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Comments

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Black holes, disk structures, and cosmological implications in *e*-dimensional space

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Abstract: We examine a modern view of the universe that builds on achieved successes of quantum mechanics, general relativity, and information theory, bringing them together in integrated approach that is founded on the realization that space itself is *e*-dimensional. The global and local implications of noninteger dimensionality are examined, and how it may have increased from the value of zero to its current value is investigated. We find surprising aspects that tie to structures in the universe, black holes, and the role of observations. © *2022 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-35.4.345]

Résumé: Nous examinons une vision moderne de l'univers qui s'appuie sur les succès obtenus de la mécanique quantique, de la relativité générale et de la théorie de l'information, en les réunissant dans une approche intégrée basée sur la réalisation que l'espace lui-même est e-dimensionnel. Les implications globales et locales de la dimensionnalité non entière sont examinées et comment elle peut avoir augmenté de la valeur de zéro à sa valeur actuelle est étudiée. Nous trouvons des aspects surprenants liés aux structures de l'univers, aux trous noirs et au rôle des observations.

Key words: Black Holes; Noninteger Dimensionality; Information Theory; Complementarity; Early Universe; Quantum Mechanics: General Relativity.

I. INTRODUCTION

Since interaction with the world is central to our construction of reality, the information resulting from the interaction must get priority over preconceived notions that likely are based on cultural biases and classical notions. This is also a central tenet of quantum mechanics (QM) the most accurate of modern scientific theories in which contextuality plays a fundamental role. When considering isolated systems at the most basic level, isolated from relevant environments, the information depends on whether one is seeking local or global variables and these form a complementary pair in QM[1,2] with applicability to many fields, including the large scale structure of the universe studied in cosmology.[3]

The complementarity view is associated with dichotomies that include those of the object and the observer in QM, space and time in the theory of relativity, and materiality and consciousness, to name a few. There is also a dichotomy associated with implicit structure and information within living systems. The complementary relationship between object and subject is most fundamental and ties various levels of reality together in information that ultimately forms what we make sense of the universe "out there." However, dichotomy does not mean division lasting beyond the contextual situation of specific observations.

The naïve notion of space is a construct of the mind. By naïve we do not imply a lesser viewpoint, rather a more contextually limited one. If the three directions of space are advanced on logical grounds, that does not fix the structure of space itself. The experience of space is tied up with change. This change is experienced in the memory bank, and it rests on models of the world that are arranged with respect to time. Time and space are, therefore, inextricably linked in our experience, and this informs our intuitions about relativity.[4,5]

The anthropic principle brought in to involve the observer, with restrictions on the nature of the universe to ensure that it can sustain sentient life, emerged from attempts by Eherenfest,[6] Whitrow,[7] and Barrow[8,9] to understand why we find space to have three dimensions. This was also related to Dirac's "large number" coincidences in cosmology[10,11] by Dicke,[12] who saw it as a means of fine tuning properties of the universe from the anthropic perspectives. Others are looking at the coincidences as making it possible for complexity to exist in the universe.[13]

As widely observed, nature privileges optimality; therefore, information in natural systems may be analyzed from the perspective of efficiency. The most basic structures related to humans constitute the alphabet that when combined in different ways will yield more complex forms quite like the way the alphabet of a language leads to words, sentences and narratives. The basic structures may be seen in terms of form or number in physical or an appropriate abstract information space. Recent work has shown that efficiency and optimality required that space is *noninteger dimensional* with the dimensional value of *e* as being optimal.[14,15]

Some initial possibilities of confirmation of this theory were obtained in the explanation for the Hubble tension,[16] which is the divergence between the expansion rates

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obtained from the early universe (in observations of the cosmic microwave background, CMB, data) and the late universe (considering the expansion and recession flows of distant matter in the universe). This approach also provides a model of asymptotic freedom of particle physics[17] for it indicates that the attractive force drops to zero when the dimensionality goes below 2. If this intuitive approach is right, then there is no need to postulate dark energy at cosmological scales, when it still remains beyond laboratory and direct observational confirmation, despite many years of attempts to detect it, to account for what appears to be an accelerating expansion of the universe,[18,19] although dark matter may still be required from galaxy observations to explain the stability of the arms of the spiral galaxies and stability of clusters of galaxies.

The approach taken before and adhered to here yields new classes of fractal structures[20,21] that at cosmological scales could be searched for in galactic structures as well as a multitude of terrestrial phenomena. Two examples of fractals over different scales are the Nautilus shell and the Whirlpool Galaxy (NGC 5194) shown below that suggest a common origin for a form that persists over scales of different orders of magnitude. One should point out that the natural structures will be mathematical structures modified by maximum entropy transformations.[22,23]

The present article explores implications of noninteger dimensional spaces to the question of the collapse of matter to black holes (BHs) and cosmological structures for which BH's are limiting cases. The universe is normally studied using general relativity (GR) as a starting point. BHs as astrophysical objects are very relevant to the universe at all scales and provide boundary limits where QM must eventually be unified with GR. As such, both QM and GR apply to levels beyond the original scales to which they applied when they were first developed. We argue that the noninteger dimensionality approach presents a new pathway to the formation of primordial BHs and black holes in binary stars and the centers of galaxies, including our own Milky Way Galaxy. There are also implications for the early universe and theoretical work in large or correspondingly small dimensionless numbers. We note here that this formalism has implications for other astrophysical objects such as pulsars and neutron stars, the most condensed matter objects. If the evolution of the universe is a consequence of an evolution of the dimensionality, then certain conclusions follow that are at total variance to the understanding of mainstream cosmology. For instance, it implies that while the universe is one-dimensional in the earliest phase, the matter will aggregate into a cosmological BH. Subsequently, there will be early one-dimensional and two dimensional structures. It predicts primordial BHs, as well as why the very early universe will have barred and spiral galaxies, all of which are at variance with mainstream cosmology.

II. OPTIMALITY AND EFFICIENCY

The proof of the assertion that e-dimensionality is optimal is elementary. Each coordinate axis in the general abstract space may be viewed as a bin. Assume a total of d bins and label them as 1, 2, 3…d. The utilization of the system would be optimal if each of the bins carries the same information or the probability of the use of each is equal to $1/d$. The information associated with each bin then equals $\ln d$.

This information increases as d increases, but this increase is obtained at the cost of the use of the larger and thus more expensive binset. The information efficiency per bin is

$$E(d) = \frac{\ln d}{d},$$

which is shown in Fig. 1. Its maximum value is obtained by taking the derivative of $E(d)$ and equating that to zero, which yields $d_{\text{opt}} = e = 2.71828\ldots$. In other words, *the optimal number of bins associated with representation is e.*

The bins may be viewed as the coordinate axes of the corresponding abstract space or be aggregated as logical classes. Quite another perspective is that this represents a fractal structure which means a space that is somehow like "cottage cheese." The noninteger nature of dimensionality can be seen in a complementary perspective as the source of the attractive force within the space. One-dimensional data would find optimal representation in e-classes. Since, our cognitions cannot do this, optimal representation will be three classes.

FIG. 1. (Color online) The Nautilus shell; M51: The Whirlpool Galaxy NGC 5194.

One needs a complementarity perspective to make sense of this. We can view it as three classes with some dependence in the data in the classes. One can also view it from the perspective of the most efficient coding of data, in which case *e* represents the radix.

It was shown earlier,[17] that for noninteger dimensionality spaces the potential $p(d, r)$ at a distance *r* from an object of unit mass varies as a function of the dimensionality *d* of the space and function *f* related to the difference between the actual dimension of the space and its ceiling integer value

$$p(d,r) = \begin{cases} \dfrac{f(2-d)}{4\pi}, & 1 \leq d \leq 2 \\ \dfrac{f(3-d)}{4\pi r d}, & 2 \leq d \leq 3. \end{cases} \tag{1}$$

For $1 \leq d \leq 2$, the potential is independent of *r* and, therefore, it may be taken to be a constant. For simplicity, we take it to be equal to zero. For $2 \leq d \leq 3$, the potential corresponds to the inverse square law, which we may on account of the Occam's razor take to be the same as the gravitational force.

This relationship between potential and dimensionality brings in another variable with fundamental implications for the evolution of the system. An obvious modification to our equations involving the gravitational constant *G* is called for because in a noninteger dimensional space, it varies with the dimension *d* as proposed in Ref. 18 (Fig. 2)

$$G(d,t) = G(d) = \begin{cases} G_0(5d - d^2 - 6); & d \leq e \\ 0.202\ G_0; & d > e. \end{cases} \tag{2}$$

If it is assumed that after the Big Bang the matter in the universe has evolved into increasing *d* and not quite reached the value *e*, one needs to consider only the first expression.

The postulation of dimensional energy and evolution of $d = 0$ space into higher dimensions provides an alternative to the standard model of cosmology.[19] This alternative narrative shows that quite like in the established big bang cosmology, the expansion goes through different stages. First, there is a very rapid expansion at a nearly instantaneous rate quite like the inflation of standard cosmological theory, followed by an inverse-square law attraction mode with two subphases (radiation-dominant and matter-dominant) where this attraction becomes increasingly larger which slows down the expansion from its initial phase, and accelerated expansion as the attraction force declines and gravitation holds steady.

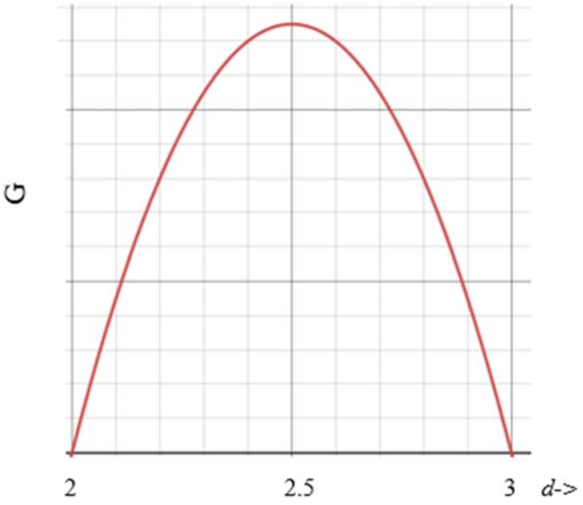

FIG. 2. (Color online) Gravitation with respect to dimensionality.

This model predicts that in the future the potential *V* will progressively decline and at a certain point it will be less than that of gravitational attraction; it will, thus, be characterized by slowing expansion followed eventually by contraction of the universe. If this is correct, evidence should be found for changing gravitational field in the past. Specifically, it should be shown that the gravitational constant, *G*, has declined by 20% from its peak estimated to $\sim 10^{10}$ years ago.[19]

We note that the issue of variation of *G* was also examined at least several decades ago, starting with Eddington,[20] Milne,[21] and Dirac,[22] including one of the present authors (see below) and others. Questions have been raised from time to time whether the gravitational constant, *G*, is varying in cosmological time[23,24] and the relationship of this change to the value of other constants.[25,26] Thus, cosmological consequences of allowing some of these constants of nature to change have been studied to evaluate the effects of time-evolution of "constants" in generalizing the framework of GR with the purpose of allowing them to become space-time variables.

III. EVOLUTION AND STRUCTURES

If one considers evolution with respect to dimension *d*, it will first rise from 0 to 1, in which process masses will be seeded at different points of the one-dimensional universe. Scale-invariance in one dimension will lead to the manifestation that may be taken to be like an optimal one-dimensional process.

When $\underline{d} \leq \underline{1}$:

For simplicity, we consider the distribution according to the random Cantor set shown in Fig. 3.[27] There are also more general fractal structures consistent with noninteger dimensionality.[28,29] For these systems, one must also apply the maximum entropy principle,[30,31] according to which Nature codes data in forms that correspond to maximum entropy. One way to see the maximum entropy principle at work is in terms of the first digit phenomenon.[32]

This phase will, thus, include barred structures with dimension in distribution corresponding to the first digit phenomenon.

FIG. 3. Cantor set.

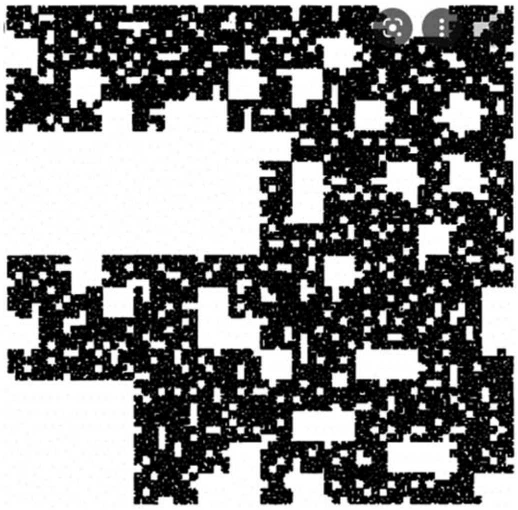

FIG. 4. A two-dimensional random fractal.

When $1 \leq d \leq 2$:

Figure 4 presents a two-dimensional generalization of the Cantor set of Fig. 3. One can propose that certain barred structures become spiral because of the expansion along the second axis.

Some structures just become disks.

When $d \geq 2$

The spiral structures have components that acquire mass along the third dimension. Broadly, this implies that barred and two-dimensional structures, which after acquiring spin become spiral, come before three-dimensional structures. In general, self-similar behavior is characterized by the first digit phenomenon and, therefore, one will see that in this case as well.

The summary of the argument is as follows:

1. Noninteger dimensionality is associated with fractal behavior and, therefore, with self-similarity.
2. Self-similarity is also associated with power-law and with the first digit phenomenon.
3. Therefore, one should expect to see the location of galaxies and other objects in space to follow the power-laws like the Newman–Benford (NB) law or the Zipf's law.[32]

Some evidence of the workings of the NB law is already available. If a counting process is uniformly distributed over the range $\{1, \ldots, S\}$, with random values of S, then the sum of a large number of these satisfies the NB law,[32] where the leading digit n ($n \in \{1, \ldots, r - 1\}$) for number to the base r, $r \geq 2$, occurs with probability as a logarithmic function[33,34]

$$P_{NB}(n) = log_r\left(1 + \frac{1}{n}\right). \tag{3}$$

When the number consists of several digits, the same law applies with n replaced by the appropriate number. NB's law is scale invariant, and if numbers in the data set are rescaled to another base, the probabilities will be adjusted for the new base.

The values for the digits to base 10 are given in Table I, which also provides the corresponding probabilities for the Zipf distribution.

The observational data from a variety of natural phenomena in Table II show that the data appear to largely conform to the frequencies expected from the NB's law but with small deviations. One can assume that these deviations have the potential to offer further insight into the nature of the data and the underlying physical process.

IV. BLACK HOLES AND ENTROPY

Consider the following thought experiment. If one throws an object up from the surface of the earth with a velocity larger than ∼11 km/s—the escape velocity—it would escape the gravitational pull of the earth. In the late 18th century, John Mitchell and Pierre-Simon Laplace, wondered if a star were to be so dense, that the escape velocity from its surface would exceed the velocity of light. In that case light would not escape from the star, and it will be dark or black. Now, given the principle that nothing can travel

TABLE I. First digit probabilities and Zipf's probabilities adjusted for peak at 0.301.

	1	2	3	4	5	6	7	8	9
1st digit, base 10	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
Zipf's freq	0.301	0.155	0.103	0.075	0.060	0.051	0.043	0.038	0.034

TABLE II. First digit data from a variety of natural phenomena.

	First Digit Frequencies percentage								
	1	2	3	4	5	6	7	8	9
NB's Law	30.1	17.6	12.5	9.69	7.92	6.69	5.80	5.12	4.58
Fund. Phys. constants	34.0	18.4	9.2	8.28	8.58	7.36	3.37	5.21	5.52
Geomagnetic field	28.9	17.7	13.3	9.4	8.1	6.9	6.1	5.1	4.5
Geomagnetic reversals	32.3	19.4	13.9	11.8	5.3	4.3	3.2	5.4	4.3
Fermi space teles. Fluxes	30.3	17.9	13.0	9.9	7.6	6.96	5.23	5.23	2.72
Pulsars rotation frequency	33.9	20.7	12.7	7.6	5.3	5.0	4.94	4.67	4.88

faster than the speed of light, no other object can also escape the star.

Let us estimate the radius of the surface—horizon—which separates the inside from where nothing can leave the star of mass M and radius R. Let the object of mass m be tossed from the surface with a velocity v. The total energy of the object is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}. \tag{4}$$

The escape velocity is achieved when it is large enough to overcome the gravitational potential. Thus, ${v_{es}}^2 = \frac{2GM}{R}$. If $v_{es} = c$, the speed of light, we obtain the critical value of R

$$R_h = \frac{2GM}{c^2}. \tag{5}$$

If the radius of the star is less than R_h, no object or light can escape. The horizon R_h is depicted by the dashed line, and the singularity is denoted by a dot in the center (Fig. 5).

Due to the uncertainty principle, $\Delta E \Delta t \le \hbar$, if a pair of virtual particles is created near the horizon, one of them will fall inside the black hole and the other will be radiated away. Hawking showed that the temperature associated with this radiation will be given by

$$T = \frac{\hbar c^3}{8\pi GM}. \tag{6}$$

Now temperature is associated with entropy, and Bekenstein[35] proposed that this entropy is proportional to the surface area of the horizon

$$A = 4\pi R_h^2, \tag{7}$$

where A is the area of the surface of the horizon.

For a system of energy E, if the energy is increased by a small amount dE, the increase in entropy will be

$$dS = \frac{dE}{T}. \tag{8}$$

For the black hole (BH) of mass M, the energy $E = Mc^2$. This gives us $dS_h = \frac{8\pi G}{\hbar c} M dM$. Integrating,

$$S_h = \frac{4\pi G}{\hbar c} M^2. \tag{9}$$

The thermodynamic entropy for a main sequence star like the Sun is about 10^{58}, whereas the entropy of Eq. (9) is about 10^{77}. This may be reconciled by assuming that the black hole entropy is with respect to the Planck dimensions.

Also note that the radiated particles from a black hole are expected to be mostly massless, mostly photons and possibly also neutrinos, and the photons will have a wavelength of $\lambda \sim R_h$, which for a solar sized black hole would be about 3 km. Black holes at the center of the galaxy have $R_h \sim 10^8$ kms, and so the wavelength of the radiation will be 10^8 km or long wavelength radio waves.

The time between successive emissions will be $\Delta t \sim \frac{R_h}{c}$ and, therefore, the photon emitted has the energy

$$E_{ph} = \frac{\hbar c}{\lambda} \sim \frac{\hbar c}{R_h}. \tag{10}$$

Since the total mass of the BH is Mc^2, the number of emitted photons is (with Planck length $l_* = \sqrt{\frac{\hbar G}{c^3}}$)

$$\mathcal{N} \sim \frac{Mc^2}{\hbar c / R_h} \sim \frac{A}{l_p^2} \sim S_h. \tag{11}$$

The total time for evaporation of the black hole will be

$$t_{evap} \sim \mathcal{N} \Delta t \sim \left(\frac{M}{m_p}\right)^3 t_p, \tag{12}$$

where the Planck mass $m_* \sim \sqrt{\frac{\hbar c}{G}} \sim 2.2 \times 10^{-5}$ g. For solar mass black hole $t_{evap} \sim 10^{63}$ years, which is much longer than the age of the universe (3×10^{10} years).

V. NONINTEGER DIMENSIONALITY AND BLACK HOLES

The expressions for $R_h(d)$ and $S_h(d)$, the horizon radius and the entropy expression will have to be modified to include the part in which G changes with respect to d

$$R_h(d) = \frac{2G_0(5d - d^2 - 6)M}{c^2}, \tag{13}$$

$$S_h = \frac{4\pi G_0(5d - d^2 - 6)}{\hbar c} M^2. \tag{14}$$

It is reasonable to assume that matter could collapse just upon itself, or and the dimensionality will also change in the process. In other words, we assume that there could be many different paths to the formation of a BH, and this could elucidate some unexplained structures in the cosmos.

Planck length and time are proportional to $\sqrt{G}$, whereas Planck mass and temperature are proportional to $\frac{1}{\sqrt{G}}$. This means that as the dimensionality of the star tends to 2, the length and time would tend to zero, whereas the mass and temperature will tend to infinity.

These considerations have implications for the formation of supermassive BH's and indeed for the universe itself.

VI. ROTATING STARS

As most stars—if not all in the universe—are rotating, when d approaches 2, the mass will flatten and assume a disklike geometry. Before approaching disklike geometry, the precursor will be a barlike structure.

The disk type structure can be of different kind, varying from simple rotation about its natural axis, to one where in order to preserve isometry, the disk will rotate rapidly about the length of the disk, so that it will appear spherical. However, this rotation of a flat disk will create a pulsing signature, and with small dimensions, the rotation could be extremely rapid.

The rotation can be in different modes like the ones below or a combination of the two.

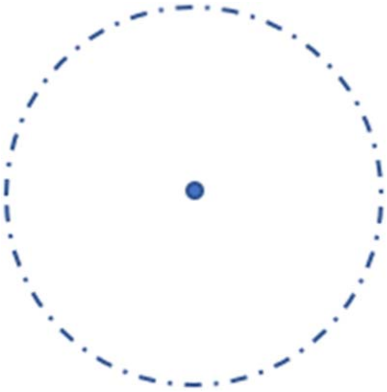

FIG. 5. (Color online) The singularity and the event horizon.

If the evolution is seen to emerge out of a linear or barred structure, then the disk will look as in Fig. 6.

One would need some ways to carry the argument further to relate the rotation speed to some other variables (Fig. 7).

VII. BARRED AND SPIRAL GALAXIES

One may also conclude that disk galaxies will be common in the early universe (as the universe evolved into the $d > 2$ phase). For $d < 2$, there would also be essentially linear or bar structures that are constituents of barred spiral galaxies (as in NGC 1672 or IC 5201) together with double-barred galaxies.[36,37] In the evolution according to dimensionality, the barred and spiral galaxies would emerge early (Fig. 8).

We also need to consider the connection of elliptical galaxies to barred spirals in the evolving universe. In this connection to be further explored, the role of central massive BHs and angular momentum which is lost in elliptical galaxies should be examined. Could the central massive BHs play a role?

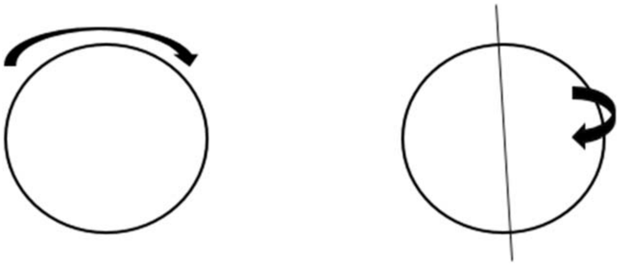

FIG. 6. Two kinds of disk rotation.

FIG. 7. (Color online) A disk with a bar structure within.

The new discovery of Galaxy DLA0817g, nicknamed the Wolfe Disk, made with the Atacama large millimeter/submillimeter array (ALMA) of a massive rotating disk galaxy, seen when the Universe was only ten percent of its current age, challenges the traditional models of galaxy formation.[38] This, the most distant rotating disk galaxy ever observed is contrary to most galaxy formation scenarios in which galaxies only start to show a well-formed disk around 6×10^9 years after the Big Bang. More recently, the galaxies called SPT-S Jo41839-4751.9 and BRI 1335–0417 only 1.4×10^9 years after the Big Bang have also been revealed to have a spiral structure.[39,40]

VIII. COSMOLOGICAL OBSERVATIONS AND CHALLENGGES

We now turn our attention to several cosmological observations involving G and other fundamental constants and discuss some of their implications as observed over cosmological scales.[41]

(a) The universe appears to be quite flat, in other words the density of the universe is very close to the so-called closure or critical density

$$\rho\text{crit} = 2 \times 10^{-29}\left(\frac{H_0}{100\ \text{kms}^{-1}\text{Mpc}^{-1}}\right)^2 \text{gr cm}^{-3}, \tag{15}$$

where H_0 is the Hubble constant defined as the apparent rate of expansion with distance and $\dot{R}/R$ and R being the scale of the universe.The observed density is not equal to the closure density when one observes regular, luminous matter. In big bang cosmology, the “Hubble constant” is actually a function of cosmic time, i.e., it is a variable. Its present-day value seems to be $\sim$75 $\text{kms}^{-1}\text{Mpc}^{-1}$. The universe appears to be close (but still off by factor of $\sim$ 10–100 from the closure limit, at present) to a flat, Euclidean, Einstein–de Sitter state as indicated from Eq. (15), and yet it is still not clear what the geometry of the universe is, i.e., whether exactly flat (which would be required by the inflationary scenario); open (yielding a forever-expanding, negatively curved space-time); or closed (yielding a maximum expansion and a positively curved space-time).

(b) If one is to assume that the universe followed an inflationary period in the distant past, then the universe must have been exactly flat to one part in 10^{50} near the time of Big Bang.[42] This is so-called *flatness* problem: This is such a remarkable requirement that the usual interpretation proposed in the early 80’s was that—early on, the universe was in an inflationary state, washing out any departures from flatness on time scales of 10^{-35} s. The inflationary model proposed by Guth[43] and others has been developed in various forms to account for the flatness of the universe and also is proposed to solve the horizon problem, or apparent homogeneity of the 2.73 K black body radiation seen by

FIG. 8. (Color online) NGC 1300, barred spiral galaxy (top); Spindle Galaxy (NGC 5866) (bottom), a lenticular galaxy in the constellation Draco.

COBE. The latter problem involves the observation that although the 2.73 K radiation was emitted $\sim 10^5$ years after the beginning, opposite sides of the sky at that time were out of causal contact, separated by $\sim 10^7$ light years. Other structures involving large-scale correlations in the universe exist as seen in the distribution of matter. These structures may be progressively hierarchical all the way to the scale of the universe itself.

(c) If the universe is indeed flat, observations indicate that baryons (or luminous matter) can only contribute at most ~ 0.05 of the closure density at present. We should ultimately be able to detect the other 90% or more of the matter required to give closure density, presumed to be in the form of cold dark matter.[43,44] Nevertheless, attempts to detect such exotic matter in the laboratory have, so far, failed. Moreover, the recent realization that the cosmological constant Λ may have to be reintroduced, to account for the possibility of an accelerating universe, has also led to the probability of Λ itself varying and other similar notions. Barrow and Magueijo[45] developed a particular theory for varying c (or α) in which the stress contributed by the cosmological constant varies through the combination Λc^2. They also showed how the observed nonzero cosmological acceleration might be linked to a varying α. According to them, the case of varying c theories is based on the fact that the effect is driven by a scalar field, coupled to the gravitational effect of pressure. The very slow

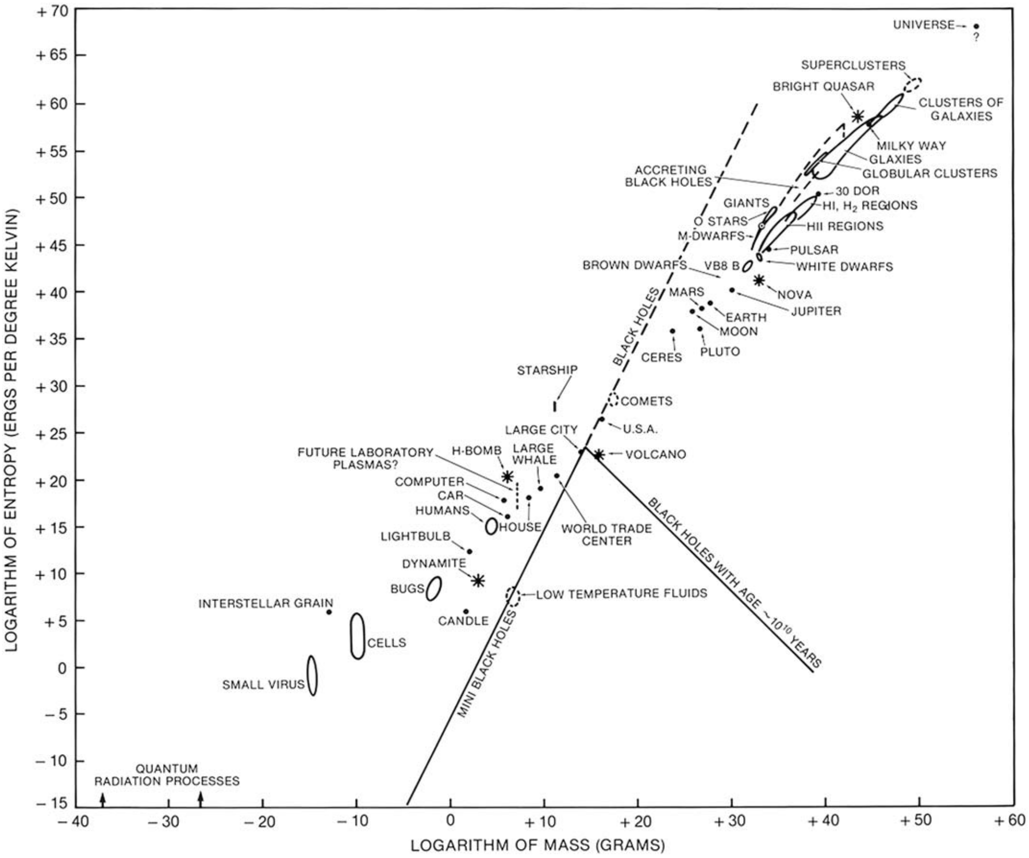

FIG. 9. Mass versus entropy diagram.

variation of the scalar field makes possible for slow variation of c, which at the radiation era converted the Λ energy density into radiation, thus preventing Λ dominance; but at the pressureless matter era the situation reversed.

This kind of theory allows variations of c or α to be $\sim 10^{-5}H_0$ at $z \sim 1$ and yet the associated Λ term can be dominant today and produce the much needed acceleration. Inflationary universe models provide a possible theoretical explanation for proximity to flatness but no explanation for the smallness of the cosmological constant itself. Nevertheless, without some direct laboratory verification or overwhelming requirements imposed by particle theory (neither of which presently exists), the nature of dark energy remains elusive. This is clearly a very unsatisfying situation.

(d) As we saw, present-day approximate flatness yields to an exact flatness in the distant past (this was one of the main reasons why the inflationary scenario was introduced to begin with). The alternative is to accept *fine tuning* in the universe. In fact, the flatness of the universe is not the only fine tuning. In considering other fundamental observed facts, the universe appears to be extremely fined tuned. As mentioned before, it was Eddington and Dirac who noticed that certain cosmic "coincidences" occur in nature linking microscopic with macroscopic quantities.

(e) A most unusual relationship is the ratio of the electric force to gravitational force (this ratio is presumably a constant in an expanding universe where the physics remains constant), or

$$\frac{e^2}{Gm_e m_p} \sim 10^{40}, \tag{16}$$

while the ratio of the observable size of the universe to the size of an elementary particle, e.g., the electron, is

$$\frac{R}{\left(\frac{e^2}{m_e c^2}\right)} \sim 10^{40}. \tag{17}$$

Here, in this relationship, the numerator is changing as the universe expands because the scale of the universe R is constantly changing in an expanding universe.

Dirac formulated the *Large Number Hypothesis*, which simply states that the two ratios in Eqs. (16) and (17) are in fact equal for all practical purposes and postulates that this is not a mere coincidence. Various attempts were made to account for the apparent equality: As we saw above, a possibility that constants such as the gravitational constant G may be varying was proposed by Dirac and others. Other ratios, such as the ratio of the size associated with an elementary particle, like the electron, to the Planck length

$$\frac{\left(\frac{e^2}{m_e c^2}\right)}{\left(\frac{\hbar G}{c^3}\right)^{1/2}} \sim 10^{20} \tag{18}$$

can also be constructed yielding to the conclusion that fine tuning is prevalent in our universe. These relationships may be indicating the existence of some deep, underlying harmonies involving the fundamental constants and linking of the microcosm to the macrocosm. Physical theory has not, however, accounted for these in a self-consistent way, waiting perhaps for the anticipated unification of all physical forces at the quantum gravity or superstring levels (as here Ref. 46).

A phenomenological and Newtonian model has been proposed by Ranada[45] to explain the recently observed cosmological variations of the fine structure constant as an effect of the quantum vacuum. He assumes a flat universe with cosmological constant Λ in the cases $(\Omega_M, \Omega_\Lambda)$ equal to (0.3, 0.7) and (1,0), respectively.

IX. NUMERICAL RELATIONS, SCALING, AND VARIATION OF PHYSICAL CONSTANTS

What we are proposing is that fractal dimensionality manifests at all levels and affects not just disk structures in the early universe, in fact it *must* be affecting the large scale structure of the universe itself, it must be affecting black holes as boundary conditions of stellar collapses to the universe itself and by extension all objects in the universe from clusters of galaxies, to galactic structures, to stars, planets, and all objects contained herein. At the other levels of scale, planetary and satellite systems, we have biological beings, molecular structures, atoms, particles and strings. The intriguing possibility is that constants of Nature such as G must also be tied to the fractal Nature. The universe would then be considered as most efficient in the large sense and when we include primordial BHs, at small scales as well. Therefore, the efficiency affects space and everything in the universe from the Hubble radius to the Planck length. The implications for the nature of reality would indeed be most profound and need to be further examined. To follow these conclusions, we notice here numerical relationships tying together all constant in nature that must eventually be studied as fractal e-dimensionality of the constants of nature themselves. We follow here some insights developed earlier.[41]

The study of the numerical relations over different scales provides surprisingly simple results involving fundamental constants. These relationships tie the structure of the universe with criticality of matter, variation of G and other constants and BHs as boundary conditions for the universe.

The critical density of the universe in Eq. (1) is defined as

$$\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G}. \tag{19}$$

Let N_p be the number of nucleons in the universe, then writing the mass of a particle in terms of cosmological quantities, we have

$$m_p = \frac{M}{N_p} = \frac{R\dot{R}^2}{2GN_p}, \tag{20}$$

where m_p and M are the mass of the nucleon and mass of the universe, respectively.

In the earlier paper[41] following some ideas of Weinberg, the masses of elementary particles, such as protons, pions, and electrons, were found to be related to the Hubble constant H_0 and other fundamental constants, namely, $\hbar, e$, G, and c with typical relationships being

$$m_p \sim \chi_{p\,\pi} \left(\frac{8\hbar^2 \left(\frac{\dot{R}}{R} \right)}{Gc} \right)^{1/3} \quad \text{with} \quad \chi_{p\,\pi} = \frac{m_p}{m_\pi}, \tag{21}$$

$$m_p \sim \chi_{\text{pe}} \left(\frac{\hbar e^2 \left(\frac{\dot{R}}{R} \right)}{Gc^2 (8\pi)^3} \right)^{\frac{1}{3}} \quad \text{with} \quad \chi_{\text{pe}} = \frac{m_p}{m_e}. \tag{22}$$

From Eq. (6) and the above relations in Ref. 41, it was shown that typical variations such as

$$G^2\hbar^2 c^{-1} \sim X_{p\pi}^{-3} \text{Np}^{-3} \frac{R^4 \dot{R}^5}{64} \tag{23}$$

result with masses of particles such as protons related to the Planck mass and other constants of nature, cf.

$$\text{m}_\text{p} = X_{p*} \sqrt{\frac{\hbar c}{G}} \tag{24}$$

for $\text{X}p_* = \frac{m_p}{m_*}$, and m_* being the Planck mass. Suffix * indicates Planck quantities. One then finds that the product of constants such as $cG\hbar$ depends on the scale of the universe and the Hubble constant. Two remarkable relations were found in Ref. 41, namely,

$$c \sim 2^{2/3} N_p^{-1/3} X_{p*}^{-4/3} X_{p\,\pi}\, \dot{R}$$

and $G\hbar = \frac{R^2 \dot{R}^3}{4} N_p^{-2} X_{p*}^{-2} \sim 3.4 \times 10^{-122}\, R^2 \dot{R}^3$ linking the speed of light to $\dot{R}$, i.e., $c = \dot{R}$ with $N_p \sim 3.7 \times 10^{79}$, which is a good estimate of the number of particle in the current universe where then $c = \dot{R}$ could be interpreted as the Hubble law $\dot{R} \sim c$ or an axiomatic approach equivalent to the Hubble law that is merely related to the constants of nature, avoiding the mysterious coincidences of Eddington and Dirac which Weinberg called "so far unexplained … a real, though mysterious significance."

It was further shown that all lengths, such as the Planck length, l_*, the classical electron radius, r_e, r_p *etc*., are all proportional to the scale of the universe. It was also shown

$$N_p^{-2} X_{p*}^{-2} \to 4 \to 1 \qquad \text{at initial conditions,}$$
$$N_p^{-2} X_{p*}^{-2}/4 \sim 3.4 \times 10^{-122} \text{ for the present universe.}$$

The limit $N_p \to 1$ indicates that *in the beginning* there was only one bubblelike object or a *cosmic egg* (similar to the ideas of Lemaitre). Moreover, in the "beginning" $R \to l_*$ and $N_p \to 1$ imply that $Xp_* \to 1$ as well, meaning that *all masses of all particles were equal to each other* in the "beginning or at "initial" conditions!

Moreover, in the beginning

$$\frac{R}{\frac{e^2}{m_e c^2}} \sim \frac{\frac{e^2}{m_e c^2}}{\frac{G}{c^3}} \sim 1$$

rather than the large values of 10^{40} and 10^{20} which these ratios are equal to today. Initially, all lengths were equal, and all masses were equal and there was only one particle or *cosmic egg* but today these ratios are not unity, as there is a very large number of particles in the universe and R is equal to $\sim 10^{28}$ cm.

Rosen and Israelit[47] proposed a cosmological model where the Universe emerges from a small bubble (*cosmic egg*) at the bounce point of a de Sitter model filled with a cosmic substrate (*prematter*). In other words, $c \equiv \dot{R}$, at the *initial time* when $N_p \to 1$ and all $\chi \to 1$, and this relationship remains invariant even in the present universe. The self-consistency is obtained by calculations for the value of N_p. This relation is a type of a scaling law and *connects the microcosm to the macrocosm*. Now, irrespective of the apparent expansion of the universe or the presence or absence of an actual expansion Hubble law the universe, R itself changing from the Planck scale to the size of the observable universe, results in the fundamental constants like G, $\hbar$ and c to all be *changing*.

Note, however, that we cannot deduce the actual variation or the initial value of c and other constants from observations: The relationship $c \equiv \dot{R}$ is not enough to tell us the actual variation or even over *how long* it takes place. It is a scale invariant relationship. It should be mentioned that though the condition $c = \dot{R}$ does not necessarily imply $c = c(t)$, they are not contradictory to each other.

Hence, it cannot be determined how c itself is varying or if it is varying. If we wanted to insist that c is *constant*, then all the other constants like G and $\hbar$ *are really constant as well*. But if c is not constant, then all the other constants are varying as well. In both cases, as the number of particles is changing, the ratios of masses are changing and the ratios of scales or lengths are also changing. An arrow of time or *evolution could*, therefore, be perceived. *This would hold even if the universe is really static!* In this picture, invariant relationships hold and from unity there is an evolution into diversity. One cannot, though, conclude how these variations are taking place, over what timescales they are taking place or even how old the universe is. The universe could be 10^{10} years old or 5×10^{-44} s (the Planck time) old, or any time in between.

Time is strictly a parameter that can be introduced in the scale-invariant relationships. It has no meaning by itself. The universe *appears to be evolving* as the number of particles and ratios are varying.

Considering how might local regions of space collapse into less than two-dimensions and thus become black holes, we propose that this mechanism will be in addition to other standard mechanisms of black hole formation.

X. FURTHER CONSIDERATIONS ON BLACK HOLES, SPACE-TIME, UNIVERSE, AND EVOLUTION

As BHs are so central to the universe, we can look at implications of noninteger dimensionality for all scales. The above Fig. 1 is a series of Universal Diagrams that one of us has developed over the years. Such diagrams indicate remarkable connectivity of physical parameters over vast dimensions of space, time, mass, and other variables. As can be observed, central roles in these diagrams are played by BHs.

A. Schwarzschild black holes

Consider the nature of the Schwarzschild singularity. It is likely of the size of Planck dimensions $\ell* \sim (\hbar\ G/c^3)^{1/2}$ and not a point singularity. As such, we also expect each of the constants that enter the Planck size to be tied to e-dimensional efficiency. The values of $\hbar$, etc., one obtains by measurements would only then be approximations. *As e is irrational, so would the values of the constants*. As we saw in Section IX, Dirac's large number hypothesis reveals ratios such the ratio of elementary particle (electron) size to Planck size, $(e^2/m_e c^2)/(\hbar G/c^3)^{1/2} \sim 10^{20}$, the ratio of electrostatic force to gravitational force, $\sim 10^{40}$, etc., one would expect that other constants such as the mass of the electron m_e, the charge e, G, etc., the mass of proton m_p would also be irrational. Finally, the Planck time itself $t_* \sim (\hbar\ G/c^5)^{1/2}$ would be tied to e-dimensionality, consistently in what we saw above that t is e-dimensional and, therefore, naturally $t \to t_*$ would be as well.

B. Kerr black holes

The more general case for a BH is a Kerr (spinning) black hole. It is generally not spinning maximally (with the speed of light) or maximum possible angular momentum but at a somewhat lower limit. (Also, it is generally assumed that charged BH's do not exist as it would be difficult to see how a charged BH would form to begin with and how it would maintain a large amount of charge.) Of course as the spin J of a Kerr BH $\to 0$, one would recover the Schwarzschild metric. As such, J itself would be tied to *e*-dimensionality.

The Kerr BHs present the intriguing possibility that such objects would connect to "other" universes through their ring-type singularities. In any case, one would have to consider the space-time paths inside a Kerr metric. There is no "flip" between r coordinate or one space coordinate and time coordinate as in the case of Schwarzschild Black Holes. Is there a new time component in Kerr BH?

In a sense, a Kerr BH is complementary to a Schwarzschild BH, in the latter case one would be constrained to be inside the BH in "this" universe, whereas in the former case one would not be. *The notion of multiverse would then apply, albeit not the usual believed from string theory*. Kerr BHs could be the gateways to the multiverse. In this case, the e-dimensionality of space could indeed be a universal law across *all* possible universe in the multiverse, with rotating BHs providing the link.

XI. SOME SPECULATIONS AND FUTURE WORK

This paper has used the noninteger dimensionality basis of space to propose a new pathway to black hole formation that can shed light on some of the puzzles of cosmology and to evidence on very early rotating barred disk galaxies that does not fit in with the standard picture of galaxy formation. What should further be explored is the role of angular momentum, of supermassive BH's and structure of barred spirals, regular spiral galaxies, and elliptical galaxies.

For a nonrotating Schwarzschild black hole inside the event horizon, the radial dimension, r, and time, t, flip. One can go back and forth in time but can only go inward in radial direction, destined to hit the singularity. Then if space is e-dimensional, one would expect that because of the above situation, time would related to e-dimensionality in some way.

Can we further conclude that there is more than one type of time? As it is the case for space, is *time also associated with information efficiency*? One would expect this to be even more natural for time, than the situation with space, as time is naturally connected to mind.

We need to explore what "information efficiency" means for time and its significance for the nature of the Schwarzschild singularity and differences with the rotating Kerr metric. Most importantly, one needs to further investigate the optimality of structures in the universe and their possible relationship to noninteger dimensionality.

In conclusion, early development of barred and disk galaxies is consistent with noninteger dimensionality, and apparent expansion of the universe with the evolution of dimensionality and the associated constants of nature. In future work, we hope to extend the noninteger dimensionality approach to other collapsed objects such as pulsars.

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