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Layout optimization and Sustainable development of waste water networks with the use of heuristic algorithms: The Luxemburgish case

By

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Declarations

Declaration of Originality

I hereby declare that the work presented in this dissertation is my own, and that everything that is not my own work has been appropriately referenced.

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Abstract

Fresh water tends to increasingly comprise a scarcity today both in arid or demographically boosted regions of the world such as large and smaller cities. On this basis, research is directed towards minimization of fresh water supply into a Waste Water Network Topology (WWNT) and maximizing water re-use. This might be composed of a cluster of agents which have certain demands for fresh water as well as waste water dependent on their daily uses and living profiles. This work is divided into two parts. In the first part, different waste water flows within a reference building unit i.e. a typical household of four (4) occupants is simulated. This type of building represents a major part of the total building stock in Luxembourg. In its first part the present study attempts to examine the optimized fresh and waste water flow pathways between water using units of the building. Between water flows two domestic treatment units are adopted. The simulation of above mentioned system is attempted by adopting different algorithm methods such as the Sequential Quadratic Programming (SQP), the interior point and meta-heuristic optimization algorithms such as the Genetic Algorithms (GA's).Suitable computational platform tools such as MATLAB and GAMS are incorporated. A comparison study on the most efficient approach is then realized on the single household unit by developing four (4) different mathematical model formulation versions. The second part of this study comprises simulation and development of the Waste Water Network Grid (WWNG) in the upscale level, such as the neighborhood level within or outside the urban context. This model encompasses all possible land uses and different kinds of buildings of different use envelopes thus demands. This range of units includes mainly building stock, agricultural and infrastructure of the tertiary sector. Integration of above mentioned model to the existing WWNG will enhance attempts to more closely reach the optimum points. The use of appropriate mathematical programming methods for the upscale level, will take place. Increased uncertainties within the built model will be attempted to be tackled by developing linear programming techniques and suitable assumptions without distorting initial condition largely. Assumptions are then drawn on the efficiency of the adopted method an additional essential task is the minimization of the overall infrastructure and network cost, which may in turn give rise to corresponding reduced waste effluents discharge off the proposed network. The case study comprises selected rural and semi-rural areas zone districts of similar living profiles outside the City of Luxembourg. Therefore a clustering of end users of similar demand will be attempted. Possible redesign of an optimized WWNG comprises a vital need within the context of large scale demographic growth of urban environments today.

Acknowledgements

Firstly I would like to thank my supervisor Professor Nilay Shah for his tireless continuous academic and personal support and encouragement, throughout my PhD engagement. His contribution on special personal and academic milestones for me was something more than crucial. My personal health ordeal that came up unexpectedly during my studies would have never allowed me to carry on completing my PhD if it was not him. Professor Shah gave me the necessary freedom to develop my research in my own interest within the instructed context, being involved and available to help me assemble the puzzles that came up in issues such as modeling as well as proper analytical skills.

I would also like to thank my colleagues at Imperial College and the MIT in my early and middle research stages who contributed vastly introducing me into modeling and mathematical programming techniques as well as fruitful discussions and research on various research domain and issues. I would also like to thank Gonzalo and James for their academic involvement and companionship during the first and later stages of my PhD at Imperial.

I would also like to pay tribute to my main engineering and business partner in Luxembourg, Klaus who put up with me professionally during my parallel engagement towards my PhD all these years as well as colleagues who helped out creating a more friendly working environment for my main professional engagement mainly in Luxembourg France and Belgium mainly. I would lastly like to thank cordially the Luxembourgish funding scheme which was actively engaged in funding the present work.

In a more personal note, the PhD experience at Imperial will always remain unforgettable scientifically as well as a personal hands-on experience as it contributed significantly to my existent way of thinking and professional and scientific development.

To my father's spirit being always around

and

My mother to whom I owe everything

List of Publications

Refereed Conference Publications and book of abstracts

- P. Broukos, (2020), “A Linearized Mathematical Formulation for the Waste Water Treatment Network Design and implementation of Benders” In: 7th International Conference on Energy, Sustainability and Climate Change (ESCC 2020), (in press), August 24-26, 2020, Skiathos, Greece;
- P. Broukos, (2019), Layout Optimization of Existing Waste Water Network: a Luxembourgish case study. In: 6th International Conference on Energy, Sustainability and Climate Change (ESCC 2019), ISBN 978-618-84403-3-3, p-48, June 3-7, 2019, Crete, Greece;
- P. Broukos, (2018), “Optimization of Waste water networks” in: 5th International Conference on Energy, Sustainability and Climate Change, (ESCC 2018), abstract reference: Q-78-19-152**,** June 4-6, 2018, Mykonos, Greece;
- P. Broukos, (2017), Optimized Wastewater Network Topology of a typical residency with the Use of Meta-heuristic Algorithm in: 4th International Conference on “Energy, Sustainability and Climate Change” (ESCC17), ISBN 978-960-949-52-7, p-54, 12-16 June 2017, Santorini, Greece.

Conference Abstracts

- N. Pnevmatikos , P. Broukos* et al (2020), “The effect of rotational component of earthquake excitation to the response of steel structures”, In: 9th European Conference on Steel and Composite Structures (Eurosteel 2020) , 9–11 September 2020, Sheffield, UK.

**(Optimization methods used in the present thesis were also engaged in the above mentioned articles towards optimized results)*

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PART I - List of Figures

PART I – Abbreviations (Main)

Symbol	Description
FWN	Fresh Water Network
FWC	Fresh Water Consumption
TP	Treatment Plant
WW	Waste Water
WWN	Waste Water Network
WWNT	Waste Water Network Topology
WWTP'(s)	Waste Water Treatment Plant(s)
WWNG	Waste Water Network Grid
WU	Water Using Unit
TU	Treatment Unit

PART II - List of Tables

PART II - List of Figures

PART II Abbreviations

W&WWN:	Water and Waste Water Network
WWTP('s):	Waste Water Treatment Plant(s)
(W&WWN's):	Water and Waste Water Networks
WWN('s):	Waste Water Network(s)
WWRM:	Waste Water Resource Management
WWTND:	Waste Water Treatment Network Design
MILV:	Mixed Integer Linear Variation
MILP:	Mixed Integer Linear Programming
MINLP:	Mixed Integer Non Linear Programming
LP:	Linear Programming
NLP:	Non Linear Programming
PE:	Population equivalent
WWOD:	Waste Water outflow demands
O&M:	operational and Maintenance
MF:	Microfiltration
RO:	Reverse Osmosis
GBD:	General Bender's Decomposition
BDM:	Bender's Decomposition Method
CBD:	Classical Bender's Decomposition
BC:	Bender's Cut
OP:	Original Problem
RMP:	Relaxed main Problem
PSP:	Primal Sub Problem
DSP:	Dual Sub Problem
UB:	Upper Bound
LB:	Lower Bound

Chapter 1. Introduction

1.1 Introduction

Existing water surface supplies tend to diminish globally. Regulations and directives have become more stringent over the recent years when it comes to irrational use especially for surface water and fresh water supply for domestic, agricultural or tertiary purposes and the contamination of surface and ground water supplies by untreated waste water. The combination of the need for more efficient management of potable water along with the intense existent demographic growth in the urban context mainly in the last two decades has given rise to new research domains. This research focuses in the attempt to optimize existing, very often obsolete, urban infrastructure networks. This piece of work deals mainly with the attempt to optimize the existing Waste Water Network Topology (WWNT) in general and in particular for the city of Luxembourg.

Significant and rapid growth in urban populations in recent years comprises a threat to the existing outdated Waste Water Networks (WWN). This threat can be justified through a series of malfunctions and outdated elements of within existing network. Some of which namely being, the poor and outdated technology of related infrastructure within large urban regions such as re-dimensioning the diameters of connections , adoption of new technology for an ageing network of pumps, the need for re-dimensioning of the plants themselves due to the increased demands of waste load, the reality of the direct discharge of non-treated waste effluent streams to rivers and water basins and in general to contamination of both surface and underground water bodies. The motivation of this work is to attempt to examine the range of margins for redistribution of fresh water supplies into a proposed network so that consequent demand for fresh water is kept to a minimum. Another essential task is to minimize waste water effluents.

The aim of this research will be developed in two parts. In the first part in the micro level scale a typical household of 4 users in Luxembourg is examined. Two domestic waste water treatment systems (DWWTS) are adopted and the effluent waste load is then taken as an input value for the upscale level thus the second part of the present work. These values will be essential as they represent estimated calculated input values of waste water influent load to the sewerage network. These represent values regarding in dependent residential building units which comprise more than 50% of the overall building stock in Luxembourgish territory. It should be noted at this point that these computed values which represent effluent of waste water coming off the household unit are distinguished from the ones within bibliography as these take into account utilization of the two DWWS's. This model of domestic flows is built with the use of non linear optimization methods and assumptions on estimation of the quantity of waste water by each typical such household is drawn.

In the second part, at the upscale level, we attempt to optimize existing Waste Water network Grids (WWNG). The optimization will be attempted through description of a selected part of the existing WWN and then simulation of selected districts or clusters of districts within the city or the whole territory will take place with the use of linear programming methods. The aim comprises minimization of fresh water that the WWN is fed with by the Water network (WN). The model aims to find an optimal number, size and location of waste water plants with a minimal cost while satisfying treated waste and regenerated water quality as well as the demand constraints. The focus of this work is to both reduce overall cost of existing and proposed possible new infrastructure. Through simulation of the models more emphasis will be shown to investigate to what extend there are margins to abolish all possible redundant plants rather than propose to build new ones. By the use of linear programming algorithms through hybrid Bender's decomposition methods (HBDM's) a rationalization of use of fresh water, recycled or regenerated waste water can be attained. This can be realized despite rapid and asymmetric growth of population mainly within urban environments and in particular in the City of Luxembourg. In addition, a more efficient ecological and rationalized management of water sources will be required in order to meet increased waste water generation demand in all end using units. This re-dimensioning of existing WWNG will also have a positive impact on sanitation of water bodies. This fact might in turn have multiple effects on decision making of a community.

However, the main research will be carried out in the second part of this work with respect to the upscale level. An attempt to simulate a synthesis of clusters of waste water network sub grids to a larger grid will be realized. From the domestic (households) up to the level a neighborhood and ultimately to the city scale and eventually include the whole territory of Luxembourg in the national scale level exceeds the range of this study. Potable water production in Luxembourg is sourced by ground waters (2/3) and surface waters (1/3). Ground waters are obtained from springs and wells (Levy, 2012). Surface waters are obtained partially by rain waters and also by treating the waste through the waste water plants. These waters are mixed before being delivered back to the customer. The main problem consists in increasing the rate of surface water in this mixture to improve the quality of the water (Levy, 2012).

Therefore one of the aims of the current work is to increase the rate of the surface water significantly. Obviously this requires greater wastewater treatment capacity (WWTC) through improvement of the existing waste water plants grid themselves or even constructing new ones. Optimization of a Waste Water Topology Network (WWTN) comprises both a continuous and a discrete optimization problem with many of the decision variables being involved to correspond to values out of a discrete design set. Many of the (Waste Water Network Systems) WWNS's structural parts are assigned to specific dimensions thus discrete ones. For the solution of non-continuous problems, especially for large scale problems which lie within the Civil Engineering domain, an appropriate algorithm can be incorporated. Thierauf and Cai, 1995 as well as M. Papadrakakis, Y. Tsompanakis et al, 1996 have presented an altered Evolutionary Strategy algorithm.

1.2 Thesis structure

The thesis is organized in two parts as follows:

Part I. comprises the following Chapters and presents the different available set up connections scenarios for 2 different wastewater treatment systems, namely being a Reverse Osmosis (RO) and a Microfiltration (MF) treatment system, taking into consideration all existent water and waste water flows within the context of a typical household.

Chapter 1 presents an introduction of the entire thesis as well as the general context of it.

In **Chapter 2** different modeling approaches of the Waste Water Network distribution are attained. Then, a general background comprises literature review on heuristic and meta-heuristic optimization algorithms. The different analytical approaches regarding reaching the optimum scenarios of problems on the most efficient set up for all waste water and water supply flows at the domestic scale (micro scale) of one single unit that is a household unit is discussed through a literature background. Specific focus is given to all different kind of Evolutionary algorithms (EA), such as Evolutionary Strategies and their other different versions as well as Genetic Algorithms (GA's) with the use of penalty functions.

Chapter 3 discusses the methods and describes the problem formulation as well as its mathematical formulation. In the first sections of this Chapter all necessary assumptions were as well as different input parameters are made in the context of mathematical formulation of the model. 4 different set up scenarios of the same problem are analyzed with the use of different connection set up of the utilized waste water systems i.e. the RO and the MF system accordingly. 4 different model scenarios were used. Sensitivity analysis was performed by configuring specific parameters of the set up model. The most efficient thus optimized output results related to reduced waste water outflows coming out of the single unit under study were then used as the additional and final input parameters for the second part of this thesis for the examined problem of the upscale context at Part II.

In **Chapter 4** additional analytical tools were introduced to known methods within the literature and a known similar problem taken from the literature is examined. The application of the GA's using penalty functions that were discussed previously within the thesis are now implemented in this Chapter. Also multiple contaminant scenarios were added to single ones which were taken from the literature. Different scheme models were discussed and new results were attained. This set of methods investigation would also be proposed as a subsequent future research work context regarding Part I of this thesis.

Part II describes and analyses a specific region and its existent waste water network profile which comprises of -different in size- clusters of units and building units of different use thus waste water demands. The objective of Part II is to come up with a feasible plan proposition including optimized spatial set up of the examined unit elements within this existing waste water infrastructure grid.

Chapter 5 sets out the general background of different waste water network set ups and formats their corresponding mathematical formulations. These set ups are firstly depicted and reviewed in mathematical formulation terms. Thereafter different versions of this general context of such a grid are also formulated mathematically.

Chapter 6 describes existing waste water distribution network in Luxembourg. This comprises actual data regarding waste water production thus actual flow charges in the grid. This Chapter also describes different possibilities for the investigated network to attain access to alternative fresh water sources such as surface and ground water sources. These comprise scenario which are formulated mathematically herein. These two model scenarios also involve consideration of storage tanks and reuse of treated waste for other purpose uses such as agriculture. Above mentioned models and their corresponding mathematical formulations are introduced but will not be used in the next Chapters. The reason for this is that the problem under study would expand in a such a degree that would go beyond the purpose of this thesis. The decision variables and parameters would entail excessive computation load and the results would be exponentially larger.

In **Chapter 7** the analysis of urban sewage systems is discussed in dependence with other system attributes which in turn contribute to the final design and the capacity of the entire system under study. Such parameters, transport of sewage, collection, transport and run off of rain water industrial wastes, water and other liquid waste from industrial processes from an urban area (town or a rural settlement) to the point of disposal and population distribution of the examined area. Increase of volume of waste water within sewer mains due to the parasitic inputs of groundwater is also taken into account in our design protocol. Future population projection to the future enable us to set off the initial design values of estimated water demands as well as waste water production and demands for their treatment within a time projection of 40 years to the future. These values comprise the set off for the estimation of optimised scenarios to be examined in detail within the following Chapters. These are the peak and their correspondent mean maximum design values which are compared to the ultimate capacity of the system under study. In addition the two kinds of sewage systems are covered here which include the so called separate and the combined system, which comprise the sewage grid and the storm network separately and its mixed version accordingly.

In **Chapter 8** the problem of Waste Water Treatment Network Design (WWTND) is examined with the use of different optimization methods. The above mentioned original problem is being approached either with –in different percentage of use for each end users cluster - or without the use of the MF and RO appliances that were incorporated in Part I of the present work. Their proven implication through the results of their contribution to the single household unit that was analyzed in Part I is being used as the initial value to examine their effect to the upscale level. This simulation comprises different linear and non linear versions through specific optimization algorithms.

Chapter 9 presents the implementation of above mentioned design and management of the system under examination to a realistic scenario taken of a selected typical Luxembourgish small village comprising of different existing clusters of buildings of various uses thus water and waste water demands. Various model assumptions are also made within this chapter and this set of assumptions are further valid for the entire simulations in the upscale level for all 20 cluster of agglomerations units examined. Furthermore the Chapter also presents various existing parameters concerning existing situation such as existing connection attributes between WWTP's units as well as end users in the upscale level of all 20 cluster units taken for our model.

In **Chapter 10,** all relevant computational results of the above examined models within the context of the second part are presented and correspondent diagrams based on these optimized results are depicted.

Chapter 11, which is the last Chapter of the present thesis, outlines the conclusions of the study as well as the future research work. The results and conclusion of the present research will enable the author to be involved with the implementation of the combination set of methods used in the present study for an even larger area of the Luxemburgish territory. This future work is prone to be granted by Luxemburgish authorities along with private research institutes funds.

The increasing population of the earth together with population movement due to immigration or urbanization leads to the necessity of redesigning existing or designing new waste water treatment networks. A lot of municipalities,.prefectures or even states like the Luxemburgish one make plans for the future network based on estimated needs. This second part studies the networks entities (pipes, treatment plants, distributed infrastructure) and offers a tool for future design and/or re-design through mathematical programming.

Relevant studies have been made in the past with (J.J. de Melo, et al. 1994) offering a detailed review of them. The different approaches include the minimization of the environmental impact and the maximization of system reliability and flexibility under uncertainty conditions. Especially, in (Z.Zhu, et al. 1988,2003) a siting model is introduced in order to locate waste water treatment facilities and the concave cost of a treatment plant is approximated by a fixed charge cost and one straight-line segment. Similarly, in (J.J. Jarvis, et. 1978) the regional waste water system is modeled as a fixed-charge network flow problem where the concave cost functions are practically linearized by using piecewise linearization. Estimating the system load in terms of population units at some target year in the future, the authors solve the linearized model for a specific region. Large gravity sewer networks are addressed in (A.A.Eliman, etal. 1989), where piecewise linearization is also applied on a nonlinear convex function relating pipeline diameter and slope.

The authors in the above mentioned article study the design only of a gravitational pipeline by combining linear programming with a diameter discretization heuristic approach.

The current work resembles these studies with the main difference being that both gravitational and pumping pipelines and distributed components are considered and all cost functions (construction and operational) of all network units are linearized by piecewise linearization.

Moreover, the optimization of the allocation and treatment of municipal wastewater sludge within an existing network, as well as the optimal location(s) for new drying facilities in this chain, are addressed in (A. De Meyer, et al. 2016). The authors apply a mixed integer linear programming model, known as OPTIMASS, whereas in the current paper the Waste Water Treatment Plants' location problem is modeled as a Mixed Integer Non Linear Problem (MINLP).

Furthermore, to deal with uncertainty, in (L.Jing, et al. 2017), the authors therein introduce a multi-scale two-stage mixed integer stochastic (MSTMIS) model. The first stage involves long-term strategic decisions (location of the Sludge Processing Center (SPC) and the type of Waste Water Treatment Plant (WWTP)) and is solved by genetic algorithm. Through a sensitivity analysis, the most influential parameters are selected and stochastic scenarios are generated in order to reach second-stage short-term decisions (amount of sludge transported from each city to the SPC, the revenue from reusing treated wastewater, and the compost sale).The implemented model withinthesis is implemented on a specific region with a 20-year projection and showed better solutions than if the long-and short-term decisions were made together using traditional optimization methods.

Some studies, such as (J.Kim, et al. 2009) and (S. R. Lim, et al. 2008), focus on the network inside an industry. The former considers wastewater and heat exchange networks design by applying a two-stage optimization approach minimizing total annual cost through a mixed integer non-linear programming (MINLP) formulation involving effluent streams containing multiple contaminants. The latter studies distributed and terminal wastewater treatment units, assessing their environmental and economic feasibility, through life cycle assessment (LCA) and life cycle costing (LCC) methods. The authors use a Non-Linear Model to compute a combined network and compare it with a conventional wastewater treatment system, where no distribution infrastructure is used. On the contrary, the present second part of the thesis addresses the combined generalized network design, including both urban and industrial clusters.

Other studies address the design of integrated water supply and wastewater collection systems. In (M.J. Naderi, et al.2017), a mixed scenario-based and probabilistic two-stage stochastic programming model is proposed and it is solved by using the sample average approximation method, the Bezdek fuzzy clustering method and an accelerated Benders decomposition algorithm. In the same way, in (A.Fatollahi, et al. 2020), in order to deal with uncertainty, the authors propose a two-stage stochastic model, solved by a Lagrangian relaxation-based algorithm. These approaches seem to be quite efficient and suitable for this complicated problem, whereas they tend to be quite complicated and not easily applicable to various real world problems, as it is the case of the present method, which addresses only the wastewater network.

Similarly, the optimal location of waste water treatment plants, along with desalination and water reclamation plants are studied in (S.Liu, et al. 2011) through a MILP minimizing the annual total costs of the network. The model is applied to two Greek islands, which lack substantial freshwater, whereas in the current paper the mathematical formulation is implemented on a larger case study in Luxembourg. Contrary to the centralized ones, distributed wastewater treatment networks are dealt within (S.Liu, et al. 2011) and (B.Galan, et al. 1998), where multi-component streams are considered in order to reduce the concentration of several contaminants in the waste water network. The authors introduce a search procedure by successively solving a relaxed linear model and the original non-convex nonlinear problem in order to yield global or near global optimal solutions.

Also, in (B.Galan, et al. 1999), in order to avoid non-convex mathematical models, a typical complex distributed network superstructure is decomposed into a set of basic network superstructures and the best treatment network design embedded in each of the basic network super structures is determined by solving a set of linear programming problems. These linear problems are generated from a structured non-convex mathematical model by fixing a small number of key problem variables. Distributed waste water treatment networks are addressed in (R.Hernandez, et al. 2004) as well, where the authors propose easily applied methods, which can handle complicated examples for both single and multiple contaminant systems.

Unlike these papers, in the current second part of the present thesis both centralized and distributed components of the network are taken into consideration. A lot of studies in the literature focus on the sewer pipeline network inside a residential area and do not consider regional design including treatment plants. As stated in (A.H Li, et al. 2018), these problems are solved using meta-heuristic methods, such as Genetic Algorithms (GA) (A. Haghighi, et al. 2017) (V.Lavric, et al. 2007), (Pan, et al. 2009), (A. Highighi, et al. 2020), -(W.H.Hassan et, al 2018), Simulated Annealing (SA),(Hassan, et al.2020), Particle Swarm Optimization(PSO)-(S.F.Yeh, et al. 2011),(Izquierdo, et al. 2008) and Tabu Search (TS) (A. Ahmadi, et al.2018). Cellular Automata (CA) (L.Y.Liang, et al. 2004), (M.Afshar et al. 2012), (Rohaniad, et al. 2015) and Ant Colony (AC) (M.M. Zaheri et al. 2020) where optimization techniques are applied in such studies, as well. However, the current thesis does not deal with the dense local pipeline network, but the regional pipes which end up to the treatment plants.

The second part of the present thesis deals with the strategic design of a Waste Water Treatment Network (WWTN) in a region for a specific future projection. It considers allocation of Waste Water Treatment Plants (WWTPs), their gravitational or pumping pipeline connection with residential and industrial areas and the potential integration of two distributed Waste Water Treatment components, Micro-Filtration (MF) and Reverse-Osmosis (RO). The problem is formulated as a Mixed Integer Non-Linear Problem (MINLP) and is linearized using piecewise linearization.This second part of the thesis is organized as follows: Firstan indicativemathematicalmodel is built which is the set off of the examined model. In a later stage the methodology is described and the non-linear cost functions included. Afterwards, the MINLP formulation and its linearization approach is presented. The case study of the examined specific area Following, where the selected method is eventually applied and the computational results are displayed. Finally, last part concludes the thesis and offers suggestions for future research.

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Chapter 2. Background and Literature Review

2.1 Modeling approach of Waste Water Topology Network (WWTN)

There are multiple uses of water in the tertiary sector and the agricultural industry. The optimum design or redesign of wastewater networks has been examined through the adoption of different computational procedures. During the 80's and the 90's, concepts such as waste water treatment, water regeneration and consequent design of the optimized water allocation, fed into the waste water network, gave rise to solutions towards reduce of the overall fresh water consumption in an existing network. In **Figure2-1** according to (Bagajewicz, 2000) some combinations of connections between the fresh water supply feed, the process water using units (denoted as "P's"), the different Treatment units (denoted as "T's") and the disposal flows can be seen. The disposal represents the post treatment of the waste. This kind of waste cannot be treated further thus is restricted to be either considered as regenerated or reused flow and be supplied back into the system thus they are disposed in the nearby water bodies. As seen in **Figure 2-1 (a)** the most straightforward scheme is examined here. All process water using units lie in parallel and dispose waste which in turn feeds the treatment units box.

Thereafter this waste is treated and disposed of. In **Figure 2-1 (b),(c)** different connection schemes can be seen. Here a slightly altered logic lies between these versions. Whereas in **Figure 2-1 (a)** version none flow of waste was allowed between the "P's", here we may notice that a ranking of water using units is established in relation to what degree of allowance is each unit "P" allowed to receive a waste flow from a higher ranked unit another unit (Valentin Ples, 2007). In **Figure 2-1 (b)** for example the P1only receives freshwater so it is the highest ranked unit. P2 may receive waste only from P1 and p3 receives the most contaminated water which comes from P1 and P2 thus it is ranked as the lowest of the three units. Of course the restrictions may apply for the connections allowed between the units and the treatment units for example in **Figure 2-1 (c)** T2 is only allowed to receive waste stream from unit P2 exclusively. Thus a similar ranking applies for treatment units given each Treatment unit's specification, capacity for treatment etc. Particularly in **Figure 2-1 (d)** we may see that it is frequent that treatment units may intervene in between two water using units according to the constrains set of each of the units. For example we may notice that T1 treatment unit lies between P1 and P2 water using unit. Regarding the problem to optimize existing waste water networks (WWN's) the P's play the role of all water using units within a household, premises, commercial, industrial agricultural recreational uses of buildings or may even represent a district or cluster of districts of households or units of different use, and generally infrastructure. On the other hand Treatment Units (TU's) may play the role of either some domestic adopted TU's which lie within the building envelopes or to be the central or peripheral Waste Water Treatment Plants (WWTP's) within the public grid of a sewerage system.

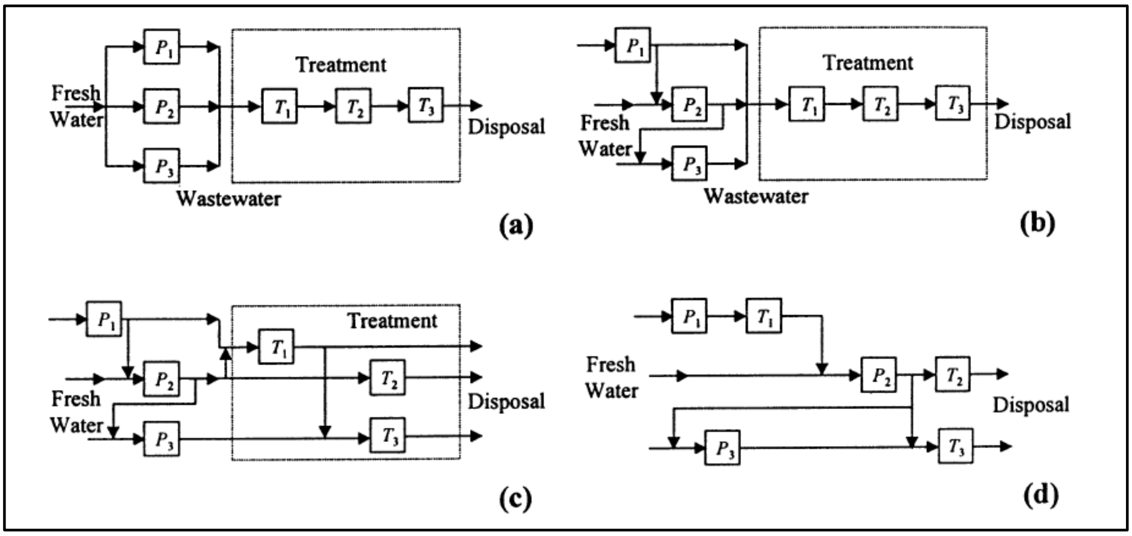


Figure 2-1
Combinations of Water utilization systems in process plants (Source: Miguel Bagajewicz, 2000)

In **Figure 2-2** the zero discharge theory is depicted. It can be seen here that there is a near closed-loop Waste Water Network (WWN) in which there is a fresh water supply to selected water using units (here denoted as Pi's) and theoretically the only discharged waste is the solid form of waste water. This can be performed through the treatment process. (In **Figure 2-3** these are denoted again as Ti' s). This network is similar to the version of **Figure 2-1** . The only difference is that the network seen in **Figure 2-2** produces reused and regenerated streams and non-liquid waste. The "Zero discharge" concept is the limit scenario according to which full regeneration of wastewater is realized within system where nearly exclusively non harmful solid waste is discharged. Of course literally 'zero' liquid waste disposal can only be approached theoretically. The theoretical idealization of this scenario cannot be easily implemented taking into account its extremely high cost. The concept of zero discharge (Bagajewicz , 2000) applies to the complete debar of the discharge of harmful contaminants for the environment and especially for the surface and underground water sources.

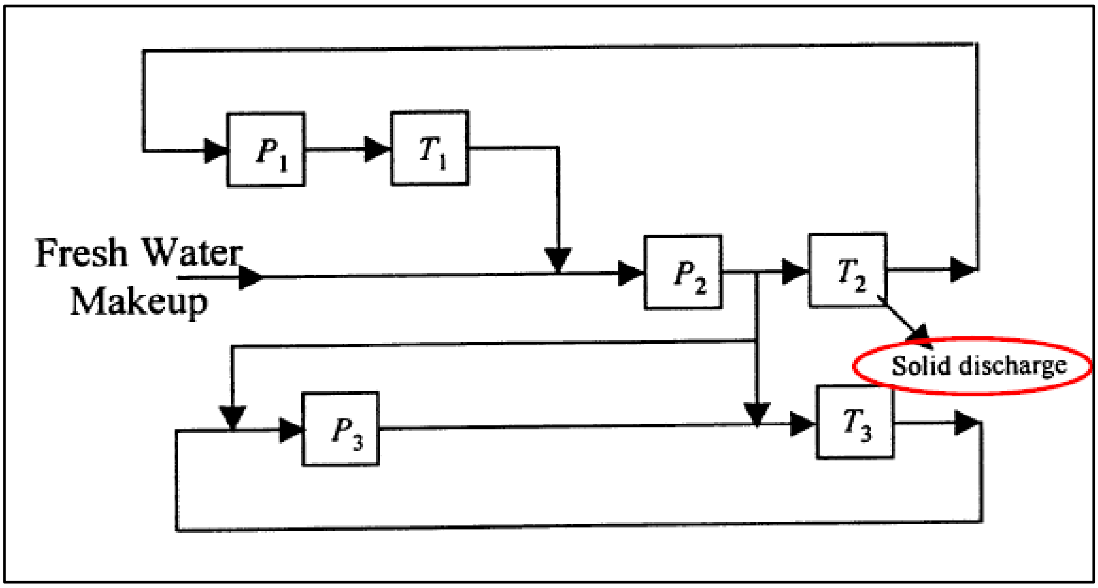


Figure 2-2: A scheme of zero liquid discharge (Source: Vasile Lavric, 2007)

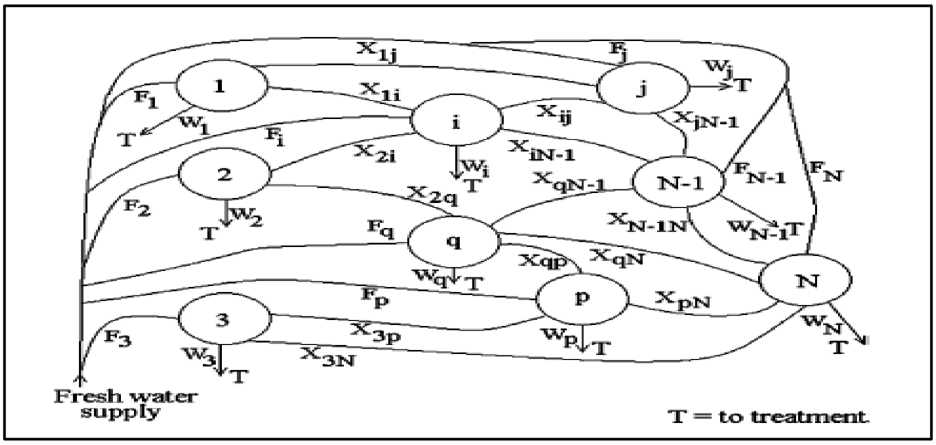


Figure 2-3

Oriented graph as a physical model of a plain Water Network (no water regeneration)

In **Figure 2-3** a so-called oriented graph can be seen. The oriented graphs are the representative illustrations of an optimized WWNT in the case of either one contaminant entering the network or several contaminant inflows. It is composed of different operation units. The usual constraint is that some of the units are restricted to be contaminant free, i.e. no influent stream coming into these units must contain contaminants. These are called free unit operations. This shape represents the classic version of such a network with no TU's among nodes. The nodes (the cyclic shapes in **Figures 2-3 and 2-4**), represent all sort of activities of the end users. **F_i' s** are the stream flows of fresh water from fresh water network (FWN) to the WWN. Numbers of nodes **1, 2,3...i, j, p, q, N-1, N** are the nodes that represent activities of end users in a network. Lines denoted with

X_i's between nodes represent all sort of internal streams of either fresh water, waste, regenerated and reused water among activities. **W_i' s** represent the waste effluent by each end user's activity. The indices denoted as **T_i' s** represent that these waste stream flows are directed to treatment in the corresponding local or central WWTP.

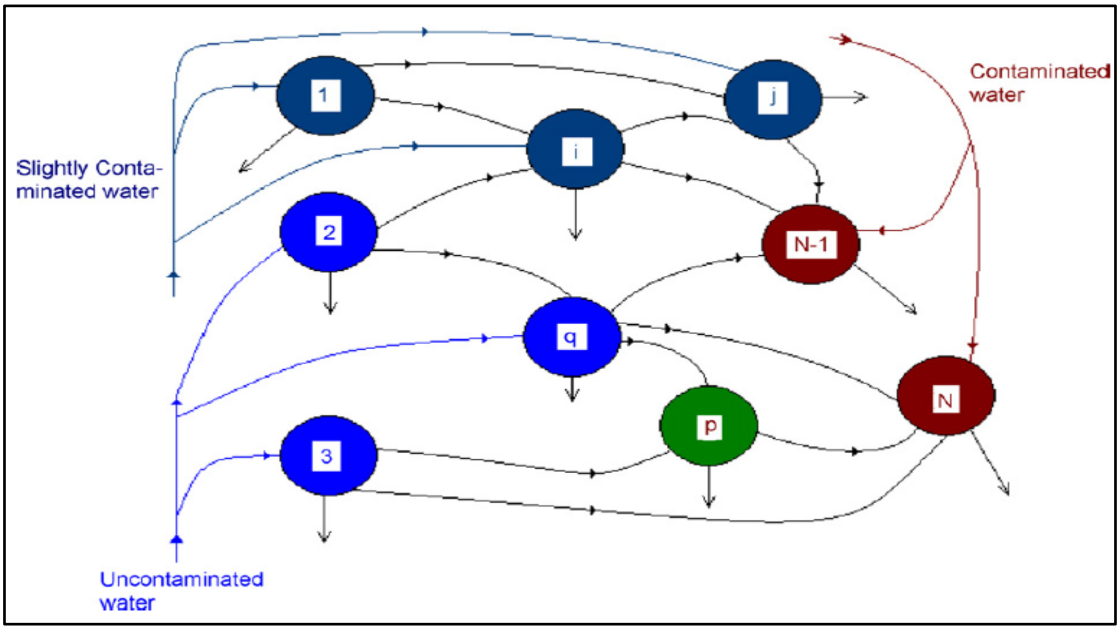


Figure 2-4
Lumping strategy for abstracting a water network supplied by several contaminated sources
(Valentin Ples, 2007)

In **Figure 2-4** there are different types of nodes such as the ones to receive only contaminants-free flows and are denoted as "2, 3, q" and receive only fresh water. In this case the water inflows to the network are of different levels of contamination which do not have an impact on the graph as it is still considered as oriented however the priority nodes are now lumped and given rank priority in relation to their incoming concentration constraints. Thus the sequence starts with the node of the most severe constraint therefore the least contaminated flow. This implies that above mentioned nodes should have priority as they are served by a non-contaminated source. Node denoted as "j" always represents the next node to be considered which receives slightly contaminated water stream. Node denoted as "i" is the previous node which also receives slightly contaminated water flows. Note that all such nodes are intermediate ones and receive also flows by previous nodes. For example the "next" node j receives both from the source as well as the previous node i. The same stands for i. However node 1 only receives from the source. These nodes thereafter send streams to the next units 'N-1' which in turn send waste to central WWTP. Note that nodes (N-1) (in deep red) and node p (in green) may be local TU's themselves of different specifications thus different capacity to treat waste and realize a first degree of treatment. A similar logic is behind the rest of the nodes in the Figure. We can also see that node 2 only receives contaminated free water load and sends flows both to node i belonging to different family of end users and to q of the same family thus user's consumption pattern.

The mathematical formulation for each node of the network can be expressed based on several conditions such as (i) overall mass load balances and (ii) influent and effluent constraints. The problem then can be solved as the number of the unknown decision variables will equal to the number of equations.

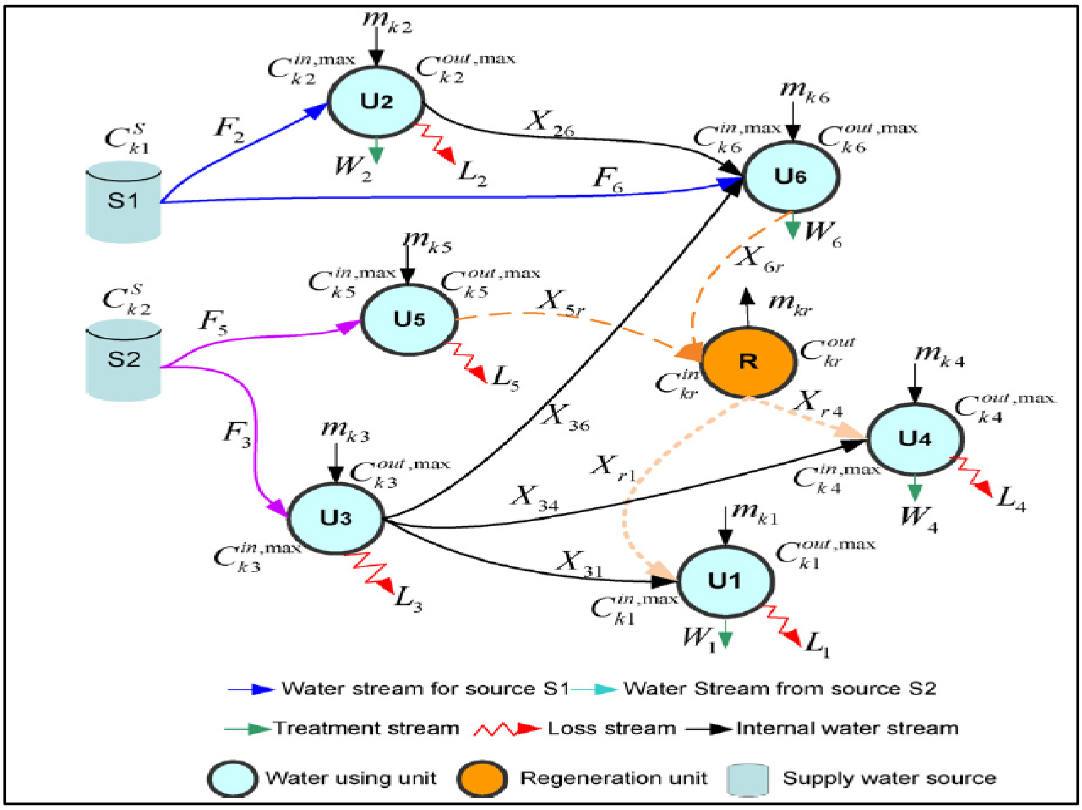


Figure 2-5
Physical model of Water Network, considering regeneration units (Source: Petrica Iancu, 2009)

Another approach is illustrated in **Figure2-5** which is the concept of reuse of Treated Municipal Wastewater (TMWW). These flows are redistributed back into central squares for park irrigation as well as for toilets of neighboring buildings and public/private premises (Aggeli, et al.2009 and Petrica Iancu, 2009). This approach could be implemented in all kinds of cases either in rural or urban ones. All sort of urban and partially urban activities such as household uses, car washing, street cleaning, irrigation and light agricultural reuse need to comply with all restrictions taken into consideration. TMWW is a relatively new concept which seems to be implemented in regions mostly in the vicinity of Waste Water Treatment Plants (WWTP's).

The usual water and waste water network regime is the multiple contaminants intake, their existence throughout the network and their effluent to sources which are considered as contaminated sources such as the surface underground waters, the rivers or the sea. The use of Genetic (GA) algorithms implies the most effective (optimum) design of this network (Vasile Lavric et al, 2005; Avila-Melgar, et al.2017). This design's efficiency is ranked according to several parameters such as the attempt to minimize the overall water supply. Constraints which relate to regulations that disallow certain streams and other uses need to also be taken into account towards the design of the optimum network. Previously mentioned oriented graphs (Valentin Plescu et al, 2007) depict such optimum designs for networks. A different approach is the combination of coupling a GA with linear programming methods (S. Shafiei et al, 2004, Ezzeldin, R et al. 2014). The first stage of this approach is to exhibit the completed sources and flow demands related network by a procedure which simulates to natural selection and adaptation.

A cost based criterion approach is defined as the optimized WWNT (Iancu et al. 2007, Bhattacharjee, K et al. 2017,) generates both a reduction capital along with the operating costs. The flows of single or multi contaminants water supply are both utilized in this approach towards optimum solution (Pellegrino, R. et al. 2017). Then, the optimum cost based approach compared to the minimum water supply approach can be evaluated through the use of a GA or its alternative versions. A GA ensures the concurrent observation of both these criteria (Iancu DA, et al. 2013 ; G Poplewski, et al. 2010 ; Verleye et al. 2013), Sheikholeslami, R et al. 2016, Shokoohi, M et al.2017, Bi, W.; et al. 2014, 2017, Avila-Melgar, et al.2017).

2.2 Reaching the best possible design with optimization tools

A family of algorithms has been proposed which are based on natural processes and these have been applied to a wide range of problems. Evolutionary Algorithms (EA) which result from this generated concept refer to a population of individual entities each of which comprise a candidate search spot within a set of contingent outputs in the context of a given question. Above mentioned algorithms espouse a sequence to conclude to the mostly well fitted entity out of the selected sets of entities along with redistribution operators. The most common known such algorithms comprise namely the (a) Evolutionary Programming (EP) (Fogel, Owens and Walsh, 1966), (b) Genetic Algorithms (GA) (Goldberg, 1989; Holland, 1975) and (c) the Evolution Strategies (ES) (Rechenberg, 1973; Schwefel, 1981, Broad, D.R et al. 2005) Back in sixties, the need of a cluster of biologists to model the natural evolution process (Barricelli et al, 1962) gave rise to the generation of the first evolutionary algorithm. GA and ES own attributes that distinguishes them from ordinary algorithms, these being: (a) while conventional algorithms use deterministic operators, the latter adopt hazard operators, i.e. mutation, selection and recombination, (b) Unlike usual algorithms which use a single design point, evolutionary algorithms adopt multiple such points within the set of decision variables and (c) continuous, discrete and mixed optimization cases may be worked out by EA. The latter attribute may enable implementation of both these EA on simultaneous computer contexts (Adeli and Cheng, 1994; Papadrakakis et al., 1998; Thierauf and Cai, 1995, Wu, W, Maier et al. 2016).

Generally, decision variables are mostly non-linearly dependent on the objective functions as well as their constraints. Gradient calculations effort involved in the mathematical programming based algorithms is significant.

According to Papadrakakis et al. (1998, 1999), the strong advantage of stochastic search algorithms is that despite the greater number of iterations required to come up with an optimum, computational efficiency is attained. Compared to mathematical programming algorithms, the former yield more economic computational analyses as no gradient data are required. In addition, global optimum would be reached through stochastic methodologies which would be attributed as more robust. The reason for that would be the random nature of their design variables. On the contrary, in mathematical programming algorithms local optima might be more likely to be found reached instead of the global one. However, mathematical programming methods tend to cope better with large numbers of constraints.

2.3 Evolutionary Optimization Strategy Algorithms (ES)

According to Rechenberg, Schwefel G. I. N. Rozvany & M. Zhou, (1993) , C. Y. Sheu & W. Prager, (1968) and Martinez-Bahena, B et al. 2017), Evolutionary Strategies (ES) were suggested to be suitable for problems dealing with parameter optimization. The different numerical depiction of design variables distinguishes GA from ES algorithms. Fixed-dimensioned byte strings are assigned to corresponding design variables, whereas ES are executed with the aid of vectors comprising real values. The use of genetic operators comprises another significant distinguish between these two methods. Both these methods adopt mutation and crossover as their basic operators. However, these operators play a different role in each one of them. In GA for instance, mutation is used to rebound lost alleles. On the contrary, in ES mutation operator is employed in an accumulating search iterative algorithm with step sizes denoted by σ or γ. Crossover operator highly contributes to both these methods towards extending existing range of values of population thus the set of variables. Furthermore, a different approach is adopted by these two methods regarding treatment against constrained optimization problems. While ES adopt the death penalty method, GA reaches a feasible output only with the aid of the Augmented Langrangian Method. Nevertheless, ES attain a high convergence performance, in relation to the GA, considering their self-adaptation search mechanisms to run real world problems (J. Holland, 1975).ES are today utilized throughout discrete as well as mixed optimization based cases (G. Thierauf & J. Cai, 1996; B. H. V. Topping & A. Bahreininejad, 1997). However, these algorithms were initially designed to run continuous optimization problems. Thierauf and Caiintroduce ES algorithms comprising the first exhibited ES algorithms to run in both regimes, either lying within discrete and/or continuous optimization set of design variables. (P. Pedersen, 1993) and (M. Papadrakakis, Y. Tsompanakis & E. Hinton & J. Sienz, 1996) first benefited from this capability to implement the concept of sizing within the domain of optimization problems of infrastructure. Furthermore different kinds of ES algorithms are put in a comparison subjected to certain tests (Coelloet al. 2006 ; M. Ehrgott and X. Gandibleux, 2008; W. K. Mashwani, 2013; C Lagos, et al. 2014).

2.4 Types of ES Algorithms

There are two kinds of Evolutionary Strategy (ES) Algorithms, These are namely the 'Two members' (2-ES) and the 'multi-members (M-ES) algorithms.

2.4.1 The two members-Evolutionary Strategies Algorithms (ES)

Mutation is the most fundamental concept to be taken as tool for working out parameter distributions. Another prominent tool for retrieving sequences of iteration comprises selection. Early version of ES consisted of population of one member.

A prominent role within all optimization algorithms related to the degree of convergence of an analytical method also plays the proper selection and further control of the step length denoted σ_ι. Depending on whether this step is picked up as too large, the output will depict only a rough estimate of the optimum whereas in the case of it being taken small then we may come up with an unnecessary large number of iterations towards optimal solution.

2.4.2 ES Algorithms composed of multiple members

Population size is one of the fundamental differences in ES. This family of algorithms comprise multiple members compared to the ones comprising only two members. In multiple members ES λ-generated members are produced out of a population which is comprised of μ source members.

All different kinds of multiple members ES's are the following:

2.4.2.1 (μ+ λ) in ES:

The fittest μ-members which are selected out of an interim population of (μ+ λ) members will form the ancestors of the $(g+1)^{th}$ iteration.

2.4.2.2 (μ, λ) in ES:

A new selection route might be attained by picking up a new cluster of λ descendants (where $\mu<\lambda$) exclusively, which in turn has been previously generated by corresponding μ ancestors.

The (μ, λ)-ES, converges faster than the type with a selection based on a single generation of descendants. This comprises an essential reason on the fact that this type of algorithm is more efficient when applied to dynamic oriented optimization problems which depend on time. According to the (μ, λ)-ES version concept, the optimum has to be preserved throughout the cycle of iterations. The weakness for this optimum to be maintained might entail possibility for the algorithm not to converge.

The process concludes as soon as the fracture $\boldsymbol{\mu_b/\mu}$ equals to $\boldsymbol{\varepsilon_d}$.

Where:

μ_b: *Number of ancestor vectors in existing batch selected to attain the closest to the optimum;*

μ : *Number of all ancestor vectors;*

ε_d: *A specific value ;*

$\tilde{s}$: *One of the interim ancestor vectors*

2.4.2.3 Contemporary ES (EC-ES). The (μ, λ, θ) ES

Generally speaking all Evolutionary Algorithms (EA's) are included within an engine of a domain called Evolutionary computation (EC). Schwefel and Rudolph (N. Olhoff, J. Rasmussen & E. Lund, 1992) have suggested a more generic ES algorithmic version. This was initially designed to be suitable to continuous problem cases, despite the fact that the aforementioned method has not been implemented to neither of the continuous or discrete cases (F. Moses, 1974).

In this kind of method, the descendants can only be produced from their ancestors one single time only within the ancestor's lifespan. In case that no descendant attains a more efficient or even same quantity of an advanced value of the objective function, then the expanded version (μ, λ, θ) implies that the ancestor is preserved throughout many iteration cycles.

The short version of notation implies a maximum generation of life span of descendants where $\theta \geq 1$ whereas the expanded version implies that the ancestor vector remains in a permanent basis throughout iterations. This kind of algorithm has $\theta = \infty$.

If $\mu \geq 1$, then $1 \leq \rho \leq \mu$

Where:

μ: *population of ancestors*

λ: *population of descendants*

p: *population of ancestors for every descendant.*

ϑ: *parameter which is dependent on the range of the ancestors vectors*.

This extended ES algorithm version has two main differences compared to the fundamental one, sawn previously, which are the following: First the basic version uses an infinite population of ancestors which participate in generation cycles with its values to range between 1 and μ and the range of the 'θ' parameter differs with respect to its range. Both Contemporary ES and Genetic Algorithms (GA) algorithms may have their selection operators similar.

2.4.2.4 Adaptive ES (A-ES)

According to the death penalty concept (Holland, 1975) all design points which do not lie within the feasible region of the problem, are debarred. This comprises handling of constraints related to the basic ES algorithm. This mechanism forces some of the proposed design values to be rejected as its solution lies just outside the feasible, thus accepted region. This leads to the loss of useful data for our model. The concept which is to be proposed within a later stage in this work is to input slacked constraint values throughout the preliminary stages of the search. As the process moves towards a global optimum point, these constraint values may start to become more stringent up to the point where they reach real values. Like the basic ES, implementation of A-ES follows the same route. First an initial population of ancestor vectors is adopted with a related design space created. A corresponding degree of violation is computed. In the case that ancestor vectors are found outside the related design set, then these ancestor vectors are forced to be adjusted to fit within feasible design region. The generated descendant vectors are then investigated whether they also fit to the feasible area for the corresponding degree of violation which has been already been determined. In every generation of descendants, a comparison of the objective functions values is realized based on the values of related ancestor and descendant vectors. The worst ones are debarred. The values of the descendant vectors remaining comprise the ancestor vector of the new iteration. This algorithm is repeated until the criterion of termination is attained.

Within this algorithm, the computation of the degree of violation is realized where the test for convergence is adopted. As seen in the inequality (2.1) convergence is attained as soon as the average value of the objective function of the defined design set converges to the realized design within current iteration. Mathematical expression for the test of convergence is expressed as follows:

$$\frac{\overline{F}^{(g)} - F_{best}^{(g)}}{\overline{F}^{(g)}} \leq \varepsilon_{ad} \qquad (2\text{-}1)$$

Where:

$\overline{F}^{(g)}$: *is the average value of the objective function*

$F_{best}^{(g)}$: *is the best value of the objective function of all ancestor vectors in the g-th generation,*

ε_{ad}=*is the upper limit space value where convergence occurs . Here set to 0.05.*

In **Figure 2-6** a flow chart of an A-ES algorithm can be seen. As seen, in this flowchart an implementation of above-mentioned algorithm into the problem of a wastewater treatment network is realized.

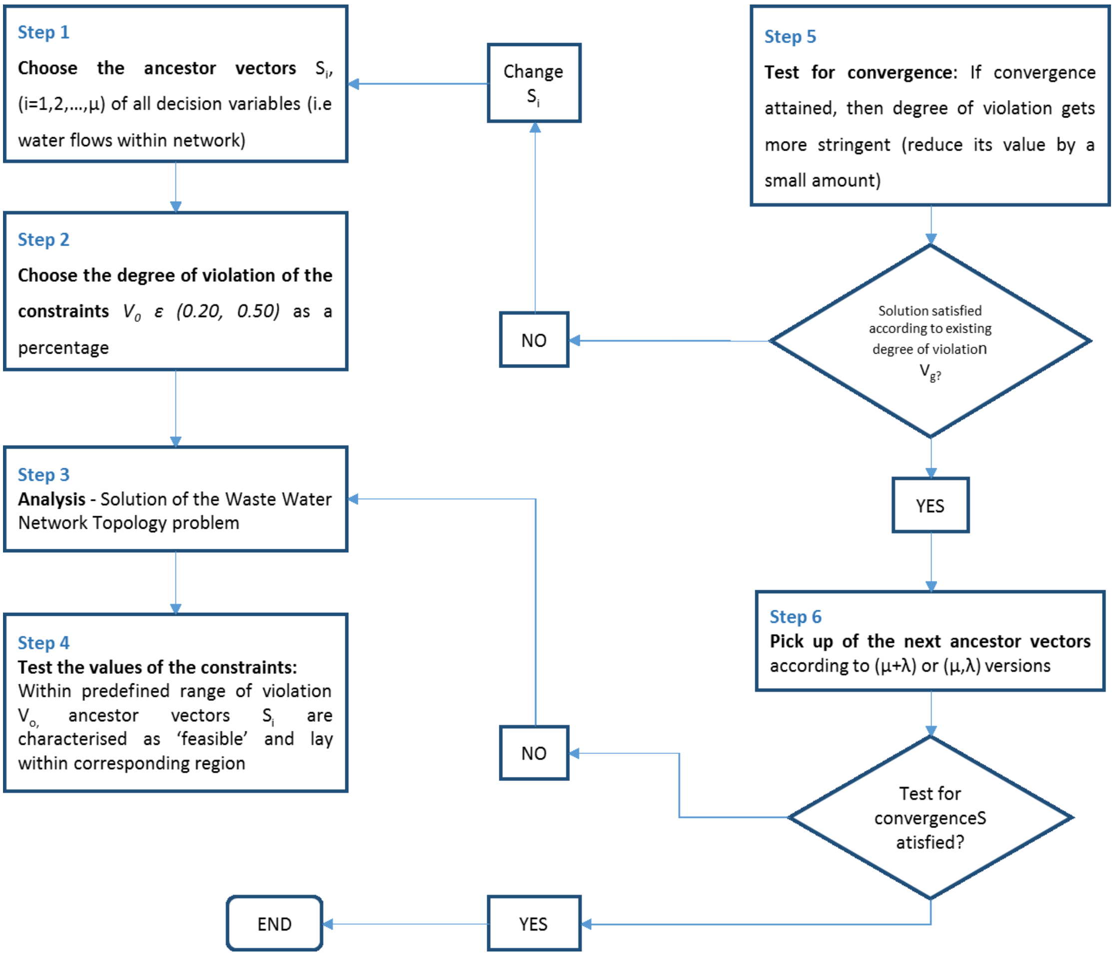


Figure 2-6 Flowchart of the Adaptive -ES algorithm implemented to a WWTN.

2.5 Genetic Algorithms (GA's)

The GA was first introduced by Holland et al. (1975). This type of Evolutionary Algorithms has gained thorough attention in the recent decades. Its mode of operation introduces a population oriented simulation which adopts different operators which actually comprises a choice out of the sample-recombination-mutation to develop. Genotype or chromosome is a term used to denote a single member-variable out of a set of variables of the sample of population with a range of values referring to real-valued bytes or binary ones. Throughout the decades since they were first introduced, GA's appeared in different forms with an aid to manage the number of constraints or downsized design variables' set of values of the whole population of variables. In this section, fundamental GA's are introduced in a theoretical basis as well as their most used versions.

The three main procedure steps of the fundamental GA's comprise:

Step 1: Initialization

The first step towards applying this algorithm is to derive the primitive population. This kick-off for creating a population is produced arbitrarily. Having completed this step, we evaluate the fitness function of each value-member of that primary population.

Step 2: Pick up

A median population is then generated out of the initial one adopting selection operators on the latter. Throughout the primary iteration we could assume that initial population would comprise the median population. In the iterations to follow the median population is considered as the primary one with the aid of selection operator and so on.

Step 3: Generation of Descendants

At this stage, a new chromosome is generated via a recombination as well as a mutation operator which is, in turns, implemented to the median population. As a result a next population is thus created. The recombination (or so called crossover) operator forms a new chromosome with similar attributes to the two parental ones whereas mutation, is also a reproduction operator which slightly reforms and eventually alters values of the parental ones towards a new formed chromosome. Above mentioned process from initial to median population is one generation within the series of development of a GA. In case that all prerequisites are met, this process ceases, otherwise it loops back to the first step.

2.5.1 The Basic Genetic Algorithms (GA's):

One of the most fundamental versions of Genetic Algorithms are the following:

a) Micro Genetic Algorithms (μ-GA)

These algorithms were first exhibited by (Krishnakumar et al., 1978). The primary aim of these algorithms is to downsize initial population. Nevertheless, the constraint of early convergence as well as weak performance that all GA's exhibit with poor sized populations, as non adequate data is provided had to be over passed. D. E. Goldberg, (1989) suggested a retriggering of the algorithm to utilize an updated population which would include optimal output already attained through the premature convergence.

As a result, the following slightly altered sequence was proposed by Krishnakumar et al. (1978).

Step 1: Initialization

There is a choice of either to utilize a random population of X magnitude or producing X-1 random strings plus 1 which has been previously picked up and this extra string is a well fitted one. It can either be selected based on the planner's judgment.

Step 2: Computation of fitness values

Evaluation of every single agent is involved towards compilation of the corresponding optimum string. The optimum one can be denoted as X and is conveyed with the same index to the next series of generated agents. The elitist strategy is therefore activated. This strategy ensures that all required preferred strings will be included in the new iterations.

Step 3: Generation of descendant vectors

The next stage is to select the X-1 remaining offspring agents of the new strings to be formed. These x-1 agents are picked up through the 'Tournament selection operator'. Crossover process is then implemented as soon as the former operator concludes.

Step 4: Test for Convergence

In case criteria are met, the algorithm terminates. If not, there are two kinds of verifications against theoretical convergence. The first is when there are binary strings present by verifying whether an eventual convergence is attained digit by digit. The latter will be a string comprising of real numbers, thus in this case check is performed by contrasting these numbers with decision variables. In case of convergence the process starts off from Step 1 otherwise it initiates by returning to Step 2.

b) Segregated GA's (S-GA's)

C. Maa & M. Shanblatt, (1992) are behind concept of the 'Segregated GA',(S-GA) which is the alternative compared to the Static penalties technique. This technique adopts utilization of more than one (two) static penalty parameters. These two values are assigned to two corresponding populations subjected to different degrees of constraints satisfaction. The most efficient agent within the groups to be examined is penalty parameter oriented. Above mentioned algorithm could be summarized in **Figure 2-7**:

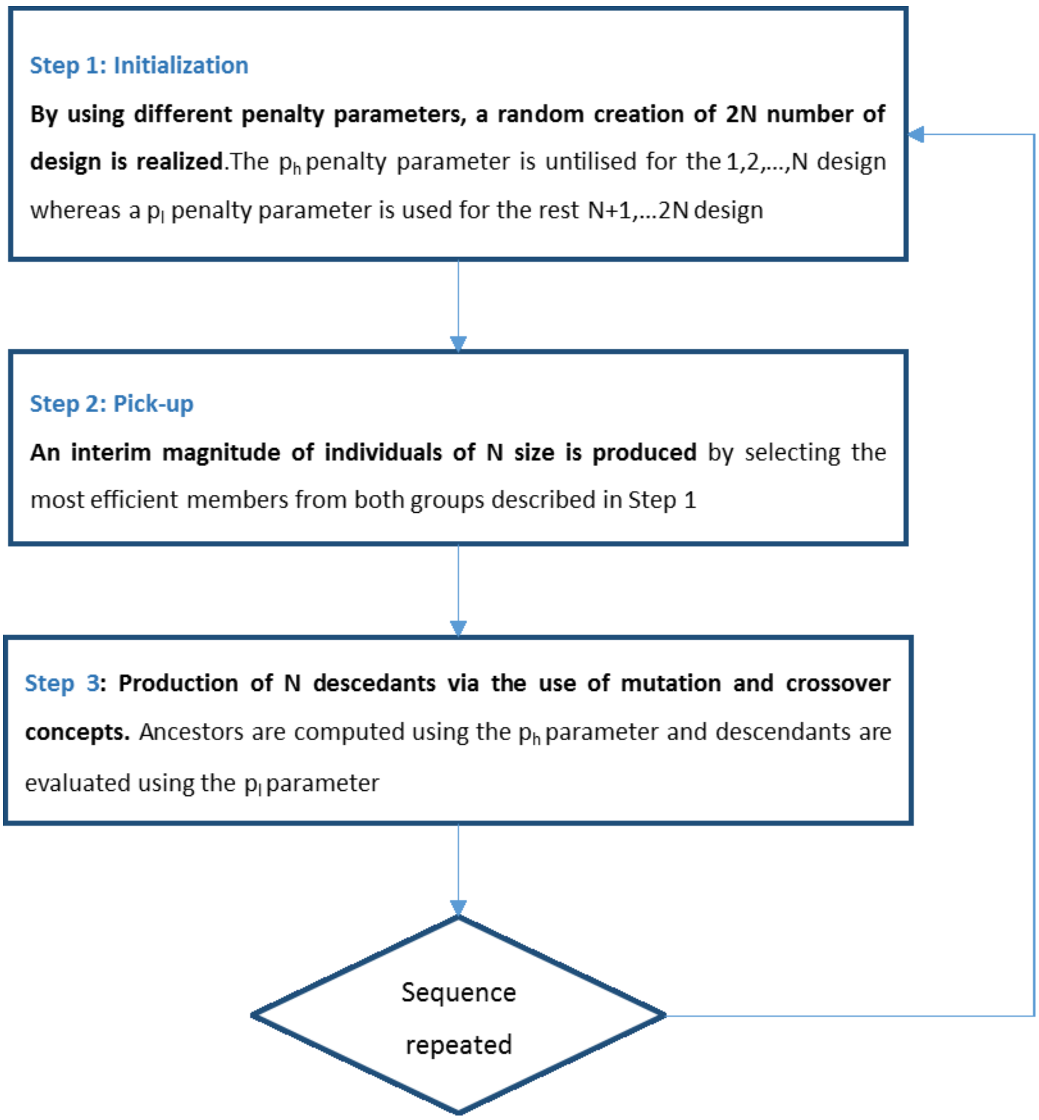


Figure 2-7 Flowchart of the Segregated Genetic Algorithm (S-GA)

2.5.2 Hybrid Optimization Algorithms

In the recent years there has been much research on optimization algorithms which are capable of utilizing both evolutionary algorithm techniques along with deterministic ones towards ~~a~~ more effective numerically optimization models where ES algorithms are combined with the sequential Quadratic Programming method (SQP). (Papadrakakis et al (2004); and (Khan,A.I., Topping,1993). In addition to this, a mix of ES programming algorithms along with the direction set methodology has also been utilized. (Waagen et al). Shape optimization cases found to be the most suitable kind of problems whereas hybrid algorithms were proved to be successful on shape optimization cases as suggested by Khan, A. I., Topping, B. H. V. and Bahreininejad, A., (1993). On the other hand, Waagen et al. suggested a different method related to non constrained test functions. According to Myung et al. an attempt to a slightly altered method than Waagen et al. was made, through constrained functions. A blend of the floating point based evolutionary concept with the method introduced by Maa and Shanblatt were used by Myung et al. The combination of these two methods was implemented to the optimum point which was extracted through the Evolutionary programming algorithm. This process is repeated until all components involved in the problem (decision variables, objective function and constraints) attain an overall balance.

An obvious attribute of the SQP oriented algorithms is to detect the fastest possible route to a local or global optimum independently of whether these algorithms attain these local or global points. The SQP method is sensitive to constraint violations thus when these occur, final convergence time of the process slows down. Unlike mathematical programming methods and their stringent mathematical constraints, EA's stochastic nature makes their process more time consuming yet more stable against possible local optima that may arise. This implies a better probability for EA's to converge in non convex optimization cases. Comparing the ES and GA's, it can be extracted that despite the fact that GA's exhibit a faster process within the non feasible region of the design set, they do not seem to always converge to feasible regions. The target of adopting a higher robustness and CPU efficiency can be achieved when both above mentioned methods are combined. This comprises a hybrid optimization approach to a problem.

There are two adopted approaches for the application of the combined algorithm methods: Regarding the first approach, the SQP is utilized with a result close to the desired optimum point. The second stage might possibly encompass the implementation of the ES algorithm which has as an effect on the acceleration of the convergence time and the avoidance of time consumed by the SQP due to its small violations around optimum and the consequent non stable nature.

The fraction correlating transition process between the two algorithms is realized under the following condition:

$$\left|\frac{f_{j+1}-f_j}{f_j}\right| \le \varepsilon \qquad (2.2)$$

Where:

fj+1 : is the value of the objective function in the next iteration

fj : is the value of the objective function in the previous iteration

ε : is the parameter correlated to algorithm convergence set to 0.01.

The above mentioned version is more effective in situations of a non convex design set. The non convexity in the design set can be noticed through the existence of an optimum point independently of the starting point in the design of the model. In a slightly altered version, the sequence of the used algorithms is reversed i.e. now the GA or an ES is used as first. By doing so, the global optimum region is first secured. In a second stage, the SQP method initiates in order that a more accurate optimum is attained.

In the altered version the transit from one algorithm to the other is realized when equation (2.2) regarding the optimum design of two consecutive iterations is getting smaller (ε=0.1). The second version comprises a more logical approach better fitted when more compound or non-convex problems arise. This kind of problem exhibits many local optima which are treated better when GA's are used as they deliver a faster convergence out of infeasible set region. As a final stage ES algorithms are also adopted after the use of the GA's so that an even improved solution is attained compared to the one found via GA's.

In relation to the higher level problem modeling there are different ways to model the water network: linear, Multi Integer Linear Programming (MILP), linear, non-linear, convex and non-convex. There are advantages and disadvantages to model one way or another. It depends on what extend are the variables interdependent and on our decision to which is taken as variables and constants. If the objective function and the set of constraints are linear functions of the decision variables, there is a high probability to a globally optimal solution reasonably quickly to be reached, given the size of the model. This is a linear programming problem; it is also a convex optimization problem (since all linear functions are convex). The Simplex Linear Programming (LP) solving method is also designed for these kinds of problems. If the objective and constraints are smooth nonlinear functions of the decision variables, solution times will be longer. If the problem is convex, there is a big chance to find a globally optimal solution. If it is non-convex, there is only a possibility to find a locally optimal solution – and even this may be hard to find.

There are also non-smooth and non-convex objective functions and constraints where only a good solution can be expected, with less chance for a locally or globally optimal solution. Thus, if the problem gets a nonlinear form, its solution quality may vary considerably upon an instance. Besides, nonlinear models are often resolved by linearization, however, this increases number of variables and constraints heavily. Therefore, if it is possible to avoid the non linear models, while making reliable assumptions, it would be recommended to stick to linear functions. Furthermore, computation time is usually less for linear models rather than that for nonlinear ones. In the second part of this project we model a selected waste water network (WWN) linearly, however a number of assumptions are made, applying it to different size of instances. There are all sorts of linear optimization solvers like CPLEX, Gurobi, Xpress and others. The history of water and waste water distribution systems their primary role in societies and today's unprecedented global water shortages present the challenge to exhibit nowadays more than ever before the need for the radical improvement of waste water treatment through the earliest stage process possible within the system's functional cycle (M.N.Kolevaa, et al. 2016). In PART I of the present work this early stage purification challenge is examined by taking into consideration four (4) common pollution indicators such as the COD, BOD, TSS and nitrate micro-pollutants within a typical household. To attain an even more realistic profile based on the earliest stage purification and optimized flow nexus among the process and treatment units within the household's scale, also two (2) common water purification appliances are incorporated i.e. a Micro-Filtration (MF) and a Reverse Osmosis (RO) unit. Non linear optimization methods are incorporated such as the SQP and interior point methods. Especially the SQP used in this work comprises a heuristic optimization method that still lacks satisfactory implementation on these kinds of problems within the early stage of design of water treatment (O.M. Awe 2019). Notions such as Resilience and the Optimal Location of Distributed Energy Systems such as waste water grid that are examined mainly in Part II of the present work are still open for research within literature as there is still not a clearly common used optimization simulation method for extended and undoubted use (Ormsbee, L.E. et al. 2006). Especially in PART II of this study, Benders Decomposition method, an optimization algorithm suitable for very large scale linear models is used as the layout optimization on systems parameters in the upscale level which is suitable for multi parametric combinatorial types of such problems . Therefore a mix of an early stage design of water treatment within the household micro-scale in combination with the set up of an upscale model using different scenarios of enhancement of MF and RO units is also examined and being attempted for the first time. Tackle of non-linearities by fostering the piecewise linearization method of the examined model are also been investigated within above mentioned problem regime.

Part I

Chapter 3. Methodologies and Solution Strategies

3.1 General

The initial phase which comprises the first part of this project is related to the mathematical model formulation of a typical domestic household in relation to the water and waste streams that comprise the overall Waste Water Network Topology (WWNT) at the domestic level of this single residential unit. Two new treatment units are selected to be used namely the Reverse Osmosis (RO) and the Microfiltration (MF) unit. These two treatment units are encompassed within household's building envelope. At a later stage, a simulation model of selected districts taking into account with different attributes and consumption profiles as well as restrictions and uncertainties within developed models will be realized. The mathematical formulation of the WNT can be described in the macroscopic view as a superstructure system which comprises multiple contaminants and different contaminated sources. As the simulated districts in our building model expand, new water and waste water mains are added to existing network. This has an obvious impact as less potable (fresh) water is supplied and apparently this may entail a reduction in the overall final outlet waste water (WW) discharged.

3.2 Problem Specification

The initial stage of this work is to undertake a comparison study of the minimum fresh water supplies of a water network system regarding a typical household of 4 occupants with either the use of linear or non linear Mathematical Programming Approach (MPA) and to generate optimum design of these netwrks via advanced algorithms such as GA's or ES's. These methods are implemented by the use of two different computing platforms, such as GAMS and MATLAB. In the case of urban network design either in the household or the greater district scale which we examine in the second part of this project, there are specific waste water constituents that must be constrained. These comprise chemical, physical and biological characteristics within water flows in the network used at as limitation functions. This work, in the second part, has a target to develop a superstructure model at the scale of a district with use of the above mentioned tools.

A comparison study will be realized comparing the outcomes which will be the agreed objective functions under existing restrictions. GA's assign every internal water flow to a gene. The whole built cluster of operation units will comprise the corresponding chromosome. According to the given constraints of the network based on their maximum or minimum flows or range of allowable contaminants' concentrations, the so called genes may be either allowed or rejected. This will depend on whether each one of these genes lies inside or outside the feasible region. With the use of the one-point crossover method as discussed earlier, the genes within the network interact based on the frequency of their random pick-up. The next step is to implement mutation on all randomly selected genes. The final step is the minimization of the objective function which comprises the overall fresh water supply to Network under study.

3.3 Model Development of a single operational Unit (household)

The model development of a wasterwater network atthe household level is realised with the use of typical data based on average user activity habits, including 5 different operational units namely being (1) the kitchen sink, (2) wash basin, (3) toilet, (4) bath and (5) shower/WC, with basic contaminant constituents such as Biochemical Oxygen Demand (**BOD**), Biochemical Oxygen Demand (**COD**), Total Suspended Solids (**TSS**), Nitrates (mainly Ammonia **NH3**) and Total Phosphate (**Total-P)**, as well as two typical domestic treatment systems such as the Reverse Osmosis (**RO**) and the Microfilatration (**MF**) units. After scaling up data in **in Tables 3-26** to **Table 3-28** (See Appendix-List of Tables Part I) this set of data are extracted and presented in **Tables 3-29 to Table3-35** (Appendix-List of Tables Part I) which are taken from different sources in the literature. At this point it is required that the nomeclature of all abreviations and symbols set of variables, constraints, parameters of the developed mathematical models which follow in the next sections is added here. At this point It should be specified that throughout Chapter 3 and 4 all flows are denoted witn indices that indicate flows coming from the denoted second index towards the first..For example xij's imply flow from j to i.Unlike Chapter 5 and on where all similar such indices imply flow coming from the first index towards the latter, for instance, the corresponding variable i.e Xij implies flow from i to j

Mathematical Models Notation

Abbreviations:

Symbol	Description
BOD	Biochemical Oxygen Demand
TSS	Total Suspended Solids
Total – P	Total Phoshate contaminants
NO3	Nitrates contaminants

Table 3-1 Abreviations

Sets:

Symbol	Description	Variables
I	{any water using operation } {1,2,3,4,5} {KITCHEN S, BATH W, WASHING M, WC }	i, j
T	{any treatment unit } {MF, RO}	t, t_1
K	{any contaminant present in the water } {BOD, TSS, Total – P, NO3}	k
Q	{any unit } $I \cup T$	q, r

Table 3-2 Sets

Design Variables:

Symbol	Subscripts	Description
F_i	$i \in I$	Fresh water flow from mains into using process i
W_i	$i \in I$	Waste water flow from using process i
$X_{q,r}$	$q, r \in Q$	Flow rate from unit r to unit q.
V_q^{IN}	$q \in Q$	Flow rate in to unit q
V_q^{OUT}	$q \in Q$	Flow rate out of unit q
$M_{q,k}^{IN}$	$q \in Q$ $k \in K$	Mass of contaminant k in to unit q
$M_{q,k}^{OUT}$	$q \in Q$ $k \in K$	Mass of contaminant k out of unit q
$C_{q,k}^{IN}$	$q \in Q$ $k \in K$	Concentration of contaminant kin to unit q
$C_{q,k}^{OUT}$	$q \in Q$ $k \in K$	Concentration of contaminant k out of unit q

Table 3-3 Design Variables

Auxiliary Variables:

Symbol	Expression	Description
R_i	$\sum_{t\in T} X_{i,t}, i \in I$	Overall flow rate from treatment units to water using operation i
T_i	$\sum_{t\in T} X_{t,i}, i \in I$	Overall flow rate from water using operation i to treatment units
R_t	$\sum_{i\in I} X_{i,t}, t \in T$	Overall flow rate from treatment unit t to all using operations
T_t	$\sum_{t\in T} X_{t,i}, t \in T$	Overall flow rate from all using operations to treatment unit t
$D_{i,j}$	$\sum_{\substack{j\in I\\ j\neq i}} X_{i,j}, i \in I$	Overall flow rate from all using operations to water using operation unit i
$D_{j,i}$	$\sum_{\substack{j\in I\\ j\neq i}} X_{j,i}, i \in I$	Overall flow rate from water using operation unit i to all using operations
C_k^T	$\frac{\sum_{t\in T}\left(\sum_{i\in I} X_{i,t}\right) C_{t,k}^{OUT}}{\sum_{t\in T}\left(\sum_{i\in I} X_{i,t}\right)}$, $k \in K$	Concentration of each contaminant k in the combined flow from the treatment units to the water using process units
$C_{i,k}^T$	$\frac{\sum_{t\in T} X_{i,t} C_{t,k}^{OUT}}{\sum_{t\in T} X_{i,t}}$, $i \in I, k \in K$	Concentration of each contaminant k in the combined flow from the treatment units to the each water using process unit i
V_i^{min}	$\max_{k\in K} \frac{M_{i,k}}{C_{i,k}^{OUT,max} - C_{i,k}^{IN,max}}$ $i \in I$	Minimum water flow rate for each water using process i

Table 3-4 Auxiliary Variables

Parameters:

Symbol	Subscripts	Description
$M_{i,k}$	$i \in I$ $k \in K$	Mass load of contaminant k at the i water using operation
$M_{t,k}$	$t \in I$ $k \in K$	Mass removal of contaminant k at the t treatment unit
$r_{t,k}$	$t \in I$ $k \in K$	Removal ratio of contaminant k at the t treatment unit
$C_{i,k}^{IN,max}$	$i \in I$ $k \in K$	Maximum inlet concentration of contaminant k into water using process i
$C_{i,k}^{OUT,max}$	$i \in I$ $k \in K$	Maximum outlet concentration of contaminant k into water using process i
C_k^F	$k \in K$	Concentration of contaminant k present in fresh water stream
$C_k^{T,max}$	$k \in K$	Concentration of contaminant k present in the regenerated water stream
$\text{Bleed} - \text{off factor}$		Factor with respect to waste water in excess

Table 3-5 Parameters

3.3.1 Developing the mathematical models

Mathematical models are developed using the flow rate balance and mass balance for each water using process unit and treatment unit. In general for a process unit we get : (See Figures 3-1 and 3-2).

$$V_i^{IN} = V_i^{OUT} = V_i \qquad (3\text{-}1)$$

$$M_{i,k}^{IN} + M_{i,k} = M_{i,k}^{OUT} \qquad (3\text{-}2)$$

$$C_{i,k}^{IN} = \frac{M_{i,k}^{IN}}{V_i^{IN}} \text{ and } C_{i,k}^{OUT} = \frac{M_{i,k}^{OUT}}{V_i^{OUT}} \quad (3\text{-}3)$$

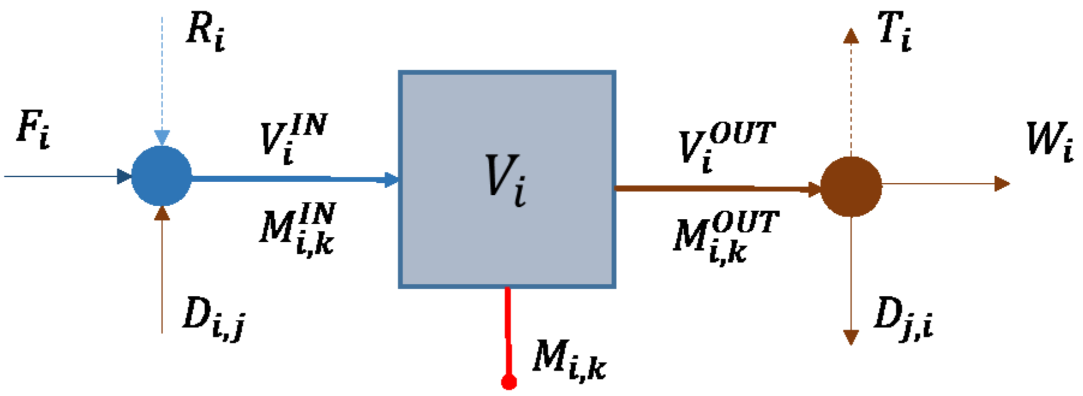


Figure 3-1 Process unit inflow and outflow schematic

For each treatment unit the equations are as follows:

$$V_t^{IN} = V_t^{OUT} = V_t \quad (3\text{-}4)$$

$$M_{t,k}^{IN} - M_{t,k} = M_{t,k}^{OUT} \quad (3\text{-}5)$$

$$C_{t,k}^{IN} = \frac{M_{t,k}^{IN}}{V_t^{IN}}, C_{t,k}^{OUT} = \frac{M_{t,k}^{OUT}}{V_t^{OUT}} \text{ and } C_{t,k}^{OUT} = (1 - r_{t,k})C_{t,k}^{IN} \quad (3\text{-}6)$$

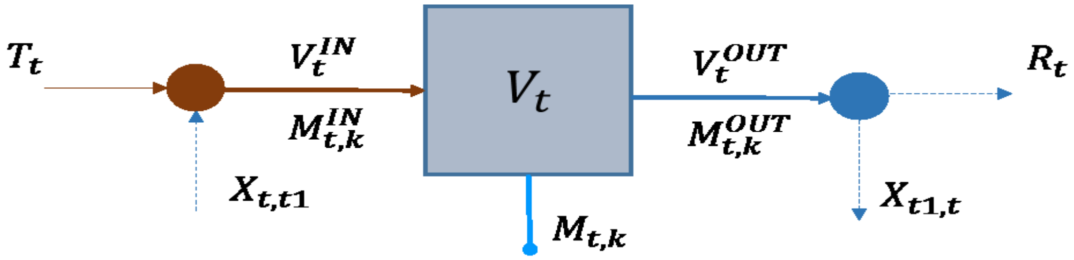


Figure 3-2 Treatment unit inflow and outflow schematic

3.3.2 Analyzing the system

Some comments can be made in order to analyze the expected behavior of the system regarding the minimum usage of fresh water. If an assumption is made that the treatment units and the process units can handle any flow rate, no matter how large, these can produce regenerated water with a concentration within the limiting inlet concentration of the process units. This is not the case for the kitchen sink process unit, for which the limiting inlet concentration for BOD and TSS contaminants is 0 indicating that there will be need for an infinite flow rate of regenerated water or 100% removal ratio of the treatment units in order to achieve zero concentration. Based on this, the kitchen sink inflow must be produced only with fresh water (BOD = 0, TSS = 0, Total-P =1 mg/l, NO_3 = 10mg/l. Furthermore, the minimum flow rate can be estimated from inlet and outlet concentrations, the minimum flow rate can be estimated from inlet and outlet concentrations in **Table 3-36** and **Table 3-37** (Appendix A- List of Tables -Part I) as well as the average mass load in **Table 3-34** (Appendix A- List of Tables -Part I) of the kitchen sink as:

$$V_{1,k}^{min} = \frac{M_{1,k}}{C_{1,k}^{OUT,max} - C_{1,k}^{IN,max}}, \quad k \in K \qquad (3\text{-}7)$$

Substituting corresponding values into the equation (3.7) we get for each contaminant the required flow rates according to **Table 3-6:**

	BOD	TSS	Total-P	NO3
Flow Rate	72.00	72.00	72.10	72.07

Table- 3-6 Flow rate contaminant requirements

From **Table-3-6** it can be seen that the flow rate in the kitchen sink should be at least 72.1 L/Day. Thus the first lower limit of fresh water usage is 72.1 L/Day.

Another constraint of the system is that the outflow of the WC is restricted to be directed to other process or treatment units, but it will be waste water. Since this stream leaves the system in each cycle, it must be replaced with fresh water. An assumption of two cases for inlet concentration of WC process unit is made. The first case will comprise inlet concentration equal to the limiting concentration of this unit whereas the second includes (theoretically) zero concentrations of all contaminants being denoted as “C” whereas total contaminants quantities containing Phosphorus P is denoted as “Total P” and all Nitrates contaminants denoted as “NO3” in the following table.

Results are presented in the following Table:

	BOD	TSS	Total-P	NO3
Flow Rate (zero C)	139.16	137.79	115.84	71.91
Flow Rate (max C)	144.00	144.00	144.14	143.93

Table 3-7 WC Flow rates (L/day)

In **Table 3-7** it can be seen that the flow rate in the WC should be at least 139.16 L/Day, even in the case of zero inlet concentrations. Since this flow is all waste water, a new lower limit of fresh water usage is 139.16 L/Day.

Furthermore, in order to control the build-up of trace contaminants in a process serviced only with recycled streams we bleed off some of the outflow of this unit. This bleed-off should be replaced with fresh water.

Finally, the maximum flow that process units and treatment units can handle must be taken in consideration in order to have a realistic estimation of the flows rates in the household unit system.

3.3.3 Model A1

As shown in **Figure 3-3** in the schematic set up the first model we developed attempts to minimize fresh water supply and wastewater discharge using two treatment units and the following assumptions:

- **A1. Treatment units are connected in serial.**
  - The overall treatment flow is the inlet of treatment unit MF.

$$V_{MF}^{IN} = \sum_{i \in I} T_i \qquad (3-8)$$

  - Outlet from treatment unit MF is the inlet of treatment unit RO.

$$V_{MF}^{OUT} = V_{RO}^{IN} \qquad (3-9)$$

- Outlet of treatment unit #2 (RO) is the regenerated flow

$$V_{RO}^{OUT} = \sum_{i \in I} R_i \qquad (3-10)$$

- No flow is redirected from treatment unit to itself or other treatment units

$$X_{t,t_1} = 0 \text{ and } X_{t,t} = 0, \qquad t, t_1 \in T \qquad (3-11)$$

- **A2. It is assumed that waste flow W_i is to equal Bleed-Off factor times the flow through each using unit.**

$$W_i = \text{Bleed Off x } V_i, \qquad i \in I \qquad (3-12)$$

- **A3. No direct reuse. No direct flow from one using unit to another.**

$$X_{ij} = 0, \qquad i, j \in I \qquad (3-13)$$

- **A4. Kitchen sink uses only fresh water**

$$R_1 = 0 \qquad (3-14)$$

- **A5. No fresh water for WC**

$$F_5 = 0 \qquad (3-15)$$

- **A6. No treatment of water flow from WC**

$$T_5 = 0 \qquad (3-16)$$

- **A7. Constant mass load on water using process**

$$M_{i,k} = \text{constant}, \qquad i \in I, k \in K \qquad (3-17)$$

- **A8. No flow rate upper limits exist for process and treatment units**

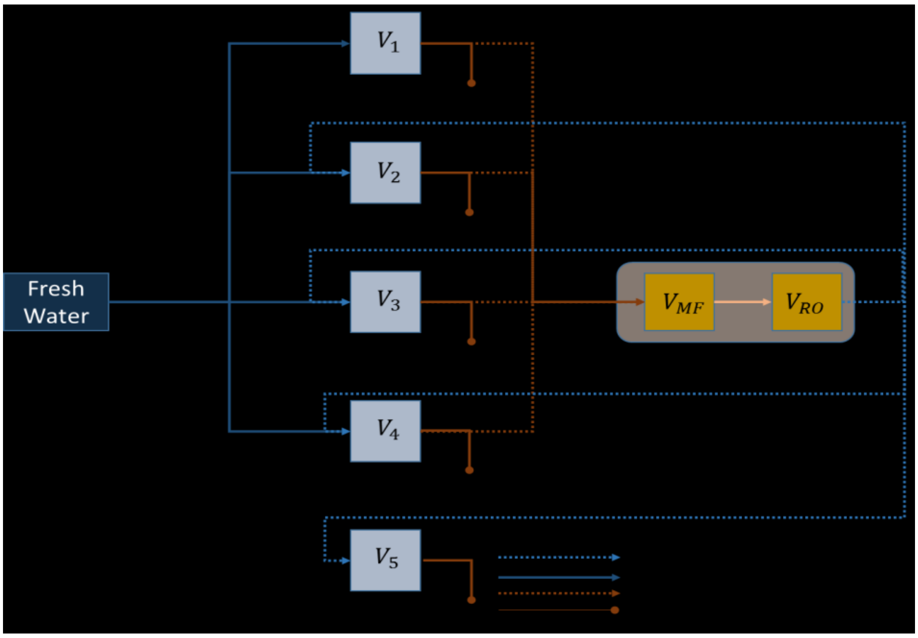


Figure 3-3 Schematic of Model A1

3.3.3.1 Formulation of the problem

According to the balance equations (3-1)

–(3-6) as well as the above assumptions (equations 3-8 – 3-17 and assumption A8) we formulate the mathematical model A1 and the selected design variables vector is:

$$\theta = [V_1 V_2 V_3 V_4 V_5 R_2 R_3 \ R_4 C_{BOD}^T \ C_{TSS}^T \ C_{Total\ -P}^T C_{NO\,3}^T] \qquad (3-18)$$

Flow rate

The inflow of each process unit is equal to the sum of fresh water and regenerated water, i.e. $V_i^{IN} = F_i + R_i = V_i$. Given the flow through each unit and the regenerated water we calculate fresh water flow at each unit as:

$$F_i = \begin{cases} V_1 & i = 1 \\ V_i - R_i & i = 2,3,4 \\ 0 & i = 5 \end{cases} \qquad (3-19)$$

Where assumptions **A4** and **A5** have been used.

Assuming that the waste flow is equal to a factor of the flow through the unit (A2) and that water flow from WC is not treated we get:

$$W_i = \begin{cases} \text{BleedOff x } V_i & i = 1,2,3,4 \\ V_i & i = 5 \end{cases} \qquad (3-20)$$

Finally, water flow to treatment unit T_i is calculated from the flow balance at each unit:

$$V_i^{IN} = V_i^{OUT} = V_i, \; i \in I \qquad (3-20a)$$

$$V_i = W_i + T_i, \quad i \in I \qquad (3-20b)$$

$$T_i = V_i - W_i, \quad i \in I \qquad (3-21)$$

Concentrations

Assuming no direct reuse between using units (A3) the inflow concentration for each unit is:

$$C_{i,k}^{IN} = \frac{F_i C_k^F + R_i C_k^T}{F_i + R_i}, \quad i \in I, k \in K \qquad (3-22)$$

If the mass loads of each unit are assumed constant (A7) then the outlet concentration at each using unit is:

$$C_{i,k}^{OUT} = C_{i,k}^{IN} + \frac{M_{i,k}}{V_i}, \quad i \in I, k \in K \qquad (3-23)$$

Then the inlet concentration at the system of treatment units is:

$$C_{MF,k}^{IN} = \frac{\sum_{i=1}^{N_{Units}} T_i C_{i,k}^{OUT}}{\sum_{i=1}^{N_{Units}} T_i}, \quad k \in K \qquad (3-24)$$

Nevertheless, assuming a pre-described removal ratio for each treatment unit then:

$$\left(1 - r_k^{RO}\right)\left(1 - r_k^{MF}\right) C_{MF,k}^{IN} = C_{RO,k}^{OUT} = C_k^T, \quad k \in K \qquad (3-25)$$

Following the above analysis the problem can be formulated as follows:

Constraints

- Concentration at treatment units. Equating $C^{IN}_{MF,k}$ at equation (3-24) and(3-25) we get:

$$\frac{\sum_{i\in I} T_i C^{OUT}_{i,k}}{\sum_{i\in I} T_i} = \frac{C^T_k}{(1-r^{RO}_k)(1-r^{MF}_k)}, \qquad k \in K \qquad \text{(3-26)}$$

- Flow balance at treatment unit

$$\sum_{i\in I} T_i = \sum_{i\in I} R_i \qquad (3-26\text{ a})$$

$$\sum_{\substack{i\in I\\ i\neq 5}} (1-\text{BleedOff})V_i + T_5 = R_1 + \sum_{\substack{i\in I\\ i\neq 5\\ i\neq 2}} R_i + R_5 \qquad (3-26\text{ b})$$

$$\sum_{\substack{i\in I\\ i\neq 5}} (1-\text{Bleed Off})V_i - V_5 - \sum_{\substack{i\in I\\ i\neq 5\\ i\neq 2}} R_i = 0 \qquad (3-27)$$

- Regenerated Water is less or equal to the inflow

$$V_i \geq R_i,, \qquad i \in I \qquad (3-28)$$

- Regenerated concentration is less than pre-described

$$C^T_k < C^{T,max}_k, \qquad k \in K \qquad (3-29)$$

- Using unit input concentration is less than maximum

$$C^{IN}_{i,k} < C^{IN,max}_{i,k}, \qquad i \in I, k \in K \qquad (3-30)$$

Since

$$C^F_k < C^{IN,max}_{i,k} \qquad (3-30\text{a})$$

$$\text{if} \quad C^{T,max}_k < C^{IN,max}_{i,k} \qquad (3-30\text{b})$$

Then

$$C^T_k < C^{T,max}_k < C^{IN,max}_{i,k}, \qquad i \in I, k \in K \qquad (3-30\text{c})$$

$$C^F_k < C^{IN,max}_{i,k}, \qquad i \in I, k \in K \qquad (3-30\text{d})$$

From equations (3.3-3.6) we have:

$$C_{i,k}^{IN} = \frac{F_i C_k^F + R_i C_k^T}{F_i + R_i} = \frac{F_i}{F_i + R_i} C_k^F + \frac{R_i}{F_i + R_i} C_k^T \qquad (3-30e)$$

$$C_{i,k}^{IN} \overset{(12)}{\leq} \frac{F_i}{F_i + R_i} C_{i,k}^{IN,max} + \frac{R_i}{F_i + R_i} C_{i,k}^{IN,max} = C_{i,k}^{IN,max} \quad (3-30f)$$

So, in order to assure that $C_{i,k}^{IN} < C_{i,k}^{IN,max}$ thus it is sufficient to have

$$C_k^{T,max} < C_{i,k}^{IN,max}, \qquad i \in I, k \in K \qquad (3.30g)$$

- Using unit out concentration is less than maximum

$$C_{i,k}^{OUT} = C_{i,k}^{IN} + \frac{M_{i,k}}{V_i} < C_{i,k}^{OUT,max}, \quad i \in I, k \in K \qquad (3.30h)$$

Since $$C_{i,k}^{IN} < C_{i,k}^{IN,max} \qquad (3.30\ i)$$

it follows that:

$$\frac{M_{i,k}}{V_i} < C_{i,k}^{OUT,max} - C_{i,k}^{IN,max} \qquad (3.30\ j)$$

Consecutive, in order to assure that

$$C_{i,k}^{OUT} < C_{i,k}^{OUT,max} \qquad (3.30\ k)$$

it is sufficient to have

$$\max_{k \in K} \frac{M_{i,k}}{C_{i,k}^{OUT,max} - C_{i,k}^{IN,max}} < V_i, \qquad i \in I \qquad (3.31)$$

Objective Function

An attempt is made to minimize the fresh water supply so the objective function will be

$$\min_{\theta} \sum_{i \in I} F_i = \min_{\theta} V_1 + \sum_{\substack{i \in I \\ i \neq 5 \\ i \neq 2}} V_i - R_i \qquad (3.32)$$

In conclusion, the problem can be stated as:

$$\min_{\theta} \sum_{i \in I} F_i = \min_{\theta} V_1 + \sum_{\substack{i \in I \\ i \neq 5 \\ i \neq 2}} V_i - R_i \qquad (3.33)$$

Where

$$\theta = \left[V_1 V_2 V_3 V_4 V_5 R_2 R_3\ R_4 C_{BOD}^{T}\ C_{TSS}^{T}\ C_{Total\ -P}^{T} C_{NO\,3}^{T}\right]$$

With respect to the following linear constraints

$$V_i > \max_{k \in K} \frac{M_{i,k}}{C_{i,k}^{OUT\,,max} - C_{i,k}^{IN,max}}, \qquad i \in I \qquad \text{(a)}$$

(b)

$$C_k^T < C_k^{T,max} < C_{i,k}^{IN,max}, \qquad i \in I, k \in K$$

(c)

$$V_i \geq R_i, i \in I$$

$$\sum_{\substack{i \in I \\ i \neq 5}} (1 - \text{Bleed Off}) V_i - V_5 - \sum_{\substack{i \in I \\ i \neq 5 \\ i \neq 2}} R_i = 0 \qquad \text{(d)}$$

And non linear constraints

$$\frac{\sum_{i \in I} T_i C_{i,k}^{OUT}}{\sum_{i \in I} T_i} = \frac{C_k^T}{\left(1 - r_k^{RO}\right)\left(1 - r_k^{MF}\right)}, \qquad k \in K \qquad \text{(e)}$$

3.3.3.2 Results

The above problem is formulated in MATLAB using the function |problem solution|

First the problem was solved by setting the Bleed Off factor to 0. The SQP algorithm of |fmincon| solver of MATLAB was used with initial values of design variables as presented in **Table 3-8**. In **Figure 3-4** the SQP solver required only 19 major iterations with 248 function evaluations to solve the problem. The estimated minimum fresh water is 144.14 L/Day where the regenerated water is 2364.95 L/Day. The regenerated water concentration is (4.20, 0.01, 0.05, 0.21) indicating that BOD concentration at its upper allowable limit, where other contaminants are far below. The flows and concentrations at each process thus water using unit for different bleed-off factors are presented in **Tables 3-36 to Table 3-37** (Appendix A List of Tables Part-I) and the estimated network is seen in **Figure 3-5 .**

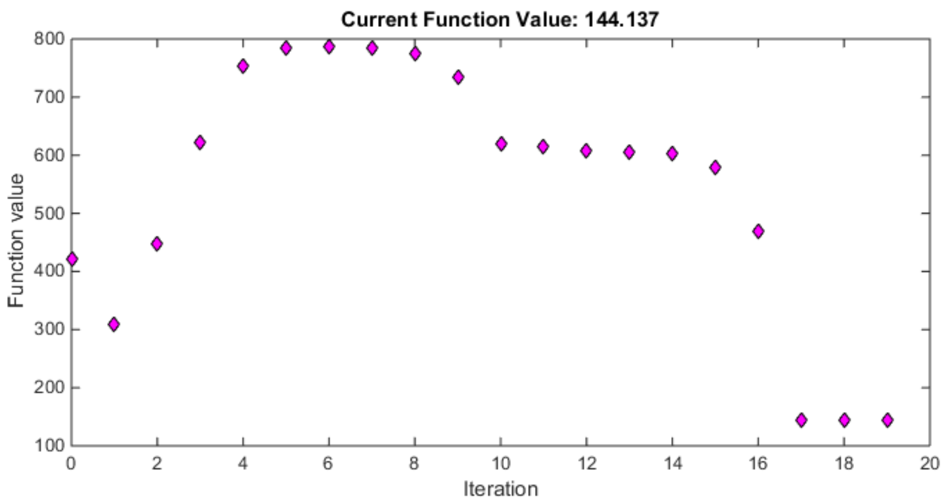


Figure 3-4 Function value for each iteration using SQP algorithm of the fmincon solver

Design variable	Initial Value	Lower Bound	Upper Bound
V_1	V_1^{min}	V_1^{min}	inf
V_2	V_2^{min}	V_2^{min}	inf
V_3	V_3^{min}	V_3^{min}	inf
V_4	V_4^{min}	V_4^{min}	inf
V_5	V_5^{min}	V_5^{min}	inf
R_2	0	0	inf

R_3	0	0	inf
R_4	0	0	inf
C_{BOD}^{T}	$C_{BOD}^{T,max}$	0	$C_{BOD}^{T,max}$
C_{TSS}^{T}	$C_{TSS}^{T,max}$	0	$C_{TSS}^{T,max}$
$C_{Total-P}^{T}$	$C_{Total-P}^{T,max}$	0	$C_{Total-P}^{T,max}$
C_{NO3}^{T}	$C_{NO3}^{T,max}$	0	$C_{NO3}^{T,max}$

Table 3-8 Initial values and bounds of the design variables for Model A1

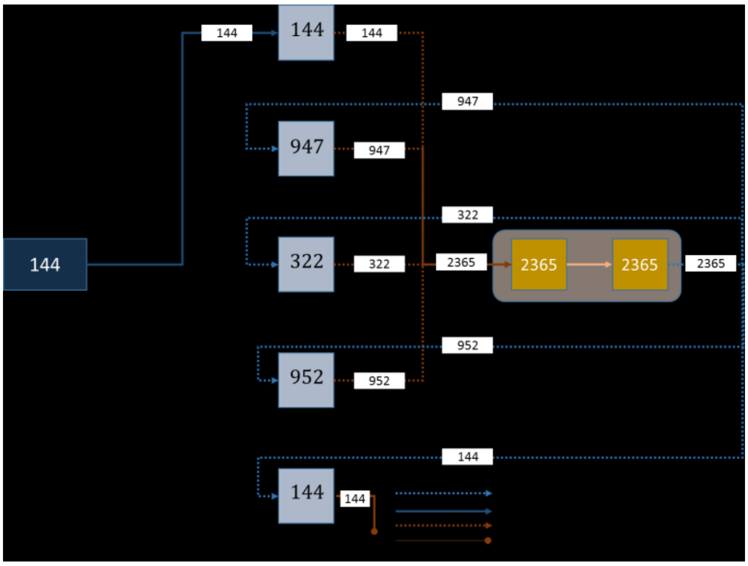


Figure 3-5 Proposed network of Model A1 for Bleed- Off = 0

From the above analysis it can be identified that the regenerated water flow rate is large even if the inlet and outlet concentrations at process units are for the most cases well below their maximum limits. This is due to the fact that the maximum allowable limit for the BOD contaminant was set at 4.2 mg/L. Thus, more regenerated water is recycled in order to have the BOD concentration below the limit. According to (Non-Potable Water substitution and reuse In the Field, Technical Information Paper, 32-002-0111), **Table 3-26 to 3-35** (Appendix, List of Tables-Part-I), the limits of BOD concentration for regenerated water can vary between 5– 25 mg/L. Since there has been no implementation of a direct non-linear constraint for the concentration of contaminants at the inlet of process units, the BOD limit of regenerated water is set to 10 mg/L (which in turn comprises the limit of inlet BOD concentration for all process units except for the kitchen sink) and the problem is resolved. In this case the estimated minimum fresh water remains at 144.14 L/Day, whereas the regenerated water is now just 985.61 L/Day.

Next, the problem is solved using a bleed–off factor of 25% (keeping BOD at 10 mg/L). In this case the estimated minimum fresh water is 385.02 L/Day, and the regenerated water is 722.64 L/Day.

The flows and concentrations at each unit for each bleed- off factor are presented in

Table 3-9

Table 3-9 (Appendix, List Of Tables –Part-I) respectively. The corresponding solution network for a bleed off of 25% is presented in **Figure 3-6** .

Using Unit	Min Flow V_i^{min}	Fresh Water F_i	Regenerated R_i	Treated T_i	Waste W_i	Balance
Kitchen sink	72.10	203.50	0.00	152.62	50.87	-0.00
Bath shower	152.08	84.78	198.69	212.60	70.87	0.00
Wash basin	32.16	0.00	181.12	135.84	45.28	-0.00
Washing Machine	164.04	96.74	198.69	221.57	73.86	0.00
Water closet	144.14	0.00	144.14	0.00	144.14	0.00
Sum	**564.51**	**385.02**	**722.64**	**722.64**	**385.02**	**0.00**

Table 3-9 Flow Results of Model A1 for Bleed Off = 25%

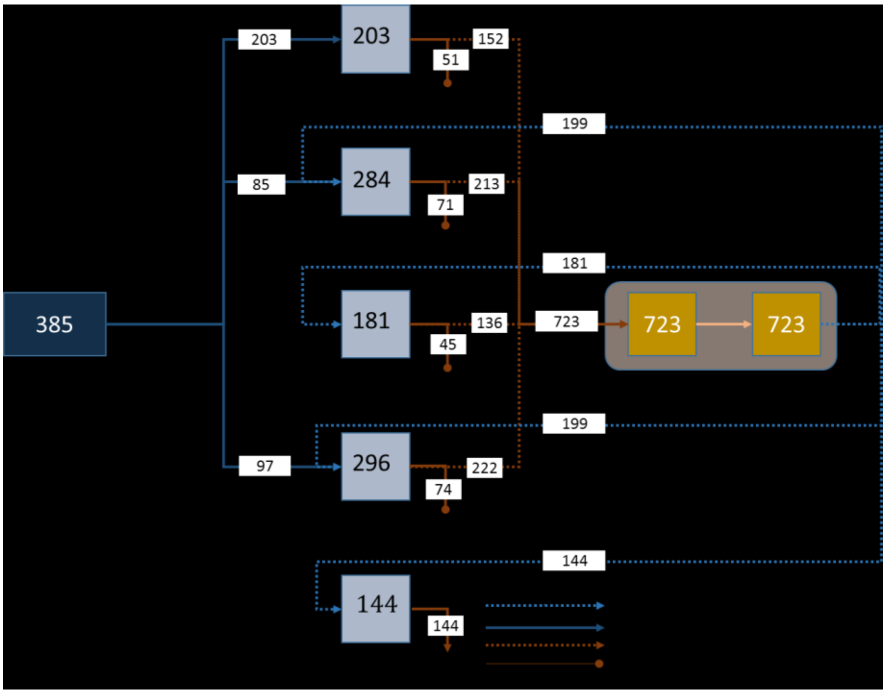


Figure 3-6 Proposed network of Model A1 for Bleed-Off = 25%

3.3.3.3 Sensitivity analysis

From the results it is clear that the maximum allowable concentration of BOD for regenerated water as well as the bleed-off percentage comprise two important parameters. So here we run a parametric analysis to estimate total fresh water and the total regenerated water for different combinations of these two parameters as shown in **Figures 3-7 and 3-8.**

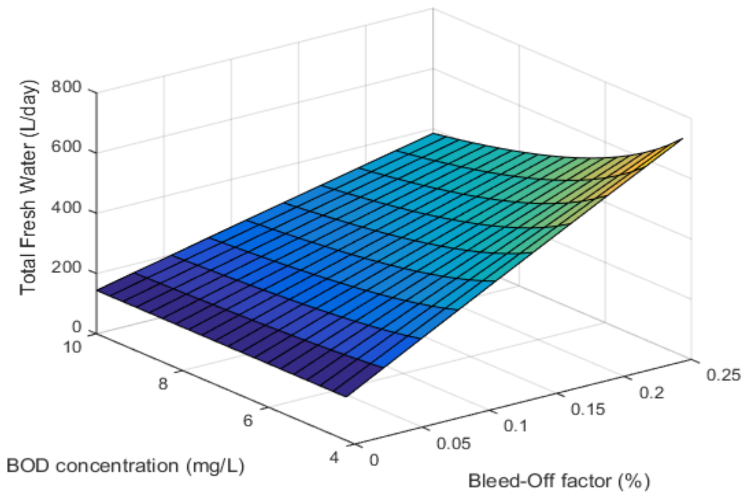


Figure 3-7
Model A1: Overall fresh water for different values of BOD concentration and Bleed-Off factor.

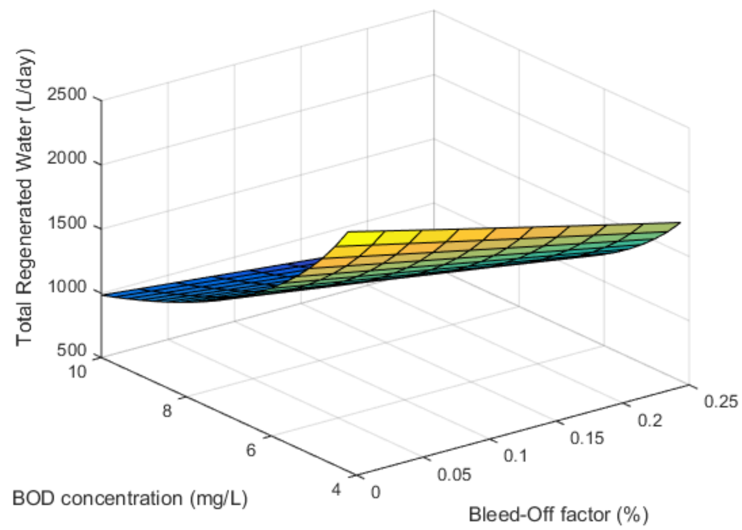


Figure 3-8
Model A1: Overall regenerated water for different values of BOD concentration and Bleed-Off factor.

From the results it can be seen that as soon as the limit for BOD concentration rises, a network using less total fresh water can be generated especially for the case of a higher bleed off factor. For example, if the bleed off factor is set to 25% the total fresh water ranges from 384.88 L/day to 719.21 L/day as seen in **Figure 3-9.** Note that, since the model assumes that no fresh water can be directed to the WC we have to consume more fresh water than without a treatment unit network (566 L/day) if the maximum BOD concentration is set below 5 mg/L. Finally due to process units inlet constraints of BOD no improvement on reduction of fresh water and treated water flows is achieved if the maximum BOD concentration is set above 10 mg/L.

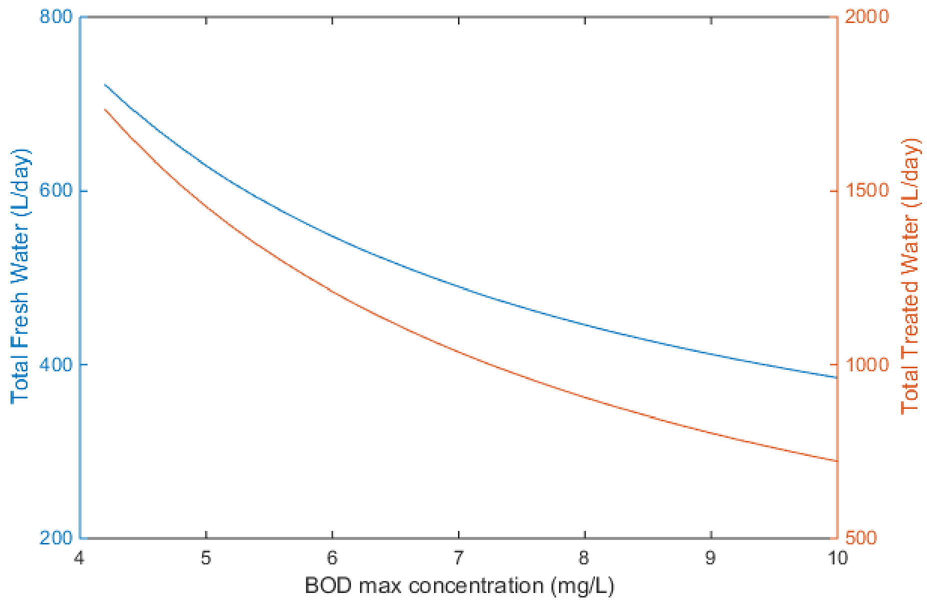


Figure 3-9 Model A1 for Bleed Off 25%: Total fresh water and treated water as a function of maximum BOD concentration.

Finally, according to **Figure 3-10**, in case that BOD concentration is set to 10 mg/L, increasing the bleed -off factor imposes the fresh water to also increase linearly consumption total treated water to decrease (**Table 3-10**).

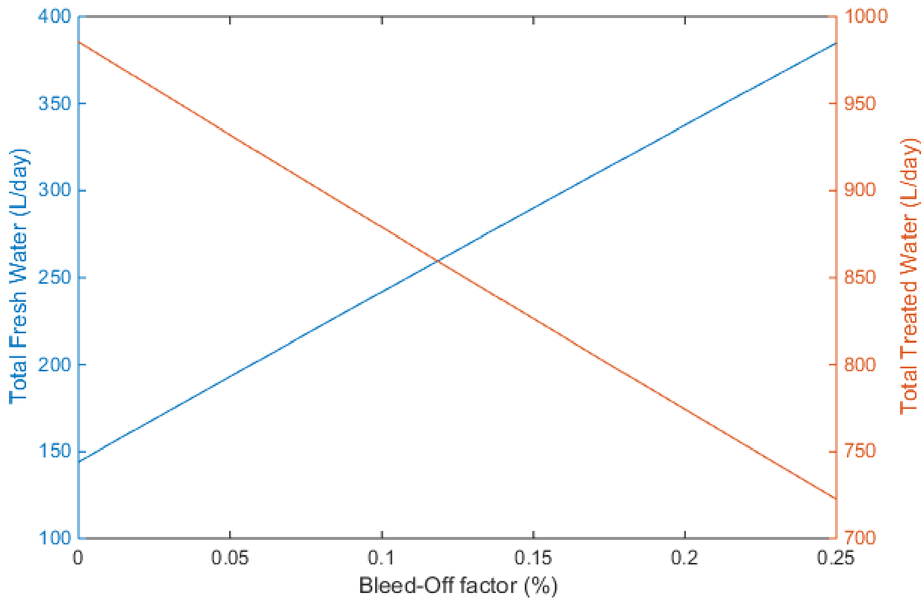


Figure 3-10
Model A1: Total fresh water and treated water as a function of Bleed-Off Factor

Bleed Off	Fresh Water	Regenerated Water
0.000	144.14	985.61
0.025	168.72	958.77
0.050	193.19	932.06
0.075	217.55	905.46
0.100	241.80	878.99
0.125	265.94	852.64
0.150	289.97	826.40
0.175	313.89	800.29
0.200	337.71	774.29
0.225	361.42	748.41
0.250	385.02	722.64

Table 3-10
Total fresh water and treated water as a function of Bleed-Off Factor. Model A1.

3.3.4 Model A2

In this model the same schematic is maintained as in Model A1 (**Figure 3-3**) and the same assumptions are made. The alteration compared to model A1 is that we relax the constraint of minimum flow of each process unit where as we set the BOD limit to 10 mg/L and instead we add a nonlinear constraint for the maximum concentration at the input and the output of each process unit. Thus Model A2 can then be formulated accordingly as follows:

$$\min_{\theta} \sum_{i \in I} F_i = \min_{\theta} V_1 + \sum_{\substack{i \in I \\ i \neq 5 \\ i \neq 2}} V_i - R_i \tag{3.34}$$

Where

$$\theta = \left[V_1 V_2 V_3 V_4 V_5 R_2 R_3 \; R_4 C_{BOD}^T \; C_{TSS}^T \; C_{Total\,-P}^T C_{NO\,3}^T \right]$$

With respect to the following linear constraints

$$C_k^T < C_k^{T,max}, \qquad k \in K \tag{a}$$

$$V_i \geq R_i, i \in I \tag{b}$$

$$\sum_{\substack{i \in I \\ i \neq 5}} (1 - \text{Bleed Off}) V_i - V_5 - \sum_{\substack{i \in I \\ i \neq 5 \\ i \neq 2}} R_i = 0 \tag{c}$$

And non linear constraints

$$\frac{\sum_{i \in I} T_i C_{i,k}^{OUT}}{\sum_{i \in I} T_i} = \frac{C_k^T}{\left(1 - r_k^{RO}\right)\left(1 - r_k^{MF}\right)}, \qquad k \in K \tag{d}$$

$$C_{i,k}^{IN} = \frac{F_i C_k^F + R_i C_k^T}{F_i + R_i} \leq C_{i,k}^{IN,max}, \qquad i \in I, k \in K \tag{e}$$

$$C_{i,k}^{OUT} = C_{i,k}^{IN} + \frac{M_{i,k}}{V_i} \leq C_{i,k}^{OUT\,,max}, \qquad i \in I, k \in K \tag{f}$$

In this case initial values and bounds are presented in

Table 3-11

Design variable	Initial Value	Lower Bound	Upper Bound
V_1	V_1^{min}	0	inf
V_2	V_2^{min}	0	inf
V_3	V_3^{min}	0	inf
V_4	V_4^{min}	0	inf
V_5	V_5^{min}	0	inf
R_2	0	0	inf
R_3	0	0	inf
R_4	0	0	inf
C_{BOD}^{T}	$C_{BOD}^{T,max}$	0	$C_{BOD}^{T,max}$
C_{TSS}^{T}	$C_{TSS}^{T,max}$	0	$C_{TSS}^{T,max}$
$C_{Total-P}^{T}$	$C_{Total-P}^{T,max}$	0	$C_{Total-P}^{T,max}$
C_{NO3}^{T}	$C_{NO3}^{T,max}$	0	$C_{NO3}^{T,max}$

Table 3-11 Initial values and bounds of the design variables for Model A2

3.3.4.1 Results

According to **Figures 3-11** and **3-12** we also notice from

Table 3-12 that as expected- the minimum fresh water usage in this case is 139.16 as was analyzed before. However in order to achieve this usage the regenerated water flow rate must be about 3,000,000 L/day. Results for all other cases are almost the same with the differences to the fresh water usage to be around 0.15 L/Day.

Nevertheless the total fresh water consumption as estimated in model A2 is less than the one estimated at model A1.

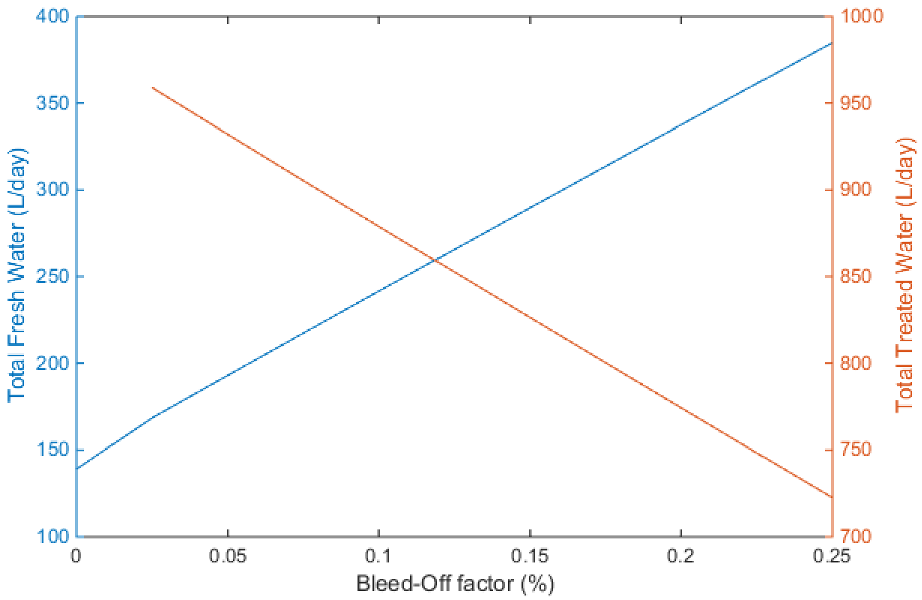


Figure 3-11 Model A2: Total fresh water and treated water as a function of Bleed off Factor

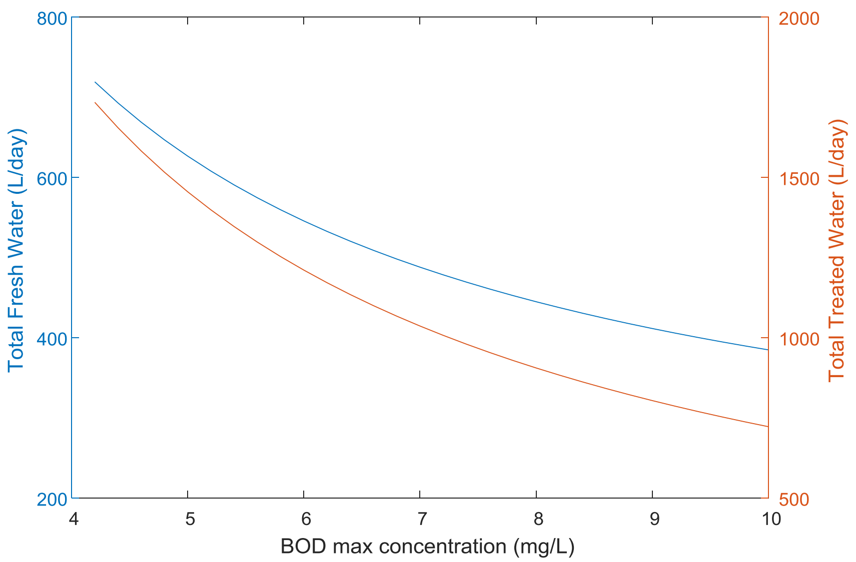


Figure 3-12 Model A2: Total fresh water and treated water as a function of BOD concentration.

Bleed Off	Fresh Water	Regenerated Water
0.000	139.16	3.07E+06
0.025	168.58	958.79
0.050	193.06	932.07
0.075	217.42	905.48
0.100	241.67	879.00
0.125	265.81	852.65
0.150	289.84	826.41
0.175	313.76	800.30
0.200	337.57	774.30
0.225	361.28	748.42
0.250	384.88	722.65

Table 3-12 Total fresh water and treated water as a function of Bleed-Off Factor. Model A2

Using Unit	Min Flow	Fresh Water	Regenerated	Treated	Waste	Balance
	V_i^{min}	F_i	R_i	T_i	W_i	
Kitchen sink	72.10	205.46	0.00	154.09	51.36	72.10
Bath shower	152.08	69.04	216.39	214.08	71.36	152.08
Wash basin	32.16	29.39	145.86	131.44	43.81	32.16
Washing Machine	164.04	81.00	216.39	223.05	74.35	164.04
Water closet	144.14	0.00	144.00	0.00	144.00	144.14
Sum	**564.51**	**384.88**	**722.65**	**722.65**	**384.88**	**564.51**

Table 3-13 Flow Results of Model A2 for Bleed-Off = 25%

The numerical results of all final flows between all water using processes are presented in **Table 3-13** and the corresponding proposed network for a bleed –Off factor 25 % Is presented in

Figure 3-13

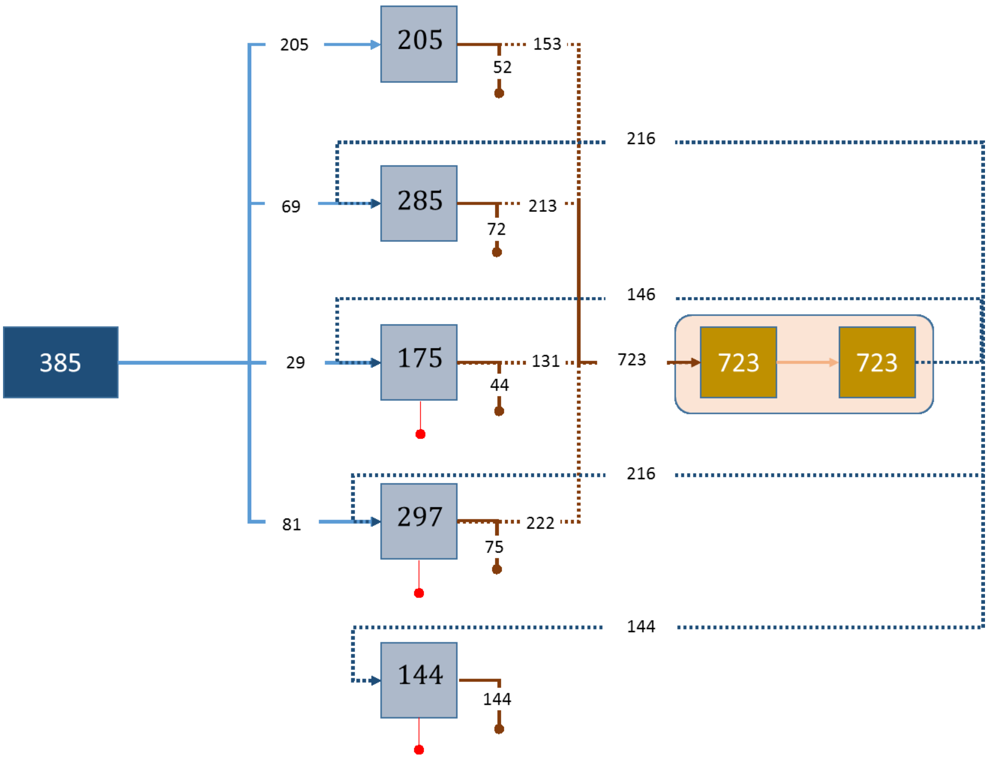


Figure 3-13 Proposed network of Model A2 for Bleed-Off = 25%

3.3.5 Model A3

This model is equivalent with model A2 and it is implemented in GAMS. The formulation is a little bit different in order to be convenient to implement it in GAMS. Assumptions are the same with Model A2. GAMS is used to analyse and solve relatively quickly the created non linear model in the household level. The motivation is to attempt a comparative study with the optimization toolbox of MATLAB.

3.3.5.1 Problem Formulation

Assuming the treatment units are connected in serial, we set as "$\boldsymbol{s_t}$" the combined treatment unit with removal ratio equal to the product of the removal ratio of each unit, i.e.

$$r_{st,k} = r_{MF,k} \text{ x } r_{RO,k} \tag{3.35}$$

The selected design variables vector in this case is

$$\theta = [F_i\ ,\ W_i, T_i, R_i, C_{i,k}^{OUT}\ , C_{st,k}^{IN}, C_{st,k}^{OUT}\ ],\ \ i \in I, k \in K \qquad (3.36)$$

In this case the problem can be stated as

$$\min_{\theta} \sum_{i \in I} F_i \qquad (3.37)$$

Where

$$\theta = [F_i\ ,\ W_i, T_i, R_i, C_{i,k}^{OUT}\ , C_{t,k}^{IN}, C_{t,k}^{OUT}\ ], \qquad i \in I, t \in T, k \in K$$

With respect to the following linear constraints

$$F_i + R_i = W_i + T_i i \in I \qquad \text{(a)}$$

$$\sum_{i \in I} T_i = \sum_{i \in I} R_i \qquad \text{(b)}$$

$$C_{st,k}^{OUT} = (1 - r_{st,k}) C_{st,k}^{IN}, \qquad k \in K \qquad \text{(c)}$$

$$C_{st,k}^{OUT} \le C_k^{T,max} \qquad \text{(d)}$$

And non linear constraints

$$F_i C_k^F + R_i C_{st,k}^{OUT} + M_{i,k} = (W_i + T_i) C_{i,k}^{OUT}\ , \qquad i \in I, k \in K \qquad \text{(e)}$$

$$F_i (C_k^F - C_{i,k}^{IN,max}) + R_i (C_{st,k}^{OUT} - C_{i,k}^{IN,max}) \le 0, \qquad i \in I, k \in K \qquad \text{(f)}$$

$$F_i (C_k^F - C_{i,k}^{OUT,max}) + R_i (C_{st,k}^{OUT} - C_{i,k}^{OUT,max}) + M_{i,k} \le 0, \qquad i \in I, k \in K \qquad \text{(g)}$$

$$\sum_{i \in I} T_i\ (C_{st,k}^{IN} - C_{i,k}^{OUT}) = 0, \qquad k \in K \qquad \text{(h)}$$

3.3.5.2 Results

Initial parameters and bounds are presented in **Table 3-14.**

The problem is formulated in GAMS_ModelA3.gms file. GAMS using SNOPT solver (SNOPT uses a sequential quadratic programming algorithm) needs 9 main iterations to solve the problem. As seen in **Tables 3-3, 3-18, 3-19** and **Figure 3-14** this model gives the same minimum flow of fresh water and regenerated water as Model A2. However flows at individual process units are different. This indicates that the problem has degenerate solutions (Bagajewicz and Miguel, 2000).

Design variable	Initial Value	Lower Bound	Upper Bound
$F_i, i \in I, i \neq 5$ F_5	V_i^{min} 0	0	inf 0
$W_i, i \in I, i \neq 5$ W_5	Bleedoff x V_i^{min} V_i^{min}	0	inf
$T_i, i \in I, i \neq 5$ T_5	$(1 - \text{Bleedoff})$ x V_i^{min} 0	0	inf 0
$R_i, i \in I, i \neq 5$ R_5	0 V_5^{min}	0	inf
$C_{i,k}^{OUT}, i \in I, k \in K$	$C_{i,k}^{OUT,max}$	0	inf
$C_{t,k}^{OUT}, t \in T, k \in K$	$C_k^{T,max}$	0	inf
$C_{t,k}^{IN}, t \in T, k \in K$	$\frac{C_k^{T,max}}{r_{st,k}}$	0	inf

Table 3-14 Initial values and bounds of the design variables for Model A3

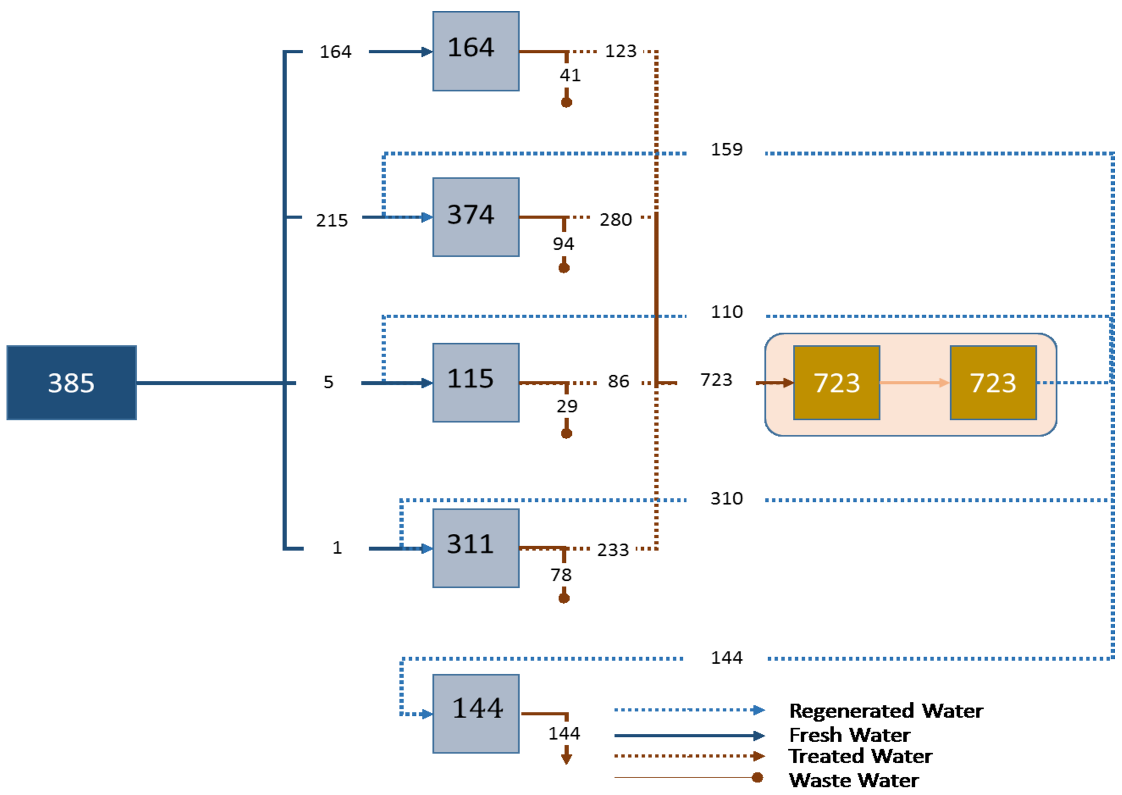


Figure 3-14 Proposed network of Model A3 (GAMS) for Bleed-Off = 25%

Using Unit	Min Flow	Fresh Water	Regenerated	Treated	Waste	Balance
	V_i^{min}	F_i	R_i	T_i	W_i	
Kitchen sink	72.10	163.51	0.00	122.63	40.88	-0.00
Bath shower	152.08	214.72	159.18	280.43	93.48	0.00
Wash basin	32.16	5.43	109.82	86.43	28.81	0.00
Washing Machine	164.04	1.23	309.65	233.16	77.72	0.00
Water closet	144.14	0.00	144.00	0.00	144.00	0.00
Sum	**564.51**	**384.88**	**722.65**	**722.65**	**384.88**	**0.00**

Table 3-15 Flow Results of Model A3 for Bleed-Off = 25%

3.3.6 Model A4

In **Figure 3-15**, a more general model is developed relaxing the assumptions about the connectivity of treatment units, thus allowing flows between using process units. The model we developed uses two treatment units and the following assumptions were made:

- A1. No fresh water for WC

$$F_5 = 0 \qquad (3.38)$$

- A2. No treatment of water flow from WC

$$T_5 = 0 \qquad (3.39)$$

- A3. No direct water flow from WC to the other water using process units

$$D_{j,5} = 0 \qquad (3.40)$$

- A4. Constant mass flows

$$M_{i,k} = \text{constant} \qquad (3.41)$$

- A5. No flow rate upper limits exist for process and treatment units

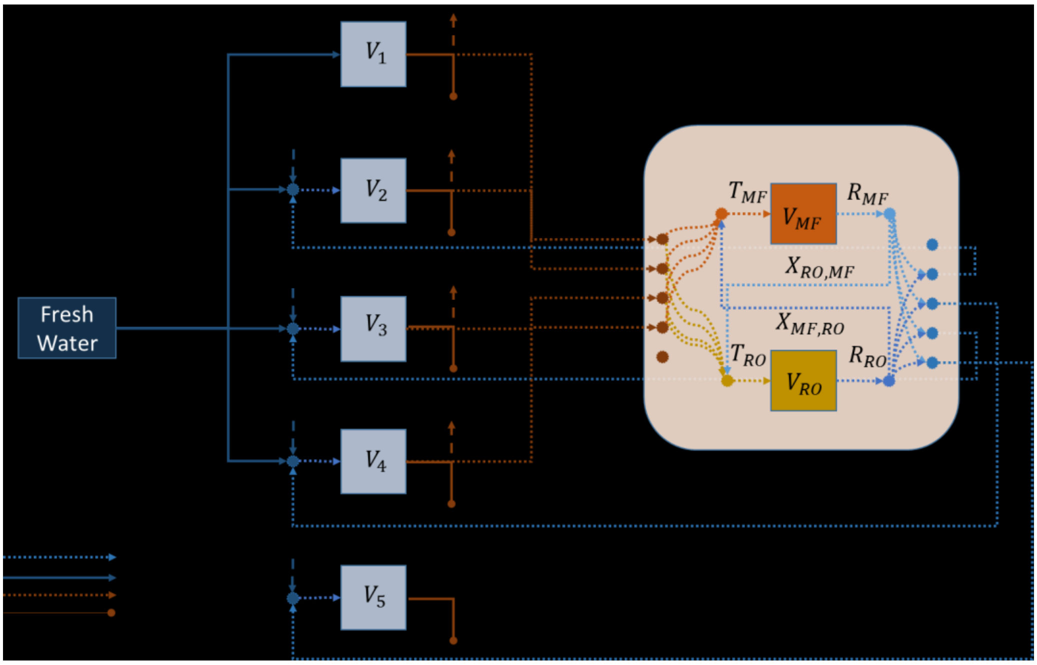


Figure 3-15 Schematic of Model A4

3.3.6.1 Problem Formulation

According to the balance equations (3.1-3.6) as well as the above assumptions (equations 3.38-3.41 and assumption A5) we formulate the mathematical model A4,

The selected design variables vector is

$$\theta = \left[F_i\,,\ W_i,\ \ X_{q,r},\ \ C_{q,k}^{OUT}\right],\ \ i \in I, q, r \in I \cup T, k \in K \qquad (3.42)$$

Resulting in a design vector of 87 parameters.

3.3.6.2 Water balance

The overall water balance for every unit (water using and treatment) states that the inflow rate equals the outflow rate or,

$$V_q^{IN} = V_q^{OUT} = V_q,\ \ q \in I \cup T \qquad (3.43)$$

In particular, the overall water balance equation can be regarding all water using processes could be written as:

$$F_i + R_i + \sum_i^j D_{i,j} = W_i + T_i + \sum_i^j D_{j,i}\,,\ \ i \in I \qquad (3.44)$$

$$F_i + \overbrace{\sum_{t \in T} X_{i,t}}^{R_i} + \overbrace{\sum_{\substack{j \in I \\ j \neq i}} X_{i,j}}^{D_{i,j}} = W_i + \overbrace{\sum_{t \in T} X_{t,i}}^{T_i} + \overbrace{\sum_{\substack{j \in I \\ j \neq i}} X_{j,i}}^{D_{j,i}}, \quad i \in I \tag{3.45}$$

or

$$F_i + \sum_{\substack{q \in (I \cup T) \\ q \neq i}} X_{i,q} = W_i + \sum_{\substack{q \in (I \cup T) \\ q \neq i}} X_{q,i}, i \in I \tag{3.46}$$

For treatment units the overall water balance equation can be written as:

$$\sum_{\substack{t_1 \in T \\ t_1 \neq t}} X_{t,t_1} + \overbrace{\sum_{j \in I} X_{t,j}}^{T_t} = \sum_{\substack{t_1 \in T \\ t_1 \neq t}} X_{t_1,t} + \overbrace{\sum_{j \in I} X_{j,t}}^{R_t}, \quad t \in T \tag{3.47}$$

or

$$\sum_{\substack{q \in (I \cup T), \\ q \neq t}} X_{t,q} = \sum_{\substack{q \in (I \cup T) \\ j \neq t}} X_{q,tg} \qquad t \in T \tag{3.48}$$

3.3.6.3 Mass balance

For each contaminant and water using process the overall mass balance equation can be written as:

$$M_{q,k}^{IN} + M_{q,k} = M_{q,k}^{OUT}, \quad q \in (I \cup T) \text{ and } k \in K \tag{3.49}$$

For a water using process

$$M_{i,k}^{IN} = F_i C_k^F + \sum_{\substack{j \in (I \cup T) \\ j \neq i}} X_{i,j} C_{j,k}^{OUT} \tag{3.50}$$

$$M_{i,k}^{OUT} = \left(W_i + \sum_{\substack{j \in (I \cup T) \\ j \neq i}} X_{i,j} \right) C_{i,k}^{OUT} \tag{3.51}$$

Thus the overall mass balance equation is

$$F_i C_k^F + \sum_{\substack{j \in (I \cup T) \\ j \neq i}} X_{i,j} C_{j,k}^{OUT} + M_{i,k} = \left(W_i + \sum_{\substack{j \in (I \cup T) \\ j \neq i}} X_{i,j} \right) C_{i,k}^{OUT}, \quad i \in I \text{ and } k \in K \tag{3.52}$$

For a treatment unit

$$M_{t,k}^{IN} = \sum_{\substack{q\in(I\cup T)\\ q\neq t}} X_{t,q} C_{q,k}^{OUT} \tag{3.53}$$

$$M_{i,k}^{OUT} = \left(\sum_{\substack{q\in(I\cup T)\\ q\neq t}} X_{t,q} \right) C_{t,k}^{OUT} \tag{3.54}$$

For treatment units the mass removal is not pre specified as a mass load but rather as removal ratio regarding inlet and outlet concentrations, i.e.

$$C_{t,k}^{OUT} = (1 - r_{t,k}) C_{t,k}^{IN} \tag{3.55}$$

$$\frac{M_{t,k}^{OUT}}{V_t^{OUT}} = (1 - r_{t,k}) \frac{C_{t,k}^{IN}}{V_t^{IN}} \tag{3.56}$$

And since $V_t^{IN} = V_t^{OUT}$

$$M_{t,k}^{OUT} = (1 - r_{t,k}) M_{t,k}^{IN} \tag{3.57}$$

And the mass balance equation can be written as:

$$\left(\sum_{\substack{q\in(I\cup T)\\ q\neq t}} X_{q,t} \right) C_{t,k}^{OUT} = (1 - r_{t,k}) \sum_{\substack{q\in(I\cup T)\\ q\neq t}} X_{t,q} C_{q,k}^{OUT}$$

$$t\in T \text{ and } k \in K \tag{3.58}$$

3.3.6.4 Maximum allowable concentrations

The maximum allowable constraints for the inlet and outlet concentrations of each single water using process are:

$$C_{i,k}^{IN} \leq C_{i,k}^{IN,max} \tag{3.59}$$

$$C_{i,k}^{IN} = \frac{M_{i,k}^{IN}}{V_i^{IN}} = \frac{F_i C_k^F + \sum_{\substack{j\in(I\cup T)\\ j\neq i}} X_{i,j} C_{j,k}^{OUT}}{F_i + \sum_{\substack{j\in(I\cup T)\\ j\neq i}} X_{i,j}} \leq C_{i,k}^{IN,max} \tag{3.60}$$

or

$$F_i \left(C_k^F - C_{i,k}^{IN,max} \right) + \sum_{\substack{j\in(I\cup T)\\ j\neq i}} X_{i,j} \left(C_{j,k}^{OUT} - C_{i,k}^{IN,max} \right) \leq 0, \qquad i \in I \text{ and } k \in K$$

$$C_{i,k}^{OUT} \leq C_{i,k}^{OUT,max}, \qquad i \in I \text{ and } k \in K \tag{3.61}$$

In order to ensure that the treated water satisfies the requirements for non-potable reuse we could either restrain the concentration of the outflow of each treatment unit (more restrictive) or the combined concentration for each using process or the combined overall.

In this model we select the more restrictive constraint that the concentration of each contaminant in each outflow from the treatment units must be less that the prescribed value for non-potable reuse water; hence we get:

$$C_{t,k}^{OUT} \leq C_k^{T,max}, \qquad t\epsilon\ T \text{ and } k \in K \qquad (3.62)$$

For reference, the concentration of each contaminant of the combined regenerated water for each water-using process can be calculated as

$$C_{i,k}^{T} = \frac{\sum_{t\epsilon T}\left(X_{i,t}C_{t,k}^{OUT}\right)}{\sum_{t\epsilon T} X_{i,t}} \leq C_k^{T,max}, \qquad i \in I \text{ and } k \in K \qquad (3.63)$$

$$\textstyle\sum_{t\epsilon T}\left(X_{i,t}\left(C_{t,k}^{OUT} - C_k^{T,max}\right)\right) \leq 0 \qquad (3.64)$$

Where the concentration of each contaminant of the overall combined regenerated water as

$$C_k^{T} = \frac{\sum_{t\epsilon T}\left(\left(\sum_{i\epsilon I} X_{i,t}\right)C_{t,k}^{OUT}\right)}{\sum_{t\epsilon T}\left(\sum_{i\epsilon I} X_{i,t}\right)} \leq C_k^{T,max}, \qquad k \in K \qquad (3.65)$$

$$\sum_{t\epsilon T}\left(\left(\sum_{i\epsilon I} X_{i,t}\right)\left(C_{t,k}^{OUT} - C_k^{T,max}\right)\right) \leq 0 \qquad (3.66)$$

According to assumptions A1, A2 and A3 the model constraints which relax the model further can be combined as follows :

No treatment of water flow from WC and no direct water flow from WC to the other water using process units

$$T_5 = 0 \qquad (3.67)$$

$$D_{j,5} = 0 \qquad (3.68)$$

Can be thus combined in the following constraint

$$X_{q,5} = 0, q \in (I \cup T) \qquad (3.69)$$

No fresh water for flushing (toilet)

$$F_5 = 0 \qquad (3.70)$$

Finally the minimum wastewater bleed-off ensuring that no trace contaminant is built up in the process in cases of water using processes served only with recycled stream

$$W_i \geq \text{BleedOff x } V_i, \ i \in I \qquad (3.71)$$

$$W_i \geq \text{BleedOff } x_i \left(F_i + \sum_{\substack{q \in (I \cup T) \\ q \neq i}} X_{i,q} \right), \qquad i \in I \qquad (3.72)$$

3.3.6.5 Objective Function

We aim to minimize the fresh water supply so the objective function will be

$$\min_{\theta} \sum_{i \in I} F_i \qquad (3.73)$$

In conclusion, the problem can be stated as

$$(3.74)$$

$$\min_{\theta} \sum_{i \in I} F_i$$

Where

$$\theta = [F_i, W_i, X_{q,r}, C^{OUT}_{q,k}], \qquad i \in I, q, r \in I \cup T, k \in K$$

With respect to the following linear constraints

$$F_i + \sum_{\substack{q \in (I \cup T) \\ q \neq i}} X_{i,q} = W_i + \sum_{\substack{q \in (I \cup T) \\ q \neq i}} X_{q,i}, i \in I \qquad \text{(a)}$$

$$\sum_{\substack{q\epsilon(I\cup T),\\ q\neq t}} X_{t,q} = \sum_{\substack{q\epsilon(I\cup T)\\ j\neq t}} X_{q,t}, \quad t\epsilon\, T \tag{b}$$

$$C_{i,k}^{OUT} \leq C_{i,k}^{OUT,max} \tag{c}$$

$$C_{t,k}^{OUT} \leq C_{k}^{T,max} \tag{d}$$

And non-linear constraints

$$F_i C_k^F + \sum_{\substack{j\epsilon(I\cup T)\\ j\neq i}} X_{i,j} C_{j,k}^{OUT} + M_{i,k} = \left(W_i + \sum_{\substack{j\epsilon(I\cup T)\\ j\neq i}} X_{i,j} \right) C_{i,k}^{OUT}, \qquad i\in I, k\in K \tag{e}$$

$$\left(\sum_{\substack{q\epsilon(I\cup T)\\ q\neq t}} X_{q,t} \right) C_{t,k}^{OUT} = (1 - r_{t,k}) \sum_{\substack{q\epsilon(I\cup T)\\ q\neq t}} X_{t,q} C_{q,k}^{OUT}, \qquad t\in T, k\in K \tag{f}$$

$$F_i\left(C_k^F - C_{i,k}^{IN,max}\right) + \sum_{\substack{j\epsilon(I\cup T)\\ j\neq i}} X_{i,j}\left(C_{j,k}^{OUT} - C_{i,k}^{IN,max}\right) \leq 0, \qquad i\in I, k\in K \tag{g}$$

3.3.6.6 Results

The above problem is formulated in MATLAB using the function |problem Full Solution|. Following the previous models results the BOD maximum concentration was set to 10 mg/L.

First we solved the problem by setting the Bleed-Off factor to 0. We used the SQP algorithm of |fmincon| solver of MATLAB with initial values of design variables as presented in **Table 3-16**. As seen in **Figure 3-20**, after 296 iterations the algorithm stopped finding a feasible solution suggesting minimum fresh water supply around 139.2 L/Day, which is about the same value as the one estimated in Model A2. In this case the maximum observed values for the flows between units were around 300,000 L/Day. In order to get more reasonable and realistic results we set an upper limit for flows between the treatment units at 5000 L/Day and flows between a water using process and any other unit at 250 L/Day. Using the above parameters the SQP algorithm converged after 336 iterations and 30765 function evaluations. The flows and concentrations at each unit i.e. the water using (process) units as well as the treatment units are presented in

Table 3-17and also **Tables 3-17, 3-18, 3-19** and **3-20** whereas the estimated networks for different bleed-off factors of 0% and 25% are shown in **Figure 3-22** respectively.

Design variable	Initial Value	Lower Bound	Upper Bound
$F_i, i \in I, i \neq 5$	V_i^{min}	0	inf(250)
$W_i, i \in I$	Bleedoff x V_i^{min}	0	inf(250)
$X_{q,r}, q, r \in Q, r \neq 5$ $X_{t,t_1}, t, t_1 \in T$	100	0	inf(250) inf(5000)
$X_{q,5}$	0	0	0
$C_{i,k}^{OUT}, i \in I, k \in K$	$C_{i,k}^{OUT,max}$	0	$C_{i,k}^{OUT,max}$
$C_{t,k}^{OUT}, t \in T, k \in K$	$C_k^{T,max}$	0	$C_k^{T,max}$

Table 3-16 Initial values and bounds of the design variables for Model A4

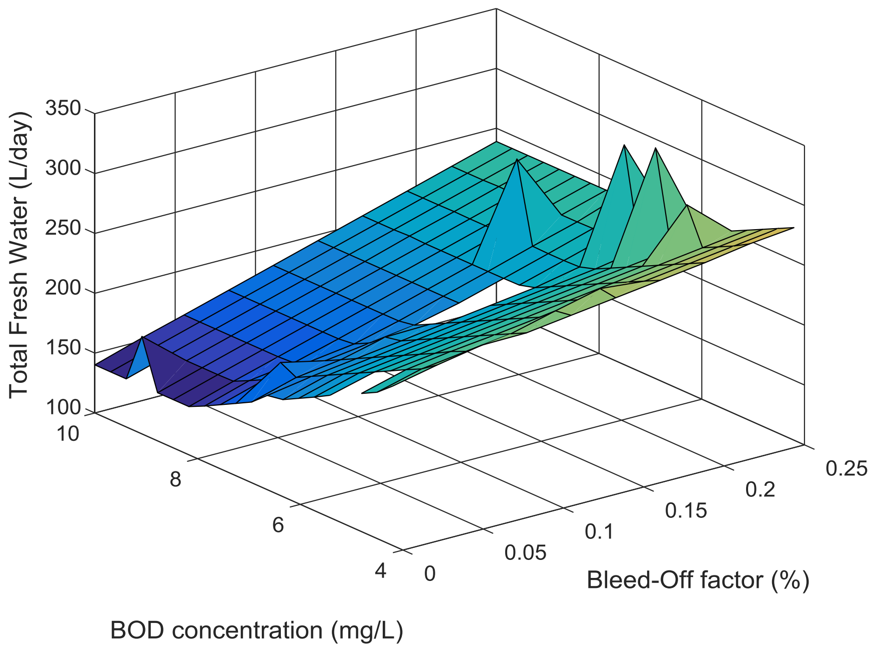


Figure 3-16 Total Fresh Water in correlation with variables of BOD concentration and Bleed-off factor

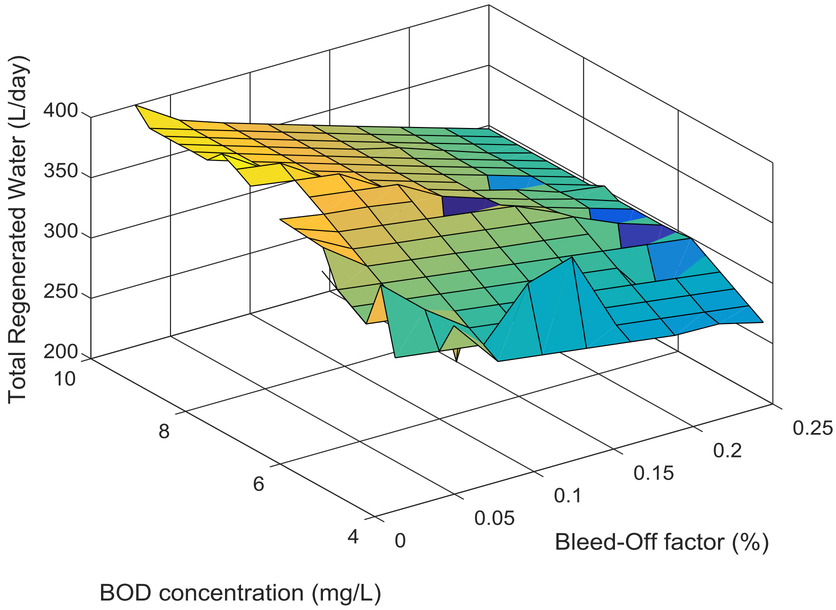


Figure 3-17 Total Regenerated Water in correlation with BOD concentration and Bleed-off factor

In **Figures 3-16** and **3-17** certain number of spikes can be detected along the BOD concentration axis as well as the Bleed-off axis . These values are due to local optima and are not taken into account in our problem. The leveled part of the total amount of fresh water (**Figure 3-16**) and the total quantity of regenarated water (**Figure 3-17**) respectively.

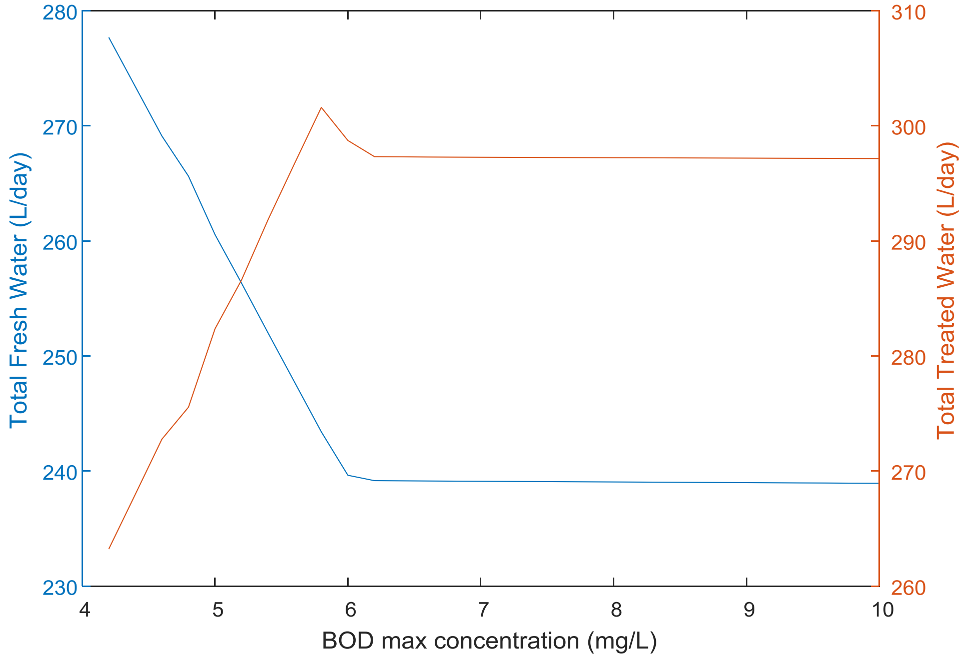


Figure 3-18 Function of Total fresh water in relation to maximum BOD and total treated Water

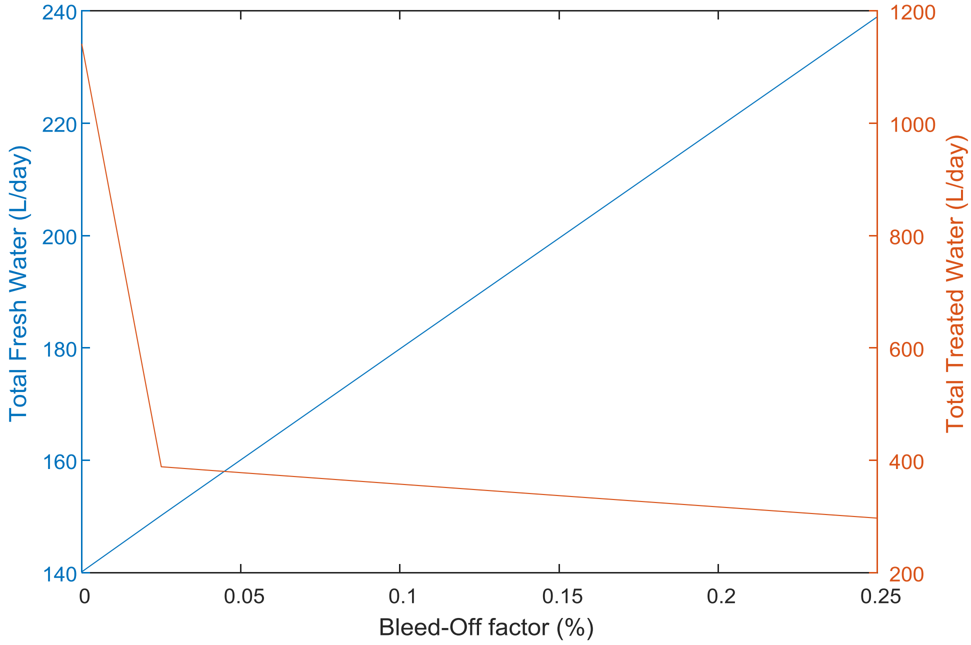


Figure 3-19 Function of Total fresh water in relation to Bleed-off factor and total treated Water

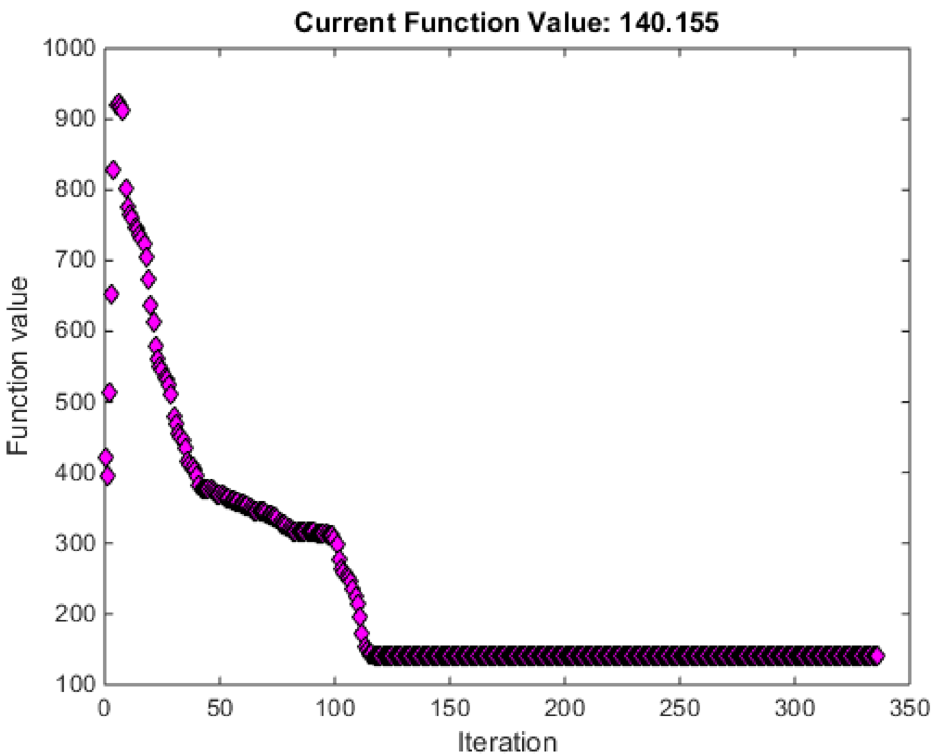


Figure 3-20

Function value for each iteration using SQP algorithm of fmincon solver and initial values of

Table 3-16

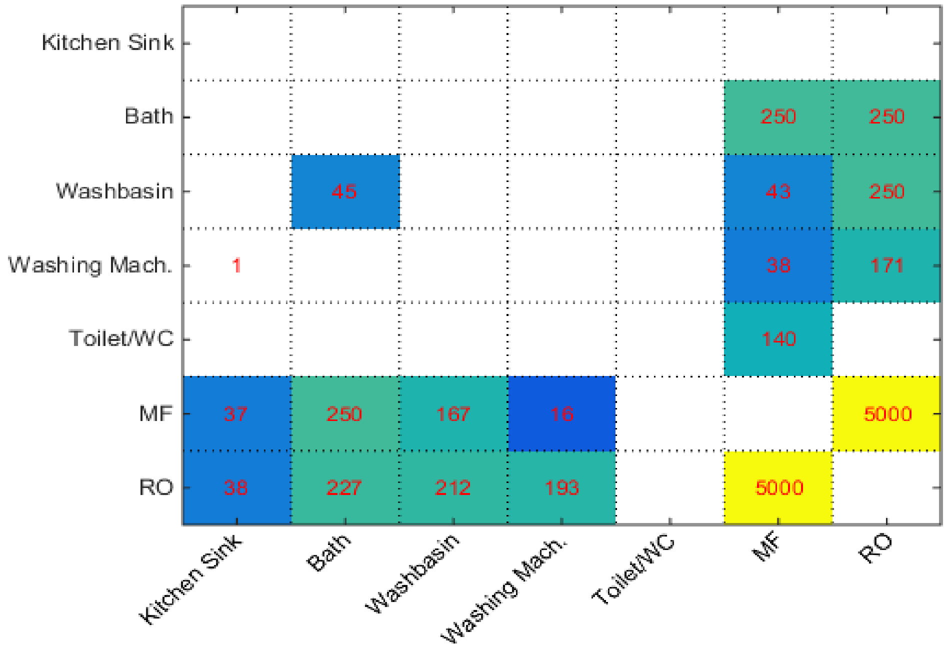


Figure 3-21 Flows between units $X_{q,r}$. Model A4 for Bleed-Off = 0%

Using Unit	Direct In Flow	Fresh Water	Regenerated	Treated	Waste	Direct Out Flow	Balance
	$D_{i,j}$	F_i	R_i	T_i	W_i	$D_{j,i}$	
Kitchen sink	0.00	76.52	0.00	0.00	75.82	0.70	0.00
Bath shower	0.00	22.22	500.00	0.00	476.92	45.30	0.00
Wash basin	45.30	41.42	292.54	0.00	379.26	0.00	0.00
Washing Machine	0.70	0.00	208.56	0.00	209.25	0.00	0.00
Water closet	0.00	0.00	140.16	140.16	0.00	0.00	0.00
Sum	46.00	140.16	1141.25	140.16	1141.25	46.00	**0.00**

Table 3-17 Flow Results for water using units. Model A4 for Bleed-Off = 0%

Treatment Unit	From Process Units	From Treat. Units	To Process Units	To Treat. Units	Balance
	T_t	X_{t,t_1}	R_t	$X_{t_1,t}$	
MF	470.49	5000.00	470.49	5000.00	0.00
RO	670.75	5000.00	670.75	5000.00	0.00
Sum	**1141.25**	**10000.00**	**1141.25**	**10000.00**	**0.00**

Table 3-18 Flow Results of treatment units. Model A4 for Bleed-Off = 0%

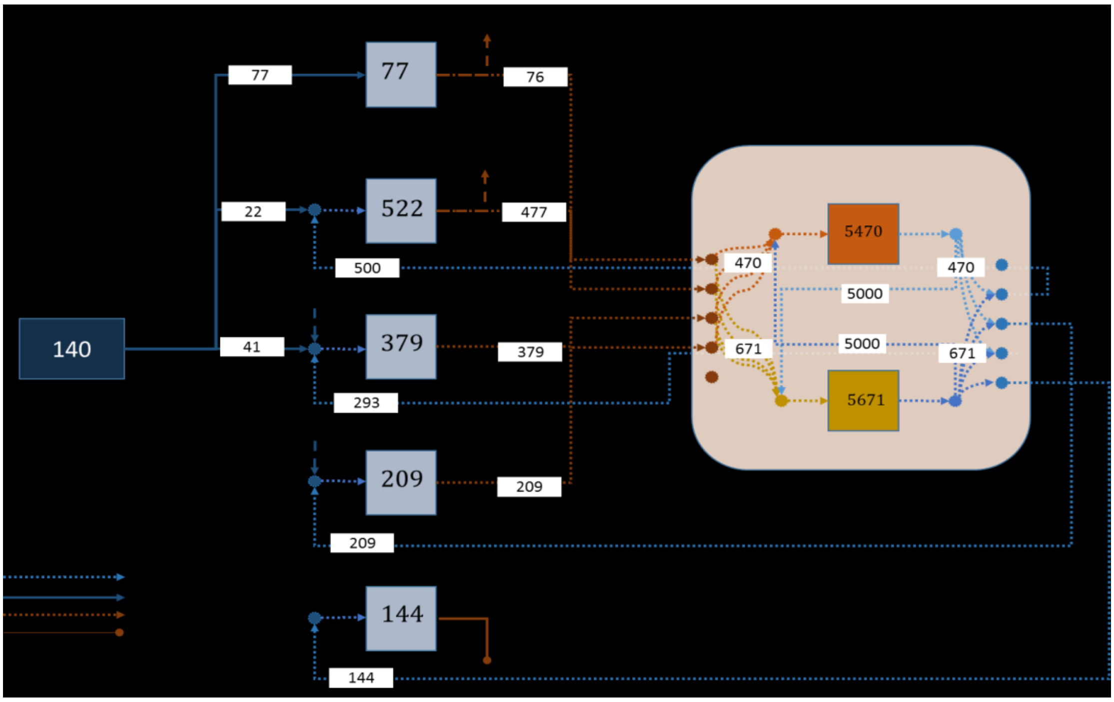


Figure 3-22 Proposed network of Model A4 for Bleed-Off = 0%

According to **Figure 3-23**, the results show minimum fresh water supply being around 140 L/Day where direct flow from water using units is only around 46 L/Day. Next we solved the problem setting the Bleed-Off factor to 25%. In this case the minimum fresh water supply around 239 L/Day is attained after 80 iterations where there is no direct flow from water using units. The flows and concentrations at each unit are presented in

Table 3-19and **Table 3-15** with the estimated networks for different bleed-off factors in **Figure 3-25**

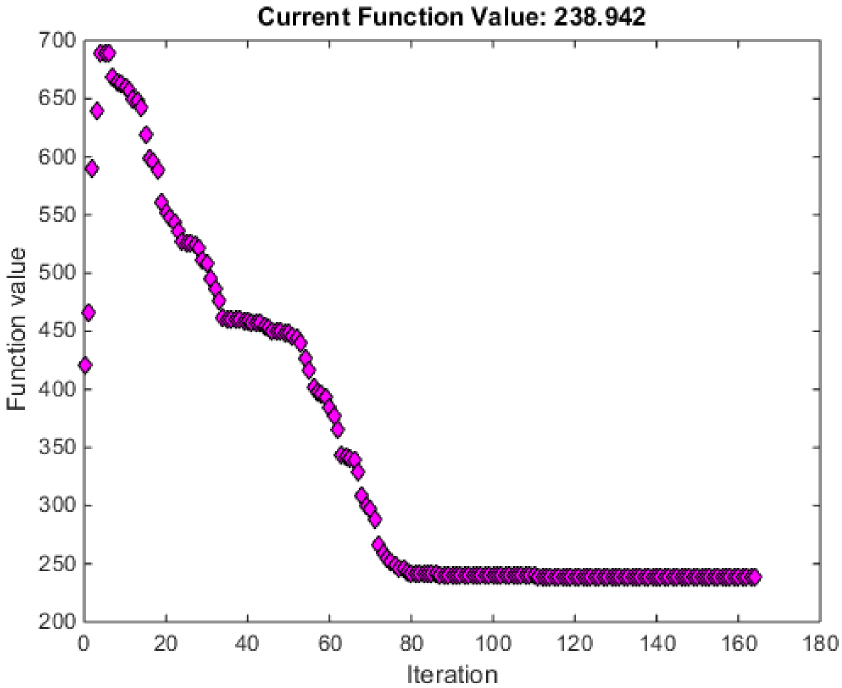


Figure 3-23

Function value for each iteration using SQP algorithm of fmincon solver and initial values of Table 3-16,Model A4. Bleed-Off = 25%.

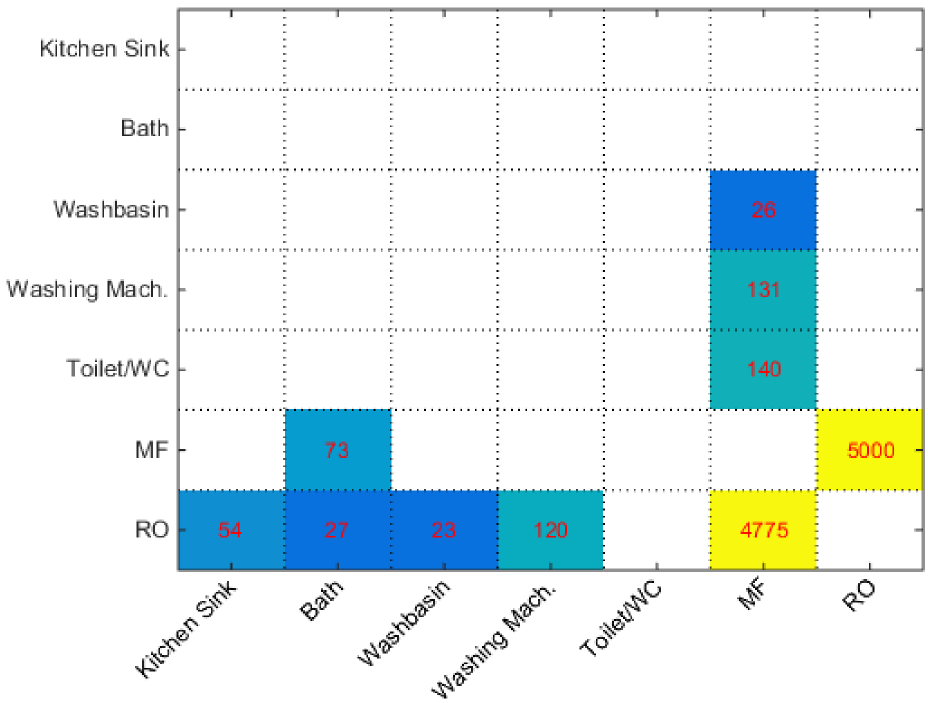


Figure 3-24 Flows between units $X_{q,r}$. Model A4 for Bleed-Off = 25%

Using Unit	Direct In Flow	Fresh Water	Regenerated	Treated	Waste	Direct Out Flow	Balance
	$D_{i,j}$	F_i	R_i	T_i	W_i	$D_{j,i}$	
Kitchen sink	0.00	72.10	0.00	18.03	54.08	0.00	0.00
Bath shower	0.00	133.30	0.00	33.33	99.98	0.00	0.00
Wash basin	0.00	4.65	26.15	7.70	23.10	0.00	0.00
Washing Machine	0.00	28.90	131.12	40.00	120.01	0.00	0.00
Water closet	0.00	0.00	139.89	139.89	0.00	0.00	0.00
Sum	**0.00**	**238.94**	**297.16**	**238.94**	**297.16**	**0.00**	**0.00**

Table 3-19 Flow Results for water using units. Model A4 for Bleed-Off = 25%

Treatment Unit	From Process Units	From Treat. Units	To Process Units	To Treat. Units	Balance
	T_t	X_{t,t_1}	R_t	$X_{t_1,t}$	
MF	72.64	5000.00	297.16	4775.48	0.00
RO	224.52	4775.48	0.00	5000.00	0.00
Sum	**297.16**	**9775.48**	**297.16**	**9775.48**	**0.00**

Table 3-20 Flow Results of treatment units. Model A4 for Bleed-Off = 25%

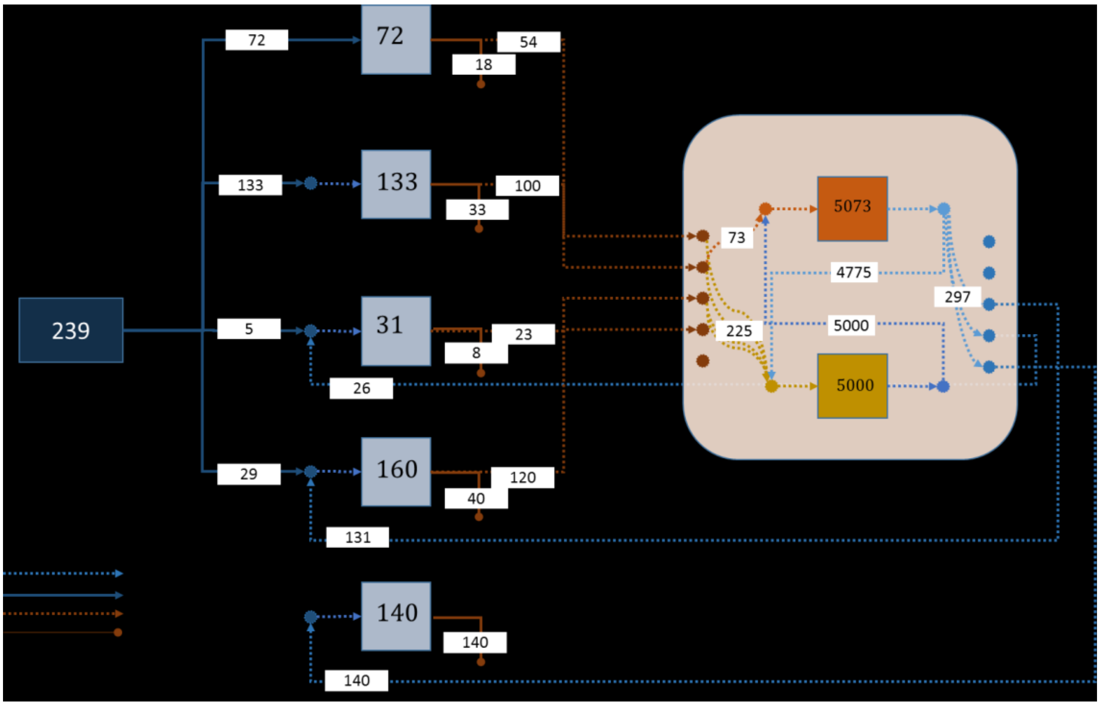


Figure 3-25 Proposed network of Model A4 for Bleed-Off = 25%

The same model was implemented and solved in GAMS using the SNOPT solver (GAMS_ModelA4.gms).The flows and concentrations in each process as well as treatment unit are presented in

Table 3-21and **3-22** respectively. From the results it can be seen that the solutions by both MATLAB and GAMS show exactly the same output values.

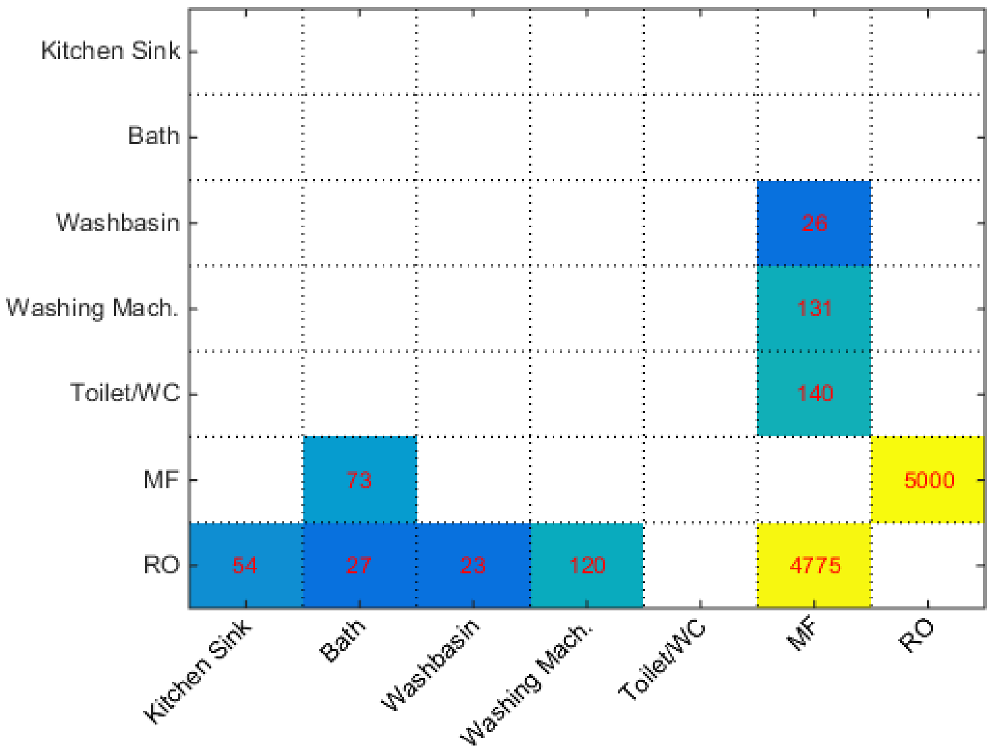


Figure 3-26 Flows between units $X_{q,r}$ .Model A4 for Bleed-Off = 25% (GAMS)

Using Unit	Direct In Flow	Fresh Water	Regenerated	Treated	Waste	Direct Out Flow	Balance
	$D_{i,j}$	F_i	R_i	T_i	W_i	$D_{j,i}$	
Kitchen sink	0.00	72.10	0.00	18.03	54.08	0.00	0.00
Bath shower	0.00	133.30	0.00	33.33	99.98	0.00	0.00
Wash basin	0.00	4.65	26.15	7.70	23.10	0.00	0.00
Washing Machine	0.00	28.90	131.12	40.01	120.02	0.00	0.00
Water closet	0.00	0.00	139.89	139.89	0.00	0.00	0.00
Sum	0.00	238.94	297.16	238.94	297.16	0.00	**0.00**

Table 3-21 Flow Results for water using units. Model A4 for Bleed-Off = 25% (GAMS)

Treatment Unit	From Process Units	From Treat. Units	To Process Units	To Treat. Units	Balance
	T_t	X_{t,t_1}	R_t	$X_{t_1,t}$	
MF	72.64	5000.00	297.16	4775.48	0.00
RO	224.52	4775.48	0.00	5000.00	0.00
Sum	297.16	9775.48	297.16	9775.48	**0.00**

Table 3-22 Flow Results of treatment units. Model A4 for Bleed-Off = 25% (GAMS)

Finally, according to **Tables 3-23** and **3-24** as shown below, GAMS/SNOPT and MATLAB/ fmincon (SQP) are compared regarding the minimum fresh water supply, the number of iterations and the required time for different values of bleed off factor. From the results it is clear that both solvers gave the same results. The major steps are from 121 iterations to 336 for MATLAB showing fast convergence of the algorithm, where for GAMS/SNOPT starts from 20 to 122 iterations showing very fast convergence in some cases, and somewhat slow in other. In most cases GAMS/SNOPT is faster than MATLAB/ fmincon (SQP), except the 5% and 7% bleed off factor cases, where computational time is about the same. This result is expected since GAMS uses analytical derivatives of the objective and constraints functions and so needs less function evaluations. MATLAB uses numerical forward or central differences at the cost of more function evaluations. However, MATLAB solution times can be improved by handling linear constraints separately (in the above results linear constraints are handled as nonlinear) or/and by supplying analytically the derivative of objective and constraints functions. In any case for this problem the computations are fast.

Bleed-Off	0%	5%	7%	10%	15%	20%	25%
Fresh Water	140.2172	160.0961	168.0050	179.8493	199.5451	219.2412	238.9424
Major Steps	24	4631	7122	20	24	140	36
Time	<1	5.5	9.5	<1	<1	<1	<1

Table 3-23 GAMS/SNOPT results for different Bleed-Off factors

Bleed-Off	0%	5%	7%	10%	15%	20%	25%
Fresh Water	140,1550	160,0961	168,0050	179,8493	199,5453	219,2412	238,9424
Iterations	336	155	223	180	199	121	164

Function Evaluations	30765	13990	20264	16194	18093	10980	14521
Time	13,5	6,1	8,9	6,8	8,1	5,2	5,6

Table 3-24 MATLAB/fmincon (SQP) results for different Bleed-Off factors

Chapter 4. Investigation on existing methodology in literature and proposal of different problem solutions

4.1 Genetic Algorithms

As we have seen, mathematical modeling of WWN's involves writing water flow balance and contaminant mass balance equations around the water - using processes and the treatment processes. Optimization of these networks concludes in minimization of fresh water with respect to the above equality constraints and several inequality constraints regarding the maximum inlet and outlet concentrations of flows of a process. In general the wastewater minimization problem of multiple contaminants can be formulated as a Non Linear Programming (NLP) problem. However, in order to reduce the freshwater usage and the cost we have to minimize two or more objective functions or formulate the problem as a mixed integer nonlinear programming problem, using binary variables representing streams between the units. For these kind of problems genetic algorithms can provide better solutions avoiding local minimum or/and providing the Pareto front.

Genetic algorithms generate and evolve solutions randomly. Even if there are techniques to handle nonlinear constraints, due to the random nature generating solutions, equality constraints are very difficult to satisfy. This is the case especially in WWN management optimization where for N water - using processes and K contaminants we have to satisfy (N x K + N) equality constraints. Until now we tried to solve the NLP household problem with standard GA algorithms in MATLAB (using default options) without any success. Runs even with large populations lead to infeasible solutions or to non-optimum solutions (usually the algorithm stacks in the first feasible solution it obtains).

In order to avoid the nonlinear equality constraints Prakotpol and Srinophakun used a special formulation splitting the variables to independent and dependent. The split depends on the number of contaminants and regeneration units. In case of one contaminant without regeneration units, the independent variables are the ones between units flow rates and dependent ones to be taken as the concentrations, fresh water and waste water [Levy,2012]. Independent variables are randomly initialized by the GA algorithm, whereas dependent variables are directly calculated so that equality constraints hold. Inequality constraints and bounds are checked against feasibility of solution. Tudor and Lavric used a different technique. Water using units are ranked according to the fresh water consumption (FWC) or maximum contaminant load (MCL). Each (WU) can receive streams from any higher ranked WU and send streams to any lower ranked WU. Treatment Units are ranked according to their maximum inlet concentration. Each TU receives streams from water using units or higher ranked TU's and send streams to lower ranked TUs or WU. The last TU sends water to WU and disposes waste water to the environment. Treatment units are assumed to have fixed outlet concentration independently of the inflow stream. Using the above formulation, the design variables are fresh water and flows between units whereas contaminants concentration and waste flow can be directly calculated through balance equations.

In order to see in what manner and to what extent can genetic algorithms be useful in waste water management optimization above methodologies were incorporated in several simple industrial systems.

4.2 Scheme 1: Comments on schemes in literature (Prakotpol and Srinophakun)

4.2.1 Problem 1

We have set up a simple industrial system with three separate water-using operations, assuming only one contaminant is diluted during transfer through operations. The limiting process data is presented in **Table 4-1**.

	Load (kg/h)	Inlet concentration restrictions (ppm)	Outlet concentration restrictions (ppm)
Unit no.			
1	3.75	0	75
2	1	50	100
3	1	75	125

Table 4-1 Limiting process data for Problem 1 (USEPA,2004)

This problem can be formulated as a Linear Programming Problem if we set the output concentrations for the contaminant for each process unit equal to the maximum allowable value. The analytic problem equations are:

Objective function

$$F = F_1 + F_2 + F_3 \qquad (4.1)$$

Mass balance around each unit operation

$$F_1 + X_{1,2} + X_{1,3} - W_1 - X_{2,1} - X_{3,1} = 0 \qquad (4.2)$$

$$F_2 + X_{2,1} + X_{2,3} - W_2 - X_{1,2} - X_{3,2} = 0 \qquad (4.3)$$

$$F_3 + X_{3,1} + X_{3,2} - W_3 - X_{1,3} - X_{2,3} = 0 \qquad (4.4)$$

Contaminant balance around each unit operation:

$$3750 + 100X_{1,2} + 125X_{1,3} - 75(W_1 + X_{2,1} + X_{3,1}) = 0 \qquad (4.5)$$

$$1000 + 75X_{2,1} + 125X_{2,3} - 100(W_2 - X_{1,2} - X_{3,2}) = 0 \qquad (4.6)$$

$$1000 + 75X_{3,1} + 100X_{3,2} - 125(W_3 - X_{1,3} - X_{2,3}) = 0 \qquad (4.7)$$

Maximum inlet concentration for each operation

$$\frac{100X_{1,2}+125X_{1,3}}{F_1+X_{1,2}+X_{1,3}} \leq 0 \qquad (4.8)$$

$$\frac{75X_{2,1}+125X_{2,3}}{F_2+X_{2,1}+X_{2,3}} \leq 50 \qquad (4.9)$$

$$\frac{75X_{3,1}+100X_{3,2}}{F_3+X_{3,1}+X_{3,2}} \leq 75 \qquad (4.10)$$

Solving this problem using a linear solver (interior – point method) in MATLAB we got the following solution as seen in **Figure 4-1**:

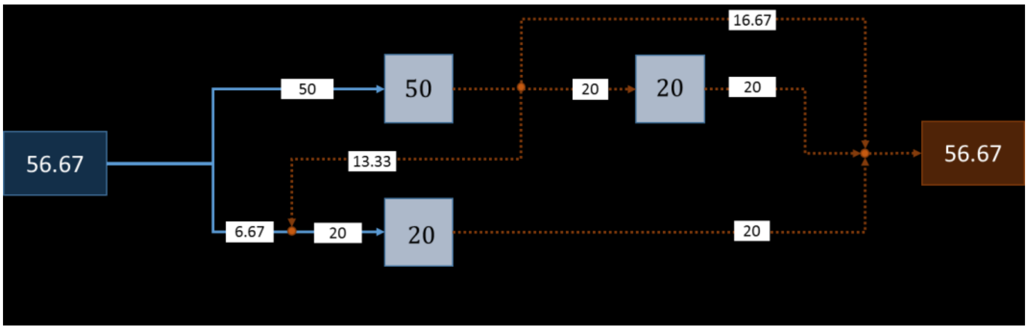


Figure 4-1 Solution of Problem 1 using linear solver

Next we used the approach of Prakotpol to solve the problem in a GA scheme. According to Prakotpol, for single contaminant problems without regeneration units, independent variables that are going to be initialized and evolve by GA are the reused streams ($X_{i,j}$) where dependent variables are the fresh water (F_i) and waste water (W_i) streams. In addition, we introduce binary variables $B_{i,j}$ indicating if the reused stream $X_{i,j}$ appears in the process. So above equations are formulated replacing $X_{i,j}$ with $B_{i,j}X_{i,j}$

We used the following penalty function

$$P = \sum_i |F_i| + \sum_i |W_i|\,(W_i < 0) + \sum_i NL_i(NL_i > 0) \qquad (4.11)$$

Where NL_i is the nonlinear inequality constraint for maximum inlet concentration for operation unit *i*. Results are the same as with the linear solver. Furthermore we can use the above scheme to set up a multi objective problem, with three objective functions namely as follows:

1. **Fresh Water**
2. **Reused water**
3. **Number of reused waters streams**

The Pareto front of the three objectives is presented in **Figure 4-2**.

Now we have a set of optimal solutions to choose depending on our goals and how we want to compromise the objectives of our design. For example we can achieve a minimum fresh flow water of 56.67 L/day using two reused water streams of total flow 33.3 L/day. On the other hand we could achieve a fresh water of 60 L/day using only one reused water stream of total flow 20. So using GA give as the ability to optimize multiple objective functions to produce a set of optimal solutions (Pareto front). In addition using GA's we can easily solve problems formulated as MINLP.

On the other hand GA's are more time consuming than gradient based methods, as SQP, and need more iterations and function evaluations to provide optimal solutions even in simple problems.

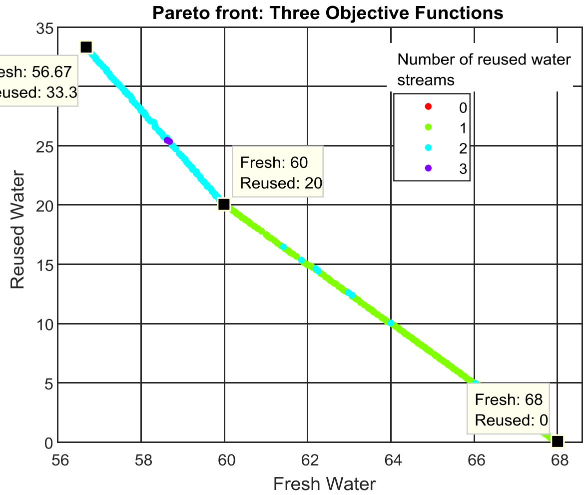


Figure 4-2 Pareto front

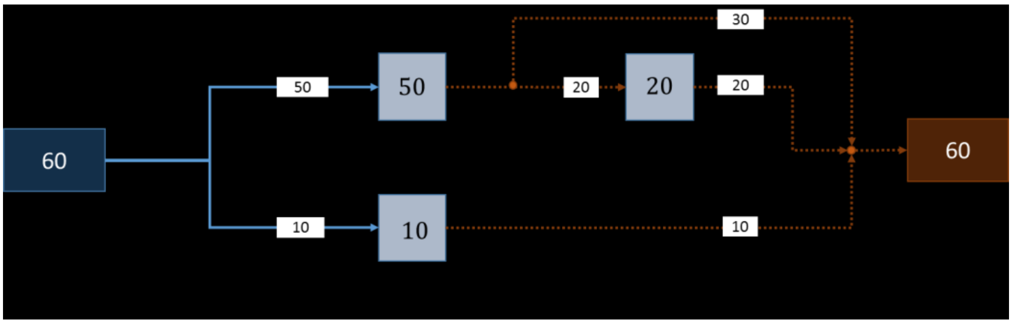


Figure 4-3 A solution from Pareto front using GA

4.2.2 Problem 2

In this problem as shown in **Figure 4-3**, we considered two separate water-using operations assuming two contaminants are diluted during transferred through those operations. Two treatment units are used. The limiting process data is presented in **Tables 4-2 and 4-3.**

Even if Prakotpol did not report results for multi-contaminant WWN's with regeneration units the scheme worked well for this small problem.

	Load (kg/h)		Inlet concentration restrictions (ppm)		Outlet concentration restrictions (ppm)	
	contaminant		contaminant		Contaminant	
Unit no.	1	2	1	2	1	2
1	3	2.4	0	0	100	80
2	4	5.6	50	20	150	160

Table 4-2 Limiting process data for Problem 2

	Removal Ratio	
	Contaminant	
Unit no.	1	2
1	0.8	0.5
2	0.5	0.9

Table 4 -3 Treatment unit removal ratio for Problem 2

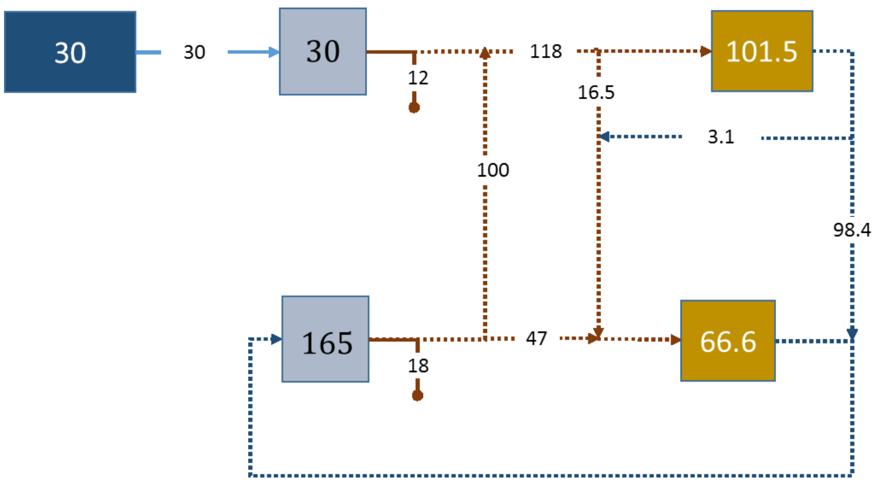


Figure 4-4 Set up with assumed fixed outflow concentrations

4.2.3 Problem 3

Finally as shown in **Figure 4-4** we tried to solve the household unit problem. Prakotpol's scheme does not accommodate the case of multi contaminant flows and regeneration units. So we developed a similar scheme for this case. In order to make the problem simpler we considered one treatment unit with fixed outflow concentration and we removed the constraints with respect to WC flows and the bleed-Off.

In our scheme all the flows are estimated in one step formulating a linear programming problem with stochastic objective function driven by the GA. In particular this step can be formulated as follows:

$$\min_{\theta} \sum_{k} B_k \, \theta_k \qquad (4.12)$$

Where

$$\theta = [F_i, X_{i,j}, R_i, W_i, T_i], \qquad i, j \epsilon I \qquad (4-13)$$

And B_k is a binary variable created and evolved by the GA algorithm

With respect to the following linear constraints

$$F_i + \sum_{j \in I} X_{i,j} + R_i = W_i + \sum_{j \in I} X_{j,i} + T_i i \epsilon I \qquad (4\text{-}14)$$

$$V_i = F_i + \sum_{j \in I} X_{i,j} + R_i > \max_{k \in K} \frac{M_{i,k}}{C_{i,k}^{OUT,max} - C_{i,k}^{IN,max}}, \qquad i \in I \qquad (4\text{-}15)$$

$$F_i, X_{i,j}, R_i, W_i, T_i > 0, \qquad i, j \in I \qquad (4\text{-}16)$$

B_k variable is created and evolved by the GA algorithm using selection, crossover and mutation functions. When B_k is equal to 1 the linear programming solver will try to make zero or at least minimize the flow associated with flow θ_k.

Once the flows have been estimated, the outflow concentration of the water-using processes can be estimated by solving a linear system produced by mass balance constraints around the water using processes, assuming the above estimated flows and specific outlet concentration for the treatment unit. In particular, the mass balance equations around the water using processes can be written as:

$$F_i C_k^F + \sum_{\substack{j \epsilon I \\ j \neq i}} X_{i,j} C_{j,k}^{OUT} + R_i C_{i,k}^T + M_{i,k} = \left(W_i + \sum_{\substack{j \epsilon I \\ j \neq i}} X_{i,j} + T_i \right) C_{i,k}^{OUT}, \qquad i \epsilon I, k \epsilon K \quad (4-17)$$

Or

$$\left(W_i + \sum_{\substack{j \epsilon I \\ j \neq i}} X_{i,j} + T_i\right) C_{i,k}^{OUT} - \sum_{\substack{j \epsilon I \\ j \neq i}} X_{i,j}\, C_{j,k}^{OUT} = F_i C_k^F + R_i C_{i,k}^T + M_{i,k} \qquad (4-18)$$

Which is an NxN linear system on water-using process outflow concentrations, $C_{i,k}^{OUT}$. Finally, after the concentrations have been estimated we check the inequality constraints regarding the maximum inlet concentrations. Maximum outlet concentrations constraints will be fulfilled since we have set a constraint in the minimum flow rate through the unit.

$$F_i\left(C_k^F - C_{i,k}^{IN,max}\right) + \sum_{\substack{j \epsilon I \\ j \neq i}} X_{i,j}\left(C_{j,k}^{OUT} - C_{i,k}^{IN,max}\right) + R_i\left(C_{i,k}^T - C_{i,k}^{IN,max}\right) \leq 0, \quad i \epsilon I, k \epsilon K \qquad (4-19)$$

The results of the algorithm for the household unit using the above scheme are presented in **Figure 4-5**:

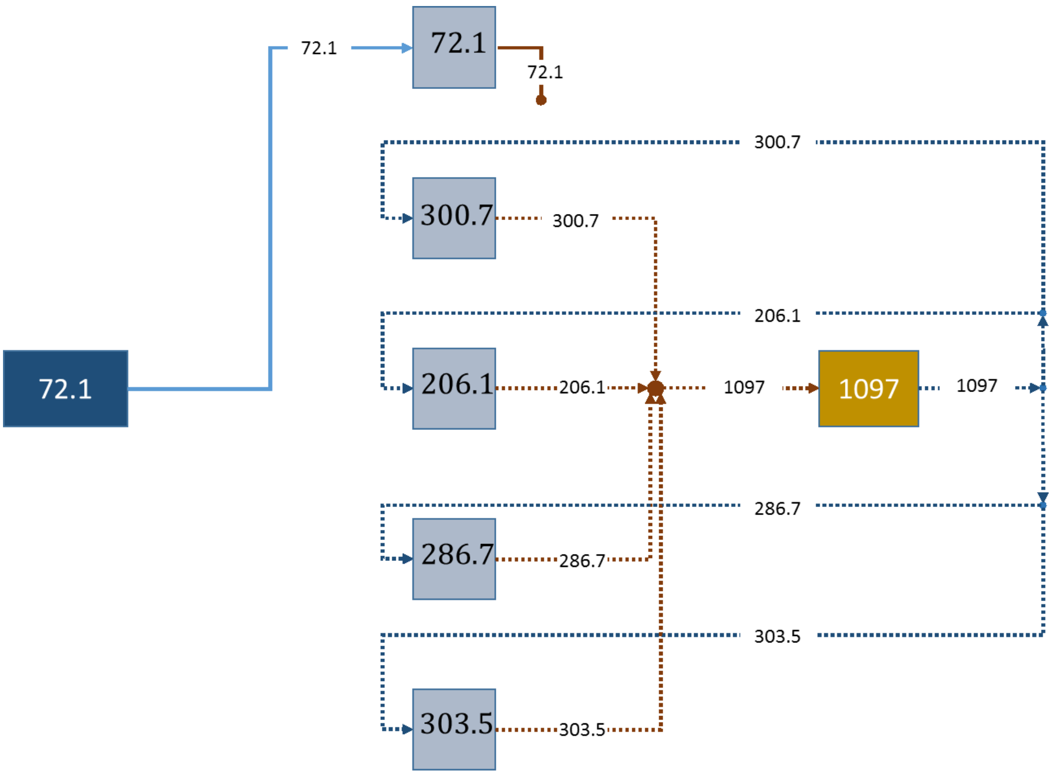


Figure 4-5
Estimated flows in one step formulating a linear programming problem with stochastic objective function driven by the GA

4.3. Scheme 2: Raluca Tudor and Vasile Lavric

We implemented the Raluca Tudor and Vasile Lavric formulation in the example they presented in their paper [Gikas, 2011]. Instead of just minimizing fresh water, we found the Pareto front solutions of minimizing fresh water and the total operating cost. We did the ranking using the MCL criteria. Example data are given in **Table 4-4** and **Table 4-5** where the estimated Pareto front is shown in **Figure 4-2**. Extreme points on the estimated front indicate a minimum fresh water usage of 14.734 tn/h with operational cost of 2,765.5 P mu[1] and minimum operational cost of 2158.9 P mu with fresh water usage of 21.356 tn/h.

	Load (kg/h)			Inlet concentration restrictions (ppm)			Outlet concentration restrictions (ppm)		
	contaminant			contaminant			Contaminant		
Unit no.	1	2	3	1	2	3	1	2	3
1	0.35	0.25	0.15	0	0	0	35	45	55
2	0.15	0.35	0.25	15	20	25	70	90	120
3	0.35	0.45	0.55	15	35	0	75	95	125
4	0.45	0.15	0.45	25	45	45	95	85	135
5	0.25	0.45	0.35	45	35	55	90	100	120
6	0.15	0.45	0.85	35	20	25	85	80	95

Table 4-4
Primary Data (Mass Loads, Inlet, and Outlet Restrictions) of the Water-Using Units of the Integrated Network (USEPA,2004)

	Inlet concentration restrictions (ppm)			Outlet concentration restrictions (ppm)		
	contaminant			Contaminant		
Treatment Unit no.	1	2	3	1	2	3
1	45	45	55	30	40	50
2	25	35	45	20	25	30
3	15	20	25	2	4	5

Table 4-5
Primary Data (Mass Loads and Inlet and Outlet Restrictions) of the Wastewater Treatment Units of the Integrated Network (USEPA,2004)

[1] mu: monetary units

The model equations are:

$$\min_{\theta} \sum_{i \in I} F_i \tag{4-20}$$

$$\theta \qquad \min P \sum_{i \in I} F_i + P \sum_{t \in T} Cost_t \left(X_{t,t-1} + \sum_{i \in I} X_{t,i} \right) \qquad (4-21)$$

Where

$$\theta = [F_i, W, \ X_{q,r}], \qquad i \epsilon I, q, r \epsilon I \cup T, k \epsilon K \qquad (4-22)$$

$$P: is\ the\ cost\ of\ fresh\ water\ in\ \frac{mu}{t}$$

$$Cost_t: is\ a\ cost\ factor\ for\ each\ \ treatment\ unit$$

With respect to the following constraints:

$$F_i + \sum_{\substack{j \epsilon I \\ i > j}} X_{i,j} + \sum_{t \in T} X_{i,t} = \sum_{\substack{j \epsilon I \\ i < j}} X_{j,i} + \sum_{t \in T} X_{t,i} \ i \epsilon I \qquad \text{(a)}$$

$$X_{t,t-1} + \sum_{i \in I} X_{t,i} = X_{t+1,t} + \sum_{i \in I} X_{i,t} \ t \epsilon T$$

(b)

$$C_{i,k}^{OUT} \leq C_{i,k}^{OUT,max}$$

$$C_{i,k}^{IN} \leq C_{i,k}^{IN,max} \qquad \text{(c)}$$

(d)

$$C_{t,k}^{IN} \geq C_{t,k}^{IN,min}$$

Where

$$C_{i,k}^{OUT} = \frac{F_i C_k^F + \sum_{\substack{j\epsilon I \\ i>j}} X_{i,j} C_{j,k}^{OUT} + \sum_{t\in T} X_{i,t} C_{t,k}^{OUT} + M_{i,k}}{\sum_{\substack{j\epsilon I \\ i<j}} X_{j,i} + \sum_{t\in T} X_{t,i}} \quad \text{(e)}$$

$$C_{i,k}^{IN} = \frac{F_i C_k^F + \sum_{\substack{j\epsilon I \\ i>j}} X_{i,j} C_{j,k}^{OUT} + \sum_{t\in T} X_{i,t} C_{t,k}^{OUT}}{F_i + \sum_{\substack{j\epsilon I \\ i>j}} X_{i,j} + \sum_{t\in T} X_{i,t}} \quad \text{(f)}$$

$$C_{t,k}^{OUT} = C_{t,k}^{OUT,max} \quad \text{(g)}$$

$$C_{t,k}^{IN} = \frac{X_{t,t-1} C_{t-1,k}^{OUT} + \sum_{i\in I} X_{t,i} C_{i,k}^{OUT}}{X_{t,t-1} + \sum_{i\in I} X_{t,i}} \quad \text{(h)}$$

4.4 Conclusion

In conclusion, GA solvers can be useful for optimizing WNNs because they can handle binary variables as well as multi objective problems. In particular we can optimize networks regarding minimum fresh water and cost as well as use binary variables to indicate if a stream appears in the network. On the other hand GA's are more time-consuming than gradient based methods, as SQP, need more iterations and function evaluations to provide optimal solutions and they need special sophisticated handling for the equality constraints.

So far the optimized proposed RO and MF system connections as well as waste and fresh water charges within the context of average demands of a typical household of 4 inhabitants were simulated. In the next chapter the upscale level of a specific geographic area within the Luxemburgish territory will be examined. Different optimization algorithms as well solution methods will be adopted. The initial nonlinear problem will be converted to a linear one as the dataset of this upscale context is significantly larger and the computational platform proposed and thus adopted within the context of this work would function accordingly through a linear model.

PART II

In Part I of the thesis the problem comprised investigation and simulation of different connection set up scenarios between the different elements of the examined system and outputs regarding optimized (reduced) fresh water inflows as well as waste water outflows of the system to the central sewage grid In this second part (PART II) of the present work the waste water network in the upscaled level is examined and then analyzed. Results of the reduced outflows from PART I of the thesis are utilized as inputs to come up with final resulting figures of this upscale model that is introduced in this second part.

Chapter 5. The Upscale Level

The present model under study describes an attempt to optimize existing waste water network of Luxembourg in the local and national scale. An optimal re-location of existing wastewater plants network for potable water supply in Luxembourg will be examined. Potable water production in Luxembourg is sourced 2/3 by ground waters and 1/3 by surface waters. Ground waters are obtained from springs and wells (Levy, 2012). Surface waters are obtained by treating the waste waters by waste water plants. These waters are mixed before being delivered to the customer. The main problem consists of increasing the rate of surface water in this mixture for improving the quality of the water (Levy, 2012). Cost for wastewater treatment plants depends on capital cost of plant construction, pipe capital and operating cost, pump capital and operating cost. The further the plant is from the customer, the longer is the pipe, the higher the operating cost. The quality of the pipe also needs to be of better material quality for larger distances. Eventually the higher is the distance more pumping energy is required to deliver the water from the plant over the pipe to the end user. Additional risk of water loss increases with the complexity of the pipeline network. However, reducing the network size might be associated with the need to the number of treatment plants, which in turn could drive cost to much higher levels as well. Therefore the aim of this work is to increase wastewater treatment capacity of selected waste water plants of a selected area like improving and/or expanding existing plant units or constructing new ones. This model aims to find an optimal number of wastewater plants and their optimal location with a minimal cost while satisfying certain demand constraints.

5.1 Water and Waste Water Network (W&WWN) mathematical Model

Here a more inclusive version of the mathematical formulation of the associated combinatorial problem that is to be utilized eventually, is introduced. An integrated holistic model is at first attempted and a complete cycle of water and waste water is represented within a network. The examined model which is eventually examined later on excludes the cycles of fresh and reclaimed potable water as well as the reclamation of waste water that is performed within a WWTP off the modeled network. This choice was taken by the author with the purpose to focus on the waste water cycle separately inside the system. This would give rise to clearer results compared to a more holistic approach to the problem.

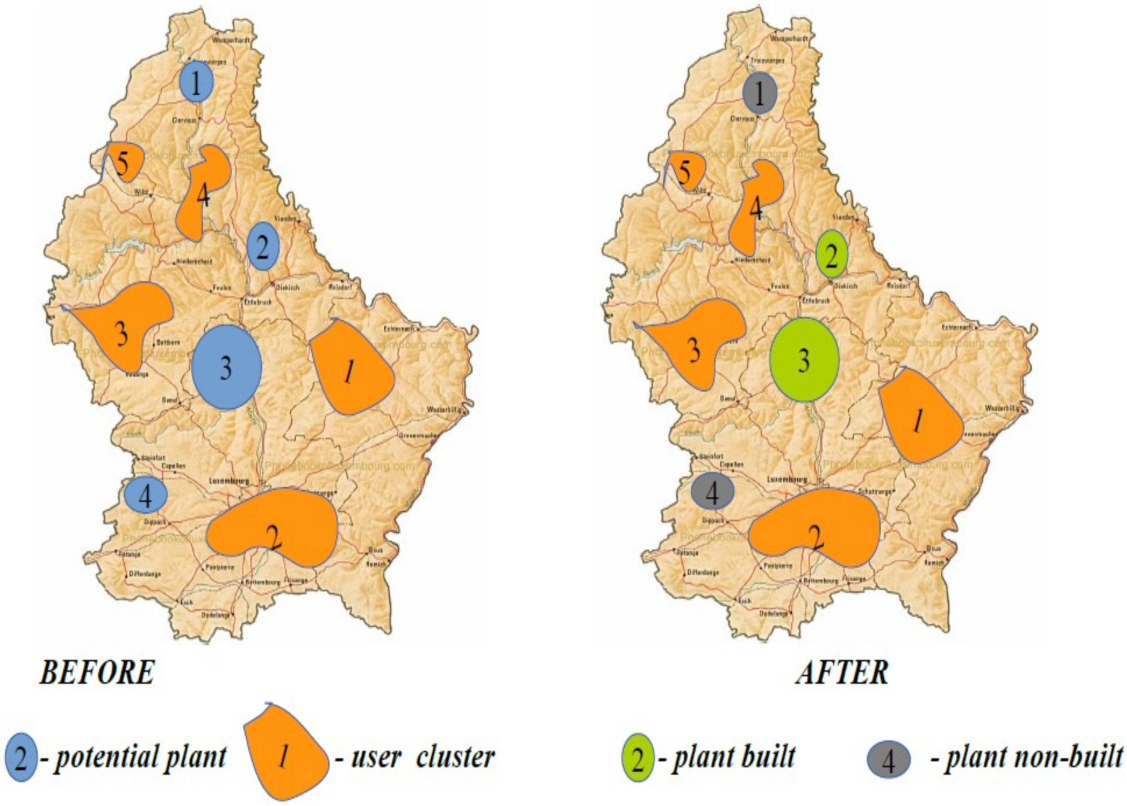


Figure 5-1 Indicative layout of intended post analysis optimized scenarios for Luxembourgish territory

The optimization problem consists of determining the actual location and size of water treatment units at each node as well as the flows between the nodes in order to satisfy the demand and to minimize the annualized total cost, including capital and operating costs. The capital cost includes the investment cost for plants, pipelines, pumps, and storage tanks, while the operating cost comprises of plant production operating cost and pumping cost.

In **Figure 5-1** an indicative layout for the optimization problem to be incorporated is presented schematically. The initial as well as the post analysis scenario of the examined optimization model will be analyzed.

In particular the variables to be determined by the problem are:

- Locations and capacities of surface water treatment, wastewater treatment, and water reclamation plants
- Pipeline main networks for fresh water, wastewater and reclaimed water, including piping diameters;
- Production volumes of surface water, treated wastewater and reclaimed water at plants;
- Main flows of fresh water, wastewater and reclaimed water during each time period;
- Number, types and operating fractions of pumps for each established link;
- Locations, number and sizes of storage tanks for potable and non-potable water;

In the optimization problem of integrated water and waste water resources management, the following are given thus comprise data set:

- Region Geometrical Parameters: regions, nodes (population centre and potential plant locations), pair-wise distances, pumping distances and elevations between the nodes;
- Region Water Parameters: Potable and non potable water demands, wastewater productions, and available groundwater;
- Plant Cost parameters: Capital investment capital cost of desalination, wastewater treatment and reclamation plants at multiple plant capacity levels;
- Unit energy consumption of surface water treatment, wastewater treatment and reclaimed water production (additional treatment after wastewater treatment), at multiple production volume levels;
- Pipe network Cost Parameters: unit costs of pipelines, dependent on pipe diameter; capital costs of storage tank, dependent on tank size; types, costs and efficiencies of pumps; unit cost of electricity.

At this point it should be noted that notations in this preliminary version of the final model refer only to this preliminary model and do not coincide with notations of variables and parameters of the final model which is eventually examined and analyzed and are set later on:

Sets

· Set **I = {i,…, N}** of end-user clusters (households, commercial , industrial, agriculture zones).

· Set **J = {j,…, K}** of possible plants. Location of each plant is pre-defined.

Parameters

It is assumed that the demand for potable water for each cluster **i** is already known, $D_{tot,i}$ is the total quantity of ground water F_i transported to the cluster **i** and total quantity of regenerated water by the clusters themselves is denoted as D_{reg}.

$\Sigma_{i \in I} X^{tr}_{i} = R_{MF}+R_{RO}$, their installation can also be considered as a binary variable as well.

$R_{MF}+R_{RO}$ is the overall quantity of the regenerated waste water treated in the MF/RO domestic treatment systems and according to PART I of the present study refers only to their installation within the private typical households of 4 users.

$D_i=D_{tot} - F_i - D_{reg}$ is the total quantity of surface water necessary to satisfy the total demand for potable surface water; X^{tr}_{ji} is the quantity of flow from the plant **j** to the cluster **i** and is known. This flow represents the treated flow, with respect to the concentration of contaminants by law measured in m3/day. The quantity of waste water flow produced in cluster **i** to the plant **j**, X^{tr}_{ij} is known. This is the contaminated waste water flow produced from users to the plant measured in **m^3/day**. For each plant P_j an associated capital cost CC_j measured in € is known. At first we consider that each plant has a necessary number of pipelines to transport water to cluster and **i**, a necessary number of pumping stations with a minimum flow rate. For each flow from plant **j** to cluster **i**, X^{tr}_{ji}, the associated capital cost CO_{ji} is also known. This cost depends on the length of the pipeline and velocity of water which in turns depends on the power of the pumping station and the diameter. The function of cost **CO_{ij}= f (length, velocity, diameter)** measured in € is given. For each flow from cluster **i** to plant **j**, X^{tr}_{ij} an associated capital cost CO_{ij} measured in € is known.

The function of cost: **CO_{ij} = f (length, velocity, diameter)** is given. Measured in €.

· Potable waste water treatment capacity for plant **j**, A_j is known;

· Potable surface water demand for cluster **i**, D_i *is* known.

· The cost of the storage tanks could also be included so this would represent a separate problem itself. It is a common problem in operations research: That is to produce “just on time of demand” this implies a -no storage- cost with a higher operational cost or to - produce and then store- scenario which gives rise to lower operational cost but a higher significant storage cost.

Variables:

· $\mathbf{P_j}$ verifies the possibility of constructing a plant **j**, $\mathbf{P_j \in \{0, 1\}}$. $\mathbf{P_j=0}$, if plant *j* is constructed, 0otherwise.

Objective function:

$$\mathbf{Min\ \Sigma_{j \in J}\, CC_j P_j + \Sigma_{i \in I}\, CO_{ij} P_j + \Sigma_{i \in I}\, CO_{ji} P_j} \quad (5\text{-}1)$$

Constraints:

The minimum demand $\mathbf{D_i}$ for potable surface water for cluster **i** should be satisfied

$$\mathbf{\Sigma_{j \in J} X^{tr}_{ji} \geq D_i} \quad (5\text{-}2)$$

- $\mathbf{D_i}$– potable surface water demand for cluster **i**;

- $\mathbf{X^{tr}_{ji}}$ - surface water flow from plant **j** to cluster **i**

Flow balance, inflow into the plant **j** is equal to the outflow from plant *j:*

$$\mathbf{\Sigma_{i \in I} X^{tr}_{ij} = \Sigma_{i \in I} X^{tr}_{ji}} \quad (5\text{-}3)$$

- $\mathbf{X^{tr}_{ij}}$- surface water flow from cluster **i** to plant **j**;

- $\mathbf{X^{tr}_{ji}}$- surface water flow from plant **j** to cluster **i**;

Total of flows from or out of plants should not exceed potable water capacity treatment for plant

$$\mathbf{j}: \quad \mathbf{\Sigma_{i \in I} X^{tr}_{ji} \leq A_j} \quad (5\text{-}4)$$

In the following tables **(Table 5-1 to Table 5-7)** all necessary input data parameters for this indicative model regarding correlation between all variables and parameters towards defining the system to be examined are presented. The applied mathematical formulation to follow in the next chapters will be partially based on above mentioned considerations for the examined area.

Input data

Cluster i\ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	1	0	0	1	1
Plant 2	1	1	1	1	1
Plant 3	0	0	0	0	0
Plant 4	1	0	0	1	0

Table 5-1 Possibility of water transport from plant j cluster i (binary):

Cluster i \ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	0	1	0	1	0
Plant 2	1	1	1	1	1
Plant 3	0	1	0	0	1
Plant 4	0	0	1	1	0

Table 5-2 Possibility of waste water transport from cluster ito plant j (binary):

Cluster i \ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 2	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 3	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 4	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$

Table 5-3 Waste water flow quantity transported from plant j to cluster i (in m3/day):

Cluster i \ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 2	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 3	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$
Plant 4	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$	$X^{tr}{}_{ji}$

Table 5-4 Waste water flow quantity transported from cluster i to plant j (in m3/day):

Plant j	CC_j
Plant 1	CC_1
Plant 2	CC_2
Plant 3	CC_3
Plant 4	CC_4

Table 5-5 Capital cost of plant j (in €)

Cluster i \ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}
Plant 2	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}
Plant 3	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}
Plant 4	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}	CO_{ji}

Table 5-6 Operating cost of flow from plant j to cluster i (in €/day)

Cluster i \ Plant j	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Plant 1	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}
Plant 2	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}
Plant 3	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}
Plant 4	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}	CO_{ij}

Table 5-7 Operating cost of flow from cluster i to plant j(in €/day)

In **Table 5-8** the decision generated as output data of the examined system is the construction decision of the potential WWTP's may be presented in the form of an array as follows:

Plant j \	Construction decision
Plant 1	1
Plant 2	0
Plant 3	1
Plant 4	0

Table 5-8 Minimum Network Cost = COST

5.2 Linear programming for Water and Waste Water Network (W&WWN's)

In the literature there are different ways to model a WWN: linear, Mixed Integer Linear Programming (MILP) linear, non-linear (MINLP), convex and non-convex. There are advantages and disadvantages to model one way or another. It depends on the degree of interdependence among variables and the selection of variables as well as constants. In case the objective function and the constraints are linear functions of decision variables, there lies the highest probability that a globally optimal solution is attained relatively quickly, given the size of the model. This is a linear programming problem;

It also comprises a convex optimization problem (as all linear functions are also convex). The Simplex LP solving method is suitably designed for these kinds of problems. In case the objective function and the set of constraints are smooth nonlinear functions of the decision variables, solution times will take longer. In case the problem is convex, there is a high probability to attain a globally optimal solution. In the scenario it is non-convex, there is only the possibility to find a locally optimal solution – and even this may be hard to find. There are also non-smooth and non-convex objective functions and constraints which may only yield a relatively good solution, with a lower probability for a locally or globally optimal solution. Thus in case the problem gets a nonlinear form, its solution quality may vary considerably upon an instance. In addition, nonlinear models are often resolved through linearization, however, this increases the number of variables and constraints significantly. Therefore, in case it is possible to avoid incorporating non linear models, while making reliable assumptions, it would be recommended to attempt to build linear functions. Furthermore, linear models' computation time is usually significantly less compared to nonlinear ones. Therefore, a first attempt might be suitable for our water network to be modeled as linear, however this can be realized when relying on a number of assumptions, applying it to different size of instances. Subsequently, switching to nonlinear methods may be required when necessary. There are plenty of linear optimization solvers such CPLEX, GAMS, Gurobi, Xpress and others. Each of these exhibit advantages and certain draw-backs and may prove to be more efficient in different types of problems. The CPLEX Optimizer was named as a suitable tool to be utilized for the Simplex method as it is mostly implemented in the C and C+ programming languages, although today it also supports other types of mathematical optimization problems and offers interfaces other than just C. Additionally, connectors to Microsoft Excel and MATLAB are provided. Finally, a stand-alone interactive optimizer executable is provided for debugging and other purposes. Here is a summary of CPLEX technology depending on the problem type as shown in **Table 5-9**. Moreover, CPLEX also offers a network optimizer aimed at a special class of linear problems with a network structure. CPLEX can optimize such problems as ordinary linear programs, but if CPLEX can extract all or part of the problem as a network, then it will apply its more efficient network optimizer to that part of your problem and use the partial solution it finds there to construct an advanced starting point to optimize the rest of the problem. (IBM 2014) besides, CPLEX has a good advantage of paralleling computing: CPLEX's speed-up in going from one to four processors is approximately 40 sec in our problem setting. CPLEX has the advantage of attaining good results in paralleling computing: CPLEX's speed-up in going from one to four processors is approximately 30% in CPU time in our problem setting.

Problem Type	No Integer Variables	Has Integer Variables	No Quadratic Terms In the Objective Function	Has Quadratic Terms In the Objective Function	Has Quadratic Terms in Constraints
lp	X		X		
qp	X			X	
qcp	X			possibly	X
milp		X	X		
miqp		X		X	
miqcp		X		possibly	X

Table 5-9 Association of algorithm methods adopted for different kind of optimization problems *(Source: IBM)*

In this section a mathematical formulation of the problem of the system is developed. The basic formulation of the problem is based in the mixed integer optimization approach for integrated water resources management derived by (Gikas, 2011).

5.3 Mass balances of Water and Waste Water Network and consideration of storage of fresh water

Flow mass balance in both potable and non potable water and waste water systems are presented accordingly in

Figure 5-2**.**

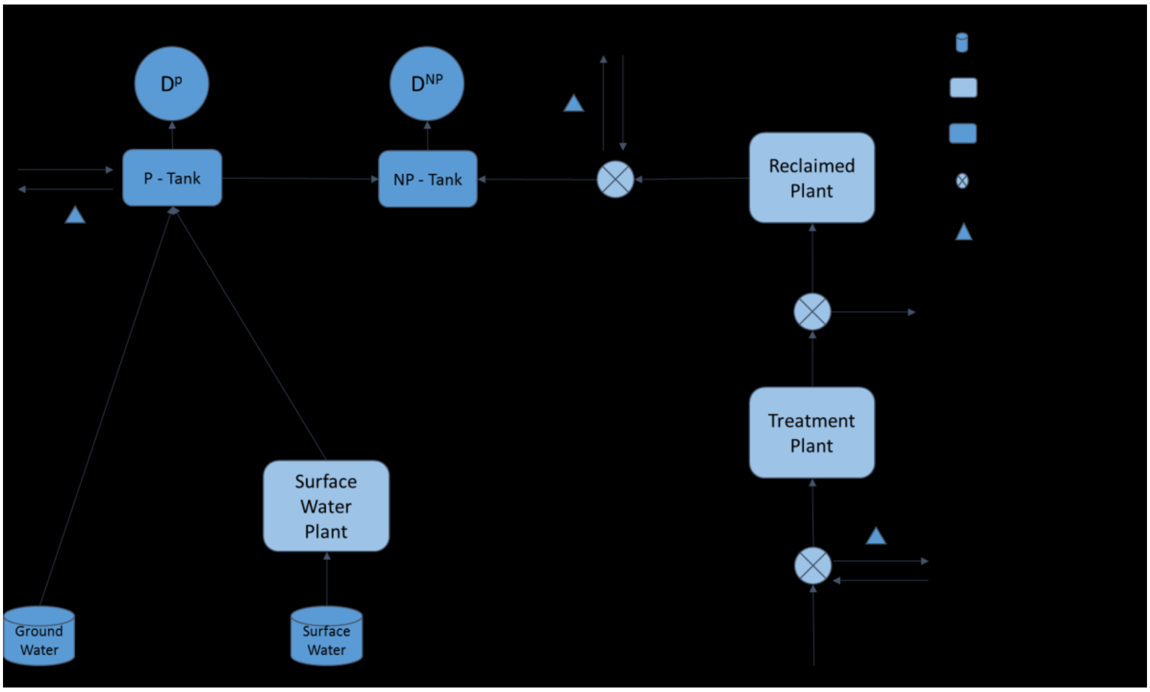


Figure 5-2 Flow mass balances in potable and non potable water systems

At any node the local potable water demand equals the ground water supply plus the treated surface water production, plus all incoming / outgoing potable water flows, minus the flows to the non-potable water system.

$$D_i^p = F_i^P = G_i + S_i + \sum_{\substack{i,j \in N \\ i \neq j}} F_{ij} - \sum_{\substack{i,j \in N \\ i \neq j}} F_{ji} - F_i^{NP} \tag{5-5}$$

And the non–potable water demands equals the flow of fresh water from the potable water tanks plus the reclaimed water from the reclaimed water plant, which equals the generated reclaimed water plus all incoming / outgoing reclaimed water flows.

$$D_i^{NP} = F_i^{NP} + R_i^{NP} = F_i^{NP} + R_i + \sum_{\substack{i,j \in N \\ i \neq j}} R_{ij} - \sum_{\substack{i,j \in N \\ i \neq j}} R_{ji} \tag{5-6}$$

The treated water production in the treatment plant equals the waste water generated by the node plus all incoming / outgoing waste water.

$$T_i = W_i + \sum_{\substack{i,j \in N \\ i \neq j}} W_{ij} - \sum_{\substack{i,j \in N \\ i \neq j}} W_{ji} \tag{5-7}$$

The reclaimed water equals the waste water minus the water disposed.

$$R_i = T_i - DS_i \tag{5-8}$$

5.3.1 Storage tanks

We assume storage tanks for the fresh water (ground water and treated surface water) and for the non – potable water. The capacity of a storage tank can be determined using the daily water demand and generation hourly profiles (Martz,1970). Using these profiles the capacity is determined as percentage of the daily demand. This percentage represents the coverage time τ a tank can cover the demands without any effluent of generated water. If C_i^P is the total capacity of fresh water tanks at node i, then the following inequality must hold

$$C_i^P \geq \tau D_i^P \tag{5-9}$$

If further we assume we have predefined size tanks andC_m^P is the capacity of tanks of type $m$and N_{im}^P is the number of fresh water tanks of type m at node i then equation can be written as:

$$\sum_{m \in ST} N_{im}^P C_m^P \geq \tau D_i^P \tag{5-10}$$

Similarly, for the non – potable water tanks

$$\sum_{m \in ST} N_{im}^{NP} C_m^{NP} \geq \tau D_i^{NP} \tag{5-11}$$

The capital cost of the storage tanks can be determined as

$$CC^{ST} = \sum_{I \in} \left(\sum_{m \in ST} N_{im}^{P} CC_m^{ST,P} + \sum_{m \in ST} N_{im}^{NP} CC_m^{ST,NP} \right) \quad (5\text{-}12)$$

Where CC_m is the cost of a storage tank of type m

5.3.2 Plant production capacity and cost

The capital cost of a plant is a function (general nonlinear) of the plant capacity (Table 5-10). Usually we use cost data from installed plants and try to fit a nonlinear or piecewise linear function of the plant capacity. For example it is assumed that the following data are known regarding the plant capital cost for different value of plant capacity as shown in

Plant Capacity (m3/day)	Plant Capital Cost (k$)
100	100
1000	650
2500	1500
5000	2300
10000	3200

Table 5-10, Typical plant capital costs (Gikas, 2011)

The linear least square technique could be utilized to fit a polynomial function, i.e. a fitted quadratic function using the above data results in the following cost function

$$Cost = -2.9472 \times 10^{-5} Capacity^2 + 0.6061\, Capacity + 74.904 \quad (5\text{-}13)$$

Another way is to express capital cost as a piecewise linear function, as

$$Cost = \begin{cases} 0.6111\, C + 38.889, & 100 \le C < 1000 \\ 0.5667\, C + 83.333, & 1000 \le C < 2500 \\ 0.3200\, C + 700 \quad , & 2500 \le C < 5000 \\ 0.1800\, C + 1400 \quad , & 5000 \le C < 10000 \end{cases} \quad (5 - 13\alpha)$$

Data and fitted functions are presented in

Figure 5-3. Usually in order to set the problem as a linear optimization problem, we use linear functions. However the piecewise form of the above function is not suitable for setting a linear programming problem. Another suitable form for expressing the above piecewise linear function is the SOS2 form. This form is used as it can be seen later on.

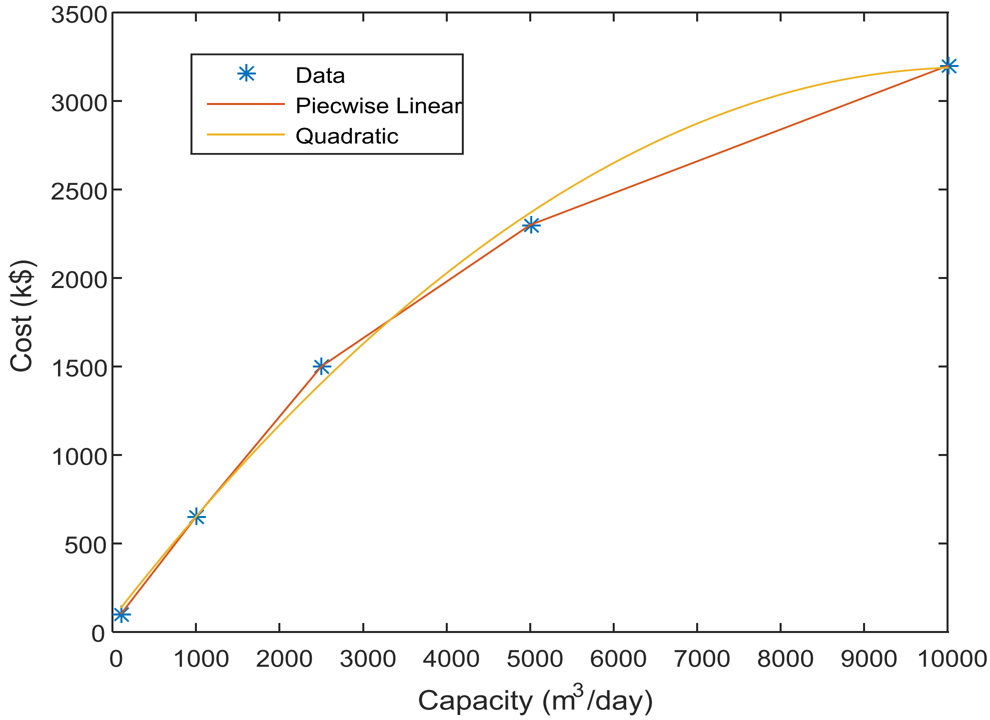


Figure 5-3 Capital cost of a plant versus capacity

Therefore using the capital cost plant break points the capacity of a treatment plant can be expressed as follows:

$$C_i^{pl} = \sum_{b \in B} C_b^{pl} \lambda_{ib}^{pl}, \qquad \forall\, pl \in PL \text{ and } \forall\, i \in I \qquad (5\text{-}14)$$

Where C_n^{pl} is the capacity of a plant at break point b and λ_{ib}^{pl} is a SOS2 variable.

The plant cost is then calculated as

$$CC_i^{pl} = \sum_{b \in B} CC_b^{pl} \lambda_{ib}^{pl}, \qquad \forall\, pl \in PL \text{ and } \forall\, i \in I \qquad (5\text{-}15)$$

Where CC_b^{pl} is the capital cost of a plant at break point b and λ_{ib}^{pl} is a SOS2 variable

Furthermore the plant production cannot exceed the plant capacity, so

$$P_i^{pl} \leq CC_i^{pl}, \forall\, pl \in PL \; and \; \forall\, i \in I \tag{5-16}$$

5.3.3 Pipeline networks. Equations & Constraints

There are three individual pipeline main networks to be determined. One for fresh water (*F*), one for the reclaimed water (*R*) and one for the waste water (W). If L_{ijp}^{n} is a binary variable indicating if there is a link of network n between nodes i and j using a pipe of type p then there can exist only one type of pipe among two nodes, or

$$\sum_{p \in P} L_{ijp}^{n} \leq 1, \forall\, n \in N, \tag{5-17}$$

In addition the pipe type for link between nodes j and i is the same with pipe type between nodes$i$and$j$.

$$L_{ijp}^{n} = L_{jip}^{n} \tag{5-18}$$

The flow rate of water in a pipe, is related to the velocity of water and the pipe diameter, and can be calculated by the following equation:

$$Q_p^n = \frac{au^n \pi d_p^2}{4}, \forall\, n \in N, p \in P \tag{5-19}$$

Then the daily flow of water through a link, if the link exists and a pump is needed, equals the flow rate of the pipe times the proportion of operating time of a pump during a day (operating fraction - f_{ij}^{n}), or

$$Q_{ij}^{n} = f_{ij}^{n} \sum_{p \in P} Q_p^n L_{ijp}^{n} = \sum_{p \in P} Q_p^n f_{ij}^{n} L_{ijp}^{n}, \forall\, n \in N \tag{5-20}$$

The term $f_{ij}^{n} L_{ijp}^{n}$ can be linearized by introducing a continuous variable

$LC_{ijp}^{n} = f_{ij}^{n} L_{ijp}^{n}$ with $LC_{ijp}^{n} \in [0,1]$. Then the nonlinear Equation is equivalent to the following set of equations

$$Q_{ij}^{n} = \sum_{p \in P} Q_{p}^{n} LC_{ijp}^{n}, \forall n \in N \qquad (5-21\alpha$$

$$f_{ij}^{n} = \sum_{p \in P} LC_{ijp}^{n}, \forall n \in N \qquad \text{(5-21b)}$$

$$LC_{ijp}^{n} \leq L_{ijp}^{n}, \forall n \in N, \forall p \in P \qquad (5-21c)$$

For the other links where no pump is needed, we use simpler constraints, which guarantee that the actual flow does not exceed the allowed flow rate in the selected pipe.

$$Q_{ij}^{n} \leq \sum_{p \in P} Q_{p}^{n} L_{ijp}^{n}, \forall n \in N \qquad \text{(5-22)}$$

If there is a link between nodes j and i and a pump is required then one type of pump should be used at most as follows:

$$\sum_{u \in U} L_{iju}^{n} \leq \sum_{p \in P} L_{ijp}^{n}, \forall n \in N \qquad \text{(5-23)}$$

The maximum flow rate of the selected pump must be no less than the flow rate on the link, or

$$\sum_{u \in U} F_{max,u}^{n} L_{iju}^{n} \geq \sum_{p \in P} Q_{p}^{n} L_{ijp}^{n}, \forall n \in N \qquad \text{(5-24)}$$

Also the summation of the maximum height of the pumps selected should not exceed the corresponding height which is called pumping elevation plus the head loss height.

$$\sum_{u \in U} \left(H_{max,u}^{n} N_{iju}^{n}\right) L_{iju}^{n} \geq H_{ij} + HL_{ij}^{n}, \forall n \in N \qquad \text{(5-25)}$$

The head loss per pipe length due to friction can be expressed using the Hazen – Williams formula as

$$\frac{HL}{l} = \frac{b}{d^{4.866}}\left(\frac{Q}{C}\right)^{1.852} \tag{5-26}$$

Where C is a design coefficient determined by the material of the pipe (the higher the factor the smoother the pipe), d is the inner hydraulic diameter of the pipe,l is the length of the pipe, Q is the flow rate and b is a unit conversion factor, equal to 10.67 for metric units. Thus the head loss for a connection between nodes I and j can be formulated as

$$HL_{ij}^{n} = b \cdot a_{ij} \cdot \sum_{p \in P} \frac{1}{d_p^{4.866}} \cdot \left(\frac{Q_p^n}{C_p}\right)^{1.852} \cdot L_{ijp}^{n} \tag{5-27}$$

The required pumping energy is equal to the energy required to pump the water to the pumping elevation plus the head loss, divided by the pump efficiency.

$$E^n = \frac{1}{\eta^n} \cdot \rho \cdot g \cdot \sum_{(i,j) \in L} \left(H_{ij} + HL_{ij}^{n}\right) \cdot Q_{ij}^{n} \tag{5-28}$$

Using Eq.(5-27), Eq. (5-28) is written as

$$E^n = \frac{1}{\eta^n} \cdot \rho \cdot g \cdot \sum_{(i,j) \in L} \left(H_{ij} + b \cdot a_{ij} \cdot \sum_{p \in P} \frac{1}{d_p^{4.866}} \cdot \left(\frac{Q_p^n}{C_p}\right)^{1.852} \cdot L_{ijp}^{n}\right) \cdot \sum_{p \in P} Q_p^n LC_{ijp}^{n} \tag{5-29}$$

Since only one pipe type can exist per link, in the summation with respect to p only one term is non zero the one corresponding to the selected for each link pipe p^*. Equation (5-29) can then be written as:

$$E^n = \frac{1}{\eta^n} \cdot \rho \cdot g \cdot \sum_{(i,j) \in L} \left(H_{ij} + b \cdot a_{ij} \cdot \frac{1}{d_{p^*}^{4.866}} \cdot \left(\frac{Q_{p^*}^n}{C_{p^*}}\right)^{1.852} \cdot L_{ij\,p^*}^{n}\right) \cdot Q_{p^*}^n LC_{ij\,p^*}^{n} \tag{5-30}$$

Or using again the summation notation

$$E^n = \frac{1}{\eta^n} \cdot \rho \cdot g \cdot \sum_{(i,j) \in L} \sum_{p \in P} \left(H_{ij} \cdot LC_{ijp}^{n} + b \cdot a_{ij} \cdot \frac{1}{d_p^{4.866}} \cdot \left(\frac{Q_p^n}{C_p}\right)^{1.852} \cdot L_{ijp}^{n} \cdot LC_{ijp}^{n}\right) \cdot Q_p^n \tag{5-31}$$

Since L_{ijp}^{n} is a binary variable, $L_{ijp}^{n} \cdot LC_{ijp}^{n} = L_{ijp}^{n} \cdot L_{ijp}^{n} \cdot f_{ij}^{n} = \left(L_{ijp}^{n}\right)^2 \cdot f_{ij}^{n} = L_{ijp}^{n} \cdot f_{ij}^{n} = LC_{ijp}^{n}$, Eq(5-31) can be written as

$$E^n = \frac{1}{\eta^n} \cdot \rho \cdot g \cdot \sum_{(i,j)\in L} \sum_{p\in P} \left(H_{ij} \cdot LC_{ijp}^{n} + b \cdot a_{ij} \cdot \frac{1}{d_p^{4.866}} \cdot \left(\frac{Q_p^n}{C_p}\right)^{1.852} \cdot LC_{ijp}^{n} \right) \cdot Q_p^n \qquad (5\text{-}32)$$

5.3.4 Capital Cost

The capital cost of the pipe network (CC^{PN}) is the pipeline capital cost (CC^{P})plus the pumping station capital costCC^{U}. The pipeline cost is determined by the unit cost of each pipe type (CC^{p}) times the length of the pipe (l_{ij})

$$CC^P = \sum_{n\in N} \sum_{(i,j)\in L} \sum_{p\in P} \left(CC^p \cdot l_{ij} \cdot L_{ijp}^{n} \right) \qquad (5\text{-}33)$$

The pumping station capital cost is determined by the cost of each operating pump ($CC^{u,n}$) plus a spare one (stand by) and the cost of the shell of the pumping station (CC^{s}),

$$CC^U = \sum_{n\in N} \sum_{(i,j)\in L} \sum_{u\in U} \left(2 \cdot CC^{u,n} + CC^{s} \right) \cdot N_{ijp}^{n} \qquad (5\text{-}34)$$

5.3.5 Operating Cost

Since no maintenance cost is taken in account the operating cost (OC^{N})is exclusively determined by the pumping energy cost (OC^{u}), which equals the daily pumping energy times the electricity cost (EC)

$$OC^U = \sum_{n\in N} OC^{u,n} = \sum_{n\in N} E^n EC \qquad (5\text{-}35)$$

Chapter 6. General Description of the Luxemburgish WWTN

6.1 General Information

The waste water treatment plants which are included in the section area that we examine may comprise centralized or decentralized plants thus local units and this is highly dependent on the users' demand profile of the section area under examination.

Both potable and non potable water demands for each type of land use and type of use of the building units could be estimated with the aid of **Tables 6-1 to 6-3**; These demands represent average values for each building profile. Above mentioned such profile could also be the estimate for the equivalent daily waste water generation of flows due to a single or a variety of uses. These establishments are adopted according to USEPA,2004 and are presented in **Tables 6-1 to 6.3.** These values are typical for a person under normal health state and normal environmental conditions. As the volume of data will be of very large quantities assumptions based on linear regression shall be taken into account.

Until 2015 there was a division of the country into 3 major regions the so called "districts". After that year this division of district was abolished.

Luxembourg is fragmented into 12 different agglomerations. These Cantons entail preliminary statutory independence within Luxemburgish territory mainly for demographic and statistical reasons. Each of these cantons comprises a specific number of sub regions i.e. municipalities called "communes". These municipalities are 106 in total throughout the whole Luxemburgish territory. The 3 major districts contain the following 12 cantons namely as follows:

In "Diekirch" district: The cantons of **"**Clervaux", Redange", " Wiltz", "Diekirch" ", "Vianden" and " Wiltz";

In **"Grevenmacher" district:** The cantons of "Remich" and **"**Grevenmacher" ;

In "Luxembourg" district: The cantons of **"**Capellen" ,"Esch-sur-Alzette", "Luxembourg" and "Mersch".

Within this study all investigated areas will be fragmented into smaller ones thus a potential generator of waste water shall be then grouped into cluster or clusters of building unit of similar user profiles i.e. similar rate production of waste flows, similar habits patterns for different urban or rural activities etc.

The largest part of Luxembourg, that is almost 98%, lies within the Rhine basin via the Moselle river and tributaries such as the Sure and Alzette areas; 2% is in the catchment called “Meuse” basin via the “Chiers” zone area. Throughout the country there are 212 mechanical and 117 biological WWTP’s in total. The above-mentioned infrastructure has an overall capacity of 925,000 population equivalent; 93.5% of the population is connected to the existing sewerage system which follows the separate system in rural areas and the mixed one within the urban context. In the following **Figure 6-1** the entire layout can be depicted.

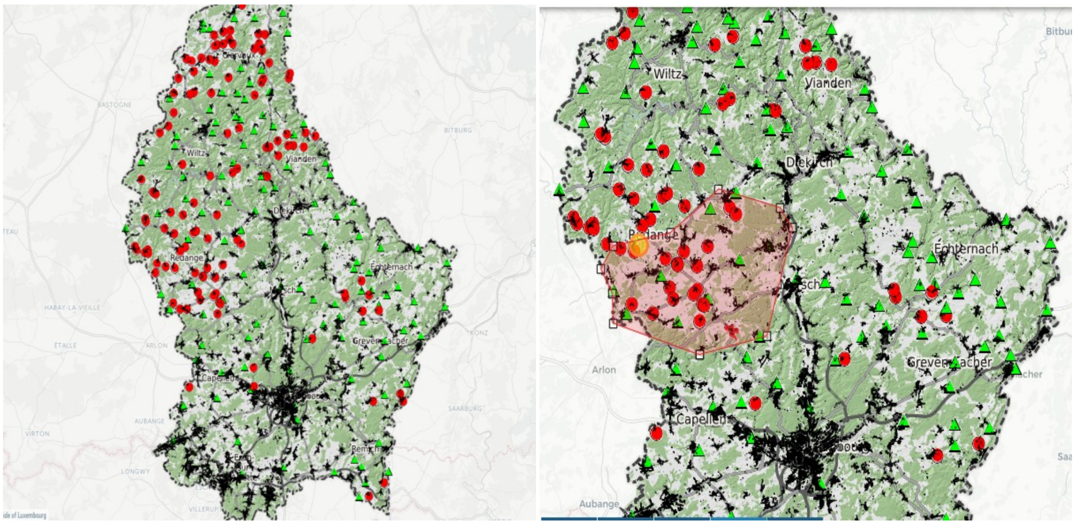


Figure 6-1 Left: General Layout of the overall biological (in green) and mechanical (in red) WWTP’s within entire Luxemburgish territory. **Right**: Area under study (Source : Lux Geoportal)

6.2. Typical waste water generation flows

In the following 4 Tables (**Tables 6-1 to 6-4**) typical waste water production are given throughout literature for building units of different uses.

Table 6-1 Typical wasteflows from commercial sources (source: USEPA, 2004)

Source	Unit	Waste Water Flow (L/Day/Unit)		
		Range		Typical
Airport	Passenger	7.9	15.1	9.8
Automobile Service Station	Vehicle Served	29.9	50.0	40.1
	Employee	34.8	59.8	50.0
Bar	Customer	4.9	20.1	7.9
	Employee	40.1	59.8	50.0
Hotel	Guest	149.9	219.6	189.6
	Employee	29.9	50.0	40.1
Industrial Building (excluding industry and cafeteria)	Employee	29.9	65.1	54.9
Laundry (self-service)	Machine	1798.1	2596.8	2195.5
	Wash	179.8	199.9	189.6
Motel	Person	90.1	149.9	120.0
Motel with Kitchen	Person	190.0	219.9	199.9
Office	Employee	29.9	65.1	54.9
Restaurant	Meal	7.9	15.1	9.8
Rooming House	Resident	90.1	189.6	149.9
Store Department	Toilet room	1601.2	2399.9	1998.7
	Employee	29.9	50.0	40.1
Shopping Center	Parking space	1.9	7.9	4.2
	Employee	29.9	50.0	40.1

Table 6-2 Typical wasteflows from insitutional sources (source: USEPA, 2004)

Source	Unit	Waste Water Flow (L/Day/Unit)		
		Range		Typical
Hospital, Medical	Bed	499.7	950.1	651.1
	Employee	20.1	60.2	40.1
Hospital, Mental	Bed	300.2	651.1	401.3
	Employee	20.1	60.2	40.1
Prison	Inmate	300.2	601.9	450.5
	Employee	20.1	60.2	40.1
Rest Home	Resident	199.9	450.5	350.2
	Employee	20.1	60.2	40.1
School Day				
With Cafeteria, Gym, Showers	Student	60.2	115.1	79.9
with Cafeteria	Student	40.1	79.9	60.2
With no Cafeteria, Gym, Showers	Student	20.1	65.1	40.1
School Boarding	Student	199.9	401.3	280.1

Table 6-3 Typical waste water flows from recreational sources (source: USEPA,2004)

Source	Unit	Waste Water Flow (L/Day/Unit)		
		Range		Typical
Apartment, Resort	Person	199.9	280.1	219.9
Cabin, Resort	Person	129.8	190.0	160.1
Cafeteria	Customer	4.2	9.8	6.1
	Employee	29.9	50.0	40.1
Campground (developed)	Person	79.9	149.9	120.0
Cocktail Lounge	Seat	50.0	99.9	75.0
Coffee Shop	Customer	15.1	29.9	20.1
	Employee	29.9	50.0	40.1
Country Club	Member Present	249.8	499.7	401.3
	Employee	40.1	60.2	50.0
Day Camp (no meals)	Person	40.1	60.2	50.0
Dining Hall	Meal Served	15.1	50.0	29.9
Dormitory, Bunkhouse	Person	75.0	174.9	149.9
Hotel, resort	Person	149.9	240.0	199.9
Laundromat	Machine	1801.9	2600.6	2199.3
Store Resort	Customer	4.9	20.1	9.8
	Employee	29.9	50.0	40.1
Swimming Pool	Customer	20.1	50.0	40.1
	Employee	29.9	50.0	40.1
Theater	Seat	9.8	15.1	9.8
Visitor Center	Visitor	15.1	29.9	20.1

Throughout the next table the pumping cost is presented as well as the cost of different treatment units adopted (Gikas et al. 2011).

Volumetric capacity (m3/day)	Desalination plant	Wastewater treatment plant	Reclamation plant
100	100	190	80
1000	650	1300	320
2500	1500	2400	800
5000	2300	5100	1200
10,000	3200	10,000	1600

Table 6-4 Plant capital costs (k$). (Gikas, et al., 2011)

The correspondent graph of the above values can be illustrated as follows in **Figure 6-2**: Capital cost can be increased radically in case a treatment plant is installed compared to the other two types of plants i.e. the desalination and the reclamation plant.

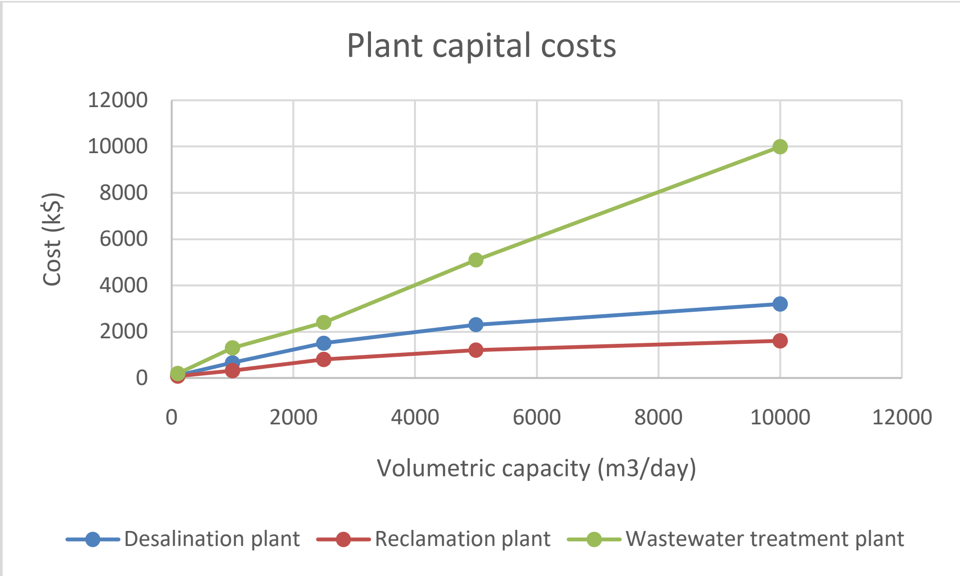


Figure 6-2 Capital costs of all three kinds of treatment plants (Gikas et al., 2011)

6.3 Estimating the population equivalent (PE)

One of the most accepted methods to estimate the capital cost of a treatment plant is through calculating its capacity measures in a unit free magnitude called population equivalent (PE).

Population equivalent or PE stated throughout literature is the estimation of the total number of existent contaminants load which is to be generated within one single day through the use of all industrial units divided by the contaminants load produced by one single user within a residential typical unit during one single day in the same time interval.

An empirical assumption is made and we let the single unit to be equal to 54 g of BOD per day. Based on this the Population equivalent can be written as follows:

$$PE = \frac{\text{BODloadfromindustry}\left[\frac{\text{kg}}{\text{day}}\right]}{0.054\ \left[\frac{\text{kg}}{\text{inhabday}}\right]} \qquad (6-1)$$

Or have it transformed

$$\mathrm{PE} = \frac{\mathrm{BOD concentration\ from\ industry}\left[\frac{\mathrm{kg}}{\mathrm{L}}\right] \cdot \mathrm{FlowRate}\left[\frac{\mathrm{L}}{\mathrm{day}}\right]}{0.054\ \left[\frac{\mathrm{kg}}{\mathrm{inhabday}}\right]} \qquad (6-2)$$

Assumingly there is an industrial production unit which daily produces 15 10^5 L of waste water. Its equivalent 5-day BOD concentration attains 360 mg/L. Above mentioned typical unit would give rise to a population equivalent (PE) as follows:

$$\mathrm{PE} = \frac{\frac{360}{10^6}\left[\frac{\mathrm{kg}}{\mathrm{L}}\right] 15\ 10^5 \left[\frac{\mathrm{L}}{\mathrm{day}}\right]}{0.054\ \left[\frac{\mathrm{kg}}{\mathrm{inhabday}}\right]} = 10{,}000 \qquad (6-3)$$

Typical values of a single treatment unit within the bibliography (Wikipedia and Martz,1970) are the following values: 60 g BOD within one single day in UK also 80 grams in the US.

Above mentioned more efficiently describe the mass load corresponding to waste water effluents arising from different end users, is called population equivalent (PE) and is denoted with PE. This could be defined as the equivalent pollutant load in grams/BOD/inhab/day or the equivalent waste water volume in litres (lt/inhab/day). A typical fixed value for each user might be applied to either change or to refer to a transitory set (i.e. commercial centers or public places of gathering of people like Airports) and is given a value of 60 gr/BOD/inhab/day and 200 lt/inhab/day. The above-mentioned values imply generation of waste by a person who lives in a normal dwelling and should result in producing 200 lt of waste which contain 60g of BOD daily. These values are directly equivalent to 1 population equivalent (PE).

Chapter 7. Analysis of urban sewage systems

7.1 General

Urban sewage[2] comprises the collection and transport of sewage and rain water from an urban area (town or a rural settlement) to the point of disposal. Urban water waste or sewage or dirty water is mixed with solid substances disposed of by normal sanitation from residential, commercial or industrial areas. Together with these, industrial wastes are also being transported, that is to say, water and other liquid waste off industrial processes. Storm water is the rainwater that flows after rainfall. Filtering is the water entering the existing drainage system in the ground. Infiltration is a category of parasitic (or additional) inflows into the grid, which, although undesirable, inevitably enter the sewer. Another category is rainfall in a pipeline designed exclusively for sewage disposal[2].

The sewer system is a collection of sewage and/or rainwater collecting and drainage pipelines, equipped with the appropriate typical specifications and accompanied with specific technical works, which facilitate the flow into the grid and allow it to be maintained at regular and specified time intervals.

The drainage network is mixed when it collects and transfers indiscriminately both rainwater and sewage. In contrast, there exists a two-grid system, the so called separate or combined system, which comprises the sewage grid and the storm network separately. In certain cases, it is possible to have a so called co-ordinated network where both these two systems co exist within the same grid (Angelakis et al., 2005). Therefore, in a part of a city (usually the oldest) there may exist the combined system whereas separate system networks might be existent in the rest of an urban area i.e. the suburbs. The main drain or main collector collects the drains of the other pipelines, which in turn, depending on their location and importance in the network, are divided into primary, secondary and tertiary ones, without this distinction being absolute. Especially in the rainfall networks, master collectors are often not built, and in their place the watercourses of the area are used, possibly after appropriate arrangement. The rainfall network is combined with a

[2] Subsequently, the term sewerage is used briefly as an equivalent to urban sewage. It is clarified that drainage of rural areas (arable land), motorways outside urban areas, etc. are particularly technological objects.

system of perimeter drainage trenches that protect the urban area from the outflow of external neighbouring natural basins.

The final recipient of sewage or rain is usually a natural water system (water stream, lake, sea) into which sewage or rain sewage is discharged. In the case of separate networks, the recipient may be different for each network. The ground may also be the network of wastewater. Final sewage effluent to the final natural recipient is attributed simply with the term 'disposal' and the pipeline system through which a 'disposal' is realised is called 'disposal pipeline'. Waste water before disposal should undergo appropriate treatment so that harmful pollutants for public hygiene can be removed. In a suitable waste water treatment plant facility there exist substances which facilitate this removal of pollutants through physical, biological or chemical treatment processes. Wastewater treatment is the subject of a particular scientific and technological area, which is not addressed in this study.

Sewage pipelines transport, urban and/or rural in many cases waste water and some extra quantities of groundwater and surface runoff entering (parasitic inflows).

Consequently, the estimation of supply of waste water generated requires the estimation of the population of the examined area served, water consumption per activity and its percentage diverted into sewerage mains, as well as parasitic infiltration of underground infiltrations for i.e. rainwater run-offs. In addition, mains often carry wastewater quantities from industrial or commercial establishments whose inflows must also be taken into account. Wastewater outflows and filtrations exhibit significant fluctuations over time, and therefore the magnitude of pipes in terms of their diameter dimensions should be sufficient to serve maximum capacity of inflows, called the design flows.

7.2 Brief historic review

The art of sewage was neglected in the Middle Age. Furthermore, the sanitary facilities were abandoned or downgraded during this period of time (e.g. a usual work trend at that time was to build ancillary toilets over castle moats), while water consumption for cleaning purposes was minimized. As a result of this situation, epidemics were plaguing medieval societies. This situation continued until recent times, and was even more acute due to the concentration of population in cities. The evolution of building activity, which led to the construction of multi-storey buildings, was not accompanied by a similar development of hygiene habits and sanitation (e.g. Versailles lacked built drains).

Therefore in many cities sewage was allowed to run freely in the yards and streets. The construction of modern drainage systems, with few exceptions, began in the middle of the 19th century. In Germany the first organised sewerage network was designed and built in Hamburg in 1842 (Martz, 1970) after a fire that destroyed the city's downtown.

The design was made by the leading English engineer W. Lindley, based on ideas and principles some of which are still adopted today. In the UK, the importance of sewage systems was recognized in 1855, following a cholera epidemic that began in 1848, and triggered the construction of a sufficient drainage network in London (Clark et al., 1977). Interestingly, the construction of sewerage works in recent years began with the purpose of removing storms, rather than household wastewater. Indeed, in several cities with a storm grid, for a long time they were forbidden to drain pipelines (Steel, 1960). In Luxembourg, there has generally been a delay in the construction of modern sewage systems.

In the city of Luxembourg, which has at its centre one of the oldest Luxemburgish sewerage networks, and in its periphery more modern segregated networks, until the 1980s most of the houses were served with cesspools. The construction of the mixed system began in 1858, although individual sewers had been built earlier. In the beginning the projects were done without systematic studies. The first study was made by the French engineer Claye.

The separate system began to be built in 1933. In 1959, the Central Sewerage Drainage (CSD) was completed, ending at title of region, leading the sewage into the sea without treatment and with surface (not underwater) mood. In 1982 began the systematic expansion of the sewerage networks of the basin under examination. At the same time, a group of key projects for the modernization of the sewer system, including the Supplemental Central Sewerage System (SCSS), the Wastewater Treatment 'title of region (about in the middle of the crossing title of region), the underwater siphon 'title of region of the plant'-WWTP, the underwater ejection duct, and the sewage treatment plant of Transformation and WWTP effluent. The latter started in 1994, initially only with primary treatment, while in 2004 the full treatment works were completed.

7.3 Design period

Waste water mains are designed with sufficient drainage capacity to meet future needs of the study area, for a given scheduled period. The pipelines' design flow capacities are estimated dependent on population conditions and water consumption demands expected at the end of that period. The various sewerage projects do not have the same programming period necessarily. The factors taken into account for choosing this period are (a) the expected life span of above mentioned projects, (b) the ease or difficulty to extend above mentioned infrastructure network, (c) the large or small uncertainty involved in the assessment of evolution of population in the examined region as well as its development of the region in general with a projection to the future (when there is great uncertainty involved, we usually take into consideration the worst possible scenario in design), (d) economic factors such as the total cost of projects and their corresponding interest rate of funding.

The life span period for drainage pipelines according to National specifications (Luxembourgish waste water service,2017) is considered to be 40-50 years for the main sewerage collectors, while for the secondary and complementary culverts, strictly the ultimate end of the design period of urban or rural development predicted is taken into account. Especially for the electromechanical equipment of the sewerage networks (pumping stations) as well as for the waste water treatment facilities the design period is less, between ranging from 20 to 25 years due to the shorter life span of this infrastructure.

7.4 Population data and Evolution

Key sources of population data are population censuses. The so called population of origin, being population at the time of study design, is a very useful tool for future population estimate. In case significant time since the last census has run, reassessment of the current (starting) population is necessary. To this purpose, various statistical indications, such as the census in schools, electricity or water consumers, subscriptions / deletions to municipalities etc. can be utilised so that interpolation for the new starting population can be realised. Predicting the future populations at the end of the design period is based on historical data. The usual methods incorporated in order that a proper estimate of the future population attained are:

1. **Assumption of linear population increase:**

$$\mathbf{P_t = P_o + at} \qquad (7\text{-}1)$$

Where:

$\mathbf{P_t}$, $\mathbf{P_o}$ are population at times t (in years) and 0 (taken as the time of origin) respectively ;

a, is a parameter correlated to population increase.

2. **Assumption of a steady percentage of annual population increase** (based on the equation of interest rate):

$$\mathbf{P_t = P_o(1 + \beta)_t} \qquad (7\text{-}2)$$

Where β expresses the percentage population in a time increase within one year;

3. **Assumption of different population increase percentages per fixed periods of time**

It is usually used for longer periods of time i.e. one decade mostly applied to large urban areas (Eurostat, Luxembourg 2017).

4. **Utilisation of the S-shaped curve,** which is characterised by a population of saturation $\mathbf{P_s}$ corresponding to the ultimate end of the designed period of future development for the examined city.

Above mentioned curve is expressed mathematically by the following relationship:

$$\mathbf{P_t = \frac{P_k}{1+me^{-nt}}} \qquad (7\text{-}3)$$

Above relationship is derived as the solution of the following differential equation

$$\mathbf{\frac{dP_t}{dt} = nP_t\left(1 - \frac{P_t}{P_k}\right)} \qquad (7\text{-}4)$$

5. **Graphical projection of variation on future population** via the use of a curve regarding known population in the past.

6. **Graphical comparison to other Luxemburgish more populated towns** demographically proves to be useful. Comparison to the period of time after which city population of more populated cities was equal to the city we examine. The choice of the method to be eventually adopted is based on the picture of its population evolution in the past. For this purpose, the graphical representation of evolution is always useful, no matter which method is chosen. In addition comparison with the evolution of the population of other cities is always useful. Changes in socio-economical conditions at the regional and local level may lead to significant changes in the evolution of the population. For instance, a serious development project in a small town can lead to a significant increase in its population.

7. **Estimate on birth and death rates and inflows due to migration /outflows of population due to other reasons (negligible).**
   Separate estimates of the non permanent population are essential as the city of Luxemburg mainly is, in large proportions populated by individuals who reside in neighbouring countries and only come into the country within the working time schedule slots. Especially in certain areas the non permanent population is in some cases larger than the permanent one and this time slot occurs from the early morning hours to late afternoon.

7.5 Population distribution

For the assessment of the benefits of individual sewage pipelines, the area under study is divided into compartments or sub-basins, the boundaries of which are determined by the topography and layout of the area's road network. The estimate of the population of each such compartment is based on its extent and the population density therein. The distribution of the population, and therefore its density, is not uniformly distributed throughout the area under study. Thus, the examined area is at first subdivided into areas depending on the activities and terms of building for each area, in such a way that each such sub-region exhibits uniformity for these two characteristics, therefore a similar population density. Based on this, the study area is at first divided into zones depending on the activities being developed in each zone (residential, commercial, industrial, public use, i.e. parks, sports facilities, municipal building units etc.). The residential zones are then further divided into sections according to the building conditions (terms and building factor) and the population density is estimated in each zone. Typical values of population density of residential areas are 35-50 inhab / ha^2 for low-building sectors (single-family houses), 100-150 inhab / ha^2 for medium-sized construction areas (double-family houses, three-storey houses) 200-400 inhab/ha^2 for high-dense regions (Eurostat, Luxembourg 2017). However, densities even much higher, of about 2500 inha / ha, have been found in extremely dense zones comprised mostly of multi-storey buildings. In relation to industrial and commercial area densities of 25-75 inhab/ ha^2 are mostly incorporated (Eurostat, Luxembourg 2017).

In general, population densities adopted must be consistent with the estimates of the total population for which the project is being designed thus along with the design capacity of infrastructure. In large cities with developed shopping malls, which exhibit significant population movements during each day, population and density of permanent population estimate must be distinguished from visitors who could in turn comprise tourists, temporary visitors or workers in that area. The City of Luxemburg falls into this category (CSS, 2019) -approximately 45 % of the total permanent population working in the city comprise a moving population permanently residing outside the borders of Luxembourg. This population comprises the working load of the country in a daily basis during working hours only.The corresponding population distribution in regions around the borders of Luxembourg can be depicted in **Figure 7-1.**

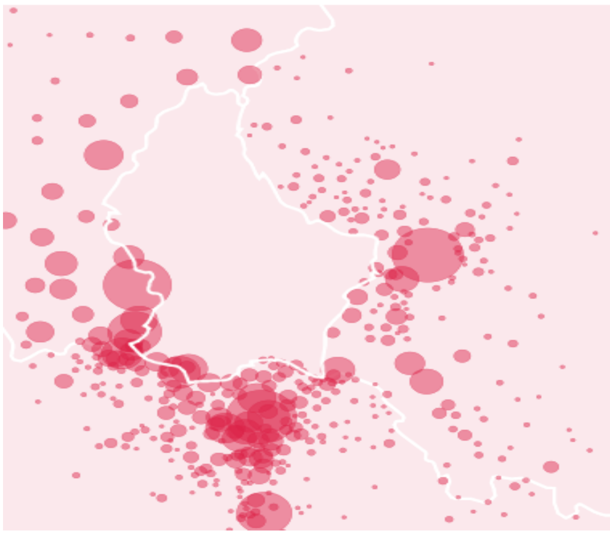


Figure 7-1 Distribution of foreigners population working in Luxembourg in a daily basis (Source: CCSS, 2019)

7.6 Water consumption

Water consumption is divided into three categories: Residential, industrial, public municipal. These consumptions exhibit significant variances dependent on the attribute of the examined area. The factors which affect this magnitude are, **a)** climate and microclimate, **b)** the level of living, **c)** the existence or not, of sewerage network, **d)** the type of commercial, **e)** industrial and touristic activities, **f)** the availability of fresh water, **g)** pressure of potable water grid, **h)** quality of fresh water, **i)** cost of fresh water and the politics of the government on water supply.

In the organised water supply systems all corresponding services in charge sustain statistical data regarding water consumption. Based on these data sets it is possible that waste water charge estimates be attained. At this point it should be noted though that a forecast of the future water demand is of great importance as improvement of level of living is realised progressively. Moreover a potential urban development and expansion of the urban environment should also be taken into consideration to this end. In addition even future network repair works might entail significant increase of water supply consumption (Eurostat, Luxembourg 2017)**.**The different components of water consumption are usually denoted with the index of the mean daily per capita consumption (L/day inhabitant).

In the Luxembourgish territory a typical household water consumption design values range from 150 L/(d inhab) for small villages up to 250 L/(d inhab) for large cities. The mean corresponding value for water consumption is 200 L/ (d inhab). Regarding touristic facilities and hospital units corresponding values can be considered as increased to the upper threshold of 300 up to 600 L/d inhab (Eurostat, Luxembourg 2017).

For the city of Luxembourg the mean yearly households design value projecting in the year 2030 has been **(**Luxemburgish waste water services, 2017**)** estimated as follows **(**Eurostat, water consumption Luxembourg, 2017):

- 70 L/(d inhab) lower/medium income urban/rural areas;
- 110 L/(d inhab) medium/high income rural/semi urban areas;
- 150 L/(d inhab) touristic/recreational and high income urban areas.

Industrial, public and municipal water consumptions are estimated separately, taking into consideration the special climatic conditions. For the sake of uniformity these conditions are interpolated and can be characterized as special water consumptions per inhabitant which in turn are in some cases integrated and considered to be included within special consumption per capita which are in turn added to existing domestic consumption (Eurostat,2017) Industrial and tertiary water demands in general comprise larger variation ranges being dependent on the corresponding actual activities.

The quantity of water supply destined for tertiary and industrial activities in Luxemburgish territory is estimated to the 10% of the total domestic demands or approximately 20L/d /inhab, whereas in areas of industrial activity this value may attain up to 100% of the overall domestic consumption (Eurostat, Luxembourg 2017). Typical values for public and municipal demand i.e. schools, hospital units, public/municipal private building units, park zones for irrigation, private and/or municipal streets washing range from **10** to **50** L/(d inhab) (Eurostat, Luxembourg 2017).

7.7 Quantity of waste and water consumption

Unless there is sufficient and reliable data from measurements in existing waste water mains, the estimates of waste water supplies are based on the respective water supplies, after deducting the quantities that do not end up in the sewers. These quantities, which in a large percent are converted into water vapor, are mainly used for watering pots, gardens and parks irrigation, cars washing and roads and house cleaning. The remaining amounts of sewage are usually estimated as a fixed percentage of overall water consumption, depending on local conditions, ranging from 60% to 80%. For the estimation of design benefits, Luxemburgish specifications require this figure to be considered as 80% **(**Luxemburgish waste water services, 2017). In order that a safety factor is maintained against failure of waste charges during period of extreme rainfalls above mentioned figure generally accounts for 85%, except for recreational zones and high income areas where it accounts for 80%.

7.8 Calculation of the design value for the waste water produced for each cluster

Before applying the mathematical formulation on a given region, preprocessing calculations are made in order to compute the model’s parameters. The preprocessing methodology is as follows:

For every examined cluster which is incorporated in the model a certain series of calculations is required. This series of calculations comprises generation of waste water of each cluster of building units in every examined municipality based on certain assumptions taken from literature such as water flow demand assumptions and dependencies on different parameters such as current population and its future projections, examined existing and future area considering possible urban and rural development. The steps for the design of waste water generation is therefore seen as follows in detail:

For the estimation of the future waste water production, the region under study is divided into clusters. Each cluster contains a community, such as a city/town, a village, smaller residential areas or industrial areas. For each cluster the following steps are made:

The current population P_{t0}(in inhabitants) of the cluster is taken based on the most recent census and the design period of $t$years is decided. The cluster population after t years is extrapolated using (7-5):(Alexandru Hening et al., 2018):

a) We find the current population P (t_o) since last accent;

b) We extrapolate (projection in the future) after the design period of t=40 years + years since the last accent;

$$\mathbf{P(t+to)=Po(1+\beta)^{t+to}} \text{ (inhabitants);} \qquad (7\text{-}5)$$

The current developed area A_{t0}(in km^2) of each cluster is considered and the full urban development area after $t$years is estimated through (7-6):(Koutsoyianis et al, 2008):

c) We find the current developed area A, exactly at the start of the design period:

d) We calculate the Area A’, when full urban development has taken place:

$$\mathbf{A' = 1.5 * A} \quad \text{(in km}^2\text{);} \qquad (7\text{-}6)$$

e) We assume from literature (Martz 1970), (Koutsoyianis et al, 2008), a fixed mean daily fresh water demand for each inhabitant at the end of the design period, q_E' = 200L/d inhab; an equivalent mean daily water demand for rural activities or light industrial activities, as q_E'' = 10L/d inhab and the mean daily water demand for municipalities (public) activities, as q_E'''= 25L/d inhab (Koutsoyianis et al, 2008). Moreover, (Koutsoyianis et al, 2008) stipulates that, the percentage of the total water quantities which enter the waste water network as waste water

Thus, the mean daily quantity of waste water produced per inhabitant is p=80% in L/day*inhabitant) (Koutsoyianis et al, 2008), Martz 1970):

f) We therefore assume a coefficient p=0.80 and this denotes the percentage of the total water quantities which enter the waste water network as waste water. It is taken from literature;

g) We calculate the mean daily quantity of waste water produced per inhabitant (Martz 1970),

$$\mathbf{q_E = p^* (q_E' + q_{E'}' + q_E''')} \quad \text{(L/d inhab);} \qquad (7\text{-}7)$$

h) We calculate the mean daily quantity of waste water produced in community: (Martz 1970),

$$\mathbf{Q_E = q_E * P\,(t+to)} \quad \text{(L/s);} \qquad (7\text{-}8)$$

i) We calculate the peak daily quantity of waste water produced in community(Martz 1970),

$$\mathbf{Q_H = \lambda_H * Q_E} \quad \text{(L/s);} \qquad (7\text{-}9)$$

j) We enhance the relationship that denotes the coefficient of instant supply peak of waste water suitable for small and medium sizes communities (0-10,000 inhabitants) (Babbitt, et al. 1948) and (Metcalf and Eddy, 1981) as this is a good fit for above mentioned space to the value of P extracted in literature and has a general application

$$\mathbf{P'} = \frac{\mathbf{5}}{(\frac{\mathbf{P}}{\mathbf{1000}})^{\mathbf{1/5}}} \quad \text{(Babbit, Gifft);} \qquad (7\text{-}10)$$

For communities of population more than 10,000 inhabitants the best fit relationship is:

$$\mathbf{P}' = \frac{\mathbf{5}}{\left(\frac{\mathbf{P}}{\mathbf{1000}}\right)^{\frac{\mathbf{1}}{\mathbf{5}}}} \quad \text{(Metcalf \& Eddy);} \qquad (7\text{-}11)$$

k) We calculate the peak instant quantity of waste water produced in a community: (Martz 1970)

$$\mathbf{Q_P = P'\,Q_H} \text{ (in L/s)} \qquad (7\text{-}12)$$

l) Extra inflows into the waste water network due to underground infiltration and rainwater (rainwater is considered by incorporating the coefficient of increase of 1.40 as within literature it is proposed to increase the share of the total amount of wastewater due to rainwater that ends up into the WWTP's by approximately 40%;(Koutsoyianis et al, 2008),

$$\mathbf{qi = 1.40} * \left(\frac{\mathbf{0.5}}{(\mathbf{A'})^{\mathbf{0.83}}}\right) \quad \text{(in L/S ha) or (m3/d km pipe) or (L/s km of pipe);} \qquad (7\text{-}13)$$

m) Total infiltration inflows taken surface area for each community at the end of designed period examined(Martz 1970):

$$\mathbf{Q_i = q_i{*}A'} \quad \text{(in L/S);} \qquad (7\text{-}14)$$

n) Total waste water production for each community at the end of designed period examined(Martz 1970)::

$$\mathbf{Q_{tot} = Q_P + Qi} \quad \text{(in L/S);} \qquad (7\text{-}15)$$

7.9 Variance of waste water flows

These flows exhibit constant changes in values which in turn are classified within the following categories:

1. Hyper annual (Seasonal) variances

These are due to the dynamic nature of population, the socio economical characteristics and level of living. The above-mentioned values tend to be incremental in general. As a consequence of this, these changes turn up to attain maximum values during the end of the design period of the infrastructure into consideration.

2. Variances during one year

These variances can be characterized mostly as periodical and random for the remaining examined design period. These take place based on corresponding potable water consumptions variations and are caused due to different climatic conditions from area to area. Based on this estimated values of water consumption are increased thus waste water flows are in consequence increased in magnitude during summer months.

3. Variances during the day

These variances are mostly deterministic in nature which and depend upon daily habits in general (reduced water demands during night and increased ones during the morning hours) as well as correlated to a random component.

In relation to sewerage our interest is mainly focused upon maximum flows. These are extracted and refer to the end of each design project period. This is essential as the whole design and the hydraulic dimensioning of infrastructure is related to these maximum consumptions. Regarding special infrastructure such as aquifers, pumping stations pipelines and WWTP's these can be properly dimensioned based on acquiring minima, maxima and mean values of waste flows in the beginning and the end of the design period of above mentioned infrastructure. The main magnitudes involved in waste water inflows which comprise the main interest of sewerage design studies and their mode of calculation are mentioned as follows:

A) Mean daily waste water flow Q_E

(Referring to per inhabitant $q_{E'}$ in the beginning or the end of design project period).

This comprises annual waste water volume, divided by duration of one single year. It is estimated as mentioned already as the fractional corresponding seasonal water flow supply Q'_E *(or* q'_E*),* for example,

$$Q_E = \rho \, Q'_E. \qquad (7\text{-}16)$$

Where

Q'_E is the corresponding water supply, p is the fractional ratio value for Luxembourg 0.80 (Eurostat, Luxembourg 2017).

B) Maximum daily waste water inflow to the grid Q_H (projected to per inhabitant q_H **)**

It is the mean daily inflow with the maximum consumption estimated based on the following relationship:

$$Q_H = \lambda_H \, Q_E \qquad (7\text{-}17)$$

Where:

λ_H is the daily peak coefficient of the maximum daily waste water inflow is usually named as the summer waste water inflow.

C) Maximum instant waste water flow (or peak flow) Q_P

It consists of the instant peak of waste water flow during the day of the maximal consumption. As per Luxemburgish standards (Eurostat, Luxembourg 2017) it can be extracted from the following relationship $$\mathbf{Q_{P}=P\ Q_{H}} \qquad (7\text{-}18)$$

Where:

P is the instant peak, a magnitude that is mentioned later on.

$\boldsymbol{\lambda_H}$ is the coefficient of instant peak

The daily peak coefficient λ_H ranges in most cases from 1.1 up to 1.5 (Eurostat, Luxembourg 2017) propose 1.15 up to 1.20 for the city of Luxemburg.

The instant peak consumption coefficient P is a statistical value and is dependent upon a) the targeted quality function of network, a, which is in turn expressed as the possibility that actual waste charge value does not overpass the designed one, b) the population that this value is referred to and c) other functional parameters.

At the end of this section, it turns out that, assumption that the supply as random variable follows a Gaussian distribution, the peak coefficient is theoretically given by the relation (Koutsoyianis et al. 2008),[

$$\mathbf{P_{=}\frac{\xi_P}{\xi_\mu}\left(1+w_a\sqrt{\frac{1-\xi_P+C_{VR}^2}{\xi_P}\frac{1}{\sqrt{\Pi}}}\right)} \qquad (7\text{-}19)$$

Where

- $\boldsymbol{\xi_\mu}$ is the average probability that an individual consumer uses the network at any time during the day;
- $\boldsymbol{\xi_P}$ is the corresponding peak probability for the corresponding peak time;
- $\mathbf{C_{VR}}$, is the variance coefficient of the supply R of a user, expressed as $\mathbf{C_{VR}}=\sqrt{\frac{\mathbf{Var\,[R\,]}}{\mathbf{E\,[R]}}}$;
- **R** is considered as the random variable representing water supply to a single user;
- $\mathbf{w_a}$ is the Gaussian distribution function value corresponding to the probability of not exceeding probability **a**.

For the extraction of (7-19) an instant response of the water supply-drainage system was considered. In reality, however, this is not correct, but on the other hand there is a delay along the water and waste water mains, which has homogeneous effects, that is, reduces the peak factor. In order to give an expression of (7-19) suitable for applications we do the following, quite conservative assumptions:

- Ratio $\frac{\xi_P}{\xi_\mu}$= **1.5**. It is noted that this reason is essentially the deterministic component of the problem and is dependent on the general habits of consumer life and hygiene (e.g general slowing of consumption late in the morning, morning increase, etc.);
- On average, a consumer charges the drainage network approximately 15 minutes per day, therefore, ξ_μ= 15 / (24 × 60) = 0.01 and ξ_P = 0.015. Note that the smaller the ξ_μ the higher the peak;
- Coefficient of variation C_{VR}= 0.25. This size does not significantly affect the peak factor (WPCF manual, 1990);
- Quality of operation $a = 1 - 10^{-5}$, (or probability of failure 10^{-5}) so w_a = 4.27.

This assumption is equivalent to adopting an acceptable failure duration of 1 minute during the summer season, which is considered a three-month period ((Martz 1970):). With these assumptions, the next relationship is extracted (see proof in the text below):

$$P = 1.5\left(1 + \frac{1.1}{\sqrt{\frac{\Pi}{1000}}}\right) \qquad (7\text{-}20)$$

In the US, different empirical relationships have been developed, corresponding to (7-20) based on statistical data. They use one total peak factor(Martz 1970):,

$$P' = \lambda_H P. \qquad (7\text{-}21)$$

Therefore, instantaneous peak supply is given in this case by(Martz 1970):

$$Q' = P' Q_E \qquad (7\text{-}22)$$

The main of these relationships are the following:

$$P' = \frac{5}{\left(\frac{\Pi}{1000}\right)^{\frac{1}{5}}} \textbf{(2.10)} \quad \text{(Babbit equation);} \qquad (7\text{-}23)$$

$$\mathbf{P'} = \frac{\mathbf{5}}{\left(\frac{\mathbf{\Pi}}{\mathbf{1000}}\right)^{\frac{1}{6}}} \qquad \text{(Gifft equation);} \qquad (7\text{-}24)$$

$$\mathbf{P'} = \mathbf{1} + \frac{\mathbf{14}}{\mathbf{4}+\sqrt{\frac{\Pi}{\mathbf{1000}}}} \qquad \text{(Harmon equation)} \qquad (7\text{-}25)$$

National specifications (State's decree, 1974) constitute the following empirical relationship, in which the peak coefficient is related to the maximum daily $\mathbf{Q_H}$ supply and not the population:

$$\mathbf{P} = \mathbf{min}\left\{\mathbf{1,5} + \frac{\mathbf{2.5}}{\sqrt{\mathbf{Q_H}}}, \mathbf{3}\right\} \text{ (Koutsoyianis et al, 2008)} \qquad (7\text{-}26)$$

Where $\mathbf{Q_H}$ is expressed in L / s. Equation (7-26) results directly from (**7-20**), in case $\Pi = \frac{Q_H}{q_H}$ substituted to the latter and consider q_H = **200 L / (d inha)**. Moreover, national water services (1985) propose that the Gifft relationship (**7-24**) is incorporated, by applying P factor on the maximum summer $\mathbf{Q_H}$ water supply instead of $\mathbf{Q_E}$ to which the original relationship refers.

Similar to (**7-26**) is also the following empirical relationship of Metcalf & Eddy (1981):

$$\mathbf{P'} = \frac{\mathbf{3.7}}{\mathbf{Q_E^{0.073}}} \qquad (7\text{-}27)$$

Where $\mathbf{Q_E}$ is the average daily flow rate in L/s. It is noted that (**7-26**) and (**7-27**) are more user-friendly than previous relationships, but also they are not well founded and have no generality of application (normally only apply to a specific value of $\mathbf{q_H}$. In addition, the restriction of P ≤ 3 in equation (**7-26**) has no physical sense. A graphical comparison of all the above coefficients estimation ratios peak is shown in **Figure 7-2**.

In this Figure a probabilistic relationship going hand in hand with Metcalf & Eddy's relationship regarding large populations and Babbit and Gifft relations for small populations is observed. It is clear that the latter two yield unacceptably small peak P-factors (even smaller than 1) for large populations. Regardless of which relationship is used, the P-factor changes from position to position and even decreases as we move downstream and population served by the pipeline grows. For the same reason, in case of pipelines 1 and 2 interference in a pipeline 3, then, $\mathbf{P_3} < \mathbf{P_1}$, $\mathbf{P_3} < \mathbf{P_2}$ and consequently $\mathbf{Q_{P3}} < \mathbf{Q_{P2}} + \mathbf{Q_{P3}}$ applies. This is not a hydraulic paradox since it is not interpreted as a non-satisfaction of the continuity equation, but is explained by the fact that the times of maximum flow in the three ducts are different.

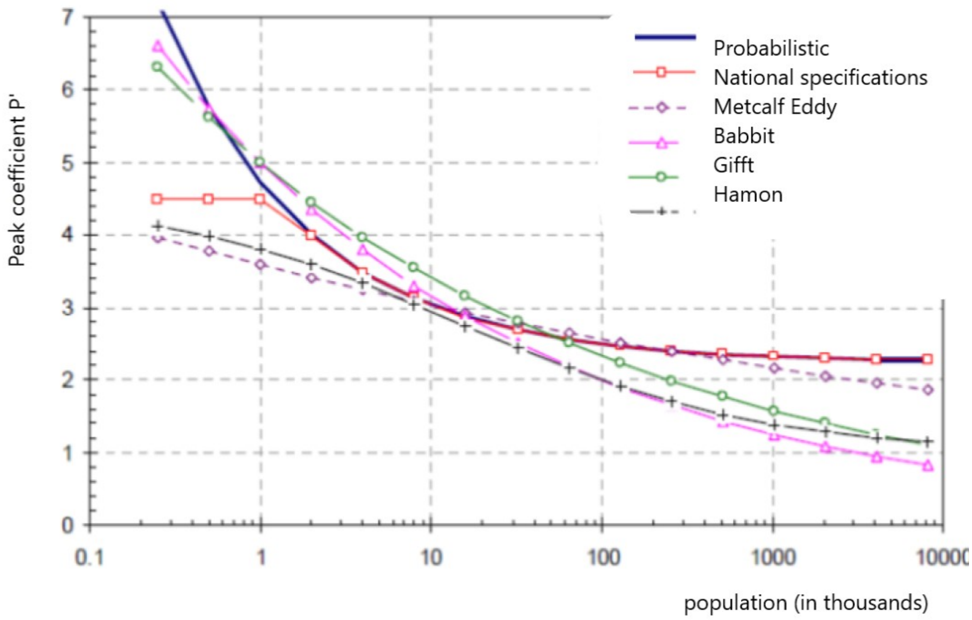


Figure 7- 2 Comparison of the total peak coefficient P = λ_H P, as calculated by various methods. It was assumed that λ_H = 1.5 and q_H = 200 L / (d inhab). (:Koutsoyiannis et al. 2008)

In a manner similar to the maximum water supplies set, the minimum daily and minimum hourly supplies can be set accordingly. As mentioned above, in some cases estimation of minimum water supplies is necessary. Their estimation methodology is identical to that of maximum ones. Thus, relations (**7-17**) and (**7-18**) or (**7-22**) are used with λ_H and P or P coefficients assumed to be equal to the inverse of the coefficients used for the maximum values.

7.10 Parasitic inflows

There is an evident increase of volume of waste water within sewer mains due to the parasitic inputs of groundwater as well as rainwater flows. The underground water enters the network through the joints and the constructional defects of waste mains and badly constructed shaft covers. This condition is called infiltration. There is also another path to cause parasitic flows into the existing sewer grid. Flow of rain water is driven to the waste water mains through the existent drainage in foundations of buildings. Most of the infiltration phenomena occur due to poor manufacturing of private sewer mains alone as well as their poor connections to the main waste collectors' network. Limitation towards eradication of the above-mentioned infiltrations is practically impossible and economically unprofitable. Regarding the separate drainage system, rainwater should be restricted from being drained into the waste water network. However, in certain instances minor amounts of these drained rain waters, come from courtyards or house roofs and eventually enter the waste water network through private autonomous unauthorised connections. These cases occur however randomly in the Luxembourgish territory thus they will be neglected in the mathematical model of the examined areas. Even smaller quantities derive from shaft covers with poor connections.

- Parasitic inflows drained into the **old** grid (WPCF manual, 1990) (Koutsoyianis et al, 2008);

$$\mathbf{q_{i}} = \mathbf{min}\left\{\frac{\mathbf{0.5}}{\mathbf{A^{0.3}}}, \mathbf{0.16}\right\} \qquad (7\text{-}28)$$

- Parasitic inflows drained into the **new** grid:

$$\mathbf{q_{i}} = \left(\frac{\mathbf{1}}{\mathbf{A^{0.25}}}\right) \qquad (7\text{-}29)$$

Where:

$\mathbf{q_i}$ in L/s km2; **A** in km2

Chapter 8. Model analysis and Computational results

8.1 Mathematical formulation (without MF and RO)

8.1.1 Non-Linear Original Problem (NLOP)

The problem of Waste Water Treatment Network Design (WWTND) is formulated as a Mixed-Integer Non-Linear Problem (MINLP) due to the non-linearity of the applied Expansion Operational and Maintenance costs (O&M) of a Waste Water Treatment Plant. For the adopted mathematical model the following notation was used:

Indices

I:	set of Clusters (index i);
J:	set of candidate locations of Waste Water Treatment Plant (index j);
P:	Set of types of WWTPs (p=0=>mechanical, p=1=>biological)

Table 8-1 Indices

Input Parameters-Data:

$\overline{WWP_i}$:	Waste Water Production of cluster i (m3/day)
$\overline{CE_p}$:	Cost of expansion of a WWTP of type p per amount of waste water treatment capacity (€/m3);
$\overline{CM_p}$:	Cost of maintenance of a WWTP of type p per amount of waste water treatment capacity (€/m3);
$\overline{CP_{ij}}$:	Cost of construction of a pipeline from cluster i to WWTP j per amount of waste water treatment capability (€/m3);
$\overline{CPM_{ij}}$:	Cost of maintenance a pipeline from cluster i to WWTP j per amount of waste water treatment capability (€/m3);
$\overline{QE_j}$:	Continuous parameter that is equal to the waste water treatment capability of the existing WWTP at location j that is the amount of waste water that can be treated in it (m3/h);
$\overline{Type_j}$:	general integer parameter that is equal to the type of the WWTP at location j.(p=0=>mechanical, p=1=>biological);
$\overline{BigM}$:	A very large number.

Table 8-2 Input parameters-Data

Decision Variables:

x_{ij}:	**Binary variable that takes the value of 1 if the i[th] cluster is decided to be connected with the j[th] WWTP and 0 otherwise (-);**
z_{ij}:	Continuous variable that takes the value of the amount of waste water transferred from cluster i to WWTP j (m3/h);
q_j:	Continuous variable that takes the value of the expansion needed to be made at WWTP at location j in terms of additional amount of waste water that can be treated in it. (m3/h);
y_j:	Binary variable that takes the value of 1 if a WWTP exists at location j and 0 otherwise (-);
r_j:	Continuous variable that takes the value of the final capability (after expansion or closure) of WWTP at location j in terms of the final amount of waste water that can be treated in it (m3/h).

Table 8-3 Decision Variables

The Non-Linear Original Problem (NLOP) is formulated as follows:

Objective function:

$$\textbf{Minimize} \sum_{i\in I}\sum_{j\in J}\overline{CP_{ij}} * x_{ij} + \sum_{i\in I}\sum_{j\in J}\overline{CPM_{ij}} * x_{ij} + \sum_{p\in P}\sum_{j\in J}\overline{CE_p} * q_j^{0.71} + \sum_{p\in P}\sum_{j\in J}\overline{CM_p} * r_j^{0.352} \qquad (8\text{-}1)$$

Subject to:

$$\sum_{j\in J} x_{ij} \geq 1, \qquad \forall i \in I, \qquad (8\text{-}2)$$

$$\sum_{j\in J} z_{ij} \geq \overline{WWP_i} \qquad \forall i \in I, \qquad (8\text{-}3)$$

$$\sum_{i\in I} z_{ij} \leq \overline{QE_j} * y_j + q_j, \qquad \forall j \in J \qquad (8\text{-}4)$$

$$z_{ij} \leq \overline{WWP_i} * x_{ij}, \qquad \forall i \in I, \quad \forall j \in J \tag{8-5}$$

$$x_{ij} \leq y_j, \qquad \forall i \in I, \quad \forall j \in J \tag{8-6}$$

$$q_j \leq \left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j} \right) * y_j, \qquad \forall j \in J \tag{8-7}$$

$$r_j \geq \overline{QE_j} * y_j + q_j, \qquad \forall j \in J \tag{8-8}$$

$$x_{ij}, \in \{0, 1\}, \qquad \forall i \in I, \quad \forall j \in J \qquad (8\text{-}9)$$

$$y_j, \in \{0, 1\}, \qquad \forall j \in J \qquad 8\text{-}10)$$

$$z_{ij} \geq 0, \qquad \forall i \in I, \quad \forall j \in J \qquad (8\text{-}11)$$

$$q_j \geq 0, \qquad \forall j \in J \qquad (8\text{-}12)$$

$$\mathrm{r_j} \geq 0, \qquad \forall \mathrm{j} \in \mathrm{J} \qquad 8\text{-}13)$$

The objective function (8-1) is a minimization of the total cost during the adopted time period of the network usage (e.g. 40 years). The first term of the objective function is the total construction cost for all the pipelines, these being either gravitational or pumped, and all the pumping stations to be required in the network. From this term the cost of the already installed pipelines and pumping stations is subtracted. The second term of the objective function is the total operational and maintenance cost for all used pipelines, either gravitational or pumping, and all pumping stations that will be considered in the network. The third term is the total non-linear expansion cost function of the Waste Water Treatment Plants (WWTPs). The fourth term is the total non-linear operational and maintenance cost of the WWTPs. Finally, the fifth term of the objective function is the total installation and operational and maintenance cost of Micro-Filtration (MF) and Reverse Osmosis (RO) systems that might be installed in the considered clusters of end users units.

Constraint (8-2) guarantees that each cluster 'i' will be linked with a WWTP 'j'.

Constraint (8-3) guarantees that for each cluster 'I' the total amount of waste water transferred to WWTPs will be at least its waste water production. Constraint (8-4) guarantees that the total capability of the WWTP will be at least as much as the total amount that is transferred to it through the pipelines. Constraint (8-5) guarantees that no amount of waste water will be transferred from i to j if a connection between them does not exist. Constraint (8-6) guarantees that no connection between i and j is established if a WWTP at j does not exist. Constraint (8-7) guarantees that no expansion is made at j if a WWTP at j does not exist. Constraint (8-8) guarantees that for each WWTP its final capacity will be at least equal to its current capacity plus its expansion. Constraints (8-9) -(8-13) declare the variables' bounds.

8.2 Piecewise Linearization

8.2.1 Introduction

In most problems water resource and waste water treatment planning ones included, many parameters and terms within a model are formulated as linear ones. Nevertheless it is quite often the case that non linear terms can be encountered. In our case where minimization of cost or even maximizing of profit by subtracting overall profit minus operating or fixed expenditures is the stake and the problem's objective we come across is the notion of economies of scale. This notion can be depicted with a non linear function as seen in **Figure 8-1**. Economies of scale is a notion behind which, the direct relationship of the magnitude of an infrastructure system is implied with its subsequent cost. That is the bigger an infrastructure built the less is the overall cost per unit of its directly associated infrastructure. This descending unit cost can be explained as per the concave nature of the examined function which is used to define this attribute.

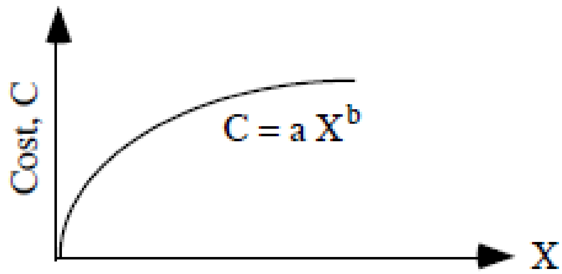


Figure 8-1 Typical Non-linear (Concave) Cost Function

In

Figure 8-1, the terms deciding the degree of non-linearity of the examined function, is the constant term b. The range of values in which b lies is for instance [0.4, 1]. When b takes the value of 1 this implies that the function f(x) becomes linear with a subsequent ascending curvature as the value of constant b descents.

In certain types of constraints sets within a mathematical formulation of a problem non linear terms may be encountered. One such example of a problem is most networked control systems and Distributed Energy Resource Systems (DERS's) (El-Hameed et al 2016), one of which is our examined problem. Numerous real world problems include variables which in turn contain constraints of non linear nature as follows where A and B are different input decision variables functions or vector containing these non-linearities.

$$\mathbf{Output} = \mathbf{f}(\mathbf{A} \bullet \mathbf{B})$$

There are different modes to handle and solve non-linearities. The one adopted here is the so called piecewise linearization and is to attain linear functions by decomposing the non linear ones into a number of linear segments. We then end up dealing with a linear model which can be handled through the use of known LP methods. Two different versions were adopted to be introduced in our specific problem (Lin, et al., 2013).

Another representative such example which comprises an overview of the principals and the applications of Non-Linear Optimization can be found at (Floudas, 1995). One of the methods to solve non-linear problems is to divide the nonlinear functions into several linear sections (piecewise linearization).The advantage of this approach is that we then have a linear problem to which any LP algorithm can be applied. For a review on the piecewise linearization methods the reader is referred to (Lin, et al., 2013). However, to address the non-linear terms in the current mathematical formulation, the piecewise-linear approximations proposed by (Misener & Floudas, 2010) were used. For the needs of the piecewise linearization, a set of variables known as a Special Ordered Set of type 1 (*SOS1*), were proposed by (Beale & Tomlin, 1969) and initially implemented by (Forrest, et al., 1974). These variables are sets with at most one nonzero component.

In the current thesis, the Special Ordered Set of variables of type 2 (*SOS2*), introduced by(Misener & Floudas, 2010), were implemented. This variant was chosen because it has the advantage that it does not rely on equal grid spacing. Thus, it allows me to create finer grids in the area where the cost function's gradient is steeper (in its smaller values) and wider grids in the area where the cost function's gradient is smoother (in its larger values).

As stated at, regarding domain partitioning in 2D by using *SOS2* variables, (Misener & Floudas, 2010) introduce a new index *t* which represents a diagonal of grid points, as shown in

Figure 8-1. The authors define *SOS2* variable set Ω_t, such that:

$$\Omega_t = \sum_i w_{i,i-N_1+t} \qquad \forall t \tag{8-14}$$

$$\sum_t \Omega_t = 1 \tag{8-15}$$

$$\Omega_t \qquad SOS2 \tag{8-16}$$

Where $w_{i,j}$ is the convex combination weight associated with grid point $\boldsymbol{x_{i,j}}$

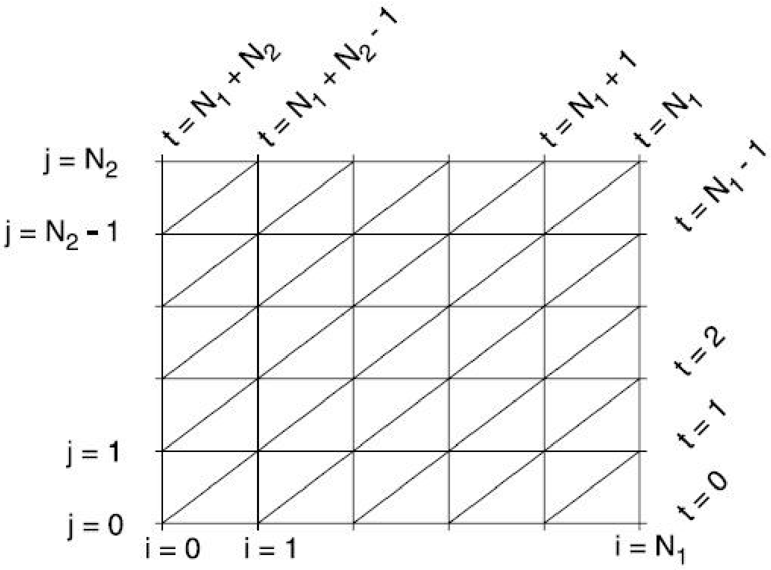


Figure 8-1: Triangulation tessellation in two dimensions (Misener & Floudas, 2010)

Taking into consideration the above constraints, the appropriate *n+1* vertices are activated for domain $X \in \mathbb{R}^n$. Thus, the interpolation of a non-linear function $f(x): X \longmapsto \mathbb{R}$ is made by the convex combination of the activated vertices. According to (Misener & Floudas, 2010), for approximation of an one dimensional variable, like the one in the current study (Equation (8-1)), the corresponding linear system of equations becomes:

$$\hat{f}(x) = \sum_{i=0}^{N_1} w_i * f(x_i) \quad (8\text{-}17)$$

$$x = \sum_{i=0}^{N_1} w_i * x_i \quad (8\text{-}18)$$

$$\sum_{i=0}^{N_1} w_i = 1 \quad (8\text{-}19)$$

$$w_i \geq 0, \qquad \forall i = 0, \ldots, N_1 \quad (8\text{-}20)$$

In the above model the variables w_i are the so-called Special Order Set of type 2 variables (SOS2). They are continuous variables between 0 and 1. **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** illustrates how the linearization of a non-linear convex function is made by using the previously described piecewise linearization method. The dashed line depicts the non-linear convex function, which resembles the one dealt with at the current study, introduced in Equation (8-1) . The solid line represents its piecewise linearization, introduced by (Misener & Floudas, 2010) and used in the current study. Moreover, in **Figure 8-3** there is an example of a non-linear cost function, similar to the one in the current thesis, which is linearized by using the previously explained unequal grid spacing. As one can notice $\overline{Q_1} \neq \overline{Q_2} - \overline{Q_1} \neq \overline{Q_3} - \overline{Q_2} \neq \overline{Q_4} - \overline{Q_3}$.

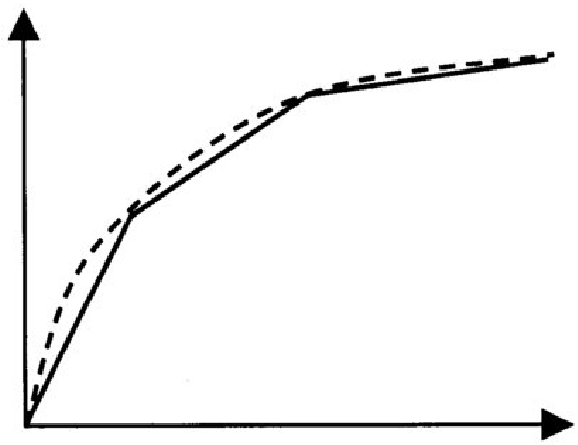


Figure 8-2: Piecewise linearization (solid line) of a non-linear convex function (dashed line). Source: (Björk, et al., 2003)

The above method guarantees a global optimum solution only for maximization problems when the function to be maximized is concave or for minimization problems when the function to be minimized is convex. However in our case, the mathematical model is a minimization of a concave objective function (8-1) and in order to better approximate it, restricted basis entry constraints are needed. As explained at (INFORMS, 2009), these constraints guarantee that at most two SOS2 variables will be positive, and if two are positive, they must be adjacent. Thus, the following constraints are added to the ones presented by (Misener & Floudas, 2010):

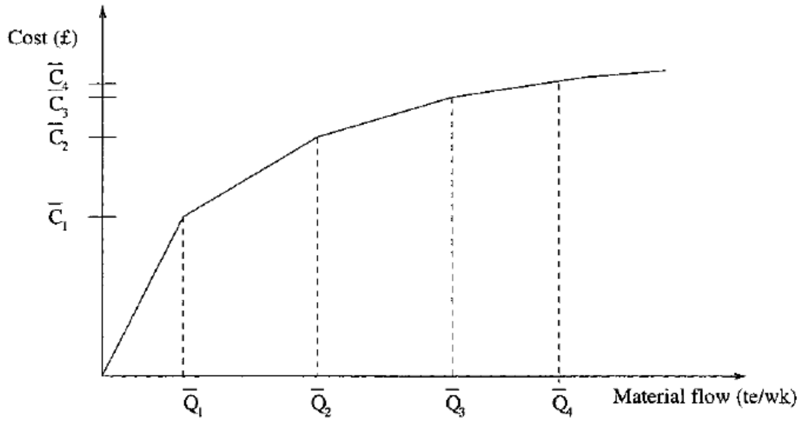


Figure 8-3: Piecewise linearization of a non-linear convex function with non-equal grid spacing. Source: (Tsiakis, et al., 2001)

$$w_i \leq z_i \forall i = 0, \dots, N_1 \tag{8-21}$$

$$\sum_{i=0}^{N_1} z_i \leq 2 \tag{8-22}$$

$$z_i + z_{j+n} \leq 1 \forall\ i = 0, \dots, N_1 - 2, \forall\ n \in [2, N_1 - j] \tag{8-23}$$

$$z_i \in \{0, 1\} \forall \forall i = 0, \dots, N_1 \tag{8-24}$$

In case of minimizing a function of concave shape to better illustrate the necessity of these constraints, if for example, we were minimizing a concave function such as shown in **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.**, the solution without restricted basis will comprise a non restricted solution set and can be written as follows:

$$x^* = w_1 a_1 + w_4 a_4 and f(x^*) = f_1 \tag{8-25}$$

However "**w's**" variables used in relationship (8-25) do not comprise complementary terms. Due to this fact, *f(x)* will not be adequately approximated by the correspondent $f(w_1, w_4)$ function therefore above written equation would not be the right one as w1 and w4 are not adjacent and therefore $f(w_1, w_4)$ is not a good approximation of $f(x)$. The restricted basis set will ban thus prevent this set of solutions whereas the correspondent optima points with the consequent optimal solution would be reformed as follows:

$$x^* = w_2 a_2 + w_3 a_3 and f(x^*) = f_2 \quad (8\text{-}26)$$

As depicted in the following **Figure 8-5**, it can be seen that ***f*₂** comprised of the blue segments the f_2 is a much more better approximation of the original f function than the previously used f_1 function since the error between the blue and the black line is much smaller than the correspondent one between the green and the black line.

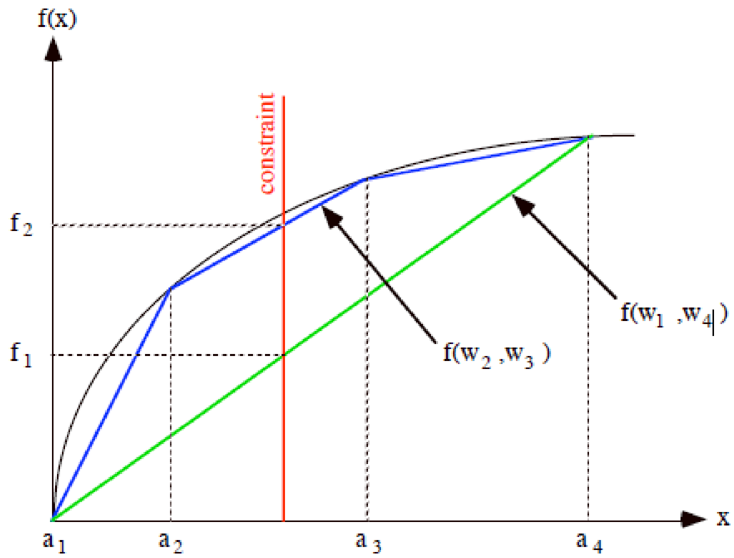


Figure 8-5 Piecewise linearization of a concave function to be minimized with (blue line) and without (green line) Restricted Basis Entry constraints

8.2.1.1 Unbounded Function Approach

This technique is introduced for the purpose of maximizing functions of a concave nature. Similarly to the formation of an optimization problem to its standard form this method would also attain the minimum of the corresponding convex of the same function in the form seen in **Figures 8-1** and **8-2** respectively.

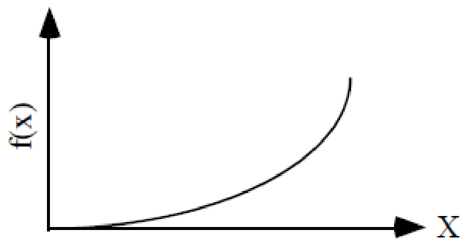


Figure 8-6 Convex function

In **Figure 8-6** our objective is that the function needs to attain a maximum value. We assume a constraint value of $X \leq 5$. In case of 2 or 3 deign variables the solution can prove to be quite obvious which in our case is X=5. In case several variables coexist within the same expression of the objective function as well as the set of constraints then this problem could not be depicted but solved analytically under the same rules of the specified method.

In place of the above, a new expression can be written as follows:

To this end an entirely updated set of data of the updated problem would be reformed as follows:

$$\textbf{Maximize } u$$

Subject to:

$$u \leq f_1 = a_1 + b_1 X$$
$$u \leq f_2 = a_2 + b_2 X$$
$$u \leq f_3 = a_3 + b_3 X$$

$$X \leq 5$$

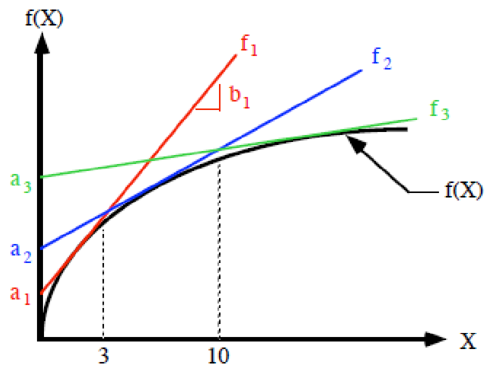


Figure 8-7 Unbounded Approach to Piecewise Linearization

In Figure 8-7 we notice that the examined non linear problem can be transformed into a LP one as each function values evaluation f(x)'s are linear approximations to the actual f(x) which is a curve. Thus we may write that $f_I \approx f(X)$ for values of x relatively very close to the examined X_o values for each such iteration. We can also see in the Figure that at the [3,10] range for x, the smallest value of the linear function is f_1, whereas the maximum value of x over this range is $u = a_2 + b_2 * 5$. In the [10,+ +∞] range, we get that $u \leq f_2(X)$ comprises the linear programming solution as $f_3(X) \leq f_2(X) \leq f_1(X)$ in above mentioned set.
Likewise a convex function might be also minimized
Based on the above a similar transformation is made accordingly:

$$\textbf{Minimize} f(X)$$

Subject to:

$g(X) \geq b$

With the use of mathematical formulated pattern as follows:

Minimize u

Subject to:

$u \geq f_1 = a_1 + b_1 X$
$u \geq f_2 = a_2 + b_2 X$
$u \geq f_3 = a_3 + b_3 X$

The above described method comprises a very good and simple estimate for global optima. It is however focused more on cases where the problem comprises constraint sets of maxima points of a concave and minima solution points of a convex shape respectively. A more inclusive method is now described. This algorithm now searches solutions that are yielded through local optima points exclusively. These points are not attained via similar set of restrictions required in the previously discussed method. By this way all these restrictions can be avoided.

8.2.1.2 Bounded Variable Approach

In **Figure 8-4**, 3 considered non linear segments in the X1 surface approximate a selected non linear $f(X_1, X_2)$ function. A similar linear approach of the $f(X_2)$ function in the X2 direction is attained with the same number of 3 linear segments. This is not shown in this Figure as the resulted shape would be getting a bit complex to depict. One could depict separately the X2 direction in the same manner as the one in the X1 direction. Three dimensional linear dimension planes are then produced out of these two directions or even linear planes of larger dimensions (i.e nxn dimensions) could be produced accordingly.

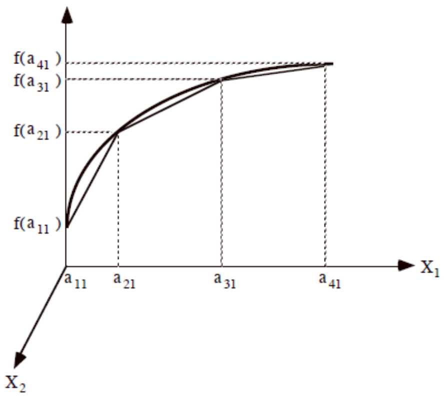


Figure 8-8 Bounded Variable Approach

The initial (original) problem is described as follows:

Maximize $Z = f(X_i)$

Subject to:

$$g_k(X_i) \leq b_k, \qquad \forall k$$

This comprises a linear problem and the following relationship describe its piecewise linearity:

Maximize $Z = \sum_i \sum_j f(a_{ji}) * W_{ji}$

Subject to:

$$\sum_j a_{ji} * W_{ji} = X_i, \qquad \forall i$$

$$\sum_j W_{ji} = 1, \qquad \forall i$$

The following constraint is only required in the case of non-linear restrictions;

$$\sum_i \sum_j g_k(a_{ji}) * W_{ji} \leq b_k, \qquad \forall k$$

In case that linear constraints are present the relationship is elaborated.

$$g_k(X_i) \leq b_k, \qquad \forall k$$

The 'W's 'terms comprise the design (decision) variables and are known in the bibliography as the 'SOS2' terms. This abbreviation stands for the 'Special Order Set of type 2' variables. These terms are similar to the binary variables with the difference that these range within a design range set of (0,1) and are assigned continuous values. In the Bounded variable approach when a maximization objective function is encountered then this function has to be a concave one necessarily. Likewise in the case of a minimization case then it has to be applied exclusively for a function of a concave nature. In certain cases though this method could be enhanced for other types of functions of other types and shapes of convexity than the ones mentioned above, when only two complementary constraints variables W_{ji} 's are also added to form the so called restricted basis set or entry.

8.3 Piecewise Linearization in WWTND

In the mathematical model presented in the previous section, the goal is to minimize a concave function without any non-linear constraints. For this reason, the most suitable piecewise linearization that was chosen to be used is the "bounded variable approach" by using additional auxiliary binary variables to express restricted basis entry constraints. (H.Lin et al, 2013) .

The non-linearity in the objective function is dealt with by applying piecewise linearization and with the use of SOS2 variables. Therefore for the Linearized Original Problem (LOP), the following notation has to be added (**Tables 8-4** to **8-6**):

Indices

B: **Set of intervals (boxes) used for the piecewise linearization (index b);**

Table 8-4 Indices

Input Parameters-Data:

$\overline{CE_{bp}}$:	**Cost of expansion of a WWTP of type p corresponding to the capability expansion $\overline{awq_b}$ in the interval b of the piecewise linearization (€);**
$\overline{CM_{bp}}$:	Cost of maintenance of a WWTP of type p corresponding to the total capability $\overline{awr_b}$ in the interval b of the piecewise linearization (€);
$\overline{awq_b}$:	general integer parameter that is equal to the value of WWTP's expansion capability at interval b (for the expansion cost of WWTPs);
$\overline{awr_b}$:	general integer parameter that is equal to the value of WWTP's total capability at block b (for the O&M cost of WWTPs)

Table 8-5 Input parameters-Data

Decision Variables:

wq_{bj}:	**Continuous variable between 0-1 (SOS2 variable) that corresponds to the expansion made of WWTP at location j in interval b of capability. (-)**
wr_{bj}:	Continuous variable between 0-1 (SOS2 variable) that corresponds to the total capability after expansion made of WWTP at location j in block b of capability.(-)
int_wq_{bj}:	Binary variable that is equal to 1 if the corresponding SOS2 variable wq_{bj} is greater than 0 and 0 otherwise (-);
int_wr_{bj}:	Binary variable that is equal to 1 if the corresponding SOS2 variable wr_{bj} is greater than 0 and 0 otherwise (-).

Table 8-6 Design variables

For example, **Table 8-7** shows how the cost of expansion of WWTP could be considered with intervals of piecewise linearization.

block b	awq_b (m3/h)	CE_bp, p=0 (mechanical) (€)	CE_bp, p=1 (biological) (€)
1	0	0.0	0.0
2	20	51841.5	15552.5
3	50	99360.8	29808.2
4	100	162534.8	48760.4

5	200	265875.1	79762.5
6	500	509583.1	152874.9
7	1000	833578.1	250073.4
8	2000	1363570.5	409071.2
9	5000	2613454.9	784036.5
10	8000	3648717.2	1094615.1

Table 8-7 Blocks of piecewise linearization for the cost of expansion of WWTP

The Linearized Original Problem (LOP) is formulated as follows:

Objective function:

$$\textbf{Minimize} \sum_{i \in I} \sum_{j \in J} \overline{CP_{ij}} * x_{ij} + \sum_{i \in I} \sum_{j \in J} \overline{CPM_{ij}} * x_{ij} + \sum_{b \in B} \sum_{p \in P} \sum_{j \in J} \overline{CE_{bp}} * wq_{bj} \ (+ \sum_{b \in B} \sum_{p \in P} \sum_{j \in J} \overline{CM_{bp}} * wr_{bj} \qquad (8\text{-}27)$$

Subject to:

Constraints (8-2) -(8-12) and the following constraints:

$$\sum_{b \in B} \overline{awq_b} * wq_{bj} = q_j, \qquad \forall j \in J \qquad (8\text{-}28)$$

$$\sum_{b \in B} \overline{awr_b} * wr_{bj} = r_j, \qquad \forall j \in J \qquad (8\text{-}29)$$

$$\sum_{b \in B} wq_{bj} = 1, \qquad \forall j \in J \qquad (8\text{-}30)$$

$$\sum_{b \in B} wr_{bj} = 1, \qquad \forall j \in J \qquad (8\text{-}31)$$

$$wq_{bj} \le int\_wq_{bj}, \qquad \forall j \in J, \quad \forall b \in B \qquad (8\text{-}32)$$

$$wr_{bj} \leq int\_wr_{bj}, \qquad \forall j \in J, \quad \forall b \in B \tag{8-33}$$

$$\sum_{b \in B} int\_wq_{bj} \leq 2, \qquad \forall j \in J \tag{8-34}$$

$$\sum_{b \in B} int\_wr_{bj} \leq 2, \qquad \forall j \in J \tag{8-35}$$

$$int\_wq_{bj} + int\_wq_{b+nj} \leq 1, \qquad \forall j \in J, \quad \forall\, b \in [1, B-2], \quad \forall\, n \in [2, B-b] \tag{8-36}$$

$$int\_wr_{bj} + int\_wr_{b+nj} \leq 1, \qquad \forall j \in J, \quad \forall b \in [1, B-1], \quad \forall\, n \in [2, B-b] \tag{8-37}$$

$$0 \leq wq_{bj} \leq 1, \qquad \forall b \in B, \quad \forall j \in J \tag{8-38}$$

$$0 \leq wr_{bj} \leq 1, \qquad \forall b \in B, \quad \forall j \in J \tag{8-39}$$

$$int\_wq_{bj} \in \{0, 1\}, \qquad \forall b \in B, \quad \forall j \in J \tag{8-40}$$

$$int\_wr_{bj} \in \{0, 1\}, \qquad \forall b \in B, \quad \forall j \in J \tag{8-41}$$

In the objective function ((8-27) , the third and fourth terms, which were previously non-linear, have been linearized by using piecewise linearization with SOS2 variables. Constraints (8-28) and (8-29)) guarantee that expansion capability and the total capability, respectively, of each WWTP will be equal to the weighted summation of the corresponding SOS2 variables. In this weighted summation each SOS2 variable is multiplied by the capability value of the corresponding interval b. Constraints (8-30) and (8-31) guarantee that for each WWTP the summation of the corresponding SOS2 variables will be equal to 1 for the expansion capability and the total capability respectively.

Constraints (8-32) and (8-33) guarantee that if a SOS2 variable is greater than 0 then the corresponding integer SOS2 variable will be equal to 1. These constraints are necessary for the following restricted basis entry constraints (8-34) -(8-37) . Constraints (8-34) and (8-35) guarantee that no more than two (2) SOS2 variables are greater than 0 and constraints (8-36) and (8-37) guarantee that these two (2) non-zero SOS2 variables are adjacent. Constraints (8-38) - (8-41) declare the variables' bounds.

Due to the high complexity of the linearized Original Problem, one can think of applying a Decomposition method so as to make it easy to solve large-scale instances. For this reason, we applied Benders Decomposition method, which is described in the following section.

8.4 The Benders Decomposition method

8.4.1 Introduction

This method was developed by a mathematician called Benders an Operation Research expert and scholar (Anon., 2018). He was the pioneer to develop the research domain of Operations Research in the Netherlands and the method developed was called after his name. He is well known for his essential contribution to mathematical programming. The 'Benders Decomposition' method comprises one of the most eminent optimization techniques ("Partitioning procedures for solving mixed-variables programming problems." (Numerische Mathematik 4.1 (1962): 238–252).

According to Scopus, 2018 above mentioned article was the trigger for its citing by more than 1,600 other papers which were published.

Ever since its initial development known as 'Benders Decomposition method' various modifications of this initial version have been made. This variations have been applied to numerous fields of Operations Research such as Mixed Integer Linear (MILP) , Stochastic, Multi-Objective (MOLP), Quadratic (QLP)and Non Linear Programming (NLP) as mentioned in (Rahmaniani, et al., 2017).

The research domain of Operations Research includes a large range of optimization problems set where some modified or extended versions of Benders decomposition are incorporated. These categories include Supply chain Network design and Network flow problem among others. These too are associated to your problem. However, despite its application in a very large variety of scientific fields, there are still open problems concerning the Benders Decomposition Algorithm (Saharidis & Fragkogios, 2018).

In today's real problems associated to networks flows such as the design of a new Waste Water Network and allocation of funds in such infrastructure distribution grids there is a necessity for the engineer of a complex such system to be able to develop new generations of optimization algorithms and methods in order that larger and more complex problems can be broken down to so called sub problems and therefore to be solved efficiently within an adequate computation time. Initial method has had an important contribution to such new generation optimization methods (Maros, 2005).

The Development of the mathematical model in the upscaled level for the case of Luxembourg in the urban or regional level before reviewing the Classical Benders Decomposition method the following set of assumptions will be made:

1) Study of the overall road map of Luxembourgish territory and possible special features it may include. Assumption that the pipeline network is tractable as it follows the road map network above it will be made;

2) Assumption that all clusters function independently with other cluster units;

3) In case that every single cluster of units is interconnected with another cluster, then we may examine some corresponding sub cases falling into this scenario;

    a) Either two districts are relatively neighboring areas or two different areas are in close proximity with one WWTP in this case we may consider it as a problem of a lower bound case;

    b) All clusters which belong to different areas are connected to each others. In such a case we shall come up with an upper bound case;

    c) We may come up with a hybrid scenario that is a combinatorial scenario of the above two where might be mostly possible that this might be the case for Luxembourgish network.

As soon as this question is resolved in this case, a cluster of units of different use demands shall be examined. Dependent on the volume of the created model (i.e. building stock) we might be able to decide on the degree of depth of the simulation of the corresponding system . Based on the latter, assumptions shall be extracted correlated to the size of the model, so that the mathematical model to be developed is tractable in terms of:

- Minimization of objective functions
- CPU time / high quality of optimal solution

The above-mentioned model may be developed using the Linear Programming or Multi Integer Linear Programming (MILP) which to optimize this cluster using the minimization of the objective function for the overall cost of the system to be developed. This overall cost corresponds to the capital cost of infrastructure, operational and Maintenance (O&M) cost of fresh water supply to the network as well as operating cost of waste water infrastructure.

The questions which may be answered by the development of the mathematical model are as follows:

1) To find optimal flows to new existing (proposed optimal location after the analysis of the model) or even new built WWTP's.

2) Allocation of new distribution plants and/or shut down of existing ones

3) Re-design of existing network.

   d) Possible optimal allocation of new Microfiltration (MF) and Reverse Osmosis (RO) systems which might serve selected single units of public or tertiary sector (Municipal or industrial buildings or cluster of units of different uses and demands (Land use , schools blocks of houses single households etc).

This is a large scale problem therefore possible scenarios to be used will be realized using one of the following methods of analysis:

5) Decomposition Method oriented methods of Analysis (for exact approach);

6) If this above mention method doe not converge heuristic algorithms such as GA's shall be utilized;

7) Third step would be the development of a hybrid method which combines the exact approaches together;

8) Eventually access to Administrative data sets that might be of public interest might be reachable. A set of different scenario cases in order to attain specific saving of resources might come up.

8.4.2 The Classical Benders Decomposition method

As previously described , Benders,(1962) introduced a wide range technique through which mixed integer programming problems (MIP) as well as mixed variable problems (MVP) are being handled efficiently. Its first implementation was for the decomposition of large or very large and compound optimization problem cases. After the development of its variation methods, there was also applied to other sort of problems such as global optimization based problems (Zhu & Kuno, 2003). Mostly used in big data where their non convexity nature has to be considered also these are used for stochastic programming based problems (Watkins, et al., 2000). Another large domain this method is applied to is the stochastic programming oriented optimization problems. In these problems one might encounter different variations of the initial problems and these might comprise multi-scenario cases, two or multi stage stochastic problem cases. The solution is attained in the following manner using above mentioned methods of Benders: The main scenarios are then broken (split) to sub-scenarios and the latter are examined as separate sub problems. This yield much reduced amount of data and subsequently reduced computational time for every single iteration. According to (Ierapetriou & Saharidis, 2009) this method is also incorporated for the simulation of mathematical prototypes where very large set of experimental data are fed into the building of a model.

Therefore one can take advantage of the decomposable attribute of a given main problem and this problem can be split to other smaller subset according to Bender's methodology notion. By doing so the solution of the smaller such sub problems can be converted accordingly. In **Table 8-8** we can notice that A is a given matrix of a main problem's constraint sets of coefficients (Saharidis et al, 2018).Furthermore the block- decomposable nature of the problem can be depicted though this matrix.

Decomposition is based on the assumption that specific design and/or constraints variables of the initial (main) problem are taken to be as the complicating ones. We then split this main problem into two other sub problems. The one is the so called Relaxed Master Problem

(RMP) and the other being the so called Primal Sub-Problem (PSP). The first comprises all engaged complicating elements previously mentioned and the latter includes all the remaining ones. The optimal final solution is eventually yielded by adopting relaxation of the initial problem by adopting the RMP after the inclusion of all its inequality constraints.

We now set an Original Problem (OP) as follows:

Original Problem (OP):

$$Min \quad c^T x + d^T y \tag{8-42}$$

$$s.t. \quad Ax + By \geq b, \tag{8-43}$$

$$x \geq 0, y \in \mathbb{Z} \tag{8-44}$$

Table 8-8 Benders Decomposition approach applied into the block-decomposable structure of a matrix of a given problem. (Fragkogios & Saharidis, 2018)

	a_{11}	a_{12}	a_{13}	…	0	0	0	…	a_{1j-2}	a_{1j-1}	a_{1j}
	a_{21}	a_{22}	a_{23}	…	0	0	0	…	a_{2j-2}	a_{2j-1}	a_{2j}
	…	…	…	…	…	…	…	…	…	…	…
	0	0	0	…	a_{nk-1}	a_{nk}	a_{nk+1}	…	a_{nj-2}	a_{nj-1}	a_{nj}
A =	0	0	0	…	a_{n+1k-1}	a_{n+1k}	a_{n+1k+1}	…	a_{n+1j-2}	a_{n+1j-1}	a_{n+2j}
	…	…	…	…	…	…	…	…	…	…	…
	0	0	0	…	0	0	0	…	a_{i-1j-2}	a_{i-1j-1}	a_{i-1j}
	0	0	0	…	0	0	0	…	a_{ij-2}	a_{ij-1}	a_{ij}

We assume that all y's variables are taken to be the complicating ones. Then the RMP is formatted as follows:

Relaxed Master Problem (RMP):

$$Min \quad z + d^T y \tag{8-45}$$

$$s.t. \quad (b - By)^T \overline{w_j} \leq z \qquad j = 1,2, \dots, p, \tag{8-46}$$

$$(b - By)^T \overline{v_i} \leq 0 \qquad i = 1,2, \dots, q, \tag{8-47}$$

$$y \in \mathbb{Z} \tag{8-48}$$

For every single $\bar{y}$ value a solution is attained within the context of the RMP. The j, p ‘s represent the iteration steps within which the space set where the PSP attains feasibility whereas the *i’s, q’s* represent the corresponding its infeasible space. Therefore the corresponding PSP comprising all non complicating variables is formatted as follows:

Primal Sub Problem (PSP($\bar{y}$)):

$$Min \quad c^T x + d^T \bar{y} \tag{8-49}$$

$$s.t. \quad Ax \geq b - B\bar{y}, \tag{8-50}$$

$$x \geq 0. \tag{8-51}$$

The following relationship represents the Primal Sub problem [PSP($\bar{y}$)] and can be reformed as per duality theory. Above mentioned problem is of a continuous linear nature therefore this reformation may be attained, regarding its dual version, as follows:

Dual Sub Problem (DSP($\bar{y}$)):

$$Max \ (b - B\bar{y})^T u + d^T \bar{y} \tag{8-52}$$

$$s.t. \quad A^T u \leq c, \tag{8-53}$$

$$u \geq 0 \tag{8-54}$$

The Classical Decomposition Method is only implemented after the decomposition of the Original Problem (OP) has been performed. In this way the optimal solution of the OP can be reached. This method is an iterative procedure. In the context of each iteration both Upper (UB) as well as Lower Bounds(LB) are recalculated and the whole algorithm converges at the time that correspondent differences of these bounds attain a value inferior to a pre value that the user defines and can be called ϵ. In the following set of relationship these updated values of these bounds are estimated:

$$LowerBound = z + d^T y \quad (8\text{-}55)$$

$$UpperBound = c^T x + d^T y \quad (8\text{-}56)$$

The algorithm is described in

Table 8-9 and **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** depicts its flowchart.

Set $LB = -\infty$ **and** $UB = +\infty$**.**
while $UB - LB > \epsilon$ do
Solve the RMP and update the $LB = z + d^T y$**, if necessary.**
With the derived master solution $\bar{y}$**, solve DSP(**$\bar{y}$**).**
If**DSP(**$\bar{y}$**) is bounded and has an optimal solution, which corresponds to an extreme point**$\bar{w}$ **of the dual feasible space,** then
Append the following optimality cut to RMP:
$(b - By)^T \bar{w} \leq z$.
Update $UB = (b - B\bar{y})^T u + d^T \bar{y}$ **if necessary.**
else if **DSP(**$\bar{y}$**) is unbounded, there exists an extreme ray** $\bar{v}$**, which corresponds to the direction of the unboundedness.** Then
Append the following feasibility cut to RMP:
$(b - By)^T \bar{v}_i \leq 0$
end if
end while

Table 8-9 The Classical Benders Decomposition algorithm (CBD)

The algorithm enhances its robustness as it incorporates the accumulation of further information regarding the OP and its correlated solution space. This further information is introduced as a new Bender's Cut (BC) and is added gradually in the RMP.

One additional attribute of the method is the following. One may notice that, during every iteration, the aforementioned objective function (8-52) is fed with information thus updated. However, the constraint set remains unmodified. This implies that the DSP comprises a solution space which remains the same until termination.

In case all BC's were made through the use of boundary points and their correspondent rays within the solution space of the DSP, then the problem would yield an extended RMP which in turn would correspond to the OP. Therefore the same optimal solution would be attained as if the OP was initially known. All boundary points as well as extreme rays of DSP of the feasible region are generally represented by the total number of BC's incorporated. This number of BC's is extremely large in general. According to (Saharidis, et al., 2010), there is no probability that a RMP at the optimum comprises a number of active constraints which exceeds the number of its corresponding decision variables. The main notion behind the Bender's Algorithm is its convergence before the integration of the total number of BC's when at the same time complying with optimality condition set.

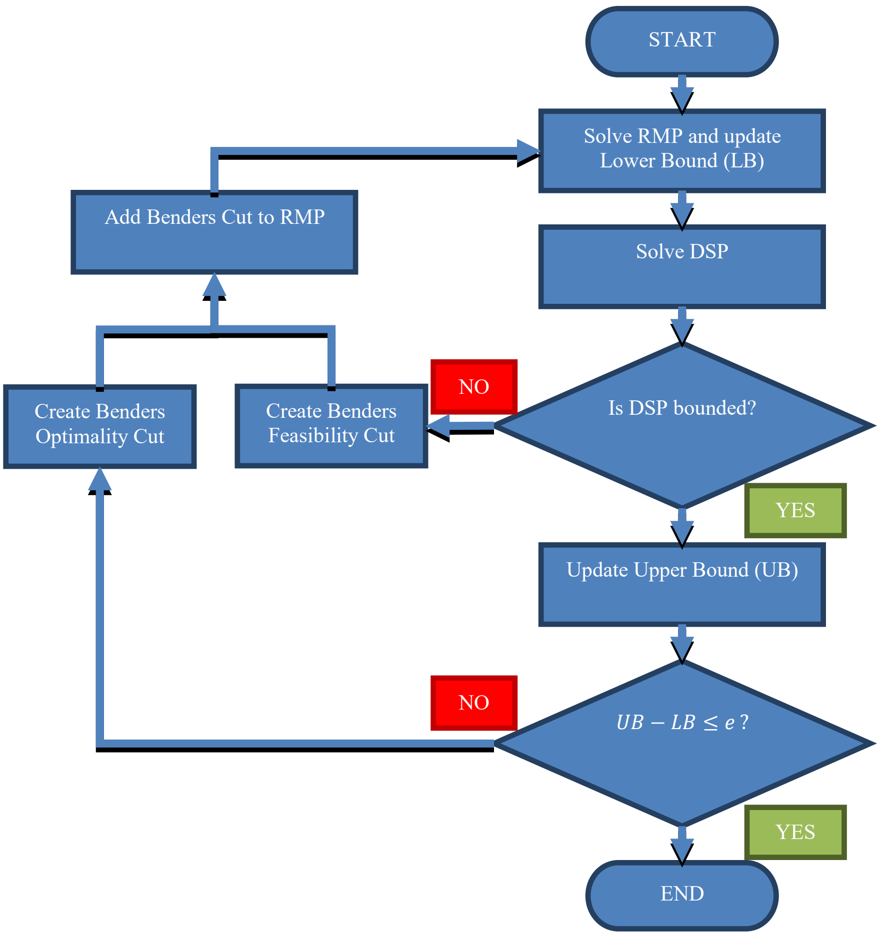


Figure 8-9 Flowchart of the Classical Benders Decomposition Algorithm. (Saharidis, et al., 2010).

As mentioned before the reason that there exist a finite algorithm converge its correspondent DSP comprises a finite number of boundary points as well as extreme rays, whereas it attains a non modified therefore stable solution space.

8.5 Waste Water Treatment Network Design by Benders Decomposition

The application of the Benders Decomposition method on the Waste Water Treatment Network Design Problem is considered in two variants. The first variant is based on the idea of “First Linearize, then Decompose”. The method can be denoted as “**BD_1**” and the decomposition is applied on the previously described already Linearized Original Problem (LOP). The second variant is based on the idea of “First Decompose, then Linearize”. The method can be denoted as “**BD_2**” and the decomposition is applied on the previously described Non-Linear Original Problem (NLOP). In this variant the piecewise linearization is applied after the decomposition. In the next sections the two variants are described:

1) First, the Non-Linear Original Problem (NLOP) was linearized by using the Piecewise Linearization method.

2) Then, the Original Linearized Problem (OLP) was decomposed by using Benders decomposition method into a Master Problem (MP1) and a Primal Sub problem (PSP1). For the decomposition, the integer variables of the OLP were considered as the complicating ones. Thus, the MP1 was a Pure Integer Linear Problem (PILP) and the PSP1 was a Continuous Linear Problem (CLP).

3) No acceleration method was applied.

8.5.1 Benders Decomposition on the Linearized Original Problem 1 (BD_1)

In the previously described Linearized Original Problem (LOP), one could consider as complicating variables the integer ones, either binary or general integers. This leads to the assumption that x_{ij}, y_j, int_wq_{bj} and int_w_{bj} are the complicate variables. Based on this assumption and according to the Benders Decomposition method described in the previous section, the Original Problem (OP) can be decomposed in the following Relaxed Master Problem 1 (RMP1) and Primal Sub Problem 1(PSP1). The RMP1 was a Pure Integer Linear Problem (PILP) and the PSP1 was a Continuous Linear Problem (CLP).

Relaxed Master Problem 1 (RMP1):

$$\textbf{Minimize} \sum_{i \in I} \sum_{j \in J} \overline{CP_{ij}} * x_{ij} + \sum_{i \in I} \sum_{j \in J} \overline{CPM_{ij}} * x_{ij} + \theta \tag{8-57}$$

$$\begin{aligned} &\sum_{i \in I} \overline{WWP_i} * \overline{u1_i} - \sum_{j \in J} \overline{QE_j} * y_j * \overline{u2_j} - \sum_{i \in I} \sum_{j \in J} \overline{WWP_i} * x_{ij} * \overline{u3_{ij}} \\ &\quad - \sum_{j \in J} \left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j} \right) * y_j * \overline{u4_j} + \sum_{j \in J} y_j * \overline{u5_j} + \sum_{j \in J} \overline{QE_j} * y_j * \overline{u6_j} \\ &\quad + \sum_{j \in J} \overline{u9_j} + \sum_{j \in J} \overline{u10_j} - \sum_{b \in B} \sum_{j \in J} \overline{int\_wq_{bj}} * \overline{u11_{bj}} \\ &\quad - \sum_{b \in B} \sum_{j \in J} \overline{int\_wr_{bj}} * \overline{u12_{bj}} - \sum_{b \in B} \sum_{j \in J} \overline{13_{bj}} - \sum_{b \in B} \sum_{j \in J} \overline{u14_{bj}} \leq \theta \end{aligned} \tag{8-58}$$

$$\begin{aligned} &\textstyle\sum_{i \in I} \overline{WWP_i} * \overline{w1_i} - \sum_{j \in J} \overline{QE_j} * y_j * \overline{w2_j} - \sum_{i \in I} \sum_{j \in J} \overline{WWP_i} * x_{ij} * \overline{w3_{ij}} - \\ &\textstyle\sum_{j \in J} \left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j} \right) * y_j * \overline{w4_j} + \sum_{j \in J} y_j * \overline{w5_j} + \sum_{j \in J} \overline{QE_j} * y_j * \overline{w6_j} + \\ &\textstyle\sum_{j \in J} \overline{w9_j} + \sum_{j \in J} \overline{w10_j} - \sum_{b \in B} \sum_{j \in J} \overline{int\_wq_{bj}} * \overline{w11_{bj}} - \sum_{b \in B} \sum_{j \in J} \overline{int\_wr_{bj}} * \overline{w12_{bj}} - \\ &\textstyle\sum_{b \in B} \sum_{j \in J} \overline{w13_{bj}} - \sum_{b \in B} \sum_{j \in J} \overline{w14_{bj}} \leq 0 \end{aligned} \tag{8-59}$$

$$\sum_{j \in J} x_{ij} \geq 1, \qquad \forall i \in I, \tag{8-60}$$

$$x_{ij} \leq y_j, \qquad \forall i \in I, \quad \forall j \in J \tag{8-61}$$

$$\sum_{b \in B} int\_wq_{bj} \leq 2, \qquad \forall j \in J \tag{8-62}$$

$$\sum_{b \in B} int\_wr_{bj} \leq 2, \qquad \forall j \in J \tag{8-63}$$

$$int\_wq_{bj} + int\_wq_{b+nj} \leq 1, \qquad \forall j \in J, \quad \forall\, b \in [1, B-2], \quad \forall\, n \in [2, B-b] \tag{8-64}$$

$$int\_wr_{bj} + int\_wr_{b+nj} \leq 1, \qquad \forall j \in J, \quad \forall b \in [1, B-1], \quad \forall\, n \in [2, B-b] \tag{8-65}$$

$$\sum_{j\in J}\sum_{b\in B} int\_wq_{bj} \geq 1 \tag{8-66}$$

$$\sum_{j\in J}\sum_{b\in B} int\_wr_{bj} \geq 1 \tag{8-67}$$

$$x_{ij}, \in \{0,1\}\ , \qquad \forall i \in I, \quad \forall j \in J \tag{8-68}$$

$$y_j, \in \{0,1\}\ , \qquad \forall j \in J, \quad \forall\, p \in P \tag{8-69}$$

$$int\_wq_{bj} \in \{0,1\}, \qquad \forall b \in B, \quad \forall j \in J \tag{8-70}$$

$$int\_wr_{bj} \in \{0,1\}, \qquad \forall b \in B, \quad \forall j \in J \tag{8-71}$$

$$\theta \geq 0 \tag{8-72}$$

Where constraints (8-58) and (8-58) are the Benders optimality and feasibility cuts respectively. In constraint (8-58) the $\bar{u}$ values correspond to the extreme points of the feasible space of the Dual Sub Problem 1(DSP1), presented below. In constraint (8-58) the $\bar{w}$ values correspond to the extreme rays of the feasible space of the DSP.

Primal Sub Problem 1 (PSP1):

$$\textbf{Minimize} \sum_{b\in B}\sum_{p\in P}\sum_{j\in J} \overline{CE_{bp}} * wq_{bj} + \sum_{b\in B}\sum_{p\in P}\sum_{j\in J} \overline{CM_{bp}} * wr_{bj} \tag{8-73}$$

$$\sum_{j\in J} z_{ij} \geq \overline{WWP_i}, \qquad \forall i \in I \tag{8-74}$$

$$-\sum_{i \in I} z_{ij} + q_j \geq -\overline{QE_j} * \overline{y_j}, \qquad \forall j \in J \tag{8-75}$$

$$-z_{ij} \geq -\overline{WWP_i} * \overline{x_{ij}}, \qquad \forall i \in I, \quad \forall j \in J \tag{8-76}$$

$$-q_j \geq -\left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j}\right) * \overline{y_j}, \qquad \forall j \in J \tag{8-77}$$

$$r_j \geq \overline{y_j}, \qquad \forall j \in J \tag{8-78}$$

$$r_j - q_j \geq \overline{QE_j} * \overline{y_j}, \qquad \forall j \in J \tag{8-79}$$

$$\sum_{b \in B} \overline{awq_b} * wq_{bj} - q_j \leq 0, \qquad \forall j \in J \tag{8-80}$$

$$\sum_{b \in B} \overline{awr_b} * wr_{bj} - r_j \leq 0, \qquad \forall j \in J \tag{8-81}$$

$$\sum_{b \in B} wq_{bj} = 1, \qquad \forall j \in J \tag{8-82}$$

$$\sum_{b \in B} wr_{bj} = 1, \qquad \forall j \in J \tag{8-83}$$

$$-wq_{bj} \geq -\overline{int\_wq_{bj}}, \qquad \forall j \in J, \quad \forall b \in B \tag{8-84}$$

$$-wr_{bj} \geq -\overline{int\_wr_{bj}}, \qquad \forall j \in J, \quad \forall b \in B \tag{8-85}$$

$$z_{ij} \geq 0, \qquad \forall i \in I, \quad \forall j \in J \tag{8-86}$$

$$q_j \geq 0, \qquad \forall j \in J \tag{8-87}$$

$$r_j \geq 0, \qquad \forall j \in J \tag{8-88}$$

$$0 \leq wq_{bj} \leq 1, \qquad \forall b \in B, \quad \forall j \in J \tag{8-89}$$

$$0 \leq wr_{bj} \leq 1, \qquad \forall b \in B, \quad \forall j \in J \tag{8-90}$$

The corresponding dual sub problem is the following:

Dual Sub Problem 1 (DSP1):

$$\begin{aligned}\textbf{Maximize} \sum_{i\in I} \overline{WWP_i} * u1_i - \sum_{j\in J} \overline{QE_j} * \overline{y_j} * u2_j - \sum_{i\in I}\sum_{j\in J} \overline{WWP_i} * \overline{x_{ij}} * u3_{ij} \\ - \sum_{j\in J}\left(\sum_{i\in I} \overline{WWP_i} - \overline{QE_j}\right) * \overline{y_j} * u4_j + \sum_{j\in J} \overline{y_j} * u5_j \\ + \sum_{j\in J} \overline{QE_j} * \overline{y_j} * u6_j + \sum_{j\in J} u9_j + \sum_{j\in J} u10_j \\ - \sum_{b\in B}\sum_{j\in J} \overline{int\_wq_{bj}} * u11_{bj} - \sum_{b\in B}\sum_{j\in J} \overline{int\_wr_{bj}} * u12_{bj} \\ - \sum_{b\in B}\sum_{j\in J} u13_{bj} - \sum_{b\in B}\sum_{j\in J} u14_{bj}\end{aligned} \quad (8\text{-}91)$$

$$u1_i \le \overline{CMFRO}, \qquad \forall i \in I \quad (8\text{-}92)$$

$$u2_j \le 0, \qquad \forall j \in J \quad (8\text{-}93)$$

$$u3_{ij} \le 0, \qquad \forall i \in I, \quad \forall j \in J \quad (8\text{-}94)$$

$$u4_j \le 0, \qquad \forall j \in J \quad (8\text{-}95)$$

$$u5_j \le 0, \qquad \forall j \in J \quad (8\text{-}96)$$

$$u6_j \le 0, \qquad \forall j \in J \quad (8\text{-}97)$$

$$u7_j \le \sum_{b\in B}\sum_{p\in P} \overline{CE_{bp}}, \qquad \forall j \in J \quad (8\text{-}98)$$

$$u8_j \le \sum_{b\in B}\sum_{p\in P} \overline{CM_{bp}}, \qquad \forall j \in J \quad (8\text{-}99)$$

$$u9_j \leq \sum_{b\in B}\sum_{p\in P}\overline{CE_{bp}}, \qquad \forall j \in J \tag{8-100}$$

$$u10_j \leq \sum_{b\in B}\sum_{p\in P}\overline{CM_{bp}}, \qquad \forall j \in J \tag{8-101}$$

$$-u11_{bj} \leq \sum_{b\in B}\sum_{p\in P}\overline{CE_{bp}}, \qquad \forall\, b \in B, \quad j \in J \tag{8-102}$$

$$-u12_{bj} \leq \sum_{p\in P}\overline{CM_{bp}}, \qquad \forall b \in B, \quad j \in J \tag{8-103}$$

$$u13_{bj} \leq \sum_{b\in B}\sum_{p\in P}\overline{CE_{bp}}, \qquad \forall\, b \in B, \quad j \in J \tag{8-104}$$

$$u14_{bj} \leq \sum_{p\in P}\overline{CM_{bp}}, \qquad \forall b \in B, \quad j \in J \tag{8-105}$$

$$u1_i, u2_j, u3_{ij}, u4_j, u5_j, u6_j, u7_j, u8_j, u11_{bj}, u12_{bj}, u13_{bj}, u14_{bj} \geq 0 \tag{8-106}$$

$$u9_j, u10_{j,} \quad free \tag{8-107}$$

Where the dual variables $u1 - u14$ correspond to the constraints (8-58) - (8-58) , (8-58) and (8-58) .

8.5.2 Benders Decomposition on the non-Linear Original Model (NLOM)

8.5.2.1 Benders Decomposition on the Non-Linear Original Model without Valid Inequalities (BD_2)

As previously stated, the second variant that is applied is based on the idea "**First Decompose, then Linearize**", meaning that in order to solve the Waste Water Treatment Network Design (WWTND) Problem by using the Benders Decomposition Method, the previously presented Non-Linear Original Problem (NLOP) is considered instead of the Linearized one. In the NLOP one could consider as complicating variables both the integer and the non-linear ones. This leads to the decomposition of NLOP by using the Benders Decomposition method into a Non-Linear Master Problem 2 (NLMP2) and a Linear Primal Sub problem 2 (LPSP2). The derived NLMP2 is a Mixed Integer Non Linear Problem (MINLP) and LPSP2 is a Continuous Linear Problem (CLP).

Non-Linear Master Problem 2 (NLMP2):

$$\textbf{Minimize} \sum_{i\in I}\sum_{j\in J}\overline{CP_{ij}} * x_{ij} + \sum_{i\in I}\sum_{j\in J}\overline{CPM_{ij}} * x_{ij} + \sum_{p\in P}\sum_{j\in J}\overline{CE_p} * q_j^{0.71} + \sum_{p\in P}\sum_{j\in J}\overline{CM_p} * r_j^{0.352} + \theta \tag{8-2}$$

Subject to the following constraints:

$$\sum_{j\in J} x_{ij} \geq 1 \qquad \forall i \in I, \tag{8-3}$$

$$\left(\frac{1}{BigM}\right)\sum_{i\in I} x_{ij} \leq y_j \qquad \forall j \in J \tag{8-4}$$

$$y_j \leq \sum_{i\in I} x_{ij} \qquad \forall j \in J \tag{8-5}$$

$$q_j \leq (\sum_{i\in I}\overline{WWP_i} - \overline{QE_j}) * y_j \qquad \forall j \in J \tag{8-6}$$

$$r_j \geq \overline{QE_j} * y_j + q_j \qquad \forall j \in J \tag{8-7}$$

$$x_{ij}, \in \{0,1\} \qquad \forall i \in I, \qquad \forall j \in J \tag{8-8}$$

$$y_j, \in \{0,1\} \qquad \forall j \in J \tag{8-9}$$

$$q_j \geq 0 \quad \forall j \in J \tag{8-10}$$

$$r_j \geq 0 \quad \forall j \in J \tag{8-11}$$

$$\theta \geq 0 \tag{8-12}$$

The above Non-Linear Master Problem 2 (NLMP2) is not in its final version, since the Benders Feasibility and Optimality Cuts are missing. The NLMP2 turns into its final formulation when linearized by using the piecewise linearization method into the following Linearized Master Problem 2 (LMP2), which is a Mixed Integer Linear Problem (MILP):

Linearized Master Problem 2 (LMP2):

$$\textbf{Minimize} \sum_{i \in I} \sum_{j \in J} \overline{CP_{ij}} * x_{ij} + \sum_{i \in I} \sum_{j \in J} \overline{CPM_{ij}} * x_{ij} + \sum_{b \in B} \sum_{p \in P} \sum_{j \in J} \overline{CE_{bp}} * wq_{bj} + \sum_{b \in B} \sum_{p \in P} \sum_{j \in J} \overline{CM_{bp}} * wr_{bj} + \theta \tag{8-13}$$

Subject to the following constraints:

$$\sum_{i \in I} \overline{WWP_i} * \overline{u1_i} - \sum_{j \in J} \overline{QE_j} * y_j * \overline{u2_j} - \sum_{i \in I} \sum_{j \in J} \overline{WWP_i} * x_{ij} * \overline{u3_{ij}} - \sum_{j \in J} \left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j} \right) * y_j * \overline{u4_j} + \sum_{j \in J} y_j * \overline{u5_j} + \sum_{j \in J} \overline{QE_j} * y_j * \overline{u6_j} + \sum_{b \in B} \sum_{j \in J} \overline{wq_{bj}} * \overline{awq_b} * \overline{u7_{bj}} + \sum_{b \in B} \sum_{j \in J} \overline{wr_{bj}} * \overline{awr_b} * \overline{u8_{bj}} \leq \theta \tag{8-14}$$

$$\sum_{i\in I}\overline{WWP_i}*\overline{w1_i}-\sum_{j\in J}\overline{QE_j}*y_j*\overline{w2_j}-\sum_{i\in I}\sum_{j\in J}\overline{WWP_i}*x_{ij}*\overline{w3_{ij}} -\sum_{j\in J}\left(\sum_{i\in I}\overline{WWP_i}-\overline{QE_j}\right)*y_j*\overline{w4_j}+\sum_{j\in J}y_j*\overline{w5_j}+\sum_{j\in J}\overline{QE_j}*y_j*\overline{w6_j} +\sum_{b\in B}\sum_{j\in J}\overline{wq_{bj}}*\overline{awq_b}*\overline{w7_{bj}}+\sum_{b\in B}\sum_{j\in J}\overline{wr_{bj}}*\overline{awr_b}*\overline{w8_{bj}}\leq 0 \tag{8-15}$$

$$\sum_{j\in J}x_{ij}\geq 1 \qquad \forall i\in I, \tag{8-16}$$

$$\left(\frac{1}{BigM}\right)\sum_{i\in I}x_{ij}\leq y_j \qquad \forall j\in J \tag{8-17}$$

$$y_j\leq\sum_{i\in I}x_{ij} \qquad \forall j\in J \tag{8-18}$$

$$\sum_{b\in B}int\_wq_{bj}\leq 2 \qquad \forall j\in J \tag{8-19}$$

$$\sum_{b\in B}int\_wr_{bj}\leq 2 \qquad \forall j\in J \tag{8-20}$$

$$int\_wq_{bj}+int\_wq_{b+nj}\leq 1\forall j\in J,\forall\, b\in[1,B-2], \qquad \forall\, n\in[2,B-b] \tag{8-21}$$

$$int\_wr_{bj}+int\_wr_{b+nj}\leq 1\forall j\in J,\forall b\in[1,B-1], \qquad \forall\, n\in[2,B-b] \tag{8-22}$$

$$\sum_{b\in B}wq_{bj}=1 \qquad \forall j\in J \tag{8-23}$$

$$\sum_{b\in B}wr_{bj}=1 \qquad \forall\, j\in J \tag{8-24}$$

$$wq_{bj}\leq int\_wq_{bj} \qquad \forall j\in J, \qquad \forall\, b\in B \tag{8-25}$$

$$wr_{bj} \leq int\_wr_{bj} \qquad \forall j \in J, \quad \forall b \in B \tag{8-26}$$

$$x_{ij}, \in \{0,1\} \qquad \forall i \in I, \quad \forall j \in J \tag{8-27}$$

$$y_j, \in \{0,1\} \qquad \forall j \in J, \quad \forall\, p \in P \tag{8-28}$$

$$int\_wq_{bj} \in \{0,1\} \qquad \forall b \in B, \quad \forall j \in J \tag{8-29}$$

$$int\_wr_{bj} \in \{0,1\} \qquad \forall b \in B, \quad \forall j \in J \tag{8-30}$$

$$0 \leq wq_{bj} \leq 1 \qquad \forall b \in B, \quad \forall j \in J \tag{8-31}$$

$$0 \leq wr_{bj} \leq 1 \qquad \forall b \in B, \quad \forall j \in J \tag{8-32}$$

$$\theta \geq 0 \tag{8-33}$$

Where constraints (8-14) and (8-15) are the Benders optimality and feasibility cuts, respectively. In constraint (8-14) the $\bar{u}$ values correspond to the extreme points of the feasible space of the DSP2, presented below. In constraint (8-15) the $\bar{w}$ values correspond to the extreme rays of the feasible space of the DSP.

Thus, the following Linear Primal Subproblem 2 (LPSP2) is derived:

$$\textbf{Minimize} - \tag{8-34}$$

$$\sum_{j \in J} z_{ij} \geq \overline{WWP_i} \forall i \in I, \tag{8-35}$$

$$-\sum_{i \in I} z_{ij} + q_j = -\overline{QE_j} * \overline{y_j} \forall j \in J \tag{8-36}$$

$$-z_{ij} \geq -\overline{WWP_i} * \overline{x_{ij}} \forall i \in I, \forall j \in J \tag{8-37}$$

$$-q_j \geq -\sum_{i \in I} \overline{WWP_i} * \overline{y_j} \forall j \in J \tag{8-38}$$

$$r_j - \sum_{i \in I} z_{ij} \geq 0 \forall j \in J \quad 8\text{-}(39)$$

$$r_j - q_j = \overline{QE_j} * \overline{y_j} \forall j \in J \tag{8-40}$$

$$q_j = \sum_{b \in B} \overline{awq_b} * \overline{wq_{bj}} \forall j \in J \tag{8-41}$$

$$r_j = \sum_{b \in B} \overline{awr_b} * \overline{wr_{bj}} \forall j \in J \tag{8-42}$$

$$z_{ij} \geq 0 \forall i \in I, \forall j \in J \tag{8-43}$$

$$q_j \geq 0 \forall j \in J \tag{8-44}$$

$$r_j \geq 0 \forall j \in J \tag{8-45}$$

One can notice that the LPSP2 has no objective function, because its variables do not take part into the objective function of the Original Problem. Thus, in fact LPSP2 is a constraint problem, which results only in Benders Feasibility cuts of the form (8-15). When all constraints of the LPSP2 are met, then a feasible solution will have been found, which will be the optimal solution of the original problem and the Benders algorithm is terminated.

The corresponding dual sub problem is the following:

Dual Sub problem 2 (DSP2):

$$\begin{aligned} \textbf{Maximize} \sum_{i\in I} \overline{WWP_i} * u1_i - \sum_{j\in J} \overline{QE_j} * \overline{y_j} * u2_j - \sum_{i\in I}\sum_{j\in J} \overline{WWP_i} * \overline{x_{ij}} * u3_{ij} \\ - \sum_{j\in J}\left(\sum_{i\in I} \overline{WWP_i} - \overline{QE_j}\right) * \overline{y_j} * u4_j + \sum_{j\in J} \overline{y_j} * u5_j \\ + \sum_{j\in J} \overline{QE_j} * \overline{y_j} * u6_j + \sum_{b\in B}\sum_{j\in J} \overline{wq_{bj}} * \overline{awq_b} * u7_{bj} \\ + \sum_{b\in B}\sum_{j\in J} \overline{wr_{bj}} * \overline{awr_b} * u8_{bj} \end{aligned} \tag{8-46}$$

$$u1_i, u3_{ij}, u4_j, u5_j \geq 0, \forall i, j \tag{8-47}$$

$$u2_j, u6_j, u7_j, u8_j \ free \ \forall j \tag{8-48}$$

Where the dual variables $u1 - u18$ correspond to the constraints (8-35)-(8-42) respectively.

8.5.2.2 Benders Decomposition on the Non-Linear Original Model with Valid Inequalities (VI) (BD_2_VI)

An augmented approach of the second variant of the proposed application of Benders Decomposition on Waste Water Treatment Network Design (WWTND) Problem is described in this subsection. In fact, due to the application of Benders decomposition, the previously presented Linearized Master Problem 2 (LMP2) has a relative small amount of information of the original problem, since some variables and constraints are moved to the Linear Primal Sub problem 2 (LPSP2). This results into the fact that the initial Lower Bounds of the Benders algorithm are quite weak (have a very low value), which leads to the need of more iterations to find the optimal solution, which in turn leads to large CPU time and slow convergence.

In order to achieve faster convergence of the Benders algorithm, a certain acceleration method is considered. Specifically, extra constraints, known as “Valid Inequalities” are added to the LMP2 in order to initialize it and result in better (higher) initial Lower Bounds of the Benders algorithm. The idea is based on the (Saharidis, et al., 2011), where valid inequalities are introduced for the application of Benders Decomposition on fixed-charge network problems. The valid inequalities are constraints that are not part of the original mathematical model, but can be derived by making valid assumptions based on the information stored in the Primal Sub problem. Thus, this information is artificially communicated within the Master Problem, in order to strengthen its initial Lower Bounds. The valid inequalities significantly restrict the solution space of the Benders master problem from the first iteration of the algorithm leading to improved convergence.

Thus, the Augmented Linearized Master Problem 2 (LMP2) is the following:

Objective function:

The same as (8-13)

Subject to:

(8-14)-(8-33)

$$\sum_{j\in J}\sum_{b\in B}\overline{awr_b} * wr_{bj} = \sum_{i\in I}\overline{WWP_i} \tag{49}$$

$$\sum_{j\in J}\sum_{b\in B}\overline{awq_b} * wq_{bj} = \sum_{i\in I}\overline{WWP_i} - \sum_{j\in J}\overline{QE_j} * y_j \tag{50}$$

Where constraint (49) denotes that the total final capability (after expansion or closure) of all Waste Water Treatment Plants which is a linearized variable will be equal to the total Waste Water Production of all clusters. Moreover, constraint (50) states that the total expansion made at all Waste Water Treatment Plants, which is a linearized variable, is equal to the total Waste Water Production of all clusters minus the total initial waste water treatment capability of the existing plants that will finally be kept in service.

Both above constraints are derived by valid assumptions, which in the original problem where expressed by constraints (8-3) ,(8-4) ,(8-7) and (8-8) . However, the application of the Benders Decomposition omitted these constraints from the Master Problem. Thus, by introducing the above two valid inequalities into the Master Problem, it includes the necessary information.

8.6 Mathematical formulation (with the use of MF and RO)

The problem of Waste Water Treatment Network Design (WWTND) is formulated as a Mixed-Integer Non-Linear Problem (MINLP) due to the non-linear costs of Expansion and of Operation and Maintenance (O&M) of a Waste Water Treatment Plant and the non-linear cost of Installation and of Operation and Maintenance of the Micro-Filtration (MF) and Reverse-Osmosis (RO). For the mathematical model the following notation (**Tables 8-10 – 8-12**)was used:

Indices:

I:	set of Clusters (index i);
J:	set of candidate locations of Waste Water Treatment Plant (index j);
P:	Set of types of WWTPs (p=0=>mechanical, p=1=>biological)

Table 8-10 Indices

Input Parameters-Data:

$\overline{WWP_i}$:	Waste Water Production of cluster i (m3/day);
$\overline{CE_p}$:	Fixed Cost of expansion of a WWTP of type p per amount of waste water treatment capability (€/m3);
$\overline{CM_p}$:	Cost of maintenance of a WWTP of type p per amount of waste water treatment capability (€/m3);
$\overline{CP_{ij}}$:	Cost of construction of a pipeline from cluster i to WWTP j per amount of waste water treatment capacity (€/m3);
$\overline{CPM_{ij}}$:	Cost of maintenance a pipeline from cluster i to WWTP j per amount of waste water treatment capability (€/m3);
$\overline{QE_j}$:	continuous parameter that is equal to the waste water treatment capability of the existing WWTP at location j (amount of waste water that can be treated in it). (m3/h);
$\overline{CMFRO}$:	Cost of installation and operation/maintenance of MF and RO per m3/h of WW saved (€ / m3*h);
$\overline{Type_j}$:	general integer parameter that is equal to the type of the WWTP at location j.(p=0=>mechanical, p=1=>biological);
$\overline{PCHouse_i}$:	Percentage of households in cluster i (-);
$\overline{PCSave}$:	Percentage of WWP that is saved if MF and RO system is applied (-);
$\overline{BigM}$:	a very large number.

Table 8-11 Input parameters-Data

Decision Variables:

x_{ij}:	**Binary variable that takes the value of 1 if the i[th] cluster is decided to be connected with j[th] WWTP and 0 otherwise (-);**
z_{ij}:	Continuous variable that takes the value of the amount of waste water transferred from cluster i to WWTP j (m3/h);
q_j:	Continuous variable that takes the value of the expansion needed to be made at WWTP at location j in terms of additional amount of waste water that can be treated in it. (m3/h);
y_j:	Binary variable that takes the value of 1 if a WWTP exists at location j and 0 otherwise (-);
r_j:	Continuous variable that takes the value of the final capacity (after expansion or closure) of WWTP at location j in terms of the final amount of waste water that can be treated in it. (m3/h) ;
p_i:	Continuous variable that equals WWP of i[th] cluster that is saved due to the installation of MF and RO (m3/h);

Table 8-12 Decision variables

The Non-Linear Original Problem (NLOP) is formulated as follows:

Objective function:

$$\textbf{Minimize} \sum_{i\in I}\sum_{j\in J}\overline{CP_{ij}} * x_{ij} + \sum_{i\in I}\sum_{j\in J}\overline{CPM_{ij}} * x_{ij} + \sum_{p\in P}\sum_{j\in J}\overline{CE_p} * q_j^{0.71} + \sum_{p\in P}\sum_{j\in J}\overline{CM_p} * r_j^{0.352} + \sum_{i\in I}\overline{CMFRO} * p_i^{0.51} \tag{8-157}$$

Subject to:

$$\sum_{j\in J} x_{ij} \geq 1 \qquad \forall i \in I, \tag{8-158}$$

$$\sum_{j\in J} z_{ij} \geq \overline{WWP_i} - p_i \qquad \forall i \in I, \tag{8-159}$$

$$\sum_{i \in I} z_{ij} \leq \overline{QE_j} * y_j + q_j \qquad \forall j \in J \tag{8-160}$$

$$z_{ij} \leq \overline{WWP_i} * x_{ij} \qquad \forall i \in I, \quad \forall j \in J \tag{8-161}$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \quad \forall j \in J \tag{8-162}$$

$$q_j \leq \left(\sum_{i \in I} \overline{WWP_i} - \overline{QE_j} \right) * y_j \qquad \forall j \in J \tag{8-163}$$

$$r_j \geq \overline{QE_j} * y_j + q_j \qquad \forall j \in J \tag{8-164}$$

$$p_i \leq \overline{PCSave} * \overline{PCHouse_i} * \overline{WWP_i} \qquad \forall i \in I, \tag{8-165}$$

$$x_{ij}, \in \{0, 1\} \qquad \forall i \in I, \quad \forall j \in J \tag{8-166}$$

$$y_j, \in \{0, 1\} \qquad \forall j \in J \tag{8-167}$$

$$z_{ij} \geq 0 \qquad \forall i \in I, \quad \forall j \in J \tag{8-168}$$

$$q_j \geq 0 \qquad \forall j \in J \tag{8-169}$$

$$r_j \geq 0 \qquad \forall j \in J \tag{8-170}$$

The objective function (8-157) is a minimization of the total cost during the time period of the network usage (e.g. for 40 years). The first term of the objective function is the total construction cost for all the pipelines, either gravitational or pumping, and all the pumping stations that will be needed in the network. From this term the cost of the already installed pipelines and pumping stations is subtracted. The second term of the objective function is the total operational and maintenance cost for all the pipelines, either gravitational or pumping, and all the pumping stations that will be needed in the network. The third term is the total non-linear expansion cost of the Waste Water Treatment Plants (WWTPs). The fourth term is the total non-linear operational and maintenance cost of the WWTPs.

Finally, the fifth term of the objective function is the total installation and operational and maintenance cost of Micro-Filtration (MF) and Reverse Osmosis (RO) systems that might be installed in the clusters. Constraint (8-158) guarantees that each cluster i will be linked with a WWTP j. Constraint (8-159) guarantees that for each cluster I the total amount of waste water transferred to WWTPs will be at least its waste water production minus the amount of waste water that is saved due to the use of MR and RO systems inside the cluster. Constraint (8-160) guarantees that the total capability of the WWTP will be at least as much as the total amount that is transferred to it through the pipelines. Constraint (8-161) guarantees that no amount of waste water will be transferred from i to j if a connection between them does not exist.

Constraint (8-162) guarantees that no connection between i and j is established if a WWTP at j does not exist. Constraint (8-163) guarantees that no expansion is made at j if a WWTP at j does not exist. Constraint (8-164) guarantees that for each WWTP its final capability will be at least equal to its current capability plus its expansion. Constraint (8-165) guarantees that for each cluster the quantity of WWP that is not dropped in the network, after the installation of MF and RO systems, is not greater than the percentage of households multiplied by the percentage of WWP that can be saved in each household. Constraints (8-166)-(8-170) declare the variables' bounds.

The non-linearity in the objective function is dealt with by applying piecewise linearization and with the use of Special Order Set (SOS) variables of type 2. Therefore concerning the linearized Original Problem (LOP), the following notation has to be added:

Indices

$\boldsymbol{B}$:	**Set of intervals (boxes) used for the piecewise linearization (index b);**

Table 8-13 Indices

Input Parameters-Data:

$\overline{\boldsymbol{CE}_{bp}}$:	**Fixed Cost of expansion of a WWTP of type p corresponding to the capacity expansion $\overline{awq_b}$ in the interval b of the piecewise linearization (€)**
$\overline{\boldsymbol{CM}_{bp}}$:	Cost of maintenance of a WWTP of type p corresponding to the total capability $\overline{awr_b}$ in the interval b of the piecewise linearization (€)
$\overline{\boldsymbol{CMFRO}_{b}}$:	Cost of installation and operation/maintenance of MF and RO per m3/h of WW saved corresponding to the capability $\overline{awp_b}$ of the MF and RO in the interval b of the piecewise linearization (€)

$\overline{awq_b}$:	general integer parameter that is equal to the value of WWTP's expansion capability at interval b (for the expansion cost of WWTPs)
$\overline{awr_b}$:	general integer parameter that is equal to the value of WWTP's total capability at block b (for the O&M cost of WWTPs)
$\overline{awp_b}$:	general integer parameter that is equal to the value of MF and RO capability at block b (for the Installation and O&M cost of MF and RO)

Table 8-14 Input parameters-Datas

Decision Variables:

wq_{bj}:	**Continuous variable between 0-1 (SOS2 variable) that corresponds to the expansion made of WWTP at location j in interval b of capability. (-)**
wr_{bj}:	Continuous variable between 0-1 (SOS2 variable) that corresponds to the total capability after expansion made of WWTP at location j in interval b of capability. (-)
wp_{bi}:	Continuous variable between 0-1 (SOS2 variable) that corresponds to the MF and RO capability at cluster i in interval b of capability. (-)
int_wq_{bj}:	Binary variable that is equal to 1 if the corresponding SOS2 variable wq_{bj} is greater than 0 and 0 otherwise (-)
int_wr_{bj}:	Binary variable that is equal to 1 if the corresponding SOS2 variable wr_{bj} is greater than 0 and 0 otherwise (-)
int_wp_{bi}:	Binary variable that is equal to 1 if the corresponding SOS2 variable wp_{bi} is greater than 0 and 0 otherwise (-)

Table 8-15 Design Variables

The introduction of the SOS2 variables has been made according to international bibliography and especially(Misener and Floudas 2010) and (INFORMS 2009).(It should be noted that in the proposed models, totally 3 SOS2 variables are integrated. In the following tables, the correspondence between the bibliography and the proposed linearized model is presented summarized (only 1 used SOS2 variables is depicted, the other 2 SOS2 variables include the respective constraints).

(Misener and Floudas 2010)	Proposed Linearized Original Problem (LOP) for Waste Water Treatment Network Design (WWTND) problem
Index of intervals/pieces: *i*	Index of intervals/pieces: *b*
SOS2 Variable: $\boldsymbol{w_i}$	SOS2 Variable: wq_{bj}
Value of the non-linear function at interval *i*: $\boldsymbol{f(x_i)}$	Value of the non-linear cost function at interval *b*: $\overline{CE_{bp}}$
Value of variable $\boldsymbol{x}$ at interval *i*: $\boldsymbol{x_i}$	Value of variable q_j at interval *b*: $\overline{awq_b}$
$\hat{f}(x) = \sum_{i=0}^{N_1} w_i * f(x_i)$	$\sum_{b \in B} \sum_{p \in P} \sum_{j \in J} wq_{bj} * \overline{CE_{bp}}$
$x = \sum_{i=0}^{N_1} w_i * x_i$	$q_j = \sum_{b \in B} wq_{bj} * \overline{awq_b} \; \forall j \in J$
$\sum_{i=0}^{N_1} w_i = 1$	$\sum_{b \in B} wq_{bj} = 1 \forall j \in J$
$w_i \geq 0, \quad \forall i = 0, \ldots, N_1$	$0 \leq wq_{bj} \leq 1 \forall b \in B, \forall j \in J$

Table 8-16 Correspondence of SOS2 variables and respective constraints between (Misener and Floudas 2010) and the proposed linearized model

(INFORMS 2009)	Proposed Linearized Original Problem (LOP) for Waste Water Treatment Network Design (WWTND) problem
Auxiliary Binary Variable: $\boldsymbol{z_i}$	Auxiliary Binary Variable: int_wq_{bj}
$w_i \leq z_i \forall i = 0, \ldots, N_1$	$wq_{bj} \leq int\_wq_{bj} \; \forall j \in J, \forall \; b \in B$
$\sum_{i=0}^{N_1} z_i \leq 2$	$\sum_{b \in B} int\_wq_{bj} \leq 2 \forall j \in J$
$z_i + z_{i+n} \leq 1 \forall \; i = 0, \ldots, N_1 - 2, \forall \; n \in [2, N_1 - i]$	$int\_wq_{bj} + int\_wq_{b+nj} \leq 1 \forall j \in J, \forall \; b \in [1, B-2], \forall \; n \in [2, B-b]$
$z_i \in \{0,1\} \forall \forall i = 0, \ldots, N_1$	$int\_wq_{bj} \in \{0,1\} \forall b \in B, \forall j \in J$

Table 8-17 Correspondence of SOS2 variables and respective constraints between (INFORMS 2009) and the proposed linearized model

From the above tables, one can easily notice that the proposed SOS2 variables/constraints are in complete correspondence with those in the international bibliography. Thus, the proposed SOS2 variables/constraints are correct and no reforming of the presentation of the SOS2 variables within existing model is required.

The Linearized Original Problem (LOP) is formulated as follows:

Objective function:

$$\textbf{Minimize} \sum_{i\in I}\sum_{j\in J}\overline{CP_{ij}} * x_{ij} + \sum_{i\in I}\sum_{j\in J}\overline{CPM_{ij}} * x_{ij} + \sum_{b\in B}\sum_{p\in P}\sum_{j\in J}\overline{CE_{bp}} * wq_{bj} + \sum_{b\in B}\sum_{p\in P}\sum_{j\in J}\overline{CM_{bp}} * wr_{bj} + \sum_{i\in I}\overline{CMFRO_b} * wp_{bi} \tag{8-171}$$

Subject to:

Constraints (8-158)-(8-169) and the following constraints:

$$\sum_{b\in B}\overline{awq_b} * wq_{bj} = q_j \qquad \forall j \in J \tag{8-172}$$

$$\sum_{b\in B}\overline{awr_b} * wr_{bj} = r_j \qquad \forall j \in J \tag{8-173}$$

$$\sum_{b\in B}\overline{awp_b} * wp_{bi} = p_i \qquad \forall i \in I \tag{8-174}$$

$$\sum_{b\in B} wq_{bj} = 1 \qquad \forall j \in J \tag{8-175}$$

$$\sum_{b\in B} wr_{bj} = 1 \qquad \forall j \in J \tag{8-176}$$

$$\sum_{b\in B} wp_{bi} = 1 \qquad \forall i \in I \tag{8-177}$$

$$wq_{bj} \le int\_wq_{bj} \qquad \forall j \in J, \quad \forall b \in B \tag{8-178}$$

$$wr_{bj} \leq int\_wr_{bj} \qquad \forall j \in J, \quad \forall b \in B \tag{8-179}$$

$$wp_{bi} \leq int\_wp_{bi} \qquad \forall\, i \in I, \quad \forall b \in B \tag{8-180}$$

$$\sum_{b \in B} int\_wq_{bj} \leq 2 \qquad \forall j \in J \tag{8-181}$$

$$\sum_{b \in B} int\_wr_{bj} \leq 2 \qquad \forall j \in J \tag{8-182}$$

$$\sum_{b \in B} int\_wp_{bi} \leq 2 \qquad \forall i \in I \tag{8-183}$$

$$int\_wq_{bj} + int\_wq_{b+nj} \leq 1 \qquad \forall j \in J, \quad \forall\, b \in [1, B-2], \quad \forall\, n \in [2, B-b] \tag{8-184}$$

$$int\_wr_{bj} + int\_wr_{b+nj} \leq 1 \qquad \forall j \in J, \quad \forall b \in [1, B-1], \quad \forall\, n \in [2, B-b] \tag{8-185}$$

$$int\_wp_{bi} + int\_wp_{b+ni} \leq 1 \qquad \forall i \in I, \quad \forall b \in [1, B-1], \quad \forall\, n \in [2, B-b] \tag{8-186}$$

$$0 \leq wq_{bj} \leq 1 \qquad \forall b \in B, \quad \forall j \in J \tag{8-187}$$

$$0 \leq wr_{bj} \leq 1 \qquad \forall b \in B, \quad \forall j \in J \tag{8-188}$$

$$0 \leq wp_{bi} \leq 1 \qquad \forall b \in B, \quad \forall i \in I \tag{8-189}$$

$$int\_wq_{bj} \in \{0,1\} \qquad \forall b \in B, \quad \forall j \in J \tag{8-190}$$

$$int\_wr_{bj} \in \{0,1\} \qquad \forall b \in B, \quad \forall j \in J \tag{8-191}$$

$$int\_wp_{bi} \in \{0,1\} \qquad \forall b \in B, \quad \forall i \in I \tag{8-192}$$

In the objective function (8-171), the third, fourth and fifth terms, which were previously non-linear, have been linearized by using piecewise linearization with SOS2 variables. Constraints (8-172) (8-172) and (8-173) guarantee that expansion capability and the total capability of each WWTP and the MF and RO capability of each cluster, respectively, will be equal to the weighted summation of the corresponding SOS2 variables. In this weighted summation each SOS2 variable is multiplied by the capability value of the corresponding interval b. Constraints (8-174), (8-175), (8-176) and (8-177) guarantee that for each WWTP the summation of the corresponding SOS2 variables will be equal to 1 for the expansion capability and the total capability respectively and that for each cluster the summation of the corresponding SOS2 variables will be equal to 1 for the MF and RO capability.

Constraints (8-58) , (8-58) and (8-58) guarantee that if a SOS2 variable is greater than 0 then the corresponding integer SOS2 variable will be equal to 1. These constraints are necessary for the following constraints (8-58) - (8-58) . Constraints(8-58) - (8-58) guarantee that no more than two (2) SOS2 variables are greater than 0 and constraints(8-58) - (8-58) guarantee that these two (2) non-zero SOS2 variables are adjacent. Constraints (8-58) -(8-58) declare the variables' bounds. Due to the high complexity of the Linearized Original Problem, one can think of applying a Decomposition method so as to make it easy to solve large-scale instances. For this reason, we applied Benders Decomposition method, described previously.

8.6.1 Cost functions

Cost functions are empirical or semi-empirical functions on which all relevant costs of the model under study is based. These comprise the following set of relationships:

The waste water network under study consists of three major components: a) The Waste Water Treatment Plants (WWTPs), b) The pipelines connecting the clusters with the WWTPs, c) distributed components and specifically Micro-Filtration (MF) and Reverse-Osmosis (RO). For all these components installation and maintenance costs are considered. The selected model coefficients herein are empirical, based on (Dekel, 2006) and (Friedler, et al. 2006), (Dekel, 2006),(Friedler, et al. 2006). derived cost functions expressing the effects of design flow and treatment level on construction costs, through the analysis of 55 municipal WWTPs in the context of a case experimental study.

8.6.1.1 Cost of expansion of a WWTP

The mechanical sewage treatment plants are usually composed by a setting pond, where the heavier wastes in the waste water ground as sewage sludge. The mud disassembles itself biochemically whereas a Biological sewage treatment plant uses natural microorganisms, usually plants or bacteria, which feed on the wastes in the water. Thus, the wastes are devoured in this way by the bacteria. For a biological WWTP, the respective overall cost is 30% of the cost of a corresponding mechanical WWTP which attains a similar degree of treatment either a secondary or a tertiary one for the purpose of comparison.(8-194) (USEPA, Region IV):

The cost of constructing a new mechanical WWTP or expanding an existing mechanical one is given by (8-193) (Dekel, 2006), Friedler, et al. 2006). :The corresponding fixed cost of expansion of a WWTP of type_p can be seen in **Table 8-18** and **Figure 8-10**.

$$\overline{CE_0} = FRC_T(int, nT) * 85{,}825\ Q_T^{0.71} \qquad \text{(8-58)}$$

$$\overline{CE_1} = 0.3 * \overline{CE_0} \qquad (8-58)$$

Where $\boldsymbol{Q_T}$: Design flow load capacity of waste water treatment of the WWTP (m3/h)

block b	awq_b	CEbp, p=0 (mechanical)	CEbp, p=1 (biological)
1	0	0.0	0.0
2	20	51841.5	15552.5
3	50	99360.8	29808.2
4	100	162534.8	48760.4
5	200	265875.1	79762.5
6	500	509583.1	152874.9
7	1000	833578.1	250073.4
8	2000	1363570.5	409071.2
9	5000	2613454.9	784036.5
10	8000	3648717.2	1094615.1

Table 8-18 Fixed Cost of expansion of a WWTP of type p (CE_bp) per amount of waste water treatment capability in each block b of expansion (€);

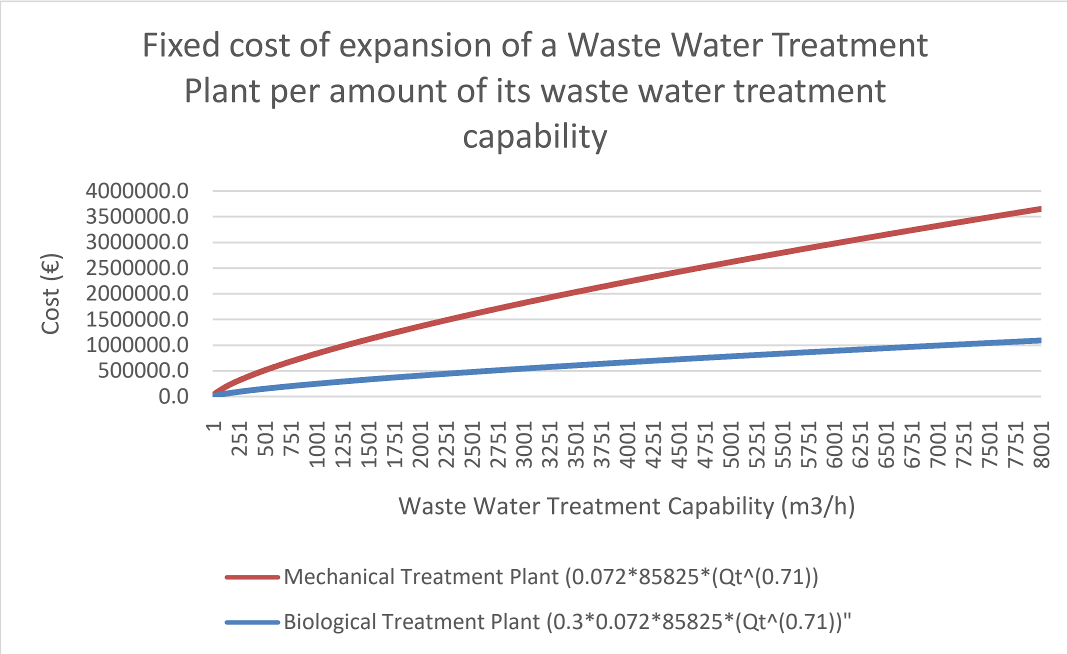


Figure 8-10
Fixed Cost of expansion of a WWTP of type p (CE_bp) per amount of waste water treatment capability in each block b of expansion (€);

8.6.1.2 Cost of maintenance of a WWTP

The annual maintenance cost of a mechanical WWTP is given by the following non-Linear Cost Function (8-195) (Dekel, 2006),:

$$\overline{CM_0} = 5{,}635 * X^{0.352} \qquad (8\text{-}58)$$

Where **X**: is the population equivalent, a measurement unit for the capacity of a WWTP. In order to compute all costs in terms of the amount of waste water that a plant can treat, the equation **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** is transformed into (8-196). (Dekel, 2006) and (Friedler, et al. 2006).

Again for a biological WWTP, the respective overall cost is approximately 30% of the cost of a mechanical WWTP (8-197).(USEPA, Region IV):

$$\overline{CM_0} = 1{,}044.77 * Q_T^{0.352} \qquad (8\text{-}58)$$

$$\overline{CM_1} = 0.3 * \overline{CM_0} \quad (8\text{-}58)$$

block of capacity	awr_b	CM_bp, p=0 (mechanical)	CM_bp, p=1 (biological)
1	0	0.0	0.0
2	20	2999.1	899.7
3	50	4140.6	1242.2
4	100	5284.7	1585.4
5	200	6745.0	2023.5
6	500	9312.4	2793.7
7	1000	11885.6	3565.7
8	2000	15170.0	4551.0
9	5000	20944.0	6283.2
10	8000	24712.1	7413.6

Table 8-19
Cost of maintenance of a WWTP of type p (CM_bp) per amount of waste water treatment capacity in each block b of capacity (€);

The form of the corresponding function representing the annual maintenance cost can be seen in **Figure 8-8.** The corresponding maintenance cost of a WWTP of type_p is seen in **Table 8-19.**

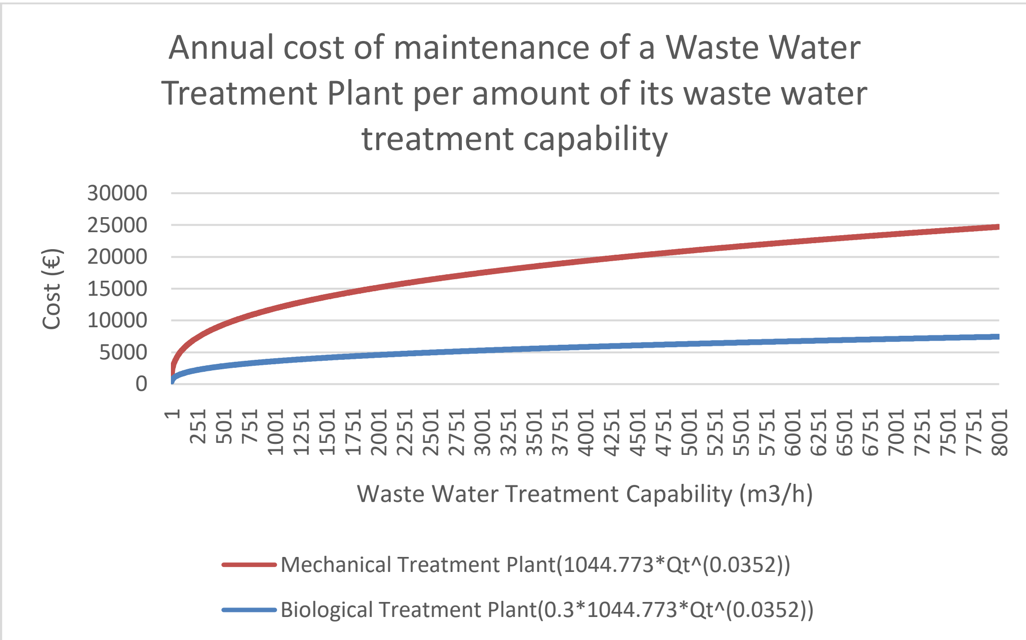


Figure 8-11
Cost of maintenance of a WWTP of type p (CM_bp) per amount of waste water treatment capacitty in each block b of capacity (€)

Figure 8-11 presents the two functions of the two kind of treatment plants that are existent in our examined model and their correlation with respect to maintenance cost per amount of waste water capacity in m3/h. We may note from the figure that mechanical comprises a more expensive choice but seems to also be exhibiting a more sudden grade from the start of its operation .

8.6.1.3 Cost of construction of a connection between cluster i to WWTP j

For the construction of a pipeline between a cluster i and a WWTP j, two types of connections are considered, the gravitational one and the pumping. For easier application, the altitude difference Δh between the two edges of the connection is considered(8-199). Thus, if the cluster i is at higher altitude than the WWTP j ($\Delta h > 0$), then the connection would be gravitational and its construction cost is as follows (8-198): (Dekel, 2006),(Friedler, et al. 2006):

For a pumping connection between i and j ($\Delta \mathrm{h} < 0$), the construction cost is the summation of the construction cost of the pumping pipeline plus the construction cost of the pumping station:

$$\overline{CP_{ij}} = FRC_{pp}(int, npp) * 382.5\, D_p{}^{1.455} L + FRC_{pu}(int, npu) * 64{,}920 \left[3.454 * \Delta h * Q + 6409(Q^{2.852} D_p^{-4.87} L)\right]^{0.33} \tag{8-58}$$

For a gravitational connection between i and j ($\Delta \mathrm{h} > 0$), the construction cost is only the construction cost of the gravitational pipeline:

$$\overline{CP_{ij}} = FRC_{pg}(int, npp) * 21.6\, D_g{}^{2.26} L + 7 \frac{H_I^2 - C_{min}^2}{2(J - J_s)} L_w \tag{8-58}$$

The aforementioned costs are assumed to be 0 for the existing pumping and gravitational connections of the network. The corresponding matrices with all relevant construction costs between clusters denoted as i's and the WWTP's denoted as j's per pipeline in EUROS concern the pipelines (gravitational or pumping ones) construction costs and can be seen in **Table 8-21** (Appendix A,List of Tables-Part II) whereas the corresponding pumping stations construction costs between clusters and the corresponding treatment plants can be seen in **Table 8-22** (Appendix A-List of Tables-Part II).

8.6.1.4 Cost of maintenance of a connection between cluster i to WWTP j

Respectively, if the cluster i is at lower altitude than the WWTP j ($\Delta h < 0$), then the connection would be pumping and its construction cost is as follows: (8-200).(Dekel, 2006),(Friedler, et al. 2006).

For a pumping connection between i and j ($\mathbf{\Delta h} < 0$), the maintenance cost is the summation of the maintenance cost of the pumping pipeline added to the maintenance cost of the pumping station (8-200):

$$\overline{CPM_{ij}} = [700 + 0.0005 * 100 * DN^2 * L] + FRC_{pu}(int, npu) * 64{,}920\left[3.454 * \Delta h * Q + 6409(Q^{2.852} D_p^{-4.87} L)\right]^{0.33} \quad (8\text{-}58)$$

For a gravitational pipeline connection between i and j ($\Delta\mathrm{h} > 0$), the maintenance cost is only the maintenance cost of the gravitational pipeline (8-201):(Dekel, 2006),(Friedler, et al. 2006)

$$\overline{CPM_{ij}} = [2{,}872 - 1.13 * 10 * DN + 0.00024 * 100 * DN^2 * L] \quad (8\text{-}58)$$

In all above equations **DN**: is the diameter of the pipe (cm)

L: is the Length of the pipe (km)

The corresponding matrices with all relevant maintenance costs between clusters denoted as i's and the WWTP's denoted as j's per pipeline in EUROS are related to the pipelines (gravitational or pumping ones) and can be seen in **Table 8-23** (Appendix A-List of Tables-Part II) whereas the cost of maintenance of the pumping stations for the afore mentioned pumping pipelines can be seen in **Table 8-24** (Appendix A-List f Tables-Part II).

8.6.1.5 Cost of MF and RO system

In the current paper two distributed network components are taken into consideration, the Micro-Filtration (MF) and Reverse-Osmosis (RO) systems. These systems can be installed in households and reduce the amount of waste water exiting each household and ending up to a WWTP by 59% as per the results of the present study. The cost of installation of such systems is given by (8-202) (Dekel, 2006),(Friedler, et al. 2006).

α) Installation cost:

$$CINST = 18{,}200 * Q^{0.51} \qquad (8\text{-}58)$$

where **Q**: is the flow rate of waste water (m3/d) through the system

In order to compute all costs corresponding to the same measurement unit of the amount of waste water ($m^3/hour$), equation **Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** is transformed into (8-203) (Dekel, 2006),(Friedler, et al. 2006):

$$CINST = 92{,}040.53 * Q^{0.51} \qquad (8\text{-}58)$$

where **Q**: Is the the same flow rate in (m3/h)as described above

β) Annual Operational and Maintenance cost:

According to the results of the present study (**Table 8-20** and **Figure 8-12**),the annual operation and maintenance cost of the MF and RO system is assumed to be 8% of the installation cost, (8-204), thus:

$$COM = 0.08 * CINST \qquad (8\text{-}58)$$

	awp_b	CMFRO_b (capital cost)	CMFRO_b (O&M annual cost)
1	0	0.0	0.0
2	0.5	64632.9	5170.6
3	2	131070.3	10485.6
4	5	209148.0	16731.8
5	10	297837.3	23827.0
6	20	424135.3	33930.8
7	50	676789.8	54143.2
8	100	963782.7	77102.6
9	200	1372474.9	109798.0
10	500	2190048.9	175203.9

Table 8-20
Cost of installation and operation/maintenance of MF and RO per m3/h of WW (CMFRO_b) saved corresponding to the capability of the MF and RO in the interval b of the piecewise linearization (in €)

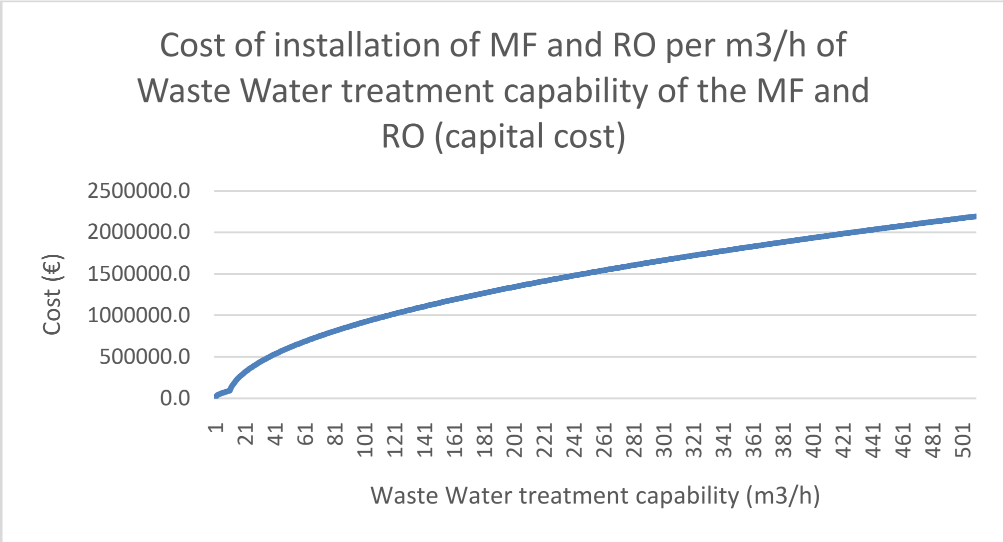


Figure 8-12
Total Installation Cost for MF and RO system taken into consideration in the mathematical model.

$$\overline{CMFRO} = CINST + years * COM \tag{8-58}$$

$$p_i \leq \overline{PCSave} * \overline{PCHouse_i} * \overline{WWP_i} \qquad \forall i \in I, \qquad (8-58)$$

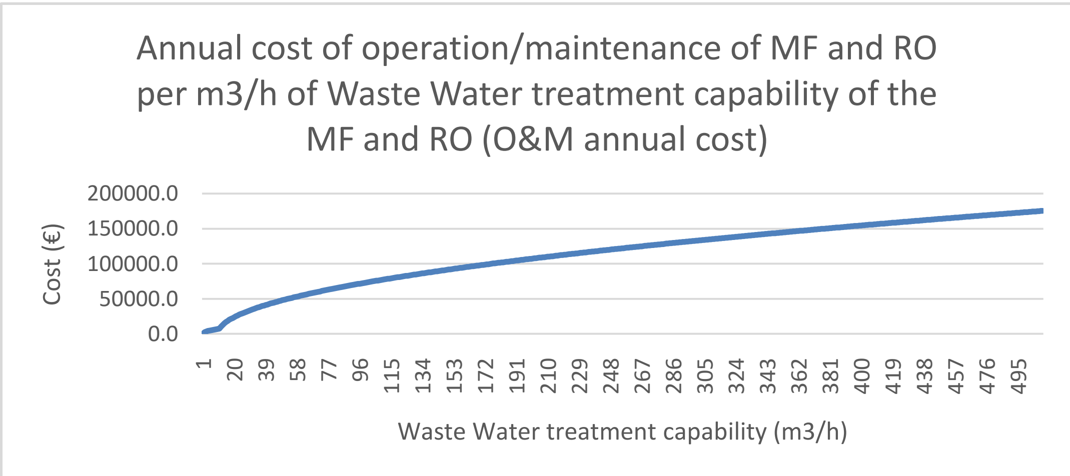


Figure 8-13 Annual operation/maintenance Cost for MF and RO system

Figure 8-13 presents the annual operation as well as maintenance expenditures of the two treatment systems which are incorporated within the examined model in m3/h . A smooth non-linear relationship is seen. As the treatment capacity of a plant increases we note a gradual non proportional reduce of the relevant operational and maintenance cost of the system.

Chapter 9. Case study

9.1 General

In this chapter the problem of water and upscale wastewater resource management (WWRM) is considered for Luxembourg.

The first step in the analysis is the division of the Luxemburg area in sub regions. Regions may be determined by population distribution, the land area, municipalities, or how distribution of waste water outflows to the Waste Water Treatment Plants (WWTP's) is considered until today.

The population is assumed to be located in each region at the relative population center, of each community (cluster) with given needs for satisfaction of waste water outflows. Waste Water outflow demands (WWOD) will be estimated directly from population as well as irrigation needs, number of industrial buildings, institutes and commercial buildings. In our described model only WWOD are considered as if each clusters comprised entirely of typical households of 4 users.

For the analysis population of a sub region is assumed to be located in the region's center, from now on called "node". In addition, twenty four (24 WWTP's are considered. These selected WWTP's are only given the possibility to either be enlarged or shut. In a future reach context several potential plant locations shall also be considered.

The optimal locations and capacities of surface water, wastewater treatment and water reclamation plants need to be determined in the problem.

The whole water system in the area is divided into non potable water and potable water systems. In the non-potable water system, wastewater is collected from all possible regions. The collected wastewater undergoes primary and secondary treatment in wastewater treatment plants according to specific quality requirements. Then, part of treated wastewater may need further treatment, at an extra cost, for reclamation, while the rest is disposed into the Mosel river. The reclaimed water could be distributed to other regions to satisfy only non-portable water demands for irrigation, industry, agriculture, etc. In the potable water system, the water from surface water treatment plants can be distributed to satisfy both potable and non-potable water demands, Groundwater may be used to satisfy, both, potable and non-potable demands, if available. We assume that there is no water loss in the processes.

The daily water demands and wastewater productions are assumed to be the same within a year period. It is assumed that both qualities of water and wastewater are allowed to be distributed to most regions, in order to satisfy all the water demands at minimum cost. Thus, the infrastructure needs for water distribution and storage, including the pipeline main network between nodes, pumping stations, and storage tanks, are also optimized in the problem. The pipeline for groundwater conveyance is assumed as existing. It should be noted that the local water distribution and storage infrastructure within each region is not considered. Between any two nodes allowed to be connected, "distances", "pumping distances" and "pumping elevations" are known (see their definitions in **Figure 9-1**). Here, the pair wise distances are used to calculate the pipe lengths and pipeline costs, while the pair wise pumping distances and elevations are required for the calculation of the pumping cost and pumping station cost. **Figure 9-1** illustrates that the pumping distances and elevations can be positive in both directions of a link.

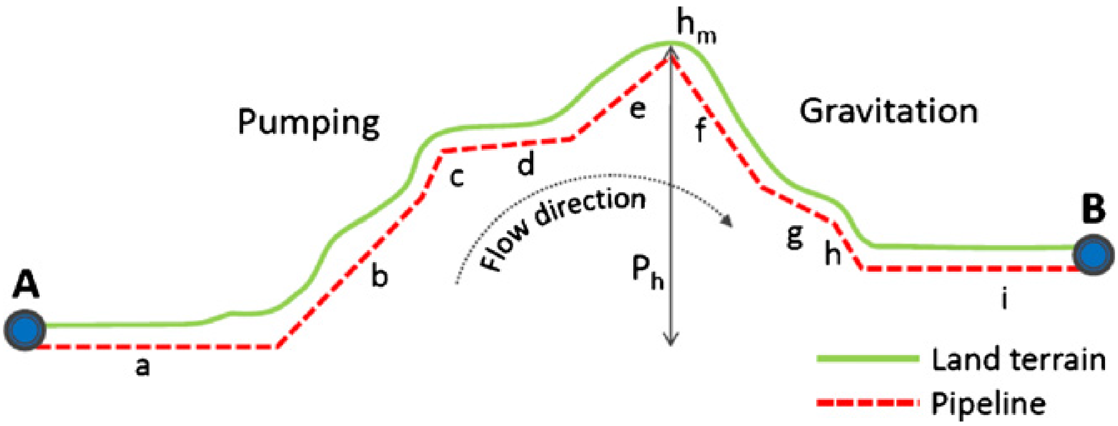


Figure 9-1 Schematic graph for the definition of the terms: "distance", "pumping distance" and "pumping elevation" between nodes "A" and "B" (Gikas et al 2011)

For flow direction A→B: the length of the pipeline between A and B is called "distance"= a+b+c+d+e+f+g+h+i, the length of the pressurised pipeline is called "pumping distance" =a+b+c+d+e, the maximum height that the liquid has to be pumped is called "pumping elevation" = Ph (Levy, 2010).

The selected examined area of Luxembourg can be mainly characterised as semirural and rural. There have been selected 20 clusters in number. These clusters comprise the communities of relatively similar size, population, set of infrastructure thus a set of different group of building units.

The existing regime is that every such cluster denoted as (i) is connected with either one (1) or even two (2) WWWTP's which are denoted in our model as (j). These selected WWTP's are twenty four in number (24) and are mainly located nearby river bodies where waste water discharge occurs easily after waste treatment. This set of data along with their explanations can be seen in Table 9-7 (Apendix-List of Tables-Part II).

It is considered that the Micro-Filtration (MF) and Reverse-Osmosis (RO) systems can be installed only at the households of a cluster. For this reason the percentage of households in a cluster is considered as presented in Table 9-1.

Cluster (i)	Aproximate total number of Buildings (-)	Aproximate total number of residentiial Buildings (-)	Percentage of households (%)
Septfontaines	197	112	57%
Tuntange	289	190	66%
Merch	2890	1345	47%
Boevange sur Atert	568	310	55%
Saeul	76	45	59%
Beckerich	459	268	58%
Oberpallen	236	125	53%
Noerdange	350	200	57%
Useldange	542	410	76%
Bissen	1417	989	70%
Colmar- Berg	850	571	67%
Vichten	789	403	51%
Preizerdaul	1080	670	62%
Ospern	456	297	65%
Reichlange	134	88	66%
Schwebach	61	29	48%
Redange sur Atert	438	201	46%
Ell	224	123	55%
Nagem	182	87	48%
Lannen	110	45	41%

Table 9-1 Number of typical households of each examined cluster i

The parameter $\overline{\boldsymbol{PCHouse_i}}$ of the mathematical model is equal to the percentage shown at column 4 in **Table 9-1**. Also, it is assumed that optimized proposed network for each single household which was examined in Part I of the study has the following effects on reduced waste water coming out of each household being around 41% of the total waste that would come out in case no MF/RO such systems would be used. This means that the savings of Waste Water Production due to MF/RO systems is 59% and this value is assigned to the parameter $\overline{\boldsymbol{PCSave}}$ of our model.

The above assumptions pose an upper bound on the amount of waste water that can be saved if the MF/RO systems are installed in all the households of each cluster. For this reason constraint (8-58) is inserted in our model.

9.2 General description of a selected examined cluster out of the 20 considered in total

We shall examine a little village community called "Schwebach" with a permanent population of 70 inhabitants (in the year 2006).Above mentioned community lies within the municipality ("commune") of Saeul and the canton of Rodange. The profile of the village is exclusively rural. The building stock is comprised almost exclusively of the following building stock: 61 buildings in total of which: 29 residential buildings (48%); 50 buildings of mixed use i.e. residential and agricultural functions together within the same building block (47%); 1 church; 2 different hotels of limited capacity; 1 trading store; 1 pharmacy store; a central building of public use; 2 buildings for agricultural use made of light construction and a water tank situated 30m off the main road. The sewer central main lies 60m from the nearest biological treatment plant. This de centralized treatment system is an autonomous unit which exhibits a capacity of population equivalent (PE) of 200 and is situated at a distance of 46 m from the nearby river called Schwebach river. Retrieving data from tables we can estimate the waste water.

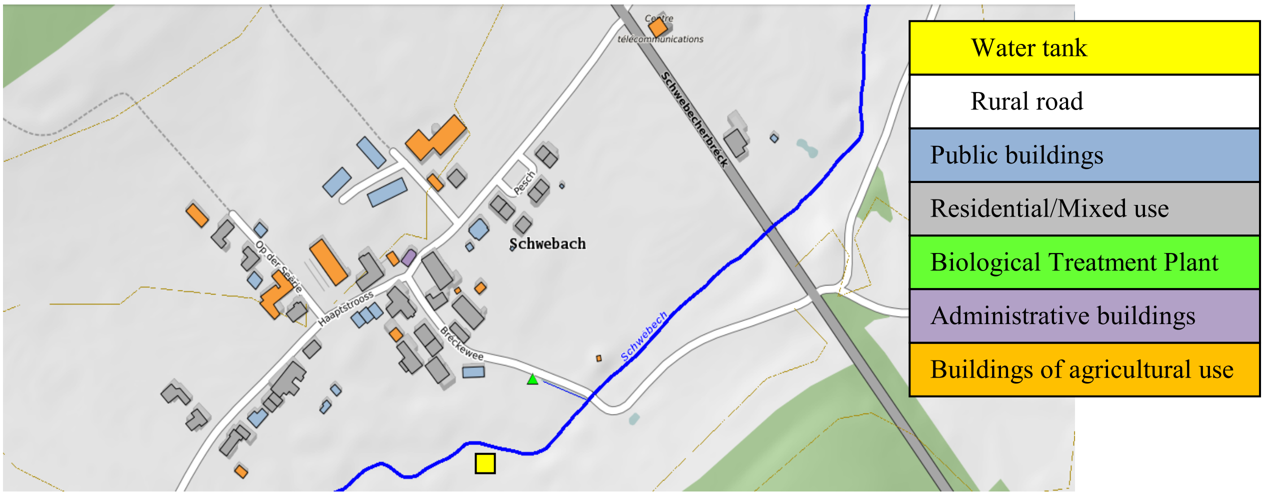


Figure 9-2 Plan view of the examined area (Geoportal Lux)

The village can be characterized as semi rural. The building stock includes a variety of activities. The main activity is agricultural. In **Figures 9-2** and **Figure 9-3** the waste water system is depicted. The sewerage network is a mixed system. A mixed sewerage system receives both household and rainfall waste in a single main. The yellow bold line represents the central main where all mixed waste (rainfall, residential and agricultural activities waste) are led to the biological waste treatment plant. The plant is represented in green and is located at a lower lever. The small square in blue represents the community water tank. The water tank receives treated waste from the plant and serves as reclamation water collector for agricultural activities mainly irrigation. Main residential and institutional units are fed partly by this water tank. More than 90% of the water provision demands of the village is satisfied by the existing fresh water network.

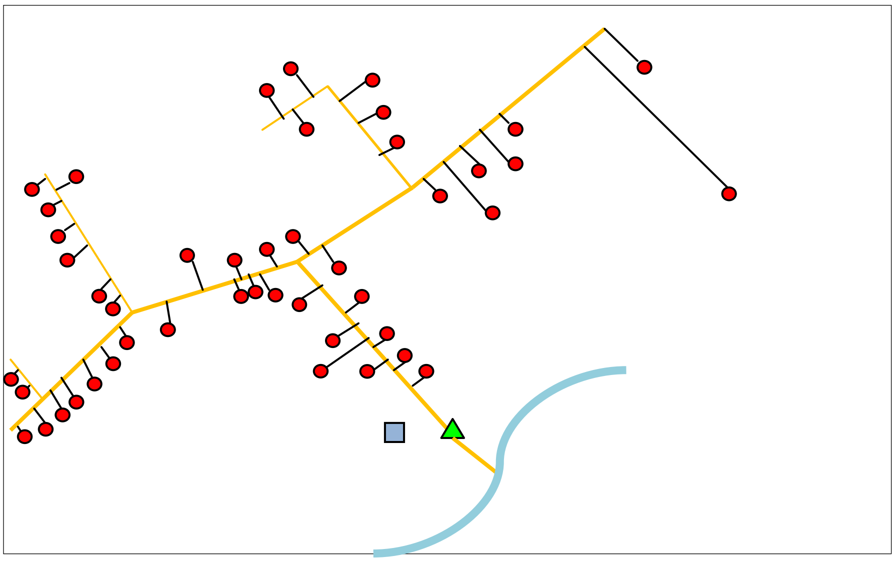


Figure 9-3 Waste water network system of Schwebach

Name of the area of WWTP	Load entering *(p.e.)	Physical Capacity (p.e.)	Treatment in place	Global compliance	BOD5	COD		Article 17
Beggen	131990	210000	More stringent treatment	Compliant	Pass	Pass		No
Esch/Schifflange	75675	90000	More stringent treatment	Compliant	Pass	Pass		No
Bleesbruck	70524	80000	More stringent treatment	Not Compliant	Pass	Pass		Yes
Bettembourg	69907	95000	More stringent treatment	Compliant	Pass	Pass		No
Petange	60901	70000	More stringent treatment	Compliant	Pass	Pass		No
Bonnevoie	49020	60000	More stringent treatment	Not Compliant	Pass	Pass		Yes
Mersch	46431	50000	More stringent treatment	Compliant	Pass	Pass		No
Uebersyren	23107	35000	More stringent treatment	Compliant	Pass	Pass		No
Hesperange	16476	36000	More stringent treatment	Compliant	Pass	Pass		No
Mamer	16265	23500	More stringent treatment	Compliant	Pass	Pass		No
Echternach	14643	36000	More stringent treatment	Compliant	Pass	Pass		No
Betzdorf	8827	10000	More stringent treatment	Compliant	Pass	Pass		No
Wiltz	6868	9000	Secondary treatment	Compliant	Pass	Pass		No

Emerange	5973	14000	More stringent treatment	Compliant	Pass	Pass		No
Clervaux	5141	4500	Secondary treatment	Compliant	Pass	Pass		No
Redange/Attert	4076	2000	Secondary treatment	Compliant	Pass	Pass		No
Medernach	3954	5000	Secondary treatment	Compliant	Pass	Pass		No
Steinfort	3933	4000	Secondary treatment	Compliant	Pass	Pass		No
Hobscheid	3766	6000	More stringent treatment	Compliant	Pass	Pass		No
Rombach/Martelange	3444	7100	More stringent treatment	Compliant	Pass	Pass		No
Kehlen	3289	5000	More stringent treatment	Compliant	Pass	Pass		No
Beaufort	3205	5000	Secondary treatment	Compliant	Pass	Pass		No
Kopstal	3168	8000	More stringent treatment	Compliant	Pass	Pass		No
Junglinster	3046	1700	Secondary treatment	Compliant	Pass	Pass		No
Vianden	2824	4500	Secondary treatment	Compliant	Pass	Pass		No
Reckange/Mess	2751	3500	Secondary treatment	Compliant	Pass	Pass		No
Troisvierges	2513	2500	Secondary treatment	Compliant	Pass	Pass		No
Biwer/Wecker	2348	3000	Secondary treatment	Compliant	Pass	Pass		No
Bissen	1819	2000	Secondary treatment	Not Compliant	Fail	Pass		Yes
Mertzig	1271	1600	Secondary treatment	Compliant	Pass	Pass		No
Erpeldange/Wiltz	112	300	Secondary treatment	Compliant	Pass	Pass		No
Perl-Besch		23000	More stringent treatment	Compliant	Pass	Pass		No

Table 9-2 (continued) Details on major WWTPS in the Luxemburgish territory (Source: EU Commission urban waste)

9.3 Existing main waste water treatment plants in Luxemburgish territory

In **Table 9-2** we may read a series of recordings which afore mention existing major and smaller in capacity WWTPs. Furthermore we may extract useful sets of data of these WWTPs directly linked with their functioning profiles as well as data presenting the degree of treatment these plants undergo. The range of values concerning the degree of treatment is either attributed of either low stringent and/or very stringent. all major WWTP's are exhibit, their corresponding actual incoming waste flows and design capacities as well as their degree of waste treatment and their compliance according to EU directive and the article 17 are as follows.

9.4 Assumptions made for the examined model

The assumptions made afore mentioning the different parameters of the built model are listed below:

1) Design period of sewage collectors: **t= 40 years;**
2) Population increase in the examined rural /semi rural area, with **a=1.5%**;
3) Development of each community in the examined zone at the end of the designed period assumed to be 50% (rural/semi rural area zone in the examined area) in average according to states urban planning projection ;thus **ω=1.50;**
4) Division of the area in zones as per activity not necessary as we are interested in the provision of waste water of central collector of each community;
5) Mean daily domestic fresh water demand per inhabitant at the end of the design period assumed to be q_E**'=200L/day/inhab**;
6) Mean daily rural and light industrial activities fresh water demand per inhabitant at the end of the design period assumed to be **qE''=10L/day/inhab**;
7) Mean daily public municipality's activities fresh water demand per inhabitant at the end of the design period assumed to be $\mathbf{q_E}$**'''= 25L/day/inh**;
8) Total amount of waste water entering the waste water grid as percentage of fresh water demands, **p=0.80**; Therefore we start off calculations off by multiplying all terms by 188;

   Q_E = 0.80*q_E = 0.80* 235 = 188L/d inhab;
9) Peak daily coefficient $\boldsymbol{\lambda_H}$ = 1.5;

10) The examined network is relatively new thus pipelines are set above existing underground water table; The relationship for infiltration will be multiplied by **α= 1.40** so that infiltration due to rainwater is taken into account towards total amounts of infiltration entering the network;

11) The amount of waste water supplied in the network follows a Gaussian distribution in case the Q (l/s) is considered as the random variable. All above relationships can yield a general purpose model of under the assumption of a Gaussian distribution;

12) **β**: assumed to be the mean value throughout the country thus **β = 1.13= 0.011;**

    where the overall waste water load is calculated as the sum of individual waste water quantities i.e. q_E= q_E' +q_E''+q_E''' = 200+10+25= 235 L/d inhab.

Table 9-3 presents all necessary set of data in order that all parameters regarding waste water flows as well as the defined area where all clusters lie, population projection and different kind of flows within the network as described above in the assumptions set as well as in the previous section which describes the algorithm of calculation of water and waste water demands for each community.

i	Name of cluster (i)	Population (2017)	Population at t=42	Initial Surface A (in km2)	Surface A' (in km2) at t=42	Mean daily waste Q_E produced in (i) (L/s)	Peak daily waste Q_H produced in (i) (L/s)	Peak Coefficient P'	Peak instant waste Q_P produced in (i) (L/S)	Extra unit inflows q_i due to infiltration (L/s km of pipe)	Extra inflows Qi due to infiltration for (i) (L/s)	WWP_i (L/s)	WWP_i ((m3/h)
1	Septfontaines	833	1319	0.19	0.29	2.9	4.3	4.7	13.6	1.0	29.1	42.6	153.5
2	Tuntange	605	958	0.62	0.93	2.1	3.1	5.0	10.5	0.7	66.5	77.0	277.4
3	Merch	9195	14558	3.98	5.97	31.7	47.5	2.9	92.7	0.4	244.5	337.2	1213.9
4	Boevange sur Atert	576	912	0.46	0.69	2.0	3.0	5.1	10.1	0.8	54.0	64.1	230.7
5	Saeul	790	1251	0.13	0.19	2.7	4.1	4.8	13.0	1.1	22.1	35.1	126.2
6	Beckerich	672	1064	0.48	0.72	2.3	3.5	4.9	11.4	0.8	55.5	67.0	241.1
7	Oberpalen	417	660	0.33	0.49	1.4	2.2	5.4	7.8	0.9	42.4	50.2	180.8
8	Noerdange	510	807	0.41	0.62	1.8	2.6	5.2	9.2	0.8	49.8	59.0	212.3
9	Useldange	622	985	0.63	0.95	2.1	3.2	5.0	10.7	0.7	67.3	78.0	280.9
10	Bissen	3,021	4783	1.54	2.31	10.4	15.6	3.7	38.1	0.5	125.8	163.8	589.8
11	Colmar- Berg	2,218	3512	1.39	2.09	7.6	11.5	3.9	29.7	0.6	117.1	146.8	528.5
12	Vichten	1,274	2017	0.79	1.18	4.4	6.6	4.3	19.1	0.7	78.6	97.6	351.4
13	Preizerdaul	1,717	2718	1.58	2.37	5.9	8.9	4.1	24.2	0.5	128.1	152.3	548.2
14	Ospern	261	413	0.59	0.89	0.9	1.3	6.0	5.4	0.7	64.6	69.9	251.8
15	Reichlange	132	209	0.22	0.33	0.5	0.7	6.8	3.1	1.0	31.9	35.0	126.1
16	Schwebach	80	127	0.06	0.09	0.3	0.4	7.6	2.1	1.5	12.7	14.8	53.1
17	Redange sur Atert	2,564	4059	0.95	1.43	8.8	13.2	3.8	33.4	0.6	89.9	123.3	443.8
18	Ell	340	538	0.34	0.51	1.2	1.8	5.7	6.6	0.9	43.9	50.5	181.8
19	Nagem	234	370	0.17	0.26	0.8	1.2	6.1	4.9	1.0	27.2	32.1	115.7
20	Lannen	106	168	0.12	0.18	0.4	0.5	7.1	2.6	1.2	20.8	23.4	84.4
													6191.4

Table 9-3 The total amount of waste water (in m^3/s) running off each community cluster

Where:

WWP_i : The total amount of waste water (in m^3/s);

β: population increase rate in Luxembourg at 2019.

9.5 Description of the examined Model

In the following sub section twenty (20) selected and examined areas of Luxemburgish territorial communities are named as clusters of buildings. These clusters in reality comprise different type of buildings use. These building units present different and mixed uses and include agricultural, commercial, private and occasional light industrial thus tertiary demands in some instances. The main assumption made in our model is that we assume that the majority of the building stock in these clusters, exhibit a similar use of a typical household of 4 users. The reason rural and semi rural area zones were selected is that this assumption simulates real uses as much as possible to above mentioned assumption. The clusters used in our model exhibit similar size and population and lie in rural and semi rural zones communities within a predefined Luxemburgish area territory of 23km X 18 km. The surface areas shown in the following figures depict the initial area of each such community aka cluster in the beginning of a 40 years design period over which all estimated design waste flows calculation are based on. Each of these examined communities as per 2017 the Luxemburgish updated urban plans are shown as follows:

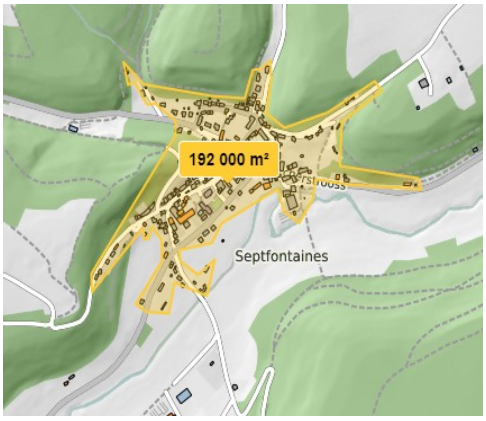


Figure 9-4 Septfontaines: 192,000 m2

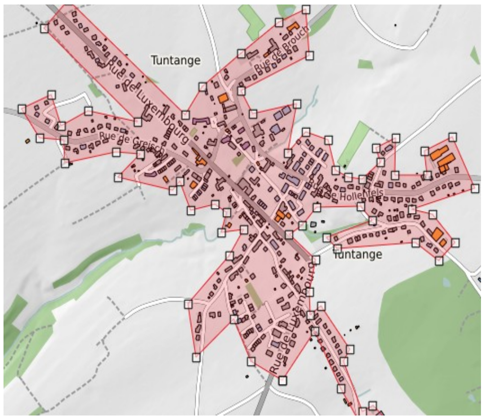


Figure 9-5 Tuntange: 617,000 m2

Figure 9-6 Merch: 3.98 km2

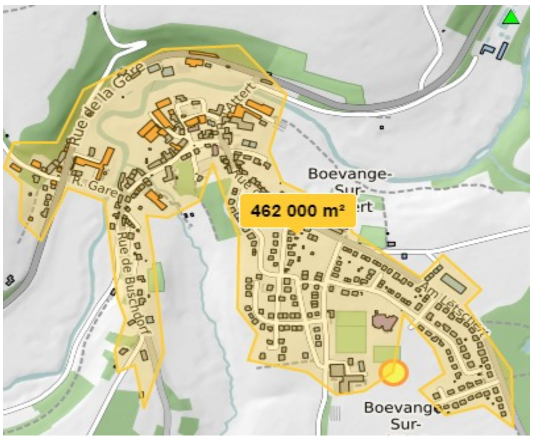


Figure 9-7 Boevange sur Atert: 462,000 m2

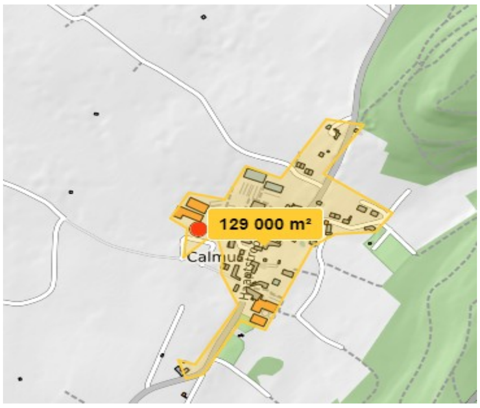


Figure 9-8 Saeul: 129,000

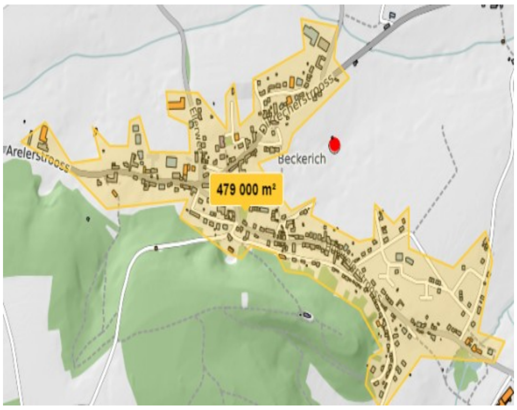


Figure 9-9 Beckerich: 479,000 m

Figure 9-10 Oberpalen: 326,000m2

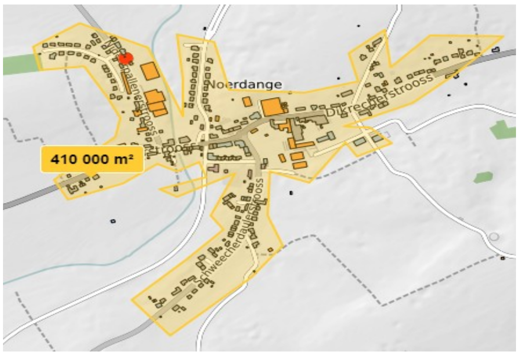


Figure 9-11 Noerdange: 410,000m2

Figure 9-12 Useldange: 630,000 m2

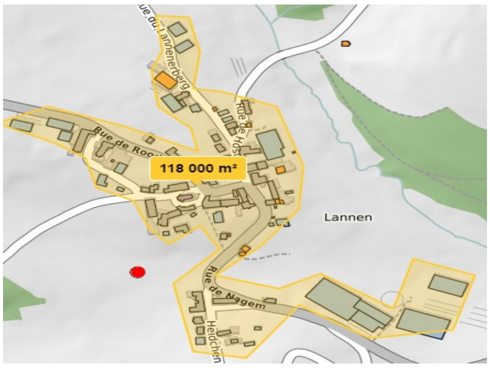


Figure 9-13 Lannen: 118,000m

Figure 9-14 Bissen: 1.54 km2

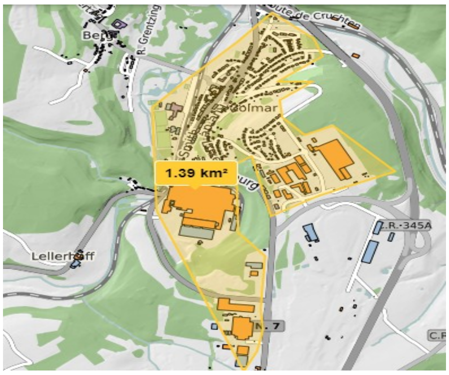


Figure 9-15 Colmar- Berg: 1.39 km2

Figure 9-16 Vichten: 786,000 m2

Figure 9-17 Preizerdaul: 1.58 km2

Figure 9-18 Ospern: 594,000m2

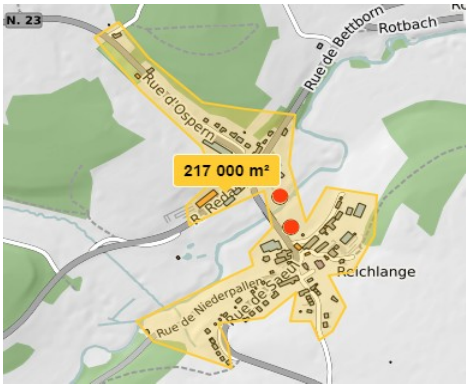


Figure 9-19 Reichlange: 217,000 m2

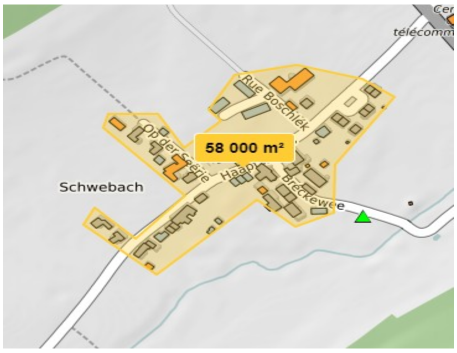


Figure 9-20 Schwebach: 58,000m2

Figure 9-21 Redange sur Attert: 953,000 m2

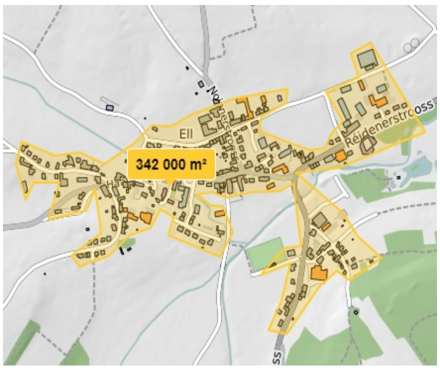


Figure 9-22 Ell: 342,000 m2

Figure 9-23 Nagem: 173,000 m2

9.5.1 Existing connections

In the following **Table 9-4 and 9-4 (continued)** the existing connection matrix of the Waste Water Plants and the clusters of building units of the examined area is represented as binary options i ,e 0 represents the non connection whereas as 1 we represent the existence of connection.

	1	2	3	4	5	6	7	8	9	10	11	12	13
Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange II	Boevange
Name of cluster (i)													
Septfontaines	1	0	0	0	0	0	0	0	0	0	0	0	0
Tuntange	0	1	0	0	0	0	0	0	0	0	0	0	0
Merch	0	1	1	0	0	0	0	0	0	0	0	0	0
Boevange sur Atert	0	0	0	1	1	0	0	0	0	0	0	0	0
Saeul	0	0	0	0	0	1	0	0	0	0	0	0	0
Beckerich	0	0	0	0	0	0	1	1	0	0	0	0	0
Oberpalen	0	0	0	0	0	0	0	0	1	0	0	0	0
Noerdange	0	0	0	0	0	0	0	0	0	1	1	0	0
Useldange	0	0	0	0	1	0	0	0	0	0	0	1	0
Bissen	0	0	0	0	0	0	0	0	0	0	0	0	1
Colmar- Berg	0	0	0	0	0	0	0	0	0	0	0	0	0
Vichten	0	0	0	0	0	0	0	0	0	0	0	0	0
Preizerdaul	0	0	0	0	0	0	0	0	0	0	0	0	0
Ospern	0	0	0	0	0	0	0	0	0	0	0	0	0
Reichlange	0	0	0	0	0	0	0	0	0	0	0	0	0
Schwebach	0	0	0	1	0	0	0	0	0	0	0	0	0
Redange sur Atert	0	0	0	0	0	0	0	0	0	0	0	0	0
Ell	0	0	0	0	0	0	0	0	0	0	0	0	0
Nagem	0	0	0	0	0	0	0	0	0	0	0	0	0
Lannen	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9-4: Existing connections between cluster and WWTP's within examined grid

1 connected; **0, otherwise**

		14	15	16	17	18	19	20	21	22	23	24
j	Name of WWTP	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
i	Name of cluster (i)											
1	Septfontaines	0	0	0	0	0	0	0	0	0	0	0
2	Tuntange	0	0	0	0	0	0	0	0	0	0	0
3	Merch	0	0	0	0	0	0	0	0	0	0	0
4	Boevange sur Atert	0	0	0	0	0	0	0	0	0	0	0
5	Saeul	0	0	0	0	0	0	0	0	0	0	0
6	Beckerich	0	0	0	0	0	0	0	0	0	0	0
7	Oberpalen	0	0	0	0	0	0	0	0	0	0	0
8	Noerdange	0	0	0	0	0	0	0	0	0	0	0
9	Useldange	0	0	0	0	0	0	0	0	0	0	0
10	Bissen	0	0	0	0	0	0	0	0	0	0	0
11	Colmar- Berg	1	0	0	0	0	0	0	0	0	0	0
12	Vichten	0	1	0	0	0	0	0	0	0	0	0
13	Preizerdaul	0	0	1	0	0	0	0	0	0	0	0
14	Ospern	0	0	0	1	0	0	0	0	0	0	0
15	Reichlange	0	0	0	0	1	0	0	0	0	0	0
16	Schwebach	0	0	0	0	0	0	0	0	0	0	0
17	Redange sur Atert	0	0	0	0	0	1	1	0	0	0	0
18	Ell	0	0	0	0	0	0	0	1	1	0	0
19	Nagem	0	0	0	0	0	0	0	0	0	1	0
20	Lannen	0	0	0	0	0	0	0	0	0	0	1

Table 9-4 (Continued) Existing connections between cluster and WWTP’s within examined grid

9.5.2 Existing physical and connection distances of WWTPS's and clusters

In **Table 9-5** and **Table 9-5 (continued**) all relevant physical distances between all existing WWTP's with all clusters are seen. All existing connection distances with pipelines are in orange whereas the Distances Dij 's (in m) are measured from center point of cluster (i) to the center point of each

j	Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
i	Name of cluster (i)												
1	Septfontaines	309	6610	12500	5000	5605	2690	3950	6550	9160	4340	5890	8090
2	Tuntange	2290	442	7270	4180	5565	3650	6170	9010	11900	4080	7210	7420
3	Merch	8280	5750	1460	9780	10950	10800	13400	15900	18800	10300	13500	10900
4	Boevange sur Atert	6570	5460	7150	3100	4680	6600	8320	10300	13100	5190	7650	4220
5	Saeul (Calmus)	6080	6580	9920	2160	2900	200	2630	5400	8390	1650	3770	5380
6	Beckerich	11200	12300	16800	6500	5405	5290	2100	2400	2840	5880	2900	6950
7	Oberpalen	14300	15400	19800	9570	8410	8660	6130	3390	500	9050	5860	9590
8	Noerdange	9500	9860	13700	3550	2320	3260	1840	2730	5700	2800	315	4160
9	Useldange	9008	8210	9520	2780	2815	5100	6170	7810	10400	3460	5240	1940
10	Bissen	10800	8550	4540	8150	9150	10300	12200	14100	16800	9000	11600	8240
11	Colmar- Berg	13600	11100	5410	11200	11900	13500	15100	17100	19700	12100	14500	10800
12	Vichten	12400	10900	9380	6720	6685	9050	10100	11200	13400	7470	8680	4550
13	Preizerdaul	13200	12600	13200	6340	5180	8170	7800	8090	9690	6650	6250	3140
14	Ospern	13500	13100	14900	6380	4975	7520	6520	5750	7060	6450	4410	3820
15	Reichlange	11600	11300	13300	4560	3195	5880	5270	5370	7370	4680	3220	2110
16	Schwebach	7270	7040	10400	82	1330	2360	3910	6150	8910	740	3660	3200
17	Redange sur Atert	13000	13200	16200	6470	5030	6820	5140	3610	4820	6230	3210	5050
18	Ell	14900	15400	18600	8780	7195	8700	6540	4200	3580	8440	5160	7400
19	Nagem	16400	16500	18500	9590	8105	10300	8540	6560	6200	9560	6540	7350
20	Lannen	17500	17700	20000	10800	9365	11400	9460	7350	6410	10700	7680	8630

Table 9-5 Existing distances of WWTPS's and clusters

j	Name of WWTP (j)	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
i	Name of cluster (i)												
1	Septfontaines	9600	12500	9900	9700	9800	8680	8580	6950	11400	8980	12100	13400
2	Tuntange	6910	9240	8500	9410	10100	8720	9650	8040	13800	11400	13200	14700
3	Merch	6740	4950	10300	12500	14400	12900	14900	13800	20100	18000	18200	19500
4	Boevange sur Atert	724	4300	3810	5950	7810	6570	8580	7650	13800	12100	11500	13000
5	Saeul (Calmus)	7340	10600	7230	7000	7210	6070	6400	4780	10200	7850	10000	11400
6	Beckerich	11200	15000	9150	7120	5730	5720	4050	3310	4980	2530	6540	7430
7	Oberpalen	14000	17700	11600	9140	7280	7810	5770	5850	2630	1580	6300	6470
8	Noerdange	8080	11800	6390	4900	4300	3540	3110	1520	7030	4930	6710	8130
9	Useldange	2960	6860	2300	3660	5330	3930	5890	5130	11200	9580	8860	10400
10	Bissen	3000	454	6600	9370	11600	10100	12300	11800	17700	16100	14900	16400
11	Colmar- Berg	6320	2900	9060	11600	13800	12800	14800	14400	20300	18800	17100	18700
12	Vichten	3130	5200	2230	4940	7060	6130	8300	8250	13700	12400	10300	11700
13	Preizerdaul	6280	9440	2520	246	2790	2710	4420	5020	9450	8720	5850	7340
14	Ospern	8320	11900	4890	2240	473	1770	2040	3220	6600	5950	3390	5120
15	Reichlange	6820	10400	3820	1540	1450	160	2200	2310	7600	6270	5060	6540
16	Schwebach	5200	8700	4910	5070	5790	4420	5510	4160	10300	8210	9050	10600
17	Redange sur Atert	9760	13400	6810	4020	2200	3000	106	2000	4630	3550	3100	4460
18	Ell	12100	15800	9030	6360	4260	5210	3160	4340	2400	2770	2400	3010
19	Nagem	11900	15300	8410	5710	3870	5240	3950	5530	4500	5320	400	1360
20	Lannen	13200	16600	9700	7060	5180	6540	5160	6700	4180	5600	1570	214

Table 9-5 (continued) Existing distances(in m) of WWTPS's and clusters

	(j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
Name of WWTP (j)	**Elevation of WWTP (m)**	309	249.73	216.25	255.54	294.655	271.91	273.63	282.39	290.66	261.02	270.58	247.74
Name of cluster (i)	**Elevation of cluster (m)**												
Septfontaines	262	-47	12.27	45.75	6.46	-32.655	-9.91	-11.63	-20.39	-28.66	0.98	-8.58	14.26
Tuntange	314.84	5.84	65.11	98.59	59.3	20.185	42.93	41.21	32.45	24.18	53.82	44.26	67.1
Merch	222.08	-86.92	-27.65	5.83	-33.46	-72.575	-49.83	-51.55	-60.31	-68.58	-38.94	-48.5	-25.66
Boevange sur Atert	263.14	-45.86	13.41	46.89	7.6	-31.515	-8.77	-10.49	-19.25	-27.52	2.12	-7.44	15.4
Saeul	291.78	-17.22	42.05	75.53	36.24	-2.875	19.87	18.15	9.39	1.12	30.76	21.2	44.04
Beckerich	298.61	-10.39	48.88	82.36	43.07	3.955	26.7	24.98	16.22	7.95	37.59	28.03	50.87
Oberpalen	291.6	-17.4	41.87	75.35	36.06	-3.055	19.69	17.97	9.21	0.94	30.58	21.02	43.86
Noerdange	273.08	-35.92	23.35	56.83	17.54	-21.575	1.17	-0.55	-9.31	-17.58	12.06	2.5	25.34
Useldange	240.95	-68.05	-8.78	24.7	-14.59	-53.705	-30.96	-32.68	-41.44	-49.71	-20.07	-29.63	-6.79
Bissen	221.38	-87.62	-28.35	5.13	-34.16	-73.275	-50.53	-52.25	-61.01	-69.28	-39.64	-49.2	-26.36
Colmar- Berg	230.27	-78.73	-19.46	14.02	-25.27	-64.385	-41.64	-43.36	-52.12	-60.39	-30.75	-40.31	-17.47
Vichten	287.46	-21.54	37.73	71.21	31.92	-7.195	15.55	13.83	5.07	-3.2	26.44	16.88	39.72
Preizerdaul	277.05	-31.95	27.32	60.8	21.51	-17.605	5.14	3.42	-5.34	-13.61	16.03	6.47	29.31
Ospern	294.03	-14.97	44.3	77.78	38.49	-0.625	22.12	20.4	11.64	3.37	33.01	23.45	46.29
Reichlange	267.44	-41.56	17.71	51.19	11.9	-27.215	-4.47	-6.19	-14.95	-23.22	6.42	-3.14	19.7
Schwebach	261.44	-47.56	11.71	45.19	5.9	-33.215	-10.47	-12.19	-20.95	-29.22	0.42	-9.14	13.7
Redange sur Atert	291.75	-17.25	42.02	75.5	36.21	-2.905	19.84	18.12	9.36	1.09	30.73	21.17	44.01
Ell	306.21	-2.79	56.48	89.96	50.67	11.555	34.3	32.58	23.82	15.55	45.19	35.63	58.47
Nagem	319.26	10.26	69.53	103.01	63.72	24.605	47.35	45.63	36.87	28.6	58.24	48.68	71.52
Lannen	355.07	46.07	105.34	138.82	99.53	60.415	83.16	81.44	72.68	64.41	94.05	84.49	107.33

Table 9-6 Existing elevation difference between centre of clusters and the centre of WWTP's in the examined model

9.5.3 Existing physical elevation differences ΔH and connection elevation distances

In **Table 9-6 and Table 9-6 (continued)** all involved differences of height (ΔH's) between the WWTP's and the center of each cluster of building users can be seen. All cells in orange represent the existing pipeline connection distances in either a positive or a negative grade. All physical elevation differences are of course positive as seen in matrix.

		Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
j	**Name of WWTP (j)**	238.02	218.77	315.76	261.98	294.28	254.9	266.53	264.13	408	281.36	303.92	340.55
i	**Name of cluster (i)**												
1	Septfontaines	23.98	43.23	-53.76	0.02	-32.28	7.1	-4.53	-2.13	-146	-19.36	-41.92	-78.55
2	Tuntange	76.82	96.07	-0.92	52.86	20.56	59.94	48.31	50.71	-93.16	33.48	10.92	-25.71
3	Merch	-15.94	3.31	-93.68	-39.9	-72.2	-32.82	-44.45	-42.05	-185.92	-59.28	-81.84	-118.47
4	Boevange sur Atert	25.12	44.37	-52.62	1.16	-31.14	8.24	-3.39	-0.99	-144.86	-18.22	-40.78	-77.41
5	Saeul	53.76	73.01	-23.98	29.8	-2.5	36.88	25.25	27.65	-116.22	10.42	-12.14	-48.77
6	Beckerich	60.59	79.84	-17.15	36.63	4.33	43.71	32.08	34.48	-109.39	17.25	-5.31	-41.94
7	Oberpalen	53.58	72.83	-24.16	29.62	-2.68	36.7	25.07	27.47	-116.4	10.24	-12.32	-48.95
8	Noerdange	35.06	54.31	-42.68	11.1	-21.2	18.18	6.55	8.95	-134.92	-8.28	-30.84	-67.47
9	Useldange	2.93	22.18	-74.81	-21.03	-53.33	-13.95	-25.58	-23.18	-167.05	-40.41	-62.97	-99.6
10	Bissen	-16.64	2.61	-94.38	-40.6	-72.9	-33.52	-45.15	-42.75	-186.62	-59.98	-82.54	-119.17
11	Colmar- Berg	-7.75	11.5	-85.49	-31.71	-64.01	-24.63	-36.26	-33.86	-177.73	-51.09	-73.65	-110.28
12	Vichten	49.44	68.69	-28.3	25.48	-6.82	32.56	20.93	23.33	-120.54	6.1	-16.46	-53.09
13	Preizerdaul	39.03	58.28	-38.71	15.07	-17.23	22.15	10.52	12.92	-130.95	-4.31	-26.87	-63.5
14	Ospern	56.01	75.26	-21.73	32.05	-0.25	39.13	27.5	29.9	-113.97	12.67	-9.89	-46.52
15	Reichlange	29.42	48.67	-48.32	5.46	-26.84	12.54	0.91	3.31	-140.56	-13.92	-36.48	-73.11
16	Schwebach	23.42	42.67	-54.32	-0.54	-32.84	6.54	-5.09	-2.69	-146.56	-19.92	-42.48	-79.11
17	Redange sur Atert	53.73	72.98	-24.01	29.77	-2.53	36.85	25.22	27.62	-116.25	10.39	-12.17	-48.8
18	Ell	68.19	87.44	-9.55	44.23	11.93	51.31	39.68	42.08	-101.79	24.85	2.29	-34.34
19	Nagem	81.24	100.49	3.5	57.28	24.98	64.36	52.73	55.13	-88.74	37.9	15.34	-21.29
20	Lannen	117.05	136.3	39.31	93.09	60.79	100.17	88.54	90.94	-52.93	73.71	51.15	14.52

Table 9-6 (continued) Existing elevation difference between centre of clusters and the centre of WWTP's in the examined model

Chapter 10. Computational Results

10.1 Results of the main model adopted

All previously descibed models incorporating the MF and RO systems in the upscale level were coded in C++ applying CPLEX 12.8 (64-bit) optimizer in Visual Studio 2017. The instance was solved in a PC with Window 10 (64-bit), Intel Core i7 and 16Gb RAM. The solution method used was the default setting of CPLEX. In this case CPLEX automatically selects an optimizer (primal simplex, dual simplex, barrier, network, sifting and concurrent optimizers), based on the characteristics of the problem. In our case, CPLEX chose the network optimizer, which is more suitable for network-flow problems. The results were obtained within 8 seconds and were exactly the same as the model without MF and RO systems. This can be easily explained by their large cost compared to the relatively small reduction of the waste water coming out. In **Table 10-1** we may notice the different scenarios of MF,RO systems in a hypothetical percentage of incorporation within each examined community or so called cluster of buildings. It should be noted that this use refers to the residential buildings which correspond to the typical household of the first Part of the study. In the left hand side of the table the reduced amounts of waste water for each cluster are shown whereas in the right hand side all their correspondent percentage reduce of waste water produced is also presented.

0%	25%	50%	100%	Unit	Clusters i	0%	25%	50%	100%
153.5	140.7	127.8	102.0	m3/h	Septfontaines	0%	-8%	-17%	-34%
277.4	250.5	223.6	169.8	m3/h	Tuntange	0%	-10%	-19%	-39%
1213.9	1130.6	1047.3	880.6	m3/h	Merch	0%	-7%	-14%	-27%
230.7	212.2	193.6	156.4	m3/h	Boevange sur Atert	0%	-8%	-16%	-32%
126.2	115.2	104.2	82.1	m3/h	Saeul	0%	-9%	-17%	-35%
241.1	220.3	199.6	158.0	m3/h	Beckerich	0%	-9%	-17%	-34%
180.8	166.7	152.6	124.3	m3/h	Oberpallen	0%	-8%	-16%	-31%
212.3	194.4	176.5	140.7	m3/h	Noerdange	0%	-8%	-17%	-34%
280.9	249.6	218.2	155.5	m3/h	Useldange	0%	-11%	-22%	-45%
589.8	529.1	468.4	346.9	m3/h	Bissen	0%	-10%	-21%	-41%
528.5	476.1	423.7	319.0	m3/h	Colmar- Berg	0%	-10%	-20%	-40%
351.4	325.0	298.5	245.5	m3/h	Vichten	0%	-8%	-15%	-30%
548.2	498.0	447.9	347.5	m3/h	Preizerdaul	0%	-9%	-18%	-37%
251.8	227.6	203.4	155.0	m3/h	Ospern	0%	-10%	-19%	-38%
126.1	113.8	101.6	77.2	m3/h	Reichlange	0%	-10%	-19%	-39%
53.1	49.4	45.7	38.2	m3/h	Schwebach	0%	-7%	-14%	-28%
443.8	413.7	383.7	323.6	m3/h	Redange sur Atert	0%	-7%	-14%	-27%
181.8	167.1	152.4	122.9	m3/h	Ell	0%	-8%	-16%	-32%
115.7	107.6	99.4	83.1	m3/h	Nagem	0%	-7%	-14%	-28%
84.4	79.3	74.2	64.0	m3/h	Lannen	0%	-6%	-12%	-24%

Table 10-1 Scenarios of MF.RO systems integration within clusters

* The percentage of households where MF/RO is used
Reduced waste water values produced corresponding to MF/RO percentage usage

Figure 10-1 depicts the corresponding results of the final waste water load reduction for different MF, and RO extracted from Table 10-1 for each community. This waste is the amount that would be then conveyed to the central Drainage systems of the Network.

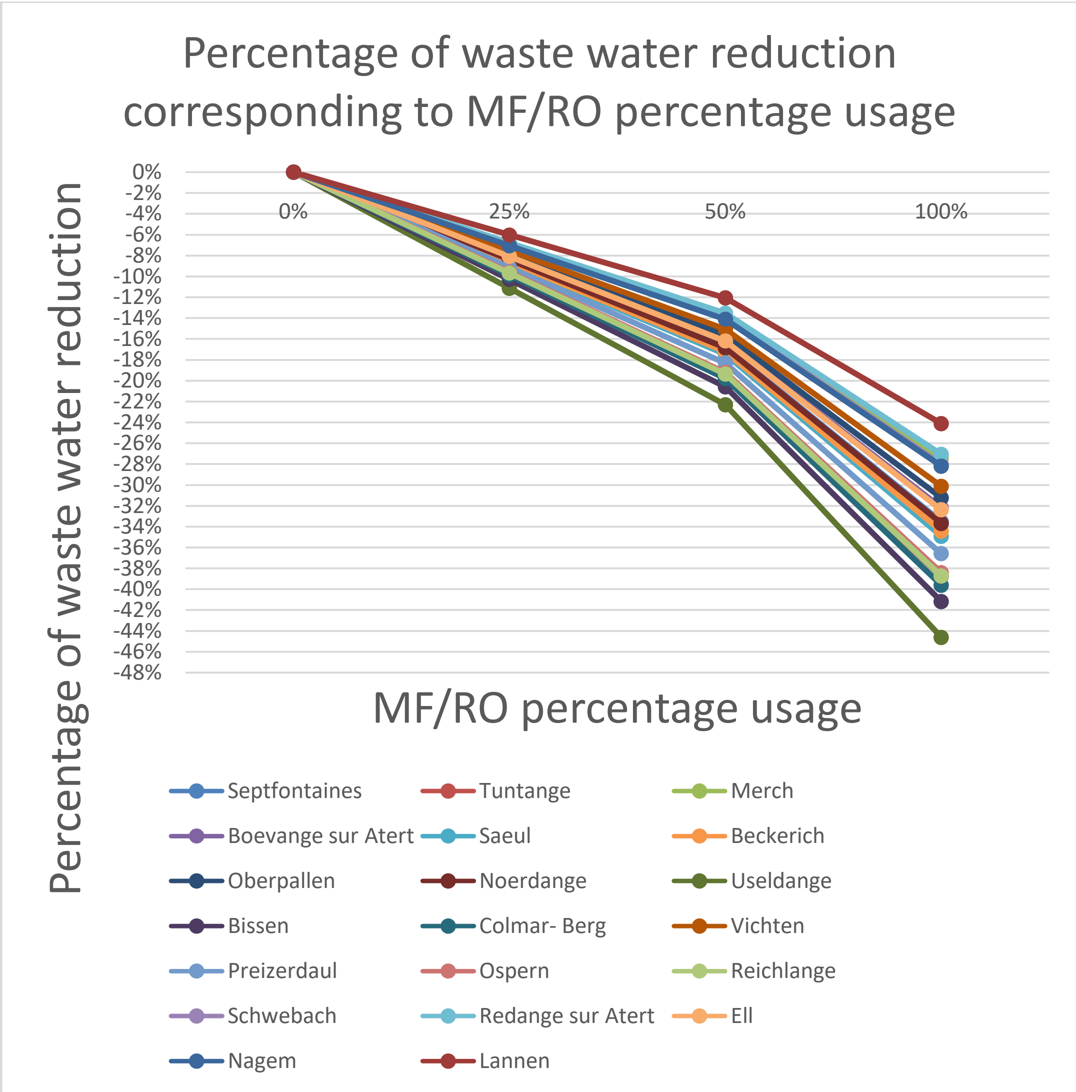


Figure 10-1 Percentage of waste water reduction for each community

In **Table 10-2** all proposed connections between all predefined clusters of buildings within communities of the area under study are presented. These potential connectivity is designated with binary variables. 0 depicted in orange cells within the matrix below represents the solution choice where existing connectivity between clusters is proposed to be abandoned whereas 1 in the deep yellow cells represents the solution choice where present connections are maintained into future grid. The 1's in the green cells represent the solution choice for the new connections required. As mentioned already the present matrix has been extracted as the solution of the newly proposed connection grid in order that the set of objective function and set of restrictions of the initial problem are satisfied.

Name of WWTP (j)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Name of cluster (i)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange II	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
Septfontaines	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tuntange	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Merch	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Boevange sur Atert	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Saeul	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Beckerich	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Oberpalen	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Noerdange	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Useldange	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Bissen	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Colmar- Berg	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Vichten	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Preizerdaul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Ospern	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Reichlange	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Schwebach	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Redange sur Atert	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Ell	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Nagem	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Lannen	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Table 10-2 Proposed future connections between clusters i and WWTPs j

Where:

Present connection maintained

Present connection abandoned

New connection

Name of WWTP (j)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Name of cluster (i)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange II	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
Septfontaines	0	0	0	153.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tuntange	0	0	0	277.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Merch	0	0	0	0	0	0	0	0	0	0	0	0	0	1213.9	0	0	0	0	0	0	0	0	0	0
Boevange sur Atert	0	0	0	230.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Saeul	0	0	0	126.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Beckerich	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	241.1	0	0	0	0	0
Oberpalen	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	180.8	0	0	0	0	0
Noerdange	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	212.3	0	0	0	0	0
Useldange	0	0	0	0	0	0	0	0	0	0	0	0	0	280.9	0	0	0	0	0	0	0	0	0	0
Bissen	0	0	0	0	0	0	0	0	0	0	0	0	0	589.8	0	0	0	0	0	0	0	0	0	0
Colmar- Berg	0	0	0	0	0	0	0	0	0	0	0	0	0	528.5	0	0	0	0	0	0	0	0	0	0
Vichten	0	0	0	0	0	0	0	0	0	0	0	0	0	351.4	0	0	0	0	0	0	0	0	0	0
Preizerdaul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	548.2	0	0	0	0	0
Ospern	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	251.8	0	0	0	0	0
Reichlange	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	126.1	0	0	0	0	0
Schwebach	0	0	0	53.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Redange sur Atert	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	443.8	0	0	0	0	0
Ell	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	181.8	0	0	0	0	0
Nagem	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	115.7	0	0	0	0	0
Lannen	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	84.4	0	0	0	0	0

Present connection maintained
Present connection abandoned
New connection

Table 10-3 Quantity of Waste Water transferred from cluster i to WWTP j (m3/h)

In **Table 10-3** like in previous **Table 10-2** the matrix of choices based on computational analysis is presented. Here the waste water loads corresponding to these choices is shown. In the green cells the quantities corresponding to the newly established connections between end users thus clusters are shown. In deep orange cells corresponding quantities regarding existing connections which are maintained based on the analysis are seen. Like mentioned above all 0's represent all existing connections which should be decided to be abandoned.

Table 10-4 exhibits all corresponding construction costs related to decisions which are extracted by the analysis. These decisions are shown in cells of different colours and either incorporate existing pumping pipelines, new pipelines for the new connections proposed, existing gravitational or newly suggested gravitational ones.

j	Name of WWTP (j)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
i	Name of cluster (i)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange II	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
1	Septfontaines	0 €	0 €	0 €	76,137 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
2	Tuntange	0 €	0 €	0 €	64,091 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
3	Merch	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	75,355 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
4	Boevange sur Atert	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
5	Saeul	0 €	0 €	0 €	33,436 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
6	Beckerich	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	61,902 €	0 €	0 €	0 €	0 €	0 €
7	Oberpalen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	87,958 €	0 €	0 €	0 €	0 €	0 €
8	Noerdange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	47,401 €	0 €	0 €	0 €	0 €	0 €
9	Useldange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	104,509 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
10	Bissen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	7,103 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
11	Colmar- Berg	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
12	Vichten	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	79,581 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
13	Preizerdaul	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	67,347 €	0 €	0 €	0 €	0 €	0 €
14	Ospern	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	31,499 €	0 €	0 €	0 €	0 €	0 €
15	Reichlange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	33,495 €	0 €	0 €	0 €	0 €	0 €
16	Schwebach	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
17	Redange sur Atert	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
18	Ell	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	48,513 €	0 €	0 €	0 €	0 €	0 €
19	Nagem	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	60,563 €	0 €	0 €	0 €	0 €	0 €
20	Lannen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	79,105 €	0 €	0 €	0 €	0 €	0 €

Table 10-4 Construction cost of connections between clusters i and WWTPs j

Existing pumping Pipeline
New pumping pipeline
Existing gravitational Pipeline
New gravitational pipeline

Table 10-5 presents corresponding costs projected to the future with a timeline of 40 years thus these are the estimated costs in 40 years from the proposed situation at t=0 years.

j	Name of WWTP (j)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
i	Name of cluster (i)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange II	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
1	Septfontaines	0 €	0 €	0 €	94,536 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
2	Tuntange	0 €	0 €	0 €	91,984 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
3	Merch	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	94,380 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
4	Boevange sur Atert	0 €	0 €	0 €	88,628 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
5	Saeul	0 €	0 €	0 €	85,704 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
6	Beckerich	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	91,580 €	0 €	0 €	0 €	0 €	0 €
7	Oberpalen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	96,932 €	0 €	0 €	0 €	0 €	0 €
8	Noerdange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	88,656 €	0 €	0 €	0 €	0 €	0 €
9	Useldange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	100,320 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
10	Bissen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	80,396 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
11	Colmar- Berg	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	88,004 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
12	Vichten	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	95,160 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
13	Preizerdaul	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	92,732 €	0 €	0 €	0 €	0 €	0 €
14	Ospern	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	85,328 €	0 €	0 €	0 €	0 €	0 €
15	Reichlange	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	85,828 €	0 €	0 €	0 €	0 €	0 €
16	Schwebach	0 €	0 €	0 €	79,240 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €
17	Redange sur Atert	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	79,312 €	0 €	0 €	0 €	0 €	0 €
18	Ell	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	88,812 €	0 €	0 €	0 €	0 €	0 €
19	Nagem	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	91,272 €	0 €	0 €	0 €	0 €	0 €
20	Lannen	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	0 €	95,032 €	0 €	0 €	0 €	0 €	0 €

Existing pumping Pipeline
New pumping pipeline
Existing gravitational Pipeline
New gravitational pipeline

Table 10-5 Operational and Management Cost of connections between cluster i and WWTP j for the total of 40 years

Table 10-6 presents the correlated expansion costs and the Operational and Maintenance costs with a 40 years future projection based on known QEJ which represents the capacity of the treated waste within each of the examined WWTP. The second cell represents the decision result for use either a mechanical or a biological treatment plant.

j	Name of WWTP	Type_j (p=0=> mechanical, p=1=> biological)	INITIAL Qej (m3/h)	FINAL Qej (m3/h)	Expansion Cost	O & M cost for 40 years
5	Rippweiler I.II	0	0.83	0	0.00 €	0.00 €
6	Calmus	0	1.00	0	0.00 €	0.00 €
8	Beckerich I	0	0.83	0	0.00 €	0.00 €
10	Kapweiler I,II	0	0.42	0	0.00 €	0.00 €
11	Noerdange	0	2.50	0	0.00 €	0.00 €
12	Everlange I, II	0	1.67	0	0.00 €	0.00 €
15	Schandel	0	1.50	0	0.00 €	0.00 €
16	Platen	0	5.00	0	0.00 €	0.00 €
17	Ospern	0	2.17	0	0.00 €	0.00 €
18	Reichlange	0	1.88	0	0.00 €	0.00 €
20	Niederpalen	0	1.67	0	0.00 €	0.00 €
22	Levelange	0	0.42	0	0.00 €	0.00 €
23	Nagem	0	0.83	0	0.00 €	0.00 €
24	Lannen	0	0.83	0	0.00 €	0.00 €
1	Dondelange	1	29.17	0	0.00 €	0.00 €
2	Hollenfels	1	7.08	0	0.00 €	0.00 €
3	Merch	1	583.33	0	0.00 €	0.00 €
4	Schwebach	1	2.08	840.9	221,124.09 €	134,304.78 €
7	Schweich	1	6.25	0	0.00 €	0.00 €
9	Oberpallen	1	12.50	0	0.00 €	0.00 €
13	Boevange	1	125.00	0	0.00 €	0.00 €
14	Bissen	1	16.67	2964.5	540,944.98 €	209,501.51 €
19	Redange	1	16.67	2386	463,674.89 €	194,181.26 €
21	Colpach-Bas	1	16.67	0	0.00 €	0.00 €
SUM			836.96	6191.4	1,225,743.96 €	537,987.54 €

Table 10-6 Current and potential capacity of each WWTP

j	Name of WWTP	Type_j(p=0 => mechanical, p=1=> biological)	X	Y	INITIAL Qej (m3/h)	FINAL Qej (m3/h)	C5 (EXPANSION COST taken by linear equation)	C6 (O & M COST 40 YEARS taken by non linear equation)	C5 (EXPANSION COSTof the model with piecewise linearization)	C6 (O & M COST 40 YEARS of the model with piecewise linearization)	Difference of piecewise linearization compared to non linear equation for the cost C5	Difference of piecewise linearization compared to non linear equation for the cost C6
5	Rippweiler I.II	0	2.1	4.7	0.83	0	0.0	0.0	0.0	0.0	0.0	0.0
6	Calmus	0	2.2	2.6	1.00	0	0.0	0.0	0.0	0.0	0.0	0.0
8	Beckerich I	0	1.1	3.15	0.83	0	0.0	0.0	0.0	0.0	0.0	0.0
10	Kapweiler I,II	0	2.3	3.8	0.42	0	0.0	0.0	0.0	0.0	0.0	0.0
11	Noerdange	0	1.6	4.15	2.50	0	0.0	0.0	0.0	0.0	0.0	0.0
12	Everlange I, II	0	2.15	6.5	1.67	0	0.0	0.0	0.0	0.0	0.0	0.0
15	Schandel	0	2.4	7.9	1.50	0	0.0	0.0	0.0	0.0	0.0	0.0
16	Platen	0	1.82	7.5	5.00	0	0.0	0.0	0.0	0.0	0.0	0.0
17	Ospern	0	1.4	7	2.17	0	0.0	0.0	0.0	0.0	0.0	0.0
18	Reichlange	0	1.7	6.6	1.88	0	0.0	0.0	0.0	0.0	0.0	0.0
20	Niederpalen	0	1.5	4.9	1.67	0	0.0	0.0	0.0	0.0	0.0	0.0
22	Levelange	0	0.6	3.8	0.42	0	0.0	0.0	0.0	0.0	0.0	0.0
23	Nagem	0	0.65	7.5	0.83	0	0.0	0.0	0.0	0.0	0.0	0.0
24	Lannen	0	0.3	7.7	0.83	0	0.0	0.0	0.0	0.0	0.0	0.0
1	Dondelange	1	3.35	0.1	29.17	0	0.0	0.0	0.0	0.0	0.0	0.0
2	Hollenfels	1	3.65	1.6	7.08	0	0.0	0.0	0.0	0.0	0.0	0.0
3	Merch	1	4.5	6	583.33	0	0.0	0.0	0.0	0.0	0.0	0.0
4	Schwebach	1	2.4	4.15	2.08	840.9	221,124.1	134,304.8	218,741.0	132,802.0	-2,383.1	-1,502.8
7	Schweich	1	1.7	2.6	6.25	0	0.0	0.0	0.0	0.0	0.0	0.0
9	Oberpallen	1	0.43	3.1	12.50	0	0.0	0.0	0.0	0.0	0.0	0.0
13	Boevange	1	3.2	7	125.00	0	0.0	0.0	0.0	0.0	0.0	0.0
14	Bissen	1	3.9	8	16.67	2,964.5	540,945.0	209501.5	527,539.0	204,316.0	-13,406.0	-5,185.5
19	Redange	1	1.3	5.9	16.67	2,386	463,674.9	194181.3	455,233.0	190,955.0	-8,441.9	-3,226.3
21	Colpach-Bas	1	0.15	4.8	16.67	0	0.0	0.0	0.0	0.0	0.0	0.0
SUM					836.96	6,191.4	1,225,744.0	537,987.5	1201513.0	528,073.0	-24,231.0	-9,914.5

Table 10-7 WWTP's treatment capacities and Model Costs breakdown in EUR

Table 10-7 presents all data that represent all waste load capacities in the proposed optimized situation as well as the corresponding C5 and C6 costs regarding overall costs calculated with the use of non linear and the onew estimated with the use of piecewise linearization for the expansion and O&M costs respectively. We may also see the difference in the cost estimates from the two methods used.

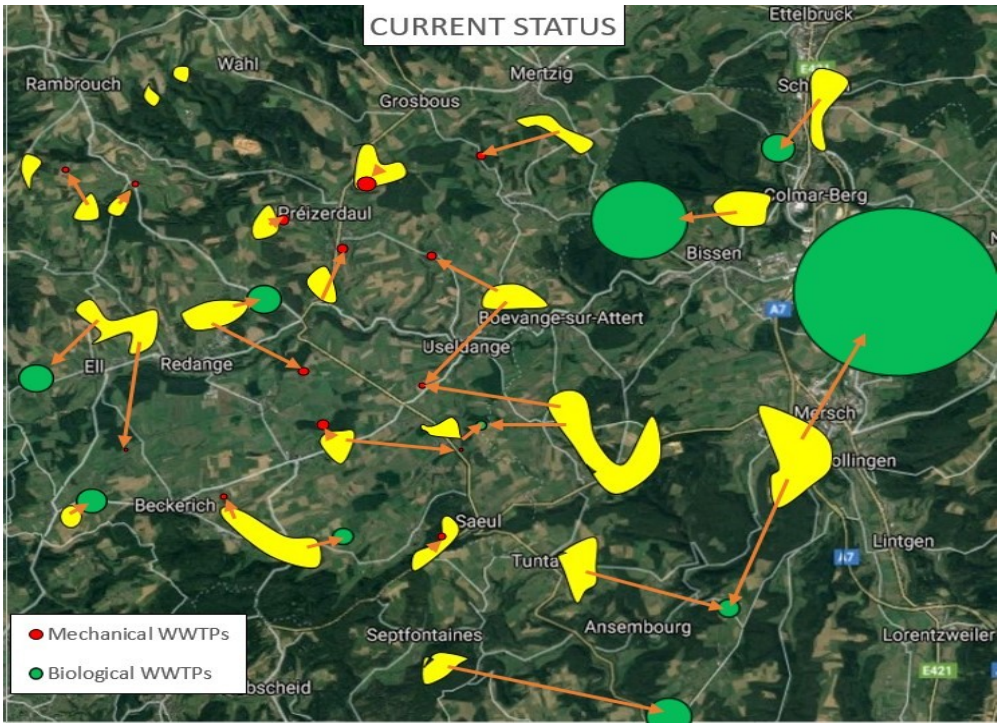


Figure 10-2
Current status of Connection grid of clusters with WWTP's in the examinedarea

Figure 10-2 depicts the examined area before the model optimisation and the implementation of the proposed methods. The small or medium sized communities are painted in yellow and their shape follows the outline of each community according to surveying maps. These are called clusters and are connected to either biological WWTP's being represented by the small green circles or mechanical WWTP's represented by the corresponding small red circles. It can be seen that in the existing regime all communities are connected to very small or small sized plants and only one community is connected to a central biological plant.

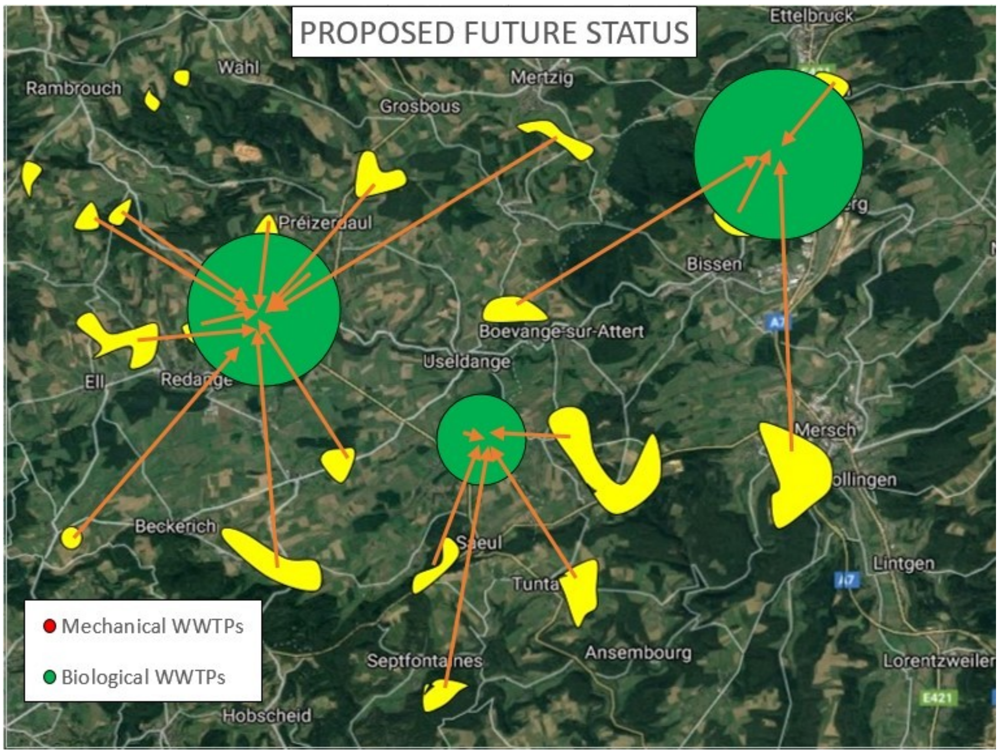


Figure 10-3
Future status of Connection grid of clusters with remaining central WWTP's (optimized).

In Figure 10-3 the future situation of the same examined area is optimised. This new connection regime suggestion is extracted based on the analysis of the mathematical model. Here it can be seen that the analysis itrepres as the most favorable thus optimized situation the grouping of neighbouring clusters-communities to be served by central treatment plants of a much larger capacity. The sizing of the green bubbles representing the future treatment plants is proportionaly to their size. The mechanical in red and biological in green represent the WWTP's of the area before the analysis and these were put I the same map intensively for the reader to be able to appreciate the difference of these two regimes.

10.2 Summarised further comments on the attained proper computational convergence

As already known, the Non-Linear Original Problem (NLOP) of the Waste Water Treatment Network Design (WWTND) problem cannot be solved by the existing commercial solvers, due to the non-linear terms in the objective function.

For this reason, the NLOP was linearized resulting in the Linearized Original Problem (LOP), which was solved by using CPLEX solver with code written in C++. In **Figure 10-4** there is a print screen of the respective code running (see APPENDIX B-Symbols and Programming Codes-Part II).

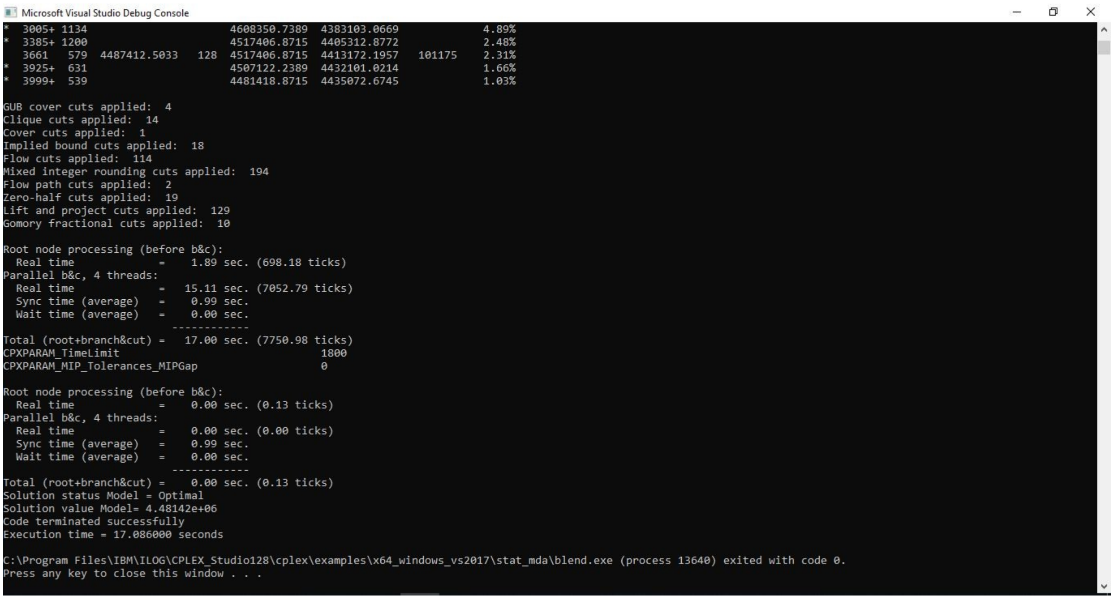


Figure 10-4 Print screen of the code running solving LOP without MF & RO systems by using CPLEX.

Moreover, the Linearized Original Problem (LOP) was enriched with extra variables and constraints in order to consider MF and RO systems. This enriched model was again solved by using CPLEX solver with code written in C++. In **Figure** Figure there is a print screen of the respective code running (see APPENDIX B-Symbols and Programming Codes-Part II).

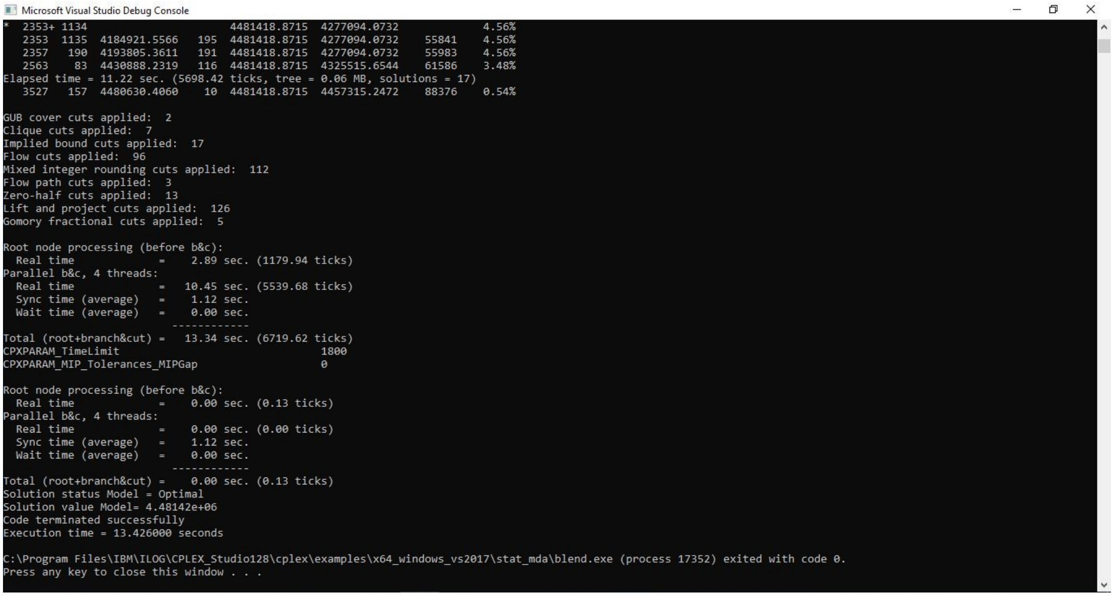


Figure 10-5 Print screen of the code running solving LOP with MF & RO systems by using CPLEX.

In order to further expand the study, the application of Benders Decomposition method was considered. This method is very frequently applied on network design problems.

In the original version of the PhD thesis, Benders Decomposition method was applied using the idea "**First Linearize, then Decompose**". The method can be denoted as "**BD_1**" and is described as follows:

1) First, the Original Non-Linear Problem (ONLP) was linearized by using the Piecewise Linearization method;
2) Then, the Original Linearized Problem (OLP) was decomposed by using Benders decomposition method into a Master Problem (MP1) and a Primal Subproblem (PSP1). For the decomposition, the integer variables of the OLP were considered as the complicating ones. Thus, the MP1 was a Pure Integer Linear Problem (PILP) and the PSP1 was a Continuous Linear Problem (CLP);
3) No acceleration method was applied;
4) The Benders algorithm applied between MP1 and PSP1 was not able to converge to an optimal solution even after 10hours running, mainly due to the fact that the initial MP1 contained only a few number of variables and constraints. Thus, the initial Lower Bound of the algorithm was too small and needed an important number of Benders Feasibility Cuts to be generated. As a result, the increased number of Benders cuts added to MP1 gradually made the MP1 a very "heavy" model to be solved after some iterations.

In the revised version of the PhD thesis, Benders Decomposition method was applied using the idea "**First Decompose, then Linearize**".The method can be denoted as "**BD_2**" and is described as follows:

1) First, the ONLP is decomposed by using the Benders Decomposition methodinto a Non-Linear Master Problem (NLMP2) and a Linear Primal Subproblem (LPSP2). For the decomposition, both the integer and the non-linear continuous variables of the ONLP are considered as complicating ones. Thus, the derived NLMP2 is a Mixed Integer Non Linear Problem (MINLP) and LPSP2 is a Continuous Linear Problem (CLP);

2) Then, the NLMP2 was linearized by using the Piecewise Linearization method and the Linearized Master Problem (LMP2) was derived, which is a Mixed Integer Linear Problem (MILP);

3) The Benders algorithm applied between LMP2and LPSP2 converged in approximately **17 minutes**, after 39 iterations, resulting in the same optimal solution as CPLEX when solving the OLP. This fact leads to the conclusion that the method is valid and correctly applied. In Figure 10-6**Σφάλμα! Το αρχείο προέλευσης της αναφοράς δεν βρέθηκε.** the total CPU time of the method BD_2 is displayed. Also, this can be seen in **Figure 10-7**, where there is a print screen of the respective code running (see APPENDIX B-Symbols and Programming Codes-Part II).

Thus, compared with the approach BD_1, which could not converge into an optimal solution, the newly developed approach BD_2 converged into an optimal solution in approximately 17 minutes. It should be noted that BD_2 is not an acceleration of BD_1, but a completely different approach of applying Benders Decomposition in order to solve the NLOP of WWTND problem.

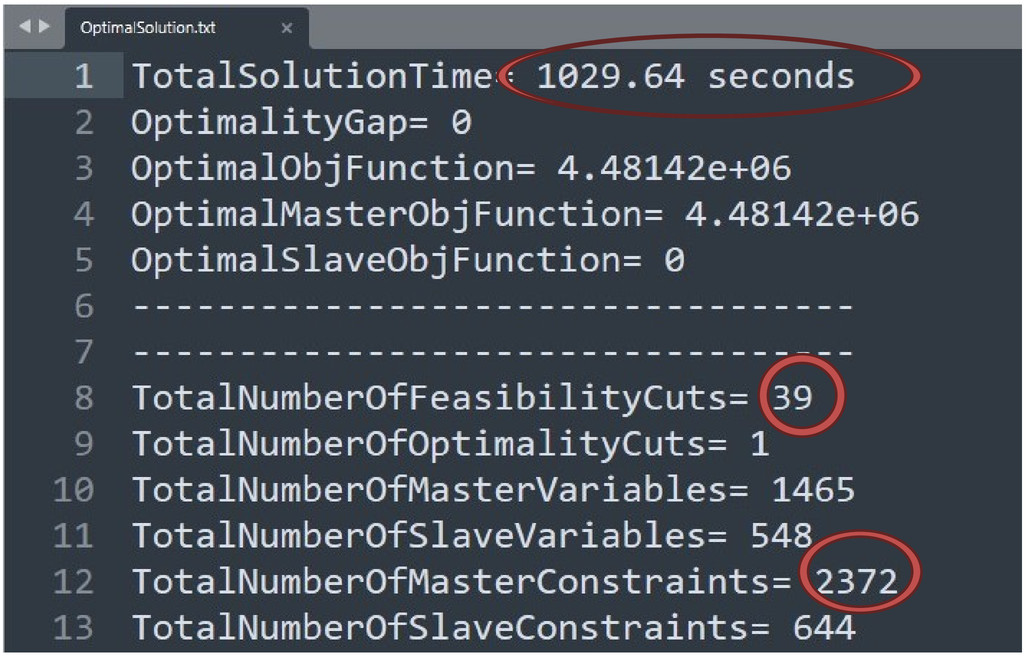


Figure 10-6 Solution time of the method BD_2 (*without Valid Inequalities*)

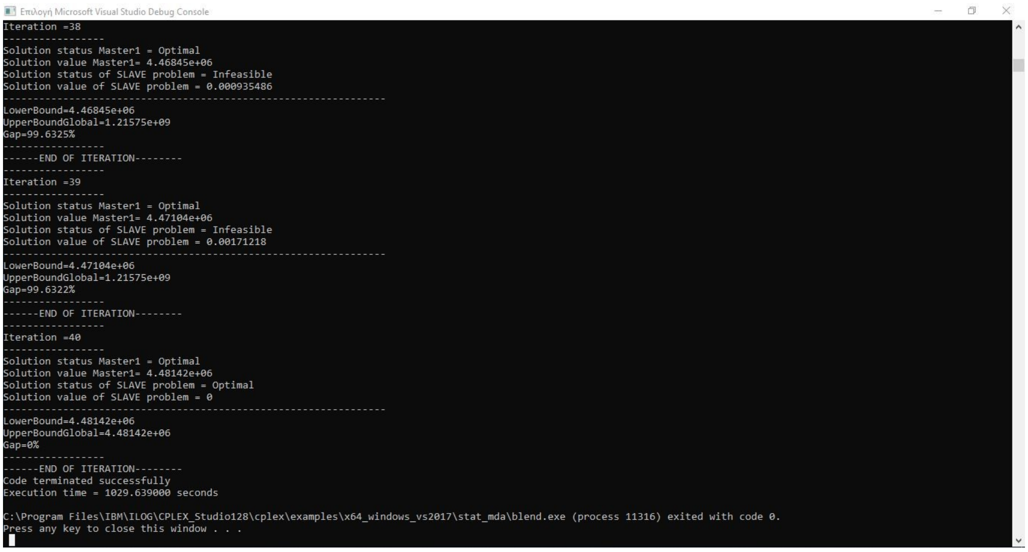
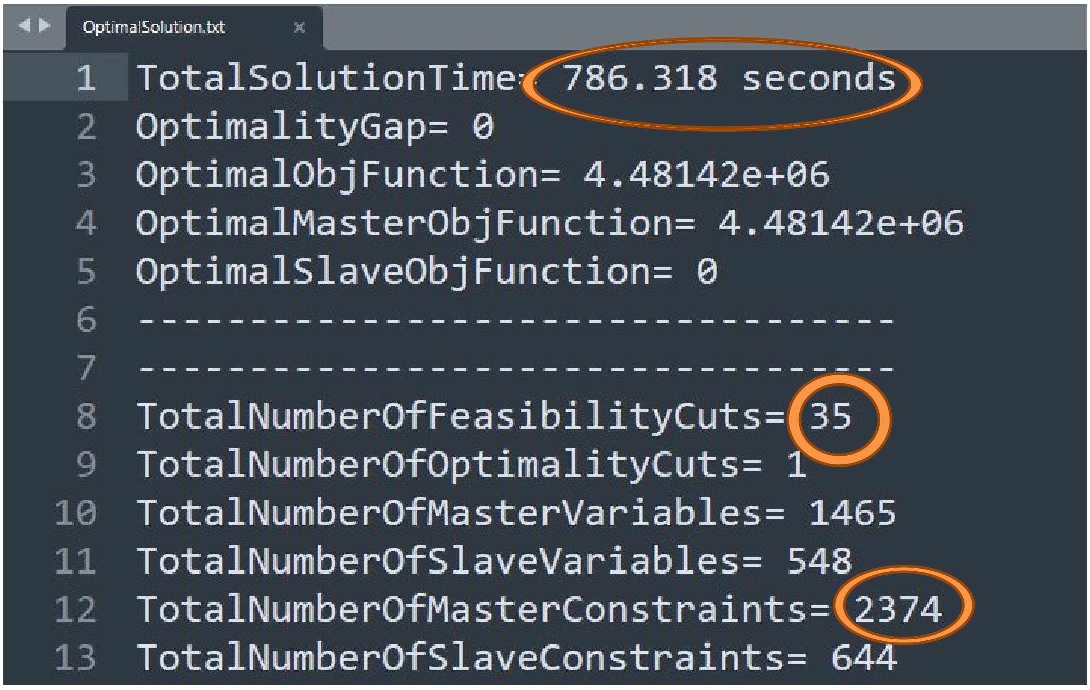


Figure 10-7 Print screen of the code running with the method BD_2.

However, the CPU time of BD_2 is considered quite large and for this reason an acceleration method is applied. Specifically, extra constraints, known as "**Valid Inequalities**" were added to the LMP2 in order to initialize it and result in better initial lower bounds of the Benders algorithm.

The idea is based on the (Saharidis, Boile and Theofanis 2011), where valid inequalities are introduced for the application of Benders Decomposition on fixed-charge network problems. The valid inequalities significantlyrestrict the solution space of the Benders master problem from the first iteration of the algorithmleading to improved convergence. This proposed method can be denoted as “**BD_2_VI**” and, as shown in **Figure 10-8** , it reaches optimal solution in only **13 minutes**, thus achieving a reduction in CPU time of 26.2% compared to BD_2. Finally, there is a reduction of about 10% in total iterations needed. (35 iterations with BD_2_VI vs 39 iterations with BD_2). Also, aprint screen of the respective code running can be seen in **Figure 10-9** (see APPENDIX B-Symbols and Programming Codes-Part II).

Thus, in the implemented updated method approach, the author managed to develop a converging version of Benders Decomposition (BD_2), also had it accelerated via the use of valid inequalities (BD_2_VI). It should be reminded that the BD_2_VI is an acceleration of the BD_2 approach.


OptimalSolution.txt

```
TotalSolutionTime= 786.318 seconds
OptimalityGap= 0
OptimalObjFunction= 4.48142e+06
OptimalMasterObjFunction= 4.48142e+06
OptimalSlaveObjFunction= 0
-----------------------------------
-----------------------------------
TotalNumberOfFeasibilityCuts= 35
TotalNumberOfOptimalityCuts= 1
TotalNumberOfMasterVariables= 1465
TotalNumberOfSlaveVariables= 548
TotalNumberOfMasterConstraints= 2374
TotalNumberOfSlaveConstraints= 644
```

Figure 10-8 Solution time of the method BD_2_VI (without Valid Inequalities)

Figure 10-9 Print screen of the code running with the method BD_2_VI

In all instances the same optimal solution was derived, which leads to the assumption that all developed formulations and applied methods are valid and coherent with each other.

In

Table the number of variables (discrete and continuous ones) and constraints of the different formulations introduced and solution methods applied in this thesis are depicted. Also, their computational results together with the CPU time and iterations needed to be solved are displayed.

Formulation-Method	Discrete Variables	Continuous Variables	Total Variables	Total Constraints	Optimal Solution	Optimality Gap	CPU Time (sec)	Iterations
Non-Linear Original Problem	504	528	1032	1078	-	100%	-	-
Linearized Original Problem (solved with CPLEX)	984	1008	1992	3356	4481420	0%	17.1	-
Linearized Original Problem with MF and RO (solved with CPLEX)	1184	1232	2416	4336	4481420	0%	13.4	-
Benders Decomposition on Linearized Original Model (BD_1)	984	1009	1993	3016	-	100%	-	-
Benders Decomposition on Non-Linear Original Model without Valid Inequalities (BD_2)	984	1009	1993	3016	4481420	0%	1029.6	39
Benders Decomposition on Non-Linear Original Model with Valid Inequalities (BD_2_VI)	984	1009	1993	3018	4481420	0%	786.3	35

Table 10-8: Model Size and Computational Results of introduced formulations-applied methods

One can notice that the Non-Linear Original Problem is the smallest one in terms of variables and constraints. However, due to its non-linearity, it cannot be solved straightforward by an optimization solver like CPLEX. For this reason, it is linearized by the use of piecewise linearization, which leads to additional variables and constraints. It can be mentioned that this larger model is easy to be solved by CPLEX in less than 20 seconds. The application of Benders Decomposition is examined in order to be able to solve large instances. The first variant of the proposed decomposition (BD_1) was based on the idea **"First Linearize, Then Decompose"** was not able to converge to an optimal solution even after 10 hours running, mainly due to the fact that the initial MP1 contained only a few number of variables and constraints. Thus, the initial Lower Bound of the algorithm was too small and needed an important number of Benders Feasibility Cuts to be generated. As a result, the increased number of Benders cuts added to MP1 gradually made the MP1 a very "heavy" model to be solved after some iterations.

This led the author to develop the second variant (BD_2) of applying Benders Decomposition, based on the idea **"First Decompose, Then Linearize"**. The Benders algorithm applied between LMP2 and LPSP2 converged in about 18 minutes (1087.19 sec), after 39 iterations, resulting in the same optimal solution as CPLEX when solving the OLP. This fact leads to the conclusion that the method is valid and correctly applied. In **Table10-8** the total CPU time of the method BD_1 is displayed. The CPU time denotes that CPLEX is 98.5% faster than BD_2. However, compared to BD_1, BD_2 managed to finally solve the example in a reasonable amount of time.

This poor result led the author to apply an acceleration method on the second variant with the introduction of Valid Inequalities into the Master Problem (BD_2_VI). This proposed method reaches optimal solution in only 13 minutes (802.41) and after 35 iterations. The CPU time denotes that again CPLEX is 97.9% faster than BD_2_VI. However, BD_2_VI outperforms BD_2 by reducing the CPU time by 26.2% and the iterations needed to reach the optimal solution by 10.2%.

The fact that throughout all different approaches implemented, both with CPLEX and the Benders Decomposition method , lead to the same result is expected as explained as follows:

1. All models in the different approaches are derived from the same model of Non-Linear Original Problem (NLOP) for the Waste Water Treatment Network Design (WWTND) problem;

2. In fact, in BD_2 method, a transformation of the NLOP has been made and it has been decomposed into 2 mathematical models (Master and Slave);

3. In BD_2_VI accelerated method, apart from the transformation and the decomposition, 2 constraints are added, which are Valid Inequalities. This means that they eliminate part of the feasible region of the Linearized Master Problem 2 (LMP2), without eliminating any feasible integer solutions. Thus, the optimal integer solution derived with the rest of the approaches is not excluded by BD_2_VI and is the one that is chosen in this approach as well. Generally, the Valid Inequalities are not known and it requires skill in order to define them in every different mathematical model;

4. Moreover, in all solvable methods, the optimality gap is 0%. This means that the globally optimal solution is found. For example, if an approach could not converge to the globally optimal solution, then it would reach a suboptimal solution, which might be different than the globally optimal one. Then the optimality gap would be strictly greater than 0% (e.g. 25%);

5. In all solvable methods, apart from the one with MF & RO systems (solved by CPLEX), the same dataset is solved. In the model with MF & RO systems (solved by CPLEX), it seems that their installation cost is higher than the rest of the network cost, as regards the amount of Waste Water reduced by them. In other words, the optimal solution "prefers" to invest money on the rest of the network (plants, pipeline) in order to treat more Waste Water instead ofreducing only a small portion of Waste Water by spending money on MF & RO, because this technique results into the minimum total cost.

Chapter 11. Conclusions and future research

The optimal solution of the model incorporates biological rather than mechanical plants due to the reason of the increased cost; Large WWTP's are preferred instead of many of smaller size due to the non linearity of the expansion and operation cost. More specifically the value of q within the corresponding third term of the corresponding equation, referring to above mentioned costs, is smaller than unity which implies that piece wise linearization was performed with an acceptable degree of error in relation to the actual non linearity.

Central larger WTTP's situated at a lower topographical height are taken so that their connection with the corresponding clusters with the use of gravitational pipelines is possible as much as possible. These gravitational instead of pumping connections are preferred by the model due to their lower cost. In general the model captures the actual regime taking into account mostly the key parameters and assumptions that have been predefined before processing.

The solution for the above small network was achieved in less than 20 seconds. This was attained with the use of a direct solution of the linearized Original Problem (LOP). However, for larger and more extended grids, the direct solution of the LOP might not be able to be solved. For this reason, the author applied the Benders Decomposition algorithm within the context of the previously described variants for the Waste Water Treatment Network Design problem. None of the variants could outperform CPLEX, however one of these managed to reach optimal solution in a reasonable amount of time. The author applied the acceleration method of Valid inequalities on the solvable variant and further improved its solution time. Again, code was developed in C++ with CPLEX 12.8. The runs were made using the same hardware, but the algorithm would not converge to a solution even after 10 hours. At this stage there were no interim solutions. Even after 10 hours no feasible solution was obtained and only feasibility cuts were generated by the Benders algorithm.For this reason, it is assumed that Benders Decomposition might not be the suitable approach for solving the linearized Original Problem (LOP) mainly due to the high complexity of the piecewise linearization variables and constraints.

For future work, the author might examine either to implement the Generalized Benders Decomposition (GBD) directly to the Non-Linear Original Problem (NLOP) as well as to further accelerate the Benders Decomposition Algorithm (BDA) applied within the solvable proposed variant for the entire Luxemburgish grid system. For this reason, accelerating methods such as Covering Cut Bundle (CCB) (Saharidis, et al., 2010), Maximum Feasible Subsystem (MFS) (Saharidis & Ierapetriou, 2010) or Maximum Density Cut (MDC) (Saharidis & Ierapetriou, 2013), either individually or combined, will be implemented in order to reduce the Benders iterations and the CPU time needed to achieve convergence of the algorithm. Thus, larger instances of the Waste Water Treatment Network Design (WWTND) problem could be solved.

This approach might lead to quicker and more accurate solutions, as the approximation error of the piecewise linearization will be avoided. Generalized Benders Decomposition (GBD), developed by (Geoffrion, 1972), is a procedure to solve certain types of Non-Linear Problems (NLP) and Mixed-Integer Non-Linear Problems. The use of this procedure has been recently suggested as a tool for solving process design problems. While analyzing the solution of non convex problems through different implementations of the GBD, it is demonstrated that in certain cases only local minima may be found, whereas in other cases not even convergence to local optima can be achieved.

In addition to the above, minimizing the of ground water flows while satisfying the maximum cost constraints of the treatment will be attempted for the entire Luxemburgish territory including the urban part (plant capital cost + flow operating costs + domestic regeneration cost).

Thus depending on the location of the treatment plants we can have different supplies of surface and ground water.

Furthermore the utilized MF and RO waste water treatment systems adopted in the typical household unit of the first Part as seen in Part I of this thesis are not linearly dependent to $\mathbf{R}_{\mathbf{MF,j}}$and $\mathbf{R}_{\mathbf{RO,j}}$ and also their treatment capacity. The reason is that they both follow the economies of scale. These systems may be either installed for each single household unit or even a cluster of neighboring households or even a cluster of buildings of mixed use and demands. In this case there is a possibility their location is optimized while minimizing their capital and operational maintenance cost. The idea is similar to optimal location of plants but only inter-cluster water networks would need to be considered at this scale.

Another way is to examine the reduce of the surface water demands from the treatment plants by installing a larger number of MF and RO systems thus a more dense grid of these which in turn are linked to some cost Therefore the problem is to find an optimal solution between different network plant-**MF/RO** scenarios, in order to reduce the overall fresh water supply to treatment plants. Finally, if the GBD cannot be implemented on the on-linearized Original Problem (NLOP) or its results are not satisfactory, the authors will examine accelerating the Classical Benders Decomposition applied on the LOP. For this reason, accelerating methods such as Covering Cut Bundle (CCB) (Saharidis, et al., 2010), Maximum Feasible Subsystem (MFS) (Saharidis & Ierapetriou, 2010) or Maximum Density Cut (MDC)(Saharidis & Ierapetriou, 2013), either individually or combined, will be implemented in order to reduce the Benders iterations and the CPU time needed to achieve convergence of the algorithm. Thus, larger scale versions or combinations of scenarios within the Waste Water Treatment Network Design (WWTND) problem could be solved. All above-mentioned problems are combinatorial and can be solved with the use of CPLEX. A brief description of the indicative selected area zones this time within the urban context thus the Luxembourg City on which simulation will be attempted in the context of future research are shown as follows:

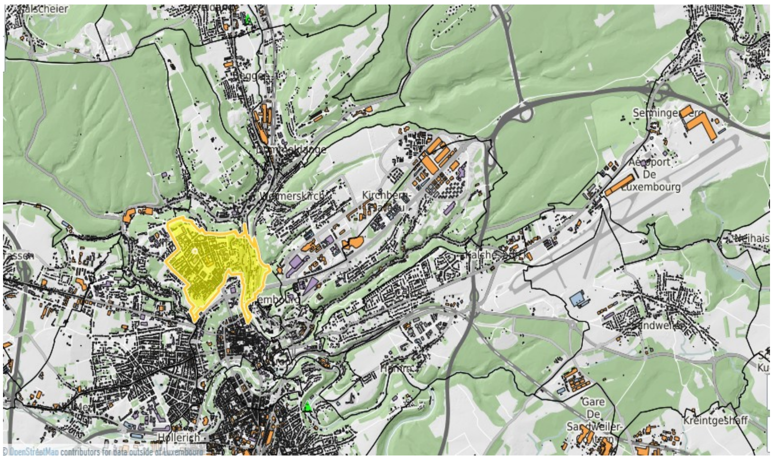


Figure 11-1 District of Limpertsberg

This is a district, depicted in **Figure 11-1**, in the north-western part of the Luxembourg City, at the centre. In the south, on the border with the main city lies the 5,000 m^{2}Glacis open air car parking. To the south east, lies the Bridge called 'Grand Duchess Charlotte' which connects Limpertsberg to the district of Kirchberg over the 'Petrusse' river. The attribute of the building stock is mainly comprised of blocks of private apartments, private stores, premises of a larger magnitude, a few medium in size supermarket units of medium capacity and a couple of medium sized school complexes.

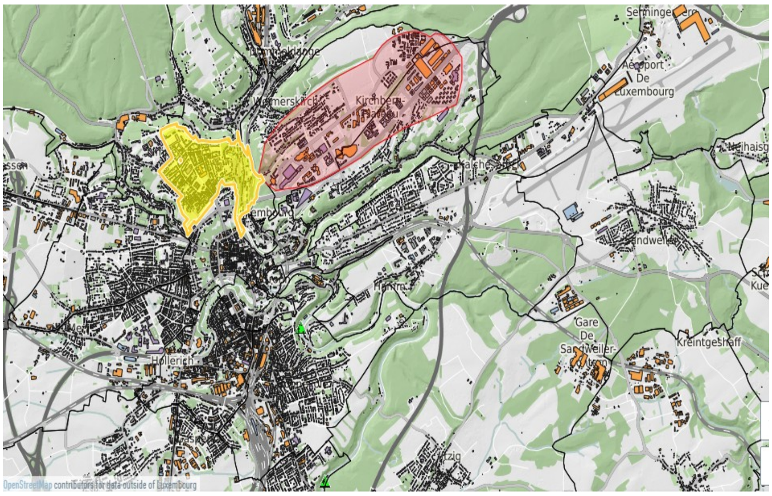


Figure 11-2 District of Kirchberg

This is a district depicted in within red area zone in **Figure 11-2** which lies in a level region called 'plateau' that is a 100,000 acres open space region in the vicinity of Limpertsberg district, comprising combined building stock profile. The rapid demographic change in Kirchberg has consisted mainly in a change from cropland into a built area. As a direct consequence of these considerable changes in land use, the region of Kirchberg has undergone significant modifications of both its effluent streams of waste water falling to the 'Petrusse' river basin and consequently the degree of contamination in its surface water .The half of this region has been a construction site in the last 15 years where 100% of the overall building stock constructed, comprise mainly premises, 2 sport malls, The European Community School , Luxembourg's main University Campus and a main market mall called 'Auchan'. This newly built region lies along the 10 km long 'Konrad Adenauer' Avenue and is built within a 500 m range in width all along the main Avenue. The other half of the region of Kirchberg, is the older part which already exists comprised of households which involve mainly private mansions and an approximately 10000 m^2 space that is entirely built in block of flats of 40 metres high each.

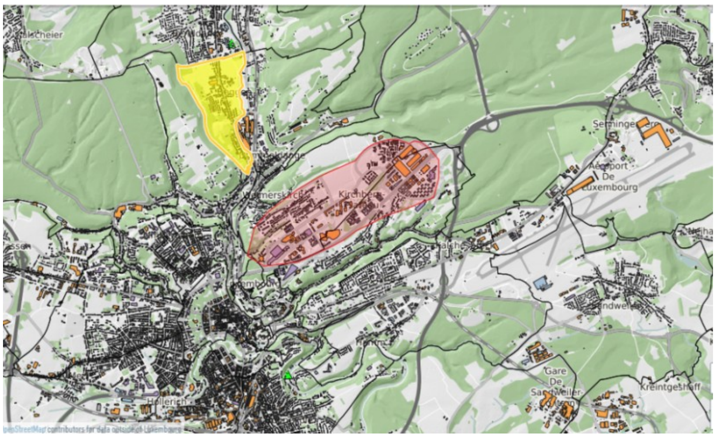


Figure 11-3 District of Beggen

This is a small suburb quarter close to Limpertsberg, depicted in **Figure 11-3** which is lies in the Eastern part of the City. This region is mainly comprised of private mansions and 30% of the households are engaged in agricultural activities and two heavy industrial units exist , thus there can be seen two important additional water flow patterns which are to be included in the creation of a typical district in this area. Until now there has been no connection between Beggen and a nearby Waste Water Treatment Plant (WWTP).During 2012, works have started to connect the nearby Plant in Pfaffental with the existing WWNS in Beggen. The target, by implementing ways to maximize recycling and reuse of water, is to come up with an optimized (Waste Water Network Design) WWND and subsequently attempt to embody water reuse, regeneration and recycling to existing water using activities or functions in the upscale level system. The hypotheses include mathematical formulation of the district scale and shall encompass two objective functions, a set of constraints which will comprise the limitation for concentration of influents contaminants at every node m (**Figure 2-5**) namely as follows:

- **Objective function 1:**Minimum fresh water supply and wastewater discharge off network;
- **Objective function 2**: Minimum capital and functioning cost per annum
- **Constraints**: Contaminants influents at nodes;
- **Design variables**: The connections between nodes. Also selection and dimensioning (sizing) of appropriate waste water treatment technology;
- **Concentration monotonicity**: At every process, the outlet concentrations are not lower than the concentration of the combined wastewater stream coming from all the operation units;
- **Maximum water influent and/or effluent concentrations.**

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APPENDICES

APPENDIX A - LIST OF TABLES (PART I)

Table 3-25 Per Capita domestic water usage (source: Butler et al, 1995)

Appliance	Flow rate in litres/head/day		Percentage of total daily usage (%)	
	Seigrist et al(1976)	Laak (1976)	Seigrist et al(1976)	Laak (1976)
Water closet	36	75	27	48
Kitchen sink	18	14	14	9
Wash basin	-	8	-	5
Bath and shower	38	32	28	21
Washing machine	41	28	31	18

Table 3-26 Per capita pollution load (mg/c/day) (Source: Butler et al,1995)

Appliance / Contaminant	WC/Toilet	Kitchen Sink	Wash Basin	Bath and Shower	Washing machine
BOD	10,720	8,340	1,860	3,090	14,810
COD	67,890	18,800	3,250	9,080	20,300
TOC	7,780	5,000	1,750	1,750	10,310
TSS	12,520	4,410	2,260	2,260	10,970
TOTAL-P	550	420	586	36	2,150
NH3-N	1,110	32.3	9	40	30.8
NO3-N	27.4	1.8	2.2	7.4	27.3

Table 3-27 Average Removal ratio concerning MF and RO treatment systems (Source: USEPA,2004)

PROCESS	BOD	COD	TOC	TSS	TOTALP	AMMONIA (NH3)	NITRATE (NO3)
MF	0.86	0.76	0.57	0.97	0	0.07	0
RO	0.4	0.91	0.94	0.99	0.99	0.96	0.96

Table 3-28 Flow rate for a typical residential dwelling of 4 residents

APPLIANCE	WATER USAGE (L/DAY)	WATER USAGE (t/day)
Water closet (WC)	144	0.144
Kitchen sink	72	0.072
Wash basin	32	0.032
Bath and shower	152	0.152
Washing machine	164	0.164

Table 3-29 Contaminant Concentration (mg/L) per appliance

	Water closet	Kitchen sink	Wash basin	Bath and shower	Washing Machine
BOD	297.78	463.33	232.5	81.32	361.22
TSS	347.78	245	59.47	59.47	267.56
Total-P*	15.28	23.33	73.25	3.947	52.44
NO3*	50.76	50.1	50.275	50.19	50.67

Table 3-30 Fresh water and prescribed concentration (mg/L) for regenerated water

		C fresh	C regenerated
Contaminant present	BOD	0	4.2
	TSS	0	5
	Total-P	1	0.5
	NO3	10	3.2

Table 3-31 Inlet concentration of streams into the water using activity

CIN		Water Using Operation				
		KITCHEN SINK*	BATH	WASHBASIN	WASHING MACHINE	Toilet/ WC
Contaminant present	BOD	0	10	10	10	10
	TSS	0	15	15	15	15
	Total-P	1	3	3	3	3
	NO3	10	25.4	25.4	25.4	25.4

Table 3-32 Outlet concentration of streams from the water using activity

COUT		Water Using Operation				
		KITCHEN SINK*	BATH	WASHBASIN	WASHING MACHINE	Toilet /WC
Contaminant present	BOD	463.33	81.32	232.5	361.22	297.78
	TSS	245	59.47	59.47	267.56	347.78
	Total-P	23.33	3.947	73.25	52.44	15.28
	NO3	50.1	50.19	50.275	50.67	50.76

Table 3-33 Average Flow rate of fresh water for water using activities

F (L/day)		Water Using Operation				
		KITCHEN SINK	BATH	WASHBASIN	WASHING MACHINE	Toilet/WC
Average		72	152	32	164	144
min		36	76	16	82	72
max		108	228	48	246	216

Table 3-34 Average Mass load of contaminants for water using activities

Mass		Water Using Operation (I)				
		KITCHEN SINK	BATH	WASHBASIN	WASHING MACHINE	Toilet/WC
Contaminant present	BOD	33.36	10.84	7.12	57.6	41.44
	TSS	17.64	6.76	1.42	41.42	47.92
	Total-P	1.61	0.14	2.25	8.11	1.77
	NO3	2.89	3.77	0.8	4.14	3.65

Table 3-35 Flow Results of Model A1 for Bleed-Off = 0

Using Unit	Min Flow	Fresh Water	Regenerated	Treated	Waste	Balance
	V_i^{min}	F_i	R_i	T_i	W_i	
Kitchen sink	72.10	144.14	0.00	144.14	0.00	0.00
Bath shower	152.08	0.00	946.61	946.61	0.00	0.00
Wash basin	32.16	0.00	321.61	321.61	0.00	0.00
Washing Machine	164.04	0.00	952.59	952.59	0.00	0.00
Water closet	144.14	0.00	144.14	0.00	144.14	0.00
Sum	**564.51**	**144.14**	**2364.95**	**2364.95**	**144.14**	**0.00**

Table 3-36 Inlet concentration at process units. Model A1 for Bleed-Off = 0

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	4.20(10.00)	0.01(15.00)	0.05(3.00)	0.21(25.40)
Wash basin	4.20(10.00)	0.01(15.00)	0.05(3.00)	0.21(25.40)
Washing Machine	4.20(10.00)	0.01(15.00)	0.05(3.00)	0.21(25.40)
Water closet	4.20(10.00)	0.01(15.00)	0.05(3.00)	0.21(25.40)

Table 3-37 Outlet concentration at process units. Model A1 for Bleed-Off = 0

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	231.45(463.33)	122.38(245.00)	12.17(23.33)	30.05(50.10)
Bath shower	15.65(81.32)	7.15(59.47)	0.20(3.95)	4.20(50.19)
Wash basin	26.34(232.50)	4.42(59.47)	7.05(73.25)	2.70(50.27)
Washing Machine	64.67(361.22)	43.49(267.56)	8.57(52.44)	4.56(50.67)
Water closet	291.70(297.78)	332.47(347.78)	12.33(15.28)	25.54(50.76)

Table 3-38 Inlet concentration at process units. Model A1 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	7.01(10.00)	0.01(15.00)	0.39(3.00)	3.42(25.40)
Wash basin	10.00(10.00)	0.02(15.00)	0.13(3.00)	0.61(25.40)
Washing Machine	6.73(10.00)	0.01(15.00)	0.42(3.00)	3.68(25.40)
Water closet	10.00(10.00)	0.02(15.00)	0.13(3.00)	0.61(25.40)

Table 3-39 Outlet concentration at process units. Model A1 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	163.94(463.33)	86.69(245.00)	8.91(23.33)	24.20(50.10)
Bath shower	45.25(81.32)	23.86(59.47)	0.88(3.95)	16.72(50.19)
Wash basin	49.31(232.50)	7.86(59.47)	12.55(73.25)	5.03(50.27)
Washing Machine	201.69(361.22)	140.22(267.56)	27.87(52.44)	17.70(50.67)
Water closet	297.50(297.78)	332.48(347.78)	12.41(15.28)	25.93(50.76)

Table 3-40 Inlet concentration at process units. Model A1 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	7.58(10.00)	0.02(15.00)	0.34(3.00)	2.88(25.40)
Wash basin	8.32(10.00)	0.02(15.00)	0.28(3.00)	2.18(25.40)
Washing Machine	7.28(10.00)	0.02(15.00)	0.37(3.00)	3.17(25.40)
Water closet	10.00(10.00)	0.02(15.00)	0.13(3.00)	0.61(25.40)

Table 3-41 Outlet concentration at process units. Model A1 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	162.37(463.33)	85.86(245.00)	8.84(23.33)	24.07(50.10)
Bath shower	45.56(81.32)	23.70(59.47)	0.83(3.95)	16.09(50.19)
Wash basin	48.95(232.50)	8.12(59.47)	13.11(73.25)	6.75(50.27)
Washing Machine	200.96(361.22)	139.29(267.56)	27.64(52.44)	17.09(50.67)
Water closet	297.78(297.78)	332.80(347.78)	12.42(15.28)	25.96(50.76)

Table 3-42 Inlet concentration at process units. Model A3 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	4.26(10.00)	0.01(15.00)	0.63(3.00)	6.00(25.40)
Wash basin	9.53(10.00)	0.02(15.00)	0.17(3.00)	1.05(25.40)
Washing Machine	9.96(10.00)	0.02(15.00)	0.13(3.00)	0.65(25.40)
Water closet	10.00(10.00)	0.02(15.00)	0.13(3.00)	0.61(25.40)

Table 3-43 Outlet concentration at process units. Model A3 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	204.03(463.33)	107.89(245.00)	10.85(23.33)	27.68(50.10)
Bath shower	33.25(81.32)	18.09(59.47)	1.00(3.95)	16.09(50.19)
Wash basin	71.31(232.50)	12.34(59.47)	19.69(73.25)	7.99(50.27)
Washing Machine	195.24(361.22)	133.26(267.56)	26.22(52.44)	13.96(50.67)
Water closet	297.78(297.78)	332.80(347.78)	12.42(15.28)	25.96(50.76)

Table 3-44 Inlet concentration at process units. Model A4 for Bleed-Off = 0%

	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	5.80(10.00)	0.09(15.00)	0.29(3.00)	0.90(25.40)
Wash basin	10.00(10.00)	1.63(15.00)	0.25(3.00)	2.22(25.40)
Washing Machine	9.99(10.00)	0.86(15.00)	0.18(3.00)	0.40(25.40)
Water closet	2.11(10.00)	0.09(15.00)	0.50(3.00)	0.91(25.40)

Table 3-45 Outlet concentration at process units. Model A4 for Bleed-Off = 0%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	0.00(10.00)	0.00(15.00)	1.00(3.00)	10.00(25.40)
Wash basin	1.31(10.00)	0.02(15.00)	0.19(3.00)	2.01(25.40)
Washing Machine	1.26(10.00)	0.02(15.00)	0.22(3.00)	2.29(25.40)
Water closet	1.54(10.00)	0.02(15.00)	0.05(3.00)	0.59(25.40)

Table 3-46 Outlet concentration at treatment units. Model A4 for Bleed-Off = 0%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	435.98(463.33)	230.54(245.00)	22.04(23.33)	47.77(50.10)
Bath shower	26.55(81.32)	13.03(59.47)	0.56(3.95)	8.12(50.19)
Wash basin	28.77(232.50)	5.37(59.47)	6.18(73.25)	4.33(50.27)
Washing Machine	285.26(361.22)	198.80(267.56)	38.94(52.44)	20.18(50.67)
Water closet	297.78(297.78)	342.00(347.78)	13.13(15.28)	26.95(50.76)

Table 3-47 Inlet concentration at process units. Model A4 for Bleed-Off = 25%

Treatment Unit	BOD	TSS	Total-P	NO3
MF	2.11(10.00)	0.09(5.00)	**0.50(0.50)**	0.91(3.20)
RO	**10.00(10.00)**	0.09(5.00)	0.02(0.50)	0.09(3.20)
Combined	6.75(10.00)	0.09(5.00)	0.22(0.50)	0.43(3.20)

Table 3-48 Outlet concentration at process units. Model A4 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	0.00(0.00)	0.00(0.00)	1.00(1.00)	10.00(10.00)
Bath shower	0.00(10.00)	0.00(15.00)	1.00(3.00)	10.00(25.40)
Wash basin	1.31(10.00)	0.02(15.00)	0.19(3.00)	2.01(25.40)
Washing Machine	1.26(10.00)	0.02(15.00)	0.22(3.00)	2.29(25.40)
Water closet	1.54(10.00)	0.02(15.00)	0.05(3.00)	0.59(25.40)

Table 3-49 Outlet concentration at treatment units. Model A4 for Bleed-Off = 25%

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	462.69(463.33)	244.66(245.00)	23.33(23.33)	50.08(50.10)
Bath shower	81.32(81.32)	50.71(59.47)	2.05(3.95)	38.28(50.19)
Wash basin	232.50(232.50)	46.13(59.47)	73.25(73.25)	27.98(50.27)
Washing Machine	361.22(361.22)	258.86(267.56)	50.90(52.44)	28.16(50.67)
Water closet	297.78(297.78)	342.58(347.78)	12.70(15.28)	26.68(50.76)

Table 3-50 Inlet concentration at process units. Model A4 for Bleed-Off = 25% (GAMS)

Treatment Unit	BOD	TSS	Total-P	NO3
MF	1.54(10.00)	0.02(5.00)	0.05(0.50)	0.59(3.20)
RO	10.00(10.00)	0.09(5.00)	0.02(0.50)	0.08(3.20)
Combined	1.54(10.00)	0.02(5.00)	0.05(0.50)	0.59(3.20)

Table 3-51 Outlet concentration at process units. Model A4 for Bleed-Off = 25% (GAMS)

Using Unit	BOD	TSS	Total-P	NO3
Kitchen sink	462.69(463.33)	244.66(245.00)	23.33(23.33)	50.08(50.10)
Bath shower	81.32(81.32)	50.71(59.47)	2.05(3.95)	38.28(50.19)
Wash basin	232.50(232.50)	46.13(59.47)	73.25(73.25)	27.98(50.27)
Washing Machine	361.22(361.22)	258.86(267.56)	50.90(52.44)	28.16(50.67)
Water closet	297.78(297.78)	342.58(347.78)	12.70(15.28)	26.68(50.76)

Table 3-52 Outlet concentration at treatment units. Model A4 for Bleed-Off = 25%

Treatment Unit	BOD	TSS	Total-P	NO3
MF	1.54(10.00)	0.02(5.00)	0.05(0.50)	0.59(3.20)
RO	10.00(10.00)	0.09(5.00)	0.02(0.50)	0.08(3.20)
Combined	1.54(10.00)	0.02(5.00)	0.05(0.50)	0.59(3.20)

LIST OF TABLES (Part II)

Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
Name of cluster (i)												
Septfontaines	0.0	100658.4	190357.5	76137.2	217311.4	135883.6	154488.4	203822.0	246329.5	66057.0	163392.3	123179.0
Tuntange	34936.7	0.0	111096.1	64091.1	84814.3	55943.7	94124.2	137242.1	181171.6	62535.8	109933.5	113227.1
Merch	549738.7	0.0	0.0	473402.8	563057.0	522006.1	561046.0	609454.1	660109.9	493118.9	556950.2	471584.9
Boevange sur Atert	272458.0	83177.0	109034.5	0.0	0.0	187978.5	212734.7	260043.6	307355.0	78999.1	192947.7	64345.8
Saeul (Calmus)	183288.9	100354.2	151226.0	33436.3	97316.8	0.0	40257.2	82239.7	127689.9	25737.3	57563.7	82152.0
Beckerich	244402.4	187324.3	255839.9	99144.6	82282.0	80676.8	0.0	0.0	43315.3	89701.1	44459.2	106016.3
Oberpalen	283003.4	234460.3	301459.4	145770.2	165789.7	131870.9	93389.5	51682.9	0.0	137843.3	89302.5	146101.6
Noerdange	280994.6	150136.6	208636.4	54192.6	179845.1	49625.2	68825.5	146579.9	204195.3	0.0	0.0	63514.7
Useldange	341748.6	214547.6	144969.8	179651.1	0.0	245390.0	259784.0	292306.7	331632.9	203157.7	244093.7	0.0
Bissen	462970.4	337288.7	69131.1	346630.1	426244.8	402983.7	426741.9	461587.0	502892.3	367947.0	414869.6	328636.3
Colmar- Berg	467381.7	330733.6	82420.6	347885.3	429630.2	408990.4	429507.7	466156.3	506745.9	371109.8	417287.9	321236.5
Vichten	315259.7	166000.8	143007.1	102429.7	202939.5	137787.5	153755.5	170465.4	254841.4	113803.1	132164.2	69538.7
Preizerdaul	389918.7	191829.3	201042.4	96601.0	260391.2	124358.4	118721.1	242154.2	298930.0	101285.8	95152.1	48100.6
Ospern	286016.7	199479.4	226935.0	97298.4	113178.0	114543.9	99331.5	87575.9	107460.5	98333.0	67293.4	58542.8
Reichlange	279555.3	172024.7	202538.9	69485.2	174076.4	137985.8	140480.0	170570.5	208766.4	71269.9	102508.9	32425.3
Schwebach	200966.5	107196.0	158421.1	0.0	127310.5	101492.8	120990.6	159379.1	198300.4	11281.0	111077.6	48843.9
Redange sur Atert	328684.9	200994.6	246700.6	98653.4	169319.5	103889.7	78342.9	55026.9	73362.0	94978.6	49073.8	77147.6
Ell	231781.8	234492.2	283231.5	133814.6	109553.0	132535.7	99698.1	64109.4	54629.1	128626.3	78760.8	112884.3
Nagem	249608.7	251251.2	281734.2	146171.2	123450.1	156907.7	130147.8	100023.9	94511.3	145696.1	99780.8	112184.9
Lannen	266416.8	269572.1	304608.5	164672.3	142740.5	173739.0	144258.8	112190.2	97889.9	163136.1	117249.7	131755.8

Table 8-21

Name of WWTP (j)	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
Name of cluster (i)												
Septfontaines	146184.1	190350.7	289069.1	147622.1	258880.2	132126.3	171822.3	137802.0	377927.8	226012.5	296302.6	349319.8
Tuntange	105535.2	140970.7	163631.9	143397.3	153777.9	132938.6	147029.3	122570.8	416063.4	173592.6	200915.5	332032.7
Merch	377097.0	75355.3	585603.6	526576.2	607976.9	517619.2	568880.4	549095.6	805330.0	635698.9	670854.0	731850.5
Boevange sur Atert	12184.2	65787.4	254032.4	90558.2	260637.8	100029.4	179350.2	146175.3	439665.4	275782.5	314664.2	375838.0
Saeul (Calmus)	111951.8	161550.3	208851.4	106674.4	138465.8	92582.0	97532.6	72940.0	331402.4	119511.9	209581.7	286432.7
Beckerich	170631.9	228460.3	245098.0	108530.5	87228.9	87308.1	61902.1	50724.1	328525.3	38732.6	172048.0	278275.8
Oberpalen	213191.3	269510.4	271064.9	139208.4	151000.0	119016.4	87958.2	89187.5	284782.9	24263.4	187298.6	259258.9
Noerdange	123113.4	179736.0	260250.3	74648.1	199103.5	54047.0	47401.1	23330.4	356652.5	164926.5	243790.3	309488.8
Useldange	45080.8	104509.3	281599.4	207786.5	285236.5	189504.6	241936.8	228422.2	449498.6	308660.3	333997.6	389014.4
Bissen	0.0	7102.5	426542.3	374013.9	452063.0	367067.2	415197.5	405146.4	630276.6	481654.9	500313.4	557756.9
Colmar- Berg	229957.2	0.0	428054.2	367788.3	449293.7	364053.8	412537.9	403181.4	632966.9	482686.2	497720.5	557131.2
Vichten	48165.5	79581.4	0.0	75354.0	205117.6	93469.5	126400.5	125649.8	464795.1	188729.2	278577.8	368859.2
Preizerdaul	95782.7	143872.6	286909.5	0.0	231181.0	41517.5	67346.9	76484.7	486088.2	244124.7	295750.0	383830.0
Ospern	126846.3	181315.8	215848.2	34570.8	0.0	27680.0	31499.2	49316.4	352726.4	90623.2	163215.4	264937.7
Reichlange	103936.9	158432.4	209903.4	23556.0	155918.0	0.0	33495.2	35203.5	320131.0	176936.6	207159.0	262226.1
Schwebach	79288.9	132568.0	183152.3	80452.6	171428.9	67316.6	116533.3	91428.5	288927.8	178345.6	214048.6	257673.6
Redange sur Atert	148720.1	204114.4	280471.7	61428.3	128587.4	46069.0	0.0	0.0	395058.8	54124.9	200275.3	307894.5
Ell	184336.4	240642.4	205711.1	97025.0	64926.0	79620.7	48513.2	66375.2	0.0	0.0	36557.1	203389.0
Nagem	181332.7	233067.8	128003.8	87236.2	59113.5	80159.0	60562.7	84494.7	248922.0	81203.2	0.0	140959.3
Lannen	201185.6	252907.3	147758.2	107887.5	79227.5	100045.3	79105.4	102421.8	196709.8	85667.3	24988.1	0.0

Table 8-21 (continued)

Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
Name of cluster (i)												
Septfontaines	0.0				161371.6	109036.5	115066.0	138450.8	154909.6		104608.1	
Tuntange												
Merch	467101.5	0.0		375795.1	453772.3	414218.4	427309.5	450766.7	472479.5	390321.4	422215.7	362799.2
Boevange sur Atert	206887.2				0.0	122108.2	129698.3	157246.1	176612.6		116598.1	
Saeul (Calmus)	122608.4				68373.8							
Beckerich	132622.6											
Oberpalen	140284.5				81855.0							
Noerdange	186181.4				156690.7		50461.7	119333.6	147307.4			
Useldange	251845.7	132609.0		151905.8	0.0	194490.3	198205.3	214360.3	227837.4	168625.7	191796.7	0.0
Bissen	355182.8	251956.8		265290.3	334924.7	300186.2	304981.8	320864.2	335222.6	278124.0	299097.7	246398.3
Colmar- Berg	331649.1	219951.8		236105.5	310864.2	274255.8	278804.5	295492.5	310133.2	250347.7	272572.9	213448.8
Vichten	191503.5				136220.9				121104.8			
Preizerdaul	258178.3				208693.0			161413.2	202220.5			
Ospern	151282.1				63525.8							
Reichlange	163783.3				142189.2	79301.4	87883.6	116976.1	135211.3		70372.3	
Schwebach	128409.5				114036.7	77939.2	81967.4	98000.0	109375.5		74549.5	
Redange sur Atert	198940.5				119118.4							
Ell	83074.7											
Nagem												
Lannen												

Table 8-22

j	Name of WWTP (j)	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
i	Name of cluster (i)												
1	Septfontaines			190263.8		161072.9		86191.0	68438.6	264151.9	136389.1	175540.5	215583.3
2	Tuntange			78799.0						278334.7			185321.7
3	Merch	309829.5		482806.2	401821.9	464260.0	388872.9	420173.4	411366.9	604725.2	456052.8	489211.9	537233.9
4	Boevange sur Atert			216007.3		182691.3		93718.9	69825.7	301936.7	155020.4	199890.3	246093.6
5	Saeul (Calmus)			136693.6		66507.6				229602.9		109778.3	172656.8
6	Beckerich			153777.9						278823.2		106776.6	204121.8
7	Oberpalen			155293.0		78343.1				258534.6		124422.4	194686.1
8	Noerdange			196475.9		156188.1				286490.7	115723.4	176822.2	228348.7
9	Useldange			258644.6	171258.4	232041.3	150281.9	183152.6	177223.1	337718.8	213048.6	245571.8	285218.9
10	Bissen	0.0		360672.1	280498.2	336291.1	266265.8	292439.4	287378.4	453624.6	320971.4	351606.3	394079.4
11	Colmar- Berg	166881.5		337632.4	252016.4	311565.0	236305.5	264828.8	259464.6	430366.0	295055.9	327056.7	370498.9
12	Vichten			0.0		134656.5				328064.4		175780.3	252089.2
13	Preizerdaul			261759.0		203335.8				391774.0	157096.1	237365.0	310574.3
14	Ospern			167044.3		0.0				286856.2		129382.1	213838.4
15	Reichlange			171778.5		141446.5				244280.4	114359.9	156658.5	196954.7
16	Schwebach			134148.9	29852.3	113642.7		61541.6	49910.3	186130.3	96407.1	123726.6	151882.0
17	Redange sur Atert			212505.6		106630.7				348849.8		169336.3	263382.2
18	Ell			115588.6						0.0			173348.2
19	Nagem									204010.5			127386.0
20	Lannen									154991.9			

Table 8-22 (continued)

j	Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
i	Name of cluster (i)												
1	Septfontaines	18370.9	2488.6	2946.6	2363.4	13921.6	4840.8	5713.7	9475.7	13055.1	2312.1	4913.5	2603.7
2	Tuntange	2152.7	2009.0	2539.9	2299.6	2407.3	2258.4	2454.4	2675.2	2899.9	2291.9	2535.2	2551.6
3	Merch	312132.8	119631.5	2088.1	162666.3	286514.2	217886.4	239616.1	281651.8	324785.4	182292.5	231180.9	146616.4
4	Boevange sur Atert	27987.2	2399.2	2530.6	2215.7	19465.6	7019.5	8364.8	13747.5	19030.9	2378.2	6494.9	2302.7
5	Saeul (Calmus)	7001.5	2486.3	2746.0	2142.6	2017.6	1990.2	2179.1	2394.5	2627.0	2102.9	2267.8	2392.9
6	Beckerich	9277.8	2931.0	3281.0	2480.0	2394.9	2386.0	2137.9	2161.2	2195.4	2431.8	2200.1	2515.0
7	Oberpalen	11047.9	3172.1	3514.2	2718.8	3575.7	2648.0	2451.3	2238.2	2013.5	2678.3	2430.3	2720.3
8	Noerdange	21270.1	2741.3	3039.9	2250.6	12332.7	2228.1	1293.6	6034.6	10947.2	2192.3	1999.1	2298.1
9	Useldange	49801.3	8791.3	2714.9	11391.2	38328.9	23259.2	24719.2	31172.9	37534.1	15339.2	22379.7	5804.7
10	Bissen	137619.5	49791.0	2327.6	57806.7	115304.9	83526.9	87831.1	102311.7	116849.5	66541.4	82848.2	46617.8
11	Colmar- Berg	112704.2	34081.9	2395.3	41681.2	92861.8	64592.1	68008.7	80840.5	93483.5	49447.5	63612.6	31281.6
12	Vichten	23442.9	2822.2	2704.0	2497.1	9123.8	2678.3	2760.0	2845.5	7989.7	2555.5	2649.6	2328.4
13	Preizerdaul	54209.5	2954.4	3001.0	2467.6	28463.6	2609.9	2581.1	14333.9	26736.2	2491.7	2460.6	2218.8
14	Ospern	13000.1	2993.3	3133.2	2470.7	2170.4	2559.4	2481.6	2421.7	2523.6	2476.2	2317.5	2271.6
15	Reichlange	15462.2	2853.3	3008.8	2329.2	9586.8	3020.9	3447.5	6171.1	9069.6	2338.5	2153.1	2138.7
16	Schwebach	8004.3	2522.0	2783.3	1981.0	5169.9	2377.1	2853.3	4358.3	5884.2	2032.1	2415.6	2223.4
17	Redange sur Atert	26086.2	3001.0	3234.3	2477.7	6380.1	2504.9	2374.3	2255.3	2349.4	2459.0	2224.2	2367.3
18	Ell	4699.7	3172.1	3420.9	2657.3	2534.1	2651.1	2483.2	2301.2	2253.0	2630.9	2375.8	2550.0
19	Nagem	3249.9	3257.6	3413.2	2720.3	2604.8	2775.5	2638.7	2484.7	2456.7	2718.0	2483.2	2546.1
20	Lannen	3335.4	3351.0	3529.8	2814.4	2702.8	2861.1	2710.2	2546.1	2473.0	2806.6	2571.8	2645.7

Table 8-23

j	Name of WWTP (j)	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
i	Name of cluster (i)												
1	Septfontaines	2721.1	2946.6	22632.5	2728.9	14531.9	2649.6	3871.4	2676.3	57610.1	9523.4	18571.3	32587.6
2	Tuntange	2511.9	2693.1	3417.9	2706.3	2760.0	2652.7	2725.0	2599.8	67467.1	2861.1	3001.0	21846.4
3	Merch	91118.2	2359.5	345151.3	199205.3	307442.0	180697.1	228076.7	213908.1	682193.7	292005.1	360402.0	477687.3
4	Boevange sur Atert	2030.9	2309.0	31211.9	2437.3	19931.8	2485.5	4406.1	2847.4	85537.8	13555.1	26182.7	47221.8
5	Saeul (Calmus)	2545.4	2798.9	9290.3	2518.9	2642.0	2446.6	2472.3	2346.3	38335.3	2585.0	6103.4	17675.5
6	Beckerich	2845.5	3141.0	12813.1	2528.3	2420.2	2419.4	2289.5	2232.0	66382.5	2171.3	5232.4	27075.9
7	Oberpalen	3063.2	3351.0	13532.6	2685.3	3195.7	2581.9	2423.3	2429.5	52712.6	2097.5	7282.3	23547.8
8	Noerdange	2602.9	2892.2	24149.8	2355.6	12543.7	2249.9	2216.4	2092.8	72279.1	5949.9	18054.3	37406.4
9	Useldange	2204.8	2508.0	52725.8	16051.7	38719.1	11247.1	19759.1	17910.9	118522.8	30920.1	46265.9	71880.8
10	Bissen	27553.8	2009.9	143372.7	68283.1	117106.9	58747.3	77663.6	73716.0	287327.5	102736.4	134199.1	188583.6
11	Colmar- Berg	15363.7	2200.1	118086.1	50319.3	93788.0	42041.2	58590.1	55184.5	245904.4	80769.7	108725.5	157356.4
12	Vichten	2218.0	2379.0	25771.7	2358.7	8929.7	2451.3	2620.0	2616.1	109161.2	2938.8	18345.9	50377.4
13	Preizerdaul	2462.9	2708.7	54671.9	1993.7	26030.8	2185.3	2318.3	2365.0	184192.3	13458.5	41452.2	91868.2
14	Ospern	2621.6	2899.9	15172.6	2148.8	894.0	2112.2	2133.2	2225.0	72482.4	2437.3	7519.1	30521.6
15	Reichlange	2504.9	2783.3	16214.5	2094.4	9171.2	1987.0	2145.7	2154.2	45361.2	6007.4	12769.5	24340.5
16	Schwebach	2379.0	2651.1	8499.9	1525.8	5847.3	2318.3	2189.7	1657.5	21383.8	4559.7	7632.9	12653.2
17	Redange sur Atert	2733.5	3016.6	30250.0	2287.2	4514.7	2207.9	1982.8	2130.1	129444.3	2250.6	15462.3	56000.4
18	Ell	2915.5	3203.2	6598.1	2469.2	2305.9	2379.7	2220.3	2312.1	46438.8	2190.0	2161.2	16501.5
19	Nagem	2899.9	3164.3	2628.6	2418.6	2275.5	2382.1	2281.8	2404.6	26559.5	2388.3	2005.7	6898.4
20	Lannen	3001.0	3265.4	2728.9	2523.6	2377.4	2483.2	2375.8	2495.6	12266.0	2410.1	2096.7	1991.2

Table 8-23 (continued)

j	Name of WWTP (j)	Dondelange	Hollenfels	Merch	Schwebach	Rippweiler I,II	Calmus	Schweich	Beckerich I	Oberpallen	Kapweiler I,II	Noerdange	Everlange I,II
i	Name of cluster (i)												
1	Septfontaines	17690.9				12383.6	3775.1	4443.8	7784.6	10941.2		3329.3	
2	Tuntange												
3	Merch	310161.5	118070.0		160451.9	284110.3	215506.8	236815.3	278446.0	321109.8	179993.9	228363.9	144220.6
4	Boevange sur Atert	26292.8				18077.5	5320.3	6386.9	11448.9	16278.7		4625.6	
5	Saeul (Calmus)	5386.6				917.8							
6	Beckerich	6833.4											
7	Oberpalen	8101.3				1583.3							
8	Noerdange	19101.1				11326.8		365.6	4962.3	9393.8			
9	Useldange	47712.0	6831.3		10310.8	37242.8	21803.0	23089.7	29277.7	35219.3	14148.7	20900.8	4860.5
10	Bissen	135239.9	47775.9		55856.4	113192.6	81228.3	85224.7	99397.5	113497.9	64453.4	80339.0	44652.9
11	Colmar- Berg	109871.0	31653.7		39236.8	90304.0	61775.1	64932.5	77440.3	89662.1	46857.3	60633.6	28902.0
12	Vichten	20804.1				7410.9				5188.9			
13	Preizerdaul	51441.1				26994.5			12393.3	24536.5			
14	Ospern	10183.1				734.4							
15	Reichlange	12953.0				8439.2	1438.3	1963.8	4671.1	7245.7		1001.5	
16	Schwebach	6196.6				4324.5	1364.8	1589.9	2732.0	3810.7		1192.7	
17	Redange sur Atert	23350.2				4935.2							
18	Ell	1655.9											
19	Nagem												
20	Lannen												

Table 8-24

j	Name of WWTP (j)	Boevange	Bissen	Schandel	Platen	Ospern	Reichlange	Redange	Niederpalen	Colpach-Bas	Levelange	Nagem	Lannen
i	Name of cluster (i)												
1	Septfontaines			20398.7		12314.3		1851.4	920.4	55133.3	7438.6	15981.1	29786.8
2	Tuntange			1410.9						64601.5			18835.0
3	Merch	89396.3		342852.7	196550.3	304479.2	177977.3	225032.9	211042.5	678307.5	288459.1	356823.6	473898.3
4	Boevange sur Atert			29964.7		18036.5		2386.1	978.1	82672.2	10964.9	23689.7	44485.8
5	Saeul (Calmus)			7489.1		843.9				36052.9		3853.4	15198.7
6	Beckerich			10700.8						64945.7		3542.9	25242.2
7	Oberpalen			11023.4		1386.3				51656.6		5631.7	21869.6
8	Noerdange			22484.6		11217.1				70510.2	4521.2	16337.3	35459.4
9	Useldange			51723.2	14828.8	37225.7	9980.4	18174.9	16449.8	116078.4	28738.2	44200.6	69566.0
10	Bissen	26437.8		141673.5	66135.2	114597.7	56481.1	75041.0	71174.4	283830.1	99498.2	131155.3	185296.8
11	Colmar- Berg	13709.9		115988.4	47810.1	90922.4	39337.6	55562.5	52221.7	241985.8	77094.1	105325.3	153697.0
12	Vichten			24780.5		7155.9				106311.8		16047.3	47852.0
13	Preizerdaul			53633.7		24948.8				182031.4	11415.9	39874.5	90049.2
14	Ospern			13750.4		187.4				70783.2		6339.9	29062.2
15	Reichlange			14965.7		8306.3				43500.0	4361.7	11319.8	22651.0
16	Schwebach			7074.5	74.5	4279.3		667.1	353.6	19085.2	2599.6	5536.8	10306.0
17	Redange sur Atert			28516.8		3528.3				128064.3		14330.1	54647.9
18	Ell			4505.3						45420.0			15383.9
19	Nagem									25200.5			6048.1
20	Lannen									10958.8			

Table 8-24 (continued)

i	Name of cluster (i)	Elevation of cluster i (in m)	Type of connected WWTP	Distance D_{ij} (in m)	(x,y) coordinates of connected WWTP (J)	elevation of WWTP (in m)	ΔH
1	Septfontaines	262	1b	309	70242E, 84573N	309	-47
2	Tuntange	314.84	2b	442	71768E, 86515N	249.73	65.11
3	Merch	222.08	2b	5,750	71768E, 86515N	249.73	-27.65
			3b	1,460	75969E, 92094N	216.25	-216.25
4	Boevange sur Atert	263.14	4b	3,100	65912E, 90050N	255.54	7.6
			4m	4,680	64339E, 90585N	296.49	-296.49
5	Saeul	291.78	5m	200	62589E, 87908N	271.91	19.87
6	Beckerich	298.61	6b	2,100	62573E, 87913N	271.63	26.98
			6m	2,400	57443E,89437N	282.39	-282.39
7	Oberpalen	291.6	7b	500	56858E, 88628N	290.66	0.94
8	Noerdange	273.08	8m (double)	2,800	65276E,89608N	261.02	12.06
			8m' (single)	315	62061E,90012N	270.58	-270.58
9	Useldange	240.95	9m	1,940	64680E,93199N	247.74	-6.79
			9m'	2,600	64803E,90874N	292.82	-292.82
10	Bissen	221.38	10b	3,000	69467E,93779N	238.02	-16.64
11	Colmar- Berg	230.27	11b	2,900	72856E,95179N	218.77	11.5
12	Vichten	287.46	12m	2,230	65959E.95106N	315.76	-28.3
13	Preizerdaul	277.05	13m	246	63213E,94609N	261.98	15.07
14	Ospern	294.03	14m	473	61257E,93906N	294.28	-0.25
15	Reichlange	267.44	15m	160	62591E,93304N	254.9	12.54
16	Schwebach	261.44	4b	82	65912E, 90050N	255.54	5.9
17	Redange sur Atert	291.75	17b	106	60633E,92287N	266.53	25.22
			17m	2000	61644E,91112N	264.13	-264.13
18	Ell	306.21	18b	2400	55445E,90902N	408	-101.79
			18m	2,770	57468E,89524N	281.36	-281.36
19	Nagem	319.26	19m	400	57767E,94561N	303.92	15.34
20	Lannen	355.07	20m	214	56149E,94839	340.55	14.52

Table 9-7

Where :

m: mechanical;

m (double) implies a second infrastructure adjacent to the initial one (thus extension of existing plant);

m': the second mechanical WWTP connected;

b:biological;

The **number in front of m, b** denotes which cluster node is assigned to be connected to the type of WWTP;

x : is the longitudinal coordinate;

y: is the coordinate of latitude;

The **m** symbol in parenthesis denotes the distance between the connected WWTP and the centre of cluster;

$\mathbf{D_{ij}}$: It is the horizontal (no gradients of Dh included) distance between centers of cluster and WWTP's;

ΔH: Head difference between centre of cluster and WWTP elevation.

APPENDIX B

SYMBOLS AND PROGRAMMING CODES (PART I)

(Notations within the context of the literature review)

Parameters

$s_p^{(g)}$	**Source member of a specific generation of members**
$s_o^{(g)}$	Generated member with small alterations in its identity compared to source
$z^{(g)}$	$z^{(g)} = \left[z_1^{(g)},\ z_2^{(g)},\ \ldots,\ z_n^{(g)}\right]^T$ is a vector hazardously selected within generation g
σ_i^2	Variance, 'average deviation from mean ranges within small values. Like in deterministic search methods, can also be called 'step length'
ξ_i	'Expected average value' for member $Z_i^{(g)}$ is zero
μ_b	Number of ancestor vectors in existing batch selected to attain the closest to the optimum.
μ	Number of all ancestor vectors
	The i^{th} member of the ancestor vector
s_a, s_b	The ancestor vectors picked up in a random manner out of total population
μ	Population of ancestors
λ	Population of descendants
p	Population of ancestors for every descendant

Constants

$\overline{F}^{(g)}$	**The average value of the objective function**
$F_{best}^{(g)}$	The best value of the objective function of all ancestor vectors in the g^{th} generation
ε_{ad}	0.05
$\varepsilon_b = \varepsilon_c$	0.0001
ε_d	A specific value
δ_{si}	Difference in values within the set of discrete variables
κ:	Random mean integer value according to the Poisson's distribution
γ :	Standard deviation
$\mu_b/\mu = \varepsilon_d$	0.5 to 0.8

Abbreviations:

Symbol	Description
FWN	Fresh Water Network
TP	Treatment Plant
WW	Waste Water
WWN	Waste Water Network
WWNT	Waste Water Network Topology
WWTP'(s)	Waste Water Treatment Plant(s)
WWNG	Waste Water Network Grid
DWWTS	Distributed Waste Water Treatment System
WU	Water Using Unit
TU	Treatment Unit

Sets:

Symbol	Description	Variables
I	{any water using operation } {1,2,3,4,5} {KITCHEN S, BATH W, WASHING M, WC }	i, j
T	{any treatment unit } {MF, RO}	t, t_1
K	{any contaminant present in the water } {BOD, TSS, Total − P, NO3}	k
Q	{any unit } $I \cup T$	q, r

Design Variables:

Symbol	Subscripts	Description
F_i	$i \in I$	Fresh water flow from mains into using process i
W_i	$i \in I$	Waste water flow from using process i
$X_{q,r}$	$q, r \in Q$	Flow rate from unit r to unit q.
V_q^{IN}	$q \in Q$	Flow rate in to unit q
V_q^{OUT}	$q \in Q$	Flow rate out of unit q
$M_{q,k}^{IN}$	$q \in Q$ $k \in K$	Mass of contaminant k in to unit q
$M_{q,k}^{OUT}$	$q \in Q$ $k \in K$	Mass of contaminant k out of unit q
$C_{q,k}^{IN}$	$q \in Q$ $k \in K$	Concentration of contaminant k in to unit q
$C_{q,k}^{OUT}$	$q \in Q$ $k \in K$	Concentration of contaminant k out of unit q

Auxiliary Variables:

Symbol	Expression	Description
R_i	$\sum_{t \in T} X_{i,t}\ , i \in I$	Overall flow rate from treatment units to using operation i
T_i	$\sum_{t \in T} X_{t,i}\ , i \in I$	Overall flow rate from using operation i to treatment units
R_t	$\sum_{i \in I} X_{i,t}\ , t \in T$	Overall flow rate from treatment unit t to all using operations
T_t	$\sum_{t \in T} X_{t,i}\ , t \in T$	Overall flow rate from all using operations to treatment unit t
$D_{i,j}$	$\sum_{\substack{j \in I \\ j \neq i}} X_{i,j}\ , i \in I$	Overall flow rate from all using operations to using operation unit i
$D_{j,i}$	$\sum_{\substack{j \in I \\ j \neq i}} X_{j,i}\ , i \in I$	Overall flow rate from using operation unit i to all using operations
C_k^T	$\frac{\sum_{t \in T}\left(\sum_{i \in I} X_{i,t}\right) C_{t,k}^{OUT}}{\sum_{t \in T}\left(\sum_{i \in I} X_{i,t}\right)},\ k \in K$	Concentration of each contaminant k in the combined flow from the treatment units to the using process units
$C_{i,k}^T$	$\frac{\sum_{t \in T} X_{i,t} C_{t,k}^{OUT}}{\sum_{t \in T} X_{i,t}},\ i \in I, k \in K$	Concentration of each contaminant k in the combined flow from the treatment units to the each using process unit i
V_i^{min}	$\max_{k \in K} \frac{M_{i,k}}{C_{i,k}^{OUT,max} - C_{i,k}^{IN,max}}\ , i \in I$	Minimum water flow rate for each water using process i

Parameters:

Symbol	Subscripts	Description
$M_{i,k}$	$i \in I$ $k \in K$	Mass load of contaminant k at the i water using operation
$M_{t,k}$	$t \in I$ $k \in K$	Mass removal of contaminant k at the t treatment unit
$r_{t,k}$	$t \in I$ $k \in K$	Removal ratio of contaminant k at the t treatment unit
$C_{i,k}^{IN,max}$	$i \in I$ $k \in K$	Maximum inlet concentration of contaminant k into water using process i
$C_{i,k}^{OUT,max}$	$i \in I$ $k \in K$	Maximum outlet concentration of contaminant k into water using process i
C_k^F	$k \in K$	Concentration of contaminant k present in fresh water stream
$C_k^{T,max}$	$k \in K$	Concentration of contaminant k present in the regenerated water stream
Bleed Off		Bleed-Off factor for the waste water in excess

Appendix B1

Problem Solution via MATLAB: Code implementing function and optimizing Model A1.
https://docs.google.com/document/d/1c4fLED4NNwMdzBZdwPI99LQ6aKu_fiKabhhzX2XQpLc/edit

Appendix B2

Problem Solution with MATLAB: Code implementing function and optimizing Model A2.
https://docs.google.com/document/d/1MZVZc3y90DaamEEZFdlhi1c9fCax__L1Af8-JbZYf4A/edit

Appendix B3

Problem Solution with the use of MATLAB: Code implementing function and optimizing Model A4
https://docs.google.com/document/d/1Jxq8qPxTuqQ1X9reR54Wdee5OGEVH3I9Ja5peg2riD8/edit

Appendix B4

Problem Solution with the use of GAMS: Code implementing function and optimizing Model A3
https://docs.google.com/document/d/1vuxI5aUCsMcVvtl7mpUX3EcWOdt4EQKB535TK-EPsLo/edit

Appendix B5

Problem solution with GAMS: Code implementing function and optimizing Model A4
https://docs.google.com/document/d/12Lp4Oj-GrQ2Ku0cmkQaQaXBlZAgp_3ncs4YoTHeHFww/edit

Symbols and Programming Codes (PART II)

Symbols (concerning the adopted and analyzed models)

$\mathbf{x_{ij}}$:

Binary variable that takes the value of 1 if the ith cluster is decided to be connected with the jth WWTP and 0 otherwise (-);

$\mathbf{z_{ij}}$:

Continuous variable that takes the value of the amount of waste water transferred from cluster i to WWTP j (m3/h);

$\mathbf{q_{j}}$:

Continuous variable that takes the value of the expansion needed to be made at WWTP at location j in terms of additional amount of waste water that can be treated in it. (m3/h);

$\mathbf{y_{j}}$:

Binary variable that takes the value of 1 if a WWTP exists at location j and 0 otherwise (-);

$\mathbf{r_{j}}$:

Continuous variable that takes the value of the final capability (after expansion or closure) of WWTP at location j in terms of the final amount of waste water that can be treated in it (m3/h);

$\boldsymbol{B}$:

Set of intervals (boxes) used for the piecewise linearization (index b);

$\overline{\boldsymbol{CE_{bp}}}$:

Cost of expansion of a WWTP of type p corresponding to the capability expansion $\overline{awq_b}$ in the interval b of the piecewise linearization (€);

$\overline{\boldsymbol{CM_{bp}}}$:

Cost of maintenance of a WWTP of type p corresponding to the total capability $\overline{awr_b}$ in the interval b of the piecewise linearization (€);

$\overline{\boldsymbol{awq_b}}$:

general integer parameter that is equal to the value of WWTP's expansion capability at interval b (for the expansion cost of WWTPs);

$\overline{awr_b}$:

general integer parameter that is equal to the value of WWTP's total capability at block b (for the O&M cost of WWTPs);

wq_{bj}:

Continuous variable between 0-1 (SOS2 variable)that corresponds to the expansion made of WWTP at location j in interval b of capability. (-);

wr_{bj}:

Continuous variable between 0-1 (SOS2 variable)that corresponds to the total capability after expansion made of WWTP at location j in block b of capability. (-);

wp_{bi}:

Continuous variable between 0-1 (SOS2 variable) that corresponds to the MF and RO capability at cluster i in interval b of capability. (-);

int_wq_{bj}:

Binary variable that is equal to 1 if the corresponding SOS2 variable wq_{bj} is greater than 0 and 0 otherwise(-);

int_wr_{bj}:

Binary variable that is equal to 1 if the corresponding SOS2 variable wr_{bj} is greater than 0and 0 otherwise (-).

Symbols (concerning indicative model for future research)

Variables

$Q_{(i,j),}$: Waste flow between nodes i and j;

$E_{(i,j)}$: Difference in hydraylic heads due to elevation difference between nodes i and j;

z_{kp}: Binary variable in case there exists a treatment plant of type p ;

QT_k: Waste load coming into treatment plant at node k;

C_{kp}: Total cost regarding installation, operation and maintenance of a treatment plant of a specific set type p at node k;

$C_{(i,j)}$: Total cost regarding installation, operation and maintenance costs of sewer mains plus pumping stations between node i and j;

$x_{(i,j)}$: Binary variable, being one in case there exists a sewer main between node i and j;

$y_{(i,j)}$: Binary variable, being one in case there exists a pump station between node i and j;

Sets

S: set of sewers;

P: set of pumping stations;

N: set of treatment plants;

N_b: set of treatment plants of type b (biological treatment);

N_m: set of treatment plants of type m (manual treatment);

R: set of rivers.

Abbreviations

ALMP: Augmented Lineraised Master Problem

BD_: Bender Decomposition

BD_2_VI: Bender's Decomposition with Valid Inequalities

BDA: Bender'sDecomposition Algorithm

CCB's: Covering Cut Bundles

WWWN: Water and Waste Water Network

WWTND: Waste Water Treatment Network Design

WWTP('s): Waste Water Treatment Plant(s)

(W&WWN's): Water and Waste Water Networks

WWN('s): Waste Water Network(s)

WWRM: Waste Water Resource Management

MDC's: MaximumDensity Cuts

MFS: MaximumBFeasible Subsystem

MILV: Mixed Integer Linear Variation

MILP: Mixed Integer Linear Programming

MINLP: Mixed Integer Non Linear Programming

CLP: Continuous Linear Problem

DSP: Dual Sub Problem

LP: Linear Programming

OP: Original Problem

LOP: Linearised Original Problem

LMP: Linearised Master Problem

LPSP: Linear Primal Sub Problem

NLP: Non Linear Programming

NLOM: Non linear Original Model

NLOP: Non Linear Original Problem

NLP's: Non Linear problems

NLMP: Non Linear Master Problem

PE: Population equivalent

PILP: Pure IntegerLinear Problem

IV's: IntegerVariants

MP: Master Problem

WWOD: Waste Water outflow demands

O&M: Operational and Maintenance

MF: Microfiltration

RO: Reverse Osmosis

GBD: Generalised Bender's Decomposition

BDM: Bender's Decomposition Method

CBD: Classical Bender's Decomposition

CSD: Central Sewerage Drainage

BC: Bender's Cut

OP: Original Problem

RMP: Relaxed Master Problem

PSP: Primal Sub Problem

DSP: Dual Sub Problem

DERS: Distributed Energy Resource Systems

UB: Upper Bound

LB: Lower Bound

SOS1: Special Order Set of Type 1

SOS2: Special Order Set of Type 2

APPENDIX B6

Programming code in Visual Basic for the excel sheet table results

https://drive.google.com/file/d/1Pq4Yr1co0Fva49ZAaCVrEU7sgMPbYddM/view

APPENDIX B7

Programming code in CPLEX for mathematical model without the use of MF/RO domestic systems (Mathematical programming-Direct solution)

https://drive.google.com/file/d/11UuhY6Dl1hbpRCqjoOkdLvWeEDjJYYGR/view

APPENDIX B8

Programming code in CPLEX for mathematical model with the use of MF/RO domestic systems (Mathematical programming-Direct solution)

https://drive.google.com/file/d/1N2Yo81yswGTBY4H5oyHvGeBDOsMNBVwr/view

APPENDIX B9

Programming code in CPLEX for mathematical model without the use of MF/RO domestic systems (Benders Decomposition method, “First Linearize, Then Decompose”,BD_1)

https://drive.google.com/file/d/1ds8SpO2n_6M89s8lTHTAQdFE6kIBAtY2/view

APPENDIX B10

Programming code in CPLEX for mathematical model without the use of MF/RO domestic systems (Benders Decomposition method, “First Decompose, Then Linearize”,BD_2)

https://drive.google.com/file/d/1MC8Rq9EEkDsjEnF4bI7W_HSc4buxmwEe/view?usp=sharing

APPENDIX B11

Programming code in CPLEX for mathematical model without the use of MF/RO domestic systems (Accelerated Benders Decomposition method with Valid Inequalities, “First Decompose, Then Linearize”,BD_2_VI)

https://drive.google.com/open?id=1brHwKQpzpHRc6JQW6h7_Db0rWdgsySAL