Multifractal analysis and wavelet leaders for structural damage detection of structures subjected to earthquake excitation

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Highlights

- Robust signal processing method, which is known as multifractal wavelet leader (MFWL) is used.
- Singularity spectrum from structural response is calculated based on multi fractal analysis.
- Quantitative and qualitative criteria for singularity spectrum in order to detect damage are proposed.

Abstract: This work is an effort to join, for the first-time, multifractal analysis and damage detection in civil structures subjected to strong ground seismic motions. Specifically, based on the singularity spectrum quantitative and qualitative criteria are proposed. The qualitative criteria are based on the concave of singularity spectrum of damage and undamaged structure. The proposed quantitative criterion is based on calculation of damage index taken the parameters of singularity spectrum. In order to achieve the above goal, a robust signal processing method, which is known as multifractal wavelet leader (MFWL) is used. The multifractal analysis is a tool to calculate fractal properties as well as scaling behavior of the structural response excited by an earthquake. The singularity spectrum is obtained from the Legendre-transformation to Holder exponents. In this paper, a parameter which is based on the shape of singularity spectrum and can identify the damage in the structure is proposed. The proposed method is an output-only approach for damage detection. Considering that the dynamic behavior of an inelastic system subjected to strong ground motion appears to be a non-stationary process, the above procedure of multifractal wavelet leader is suitable to retrieve the simulation response data. The findings from the analysis show that the MFWL is an appropriate scheme for structural damage detection.

Keywords: multifractal wavelet leader; damage detection; singularity spectrum; earthquake engineering; structural safety.

1. Introduction

In order to understand the nature's complicated dynamics in time and space the concept of fractal geometry is a suitable tool that was introduced by Mandelbrot [1]. A comprehend investigation into various directions found multifractal phenomena everywhere in nature [2].

Fractal systems can be classified into one of two categories: either monofractal or multifractal. They are characterized with the power law with actual scaling exponents. Monofractal systems are designated by a singular single scaling exponent, contrary to the multifractal systems that are labelled by a variety of scaling exponents. The multifractal wavelet leader technique has been developed by

Jaffard et al. [3]. Wendt et al. [4], proposed a new multifractal formalism which is based on wavelet leaders (MFWL). The coefficients of a 2D Discrete Wavelet Transform (DWT) are used to construct the wavelet leaders and therefore take advantages from low computational effort and a simplified execution. Other structural health monitoring and damage detection methods of structures are presented in the work of Chen [5].

In this work a correlation of multifractal analysis with damage detection of buildings subjected strong ground motion excitation is demonstrated. The proposed methodology is an output only signal processing. The response output of damaged and undamaged structure is measured and the singularity spectrum is obtained for both structures. Based on singularity spectrum a proposed damage index is calculated for both structures. The damage index is calculated using the parameters of the shape of singularity spectrum. The damage detection strategy is based only on its output response of structure. Based on the shape of singularity spectrum and the range of values of the damage index of the output response of structure conclusions regarding on damage or not for a structure can be done.

2. Theoretical Background

Wavelet analysis makes available a great implement to portray local features for a signal. In contrast to Fourier transform, where the function examined as the heart of decomposition is every time a wave of sinusoidal type, further beginning functions can be carefully chosen for the wavelet shape in line with the types of the signal. In time/frequency domain, the MFWL technique shares the signal into stretched and translated wavelet that should be orthogonal and gives waveform with zero mean fast decaying. The basis function for wavelet analysis is well-defined by two factors: translation and scale. These characteristics cause a multi-resolution representation for the case of non-stationary signals. The stretched and translated form of the wavelet can be expressed as:

$$, \quad ()= � \quad \frac{-'}{} � \tag{1}$$

The convolution operation, f(t), for the signal is applied to obtain the wavelet leader, w(i, s). The latter signifies the suprema for the coefficient of wavelet, and is calculated as

$$'< \quad 1 \quad \infty \quad \frac{-'\ '}{} \quad � \quad � \tag{2}$$

which is based on wavelet leader, can be expressed as:

The scaling function, ζ(

$$(,)=� \quad \frac{1}{} �\{ (,)\}�^{1�} \quad =1 \tag{3}$$

where q is the order and N_w has to do with the number of the wavelet leaders. In order to compute the best exact fit for multifractal analysis, one can use the least sum for the squared errors, SSE, i.e.,

$$=��[(,)- 0(-)-0]^2 \tag{4}$$

In Eq. 4, log denotes the logarithm and L the largest scale. Furthermore, the scaling behavior can be observed through determining the q order Hurst exponent, H(q), that can be achieved through the computation of regression lines' slope, which shows the multifractal structure parameterization of time series data. The multifractal Hurst exponents, H, and the Holder exponents, h, are connected with scaling exponents, τ, by:

$$h= \frac{()}{} \quad = \frac{(\quad ())}{-1} \tag{5}$$

In this study, the exponents of Hurst can be examined in order to separate the progressive structure of data for time series and recognized the multifractality level where a monofractal scheme has a constant Hurst exponent, although the multifractal signals presenting a noteworthy reliance of the Hurst's exponent on the q-order.

Multifractal spectrum is examined to arrest the multifractality degree. A meter to the multiscaling procedure is the multifractal spectrum width. The execution of Legendre-transformation for the Holder exponents' singularity or the spectral singularity is computed.

$$(h) = \inf � \cdot h - ( )� \tag{6}$$

Such a singularity spectrum is shown in Figure 1. As one can observe singularity spectrum depicts a concave parabolic shape function. The shape and the width of the spectrum arch has to do with±∞representative data of the response. The singularity spectrum is constrained by 2 bounds at q= .

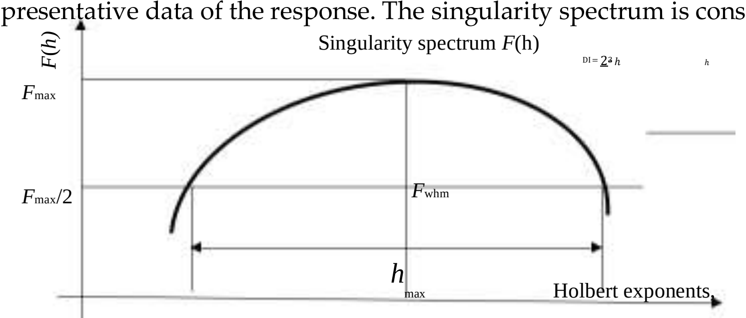


Figure 1. The singularity spectrum, a concave parabolic shape function and representative parameters for calculating the damage indicator.

The following parameter, DI, is a mixture of the full width at half extreme of the spectrum and the Holder exponent at the supreme of multifractal spectrum. This parameter is used to distinguish the response signal data of damaged structure to the response signal data of undamaged structure where both structures subjected to earthquake excitation and can serve as the damage index, DI.

$$= \frac{{}^{2h} \qquad h}{3} \tag{7}$$

Regarding the qualitative characteristic of the shape of singularity spectrum as concave is the spectrum as the response data are represent an undamaged structure which is subjected to earthquake excitation. Furthermore, as bigger is the range of spectrum in F(h) axis as more the spectrum response data are representing a damaged structure which is subjected to earthquake excitation.

As far as the quantitative characteristic of singularity spectrum as lower the DI value is as more undamaged the structure that the response data come from. The above quality and quantity characteristic of two singularity spectrum that calculated from response date coming from undamaged and damage structure is shown in Figure 2.

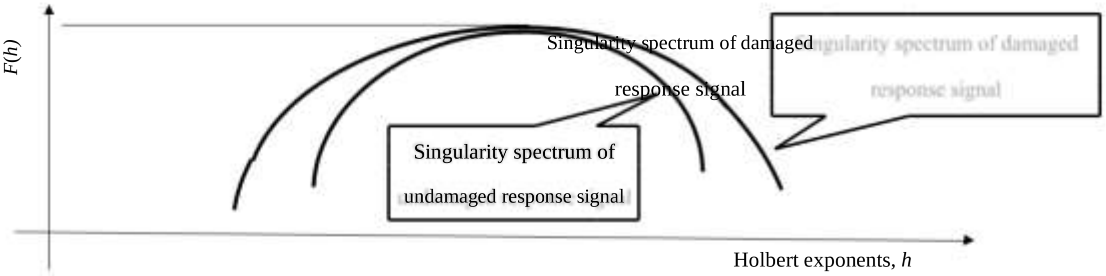


Figure 2. Comparison of singularity spectrums of two signals, of damaged and undamaged structural response signal.

3. Case study – Numerical Results and Discussions

The above singularity spectrum has been calculated for the three-story building subjected to earthquake excitation. The numerical model of building is represented as a dynamical system with concentrated mass and stiffness. The story mass is 10t while the elastic stiffness 10_4 kN/m. The columns exhibit nonlinear behavior elastic perfectly plastic with yield strength 100 kN. A kinematic hysteretic model was chosen for simulating the nonlinear behavior of columns. The layout, parameters and structural properties are shown in Figure 3. Athens 1999 earthquake record was applied at the base of the structure. Non-linear dynamic analysis has been executed using the software program SAP2000nl. The response at each mass and the shear force at each column were calculated. With appropriate scale of the earthquake excitation signal, the linear or undamaged (healthy) response, $u_{undamaged}(t)$ and non-linear, damaged (unhealthy), $u_{damaged}(t)$, response was calculated.

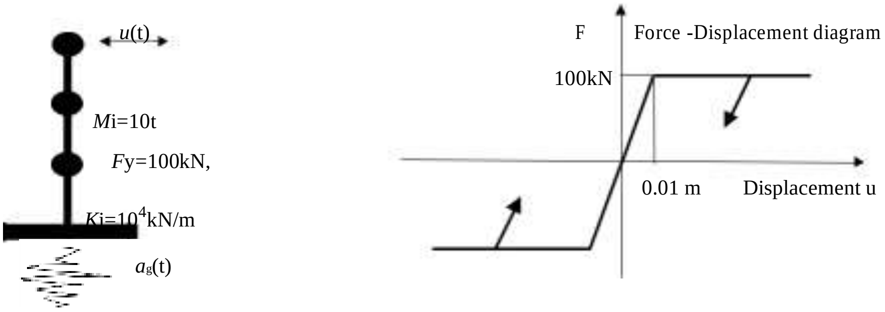
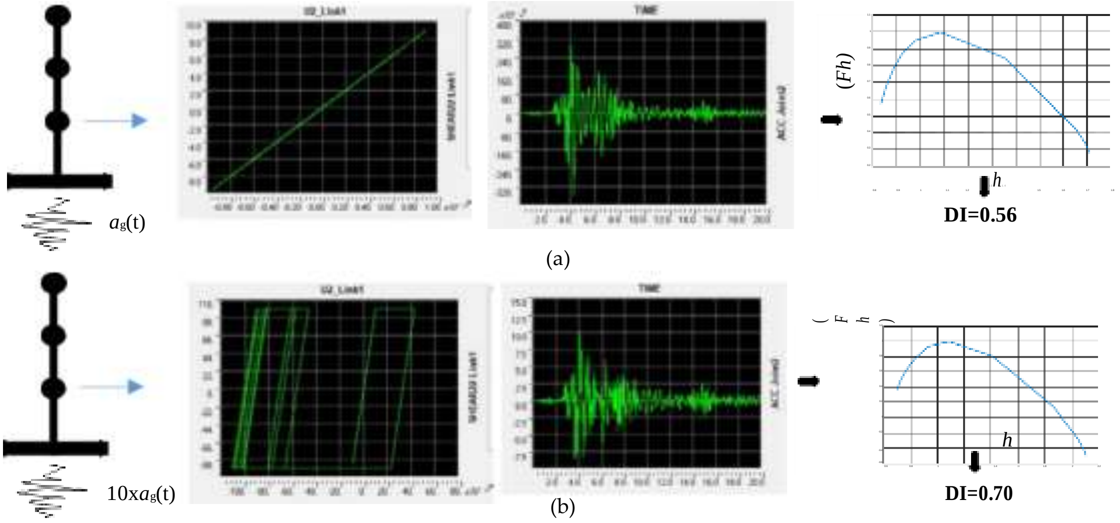


Figure 3. Characteristic structural parameters and Force -Displacement diagram of columns.

The response was presented in terms of acceleration, velocity and displacement of each mass. The response and the shear force vs displacement for damaged and undamaged structure are shown in figure 5.

Based on the above response data, the multifractal spectrum or the singularity spectrum for the damaged and undamaged response signal at the first floor and the corresponding damage indicator, DI, has been calculated using MATLAB software. Multifractal spectrum and damage indicator, DI, for the damaged and undamaged structures is also shown in Figure 4.



(a)

(b)

Figure 4. Shear force vs displacement at base column, acceleration at first floor, and the corresponding singularity spectrum for undamaged, (a), and damaged structure, (b).

Based on the above numerical results a comparison of multifractal spectrum for damage and undamaged structure have been done. The multifractal spectrum for acceleration, velocity and displacement response of first floor is shown in figures 6 to 8. In all spectrums it seems that the shape of singularity spectrum of the undamaged structure as more concave compare to the singularity spectrum of damaged structure. Furthermore, the range of spectrum in F(h) axis for damaged structure is bigger than the range of spectrum in F(h) axis for undamaged structure which is subjected to earthquake excitation.

The damage index, DI, is calculated in the case of acceleration response and it was found 0.6 for the undamaged structure and 0.7 for damaged structure.

4. Conclusions

This work provides with an effort to join, for the first-time, multifractal analysis and damage detection in civil structures under the action of strong earthquakes. From the above analysis it seems that the proposed connection between multifractal analysis and damage detection of civil structures is a promising area of research. Singularity spectrum is obtained with an application of multifractal wavelet leader, to the response signal of the damaged and undamaged structure. Based on singularity spectrum a proposed damage index is calculated. The damage index is extracted based on the shape of singularity spectrum and can identify the damage or not to the structure.

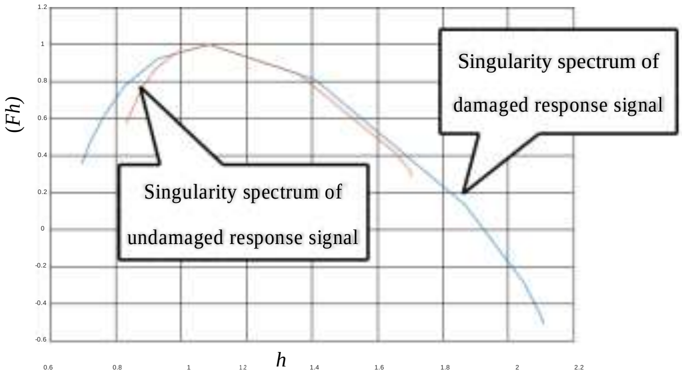


Figure 6. Comparison of singularity spectrums from damaged and undamaged structural acceleration signal in first floor.

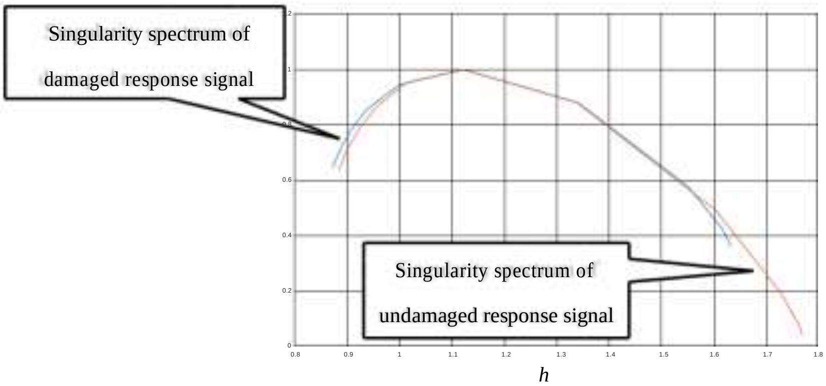


Figure 7. Comparison of singularity spectrums from damaged and undamaged structural velocity signal in first floor.

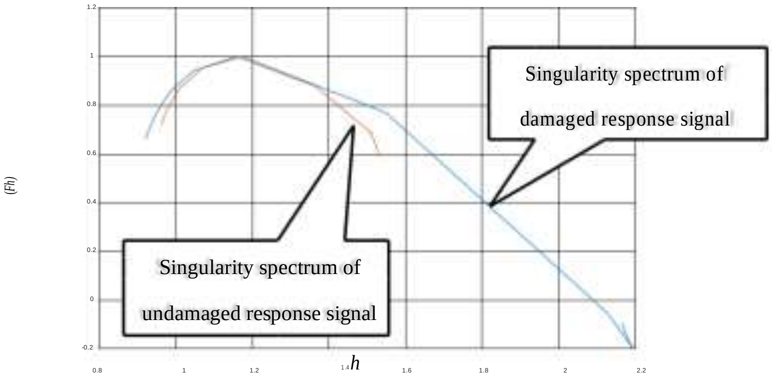


Figure 8. Comparison of singularity spectrums from damaged and undamaged structural displacement signal in first floor.

The proposed method is an output-only damage detection method. The numerical results indicate that the shape of singularity spectrum obtained from undamaged structure is more concave than in singularity spectrum obtained from damaged structure. This holds for displacement, velocity and acceleration response. Furthermore, regarding the shape of singularity spectrum, the range in values in y axis is wider for the damaged structure compared the range of values in y axis for the undamaged structure. As far as the damage index concerns it was found that this index is lower for the undamaged structure compared to the damaged. The numerical results indicate that the proposed method based on singularity response is an appropriate method for structural damage detection.

5. References

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