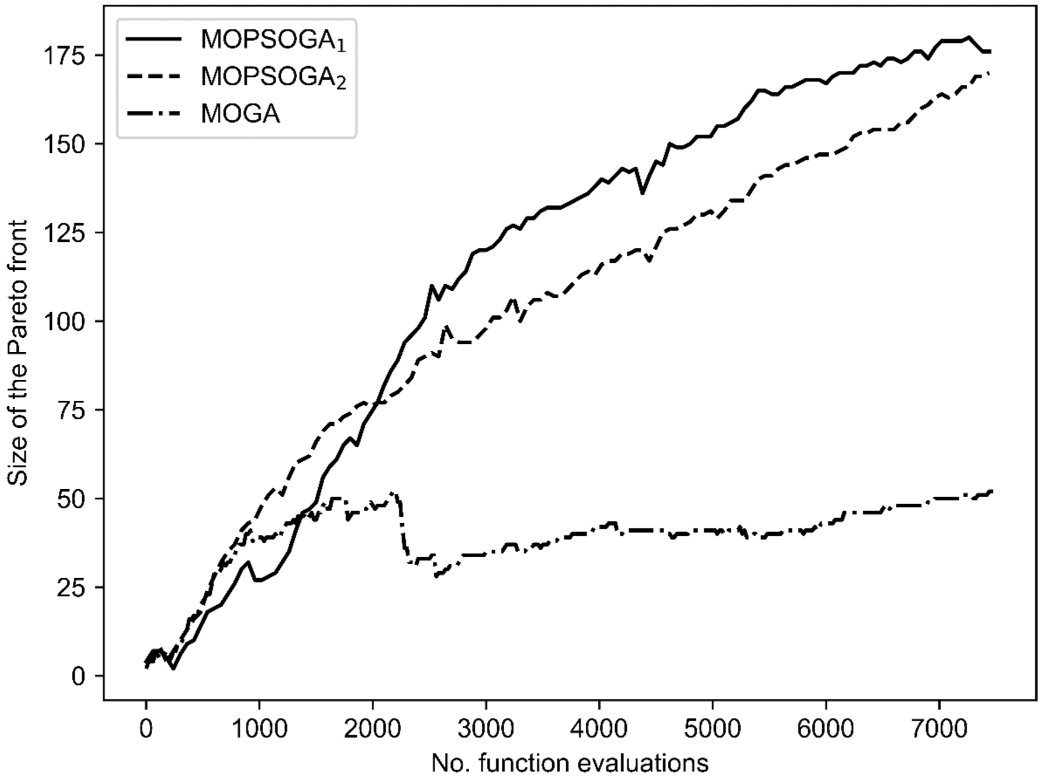

Figure 38. Variation of the size of the Pareto front resulted from the application of MOPSOGA$_1$, MOPSOGA$_2$ and MOGA, at different stages of the optimization process

MOPSOGA method can be applied with tuned population sizes. MOPSOGA$_1$ can have different sizes for both PSO and GA populations. Figure 39 shows the Pareto fronts resulting from the application of MOPSOGA1 with different PSO population sizes, the total population size of 60 particles. The number of function evaluations is the same for all the tests, so the computational time effects have fewer effects. It shows that MOPSOGA$_1$ performs better with fewer PSO particles for the left side of the Pareto front but using more PSO particles leads to better exploitation on the right side of that front.

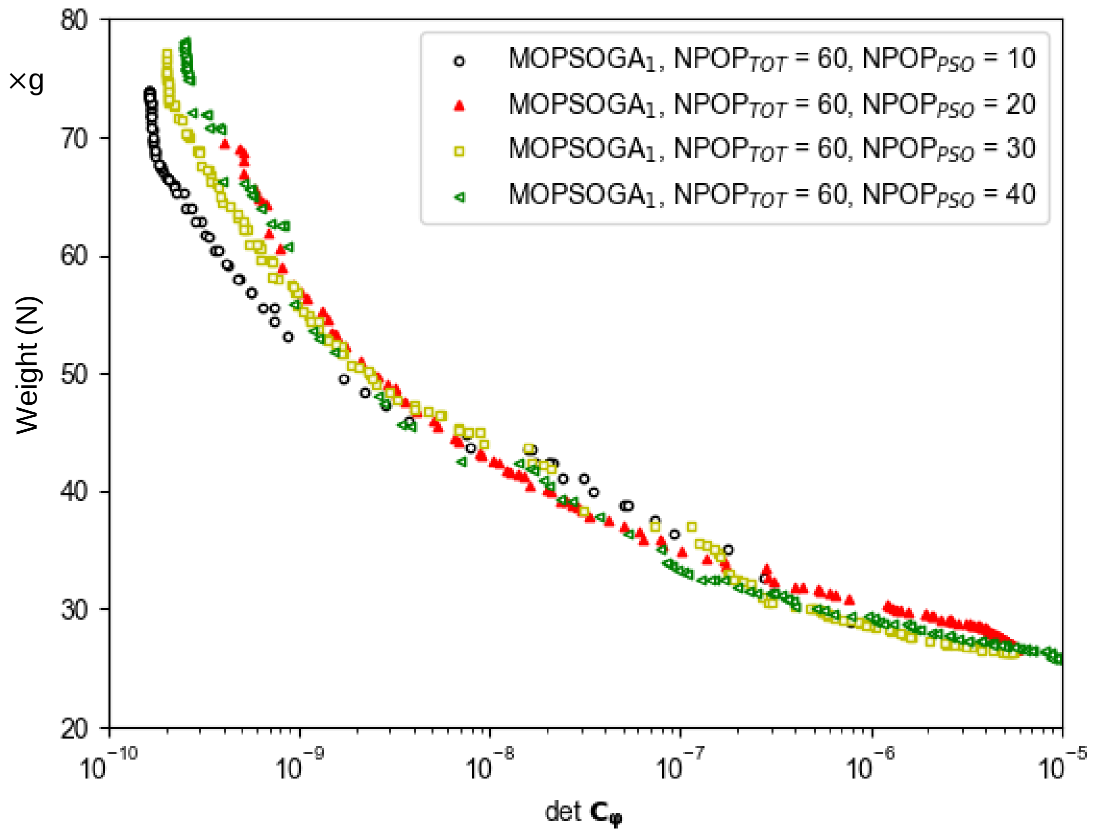

Figure 39. Pareto front resulted from the application of MOPSOGA$_1$ with a total population size of 60 individuals, for different PSO sub-population sizes

The same occurs in Figure 40, which also represents the Pareto fronts for different PSO population sizes but uses a total population size of 40 particles.

Verifying both figures and considering the obtained results of MOPSOGA compared to MOGA, one may reach a conclusion. It is related to the zone of the Pareto front that each method performs better. The results suggest that, when using the PSO technique, the right side of the front achieves several more solutions due to the exploitation of the method, but GA technique reaches to better solutions on the left side. The middle zone requires a hybridization method to achieve more solutions and fewer discontinuities, but deeper investigations must be carried out in the future.

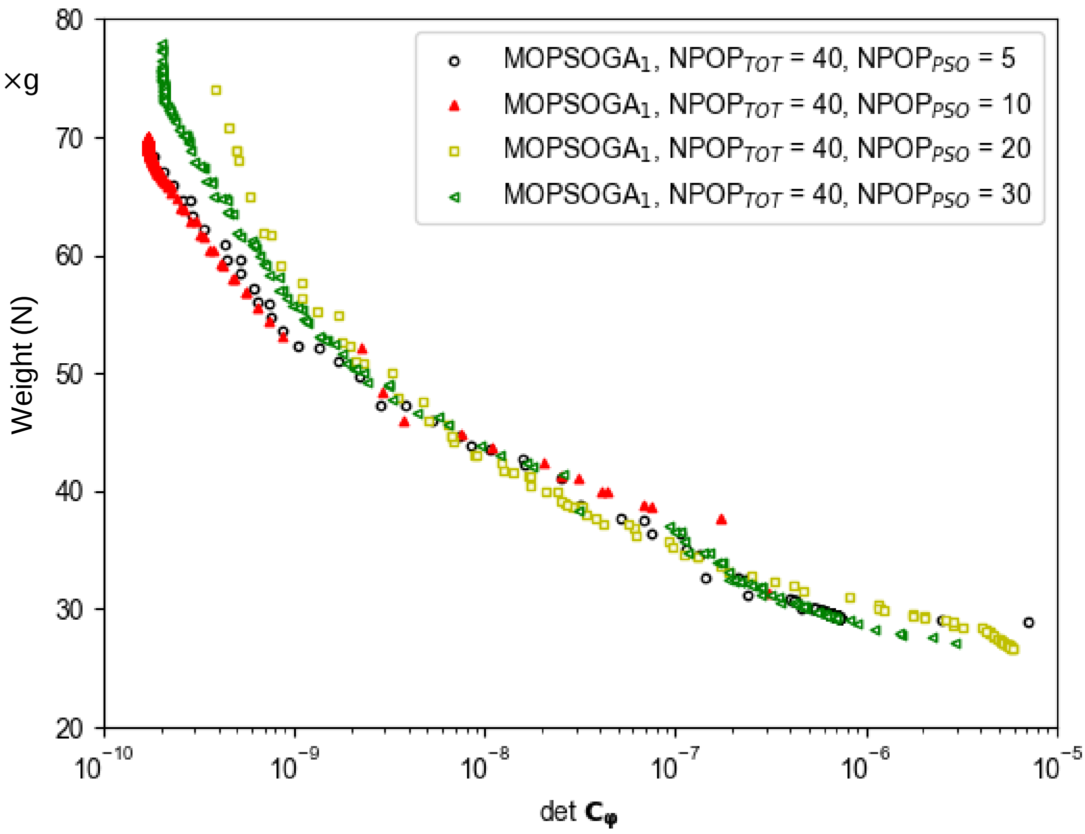

Figure 40. Pareto front resulted from the application of $MOPSOGA_1$ with a total population size of 40 individuals, for different PSO sub-population sizes

6 Conclusions and Future Works

Optimization is widespread in Engineering problems. They are usually complex and require more recent optimization techniques, such as bio-inspired algorithms. PSO and GA are very popular and are used on structural optimization problems, such as composite design optimization problems. A multi-objective GA has already been applied in minimizing the weight and maximizing the robustness of a laminated structure. This work is aimed to introduce three distinct novelties: 1. It presents a literature review of the most recent applications and developments concerning structural optimization problems; 2. It presents a novel multi-objective PSO adapted to the relative fitness assignment methodology that is implemented to improve the Pareto set, and it is applied to minimize the weight and maximize the robustness of a shell laminated structure; 3. It presents a novel multi-objective PSO hybridized with GA adapted to the referred fitness assignment and applied to the aforementioned structural bi-objective optimization problem. Moreover, this work also introduces a numerical example of PSO to make the application of such a technique easier for the reader.

The developed review has conducted to some conclusions. Plenty of algorithmic developments and structural engineering applications are rapidly being performed; achieving the minimum weight is a prevalent objective among all the structural optimization studies reviewed; size optimization is the most common type of structural optimization in the review. Topology optimization is widespread in composite structure optimization problems at the level of material design. The first may be related to the ease of computational implementation and the lower computational cost. The second may be because composite materials are complex, and stacking sequence optimization is frequently performed to achieve significant weight and buckling load improvements. Moreover, GA is used significantly more often than PSO.

However, from the review, some gaps have been identified. Firstly, there is a lack of evidence on when or why GA or PSO should be applied. For example, studies on composite structures have been mentioned that are very similar in terms of objective functions and design variables. Little to no comparative studies are done between the techniques when different methodologies are used. From another point of view, and according to the well-known “No Free Lunch Theorem” (Wolpert and Macready 1997), one may have to choose which technique can better perform a specific set of problems. This gap had been already previously identified by Mei and Wang (Mei and Wang 2021). Moreover, stacking sequence optimization is very usually performed in laminated composite plates, sometimes with the same discrete design variables.

The review has three significant contributions: the literature review of a great part of the applications on structural optimization; statistical analysis of different optimization-related categories, such as the usual type of structural optimization and the most used metaheuristic; a small literature review more focused on composite structures. Lastly, it has filled the lack of reviews for the generic topic of structural optimization in the optimization perspective.

The second contribution of the current work is developing a novel multi-objective PSO approach to solve the optimization problem of minimizing the weight of a shell laminated structure. As concluded from the literature review, there is neither a novel single nor multiple objective PSO method to minimize structures. However, the methodology used by Antonio and Hoffbauer (António and Hoffbauer 2017) to score merit to each individual of the local population can be only used to compare the performance of the individuals of the same generation. Since GA uses an elitism operation to preserve the best individuals, comparing the rest of the population with such individuals is already done at the local population level, and no further extra operations must be considered. However, initially, the PSO technique does not necessarily preserve all the elements of the population. Instead, it uses an extra population to store the best positions; the best element of such a population is the global best position. Since the presented fitness assignment methodology can only be used to evaluate solutions of the local population, both the local population and best-positions population are combined, and an original algorithm is developed to preserve the original principles of PSO.

The results of the application of MOPSO have been shown a success due to the following reasons: many more solutions have been found for the same number of function evaluations when compared to MOGA (C. C. António and Hoffbauer 2017); improved solutions regarding both weight and robustness objectives are achieved; Pareto front is more evident represented when compared to the Pareto front achieved by applying MOGA. Due to the inertia coefficient variation, MOPSO can also introduce more exploitation in the middle of the convergence process to find more solutions.

When varying the local population size, the results suggest that the larger the local population, the better the Pareto front in terms of robust improvement of heavier structure solutions. Regarding the lighter solutions, little to no difference is observed.

The third contribution consisted of developing PSO hybridization with GA, called MOPSOGA methods, of exploring and improving the results of MOGA and MOPSO. MOPSOGA methods consist of two main approaches: 1. the PSO and GA operations onto two different sub-populations separately; 2. PSO operates in one of both sub-populations, whereas GA crossover, applied and presented by Antonio and Hoffbauer (António and Hoffbauer 2017), is considered after PSO application in the other sub-populations. The methods have been challenging to develop due to the following: 1. PSO does not preserve the best solution for its local population, so the best-particle populations must be combined with the local population, as carried out in MOPSO; 2. GA and PSO operate with different types of variables, so decimal values are temporarily converted to binary to allow GA crossover to be performed; 3. The solutions are ranked at the end of the fitness assignment, but a rearrangement process must be carried out so that the principles of both PSO and GA techniques remain valid.

The results are promising since the number of solutions of the Pareto set is more significant than that achieved by MOGA. The variation of the PSO sub-population sizes allows, in general, the exploration of better solutions in different zones of the Pareto front.

Then, combining all the solutions of the three approaches MOGA, MOPSO and MOPSOGA is done. The results in terms of the design variables are in general consensual: the ply angle is between 89.5º and 90º; the thickness of the third macro element is suggested to be the minimum allowed value of 5 mm of the side constraints; the fourth macro element has the maximum value, being this value varying the most among all the selected cases.

Although, the proposed methods and the considered results have some limitations. Other parameters are considered fixed despite the varied population size along with the experiments. For instance, the inertia weight constantly changes from 0.9 to 0.4 in a descent order. Still, no experiments are carried out with other different values to understand its influence on the size of the Pareto front. Moreover, the population size of the MOGA approach is not varied during the experiments, which cannot allow the best comparison between the proposed methods and MOGA.

In the future, a deeper study on the considered parameters and their influence on the results must be carried out. Moreover, other optimization techniques are being developed in the literature and thus they have been found suitable for composite structures optimization. Therefore, such methods can be used in the future for implementation. Moreover, surrogate models such as Neural Networks and similar approaches can be used to substitute the FEM calculations here.

The author is confident that the developed contributions allowed the reader to understand the importance of the optimization techniques in structural problems, how to apply PSO numerically, and that the results of the optimization problem of the minimization of the weight of a laminated shell can be improved with PSO.

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