

Positivity Bounds on Dark Energy: When Matter Matters

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ABSTRACT: Positivity bounds—constraints on any low-energy effective field theory imposed by the fundamental axioms of unitarity, causality and locality in the UV—have recently been used to constrain scalar-tensor theories of dark energy. However, the coupling to matter fields has so far played a limited role. We show that demanding positivity when including interactions with standard matter fields leads to further constraints on the dark energy parameter space. We demonstrate how implementing these bounds as theoretical priors affects cosmological parameter constraints and explicitly illustrate the impact on a specific Effective Field Theory for dark energy. We also show in this model that the existence of a standard UV completion requires that gravitational waves must travel superluminally on cosmological backgrounds.

KEYWORDS: Positivity Bounds, Effective Field Theory, Dark Energy

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1 Introduction

Challenging our understanding of the Universe and General Relativity (GR) is a central goal of modern cosmology. Despite its many successes, GR cannot be the fundamental description of our Universe—for instance, it is only an effective description of gravity at low energies (breaking down at least at the Planck scale, if not below). In parallel, accounting for the late-time acceleration of the Universe leads to the well-known cosmological constant problem. GR may therefore require modifications on both theoretical and phenomenological grounds. Fortunately, in recent years there has been significant progress in developing model-independent parameterised approaches that allow for a systematic exploration of dark energy/modified gravity effects in a (linear) cosmological setting [1–12], resulting in a variety of cosmological parameter constraints on deviations from GR from (current and forecasted) experimental data, see e.g. [13–34].

As with any effective field theory (EFT), these parameterised approaches remain agnostic about the nature of the underlying UV completion. While this greatly improves efficiency (allowing the translation of observations into model-independent constraints), it introduces the risk that certain regions of parameter space may be secretly unphysical or “unstandard”—what could seem a perfectly consistent EFT may not have any healthy UV (high-energy) completion, or may not enjoy any local UV completion compatible with standard axioms. To ensure that the underlying UV theory respects fundamental properties—such as unitarity, causality and locality—the low-energy EFT must satisfy various constraints, known as “positivity bounds”. Following many recent advances in these EFT bounds and their consequences for dark energy and modified gravity [35–49], it is now more important than ever to

incorporate these constraints when performing precision tests for cosmology and for physics beyond GR.

In a cosmological setting, one often separates the matter fields from the gravitational sector and may even model them completely separately (e.g. as a perfect fluid, rather than as dynamical degrees of freedom). However, since all fields couple gravitationally, they ultimately mediate scattering processes which must obey positivity arguments if the EFT is to have a standard (unitary, causal, local) UV completion. This applies to light, baryonic matter as well as any other matter field living in a dark or decoupled sector, irrespectively of whether or not those fields are relevant to the dynamics or phenomenology of the EFT in question. We will therefore state the following very explicitly,

A low-energy EFT has no standard UV completion if it violates the positivity bounds for scattering between *any* of its low-energy degrees of freedom.

This observation is widely appreciated, but we emphasise it nonetheless since it strictly strengthens the power of positivity constraints on any given EFT. Rather than applying bounds to the scattering of one particular species (or a small number of species), the previous statement is making explicit the potential constraining power in scattering all possible combinations of all possible fields—including, in the case of cosmology, scattering quantized matter fields with each other and with fields in the gravitational sector. In some situations, accounting for these additional positivity bounds can lead to constraints that are orthogonal to what was otherwise considered as common wisdom (eg. see [50–52]).

At this stage it may be worth commenting on the notion of “standard” UV completion which is implicit in the applicability of the positivity bounds. By standard UV completion, we have in mind the EFT to be the low-energy limit in the Wilsonian sense of a local, unitary, Lorentz invariant and causal weakly-coupled high-energy completion in which the Froissart bound is satisfied. Note that the assumption of weak coupling does not require a tree-level completion as is sometimes further implicitly assumed in the literature. We refer to the violation of any of these assumptions as a “non-standard” UV completion, [53–59]. Such non-standard UV completions may either be non-weakly coupled, or may for instance include a small violation of locality or micro-causality.

Gravitational scalar field theory: Throughout we will be working within the context of a gravitational EFT that contains a light scalar degree of freedom ϕ [5] that may for instance play the role of dark energy. Moreover, we shall, for simplicity, restrict ourselves to a shift-symmetric Horndeski theory [60, 61], also known as Weakly Broken Galileons [62]. Specifically, we will focus on

$$S = \int d^4x \sqrt{-g} \left\{ \Lambda_2^4 G_2(X) + M_{\text{Pl}}^2 G_4(X) R + \Lambda_2^4 G_{4,X}(X) ([\Phi]^2 - [\Phi^2]) + \mathcal{L}_{\text{matter}}(\psi, g) \right\}, \quad (1.1)$$

where the dimensionless matrix $\Phi_\mu^\nu \equiv \nabla_\mu \nabla^\nu \phi / \Lambda_3^3$ (with $[\Phi^n]$ denoting the trace, e.g. $[\Phi^2] \equiv \nabla^\mu \nabla_\nu \phi \nabla^\nu \nabla_\mu \phi / \Lambda_3^6$), G_2 and G_4 are functions of the dimensionless $X \equiv -\frac{1}{2\Lambda_2^4} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$,

and $M_{\text{Pl}} \gg \Lambda_2 \gg \Lambda_3$ are constant scales which characterise the EFT. $\mathcal{L}_{\text{matter}}(\psi, g)$ indicates the Lagrangian for all the matter fields ψ which, in this theory, are assumed to be minimally coupled to the metric $g_{\mu\nu}$. Introducing non-minimal couplings between a matter field and the metric $g_{\mu\nu}$ or with the scalar field ϕ would further affect the EFT. The power counting in (1.1) ensures that the operators appearing at the scale Λ_3 are protected by Galileon invariance, and although this is broken by gravitational corrections, since graviton exchange is suppressed by at least one factor of M_{Pl} this ensures that a hierarchy $\Lambda_2^4 \sim M_{\text{Pl}}\Lambda_3^3$ is radiatively stable (see [63–67])¹.

The scalar-tensor theory (1.1) corresponds to a particular shift-symmetric subset of the Horndeski scalar-tensor theory [60, 61], in which cubic and quintic interactions have been turned off. This is the same example theory previously explored in Ref. [71] and has the particularly nice feature that positivity constraints are easily mapped onto constraints on the effective parameters controlling linearised cosmological perturbations [4]. Here we expand on the analysis of [71], where positivity bounds from the scattering of dark energy scalars were used to constrain cosmological parameters, by deriving and applying additional positivity bounds that arise in the presence of matter degrees of freedom. In particular, we focus on the subset of Horndeski theories for which the functions $G_2(X)$ and $G_4(X)$ obey the following two conditions:

- (i) in addition to a cosmological solution for ϕ , there is also a stable Minkowski solution $\phi = 0$, $g_{\mu\nu} = \eta_{\mu\nu}$,
- (ii) the effective couplings $G_{4,X}$ and $G_{4,XX}$ do not receive large corrections from the cosmological background so are comparable in both backgrounds.

For this subset of scalar-tensor theories, Lorentz-invariant positivity bounds around the stable Minkowski solution can be imported to the cosmological solution and compared with data. While there are interesting theories which may violate one or both of the above assumptions, by focussing on this particular subset we are able to demonstrate explicitly that positivity constraints from the UV can have important consequences for how we analyse data.

Constraints from speed of gravitational waves: Following the direct detection of gravitational waves from the Neutron star merger GW170817 with optical counterpart, [72–74], the speed of gravitational waves at LIGO frequencies is constrained to be luminal within one part in 10^{15} . Trusting EFTs of dark energy at order 10^2Hz would then lead to ruling out any model for which the speed of gravitational waves differs from unity, including the model considered in (1.1) [75–83]. Note however that since the theory breaks down at or below the cutoff $\Lambda_3 \sim 10^2\text{Hz}$, we do not expect that EFT to be meaningful on those scales [84] and in what follows we contemplate the possibility that (1.1) remains an acceptable low-energy

¹Note that the radiatively stable nature of such theories can be maintained in the presence of at least some specific sets of shift-symmetry-breaking interactions – see [68–70] for examples and a more detailed discussion of this point.

EFT at sufficiently low energies relevant for dark energy and the late-time acceleration of the Universe.

Positivity bounds: Unitarity implies the positivity of the coefficients of the partial wave expansion of the elastic 2–2 amplitude \mathcal{A} , between two massive particles on a flat background. The simplest bounds use the positivity of the first coefficients and the $s \leftrightarrow u$ crossing symmetry, [35] (see also [85–88] or earlier discussion of this constraint in chiral perturbation theory), assuming a causal (analytic in energy), and local (polynomially bounded growth at high energies) UV completion places constraints on the Wilson coefficients appearing in \mathcal{A} (see also [36, 89]).

There is, however, an infinite number of bounds that can be derived from the requirement of unitarity [44–46], and all bounds can further be improved by appropriate subtraction of the light loops contributions up to the cutoff of the EFT [47–49]. Lorentz invariance however implies full $s \leftrightarrow u \leftrightarrow t$ crossing symmetry and this information was seldom used until recently. Indeed full crossing symmetry was recently implemented directly at the level of the positivity bounds [90–93], where it was shown to further constrain massive Galileons [48] and other scalar field theories with weakly broken symmetries. Typically, the direct implementation of these bounds to the gravitational context is challenging, most notably due to the presence of a t -channel pole and concrete models are known to slightly violate the positivity bounds in the gravitational setup [52, 94], with a resolution provided in [95]. In this work, we shall be working in a decoupling limit $M_{\text{Pl}} \rightarrow \infty$ where issues related to the t -channel pole may be evaded. Moreover, we will focus on one of the simplest Lorentz-invariant bounds from unitarity and $s \leftrightarrow u$ crossing symmetry. This will be sufficient to illustrate our main point: that the coupling to matter inevitably generates additional bounds from dark energy-matter scattering.

The positivity bounds we shall consider will require Lorentz invariance. We shall therefore first consider the scattering of small fluctuations about the trivial Minkowski background $\phi = 0$, which ensures a Lorentz-invariant scattering amplitude and then import the resulting constraints to a cosmological background, invoking the covariant nature of (1.1). Such an approach is justified when assuming that the cosmological solutions we consider here smoothly connect with a trivial Minkowski vacuum. One could in principle go further, in particular there has been recent developments in establishing positivity bounds directly to Lorentz-breaking backgrounds [96], but we leave these considerations for further studies.

In the case we shall be interested in, when expanded in powers of the center of mass energy, s , and the momentum transfer, t and taking the $M_{\text{Pl}} \rightarrow \infty$, the amplitude takes the form

$$\mathcal{A}(s, t) = c_{ss} \frac{s^2}{\Lambda_2^4} + c_{sst} \frac{s^2 t}{\Lambda_3^6} + \dots, \quad (1.2)$$

and UV requirements then demand that $c_{ss} > 0$. Going beyond the forward limit, we will

use positivity in the form,

$$c_{sst} \geq -\frac{3\Lambda_3^4}{2\Lambda_2^4}c_{ss}, \quad (1.3)$$

first given in [44, 45] (see also [38, 47, 87, 97, 98] for earlier discussion of positivity at finite t), assuming² that the EFT can be used to subtract the contribution from light loops up to the scale Λ_3 . Notionally, these bounds on c_{ss} and c_{sst} are diagnosing whether it is possible (even in principle) for some new physics to enter at the scales Λ_3 and Λ_2 to restore unitarity in the full UV amplitude. If these bounds were violated, it would indicate that this new high energy physics is of a non-standard type, as indicated earlier, either due to the violation of the weakly coupled completion, violation of micro-causality or mild violation of locality [53–59]. By itself, observing a violation of the positivity bounds would have groundbreaking consequences for our understanding of high-energy physics.

For the scalar-tensor theory (1.1), we show below that, in addition to the positivity bounds from scalar-scalar scattering found in [71],

$$\frac{\bar{G}_{4,X}}{\bar{G}_4} \geq -2\bar{G}_{2,XX}, \quad \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq -\bar{G}_{4,XX}, \quad (1.4)$$

there are separate bounds from scalar-matter scattering,

$$\bar{G}_{4,X} \geq 0, \quad (1.5)$$

where the \bar{G}_n are the functions evaluated on the Minkowski background $\phi = 0$. Interestingly, the new bound (1.5) places a qualitatively different restriction on the low-energy EFT, and in particular requires that the speed of tensor modes (to which matter fields – including light – couple minimally) is strictly *greater* than the speed of any matter field (including photons)—gravitational waves are superluminal [50, 51].

When compared with observational data, we show that the bounds (1.4) and (1.5) can be implemented as priors to improve parameter estimation (assuming that $G_{2,X} > 0$ so that the flat vacuum is stable). Interestingly, we find that once the prior (1.4) is imposed, the resulting observational constraints are already highly consistent with positivity in the matter sector (1.5). We also show how this outcome would have been dramatically different had one instead imposed other priors that do not rely on the same assumptions of causality and unitarity.

The rest of the manuscript is organized as follows: In section 2, we review the positivity bounds inferred from scalar-scalar scattering in a limit where gravity decouples and issues from the t -channel pole are irrelevant. We then derive the new positivity bounds inferred from

²Note that if the EFT breaks down at some low scale, $\epsilon\Lambda_3 \ll \Lambda_3$, then the bound (1.3) would be $c_{sst} \geq -\frac{3\Lambda_3^4}{2\epsilon^2\Lambda_2^4}c_{ss}$. Neglecting the small ratio $(\Lambda_3/\Lambda_4)^4 \sim (H/M_{\text{Pl}})^{2/3}$ only requires that the EFT be valid at scales much above $(H/M_{\text{Pl}})^{1/3}\Lambda_3 \sim H$, and so this assumption is equivalently a necessary condition for describing the expanding spacetime background.

scalar-matter scattering. In section 3 we show how the positivity priors affect the outcome of observational constraints on the parameters of the dark energy EFT. In particular we show how positivity priors differ from standard stability and subluminality criteria. We end with a discussion and outlook in section 4 and leave some of the technical details to Appendix A.

2 Positivity bounds

Positivity of scalar-scalar scattering: As shown in [71], (see also Appendix A), expanding (1.1) about a flat background ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\text{Pl}}$) with zero-vev for the scalar field ($\phi = 0 + \varphi$), then canonically normalizing φ such that $\bar{G}_{2,X} = 1/2$, the tree-level scattering amplitude for $\varphi\varphi \rightarrow \varphi\varphi$ takes the form (1.2), with,

$$c_{sst} = -6 (\bar{G}_{4,XX} + \bar{G}_{4,X}^2/\bar{G}_4) , \quad c_{ss} = 2\bar{G}_{2,XX} + \bar{G}_{4,X}/\bar{G}_4 , \quad (2.1)$$

where an overbar indicates that the function is evaluated on the flat background. The bounds $c_{ss} > 0$ and (1.3) therefore become,

$$\frac{\bar{G}_{4,X}}{\bar{G}_4} \geq -2\bar{G}_{2,XX} , \quad \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq -\bar{G}_{4,XX} , \quad (2.2)$$

where we have assumed $\Lambda_2 \gg \Lambda_3$. The other elastic amplitudes, $\varphi h \rightarrow \varphi h$ and $hh \rightarrow hh$, vanish at $\mathcal{O}(1/M_{\text{Pl}})$ (with Λ_3 fixed), and so scattering with external gravitons does not impose any additional constraints in the $M_{\text{Pl}} \rightarrow \infty$ decoupling limit. If one goes beyond this decoupling limit by including the subleading $\mathcal{O}(1/M_{\text{Pl}})$ corrections, the massless t -channel pole affects these bounds [52, 94, 95]. Our bounds (2.2) can nonetheless be consistently applied with the understanding that they are subject to small corrections of order $\mathcal{O}(\Lambda_3/M_{\text{Pl}})$, (such corrections are of the same order as the $\mathcal{O}(\Lambda_3^4/\Lambda_2^4)$ corrections in (1.3) which we have already neglected, see Appendix A for details).

Note that the above amplitudes have been derived on a flat background. We will assume that these bounds are still applicable on cosmological backgrounds. This amounts to assuming that $G_{2,X}$ remains positive for fluctuations about both backgrounds, where we implicitly invoke the fully covariant nature of (1.1) to suggest that such backgrounds can be smoothly connectable.

Positivity of scalar-matter scattering: Having considered positivity bounds from scalar-scalar scattering, we now move on to the matter sector, assuming a universal coupling to matter, $h_{\mu\nu}T^{\mu\nu}/2M_{\text{Pl}}$, where $T^{\mu\nu}$ is the stress-tensor for all matter fields. We focus on one such field, which we call χ , but emphasize that there is no implicit restriction on the nature of χ . In principle χ could be designating any light Standard Model field, including light or baryonic matter, as well as any other dark sector, or other particle that may exist in our Universe. Within the low-energy EFT, by “light”, we typically mean a particle with a mass below Λ_3 , which is typically close to $(M_{\text{Pl}}H_0^2)^{1/3} \approx 10^{-12}\text{eV}$ for dark energy, and this includes the photon. For the other massive particles of the Standard Model, the incoming

	c_B	c_M	c_T
no priors	$0.71^{+0.90}_{-0.71}$	$-0.02^{+1.32}_{-0.89}$	$-1^* \leq c_T < 0.25$
$\varphi\chi$ prior	$0.54^{+0.77}_{-0.62}$	$0.47^{+1.17}_{-0.88}$	$0^* \leq c_T < 0.78$
$\varphi\varphi$ prior	$0.26^{+0.46}_{-0.46}$	$0.67^{+0.97}_{-0.58}$	$0.46^{+0.64}_{-0.41}$
both priors	$0.26^{+0.46}_{-0.46}$	$0.67^{+0.97}_{-0.58}$	$0.46^{+0.64}_{-0.39}$

Table 1. Posteriors on the dark energy/modified gravity c_i parameters (3.5) for the quartic Horndeski theory (1.1) as displayed in Fig. 1, i.e. following from different combinations of positivity priors – see (3.3) and (3.4). Uncertainties shown denote the 95% confidence level. The distribution for c_T can be strongly skewed. We therefore do not give a mean value in such cases and denote limit values due to prior boundaries (when there is an excellent fit to the data on that boundary) with an asterisk. As expected from fig. 1, the constraints for the last two rows are near identical, with only a minimal modification in the lower bound for c_T .

energy s would necessarily be larger than Λ_3^2 in the $\varphi\chi \rightarrow \varphi\chi$ scattering process. Trusting the calculation of this amplitude for a field of mass larger than Λ_3 may require trusting the EFT beyond its regime of validity, so in all what follows we will have in mind a field χ lighter than Λ_3 , for instance the photon³. As shown in Appendix A, the specific spin of the field χ has no impact on our conclusions. In the presence of such a matter field χ , we can then derive additional positivity bounds from $\varphi\chi \rightarrow \varphi\chi$ scattering. Canonically normalizing the field χ , the amplitude takes the form (1.2) with,

$$c_{ss} = \bar{G}_{4,X} \quad (2.3)$$

and positivity requires $\bar{G}_{4,X} \geq 0$. This bound is qualitatively different from scattering without matter, and as we shall discuss below, requires in particular that the speed of tensors is always larger than that of χ whenever considering a profile that spontaneously breaks Lorentz invariance [50].

Speed of gravitational waves: We now consider cases in which the $\phi = 0$ Minkowski solution in the EFT (1.1) smoothly connects to a cosmological background, or in fact to any other background that spontaneously breaks Lorentz invariance (even if very softly). As soon as we investigate a background on which $\partial_\mu \bar{\phi}$ is no longer null, i.e. for which $\bar{X} \neq 0$, then the speed c_{GW} of gravitational waves on that background is given by,

$$\frac{c_{\text{GW}}^2}{c_m^2} = \left(1 - 2\bar{X} \frac{\bar{G}_{4,X}}{\bar{G}_4} \right)^{-\text{sign}\bar{X}}, \quad (2.4)$$

³Note however that the positivity bounds are not applied in the physical region but rather in the small s region. So while the massive particles of the Standard Model are considered as heavy field in terms of this EFT, they may still affect the bounds of the EFT coefficients at low-energy.

where we have included the speed c_m of light or of any other matter field, which in the frame we consider in (1.1) is minimally coupled to the metric $g_{\mu\nu}$ and is hence exactly luminal. Since \bar{G}_4 ought to be positive for that background to make sense (stable tensor modes), it follows that the corrections to the speed of gravitational waves as compared to that of light is always determined by the coefficient of $\bar{G}_{4,X}$ (independently of the sign of \bar{X}), which is precisely the same coefficient that is bounded by the scalar-matter positivity bound. It follows that the scalar-matter positivity bounds always impose the speed of gravitational waves to be larger than that of any other field χ minimally coupled to the metric $g_{\mu\nu}$ in (1.1).

In appendix A, we show how these results hold in more generic dark energy EFTs, including for instance the quintic Horndeski term, and is a general consequence of positivity applied to any disformal matter-coupling in the Einstein frame at leading order in derivatives.

3 Comparison with observational constraints

Equipped with the positivity bounds of the previous section, we are now in a position to use them as theoretical priors and to investigate the impact of such priors on cosmological constraint analyses. We will closely follow the approach of [71] here, with a focus on integrating the novel positivity priors discussed above.

Linear cosmology: The dynamics of linear perturbations around cosmological backgrounds (following the approach of Refs. [14, 20], we will assume this to be a Λ CDM background) for Horndeski theories is controlled by four background functions (in addition to the Hubble scale, which controls the background expansion itself): the so-called α_i [4]. These are the running of the effective Planck mass α_M , the kineticity α_K that contributes to the kinetic energy of scalar perturbations (effectively unconstrained by linear cosmological perturbations [14, 20], so we will omit it from the subsequent analysis), the braiding α_B that quantifies the strength of kinetic mixing between scalar and tensor perturbations, and the tensor speed excess α_T , which is related to the speed of sound of tensor perturbations c_{GW} via $c_{\text{GW}}^2 = 1 + \alpha_T$. In terms of the model functions G_i , and for our specific example (1.1), these are given by [71],

$$\alpha_M = -\frac{2\dot{X}}{HM^2}(G_{4,X} + 2XG_{4,XX}), \quad \alpha_B = \frac{8X}{M^2}(G_{4,X} + 2XG_{4,XX}), \quad \alpha_T = \frac{4X}{M^2}G_{4,X}, \quad (3.1)$$

where $M^2 = 2(G_4 - 2XG_{4,X})$. Here it is useful to re-arrange two of the above expressions and instead write them as

$$\alpha_B = 2\alpha_T + 16\frac{X^2}{M^2}G_{4,XX}, \quad \alpha_M = -\frac{1}{4}\frac{\dot{X}}{HX}\alpha_B. \quad (3.2)$$

Note that all functions in (3.1) and (3.2) are un-barred, since we are working on a cosmological background here (recall the bar denoted evaluation on a flat Minkowski background). Having derived the positivity bounds (2.2) and (2.3) on the derivatives of $G_4(X)$, we would now like to re-cast them in a form more directly applicable to cosmological constraint analyses. Specifically, this involves relating the above bounds to the α_i used in this context.

$\varphi\varphi$ scattering prior: We can now translate the positivity bounds (2.2) and (2.3) into priors on the α_i . For the bounds derived from (2.2) this is discussed in detail in [71] – here we quickly summarise the key outcomes relevant to this section. The c_{ss} bound on $\bar{G}_{2,XX}$ is not particularly constraining at this level, since none of the α_i in (3.1) depend on G_2 (only α_K depends on this and, as mentioned above, α_K is essentially unconstrained by observational constraints on linearly perturbed cosmologies). However, the c_{sst} bound is highly constraining, since in an expanding universe it demands,

$$\varphi\varphi \text{ prior : } \quad \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq -\bar{G}_{4,XX} \quad \Rightarrow \quad \alpha_B \leq \frac{2\alpha_T}{1 + \alpha_T}, \quad (3.3)$$

where we have used (3.2) as well as the fact that $M^2 = 2G_4/(1 + \alpha_T)$. Note that this expression for M^2 holds for the specific model under consideration here, not in general.

$\varphi\chi$ scattering prior: The bound from $2 \rightarrow 2$ matter-scalar scattering additionally imposes

$$\varphi\chi \text{ prior : } \quad \bar{G}_{4,X} \geq 0 \quad \Rightarrow \quad \alpha_T \geq 0, \quad (3.4)$$

where χ is an arbitrary matter field that may live in an entirely separate sector, as discussed above. Phrased in a different way, this bound imposes that the speed of gravitational waves in the EFT (1.1) is equal to or larger than the speed of light. Importantly, note that this bound is therefore orthogonal to some of the subluminality priors that have been considered previously – see e.g. the seminal work of [99] and the more recent [71]. Again we emphasise that both positivity bounds used here were derived on a Lorentz-invariant Minkowski background ($\phi = 0$) – hence the bars on the left of (3.3) and (3.4) – while we ultimately rephrase these as bounds on the α_i , i.e. as bounds on functions of the cosmological background. As discussed above, we therefore assume that one can port constraints from one background to the other, invoking the covariant nature of (1.1) in the process.

Cosmological parameter constraints: We can now apply the theoretical priors (3.3) and (3.4) to cosmological parameter constraints. Doing so in the presence of free functions, such as $G_4(X)$ in (1.1), requires choosing a parametrisation for the freedom encoded in these functions. Instead of choosing a particular ansatz for these functions in the Lagrangian (e.g. a truncated expansion in powers of X), we here follow the approach employed by several current Einstein-Boltzmann solvers (see e.g. [15, 19]) and parameterise at the level of the α_i . Numerous such parameterisations exist – see [13] and references therein for how these affect constraints and for discussions of their relative merits – but here we will pick arguably the one most frequently used [4] for illustration

$$\alpha_i = c_i \Omega_{\text{DE}}. \quad (3.5)$$

This parameterises each α_i in terms of just one constant parameter, c_i , and is known to accurately capture the evolution of a wide sub-class of Horndeski theories [100, 101]. Note that, when imposing priors, we will require them to be satisfied at all times, i.e. dynamically

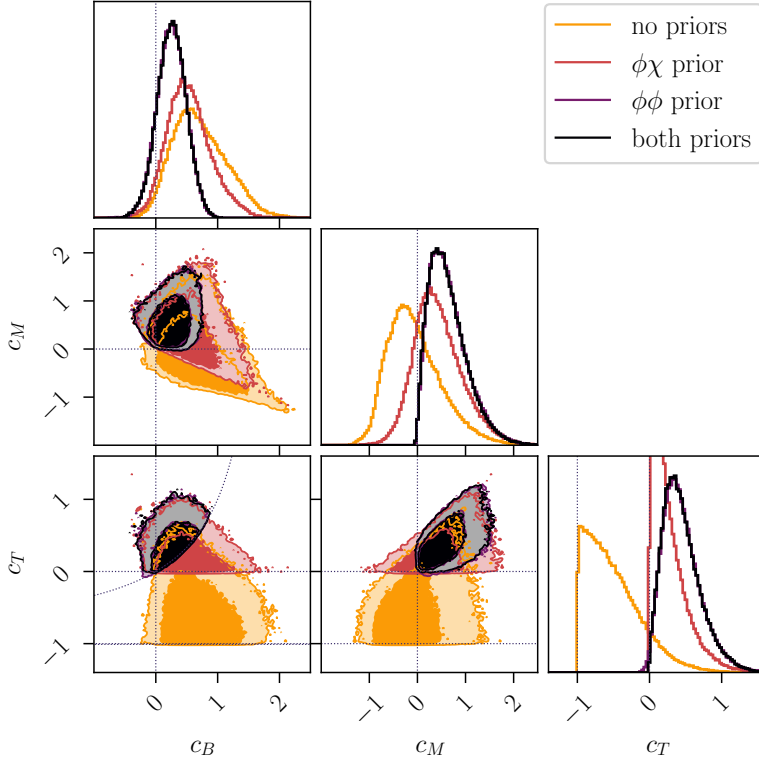


Figure 1. Cosmological parameter constraints for the EFT of dark energy considered in (1.1), using different combinations of positivity priors – see (3.3) and (3.4). The priors are derived from $\varphi\varphi \rightarrow \varphi\varphi$ and $\varphi\chi \rightarrow \varphi\chi$ scattering, where χ denotes matter fields and φ the fluctuation of the dark energy field. Contours mark 68% and 95% confidence intervals, computed using CMB, RSD, BAO and matter power spectrum measurements, and we use $\alpha_i = c_i\Omega_{\text{DE}}$ (3.5). Dotted lines mark $c_i = 0$ (the GR value), $c_T \geq -1$ (real GW speed) and $c_B < 2c_T/(1 + c_T)$ (late-time bound from $\varphi\varphi$ scattering). Note that the $\varphi\varphi$ prior constraint is only very marginally enhanced by adding the $\varphi\chi$ prior, see e.g. the $c_T - c_B$ plot for negative c_T . Note that the speed of gravitational waves is given by $c_{\text{GW}}^2 = 1 + c_T\Omega_{\text{DE}}$.

throughout the evolution until today *as well as* at late times, when $\Omega_{\text{DE}} \rightarrow 1$ on our ΛCDM -like background. In the context of (3.5), this late time limit yields the strongest bounds on the c_i , given the above priors on the α_i .

We now compute constraints on the α_i by performing a Markov chain Monte Carlo (MCMC) analysis, using Planck 2015 CMB temperature, CMB lensing and low- ℓ polarisation data [102–104], baryon acoustic oscillation (BAO) measurements from SDSS/BOSS [105, 106], constraints from the SDSS DR4 LRG matter power spectrum shape [107] and redshift space distortion (RSD) constraints from BOSS and 6dF [108, 109]. Note that the constraints presented here are strongly driven by Planck and RSD data, whereas the other

data sets do not add significant extra constraining power in our context – see [13] for details regarding the implementation of the relevant likelihoods and related theoretical and observational modelling details. Using the parametrisation (3.5), we can therefore compute constraints on the modified gravity/dark energy parameters c_B, c_M and c_T , marginalising over the standard Λ CDM parameters $\Omega_{\text{cdm}}, \Omega_b, \theta_s, A_s, n_s$ and τ_{reio} . The results are shown in Fig. 1 and in Table 1.

Focusing on the results as presented in Fig. 1, we recover the result of [71] that applying the $\varphi\varphi \rightarrow \varphi\varphi$ positivity prior significantly reduces the 2σ volume in parameter space. It effectively rules out $c_B > 1$ and $c_M < 0$, i.e. the running of the Planck mass is always zero or positive here and the amount of ‘braiding’ present is limited. Key for our discussion is that the $\varphi\varphi \rightarrow \varphi\varphi$ positivity prior also already introduces a very strong preference for $c_T > 0$ (gravitational waves propagating faster than light). A very small region of parameter space with $c_T < 0$ and $c_B < 0$ is still a good fit to the data, but is dwarfed in volume by the other regions within the 2σ volume. As such, the added inclusion of the $\varphi\chi \rightarrow \varphi\chi$ positivity prior from scalar-matter scattering only has a very small effect on the combined constraints. Since this prior effectively rules out $c_T < 0$, it only modifies the previous results by ruling out the aforementioned very small (previously remaining) region of parameter space with negative c_T . We present results for the marginalised 1D distributions of the c_i parameters in Table 1, where we accordingly observe nearly identical such posteriors when both priors are applied and when only the $\varphi\varphi \rightarrow \varphi\varphi$ prior is applied, with only a very small noticeable difference between the two cases for the c_T distribution. Crucially, however, the scalar-matter prior makes it clear that (assuming a standard UV completion) applying a different subluminality prior on the tensor modes here is not consistent with positivity bounds at large. Taken together the two positivity priors are therefore remarkably consistent here, with the $\varphi\varphi \rightarrow \varphi\varphi$ clearly the stronger bound in the present context.

It is instructive to quantify the relative strength of the (different subsets of) positivity-induced bounds more precisely. We will do so by using $\Delta \equiv \Delta c_B \Delta c_M \Delta c_T$ as a rough measure of the ‘volume’ in parameter space allowed by a given set of constraints, where Δc_i denotes the 95% confidence interval for the posterior of that c_i – for example, with no priors we have $\Delta c_B = 0.90 + 0.71 = 1.61$. This simple measure is of course not unique, but it will be perfectly sufficient to roughly quantify and compare the relative strength of different combinations of constraints. From table 1 we can then see that the viable parameter space volume Δ shrinks by close to 70% when comparing constraints without any positivity-induced priors vs. constraints where these priors are applied. In addition, there is a noticeable shift towards larger (positive) c_M , smaller c_B and larger (positive) c_T . That reduction in parameter space volume is near-identical with the one achieved when only applying the $\varphi\varphi \rightarrow \varphi\varphi$ positivity prior and is to be compared with a roughly 50% reduction in parameter space volume Δ , when only applying the $\varphi\chi \rightarrow \varphi\chi$ positivity prior. Importantly, the prior from matter-scalar scattering therefore has a significant effect by itself, e.g. preferring larger c_M values than without positivity priors (in addition to the more obvious effect on c_T). So, in its own right, it remains a powerful prior to use for computing cosmological data constraints. We therefore

caution against ignoring this second prior, because it effectively being pre-empted by the $\varphi\varphi \rightarrow \varphi\varphi$ prior here may be a consequence of the specific example model we have chosen. This is especially important, since we expect the scalar-matter prior to generically be linked to α_T (see the related discussion in the appendix), while the nature of the scalar-scalar prior is likely to change more model-dependently. For a related discussion see [71].

Finally, we should point out a number of ways in which the present analysis can be strengthened and extended going forward. First, note that there can be an interesting interplay between the choice of parametrisation for the α_i and the constraining power of theoretical priors in the present context [110]. Increasingly physically well-motivated and theoretically constrained parameterisations for the α_i should remedy (some of) this modelling uncertainty in the future. In addition, the present analysis can be improved by incorporating further observations and constraints. Examples include adding weak lensing constraints to the analysis along the lines described in [29] or incorporating additional constraints from solar system scales, e.g. recent bounds on α_M using lunar laser ranging [111].

Comparison with priors relying solely on stability and subluminality: To highlight the strength of positivity priors when putting constraints on cosmological parameters, it is instructive to compare with what one would have inferred, had we not used any knowledge from the implementation of our low-energy EFT within its high-energy completion, and had we instead based our priors solely on the stability of the low-energy EFT and subluminality of the dark energy scalar field as well as subluminality of tensor modes.

• **Stability** of the low-energy EFT requires absence of ghosts and gradients instabilities for both scalar and tensor modes – see [112] for general expressions for these requirements in Horndeski scalar-tensor theories. In terms of the α_i , these conditions amounts to [4, 19]

$$\begin{aligned} \text{scalar modes :} \quad & \text{no ghost : } \mathcal{D} \equiv \alpha_K + \frac{3}{2}\alpha_B^2 > 0, & \text{grad. stab. : } c_s^2 \geq 0, \\ \text{tensor modes :} \quad & \text{no ghost : } M^2 > 0, & \text{grad. stab. : } \alpha_T \geq -1, \end{aligned} \quad (3.6)$$

where

$$\mathcal{D}c_s^2 = (2 - \alpha_B) \left(\frac{1}{2}\alpha_B(1 + \alpha_T) + \alpha_M - \alpha_T - \frac{\dot{H}}{H^2} \right) + \frac{\dot{\alpha}_B}{H} - \frac{3(\rho_{\text{tot}} + p_{\text{tot}})}{H^2 M^2}. \quad (3.7)$$

Here ρ_{tot} and p_{tot} are the total energy density and pressure in the universe. Ghost and gradient stability priors are uniformly applied for all the constraints we are showing throughout this paper⁴. We note in passing that, while the positivity bound on $G_{4,X}$ requires gravitational waves to travel faster than matter in this model, the scalar fluctuations may travel faster or slower than matter.

⁴As an aside, note that these priors are effectively imposed automatically for the setup we are considering, which importantly includes the ansatz (3.5). This is because the data here exclude almost all regions, where such instabilities are present, by themselves, regardless of whether ghost- and gradient-stability priors are explicitly imposed or not [13, 69].

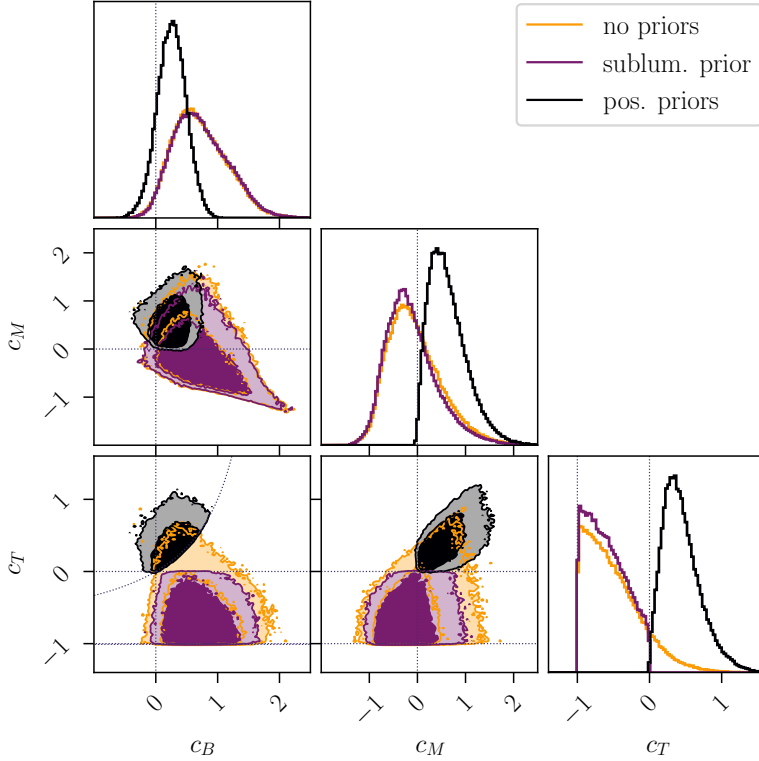


Figure 2. Comparison of constraints on the cosmological parameters for the EFT (1.1), where $\alpha_i = c_i \Omega_{\text{DE}}$ and we are using either priors inferred from positivity bounds (related to causality, locality and unitarity) or a subluminality prior (which is *not* related to causality here). Note that, as we are comparing other priors here, gradient and ghost stability priors are applied uniformly for all cases (including 'no priors'). We emphasise that the subluminality prior is incompatible with the positivity bounds shown here. So requiring subluminality (with or without explicitly requiring further stability bounds) leads to drastically different conclusions compared with requiring positivity bounds to be satisfied which include in themselves causality requirements. Note that the speed of gravitational waves is given by $c_{\text{GW}}^2 = 1 + c_T \Omega_{\text{DE}}$.

Note that the above ghost and gradient stability criteria are derived by considering scalar and tensor modes propagating on an FLRW background. One may of course complement this by specifying other backgrounds/vacua the EFT in question should also describe. Here we are implicitly requiring this to be the case for the flat Minkowski vacuum, given that we are porting positivity bounds from there. Another example of interest is demanding the stability of propagating scalar modes on backgrounds sourced by massive binary systems [113] – the resulting theoretical priors can then have a strong effect on cosmological parameter constraints as well [34]. We leave a further exploration of these issues for future work, but stress that the interplay between more exhaustive (future) sets of stability and positivity priors is likely

to allow us to extract increasingly tight constraints.

• **Subluminality vs Causality.** It is worth emphasizing from the outset that the notion of subluminality in this gravitational EFT is not directly linked with that of causality and may sometimes be orthogonal to it. Indeed the notion of causality is intrinsically linked to the speed of propagation of information which is related to the front velocity, i.e. the high (or even infinite) frequency limit of the phase velocity [114–116]. Implementing conditions related to causality therefore requires some knowledge of the high-energy behaviour (by definition beyond the regime of validity of the EFT) which is precisely what is encoded in the positivity bounds. Within a low-energy EFT, the notion of causality is much more subtle to identify [50, 51, 117]. In the EFT (1.1), from (3.7), subluminality of the scalar mode can always be achieved by requiring a sufficiently large kineticity α_K (and hence \mathcal{D}). Since (as discussed above) α_K is effectively unconstrained by observations here, this means a subluminality prior for the speed of scalar perturbations does not lead to any significant constraints in our context⁵. Requiring (sub)luminality of the tensor modes, on the other hand, would impose $\alpha_T \leq 0$. Taken together with the gradient stability prior for tensor modes, at late times when $\Omega_{\text{DE}} \rightarrow 1$, this means the prior boundaries would effectively become $-1 \leq c_T \leq 0$.

Given the exact same cosmological data as used previously, a (sub)luminality prior would therefore lead to rather different conclusions than the priors from positivity we discussed above. This is illustrated in Fig. 2, where constraints using our positivity priors (that include the requirement of causality) are compared with the constraints one would have inferred had we instead focused on (sub)luminality priors (which in this case do not go hand in hand with causality). We again emphasise that ghost and gradient stability priors are uniformly implemented for all cases, so differences between the cases shown are solely due to the other priors as discussed above. This illustrates the importance of carefully choosing priors based on theoretical consistency when constraining EFT parameters from data.

4 Discussion

In this paper, we have considered the effects of incorporating a novel positivity bound into the analysis of cosmological scalar-tensor theories. Specifically, this results from including bounds derived from interactions with any other matter fields (which are known to interact at the very least gravitationally). In order to illustrate the effect of additional positivity priors derived in the presence of any matter fields, we extended the analysis of [71], considering a specific (quartic and shift-symmetric) dark energy EFT and computing the corresponding positivity priors derived from scalar-scalar and scalar-matter scattering. While the qualitatively new prior from matter-scalar scattering does significantly tighten constraints in comparison with those derived using observational data only, we also find that for the specific theory considered here, this new prior is effectively pre-empted by the known one from scalar-scalar scattering.

⁵It does place one rather weak constraint. Namely, a lower bound on α_K , which from (3.7) itself depends on the other α_i .

Phrased differently, this means the two priors are highly consistent with one another here. We emphasise, however, that both priors in principle constrain the parameter space in completely different ways – the matter-scalar bound requires that the speed of tensor modes is strictly greater than the speed of any matter field (including photons) – so should both be taken into account when going beyond the example theory used here in the future.

With an eye on such future surveys, it will be very interesting to see whether additional positivity bounds on dark energy (continue to) display a high degree of consistency, both for the example model considered here and for more generic EFTs. At the moment, the high level of consistency between the two sets of positivity bounds appears to be suggestive that once priors from a causal and unitary high energy completion are accounted for, data naturally folds itself in a region of parameter space that then remains highly consistent with other such considerations. However, it is possible that in the future, additional positivity bounds could restrict the parameter space in highly orthogonal ways, which would then provide even more powerful constraints, effectively ruling out large classes of dark energy candidates.

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A Scattering amplitudes

In this Appendix we review the amplitudes which lead to the constraints (2.2) and (2.3) comparing the positivity requirement with relative speed of tensors to matter fields, c_{GW}^2/c_m^2 . We start by expanding the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}/M_{\text{Pl}}$ and the scalar field $\phi = \bar{\phi} + \varphi$ in (1.1), and use an overbar \bar{G}_n to denote functions evaluated on the flat background. χ is used throughout to denote matter fields.

A.1 Scattering on a flat background

We first consider the amplitudes about a flat background and without loss of generality, we set $\bar{G}_{2,X} = 1/2$ to canonically normalise the φ propagator. The most important point to notice is that for the theory considered in (1.1), the coupling between h and φ to leading

order in the vertex $h\varphi\varphi$ has the precise same structure as the graviton kinetic's term, so that whenever the emission of two φ 's is mediated by $h_{\mu\nu}$ there is no pole in the amplitude (up to the $\mathcal{O}(1/M_{\text{Pl}})$ at which we are working). There is therefore no t -channel pole to affect the applicability of our bounds to this order.

• $\varphi\varphi \rightarrow \varphi\varphi$ **scattering**

The leading contributions to the $\varphi\varphi \rightarrow \varphi\varphi$ amplitude are given by

where dashed lines indicate the scalar field φ while wiggly lines indicate the graviton propagator. To leading order, the resulting amplitude to the $\varphi\varphi \rightarrow \varphi\varphi$ process is therefore,

$$\mathcal{A}^{\varphi\varphi \rightarrow \varphi\varphi} = 4! \frac{\bar{G}_{4,XX} + \bar{G}_{4,X}^2/\bar{G}_4}{\Lambda_3^6} \frac{stu}{4} + \frac{\bar{G}_{2,XX}}{\Lambda_2^4} (s^2 + t^2 + u^2) - \frac{\bar{G}_{4,X}}{\bar{G}_4 \Lambda_2^4} (su + st + ut), \quad (\text{A.1})$$

which at large s takes the form

$$\mathcal{A}(s, t) = c_{ss} \frac{s^2}{\Lambda_2^4} + c_{sst} \frac{s^2 t}{\Lambda_3^6} + \dots, \quad (\text{A.2})$$

with,

$$c_{sst} = -6 (\bar{G}_{4,XX} + \bar{G}_{4,X}^2/\bar{G}_4), \quad c_{ss} = 2\bar{G}_{2,XX} + \bar{G}_{4,X}/\bar{G}_4. \quad (\text{A.3})$$

This gives the two positivity bounds studied in [71],

$$\bar{G}_{2,XX} \geq -\bar{G}_{4,X} \frac{\bar{G}_{2,X}}{\bar{G}_4}, \quad \bar{G}_{4,XX} + \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq 0. \quad (\text{A.4})$$

• $\varphi h \rightarrow \varphi h$ **scattering**

For the $\varphi h \rightarrow \varphi h$ amplitude, the only non-zero diagrams are,

which exactly cancel, so $\mathcal{A}_{\varphi h \rightarrow \varphi h} = 0$ at this order.

• $\varphi\chi \rightarrow \varphi\chi$ scattering

Including matter fields, χ , now gives an additional amplitude for $\varphi\chi \rightarrow \varphi\chi$. In the frame in which (1.1) is defined, there is no direct coupling between any of the matter fields and the dark energy field φ , but a scattering process is mediated by gravitational exchange,

$$T^{\mu\nu} \otimes \begin{array}{c} \nearrow p_\mu \\ \text{---} h_{\mu\nu} \text{---} \\ \searrow \end{array} \varphi = \frac{2\bar{G}_{4,X}}{G_4 M_{\text{Pl}} \Lambda_3^3} T^{\mu\nu} p_\mu p_\nu$$

Again we see the graviton propagator cancelling against the h derivatives in the vertex factor, leaving only an effectively contact coupling between the current $T^{\mu\nu}$ of χ and the momentum p_μ of one of the φ 's (by momentum conservation, the momentum of the other φ can be written as $-p_\mu - p_\mu^{(T)}$, but $p_\mu^{(T)} T^{\mu\nu} = 0$ since $T^{\mu\nu}$ is conserved). For example for a light scalar matter field (with canonical kinetic term and neglecting self-interactions),

$$T^{\mu\nu} = \nabla^\mu \chi \nabla^\nu \chi - \frac{1}{2} g^{\mu\nu} \nabla^\alpha \chi \nabla_\alpha \chi , \quad (\text{A.5})$$

this gives a single (t -channel) diagram,

$$\begin{array}{c} \chi(p_1) \\ \chi(p_3) \end{array} \begin{array}{c} \nearrow \\ \text{---} h_{\mu\nu} \text{---} \\ \searrow \end{array} \begin{array}{c} \varphi(p_2) \\ \varphi(p_4) \end{array} = \frac{\bar{G}_{4,X}}{2\Lambda_2^4} (s^2 + u^2 - t^2)$$

and so the amplitude $\mathcal{A}_{\varphi\chi \rightarrow \varphi\chi}$ takes the form (A.2) with,

$$c_{ss} = \bar{G}_{4,X} . \quad (\text{A.6})$$

This gives a new positivity bound on the function G_4 (expanded about a flat background),

$$\bar{G}_{4,X} \geq 0 , \quad (\text{A.7})$$

which is qualitatively different to the previous bounds found from $\varphi\varphi$ (and φh) elastic scattering without matter fields.

Note that one obtains the same bound for vector matter fields, χ_μ , since,

$$T^{\mu\nu} = F^{\mu\alpha} F^\nu{}_\alpha - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} , \quad (\text{A.8})$$

where $F_{\mu\nu} = \partial_{[\mu} \chi_{\nu]}$ (we assume that χ is Abelian and minimally coupled to gravity) gives forward-limit helicity amplitudes,

$$\mathcal{A}_{0\pm \rightarrow 0\pm}^{\varphi\chi \rightarrow \varphi\chi} = + \frac{4\bar{G}_{4,X}}{G_4 \Lambda_2^4} s^2 . \quad (\text{A.9})$$

When both a scalar and vector (ingredients of the Standard Model) couple minimally to the metric indicated in (1.1) then positivity requires (A.7)⁶.

Sound speeds on a cosmological background

On a non-trivial background, the leading contributions to the second derivatives of the metric perturbations in de Donder gauge and focusing on the tensor modes, the relevant contributions are of the form

$$\mathcal{L}_{hh}^2 \sim \bar{G}_4 h^{\mu\nu} \square h_{\mu\nu} + \frac{\Lambda_2^4 \bar{G}_{4,X}}{\Lambda_3^6 M_{\text{Pl}}^2} \partial^\alpha \bar{\phi} \partial^\beta \bar{\phi} h^{\mu\nu} \partial_\alpha \partial_\beta h_{\mu\nu} + \dots \quad (\text{A.10})$$

Diagonalizing the metric at a point, then for a time-like background where $\partial \bar{\phi} \sim \delta_\mu^0$ so that $\bar{X} > 0$, the speed of the tensor modes is given by

$$\frac{c_{\text{GW}}^2}{c_m^2} = \frac{\bar{G}_4}{\bar{G}_4 - 2\bar{X}\bar{G}_{4,X}}, \quad (\text{A.11})$$

where we have included the speed c_m of matter fields minimally coupled to the metric $g_{\mu\nu}$. In this frame, $c_m = 1$. We conclude that the speed of gravitational waves is larger than the speed c_m of matter fields minimally coupled to the metric $g_{\mu\nu}$ if $\bar{G}_{4,X}$ is positive. Had we instead consider a space-like background where $\partial \bar{\phi} \sim \delta_i^0$ so that $\bar{X} < 0$ (assuming again we diagonalized the metric at that point), then the speed of the tensor modes is given by

$$\frac{c_{\text{GW}}^2}{c_m^2} = \frac{\bar{G}_4 - 2\bar{X}\bar{G}_{4,X}}{\bar{G}_4}, \quad (\text{A.12})$$

which is again superluminal if $\bar{G}_{4,X}$ is positive.

More generally, had we included the quintic Horndeski term,

$$\mathcal{L}_5 = M_{\text{Pl}}^2 G_5 G_{\mu\nu} \Phi^{\mu\nu} - \frac{1}{6} \Lambda_2^4 G_{5,X} ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]), \quad (\text{A.13})$$

the tensor sound speed on a cosmological background would then have been,

$$c_{\text{GW}}^2 = 1 + \frac{2\bar{X}}{M^2} (2\bar{G}_{4,X} - \bar{G}_{5,\phi} + (H\partial_t \bar{\phi} - \partial_t^2 \bar{\phi})\bar{G}_{5,X}), \quad (\text{A.14})$$

where now $M^2 = 2\bar{G}_4 + \bar{X}(-4\bar{G}_{4,X} + \bar{G}_{5,\phi} - 2H\partial_t \bar{\phi}\bar{G}_{5,X})$ is the effective Planck mass (assumed positive). On the other hand, $\varphi\chi$ scattering on a flat background is not sensitive to H , with an amplitude given by,

$$c_{ss} = \bar{G}_{4,X} - \frac{1}{2}\bar{G}_{5,\phi} \quad (\text{A.15})$$

and so positivity again requires $c_{\text{GW}}^2 \geq c_m^2$, where c_m is the speed of matter fields minimally coupled to the metric $g_{\mu\nu}$, up to $\mathcal{O}(H)$ corrections for Horndeski-type theories.

⁶Coupling higher spin fields to gravity is difficult (e.g. when the spin is $\geq 5/2$, the gauge symmetry required to decouple longitudinal modes is deformed non-trivially by the curvature [124–126]), and for half-integer spin requires a spin connection—but the speeds of spin-0 and spin-1 fields are constrained to be $\leq c_T$. What matters for us is that, as soon as there is a single spin-0 or spin-1 matter field in the universe coupled to gravity like $+h_{\mu\nu}T^{\mu\nu}/2M_{\text{Pl}}$, then $\bar{G}_{4,X}$ must be positive.

A.2 Einstein frame

It is instructive to show how the same positivity constraints arise in the Einstein frame. This is both a useful consistency check (in particular it removes the need to ever discussing about the graviton pole), and it also makes it clear why it is the $\varphi\chi$ amplitude which is responsible for the previous sound speed relation (since in the Einstein frame $c_{\text{GW}}^2 = 1$ the effect of the $\bar{\phi}$ background is to lower c_m , which is precisely the statement encoded in the $\varphi\chi$ amplitude).

The equations of motion for ϕ and for $h_{\mu\nu}$ from $S[h_{\mu\nu}, \phi] + S_m[h_{\mu\nu}, \chi]$ can be written as,

$$2\bar{G}_{2,X}\phi_\mu^\mu - 4\frac{g_4}{\Lambda_3^6}\phi_\mu^{[\mu}\phi_\alpha^\alpha\phi_\rho^{\rho]} = +\frac{2\bar{G}_{4,X}}{\bar{G}_4\Lambda_2^4}T^\mu{}_\nu\phi_\mu^\nu \quad (\text{A.16})$$

$$\delta_{\nu\beta\sigma}^{\mu\alpha\rho}\nabla_\alpha\nabla^\beta\left[h_\rho{}^\sigma - \frac{2\bar{G}_{4,X}}{\bar{G}_4\Lambda_3^3}\phi_\rho\phi^\sigma\right] = -\frac{1}{\bar{G}_4M_{\text{Pl}}}\mathcal{T}_\nu^\mu \quad (\text{A.17})$$

where $g_4 = \bar{G}_{4,XX} + \bar{G}_{4,X}^2/\bar{G}_4$. This shows that the mixing between ϕ and $h_{\mu\nu}$ can be removed at this order by redefining the metric fluctuation,

$$h_{\rho\sigma} = h_{\rho\sigma}^{(E)} + \frac{2\bar{G}_{4,X}}{\bar{G}_4\Lambda_3^3}\varphi_\rho\varphi_\sigma, \quad (\text{A.18})$$

and thus $2\bar{G}_{4,X}/\bar{G}_4$ plays the role of a disformal coupling in the Einstein frame. In particular, from the canonical $h_{\mu\nu}T^{\mu\nu}$ coupling in the Jordan frame, the matter is now coupled to,

$$\frac{1}{2M_{\text{Pl}}}\mathcal{T}^{\mu\nu} = \frac{1}{2M_{\text{Pl}}}h_{\mu\nu}^{(E)}\mathcal{T}^{\mu\nu} + \frac{\bar{G}_{4,X}}{\bar{G}_4\Lambda_2^4}\varphi_\mu\varphi_\nu\mathcal{T}^{\mu\nu}, \quad (\text{A.19})$$

in the Einstein frame.

Scattering on a flat background

In this frame, $h_{\mu\nu}^{(E)}$ does not couple directly to φ at this order, but there is instead a contact interaction $T^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$ which is responsible for the $\varphi\chi$ scattering,

$$\begin{array}{c} p_\mu \\ \nearrow \\ T^{\mu\nu} \otimes \varphi \\ \searrow \\ \varphi \end{array} = \frac{2\bar{G}_{4,X}}{\bar{G}_4\Lambda_2^4}T^{\mu\nu}p_\mu p_\nu.$$

For instance, for the scalar matter field (A.5), there is again a single diagram,

$$\begin{array}{c} \chi(p_1) \quad \varphi(p_2) \\ \diagdown \quad \diagup \\ \chi(p_3) \quad \varphi(p_4) \end{array} = \frac{G_{4,X}}{2\Lambda_2^4}(s^2 + u^2 - t^2),$$

which reproduces the amplitude (A.6) and positivity bound (A.7) (as obviously expected, since a field redefinition like (A.18) leaves S -matrix elements unchanged).

Sound speeds on a cosmological background

In the Einstein frame, the equation of motion for $h_{\mu\nu}^{(E)}$ is unaffected by the background of $\bar{\phi}$, and tensor modes therefore propagate luminally just as in GR, $c_{\text{GW}} = 1$. However, matter now couples to an effective metric, which on this background reads,

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{(E)} + \frac{2\bar{G}_{4,X}}{\bar{G}_4\Lambda_2^4} \partial_\mu \bar{\phi} \partial_\nu \bar{\phi}, \quad (\text{A.20})$$

and consequently the sound speed of all matter fields is now shifted by $\bar{\phi}$ relative to c_{GW} , by an amount

$$\frac{c_m^2}{c_{\text{GW}}^2} = 1 - \frac{2\bar{X}\bar{G}_{4,X}}{\bar{G}_4}. \quad (\text{A.21})$$

where $\bar{X} = (\partial_t \bar{\phi})^2 / \Lambda_2^4$ on a time-like background.

In fact, for any constant disformal coupling to matter in the Einstein frame,

$$g_{\mu\nu} = g_{\mu\nu}^{(E)} + \frac{\partial_\mu \bar{\phi} \partial_\nu \bar{\phi}}{\mathcal{M}^4} \quad (\text{A.22})$$

positivity demands that $\mathcal{M}^4 \geq 0$, and therefore the speed of matter relative to the speed of $\delta g_{\mu\nu}^{(E)}$ fluctuations,

$$\frac{c_m^2}{c_{\text{GW}}^2} = 1 - \frac{\bar{X}}{\mathcal{M}^4} \quad (\text{A.23})$$

is constrained to be less than one on time-like backgrounds for $\bar{\phi}$ (for which $\bar{X} \geq 0$). Similarly as in the previous argument, when dealing with a space-like background, the relation (A.21) is inverted and we still recover the same outcome $c_m^2 \leq c_{\text{GW}}^2$.

References

- [1] G. Gubitosi, F. Piazza and F. Vernizzi, *The Effective Field Theory of Dark Energy*, *JCAP* **1302** (2013) 032, [[1210.0201](#)].
- [2] J. K. Bloomfield, E. E. Flanagan, M. Park and S. Watson, *Dark energy or modified gravity? An effective field theory approach*, *JCAP* **1308** (2013) 010, [[1211.7054](#)].
- [3] J. Gleyzes, D. Langlois and F. Vernizzi, *A unifying description of dark energy*, *Int. J. Mod. Phys. D* **23** (2015) 1443010, [[1411.3712](#)].
- [4] E. Bellini and I. Sawicki, *Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity*, *JCAP* **1407** (2014) 050, [[1404.3713](#)].
- [5] J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, *Essential Building Blocks of Dark Energy*, *JCAP* **08** (2013) 025, [[1304.4840](#)].
- [6] R. Kase and S. Tsujikawa, *Effective field theory approach to modified gravity including Horndeski theory and Horava-Lifshitz gravity*, *Int. J. Mod. Phys. D* **23** (2015) 1443008, [[1409.1984](#)].

- [7] A. De Felice, K. Koyama and S. Tsujikawa, *Observational signatures of the theories beyond Horndeski*, *JCAP* **1505** (2015) 058, [[1503.06539](#)].
- [8] D. Langlois, M. Mancarella, K. Noui and F. Vernizzi, *Effective Description of Higher-Order Scalar-Tensor Theories*, *JCAP* **1705** (2017) 033, [[1703.03797](#)].
- [9] N. Frusciante and L. Perenon, *Effective field theory of dark energy: A review*, *Phys. Rept.* **857** (2020) 1–63, [[1907.03150](#)].
- [10] C. Renevey, J. Kennedy and L. Lombriser, *Parameterised post-Newtonian formalism for the effective field theory of dark energy via screened reconstructed Horndeski theories*, *JCAP* **12** (2020) 032, [[2006.09910](#)].
- [11] M. Lagos, T. Baker, P. G. Ferreira and J. Noller, *A general theory of linear cosmological perturbations: scalar-tensor and vector-tensor theories*, *JCAP* **1608** (2016) 007, [[1604.01396](#)].
- [12] M. Lagos, E. Bellini, J. Noller, P. G. Ferreira and T. Baker, *A general theory of linear cosmological perturbations: stability conditions, the quasistatic limit and dynamics*, *JCAP* **1803** (2018) 021, [[1711.09893](#)].
- [13] J. Noller and A. Nicola, *Cosmological parameter constraints for Horndeski scalar-tensor gravity*, *Phys. Rev. D* **99** (2019) 103502, [[1811.12928](#)].
- [14] E. Bellini, A. J. Cuesta, R. Jimenez and L. Verde, *Constraints on deviations from Λ CDM within Horndeski gravity*, *JCAP* **2** (Feb., 2016) 053, [[1509.07816](#)].
- [15] B. Hu, M. Raveri, N. Frusciante and A. Silvestri, *Effective Field Theory of Cosmic Acceleration: an implementation in CAMB*, *Phys. Rev. D* **89** (2014) 103530, [[1312.5742](#)].
- [16] M. Raveri, B. Hu, N. Frusciante and A. Silvestri, *Effective Field Theory of Cosmic Acceleration: constraining dark energy with CMB data*, *Phys. Rev. D* **90** (2014) 043513, [[1405.1022](#)].
- [17] J. Gleyzes, D. Langlois, M. Mancarella and F. Vernizzi, *Effective Theory of Dark Energy at Redshift Survey Scales*, *JCAP* **1602** (2016) 056, [[1509.02191](#)].
- [18] C. D. Kreisch and E. Komatsu, *Cosmological Constraints on Horndeski Gravity in Light of GW170817*, [1712.02710](#).
- [19] M. Zumalacárregui, E. Bellini, I. Sawicki, J. Lesgourgues and P. G. Ferreira, *hi-class: Horndeski in the Cosmic Linear Anisotropy Solving System*, *JCAP* **1708** (2017) 019, [[1605.06102](#)].
- [20] D. Alonso, E. Bellini, P. G. Ferreira and M. Zumalacárregui, *Observational future of cosmological scalar-tensor theories*, *Phys. Rev. D* **95** (2017) 063502, [[1610.09290](#)].
- [21] S. Arai and A. Nishizawa, *Generalized framework for testing gravity with gravitational-wave propagation. II. Constraints on Horndeski theory*, *Phys. Rev. D* **97** (2018) 104038, [[1711.03776](#)].
- [22] N. Frusciante, S. Peirone, S. Casas and N. A. Lima, *The road ahead of Horndeski: cosmology of surviving scalar-tensor theories*, [1810.10521](#).
- [23] R. Reischke, A. S. Mancini, B. M. Schafer and P. M. Merkel, *Investigating scalar-tensor-gravity with statistics of the cosmic large-scale structure*, [1804.02441](#).

- [24] A. Spurio Mancini, R. Reischke, V. Pettorino, B. M. Schafer and M. Zumalacárregui, *Testing (modified) gravity with 3D and tomographic cosmic shear*, *Mon. Not. Roy. Astron. Soc.* **480** (2018) 3725, [[1801.04251](#)].
- [25] G. Brando, F. T. Falciano, E. V. Linder and H. E. S. Velten, *Modified gravity away from a Λ CDM background*, *JCAP* **11** (2019) 018, [[1904.12903](#)].
- [26] R. Arjona, W. Cardona and S. Nesseris, *Designing Horndeski and the effective fluid approach*, *Phys. Rev. D* **100** (2019) 063526, [[1904.06294](#)].
- [27] M. Raveri, *Reconstructing Gravity on Cosmological Scales*, *Phys. Rev. D* **101** (2020) 083524, [[1902.01366](#)].
- [28] L. Perenon, J. Bel, R. Maartens and A. de la Cruz-Dombriz, *Optimising growth of structure constraints on modified gravity*, *JCAP* **06** (2019) 020, [[1901.11063](#)].
- [29] A. Spurio Mancini, F. Köhlinger, B. Joachimi, V. Pettorino, B. M. Schäfer, R. Reischke et al., *KiDS + GAMA: constraints on horndeski gravity from combined large-scale structure probes*, *Mon. Not. Roy. Astron. Soc.* **490** (2019) 2155–2177, [[1901.03686](#)].
- [30] A. Bonilla, R. D’Agostino, R. C. Nunes and J. C. N. de Araujo, *Forecasts on the speed of gravitational waves at high z* , *JCAP* **03** (2020) 015, [[1910.05631](#)].
- [31] T. Baker and I. Harrison, *Constraining Scalar-Tensor Modified Gravity with Gravitational Waves and Large Scale Structure Surveys*, *JCAP* **01** (2021) 068, [[2007.13791](#)].
- [32] S. Joudaki, P. G. Ferreira, N. A. Lima and H. A. Winther, *Testing Gravity on Cosmic Scales: A Case Study of Jordan-Brans-Dicke Theory*, [2010.15278](#).
- [33] J. Noller, L. Santoni, E. Trincherini and L. G. Trombetta, *Scalar-tensor cosmologies without screening*, *JCAP* **01** (2021) 045, [[2008.08649](#)].
- [34] J. Noller, *Cosmological constraints on dark energy in light of gravitational wave bounds*, *Phys. Rev. D* **101** (2020) 063524, [[2001.05469](#)].
- [35] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, *Causality, analyticity and an IR obstruction to UV completion*, *JHEP* **10** (2006) 014, [[hep-th/0602178](#)].
- [36] A. Jenkins and D. O’Connell, *The Story of O: Positivity constraints in effective field theories*, [hep-th/0609159](#).
- [37] A. Adams, A. Jenkins and D. O’Connell, *Signs of analyticity in fermion scattering*, [0802.4081](#).
- [38] A. Nicolis, R. Rattazzi and E. Trincherini, *Energy’s and amplitudes’ positivity*, *JHEP* **05** (2010) 095, [[0912.4258](#)].
- [39] B. Bellazzini, L. Martucci and R. Torre, *Symmetries, Sum Rules and Constraints on Effective Field Theories*, *JHEP* **09** (2014) 100, [[1405.2960](#)].
- [40] B. Bellazzini, C. Cheung and G. N. Remmen, *Quantum Gravity Constraints from Unitarity and Analyticity*, *Phys. Rev.* **D93** (2016) 064076, [[1509.00851](#)].
- [41] D. Baumann, D. Green, H. Lee and R. A. Porto, *Signs of Analyticity in Single-Field Inflation*, *Phys. Rev.* **D93** (2016) 023523, [[1502.07304](#)].
- [42] C. Cheung and G. N. Remmen, *Positive Signs in Massive Gravity*, *JHEP* **04** (2016) 002, [[1601.04068](#)].

- [43] J. Bonifacio, K. Hinterbichler and R. A. Rosen, *Positivity constraints for pseudolinear massive spin-2 and vector Galileons*, *Phys. Rev.* **D94** (2016) 104001, [[1607.06084](#)].
- [44] C. de Rham, S. Melville, A. J. Tolley and S.-Y. Zhou, *Positivity bounds for scalar field theories*, *Phys. Rev.* **D96** (2017) 081702(R), [[1702.06134](#)].
- [45] C. de Rham, S. Melville, A. J. Tolley and S.-Y. Zhou, *UV complete me: Positivity Bounds for Particles with Spin*, *JHEP* **03** (2018) 011, [[1706.02712](#)].
- [46] C. de Rham, S. Melville, A. J. Tolley and S.-Y. Zhou, *Positivity Bounds for Massive Spin-1 and Spin-2 Fields*, [1804.10624](#).
- [47] B. Bellazzini, *Softness and amplitudes' positivity for spinning particles*, *JHEP* **02** (2017) 034, [[1605.06111](#)].
- [48] C. de Rham, S. Melville, A. J. Tolley and S.-Y. Zhou, *Massive Galileon Positivity Bounds*, *JHEP* **09** (2017) 072, [[1702.08577](#)].
- [49] C. de Rham, S. Melville and A. J. Tolley, *Improved Positivity Bounds and Massive Gravity*, *JHEP* **04** (2018) 083, [[1710.09611](#)].
- [50] C. de Rham and A. J. Tolley, *Speed of gravity*, *Phys. Rev. D* **101** (2020) 063518, [[1909.00881](#)].
- [51] C. de Rham and A. J. Tolley, *Causality in curved spacetimes: The speed of light and gravity*, *Phys. Rev. D* **102** (2020) 084048, [[2007.01847](#)].
- [52] L. Alberte, C. de Rham, S. Jaitly and A. J. Tolley, *Positivity Bounds and the Massless Spin-2 Pole*, [2007.12667](#).
- [53] G. Dvali, G. F. Giudice, C. Gomez and A. Kehagias, *UV-Completion by Classicalization*, *JHEP* **08** (2011) 108, [[1010.1415](#)].
- [54] G. Dvali and D. Pirtskhalava, *Dynamics of Unitarization by Classicalization*, *Phys. Lett.* **B699** (2011) 78–86, [[1011.0114](#)].
- [55] G. Dvali, *Classicalize or not to Classicalize?*, [1101.2661](#).
- [56] G. Dvali, C. Gomez and A. Kehagias, *Classicalization of Gravitons and Goldstones*, *JHEP* **11** (2011) 070, [[1103.5963](#)].
- [57] A. Vikman, *Suppressing Quantum Fluctuations in Classicalization*, *EPL* **101** (2013) 34001, [[1208.3647](#)].
- [58] A. Kovner and M. Lublinsky, *Classicalization and Unitarity*, *JHEP* **11** (2012) 030, [[1207.5037](#)].
- [59] L. Keltner and A. J. Tolley, *UV properties of Galileons: Spectral Densities*, [1502.05706](#).
- [60] G. W. Horndeski, *Second-order scalar-tensor field equations in a four-dimensional space*, *Int. J. Theor. Phys.* **10** (1974) 363–384.
- [61] C. Deffayet, G. Esposito-Farese and A. Vikman, *Covariant Galileon*, *Phys. Rev.* **D79** (2009) 084003, [[0901.1314](#)].
- [62] D. Pirtskhalava, L. Santoni, E. Trincherini and F. Vernizzi, *Weakly Broken Galileon Symmetry*, *JCAP* **1509** (2015) 007, [[1505.00007](#)].
- [63] M. A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the DGP model*, *JHEP* **09** (2003) 029, [[hep-th/0303116](#)].

- [64] A. Nicolis and R. Rattazzi, *Classical and quantum consistency of the DGP model*, *JHEP* **06** (2004) 059, [[hep-th/0404159](#)].
- [65] C. de Rham and A. J. Tolley, *DBI and the Galileon reunited*, *JCAP* **05** (2010) 015, [[1003.5917](#)].
- [66] C. Burrage, C. de Rham, D. Seery and A. J. Tolley, *Galileon inflation*, *JCAP* **01** (2011) 014, [[1009.2497](#)].
- [67] C. de Rham and R. H. Ribeiro, *Riding on irrelevant operators*, *JCAP* **1411** (2014) 016, [[1405.5213](#)].
- [68] C. de Rham, G. Gabadadze, L. Heisenberg and D. Pirtskhalava, *Nonrenormalization and naturalness in a class of scalar-tensor theories*, *Phys. Rev. D* **87** (2013) 085017, [[1212.4128](#)].
- [69] J. Noller and A. Nicola, *Radiative stability and observational constraints on dark energy and modified gravity*, *Phys. Rev. D* **102** (2020) 104045, [[1811.03082](#)].
- [70] L. Heisenberg, J. Noller and J. Zosso, *Horndeski under the quantum loupe*, *JCAP* **10** (2020) 010, [[2004.11655](#)].
- [71] S. Melville and J. Noller, *Positivity in the Sky: Constraining dark energy and modified gravity from the UV*, *Phys. Rev. D* **101** (2020) 021502, [[1904.05874](#)].
- [72] LIGO SCIENTIFIC, VIRGO collaboration, B. Abbott et al., *GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral*, *Phys. Rev. Lett.* **119** (2017) 161101, [[1710.05832](#)].
- [73] LIGO SCIENTIFIC, VIRGO, FERMI-GBM, INTEGRAL collaboration, B. P. Abbott et al., *Gravitational Waves and Gamma-rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A*, *Astrophys. J.* **848** (2017) L13, [[1710.05834](#)].
- [74] B. P. Abbott et al., *Multi-messenger Observations of a Binary Neutron Star Merger*, *Astrophys. J.* **848** (2017) L12, [[1710.05833](#)].
- [75] L. Lombriser and A. Taylor, *Breaking a Dark Degeneracy with Gravitational Waves*, *JCAP* **1603** (2016) 031, [[1509.08458](#)].
- [76] L. Lombriser and N. A. Lima, *Challenges to Self-Acceleration in Modified Gravity from Gravitational Waves and Large-Scale Structure*, *Phys. Lett.* **B765** (2017) 382–385, [[1602.07670](#)].
- [77] P. Creminelli and F. Vernizzi, *Dark Energy after GW170817 and GRB170817A*, *Phys. Rev. Lett.* **119** (2017) 251302, [[1710.05877](#)].
- [78] J. Sakstein and B. Jain, *Implications of the Neutron Star Merger GW170817 for Cosmological Scalar-Tensor Theories*, *Phys. Rev. Lett.* **119** (2017) 251303, [[1710.05893](#)].
- [79] J. M. Ezquiaga and M. Zumalacárregui, *Dark Energy After GW170817: Dead Ends and the Road Ahead*, *Phys. Rev. Lett.* **119** (2017) 251304, [[1710.05901](#)].
- [80] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller and I. Sawicki, *Strong constraints on cosmological gravity from GW170817 and GRB 170817A*, *Phys. Rev. Lett.* **119** (2017) 251301, [[1710.06394](#)].
- [81] Y. Akrami, P. Brax, A.-C. Davis and V. Vardanyan, *Neutron star merger GW170817 strongly constrains doubly coupled bigravity*, *Phys. Rev.* **D97** (2018) 124010, [[1803.09726](#)].

- [82] L. Heisenberg and S. Tsujikawa, *Dark energy survivals in massive gravity after GW170817: $SO(3)$ invariant*, *JCAP* **1801** (2018) 044, [[1711.09430](#)].
- [83] J. Beltrán Jiménez and L. Heisenberg, *Non-trivial gravitational waves and structure formation phenomenology from dark energy*, *JCAP* **1809** (2018) 035, [[1806.01753](#)].
- [84] C. de Rham and S. Melville, *Gravitational Rainbows: LIGO and Dark Energy at its Cutoff*, *Phys. Rev. Lett.* **121** (2018) 221101, [[1806.09417](#)].
- [85] T. Pham and T. N. Truong, *Evaluation of the Derivative Quartic Terms of the Meson Chiral Lagrangian From Forward Dispersion Relation*, *Phys. Rev. D* **31** (1985) 3027.
- [86] B. Ananthanarayan, D. Toublan and G. Wanders, *Consistency of the chiral pion pion scattering amplitudes with axiomatic constraints*, *Phys. Rev. D* **51** (1995) 1093–1100, [[hep-ph/9410302](#)].
- [87] M. Pennington and J. Portoles, *The Chiral Lagrangian parameters, l_1 , l_2 , are determined by the rho resonance*, *Phys. Lett. B* **344** (1995) 399–406, [[hep-ph/9409426](#)].
- [88] J. Comellas, J. I. Latorre and J. Taron, *Constraints on chiral perturbation theory parameters from QCD inequalities*, *Phys. Lett. B* **360** (1995) 109–116, [[hep-ph/9507258](#)].
- [89] G. Dvali, A. Franca and C. Gomez, *Road Signs for UV-Completion*, [1204.6388](#).
- [90] B. Bellazzini, J. Elias Miró, R. Rattazzi, M. Riembau and F. Riva, *Positive Moments for Scattering Amplitudes*, [2011.00037](#).
- [91] A. J. Tolley, Z.-Y. Wang and S.-Y. Zhou, *New positivity bounds from full crossing symmetry*, [2011.02400](#).
- [92] S. Caron-Huot and V. Van Duong, *Extremal Effective Field Theories*, [2011.02957](#).
- [93] A. Sinha and A. Zahed, *Crossing Symmetric Dispersion Relations in QFTs*, [2012.04877](#).
- [94] L. Alberte, C. de Rham, S. Jaitly and A. J. Tolley, *QED positivity bounds*, [2012.05798](#).
- [95] S. Caron-Huot, D. Mazac, L. Rastelli and D. Simmons-Duffin, *Sharp Boundaries for the Swampland*, [2102.08951](#).
- [96] T. Grall and S. Melville, *Positivity Bounds without Boosts*, [2102.05683](#).
- [97] L. Vecchi, *Causal versus analytic constraints on anomalous quartic gauge couplings*, *JHEP* **11** (2007) 054, [[0704.1900](#)].
- [98] A. V. Manohar and V. Mateu, *Dispersion Relation Bounds for $\pi\pi$ Scattering*, *Phys. Rev. D* **77** (2008) 094019, [[0801.3222](#)].
- [99] V. Salvatelli, F. Piazza and C. Marinoni, *Constraints on modified gravity from Planck 2015: when the health of your theory makes the difference*, *JCAP* **09** (2016) 027, [[1602.08283](#)].
- [100] O. Pujolas, I. Sawicki and A. Vikman, *The Imperfect Fluid behind Kinetic Gravity Braiding*, *JHEP* **11** (2011) 156, [[1103.5360](#)].
- [101] A. Barreira, B. Li, C. Baugh and S. Pascoli, *The observational status of Galileon gravity after Planck*, *JCAP* **1408** (2014) 059, [[1406.0485](#)].
- [102] Planck Collaboration, *Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters*, *Astronomy and Astrophysics* **594** (Sept., 2016) A11, [[1507.02704](#)].

- [103] Planck Collaboration, *Planck 2015 results. XV. Gravitational lensing*, *Astronomy and Astrophysics* **594** (Sept., 2016) A15, [[1502.01591](#)].
- [104] Planck Collaboration, *Planck 2015 results. XIII. Cosmological parameters*, *Astronomy and Astrophysics* **594** (Sept., 2016) A13, [[1502.01589](#)].
- [105] L. Anderson et al., *The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: baryon acoustic oscillations in the Data Releases 10 and 11 Galaxy samples*, *MNRAS* **441** (June, 2014) 24–62, [[1312.4877](#)].
- [106] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden and M. Manera, *The clustering of the SDSS DR7 main Galaxy sample - I. A 4 per cent distance measure at $z = 0.15$* , *MNRAS* **449** (May, 2015) 835–847, [[1409.3242](#)].
- [107] M. Tegmark et al., *Cosmological constraints from the SDSS luminous red galaxies*, *Phys. Rev. D* **74** (Dec., 2006) 123507, [[astro-ph/0608632](#)].
- [108] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, G. B. Poole et al., *The 6dF Galaxy Survey: $z \approx 0$ measurements of the growth rate and σ_8* , *MNRAS* **423** (July, 2012) 3430–3444, [[1204.4725](#)].
- [109] L. Samushia et al., *The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: measuring growth rate and geometry with anisotropic clustering*, *MNRAS* **439** (Apr., 2014) 3504–3519, [[1312.4899](#)].
- [110] J. Kennedy and L. Lombriser, *Positivity bounds on reconstructed Horndeski models*, *Phys. Rev. D* **102** (2020) 044062, [[2003.05318](#)].
- [111] C. Burrage and J. Dombrowski, *Constraining the cosmological evolution of scalar-tensor theories with local measurements of the time variation of G* , *JCAP* **07** (2020) 060, [[2004.14260](#)].
- [112] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Generalized G -inflation: Inflation with the most general second-order field equations*, *Prog. Theor. Phys.* **126** (2011) 511–529, [[1105.5723](#)].
- [113] P. Creminelli, G. Tambalo, F. Vernizzi and V. Yingcharoenrat, *Dark-Energy Instabilities induced by Gravitational Waves*, *JCAP* **05** (2020) 002, [[1910.14035](#)].
- [114] P. Milonni, *Fast Light, Slow Light and Left-Handed Light*, (*Series in Optics and Optoelectronics*), *Fast Light, Slow Light and Left-Handed Light*(2004) 262 pages, [([Taylor & Francis](#))].
- [115] L. Brillouin, *Wave Propagation and Group Velocity (Series in Pure & Applied Physics)* , *Wave Propagation and Group Velocity*(1960) 154 pages, [([Academic Press](#))].
- [116] C. de Rham, *Massive Gravity*, *Living Rev. Rel.* **17** (2014) 7, [[1401.4173](#)].
- [117] H. S. Reall, *Causality in gravitational theories with second order equations of motion*, [2101.11623](#).
- [118] D. Blas, J. Lesgourgues and T. Tram, *The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes*, *JCAP* **1107** (2011) 034, [[1104.2933](#)].
- [119] D. Foreman-Mackey, *corner.py: Scatterplot matrices in python*, *The Journal of Open Source Software* **24** (2016) .

- [120] E. Bellini, I. Sawicki and M. Zumalacárregui, *hi_class: Background Evolution, Initial Conditions and Approximation Schemes*, *JCAP* **02** (2020) 008, [[1909.01828](#)].
- [121] B. Audren, J. Lesgourgues, K. Benabed and S. Prunet, *Conservative Constraints on Early Cosmology: an illustration of the Monte Python cosmological parameter inference code*, *JCAP* **1302** (2013) 001, [[1210.7183](#)].
- [122] T. Brinckmann and J. Lesgourgues, *MontePython 3: boosted MCMC sampler and other features*, [1804.07261](#).
- [123] J. M. Martín-García, *xAct 2002-2014*, <http://www.xact.es/> .
- [124] C. Aragone and S. Deser, *Consistency Problems of Hypergravity*, *Phys. Lett. B* **86** (1979) 161–163.
- [125] M. Porrati, *Massive spin 5/2 fields coupled to gravity: Tree level unitarity versus the equivalence principle*, *Phys. Lett. B* **304** (1993) 77–80, [[gr-qc/9301012](#)].
- [126] A. Cucchieri, M. Porrati and S. Deser, *Tree level unitarity constraints on the gravitational couplings of higher spin massive fields*, *Phys. Rev. D* **51** (1995) 4543–4549, [[hep-th/9408073](#)].