Journal of Computational Design and Engineering, 2020, 7(6), 816–829

doi: 10.1093/jcde/qwaa059
Journal homepage: www.jcde.org
Advance Access Publication Date: 12 August 2020

RESEARCH ARTICLE

A fuzzy-based framework to support multicriteria design of mechatronic systems

Abolfazl Mohebbi*, Sofiane Achiche and Luc Baron

Department of Mechanical Engineering, Polytechnique Montreal, Montreal, Quebec, Canada, H3T 1J4.

*Corresponding author. E-mail: abolfazl.mohebbi@polymtl.ca http://orcid.org/0000-0003-2101-9651

Abstract

Designing a mechatronic system is a complex task since it deals with a high number of system components with multidisciplinary nature in the presence of interacting design objectives. Currently, the sequential design is widely used by designers in industries that deal with different domains and their corresponding design objectives separately leading to a functional but not necessarily an optimal result. Consequently, the need for a systematic and multiobjective design methodology arises. A new conceptual design approach based on a multicriteria profile for mechatronic systems has been previously presented by the authors, which uses a series of nonlinear fuzzy-based aggregation functions to facilitate decision-making for design evaluation in the presence of interacting criteria. Choquet fuzzy integrals are one of the most expressive and reliable preference models used in decision theory for multicriteria decision-making. They perform a weighted aggregation by the means of fuzzy measures assigning a weight to any coalition of criteria. This enables the designers to model importance and also interactions among criteria, thus covering an important range of possible decision outcomes. However, specification of the fuzzy measures involves many parameters and is very difficult when only relying on the designer's intuition. In this paper, we discuss three different methods of fuzzy measure identification tailored for a mechatronic design process and exemplified by a case study of designing a vision-guided quadrotor drone. The results obtained from each method are discussed in the end.

Keywords: mechatronic system; multicriteria design; decision support; interacting objectives; fuzzy measures

List of symbols

MMP:	Mechatronic multicriteria profile
GCS:	Global concept score
MIQ:	Machine intelligence quotient
RS:	Reliability score
CX:	Design complexity
FX:	Design flexibility
CT:	Cost of manufacture and production
m_i:	Criteria values
ϕ_i:	Normalized sub-criteria values
μ:	Fuzzy measures
λ:	Sugeno measure
I:	Interaction index
ϕ:	Importance index
C_μ:	Choquet integral
E:	Error criterion
$\mathbf{u}$:	A vector containing all the coefficients of fuzzy measures

1. Introduction

Multidisciplinary systems that include synergetic integration of mechanical, electrical, electronic, and software components are known as mechatronic systems (Rzevski, 2014). Because of the high number of the constituent components, the multiphysical aspect of the subsystems, and the couplings between the different engineering disciplines involved, the design of mecha-

Received: 19 May 2020; **Revised:** 28 June 2020; **Accepted:** 11 July 2020

tronic systems can be rather complex and it requires a concurrent approach to obtain optimal solutions (Torry-Smith et al., 2013; Mohebbi, Baron, Achiche, & Birglen, 2014c; Mohebbi, Baron, Achiche, & Birglen, 2014d). In a similar manner to other systems, the design of mechatronic devices includes three major phases: conceptual design, detailed design, and prototyping and improvements. Several problems and limitations are encountered when the design is at its early stages, as it requires choosing the "Elite Set" that is the selection of components such as mechanical, electrical, software, and control strategies. This practice creates challenges due to insufficient support of a multicriteria approach for mechatronic system design, which calls for a decision-making algorithm across various disciplines. In such cases, design engineers tend to choose the first and the best components from what they see as available and feasible to meet their design requirements. Such decisions can often lead to a functional design, but rarely to an optimal one. This ill decision-making generally occurs due to improper selection and assessment of performance criteria and lack of knowledge about the co-influences among criteria.

The goal of concept evaluation is to compare the generated concepts based on the design requirements and to select the best alternative for further device and then product development. To this end, the authors have previously presented a new approach based on a newly introduced "multicriteria mechatronic profile" (MMP) toward a better concept evaluation process during the conceptual design phase (Mohebbi, Achiche, & Baron, 2014a). The MMP included five main elements: machine intelligence, reliability, flexibility, complexity, and cost, while each main criterion has several sub-criteria. To facilitate the decision-making process in the presence of interacting criteria, the concept of fuzzy measures and Choquet integrals were utilized in this approach. However, specifying each fuzzy measure associated with the design criteria is difficult by only relying on the intuition of decision makers (DMs). In this paper, we introduce three methods of fuzzy measure identification customized for the mechatronic design process and exemplified by a case study focused on designing a vision-guided quadrotor drone. With the proposed approaches, we aim to support the designers with using fuzzy-based multicriteria decision-making (MCDM) tools by facilitating the identification process of the parameters involved. Ultimately, by helping the designers with choosing the appropriate identification method, we plan to encourage the application of MMP in mechatronic design cases.

This article is organized as follows. Section 2 reviews the available literature on the fuzzy-based multicriteria design and the identification of the fuzzy measures. Section 3 gives a brief overview of the conceptual design of mechatronic systems and the previously developed methodology based on the MMP as a design evaluation index. Fuzzy decision support and the Choquet aggregation technique are described in Section 4 alongside the necessary definitions of fuzzy measures and integrals, illustrated with some properties. Section 5 describes three different algorithms for elicitation and identification of fuzzy measures with their philosophy, while Section 6 reports the results of a case study to incorporate and compare all the design evaluation attempts. Finally, Section 6 discusses the concluding remarks of the presented research.

2. Literature Review

Tomiyama et al. (2009) presented a comprehensive description of the design theory and methodology and an evaluation of its application in practical scenarios. Ullman (1992) has analyzed four concept evaluation methods. All of these methods provide qualitative frameworks to evaluate candidate solutions. The results of these comparisons highly depend on the experience of the design engineer. Novice designers would make decisions easier if quantitative evaluation methods are available for them. To this effect, an evaluation index can be used to rank the generated feasible solutions and therefore more easily choose between design alternatives. Moulianitis, Aspragathos, and Dentsoras (2004) introduced a mechatronic index that characterizes the mechatronic designs by their control performance, complexity, and flexibility. The overall evaluation was formulated based on the averaging operators and weight factors were manually applied to highlight the importance of each criterion. They did not, however, consider the interactions between design criteria. Behbahani and de Silva (2007) proposed a framework for the design of mechatronic systems in which the performance requirements were represented by a mechatronic design quotient (MDQ). Correlations between design criteria have been taken into account by using fuzzy functions. MDQ was implemented in some case studies (Behbahani & de Silva, 2008), and was claimed to be efficient; however, the assessment of criteria was very qualitative and no systematic measurement approach has been presented nor implemented, which puts the burden on the engineering designers.

Mohebbi et al. (2014a) presented a new approach based on their newly introduced MMP for the conceptual design stage. The MMP included five main elements: machine intelligence, reliability, flexibility, complexity, and cost, while each main criterion has several sub-criteria. To facilitate fitting the intuitive requirements for decision-making in the presence of interacting criteria, three different criteria aggregation methods were proposed and inspected using a case study of designing a vision-guided quadrotor drone and also a robotic visual servoing system. These methods benefit from three different aggregation techniques, namely: Choquet integral, Sugeno integral (Mohebbi et al., 2014a, Mohebbi, Achiche, Baron, & Birglen, 2014b), and a fuzzy-based neural network (Mohebbi et al., 2014c). These techniques proved to be more precise and reliable in multicriteria design problems where interaction between the objectives cannot, and should not, be overlooked (Moghtadernejad, Chouinard, & Saeed Mirza, 2018, 2020). There are also various examples of using fuzzy measures and Choquet integrals in MCDM cases in economics and enterprise evaluations (Liu & Tang, 2016, 2018). The Choquet integral is one of the most expressive preference models used in decision theory. It performs a weighted aggregation of criteria using a capacity function assigning a weight to any coalition of criteria. This enables the expression of both positive and negative interactions and covering an important range of possible decision dilemmas, which is generally ignored in other MCDM methods (Grabisch, 1996, 1997). A 2-additive Choquet integral has been used in the work of Mohebbi et al. (2014a), which only uses relatively simple quadratic complexity and enables the modeling of the interaction between pairs of criteria.

Despite the modeling capabilities, the specification of the fuzzy measures has been always a place for various challenges, which makes the practical use of such aggregation techniques difficult. While the definition of a simple weighted sum operator with n criteria requires $n-1$ parameters, the definition of the Choquet integral with n criteria requires setting of 2^n-2 capacities (measures), which can become quickly unmanageable even for low values of n and even for an expert who can assess the coefficients based on semantical considerations. Most of the previous works on the capacity specification for Choquet integral-based decision analysis consider a static preference database as input (learning set) and focuses on the determination of a set of measures that best fits the available preferences (Marichal & Roubens, 2000). For example, a quadratic error between Choquet values and target utility values prescribed by the DM can be minimized on a sample of reference alternatives (Meyer & Roubens, 2006). Generally, questions are asked to the decision maker, and the information obtained is represented as linear constraints over the set of parameters. An optimization problem is then solved to find a set of parameters that minimizes the error according to the information given by the DM (Grabisch, 1995). In Marichal and Roubens (1998), it is supposed that an expert is able to tell the relative importance of criteria and identify the type of interaction between them if any. These relations can be expressed as a partial ranking of the alternatives on a global basis: partial ranking of the criteria, partial ranking of interaction indices and also the type of interaction between some pairs of criteria. These approaches differ with respect to the optimization objective function and the preferential information they require as input. Rowley, Geschke, and Lenzenb (2015) and Moghtadernejad, Saeed Mirza, and Chouinard (2019) proposed methods to extract the fuzzy measures using the principal component analysis. The method is based on identifying a measure of independence among design criteria. The major problems of the aforementioned approaches are the lack of transparency on how the measures are made, the lack of robustness, and the lack of reproducibility (Timonin, 2013). Another alternative seems to be appropriate when using an optimization algorithm alongside a minimal intuitive determination by the DM. These approaches take advantage of the lattice structure of the coefficients (Mori & Murofushi, 1989).

While most of these methods are developed within a pure mathematical framework, some others were reflected in a limited number of applications such as computer vision, pattern recognition, software engineering, and website design. To our knowledge, none of the developed approaches are applied to an inherently cross-disciplinary engineering design problem with multiple design objectives, e.g. mechatronics design. In this paper, we will explore various approaches of fuzzy measure identification applied to a mechatronic design problem. A Choquet integral aggregation was previously used by the authors for the multicriteria design of a mechatronic system in Mohebbi et al. (2014a) and Mohebbi, Achiche, and Baron (2019) where the measures were determined intuitively by the authors and a group of 30 researchers (all specialized in system design and mechatronics) through a questionnaire.

3. Multicriteria Design of Mechatronic Systems

3.1. Conceptual design

Conceptual design is an early stage of design in which the designers generally choose among the concepts that fulfill the design requirements and then decide how to interconnect these concepts into system architectures. Usually, at the beginning of every conceptual design process, a large number of candidate concepts exist for a given design problem. Consequently, a considerable amount of uncertainty arises about which of these solutions will be best fitted to the given requirements and objectives. This is more evident when the designer has to meet highly dynamic and interconnected design requirements. It is crucial to abandon the traditional end-to-end and sequential design process and to consider all aspects of a design problem concurrently. This is particularly necessary for multidisciplinary systems such as mechatronic systems where mechanical, control, electronic, and software components interact and a high-quality design cannot be achieved without simultaneously considering all domains (Rzevski, 2003).

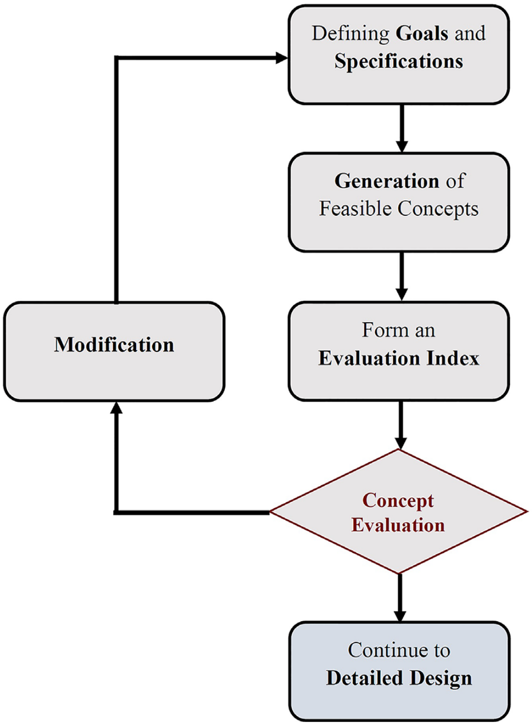

Figure 1: Process of concept evaluation in design.

3.2. Concept evaluation

To achieve more optimal mechatronics designs, one requires a systematic evaluation approach to choose among the candidate design solutions. This evaluation includes both comparison and decision-making (Coelingh, de Vries, & Koste, 2002). In other words, decision-making is achieved by selecting the "best" alternatives by comparison. It is crucial to take into account both correlation between system requirements and also interactions between the multidisciplinary subsystems. The candidate solutions are generated based on a series of design specifications. The goal of concept evaluation is to compare the generated concepts against the requirements and to select the best one for the detailed design and optimization stages. This process is illustrated in Fig. 1.

3.3. Mechatronic multicriteria profile (MMP)

One important challenge faced during conceptual design is to find the right set of criteria to concurrently evaluate and synthesize the designs. Generally, making design decisions with multiple criteria is often performed using a Pareto approach. Without the identification of the system performance parameters and the full understanding of their co-influences, it is unrealistic to expect achieving optimal solutions. In order to form an integrated and systematic evaluation approach, the most significant criteria for mechatronic design and their related sub-criteria have been quantified by the authors in Mohebbi, Achiche, and Baron (2018) to form an index vector of five normalized elements

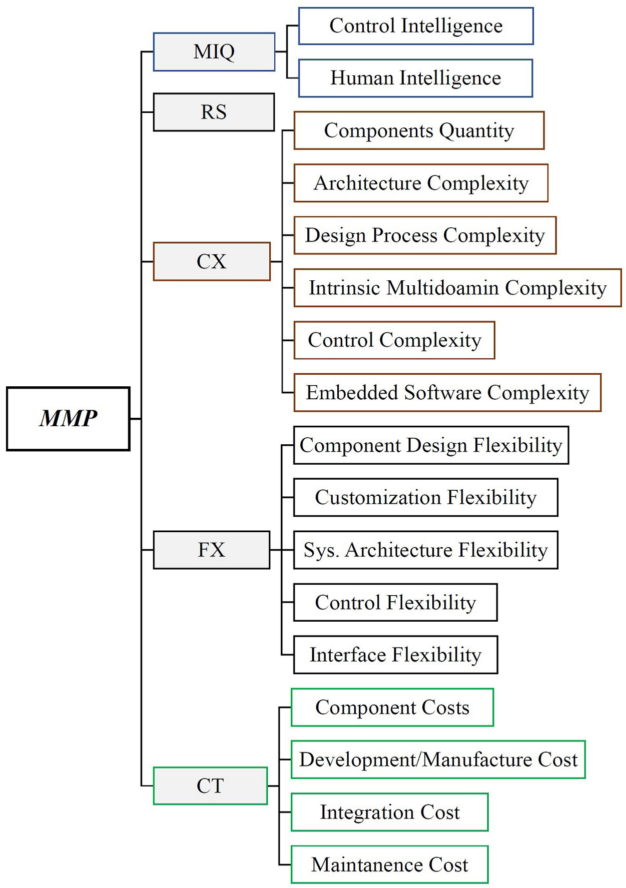

Figure 2: MMP and all sub-criteria.

called MMP as follows:

$$\mathrm{MMP} = [\mathrm{MIQ}, \mathrm{RS}, \mathrm{CX}, \mathrm{FX}, \mathrm{CT}]^{\mathrm{T}} \quad (1)$$

where MIQ is the machine intelligence quotient, RS is the reliability score, CX is the design complexity, FX is the flexibility, and CT is the cost of manufacture and production. Figure 2 describes the MMP with all corresponding sub-criteria. MMP will be used in this paper. We also define x_i as the parameters used in calculating a criterion i, using which the criteria values are calculated using a function f and $0 \leq f(x_i) \leq 1$. After determination and normalizing each sub-criterion, and by using a linear summation of weighted factors, the value of each main criterion will be assessed as follows:

$$f(x_i) = \sum_{j=1}^{n} w_j \bar{\rho}_i, \quad (2)$$

where $\bar{\rho}_i$ is the calculated value for each sub-criterion, n is the total number of sub-criteria, and w_j are the assigned-by-designer weights associated with each sub-criterion.

3.4. Detailed design

Preliminary features of a structure and the architecture of the mechatronic system are decided in the conceptual design stage where the components and subsystems of the product are specified. The control scheme is also selected in this stage without specifying its parameters. Subsequently, the calculation and specification of design parameters are done in the detailed design stage. Some of the design parameters can be specified or

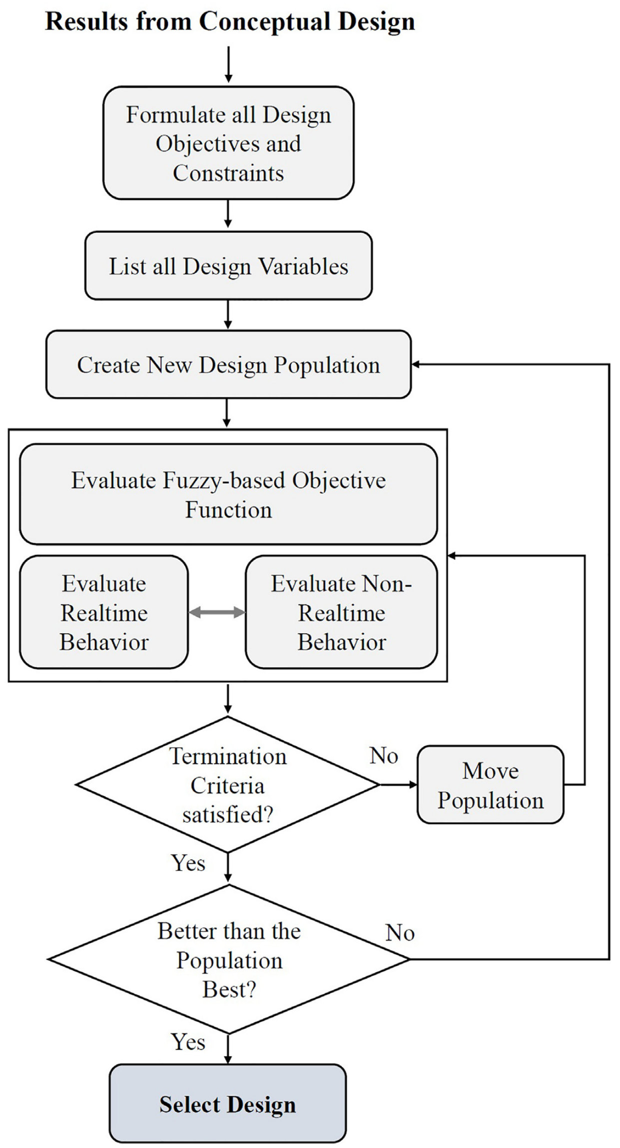

Figure 3: Proposed detailed design procedure.

tuned after the machine is built (real-time parameters) and some others are not (non-real-time parameters). Regardless of these categories, all design variables should be computed and optimized in a concurrent and integrated manner concerning multiple criteria that affect the performance of the system. We previously proposed an integrated approach for the detailed design of mechatronic systems formulated in a multiobjective cross-disciplinary design optimization problem in which the design objectives of all subsystems are considered alongside the corresponding constraints (Mohebbi et al., 2019). This approach is summarized in Fig. 3.

4. Fuzzy Decision Support and Aggregation

4.1. Criteria aggregation

The problem of aggregating criteria functions to form overall decision functions is of considerable importance in many disciplines. A primary factor in the determination of the structure of such aggregation functions is the relationship between the criteria involved. Choquet integral is a nonlinear fuzzy integral

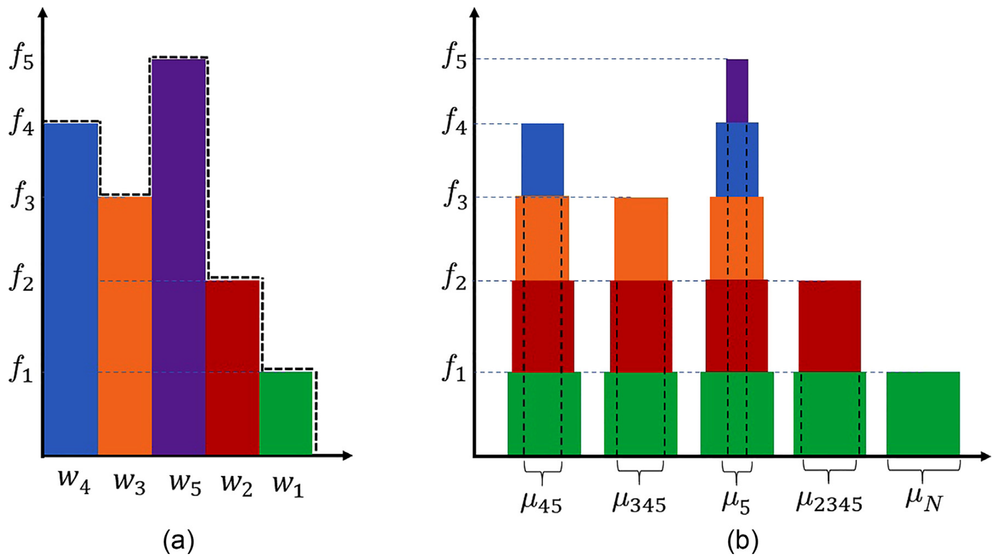

Figure 4: Graphical illustration of (a) weighted sum and (b) Choquet integral.

that has been successfully used for the aggregation of criteria in the presence of interactions. For mechatronics design and after quantifying all MMP elements and corresponding subsets, an effective comparison algorithm is needed. A global concept score (GCS) as a multicriteria evaluation index can be defined to enable the designers to compare the feasible generated design concepts. GCS can be expressed as follows:

$$GCS = S(m_1^*, m_2^*, \ldots, m_n^*) . \prod_{i=1}^{m} g(m_i), \tag{3}$$

where m_i^* are the normalized criteria values, S(.) represents an aggregation function which, in this paper, is the Choquet integral, and $g(m_i)$ indicates whether a design constraint has been met (binary value).

4.2. Fuzzy measures and Choquet integrals

Choquet integral provides a weighting factor for each criterion, and also for each subset of criteria. Using Choquet integrals is a very effective way to measure an expected utility when dealing with uncertainty, which is the case in design in general and mechatronics design in particular. The main advantage of using this technique over other methods, such as weighted mean, is that by defining a weighting factor for each subset of criteria, the interactions between multiple objectives and criteria can be easily taken into account as well as their individual importance. To help a better understanding of the proposed solution, we will state some definitions in the following paragraphs.

Definition 1: The weighting factor of a subset of criteria is represented by a fuzzy measure on the universe N satisfying the following fuzzy measure (μ) equations:

$$\mu(\phi) = 0, \ \mu(N) = 1, \tag{4}$$

$$A \subseteq B \subseteq N \rightarrow \mu(A) \leq \mu(B), \tag{5}$$

where A and B represent the fuzzy sets (Sugeno, 1975). Equation (4) represents the boundary conditions for fuzzy measures while Equation (5) is also called the monotonicity property of fuzzy measures.

Definition 2: Let μ be a fuzzy measure on vector X, whose n elements are denoted by $x_1, x_2, \ldots, x_n$. The discrete Choquet integral of a function $f: X \rightarrow \mathbb{R}^+$ with respect to μ is defined by

$$C_\mu(f) = \sum_{i=1}^{n} (f(x_i) - f(x_{i-1}))\, \mu(A_{(i)}), \tag{6}$$

where indices have been permuted so that $0 \leq f(x_1) \leq f(x_2) \leq \ldots \leq f(x_n)$ and $A_{(i)} = \{(i), \ldots (n)\}$, and $A_{(n+1)} = \emptyset$ while $f(x_0) = 0$. Figure 4 gives a graphical illustration of Choquet integral compared to a weighted sum while Table 1 shows the most common semantic interactions among criteria pairs and the corresponding fuzzy measures. The difference between $\mu(i, j)$ and $\mu(i) + \mu(j)$ reflects a degree of interaction between criteria i and j. If $\mu(i, j) = \mu(i) + \mu(j)$, there is no interaction between two criteria; if $\mu(i, j) < \mu(i) + \mu(j)$, there is a redundancy (positive correlation); and when $\mu(i, j) > \mu(i) + \mu(j)$, there is a synergy (negative correlation).

A lattice representation can be used for describing fuzzy measures in the case of a finite number of criteria. Figure 5gives an illustration when $n = 4$. Please note that for simplicity we use μ_{ij} instead of $\mu(\{i, j\})$.

Definition 3: Let μ be a fuzzy measure. The interaction index $I(\mu, ij)$ for any pair of criteria i and j is defined as follows (Marichal, 2002):

$$I(\mu, ij) = \sum_{T \subseteq N \backslash i,j} \frac{(n-t-2)!t!}{(n-1)!} \times [\mu(T \cup ij) - \mu(T \cup i) - \mu(T \cup j) + \mu(T)], \tag{7}$$

where T is a subset of criteria. The interaction index ranges in [−1, 1].

Definition 4: The importance index $\phi(\mu, i)$ for a criterion i is computed by the Shapley value (ϕ) (Marichal, 2002), which is defined as

$$\phi(\mu, i) = \sum_{T \subseteq N \backslash i} \frac{(n-t-1)!t!}{n!} [\mu(T \cup i) - \mu(T)]. \tag{8}$$

Table 1: Fuzzy interactions and measurements.

#	Description of the interaction	Fuzzy relation
I	Negative correlation (synergy)	$\mu(i,\ j) > \mu(i) + \mu(j)$
II	Positive correlation (redundancy)	$\mu(i,\ j) < \mu(i) + \mu(j)$
III	Substitution	$\mu(T)_{T\subseteq N\setminus i,j} < \{ {\mu(T\cup i) \atop \mu(T\cup j)} \simeq \mu(T\cup i\cup j)$
IV	Veto effect	$\mu(T) \approx 0 \text{ if } T \subset N, i \notin T$
V	Pass effect	$\mu(T) \approx 1 \text{ if } T \subset N, i \in T$
VI	Complementary	$\mu(T)_{T\subseteq N\setminus i,j} \simeq \{ {\mu(T\cup i) \atop \mu(T\cup j)} \simeq \mu(T\cup i\cup j)$

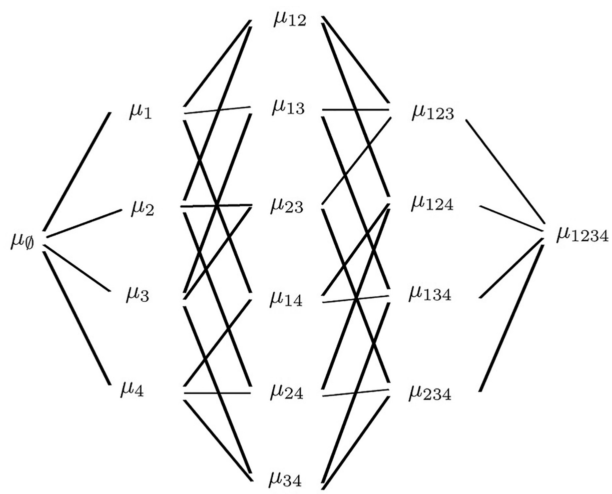

Figure 5: Lattice of the coefficients of a fuzzy measure ($n = 4$).

The Shapley value ranges between [0, 1] and represents a true sharing of the total amount $\mu(N)$, since

$$\sum_{i=1}^{n} \phi(\mu, i) = \mu\ (N) = 1. \tag{9}$$

It is convenient to scale these values by a factor n, so that an importance index greater than 1 indicates an attribute more important than the average.

Lemma 1. If the coefficients $\mu(\{i\})$ and $\mu(\{i,\ j\})$ are given for all $i,\ j\ \in N$, then the necessary and sufficient conditions that μ is a 2-additive measure are

$$\sum_{\{i,\ j\}\subseteq N} \mu(\{i,\ j\}) - (n-2)\sum_{i\in N} \mu(\{i\}) = 1 \quad \text{(Normality)} \tag{10}$$

$$\begin{matrix} \mu(\{i\}) \geq 0,\ \forall i \in N \\ \forall A \subseteq N,\ \ |A| \geq 2,\ \forall k \in A, \end{matrix} \quad \text{(Non}-\text{negativity)} \tag{11}$$

$$\sum_{i\in A\setminus\{k\}} (\mu(\{i, k\}) - \mu(\{i\})) \geq (|A| - 2)\,\mu(\{k\}) \quad \text{(Monotonicity)}. \tag{12}$$

The expression of the 2-additive Choquet is

$$C_\mu(f) = \sum_{i=1}^{n} \phi(\mu,\ i)\, f(x_i) - \frac{1}{2}\sum_{\{i,\ j\}\subseteq N} I(\mu, ij)\,|f(x_i) - f(x_j)|. \tag{13}$$

Here, $I(\mu,\ ij) = 0$ means criteria i and j are independent while $I(\mu,\ ij) > 0$ means there is a complementary among i and j and that for the DM; both criteria have to be satisfactory in order to get a satisfactory alternative. If $I(\mu,\ ij) < 0$, then there is sub-

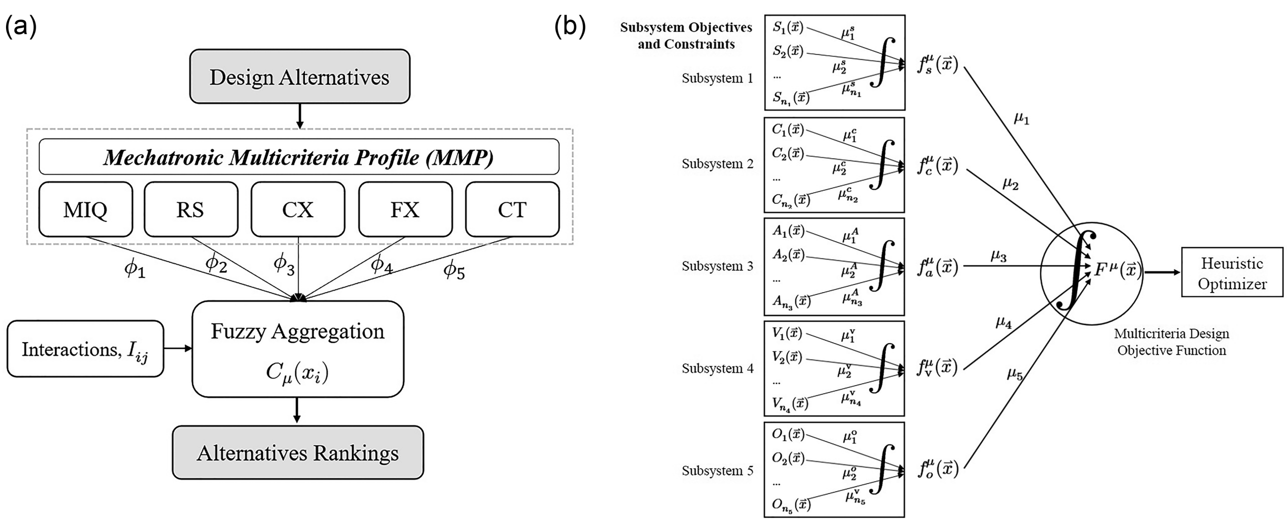

Figure 6: Fuzzy-based design of a mechatronic system for (a) concept evaluation, and (b) detailed design using a multiobjective optimization scheme.

stitutability or redundancy among i and j. This means that for the DM, the satisfaction of one of the two criteria is sufficient to have a satisfactory alternative. It is worthy to note that a *positive correlation* leads to a *negative interaction index*, and vice versa. The fuzzy measures should be specified in such a way that the desired overall importance and the interaction indices are satisfied.

4.3. Fuzzy-based design schemes

Using the formulations described in Sections 4.1 and 4.2 for aggregation of interacting criteria, the procedure of conceptual and detailed design can now be illustrated as Fig. 6a and b. In the conceptual stage and using the assessed MMP, the fuzzy measures are used to specify the weight of importance and interactions among design criteria. Then, each design alternative is evaluated by incorporating a Choquet aggregation function and a rank on the elite set of concepts is provided to port to the detailed design stage. In detailed design, a multiobjective optimization process is considered to concurrently design for real-time and non-real-time variables that correspond to the optimal behavior of the overall system. In order to provide the optimization algorithm with an interactive objective function that includes all the design requirements from various disciplines, a cascade Choquet integral-based aggregation is used. This takes into account all the interactions among design objectives and also their relative importance in the design process.

5. Identification of Fuzzy Measures

As shown in Fig. 6, in both stages, identification of fuzzy measures is a crucial stage that should be carefully done to correctly reflect on the decision-making process. We now address the problem of identification of $(2^n - 2)$ fuzzy measures, μ, taking into account the monotonicity relations between the coefficients and the preferences specified by requirements and the DMs. Four different approaches are essentially discussed here.

5.1. Identification using Sugeno measures

As the number of criteria, n, grows specifications of the fuzzy measures using the aforementioned methods become more and more difficult. Sugeno (1975) created a way to automatically generate the entire lattice based on just the singleton μ_i densities, thus $(2^n - 2 - n)$ values. The Sugeno λ–fuzzy measure has the following additional property: If $A,\ B \in \Omega$ and $A \cap B = \emptyset$,

$$\mu\ (A \cup B) = \mu\,(A) + \mu\,(B) + \lambda\mu\,(A)\mu(B)\,. \tag{14}$$

It is proven that a unique λ can be found by solving the following equation:

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda\mu_i), \quad -1 < \lambda < \infty,\ \lambda \neq 0, \tag{15}$$

where $\mu_i = \mu\,\{x_i\}$. Thus, the n densities determine the 2^n values of a Sugeno measure. There are three cases with regard to the singleton measures:

$$\text{If } \sum_{i=1}^{n} \mu_i > \mu(N), \text{ then } \quad -1 < \lambda < \infty. \tag{16}$$

$$\text{If } \sum_{i=1}^{n} \mu_i = \mu(N), \text{ then } \lambda = 0. \tag{17}$$

$$\text{If } \sum_{i=1}^{n} \mu_i < \mu(N), \text{ then } \lambda > 0. \tag{18}$$

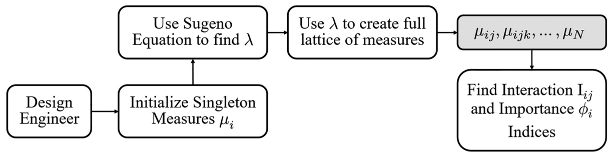

Figure 7: Identification of fuzzy measures using Sugeno process.

The process of using the Sugeno method to identify the full lattice of fuzzy measures is summarized in Fig. 7.

5.2. Identification based on learning data

Having a set of learning data in hand, the parameters of a Choquet integral model can be identified by minimizing an error criterion. Suppose that $(f_k,\ y_k)$, $k = 1,\ 2,\ \dots,\ l$ are learning data where $f_k = [f^k(x_1),\ \dots,\ f^k(x_n)]^T$ is an n-dimensional input vector, containing the degrees of satisfaction or quantified assessment values of alternative (concept) k with respect to criteria 1 to n, and y_k is the global evaluation of object k (not necessarily an aggregated value). There must be at least $l = \frac{n!}{[(\frac{n}{2})!]^2}$ (when n is even) or $l = \frac{n!}{[\frac{n-1}{2}]![\frac{n+1}{2}]!}$ (when n is odd) sets of learning data (Grabisch, Nguyen, & Walker, 2013). Then, one can try to identify the best fuzzy measure $\mu*$ so that the squared error criterion (E) is minimized (Grabisch, 1996).

$$E^2 = \sum_{k=1}^{l} \left[C_\mu\left(f^k(x_1), \dots,\ f^k(x_n)\right) - y_k\right]^2 \tag{19}$$

Under a quadratic program form, we have

$$\min\left[\ E^2 = \left(\frac{1}{2}\mathbf{u}^t\mathbf{D}\mathbf{u} + \mathbf{c}^t\mathbf{u}\right)\right], \tag{20}$$

where $\mathbf{u}$ is a $(2^n - 2)$ dimensional vector containing all the coefficients of the fuzzy measure μ, except for $\mu_\emptyset = 0$ and $\mu_N = 1$, as follows:

$$\mathbf{u} = [[\mu_i],\ [\mu_{ij}],\ [\mu_{ijk}],\ [\mu_{ijkl}],\ \dots]^T\,. \tag{21}$$

It is important to note that the components of $\mathbf{u}$ are not independent of each other because fuzzy measures must satisfy a set of monotonicity relations. Moreover, $\mathbf{D}$ is a symmetric $(2^n - 2)$ dimensional matrix and $\mathbf{c}$ is a $(2^n - 2)$ dimensional vector. The first set of constraints contains the measures monotonicity constraints described as follows:

$$\mathbf{A}\mathbf{u} + \mathbf{b} \le 0, \tag{22}$$

where matrix $\mathbf{A}$ is a $n(2^{n-1} - 1) \times (2^n - 2)$ dimensional matrix and $\mathbf{b}$ is a $n(2^{n-1} - 1)$ vector defined by

$$\mathbf{b} = \left[0,\ \dots,\ 0,\ \underbrace{-1,\ \dots,\ -1}_{n}\right]^T. \tag{23}$$

More precisely for Equation (19) we have:

$$C_\mu\,(f_k) = \mathbf{c}_k^t\,.\mathbf{u} + f^k(x_1)\,, \tag{24}$$

where $\mathbf{c}_k$ is a $(2^n - 2)$ dimensional vector containing the differences $f(x_i) - f(x_{i-1})$, $i = 2,\ \dots,\ n$, so that there are at most $(n - 1)$ non-zero terms in it, which are all positive. Accordingly, we attain

$$\mathbf{c} = 2\sum_{k=1}^{l} (f^k(x_1) - y_k)\,\mathbf{c}_k. \tag{25}$$

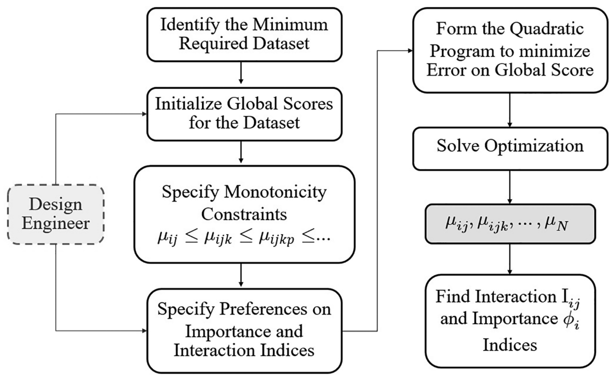

Figure 8: Identification of fuzzy measures using learning data and quadratic programming.

Additionally, D_k is a $(2^n - 2)$ dimensional square matrix where

$$\mathbf{D} = 2\sum_{k=1}^{l} \mathbf{D}_k = 2\sum_{k=1}^{l} \mathbf{c}_k \mathbf{c}_k^T. \tag{26}$$

Thus, we can rewrite the program in Equation (20) as

$$\min\left[E^2 = 2\sum_{k=1}^{l} \mathbf{u}^T \mathbf{c}_k \mathbf{c}_k^T \mathbf{u} + 2\sum_{k=1}^{l} \mathbf{c}_k^T.\mathbf{u}\ \left(f^k(x_1) - y_k\right)\right]. \quad \text{Subj. to } \mathbf{Au} + \mathbf{b} \leq 0 \tag{27}$$

Since $u^T\mathbf{D}u$ consists of a sum of squares, thus for all $u \geq 0$, $u^T\mathbf{D}u \geq 0$ and $\mathbf{D}$ is positive semi-definite. The above quadratic program has a unique (global) minimum since the criterion to be minimized is convex. This solution can be a point or a convex set in $[0, 1]^{2n-2}$. This program can be solved by any standard method of quadratic optimization, although matrix $\mathbf{D}$ may be ill-conditioned ($rank < 2^n - 2$) since based on the definition of vector $\mathbf{c}_k$, matrix $\mathbf{D}$ contains columns and rows of zeroes. Obviously, this effect will disappear if the number of training data increases.

Now, we can take into account the decision maker's preferences with regard to the importance of criteria and interactions among criterion pairs as constraint relations.

$$\mu(A \cup i) - \mu(A) \geq 0,\ \forall i \in N,\ \forall A \in N \backslash i \tag{28}$$

$$C_\mu(f) - C_\mu\left(f'\right) \geq \delta_C \tag{29}$$

$$\phi(\mu, i) - \phi(\mu, j) \geq \delta_\phi \tag{30}$$

$$\text{Constraints on } I(\mu, ij) \tag{31}$$

The process of using learning data in addition to the designer's preferences to identify the fuzzy measures is summarized in Fig. 8.

5.3. Identification based on fuzzy measure semantics and learning data

To reduce the complexity and provide better guidelines for the identification of measures, the combination of semantical considerations with learning data can lead to a more efficient algorithm. With this approach, the objective would be to minimize the distance to the additive equidistributed fuzzy measure defined by $\mu_j = 1/n$. Consequently, instead of trying to minimize the sum of the squared errors between model output and data, we try to minimize the distance to the additive equidistributed measure set $\mathbf{u_o}$. Thus, we can have the following quadratic form.

$$\text{Min } J = \tfrac{1}{2}(u - u_0)^T (u - u_0) \quad \text{Subj. to } \mathbf{Au} + \mathbf{b} \leq 0 \tag{32}$$

Here, training data are no longer in the objective function, but are used as the second set of constraints.

$$y_k - \delta_k \leq \mathbf{c}_k^t.\mathbf{u} + f(x_1) \leq y_k + \delta_k \tag{33}$$

Moreover, the DM needs to express some preferences about the fuzzy measures on sets A and B.

$$\mu(A) \leq \eta\mu(B) \tag{34}$$

If the DM considers a positive correlation (redundancy) between A and B, then $\mu(A \cup B) < \mu(A) + \mu(B)$ and this interaction can be modeled by

$$\mu(A \cup B) = \mu(A) + \lambda\mu(B), \tag{35}$$

where $\mu_A \geq \mu_B$ and η defines the degree of the relative importance of A with respect to B. For the interactions between criteria A and B, $\lambda \in [0, 1]$ and A and B are fully dependent when $\lambda = 0$, and independent when $\lambda = 1$. The correlation between A and B can be modeled by

$$\mu(A \cup B) = \mu(A) + \mu(B) + \gamma\left[1 - \mu(A) - \mu(B)\right], \tag{36}$$

where γ specifies the level of support between criteria pairs. All these constraints based on the DM's preferences can be used to modify the initial monotonicity constraint by adding to the initial $\mathbf{A}$ and $\mathbf{b}$ and form a new constraint as

$$\mathbf{A}'\mathbf{u} + \mathbf{b}' \leq 0. \tag{37}$$

In order to use Equations (32–36) for modeling the relations between criteria pairs, we define the proper linguistics as described in Tables 2–4.

Table 2: Linguistic representation of the relative importance of criteria.

Relative importance	Value
Same level	$0.9 \leq \eta \leq 1.1$
A is a little more important than B	$1.1 \leq \eta \leq 1.3$
A is more important than B	$1.3 \leq \eta \leq 1.7$
A is quite more important than B	$1.7 \leq \eta \leq 1.9$

Table 3: Linguistic representation of dependence between criteria.

Criteria dependence	Value
Highly dependent	$\lambda = 0.0$
Dependent	$0.0 \leq \lambda \leq 0.5$
A little dependent	$0.5 \leq \lambda \leq 1.0$
Independent	$\lambda = 1.0$

Table 4: Linguistic representation of support between criteria.

Criteria synergy	Value
High support	$\gamma = 1.0$
Support	$0.5 \leq \gamma \leq 1.0$
A little support	$0.0 \leq \gamma \leq 0.5$

Table 5: Design alternatives adopted from Mohebbi et al. (2016a).

Components	Concept I	Concept II	Concept III	Concept IV
Frame structure	X-shape	H-shape	X-shape	H-shape
Material	AL	AL	Poly	Poly
Motors	Brushed DC	Brushed DC	Brushless DC	Brushless AC
Motor encoder	Optical	Magnetic	Optical	Magnetic
Visual servo	PBVS	PBVS	IBVS	IBVS
Camera config	Mono	Stereo	Stereo	Mono
Motion control	PID	LQR	PID	LQR
Position sensor	GPS + Accel	Motion Cam	GPS + Accel	GPS + Accel
Battery	Li-ion	Li-poly	Li-ion	Li-poly

Table 6: Estimated design parameters for generated concepts.

Requirements	Concept I	Concept II	Concept III	Concept IV
Power (W)	450	500	350	400
Max inertia moment (kg.m^2)	5E-3	5.2E-3	4E-3	4.5E-3
Bandwidth (Hz)	70	70	60	60
Payload (Kg)	0.5	0.5	0.6	0.6
Cost (unit) (normal)	0.8	1	0.7	0.7

6. Case Study: Conceptual Design of a Vision-Guided Quadrotor Drone

Recently, the quadrotors are being deployed as highly maneuverable aerial robots that have the ability of easy hover, take off, fly, and land in small and remote areas (Mohebbi, Achiche, & Baron, 2015). Recent technological advances in energy storage devices, sensors, actuators, and information processing have boosted the development of unmanned aerial vehicle platforms with significant capabilities. Unmanned quadrotor helicopters are excellent examples of highly coupled mechatronic systems where the disciplines of aerodynamics, structures, materials, flight mechanics, and control are acting upon each other in a typical flight condition. Moreover, the integration of vision sensors with robots has helped solve the limitation of operating in non-structured environments (Mohebbi, Keshmiri, & Xie, 2016b).

Here, the discussed fuzzy measure identification methods are utilized in a conceptual design process using the MMP for a vision-guided quadrotor unmanned aerial vehicle. From our previous work (Mohebbi, Achiche, & Baron, 2016a), we have chosen four concepts to study the proposed design method. Table 5 shows the design alternative and the corresponding subsystems and components. Based on the material used, the frame structure and subsystems selected for one specific concept, the total mass, required power, payload, maximum allowable inertia moment, force, and bandwidth can be also easily estimated. An approximation of the total cost can also be calculated based on the components and manufacturing process.Table 6 briefly gives the results for the estimated values for the proposed concepts.

Ultimately, by using a set of intuitive Choquet fuzzy measures, the evaluations for all concepts and corresponding design criteria are listed in Table 7. Details of the criteria assessment and calculations are thoroughly discussed and exemplified in our previous work introducing the MMP (Mohebbi et al., 2014a, 2016a). The fuzzy measures used in the previous study were obtained intuitively by the authors and a group of 30 researchers (all specialized in system design and mechatronics) through a questionnaire. In this questionnaire, the participants were asked to reflect their intuitive idea about the importance of each criterion in designing a good mechatronic product in terms of a score between 1 and 10. Moreover, the degree of correlation between each pair of criteria or the effect of increasing criterion i on criterion j was also asked and reflected in terms of a score between −10 and 10. Then, the obtained values were transformed into fuzzy measures that fit the requirements discussed in Equations (4, 5, 10–12). These measures are shown in Table 8:

$$\Phi = [\phi_1, \ \phi_2, \ \phi_3, \ \phi_4, \ \phi_5] \\ = [0.2085, \ 0.2612, \ 0.1598, \ 0.1431, \ 0.2020]. \tag{38}$$

We remind that in order to calculate a Choquet integral and its corresponding measures, a permutation on the criteria values should be initially performed in such a way that $0 \leq f(x_1) \leq f(x_2) \leq \ldots \leq f(x_n)$. However, throughout our case study and to avoid any confusion, we reshape the outputs for measures and also importance indices at the end of the identification algorithm so that the following order always persists:

$$\Phi = [\phi_1, \ \phi_2, \ \phi_3, \ \phi_4, \ \phi_5] \\ = [\phi_{MIQ}, \ \phi_{RS}, \ \phi_{CX}, \ \phi_{FX}, \ \phi_{CT}]. \tag{39}$$

6.1. Identification using Sugeno measures

Based on Equations (14 and 15) for five criteria illustrated in Table 7, we have

$$\lambda + 1 = (\lambda\mu_1 + 1)(\lambda\mu_2 + 1)(\lambda\mu_3 + 1)(\lambda\mu_4 + 1)(\lambda\mu_5 + 1) \\ -1 < \lambda < \infty, \ \lambda \neq 0, \tag{40}$$

where for μ_i we use the values from Table 8. The solution of the above equation yields $\lambda = 0.0255$ and consequently, we attain the results for fuzzy measures listed in Table 9.

The fuzzy measures obtained by the Sugeno λ−method yield the following importance indices:

$$\Phi = [\phi_1, \ \phi_2, \ \phi_3, \ \phi_4, \ \phi_5] \\ = [0.2221, \ 0.2422, \ 0.1718, \ 0.1617, \ 0.2020]. \tag{41}$$

Table 7: Concept evaluations for design alternatives (Mohebbi et al., 2014a, 2016a).

MMP	Concept I	Concept II	Concept III	Concept IV
MIQ	0.84	0.84	1	1
RS	0.86	0.91	0.93	1
CX	0.85	0.69	0.93	0.89
FX	0.91	0.96	0.91	0.88
CT	1	0.78	0.94	0.91
GCS$_\mu$	0.89	0.83	0.96	0.94

6.2. Identification using a learning set

As mentioned before, in order to identify the fuzzy measures, it is possible to employ a "learning set"—a number of objects whose assessment is manually performed by the DM. According to Grabisch et al. (2013), the minimum number of data sets we need to solve the squared error minimization program (20) is equal to

$$l = \frac{n!}{\left[\frac{n-1}{2}\right]!\left[\frac{n+1}{2}\right]!} = \frac{5!}{\left[\frac{5-1}{2}\right]!\left[\frac{5+1}{2}\right]!} = 10. \tag{42}$$

Accordingly, we need to provide 10 sets of criteria evaluation and corresponding GCSs. The vector of variables contains the 30 fuzzy measures and as for the monotonicity constraints described in Equation (22) we have the following matrices:

$$\mathbf{A}_{[75\times30]}, \mathbf{u}_{[30\times1]}, \mathbf{b} = \left[0,\ \ldots,\ 0,\ \underbrace{-1,\ \ldots,\ -1}_{5}\right]^T_{[75\times1]}, \tag{43}$$

in which we describe all 75 monotonicity relations such as

$$\begin{aligned} &\mu_1 \le \mu_{12},\ \ldots,\ \mu_5 \le \mu_{45},\\ &\mu_{12} \le \mu_{123},\ \ldots,\ \mu_{45} \le \mu_{345},\\ &\mu_{123} \le \mu_{1234},\ \ldots,\ \mu_{345} \le \mu_{2345},\\ &\mu_{1234} \le 1,\ \ldots,\ \mu_{2345} \le 1. \end{aligned} \tag{44}$$

In order to form the objective function from Equation (20), we also need to form the matrix **D** and vector **c**, which have the following format:

$$\begin{aligned} &\mathbf{D}_{[30\times30]},\ \mathbf{c}_{[30\times1]},\ \mathbf{c}_{k[30\times1]}\\ &\mathbf{c} = 2\sum_{k=1}^{10}\left(f^k(x_1) - y_k\right)\mathbf{c}_k, \end{aligned} \tag{45}$$

$$\mathbf{D} = 2\sum_{k=1}^{10}\mathbf{D}_k = 2\sum_{k=1}^{10}\mathbf{c}_k\mathbf{c}_k^T, \tag{46}$$

in which $\mathbf{c}_k$ is a 30D vector containing the differences $f(x_i) - f(x_{i-1})$, $i = 2, \ldots, 5$ so that there are at most four non-zero terms in it, which are all positive. Consequently, we get

$$\begin{aligned} \mathbf{c}_k\ (5) &= f^k\ (x_5) - f^k(x_4),\\ \mathbf{c}_k\ (15) &= f^k\ (x_4) - f^k(x_3),\\ \mathbf{c}_k\ (25) &= f^k\ (x_3) - f^k(x_2),\\ \mathbf{c}_k\ (30) &= f^k\ (x_2) - f^k(x_1),\\ \mathbf{c}_k\ (i) &= 0,\ \ (\forall i \ne 5,\ 15,\ 25,\ 30). \end{aligned} \tag{47}$$

Finally, the DM's preferences can be taken into account using the constraints listed in Table 10.

The above problem will be solved here using MATLAB quadratic programming from the optimization toolbox and the method of "interior-point-convex." Table 11shows the resulting values for the fuzzy measures.

The above results will lead to the following importance indices:

$$\begin{aligned} \Phi &= [\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5]\\ &= [0.2145,\ 0.2535,\ 0.1701,\ 0.1597,\ 0.1967]. \end{aligned} \tag{48}$$

6.3. Identification based on fuzzy measure semantics and learning data

The linguistics described in Tables 2–4 in addition to the monotonicity conditions are translated into the constraints as the DM's preferences as described in Table 12.

This approach can also include an interactive dialogue between the DM and the fuzzy measure identifying system. Solutions are presented to the DM, who can refine them by specifying or modifying the relative importance and interaction between criteria if he is not satisfied with the solution. As an example, here we use the concept evaluation data from our previous work. As for the additive equidistributed singleton fuzzy measures, we have

$$M_0 = [0.2\ 0.2\ 0.2\ 0.2\ 0.2]\ . \tag{49}$$

Moreover, we use the 10 training data sets from the previous section to form the following second set of constraints based on Equation (37) with $\delta_k = 0.35$:

$$\begin{aligned} &0.54 \le c_1^T\mathbf{u} + 0.84 \le 1.24, && 0.47 \le c_6^T\mathbf{u} + 0.64 \le 1.17,\\ &0.48 \le c_2^T\mathbf{u} + 0.69 \le 1.18, && 0.19 \le c_7^T\mathbf{u} + 0.45 \le 0.89,\\ &0.61 \le c_3^T\mathbf{u} + 0.91 \le 1.31, && 0.53 \le c_8^T\mathbf{u} + 0.75 \le 1.23,\\ &0.59 \le c_4^T\mathbf{u} + 0.88 \le 1.29, && 0.58 \le c_9^T\mathbf{u} + 0.85 \le 1.28,\\ &0.44 \le c_5^T\mathbf{u} + 0.72 \le 1.14, && 0.07 \le c_{10}^T\mathbf{u} + 0.35 \le 0.77, \end{aligned} \tag{50}$$

where c_k^T is a $[1 \times 30]$ vector and can be calculated from Equation (33), while for **u** we have

$$\mathbf{u}_{[30\times1]} = [[\mu_i],\ [\mu_{ij}],\ [\mu_{ijk}],\ [\mu_{ijkl\ldots}],\ \ldots]^T. \tag{51}$$

By combining all the constraints in Equation (50), Table 13 and also the monotonicity constraints, we can formulate a new linear constraint as $A'u + b' \le 0$ and solve the quadratic program

Table 8: Fuzzy measures for the conceptual design of a Quadrotor drone equipped with a visual servoing system.

$\mu_1 = 0.23$	$\mu_{12} = 0.45$	$\mu_{13} = 0.47$	$\mu_{14} = 0.34$	$\mu_{15} = 0.51$
$\mu_{123} = 0.61$	$\mu_2 = 0.29$	$\mu_{23} = 0.52$	$\mu_{24} = 0.42$	$\mu_{25} = 0.56$
$\mu_{124} = 0.60$	$\mu_{135} = 0.69$	$\mu_3 = 0.17$	$\mu_{34} = 0.35$	$\mu_{35} = 0.33$
$\mu_{125} = 0.67$	$\mu_{145} = 0.67$	$\mu_{245} = 0.73$	$\mu_4 = 0.16$	$\mu_{45} = 0.41$
$\mu_{134} = 0.63$	$\mu_{234} = 0.68$	$\mu_{345} = 0.49$	$\mu_{235} = 0.62$	$\mu_5 = 0.22$
$\mu_{1234} = 0.77$	$\mu_{1235} = 0.84$	$\mu_{1345} = 0.84$	$\mu_{2345} = 0.78$	$\mu_{1245} = 0.82$

Table 9: Fuzzy measures identified using Sugeno λ-measures.

$\mu_1 = 0.22$	$\mu_{12} = 0.4613$	$\mu_{13} = 0.3910$	$\mu_{14} = 0.3809$	$\mu_{15} = 0.4211$
$\mu_{123} = 0.6333$	$\mu_2 = 0.24$	$\mu_{23} = 0.4110$	$\mu_{24} = 0.4010$	$\mu_{25} = 0.4412$
$\mu_{124} = 0.6232$	$\mu_{135} = 0.5930$	$\mu_3 = 0.17$	$\mu_{34} = 0.3307$	$\mu_{35} = 0.3709$
$\mu_{125} = 0.6637$	$\mu_{145} = 0.5828$	$\mu_{245} = 0.6030$	$\mu_4 = 0.16$	$\mu_{45} = 0.3608$
$\mu_{134} = 0.5526$	$\mu_{234} = 0.5727$	$\mu_{345} = 0.5324$	$\mu_{235} = 0.6131$	$\mu_5 = 0.20$
$\mu_{1234} = 0.7959$	$\mu_{1235} = 0.8366$	$\mu_{1345} = 0.7554$	$\mu_{2345} = 0.7756$	$\mu_{1245} = 0.8264$

Table 10: DM's preferences on criteria relations.

Maximum separation of alternatives	
$C_\mu(f) - C_\mu(f') \geq \delta_c$ ($\delta_c = 0.05$)	
Preferences on the importance of criteria	
$\phi_2 - \phi_1 \geq \in$	$\phi_2 - \phi_5 \geq \in$
$\phi_1 - \phi_3 \geq \in$	$\phi_5 - \phi_3 \geq \in$
$\phi_1 - \phi_4 \geq \in$	$\phi_5 - \phi_4 \geq \in$
$\phi_2 - \phi_3 \geq \in$	$\phi_1 = \phi_5$
$\phi_2 - \phi_4 \geq \in$	$\phi_3 = \phi_4$
Preferences on the interactions between criteria pairs	
$I(1.5) - I(1.3) \geq \in$	$I(4.5) - I(3.4) \geq \in$
$I(2.5) - I(2.3) \geq \in$	$I\ (2.4) = I(3.4)$
$I(1.3) - I(2.4) \geq \in$	$I\ (1.4) = I(3.5)$

in Equation (32). Again, by using MATLAB quadratic programming and the interior-point-convex algorithm, we attain the following results.

Accordingly, we get the following Shapley values:

$$\Phi = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5] = [0.2085, 0.2612, 0.1598, 0.1431, 0.2020]. \quad (52)$$

7. Discussion and Comparison

Figure 9 describes the evolution of the full lattice of the fuzzy measure identified using the three methods discussed in this paper. Sugeno measures are among the most widely used fuzzy measures (Tahani & Keller, 1990). Using λ-measures is an abstract and efficient way when there is not enough information about DM's preferences or the order of preference on alternatives or interaction and importance indices. It can rapidly generate the entire lattice of fuzzy measures based on just the singleton densities. However, not all expert reasoning can be described by these measures and guessing the μ_i values intuitively is not a trivial process. In that case, this method can be also regarded as an optimization problem with all the preferences as constraints. Further information can be found in Lee and LeeKwang (1995) since a complete identification process on λ- measures was not in the scope of this paper.

The identification based on learning data that uses minimization of the squared error needs only a global score, which can be provided by a ranking of the acts through a suitable mechanism. Besides the fuzzy measure, the output also provides an estimation of the model error. One important advantage of using this method is that having a proper optimization solver; it always provides a solution, which fits the given global scores. Moreover, the method does not need any information on the decision strategy (importance and interaction). It is perfectly suitable for identifying hidden decision behavior. However, it may temper with the concept rankings provided by the DM.

In the identification based on combined fuzzy semantics and learning data, we need a ranking of the acts, not necessarily the global scores, a ranking on the importance of the criteria, and possibly some information on the interactions. There is no notion of model error in this approach in the sense that either there is a solution satisfying the constraints, or there is not.

Table 11: Results for fuzzy measures identified using a learning set.

$\mu_1 = 0.3292$	$\mu_{12} = 0.4502$	$\mu_{13} = 0.6366$	$\mu_{14} = 0.2985$	$\mu_{15} = 0.6416$
$\mu_{123} = 0.7983$	$\mu_2 = 0.2829$	$\mu_{23} = 0.5137$	$\mu_{24} = 0.5615$	$\mu_{25} = 0.5332$
$\mu_{124} = 0.4398$	$\mu_{135} = 0.7296$	$\mu_3 = 0.1901$	$\mu_{34} = 0.4698$	$\mu_{35} = 0.1789$
$\mu_{125} = 0.8048$	$\mu_{145} = 0.6610$	$\mu_{245} = 0.8620$	$\mu_4 = 0.2584$	$\mu_{45} = 0.5167$
$\mu_{134} = 0.6273$	$\mu_{234} = 0.8137$	$\mu_{345} = 0.5088$	$\mu_{235} = 0.5446$	$\mu_5 = 0.2082$
$\mu_{1234} = 0.8093$	$\mu_{1235} = 0.9334$	$\mu_{1345} = 0.8093$	$\mu_{2345} = 0.8093$	$\mu_{1245} = 0.8444$

Table 12: DM's preferences as linear constraints.

	Relative importance of criteria	
$\mu_2 \leq 1.3\mu_1$		$0.9\mu_4 \leq \mu_3 \leq 1.1\mu_4$
$\mu_1 \leq 1.3\mu_4$		$0.9\mu_3 \leq \mu_5 \leq 1.1\mu_3$
$\mu_2 \leq 1.7\mu_4$		$0.9\mu_5 \leq \mu_1 \leq 1.1\mu_5$
$\mu_2 \leq 1.7\mu_3$		$0.9\mu_5 \leq \mu_1 \leq 1.1\mu_5$
	Dependence between criteria pairs	
$\mu_2 + 0.5\mu_3 \leq \mu_{23} \leq \mu_2 + \mu_3$		$\mu_3 + 0.8\mu_4 \leq \mu_{34} \leq \mu_3 + \mu_4$
$\mu_2 + 0.5\mu_4 \leq \mu_{24} \leq \mu_2 + \mu_4$		$\mu_4 + 0.5\mu_5 \leq \mu_{45} \leq \mu_4 + \mu_5$
$\mu_2 + 0.5\mu_5 \leq \mu_{25} \leq \mu_2 + \mu_5$		
	The synergy between criteria pairs	
	$\mu_1 + \mu_4 + 0.3(1 - \mu_1 - \mu_4) \leq \mu_{14} \leq \mu_1 + \mu_4 + 0.7(1 - \mu_1 - \mu_4)$	
	$\mu_3 + \mu_5 + 0.3(1 - \mu_3 - \mu_5) \leq \mu_{35} \leq \mu_3 + \mu_5 + 0.7(1 - \mu_3 - \mu_5)$	
	$\mu_1 + \mu_2 \leq \mu_{12} \leq \mu_1 + \mu_2 + 0.3(1 - \mu_1 - \mu_2)$	

Table 13: Fuzzy measures identified using a learning set and design semantics.

$\mu_1 = 0.3243$	$\mu_{12} = 0.4860$	$\mu_{13} = 0.6160$	$\mu_{14} = 0.2441$	$\mu_{15} = 0.6318$
$\mu_{123} = 0.8198$	$\mu_2 = 0.2615$	$\mu_{23} = 0.4741$	$\mu_{24} = 0.5514$	$\mu_{25} = 0.5090$
$\mu_{124} = 0.4258$	$\mu_{135} = 0.7174$	$\mu_3 = 0.1705$	$\mu_{34} = 0.4618$	$\mu_{35} = 0.1748$
$\mu_{125} = 0.8307$	$\mu_{145} = 0.6068$	$\mu_{245} = 0.8540$	$\mu_4 = 0.2700$	$\mu_{45} = 0.5354$
$\mu_{134} = 0.5572$	$\mu_{234} = 0.7854$	$\mu_{345} = 0.5212$	$\mu_{235} = 0.5446$	$\mu_5 = 0.2104$
$\mu_{1234} = 0.7810$	$\mu_{1235} = 0.9584$	$\mu_{1345} = 0.7810$	$\mu_{2345} = 0.7810$	$\mu_{1245} = 0.8255$

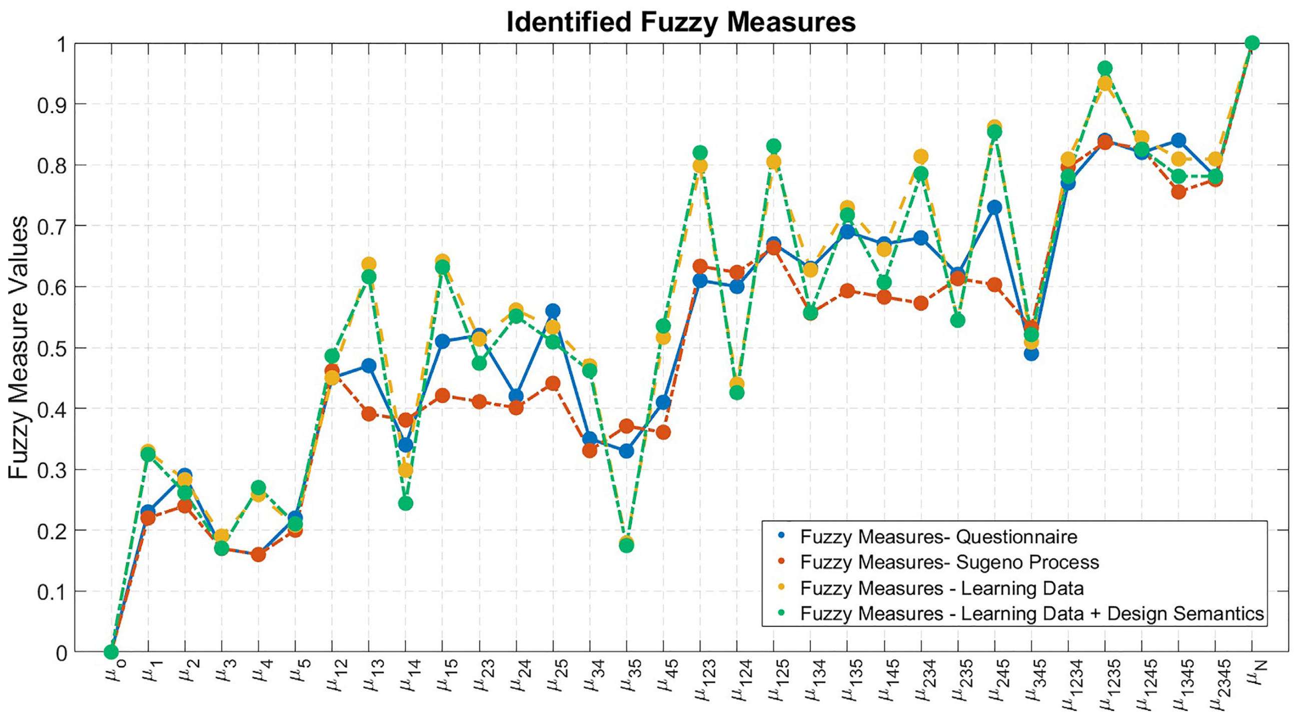

Figure 9: Identification of fuzzy measures using learning data and quadratic programming.

Table 14: GCSs calculated using the identified fuzzy measures.

Identification method	Global concept scores (GCS)			
	Concept 1	Concept 2	Concept 3	Concept 4
Questionnaire	0.89	0.83	0.96	0.94
Sugeno	0.90	0.86	0.92	0.95
QP-LD	0.87	0.81	0.90	0.96
QP-SLD	0.88	0.80	0.91	0.95

This method only requires a piece of ordinal information on the alternative and more importantly does not violate the ranking provided by the DM. However, the method ideally needs some information on the decision strategy. For example, one may use the method without any information on constraints but only the ranking of the relations. This makes the space of feasible solutions very big that the solution chosen may not have a real interpretation in terms of decision strategy. This method is more suitable when we need to define or build a decision strategy in terms of importance and interaction.

Table 14 shows the calculated concept scores using the identified measures by the questionnaire, Sugeno method, quadratic program based on learning data (QP-LD), and quadratic program based on measure semantics and learning data (QP-SLD). The GCSs calculated using the three methods proposed in this paper choose a different concept as the designer's decision compared to the method that has used a questionnaire among designers to identify fuzzy measures. Evidently, the order of the design alternatives has been changed using these methods. The discrepancy between the scores calculated based on the Sugeno method is larger than the differences between the scores calculated using the QP-LD and QP-SLD methods. However, the order of the alternatives remains the same.

In order to support the designers to choose a proper identification method, Table 15 provides a summary of the use case, inputs from the decision maker, and advantages and shortcomings of each identification process.

8. Conclusions

Mechatronic systems are a combination of cooperative mechanical, electronics, and software components aided by various control strategies. They are often highly complex, because of the high number of their components, their multiphysical aspect, and the couplings between the different engineering domains involved, which complexify the design task. Therefore, to achieve a better design process as well as a better final product more efficiently, these couplings need to be considered in the early stages of the design process.

Table 15: Specifications of the proposed fuzzy measure identification methods.

Questionnaire	DM input	1) Survey questions 2) Analytical interpretation of the semantics
	Advantages	1) Simple calculations
	Shortcomings	1) Not suitable for large numbers of criteria 2) Time consuming
	Use case	1) A limited number of criteria are involved in the design process 2) A sufficient number of design experts are available
Sugeno	DM input	1) Accurate assignment of singleton measures (μ_i)
	Advantages	1) Simple calculations 2) Fast process
	Shortcomings	1) Inaccurate estimation of interaction indices 2) Unable to interpret the decision maker's semantics
	Use case	1) No information about the decision maker's preferences on the interaction and importance of coalitions of criteria is available
QP-LD	DM input	1) Initial values for scores and rankings of the design alternatives 2) Preference information in terms of constraints on interaction and importance indices
	Advantages	1) Does not require fuzzy measure values of any kind 2) Implementable in an automated algorithm
	Shortcomings	1) Large data sets should be available when a large number of criteria are involved in the design activity 2) Complex computation
	Use case	1) Only the relative global scores on design alternatives are available 2) A data set from previous design cases or available databases is available
QP-SLD	DM input	1) Preference information in terms of constraints on fuzzy measures
	Advantages	1) Does not require scores and rankings of the design alternatives 2) Implementable in an automated algorithm
	Shortcomings	1) Multiple parameters are involved in the optimization which require tuning
	Use case	1) No information about the global scores and the rankings on design alternatives is available 2) Preferences are not expressed in terms of interaction and importance indices

The concept of the MMP has been previously introduced to facilitate fitting the intuitive requirements for decision-making in the presence of interacting criteria in conceptual design. The MMP includes five main elements: machine intelligence, reliability, flexibility, complexity, and cost. Each main criterion has several sub-criteria. The design process using MMP includes a fuzzy aggregation function based on Choquet fuzzy integrals that can efficiently model the interdependencies between a subset of criteria. However, the main difficulty of the Choquet method is the identification of its fuzzy measures that exponentially increase by the number of design objectives.

The objective of this study was to provide a framework to support the designers with the identification of fuzzy measures based on various available information and design preferences. We discussed three different methods of fuzzy measure identification applied to a case study of the conceptual design of a vision-guided quadrotor drone. These methods include using a Sugeno fuzzy model, a learning data set, and fuzzy semantics. The results obtained from each method have been presented in the case study section and finally, a discussion on each method and their applications was carried out. From the implementation and results, we infer that in the case that there is not enough information about the design preferences or the interaction and importance of coalitions of criteria, using Sugeno λ-measures can be an abstract and efficient way. When only the relative global scores on each design alternative are available, the identification based on learning data is shown to be effective. However, this method requires information about the DM preferences on the importance and interaction indices. The data sets can be obtained from previous design cases or from an available database. This suggests an interesting subject of future work where the implementation of a web-based integrated platform connecting various design projects would be explored. In the absence of the global scores, the method combining the fuzzy measure semantics and learning data can be used. This method calls only for ordinal information on the alternatives and their importance of the criteria.

References

Behbahani, S., & de Silva, C. W. (2007). Mechatronic design quotient as the basis of a new multicriteria mechatronic design methodology. *IEEE/ASME Transactions on Mechatronics*, *12*(2), 227–232.

Behbahani, S., & de Silva, C. W. (2008). System-based and concurrent design of a smart mechatronic system using the concept of mechatronic design quotient (MDQ). *IEEE/ASME Transactions on Mechatronics*, *13*(1). 14–21.

Coelingh, E., de Vries, T. J. A., & Koste, R. (2002). Assessment of mechatronic system performance at an early design stage. *IEEE/ASME Transactions on Mechatronics*, *7*(3), 269–279.

Grabisch, M. (1995). A new algorithm for identifying fuzzy measures and its application to pattern recognition. *Proceedings of the International Joint Conference of the 4th IEEE International Conference on Fuzzy Systems and the 2nd International Fuzzy Engineering Symposium* (pp. 145–150), Yokohama, Japan.

Grabisch, M. (1996). The application of fuzzy integrals in multicriteria decision making. *European Journal of Operational Research*, *89*(3), 445–456.

Grabisch, M. (1997). K-order additive discrete fuzzy measures and their representation. *Fuzzy Sets and Systems*, *92*(2), 167–189.

Grabisch, M., Nguyen, H. T., & Walker, E. A. (2013). *Fundamentals of uncertainty calculi with applications to fuzzy inference*. Vol. 30. Springer Science & Business Media.

Lee, K.-M., & LeeKwang, H. (1995). Identification of λ-fuzzy measure by genetic algorithms. *Fuzzy Sets and Systems*, *75*(3), 301–309.

Liu, P., & Tang, G. (2016). Multi-criteria group decision-making based on interval neutrosophic uncertain linguistic variables and Choquet integral. *Cognitive Computation*, *8*(6), 1036–1056.

Liu, P., & Tang, G. (2018). Some intuitionistic fuzzy prioritized interactive Einstein Choquet operators and their application in decision making. *IEEE Access*, *6*, 72357–72371.

Marichal, J.-L. (2002). Aggregation of interacting criteria by means of the discrete Choquet integral. In *Aggregation operators, Studies in fuzziness and soft computing*(pp. 224–244), vol. 97. Heidelberg: Physica.

Marichal, J.-L., & Roubens, M. (1998). Dependence between criteria and multiple criteria decision aid. In *Proceedings of the 2nd International Workshop on Preferences and Decision (TRENTO'98)*. Università di Trento.

Marichal, J.-L., & Roubens, M. (2000). Determination of weights of interacting criteria from a reference set. *European Journal of Operational Research*, *124*(3), 641–650.

Meyer, P., & Roubens, M. (2006). On the use of the Choquet integral with fuzzy numbers in multiple criteria decision support. *Fuzzy Sets and Systems*, *157*(7), 927–938.

Moghtadernejad, S., Chouinard, L. E., & Saeed Mirza, M. (2018). Multi-criteria decision-making methods for preliminary design of sustainable facades. *Journal of Building Engineering*, *19*, 181–190.

Moghtadernejad, S., Chouinard, L. E., & Saeed Mirza, M. (2020). Design strategies using multi-criteria decision-making tools to enhance the performance of building façades. *Journal of Building Engineering*, *30*, 101274.

Moghtadernejad, S., Saeed Mirza, M., & Chouinard, L. E. (2019). Determination of the fuzzy measures for multicriteria and optimal design of a building façade using Choquet integrals. *Journal of Building Engineering*, *26*, 100877.

Mohebbi, A., Achiche, S., & Baron, L. (2014a). Mechatronic multicriteria profile (MMP) for conceptual design of a robotic visual servoing system. In *Proceedings of the ASME 2014 12th Biennial Conference on Engineering Systems Design and Analysis*, Copenhagen, Denmark. American Society of Mechanical Engineers (ASME).

Mohebbi, A., Achiche, S., & Baron, L. (2015). Integrated design of a vision-guided quadrotor UAV: A mechatronics approach. In *Proceedings of the 2015 CCToMM Symposium on Mechanisms, Machines, and Mechatronics*, Canadian Committee for the Theory of Machines and Mechanisms.

Mohebbi, A., Achiche, S., & Baron, L. (2016a). A multicriteria fuzzy decision support for conceptual evaluation in design of mechatronic systems: A quadrotor design case study. *Springer Journal of Research in Engineering Design*, *29*, 329–349.

Mohebbi, A., Achiche, S., & Baron, L. (2018). Multi-criteria fuzzy decision support for conceptual evaluation in design of mechatronic systems: A quadrotor design case study. *Research in Engineering Design*, *29*(3), 329–349.

Mohebbi, A., Achiche, S., & Baron, L. (2019). Integrated and concurrent detailed design of a mechatronic quadrotor system using a fuzzy-based particle swarm optimization. *Engineering Applications of Artificial Intelligence*, *82*, 192–206.

Mohebbi, A., Achiche, S., Baron, L., & Birglen, L. (2014b). Fuzzy decision making for conceptual design of a visual servoing system using mechatronic multi-criteria profile (MMP). In *Proceedings of the ASME 2014 International Mechanical Engineering Congress and Exposition*. American Society of Mechanical Engineers (ASME).

Mohebbi, A., Baron, L., Achiche, S., & Birglen, L. (2014c). Neural network-based decision support for conceptual design of a mechatronic system using mechatronic multi-criteria profile (MMP). In *Proceedings of the 2014 International Conference on Innovative Design and Manufacturing (ICIDM)*, Montreal, QC, Canada.

Mohebbi, A., Baron, L., Achiche, S., & Birglen, L. (2014d). Trends in concurrent, multi-criteria and optimal design of mechatronic systems: A review. In *Proceedings of the 2014 International Conference on Innovative Design and Manufacturing (ICIDM)*, Montreal, QC, Canada.

Mohebbi, A., Keshmiri, M., & Xie, W.-F. (2016b). A comparative study of eye-in-hand image-based visual servoing: Stereo vs. mono. *Journal of Integrated Design and Process Science*, *19*(3), 25–54.

Mori, T., & Murofushi, T. (1989). An analysis of evaluation model using fuzzy measure and the Choquet integral. In *Proceedings of the 5th Fuzzy System Symposium*, Kobe, Japan. Japan Society for Fuzzy Sets and Systems.

Moulianitis, V., Aspragathos, N. A., & Dentsoras, A. J. (2004). A model for concept evaluation in design—An application to mechatronics design of robot grippers. *Mechatronics*, *14*(6), 599–622.

Rowley, H. V., Geschke, A., & Lenzenb, M. (2015). A practical approach for estimating weights of interacting criteria from profile sets. *Fuzzy Sets and Systems*, *272*, 70–88.

Rzevski, G. (2003). On conceptual design of intelligent mechatronic systems. *Mechatronics*, *13*(10), 1029–1044.

Rzevski, G. (2014). *Mechatronics: Designing intelligent machines Volume 1: Perception, cognition and execution*. Newnes.

Sugeno, M. (1975). Theory of fuzzy integrals and its applications. Ph.D. Thesis. Tokyo Institute of Technology.

Tahani, H., & Keller, J. M. (1990). Information fusion in computer vision using the fuzzy integral. *IEEE Transactions on systems, Man, and Cybernetics*, *20*(3), 733–741.

Timonin, M. (2013). Robust optimization of the Choquet integral. *Fuzzy Sets and Systems*, *213*, 27–46.

Tomiyama, T., Gu, P., Jin, Y., Lutters, D., Kind, Ch., & Kimura, F. (2009). Design methodologies: Industrial and educational applications. *CIRP Annals-Manufacturing Technology*, 58(2), 543–565.

Torry-Smith, J. M., Qamar, A., Achiche, S., Wikander, J., Mortensen, N. H., & During, C. (2013). Challenges in designing mechatronic systems. *Journal of Mechanical Design*, *135*(1), 011005.

Ullman, D. G. (1992). *The mechanical design process*. New York: McGraw-Hill.