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Bouncing alternatives to inflation

Rebond primordial comme alternative à l'inflation

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A B S T R A C T

Although the inflationary paradigm is the most widely accepted explanation for the current cosmological observations, it does not necessarily correspond to what actually happened in the early stages of our Universe. To decide on this issue, two paths can be followed: first, all the possible predictions it makes must be derived thoroughly and compared with available data, and second, all the imaginable alternatives must be ruled out. Leaving the first task to all other contributors of this volume, we concentrate here on the second option, focusing on the bouncing alternatives and their consequences.

R É S U M É

Quoique le paradigme inflationaire soit maintenant communément accepté comme représentant la meilleure explication des données cosmologiques, il n'est pas pour autant possible de dire qu'une telle phase soit avérée. Pour s'approcher d'une telle conclusion, on peut suivre deux chemins différents : on peut explorer les conséquences de l'inflation pour la pousser dans ses derniers retranchements, ou bien, au contraire, étudier en détail les alternatives possibles. La première option faisant l'objet de la plupart des contributions de ce volume, nous nous concentrons ici sur la seconde, et présentons les modèles dans lesquels une phase de contraction est suivie d'un rebond conduisant à notre époque d'expansion.

1. Introduction

Starting out in a dense state some 13.8 billion years ago, our Universe and its evolution since this initial time are well understood, with an initially almost scale-invariant, but not quite, spectrum of primordial perturbations condensing into the presently observed large-scale structures by means of gravitational collapse. The very high densities of the early stages provide initial conditions to explain the relative amounts of different nuclei, and the ensuing phases, being controlled by well-known physical mechanisms, permit to reconstruct, from the cosmic microwave background (CMB) observations, the

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properties of the last scattering surface. We have arrived at the point [1,2] where cosmological data can be used to probe the earliest conceivable phases.

The most widely accepted paradigm for describing the earliest phases of the Universe, when the energy density was a mere few orders of magnitude below the Planck scale, is inflation [3,4]. Easily implemented by means of a scalar field, this almost exponentially expanding era rapidly leads to a flat Friedmann–Lemaître (FL) spacetime with a very slightly reddish spectrum of initial perturbations, from which the rest of the history of the Universe ensues. As is, such a scenario is compatible with all currently available data.

This contribution reviews some properties of some non-inflationary bouncing models. The first natural question that comes to mind before going any further is: why should we bother with possible alternatives to a working scenario? There are in fact many reasons, the first of which being that the phase of inflation is silent relative to the primordial singularity, as we discuss in Section 2 below. The second is that there is no way we will ever be able to assert that a phase of inflation did actually take place, except through its presently observable consequences. But then the question arises as to whether other competing theories could induce similar consequences. Thus, examining all plausible scenarios in detail seems to be the only way to assert whether inflation is the unique possibility leading to our observable Universe. In the end, ruling out alternatives, or not, increases or decreases our level of confidence in inflation until it becomes, if ever, recognized as valid beyond any reasonable doubt. As we shall see in Section 3, there are bouncing alternative explanations to the standard model puzzles of homogeneity, flatness, isotropy, horizon and the overproduction of relics, as well as many models, some of which are listed in Section 4, in which those bounces can be implemented.

Getting a background-compatible model is however not the end of the story: the recently released Planck data [5,6] confirm what was suggested by previous experiments, namely that the spectrum of primordial perturbations was almost scale invariant: slightly red, with a spectral index $n_{\mathrm{s}} = 0.9639 \pm 0.0047$, excluding exact scale invariance at the 5σ level. The level of non-gaussianity is compatible with zero, and the contribution of tensor modes remains below the $\sim$10% limit relative to the scalar amplitude. All these facts are compatible with the perturbations having been produced by quantum vacuum fluctuations of a single scalar degree of freedom, a natural consequence of slow-roll single-field inflation. Can a non-inflationary bouncing model reproduce such results? As of now, there is no definite answer to this question. For this reason, and for lack of space in the present article, we shall not discuss these points below, and instead refer the reader to a recent review [7] in which all the relevant constraints for the models exhibited below are derived.

2. The singularity

The fact that cosmology, or at least its classical implementation in terms of general relativity (GR), always leads to the existence of singularities stems from the well-known singularity theorems [8]. A general argument was proposed in Ref. [9]: in an FL spacetime with metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}^{\mathcal{K}}(\boldsymbol{x})\,\mathrm{d}x^i\mathrm{d}x^j = a^2(\eta)\left[-\mathrm{d}\eta^2 + \gamma_{ij}^{\mathcal{K}}(\boldsymbol{x})\,\mathrm{d}x^i\mathrm{d}x^j\right] \tag{1}$$

with $\gamma_{ij}^{\mathcal{K}}$ the constant-curvature ($\mathcal{K} = 0, \pm 1$) spatial metric, let $\mathcal{U}^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}\lambda$, with λ an affine parameter, be a lightlike tangent to a geodesic curve, i.e. $\mathcal{U}^\mu\mathcal{U}_\mu = \gamma_{ij}^{\mathcal{K}} a^2 \mathcal{U}^i\mathcal{U}^j - \left(\mathcal{U}^0\right)^2 = 0$ and $\mathcal{U}^\mu\nabla_\mu\mathcal{U}^\alpha = 0$. Expanding the geodesic equation in terms of the connections associated with the metric (1) and taking into account the lightlike character of $\mathcal{U}$, one finds that

$$\frac{\mathrm{d}\mathcal{U}^0}{\mathrm{d}\lambda} + H\left(\mathcal{U}^0\right)^2 = 0$$

which implies that

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\mathrm{d}t}{\mathrm{d}\lambda}\right) + H\left(\frac{\mathrm{d}t}{\mathrm{d}\lambda}\right)^2 = 0 \tag{2}$$

where the Hubble scale is $H = \mathrm{d}\ln a/\mathrm{d}t$. Eq. (2) is solved by choosing the affine parameter λ to satisfy $\mathrm{d}\lambda = [a(t)/a(t_{\mathrm{f}})]\mathrm{d}t$, with t_{f} a reference time, today say. We now assume our spacetime to begin at some initial coordinate time t_{i}, which can take any value between 0 say, to $-\infty$; this depends on the actual cosmological realization. The average Hubble rate along the geodesic parameterized by λ is found to be

$$H_{\text{average}} \equiv \frac{1}{\lambda(t_{\mathrm{f}}) - \lambda(t_{\mathrm{i}})} \int_{\lambda(t_{\mathrm{i}})}^{\lambda(t_{\mathrm{f}})} H(\lambda)\mathrm{d}\lambda = \frac{1}{\lambda(t_{\mathrm{f}}) - \lambda(t_{\mathrm{i}})}\left\{1 - \frac{a[\lambda(t_{\mathrm{i}})]}{a[\lambda(t_{\mathrm{f}})]}\right\} \le \frac{1}{\lambda(t_{\mathrm{f}}) - \lambda(t_{\mathrm{i}})} \tag{3}$$

so that in order for H_{average} to be strictly positive, a condition that is generally satisfied in inflationary models, one finds that the interval in affine parameter must be finite, and therefore that the spacetime under consideration is not geodesically complete. This argument can be extended to timelike geodesics and more arbitrary cosmological models, i.e. with no specific assumptions regarding homogeneity and isotropy. This requires the definition of a local expansion rate that is not dependent on the special FL metric solution; in this case, it is the deviation between neighboring geodesics that needs to be used

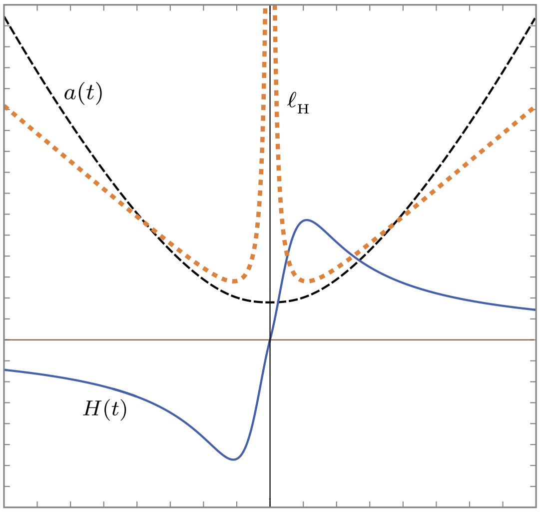

Fig. 1. (Color online.) Typical time evolution of the scale factor $a(t)$ (dashed line), Hubble rate $H(t) \equiv \dot{a}/a$ (full line) and Hubble length $\ell_H(t) = H^{-1}$ (dotted line) for a bouncing scenario. For large negative times (conventionally setting the bounce at $t = 0$), the scale factor decreases in a non-accelerated way, then it curves up, accelerating and rendering the curve convex, finally connecting, not necessarily in a symmetric way, to a more standard non-accelerating expansion. The Hubble rate starts vanishingly small, then decreases to large negative values, passes through the bounce almost linearly increasing, reaches a maximum and then decreases back to its usual behavior. The Hubble length is originally very large, reaches a minimum and diverges at the bounce point: there is no super-Hubble scale at the bounce!

explicitly to define the expansion rate (in the highly symmetric FL universe, the geodesic deviation is given by the expansion only, as this is the only relevant observable). The conclusion then is that, regardless of any energy condition, inflating spacetimes are past incomplete.

An obvious way out of this problem consists in allowing the average Hubble rate to be negative. This corresponds to having some amount of contraction, and hence, given that we observe the Universe to be currently expanding, that it must have bounced. In the framework of GR however, this is not always easy.

Using the metric (1) and a fluid stress-energy tensor $T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$ with energy density ρ, pressure P, and u_μ a timelike vector, the Einstein equations read

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{1}{3}\rho, \qquad \dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P) \tag{4}$$

leading to

$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}(\rho + P) \tag{5}$$

(we use natural units where $\hbar = c = 8\pi G_N \equiv 1$ so that the Planck mass $M_{Pl} \equiv G_N^{-1/2}$ is dimensionless). Although having an inflationary phase with $\ddot{a} > 0$ merely demands the violation of the Strong Energy Condition (SEC: $\rho + 3P > 0$), a bounce, requiring $H \to 0$ while $\dot{H} > 0$ at the same time, implies that either the spatial sections must be positively curved ($\mathcal{K} > 0$) or the Null Energy Condition (NEC: $\rho + P > 0$) must be violated. In the former case, the scale factor at the bounce a_B is obtained as the solution to $3\mathcal{K}/a_B^2 = \rho(a_B)$ and must satisfy $\mathcal{K}/a_B^2 > -P(a_B)$. This condition is for instance fulfilled in the very simple case in which a single scalar field evolves in a potential with a local maximum [10–12]. The bouncing solution, seen generically as indicated in Fig. 1, does have an accelerating phase, the scale-factor curve being convex at the bounce; although this technically implies the SEC to be also violated during a bouncing epoch, this cannot be understood as an inflating phase since the accelerating phase is not associated with a large increase of the size of the Universe.

In the more familiar (to inflation-oriented cosmologists) case of vanishing or negligible spatial curvature,[1] as mentioned above, Eqs. (4) imply a much more stringent constraint, namely that the NEC be violated; as discussed in Section 4, this often leads to various instabilities that then need to be tamed in order for the model to make any sense at all.

[1] Note that this is an assumption that can only be checked a posteriori: given a material content with positive and negative energy components, one must first solve the Friedmann equation for the minimum scale factor a_B, and then verify that the curvature term $\mathcal{K}/a_B^2$ is indeed negligible with respect to all other contributions.

3. Standard model puzzles, bouncing solutions – new issues

The reason why the inflationary scenario is so fashionable stems from its successes in solving the standard hot big bang puzzles in a unified way, while at the same time providing a means of producing perturbations whose spectrum can be made to agree with all known data. Can a bouncing scenario, on top of naturally avoiding the singularity, propose satisfying solutions to the standard hot big bang puzzles? If it is the case, can a bouncing scenario provide a means to generate cosmological perturbations whose statistics agree with observations? As mentioned before, we refer the reader to Ref. [7] for a detailed discussion of this latter question, and focus in the remainder of this review on bouncing solutions to the background cosmological problems and on a review of existing bouncing models.

3.1. Horizon and flatness puzzles

The standard hot big bang model suffers from a few puzzling problems, and we will treat in this section how a bounce, which implies a contracting phase preceding the current expansion, deals with the two most important, namely the horizon and flatness problems. We refer the reader to Refs. [7,13] for more details relative to the other commonly addressed puzzles.

— Horizon

The horizon problem relates to the inability to explain the quasi-homogeneity of the observable universe within the context of standard cosmology in which the entire evolution of the universe consists in decelerated expansion during the radiation- and matter-dominated epochs. In the context of inflation, the necessity of a period of accelerated expansion for a period lasting a minimum of $N \sim 60$ e-folds can be phenomenologically understood by computing the solid angle subtended by causally connected regions. Assuming an expansion history in which the universe is initially radiation-dominated, then dominated by the inflaton, represented by a fluid X with equation of state parameter w_X until a redshift $z_{\rm e}$, then once again radiation-dominated until the last scattering surface at $z_{\rm lss}$, and finally matter-dominated till today, we find that [14]

$$\Delta\Omega = \frac{1}{2}\left[1-(1+z_{\rm lss})^{-1/2}\right]^{-1}(1+z_{\rm lss})^{-1/2}\left\{1+\frac{1-3w_X}{1+3w_X}\frac{1+z_{\rm lss}}{1+z_{\rm e}}\left[1-{\rm e}^{-N(1+3w_X)/2}\right]\right\} \tag{6}$$

where $N = \ln(a_{\rm e}/a_{\rm i})$ with $a_{\rm i}$ the scale factor at the onset of the X-dominated period. If we assume that $N = 0$, we recover standard cosmology and $\Delta\Omega \sim 0.85$ degrees. This corresponds to a total of about 10^6 causally disconnected regions in which, strangely enough, the CMB is everywhere the same up to 1 part in 10^5. Increasing $\Delta\Omega$ is easily achieved in the context of inflation by requiring $w_X < -1/3$ and large positive N.

A similar calculation can be done in the context of bouncing cosmology. Here again, we shall assume a phase dominated by a fluid with equation of state parameter w_X during which the Universe first contracts. The bounce is assumed non-singular, occurring at a redshift $z_{\rm b}$ and short enough that we can ignore its contribution. It is followed by the standard radiation- and matter-dominated phases. The solid angle subtended by causally connected regions is then

$$\Delta\Omega = \frac{1}{2}\left[1-(1+z_{\rm lss})^{-1/2}\right]^{-1}(1+z_{\rm lss})^{-1/2}\left\{1+\frac{1+z_{\rm lss}}{1+z_{\rm b}}\left[\frac{3(1+w_X)}{1+3w_X}{\rm e}^{-N(1+3w_X)/2}-\frac{2(2+3w_X)}{1+3w_X}\right]+1\right\} \tag{7}$$

In this expression, $N = \ln(a_{\rm i}/a_{\rm B}) \leq 0$, with $a_{\rm i}$ and $a_{\rm B}$ the scale factors at the onset of the X-dominated contraction and at the bounce respectively. Here, large values of $\Delta\Omega$ are obtained for $w > -1/3$ and large values of $|N|$. Thus, in contrast with inflation, the horizon problem can be solved using a fluid that satisfies all energy conditions.

— Flatness

The flatness problem is easily understood by working with the ratio of the Friedmann equation with the critical density $\rho_{\rm crit} = 3H^2$, which is the energy density the Universe would have if it had exactly flat spatial sections. In the presence of the spatial curvature term, the Friedmann equation (4) takes the form

$$\frac{\mathcal{K}}{a^2H^2} = \sum_{i=1}^{N}\Omega_i - 1 = \Omega_{\rm T} - 1 \tag{8}$$

with

$$\Omega_i = \frac{\rho_i}{\rho_{\rm crit}} = \frac{H^2(t_0)}{H^2(t)}\Omega_i(t_0)\left(\frac{a}{a_0}\right)^{-3(1+w_i)} \tag{9}$$

From observations, we know that the total density parameter today is $\Omega_{\rm T}(t_0) \simeq 1$. It is easy to recast Eq. (8) in the convenient form [14]

$$\Omega_{\rm T}(a) = \sum_{j=1}^{N}\Omega_j(t_0)\left(\frac{a}{a_0}\right)^{-3(1+w_j)}\left\{\sum_{i=1}^{N}\Omega_i(t_0)\left(\frac{a}{a_0}\right)^{-3(1+w_i)} - [\Omega_{\rm T}(t_0)-1]\left(\frac{a}{a_0}\right)^{-2}\right\}^{-1} \tag{10}$$

At early times (small scale factor), the Universe is radiation dominated, and Eq. (10) simplifies to

$$\Omega_{\mathrm{T}}(t)-1 \simeq \frac{\Omega_{\mathrm{T}}(t_0)-1}{\Omega_{\mathrm{rad}}(t_0)}\left(\frac{1}{1+z}\right)^2 \tag{11}$$

For $z \gg 1$, $\Omega(t)-1$ must be much less than 1. For instance, taking $z_{\mathrm{nucl}}=3\times 10^8$, $\Omega_{\mathrm{rad}}(t_0)=10^{-4}$, and $\Omega_{\mathrm{T}}(t_0)-1=0.01$, one finds $\Omega_{\mathrm{T}}(t_{\mathrm{nucl}})-1 \sim 10^{-15}$. The value of the total density parameter at nucleosynthesis required to satisfy today's observed value $\Omega_{\mathrm{T}}(t_0)\sim 1$ is highly fine-tuned and thus highly improbable. This embodies the flatness problem of standard cosmology.

Let us now consider the case of a bouncing universe that contracts in a phase dominated by a fluid of equation of state parameter w_X, bounces and then expands according to the standard scenario. Note that the set of equations above do not apply at the bounce point where $H=0$. In fact, in the presence of a spatial curvature term, Ω_{T} diverges at the bounce.

For a universe dominated by some fluid X, one has

$$\Omega_{\mathrm{T}}(t)=\frac{\Omega_X(t_{\mathrm{i}})}{\Omega_X(t_{\mathrm{i}})-[\Omega_{\mathrm{T}}(t_{\mathrm{i}})-1]\,(a/a_{\mathrm{i}})^{1+3w_X}} \tag{12}$$

The total density parameter at the end of the contracting phase at t^- is given by

$$\Omega_{\mathrm{T}}(t_-)-1 \simeq \frac{\Omega_{\mathrm{T}}(t_{\mathrm{i}})-1}{\Omega_X(t_{\mathrm{i}})}\left(\frac{a^-}{a_{\mathrm{i}}}\right)^{1+3w_X} \tag{13}$$

while it is given by Eq. (11) at the beginning of the expanding phase, for $t=t_+$ and $z=z_+$. The difference

$$\Delta\Omega_{\mathrm{T}}=[\Omega_{\mathrm{T}}(t_+)-1]-[\Omega_{\mathrm{T}}(t_-)-1] \tag{14}$$

can be computed using Eq. (8) and the Taylor expansion of the scale factor close to the bounce,

$$a(t)=a_{\mathrm{B}}\left[1+\left(\frac{t}{t_{\mathrm{c}}}\right)^2+\beta\left(\frac{t}{t_{\mathrm{c}}}\right)^3+\ldots\right] \tag{15}$$

One finds

$$\Delta\Omega_{\mathrm{T}} \simeq -\frac{3\beta}{2}\left(\frac{t_{\mathrm{c}}}{a_{\mathrm{B}}}\right)^2 \tag{16}$$

Generically, in the absence of any fine-tuning, one should assume $\Omega_{\mathrm{T}}(t_{\mathrm{i}})-1$ and $\Omega_X(a_{\mathrm{i}})$ take values of $\mathcal{O}(1)$ while it is known that $\Omega_{\mathrm{rad}}(t_0)\simeq 10^{-4}$ and $\Omega_{\mathrm{T}}(t_0)-1\lesssim 10^{-2}$. Hence, we have:

$$\left(\frac{a_-}{a_{\mathrm{i}}}\right)^{1+3w_X}-\frac{3\beta}{2}\left(\frac{t_{\mathrm{c}}}{a_{\mathrm{B}}}\right)^2 \lesssim 10^6\times z_+^{-2} \tag{17}$$

Taking as before $z_+\simeq 10^{28}$, and with $w=1/3$, we have

$$\mathrm{e}^{2N}-\frac{3\beta}{2}\left(\frac{t_{\mathrm{c}}}{a_{\mathrm{B}}}\right)^2 \lesssim 10^{-50} \tag{18}$$

where $N<0$. Thus, for $N\lesssim -60$, and β of order 1, a bounce with a short characteristic timescale and a large value of the scale factor at the bounce such that $t_{\mathrm{c}}/a_{\mathrm{B}}\lesssim 10^{-25}$ can satisfy current constraints on the spatial curvature of the universe.

3.2. Shear/BKL instability

In a bouncing scenario, the standard puzzles find natural solutions because of the contracting phase. However, such a phase can also induce another problem: the fate of any initial amount of anisotropy. To focus on this question, we consider a spatially flat model whose dynamics derives from the Bianchi I metric $\mathrm{d}s^2=-\mathrm{d}t^2+a^2(t)\mathrm{d}\boldsymbol{x}^2$, whose spatial part reads

$$\mathrm{d}\boldsymbol{x}^2=\mathrm{e}^{2\theta_x(t)}\mathrm{d}x^2+\mathrm{e}^{2\theta_y(t)}\mathrm{d}y^2+\mathrm{e}^{2\theta_z(t)}\mathrm{d}z^2 \tag{19}$$

with $\sum_i\theta_i\equiv\theta_x+\theta_y+\theta_z=0$. Plugging (19) into the Einstein equations generalizes (4) to

$$H^2\equiv\left(\frac{\dot a}{a}\right)^2=\frac{1}{3}\rho+\frac{1}{6}\sum_i\dot\theta_i^2\equiv\frac{1}{3}(\rho+\rho_\theta) \tag{20}$$

and

$$\dot{H} = -\frac{1}{2}(\rho + P) - \frac{1}{2}\sum_i \dot{\theta}_i^2 \tag{21}$$

where we have identified the shear energy density ρ_θ contained in the anisotropy stemming from the functions θ_i: Eqs. (20) and (21) imply that $\ddot{\theta}_i + 3H\dot{\theta}_i = 0$, and therefore $\rho_\theta \propto a^{-6}$.

With dust and radiation scaling as $\rho_{\rm m} \propto a^{-3}$ and $\rho_{\rm r} \propto a^{-4}$, respectively, the above result is a catastrophe: as the universe contracts, any initial anisotropy, however small,[2] will grow until it eventually dominates the dynamics. This was shown [17] by Belinsky, Khalatnikov, and Lifshitz (BKL) to induce an instability sufficient to spoil the bounce.

One way out of the shear problem is to add an extra component, usually a scalar field in a potential satisfying specific constraints, with large effective equation of state $w_\phi \gg 1$, so that the resulting Friedmann equation reads:

$$H^2 = \frac{1}{3}\left[-\frac{3\mathcal{K}}{a^2} + \frac{\rho_{\rm m0}}{a^3} + \frac{\rho_{\rm r0}}{a^4} + \frac{\rho_{\theta 0}}{a^6} + \frac{\rho_{\phi 0}}{a^{3(1+w_\phi)}}\right] \tag{22}$$

If this so-called ekpyrotic phase [18] lasts long enough, it eventually comes to dominate over all other constituents when $a \to 0$, including the shear contribution. The Universe then bounces and starts expanding again while in a fully symmetric FL phase, a condition absolutely required to explain the observational data.

The bounce itself is another matter, which we now turn to.

4. Existing models

There exist a large number of bouncing cosmological models in the literature; we shall refer the reader to Ref. [7] for an exhaustive review and all the relevant references. We will instead focus here on a few models and give concrete examples.

4.1. Classical bounces

Bouncing models predate by many decades the inflationary paradigm, as they were first introduced in the 1930s. Classical models involve unconventional perfect fluids or scalar fields with possibly non-standard kinetic terms, or various combinations of those. The most conservative setup that may be used to obtain a bounce is to introduce spatial curvature and to violate the strong-energy condition. In such a setup, scalar field matter is required in order to achieve $\rho + 3P < 0$ and either a quasi-symmetric bounce or a post-bounce phase of inflation is needed to drive $\Omega_\mathcal{K}$ towards zero.

• Perfect fluids

In a theory restricted to GR and FL spacetime, generically, for $\mathcal{K} \neq +1$, the null-energy condition has to be violated in order to obtain a bounce, as discussed below Eq. (5). Exotic hydrodynamical fluids that violate the null-energy condition, and thus all other energy conditions, are *a priori* allowed, and models using those can be built in the framework of an FL spacetime. Assuming an expansion for the scale factor of the form

$$a = a_0 + b\eta^{2n} + d\eta^{2n+1} + e\eta^{2n+2} \tag{23}$$

where $n \geq 1$, and (a_0, b, d, e) constant, it is possible to compute, in a fully analytic way, the evolution of *adiabatic* perturbations around the bounce [19]. The possible choices for n and $\mathcal{K}$ are: (i) $n > 1$ and $\mathcal{K} \neq 0$; (ii) $n > 1$ and $\mathcal{K} = 0$; (iii) $n = 1$ and $d \neq 0$ $\forall \mathcal{K}$; (iv) $n = 1$ and $d = 0$ $\forall \mathcal{K}$; setting $d = 0$ restricts to a symmetric bounce. In the first three cases, the Bardeen potential Φ describing the gauge-invariant perturbations turns out to be singular at the bounce, while in case (iv), although Φ is well behaved, the NEC needs to be violated, even if $\mathcal{K} = +1$. Given that at late times, it has to be satisfied, there must exist a time t_* at which $\rho(t_*) + P(t_*) = 0$. It turns out that, at this *NEC transition*, the growth of Φ is unlimited, raising potential questions on the perturbative expansion through a bouncing phase.

When entropy perturbations are considered in addition to the adiabatic ones, they are found to be sourced by the interaction of the hydrodynamical fluids involved in the cosmological evolution. This implies that the fluids do not evolve independently. Inclusion of entropy perturbations has the effect of regularizing the Bardeen potential and its derivatives at the NEC transition. It may thus be concluded that perfect-fluid-dominated bouncing models in which both adiabatic and entropy perturbations are taken into account are regular and do not necessarily lead to strong backreaction effects of the perturbations onto the background geometry [20].

When it comes to violating energy conditions, it is tempting to make use of scalar fields: most implementations of the inflationary paradigm are based on scalar fields, and the bounce does no better in that respect! Bouncing models powered

[2] Actually, the problem only arises in the presence of primordial classical shear: it has been shown that if the primordial shear is generated by quantum vacuum fluctuations, scalar and vector perturbations remain comparable [15,16].

by such fields can be broadly distinguished in two categories, depending on the coupling (minimal or extended) with the geometry. We now discuss both these possibilities.

- **Minimally coupled scalar fields theories**

Theories using minimally coupled scalar fields can be separated in two categories. First, an ordinary scalar field, with a standard kinetic term and a potential. In this case, in order to obtain a bouncing cosmology and preserve the weak energy condition, one needs $\mathcal{K} = +1$. We shall not dwell here with such cases, which then demand the curvature problem to be addressed independently, and may result in perturbations being large and potentially highly non-Gaussian [21].

The second category naturally involves non-standard kinetic terms; those can be generalized to the galileon theories (non-minimal coupling, see below). Their advantage over the standard kinetic terms is that those can be implemented in a flat FL universe.

— Ghost condensates

The simplest possible example of a non-standard kinetic terms consists in merely switching its sign, making it a so-called ghost field, which yields an untenable theory because instabilities, both classical and quantum, will immediately develop and ruin any configuration. Although one can use a ghost as an effective means to initiate a bounce with $\mathcal{K} = 0$ [22], it makes more sense to induce NEC violations with dynamical ghosts. This can be achieved in higher-derivative theories with second-order equations of motion that by construction prevent gradient or ghost instabilities. This is the ghost condensate mechanism [23], whose features we can sketch with the Lagrangian

$$\mathcal{L} = P(X) \text{ where } X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \tag{24}$$

and where the pressure P is an arbitrary function of the kinetic energy X. Eq. (24) in a flat FL metric yields

$$\frac{\mathrm{d}}{\mathrm{d}t}(a^3 P_{,X} \dot{\phi}) = 0 \tag{25}$$

where $P_{,X} \equiv \mathrm{d}P/\mathrm{d}X$.

If X is a constant and $P_{,X} = 0$ at $X = X_\mathrm{c}$, the equation of motion yields the solution

$$\phi = \sqrt{2X_\mathrm{c}}\, t \tag{26}$$

Given that

$$\rho + P = 2XP_{,X} \tag{27}$$

and the constraint $X > 0$, a violation of the NEC can take place if $P_{,X} < 0$ in some interval of the values of X. This, the condition that $P_{,X} = 0$ at $X = X_\mathrm{c}$ and requiring that $P_{,XX} > 0$ at $X = X_\mathrm{c}$ in order to prevent the existence of ghosts implies that the function $P(X)$ should have a local minimum at X_c, (see Fig. 2). This construction was employed in the so-called $\mathcal{K}$–bounce [24], new Ekpyrotic [25] and matter bounce [26] scenarios. Unfortunately, the ghost condensate phase in models such as this cannot be smoothly connected with a branch $P_{,X} > 0$ at $X = 0$. Ghost-condensate-type models therefore do not admit a stable Poincaré-invariant vacuum state and are thus severely flawed. Consequences of this instability were for instance demonstrated explicitly in the case of the new Ekpyrotic scenario in Refs. [27,28].

— Ekpyrotic potential and ghost condensation

As was mentioned earlier, the contracting phase of a bouncing cosmology is generically unstable under the growth of anisotropies and leads to chaotic mixmaster oscillations unless a period of ekpyrotic contraction with $w > 1$ is invoked or if the contraction is sufficiently brief. The smooth transition from ekpyrotic contraction to expansion through a non-singular bouncing phase relying on the NEC violation (with $w < 1$) was first studied in the new Ekpyrotic model [25]. It is obtained with a single scalar field rolling down a steep negative potential during the ekpyrotic phase and then undergoing ghost condensation. In this approach, the ghost-condensate Lagrangian is thus supplemented with a potential term $V(\phi)$. The function $P(X)$ realizing the ghost-condensate phase, and the ekpyrotic potential $V(\phi)$ are depicted in Fig. 2 As shown for instance in [29], however, this model suffers from a gradient instability and from the regrowth of the initial anisotropy during the bouncing phase. In addition, the absence of a Lorentz-invariant vacuum remains, as in the ghost-condensate model. It also predicts a blue spectrum of curvature perturbations.

- **Conformal galileons**

Instead of realizing NEC-violations by relying on Lagrangians that are restricted to general functions of the *kinetic term* only, it is possible to construct yet more general NEC-violating theories with Lagrangians that exhibit couplings between various

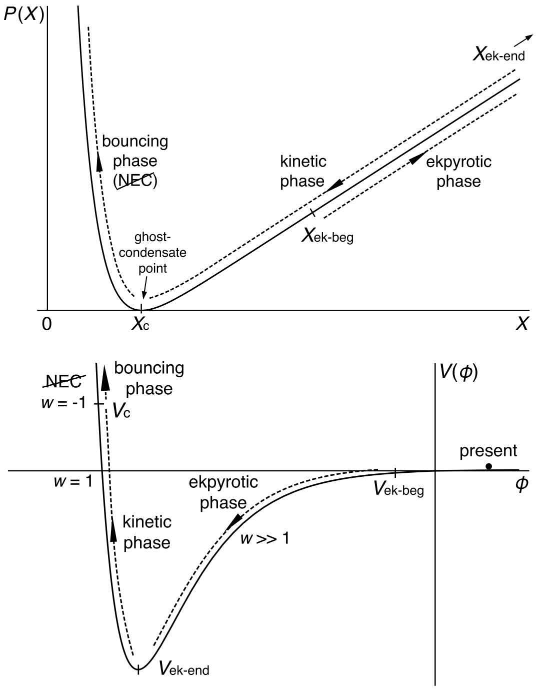

Fig. 2. Top: ghost condensate kinetic function $P(X)$. Bottom: ekpyrotic potential $V(\phi)$. Plots obtained from Ref. [29].

scalar-field derivative terms [30]. Such scalar-field theories are called galileon theories and have drawn much interest due to the fact that they naturally admit self-accelerating solutions. In galileon theories, the scalar Lagrangian involves higher derivative interactions with at most second-order derivatives in the equations of motion and is invariant under (possibly conformal or DBI-conformal) Galilean transformations. Galileon theories are a subclass of the theory of generalized galileons, which are described by the most general scalar-tensor (Horndeski) action leading to second-order equations of motion. Denoting the scalar field by $\pi(x)$, it reads

$$S_{\mathrm{H}} = \int \mathrm{d}^4x\sqrt{-g}\Big\{P(\pi,X) + G_{\Box}(\pi,X)\Box\pi + G_{\mathcal{R}}(\pi,X)\mathcal{R} + G_{\mathcal{R},X}\left[(\Box\pi)^2 - \left(\nabla_\mu\nabla_\nu\pi\right)^2\right]$$
$$+ G_G(\pi,X)\,G_{\mu\nu}\nabla^\mu\nabla^\nu\pi - \frac{G_{G,X}}{6}\left[(\Box\pi)^3 - 3\Box\pi\left(\nabla_\mu\nabla_\nu\pi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\pi\right)^3\right]\Big\} \tag{28}$$

with $\mathcal{R}$ the scalar curvature, the functions $G_{\Box}$, $G_{\mathcal{R}}$ and G_G being arbitrary functions of the field π and its kinetic energy X; the last two functions make manifest the non-minimal couplings with the gravitational sector.

Conformal formulations of galileon theories are particularly advantageous because the 4D conformal group reduces to the Anti de Sitter, Minkowski and de Sitter symmetry groups for particular solutions to the dilaton equation of motion [31]. More importantly, whereas superluminal propagation of perturbations is common in higher derivative theories such as galileons, in conformal galileon theories [31], perturbations of the scalar can be shown to travel with a speed at most equal to the speed of light in the entire phase space as long as matter fields are excluded [32]. In [31], a simple and almost viable example is provided, with the theory described by the Lagrangian

$$\mathcal{L} = f^2\mathrm{e}^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(1+\alpha)(\partial\pi)^4 \tag{29}$$

where f, Λ and α are constant. Provided $\mathcal{L}$ features a negative kinetic term, as is the case in Eq. (29), this theory admits a time-dependent de Sitter solution,

$$e^{\pi} = \frac{1}{-H_0 t}, \qquad \text{with} \quad H_0 = \frac{2}{3}\frac{1}{1+\alpha}\frac{\Lambda^3}{f} \tag{30}$$

Here, Λ is a strong coupling scale. This theory violates the NEC. The sound speed is subluminal for $0 < \alpha < 3$, but the NEC-violating solution, as is the case for the ghost condensate, cannot be smoothly connected with a Lorentz-invariant vacuum solution. This theoretical setup nevertheless does bare a particularly interesting aspect, which was dubbed the 'galileon genesis", namely that the cosmological solution displayed here is an attractor solution. This allows the possibility to have either emerging cosmological evolution or bouncing solutions.

4.2. Semi-classical and quantum bounces

Semi-classical models are those involving quantized scalar fields in classical spacetimes. The vacuum state being ill defined on a curved background, except in the adiabatic limit of slowly varying scale factor, a regularization or renormalization scheme is required to cure such semi-classical theories of the infinities that arise in the formal expression for the stress-energy tensor. These infinities are associated with the inability to properly define the creation and annihilation operators and thereby unambiguously remove the infinite vacuum term in the usual way. Renormalization results in the inclusion of higher order curvature (counter-) terms in the Einstein–Hilbert action leading to new terms in the Friedmann equations and to the possibility of constructing singularity-avoiding cosmologies.

A more ambitious approach to describe quantum gravitational effects is string theory. The full action of superstring theory possesses scale factor duality and time reversal symmetry, which can be used to construct non-singular cosmologies. These require a *branch change* that smoothly interpolates between contracting and expanding spacetimes. This is the well-known *pre-big-bang* cosmology[3] [33]. In its original version, this model consists of only the dilaton field and the metric. At tree level, it can be shown that a contracting cosmology in the string frame corresponds to an expanding cosmology in the Einstein frame. These two frames are simply related by a conformal transformation. It is therefore not clear whether to identify such a tree-level cosmology with a *bouncing* cosmology *per se*. However, with the inclusion of loop corrections, it is possible to show that the cosmology is indeed non-singular, and it is then plausible that one identifies this non-singular evolution with *branch changing*.

Another possibility to smooth out the curvature singularity is the inclusion of a coupling to a matter or radiation fluid in the tree-level effective 4D action of string theory [34,35]. In order to study the propagation of cosmological perturbations through a bounce, it can either be modeled by a discontinuity across a spacelike hypersurface, in which case it is singular, and the behavior of cosmological perturbations transferred through the bouncing phase depends on how the Israel junction conditions are implemented, or, alternatively, if one smooths the curvature singularity by including higher-order corrections in the effective action, then it becomes possible to actually follow perturbations through the bounce.

Another string way to a non-singular cosmology involves the motion and interactions of higher dimensional (mem)branes in yet higher dimensional bulk geometries. While initially, models were based on branes embedded in, e.g., 5-dimensional bulk geometries, more recent models, based on either heterotic M-theory or on the compactification of 10-dimensional type-II A/B superstring theory manifolds, and on the stabilization of the "moduli" fields,[4] have led to interesting brane dynamics in warped parts of the geometry. In general, these constructions lead to additional terms in the Friedmann equations or unconventional kinetic terms for the inflaton field that can lead to a period of inflation, to bouncing branes and to cyclic cosmologies. The *ekpyrotic* model [36] is one realization of brane cosmology based on heterotic M-theory, while examples of bouncing cosmologies in warped string compactifications can be found in, e.g., [37,38].

A final option, also fully quantum, consists in assuming the energy scale at which the bounce takes place to be sufficiently small that a Wheeler de Witt treatment of quantum cosmology would be appropriate. Although one naturally faces measurement questions in such a context, there exists ways to treat both background and perturbations on an equal footing; a full set of predictions to compare with current data is however not yet available, since only toy models have been written down, but it would seem that a consistent model should be attainable in the near future (see Ref. [39] and references therein).

5. Conclusions

Although inflation appears to largely dominate the field of primordial cosmology, due, in particular, to the fact that many implementations have predicted consequences rather similar to the presently available observations, bouncing alternatives are not entirely ruled out. In addition, a contracting phase followed by a bounce can solve the primordial singularity problem, which renders such models attractive and worth investigating. It must be conceded however that, currently, most bouncing models have difficulties, either because they demand a complicated theoretical framework or because their predictions disagree with cosmological data.

From a purely theoretical standpoint, one might argue that bouncing cosmologies relying on either ghost condensates or unknown quantum gravitational effects [7] in order to successfully avoid the classical singularity should, in view of Occam's razor, be disfavored when compared to much simpler inflationary models involving perfectly well-behaved scalar fields [4]. One should nevertheless remember that the singularity problem of big-bang cosmology will need to be addressed at some stage, and its solution, however contrived it may look from our perspective, may end up being quite natural within a few decades. In other words, one should not refute a theory or a paradigm on philosophical grounds, but instead on whether it is able to answer as of yet unanswered physical questions and whether it agrees with the data or not.

From an observational perspective, given increasingly stringent constraints imposed by present-day cosmological data, many bouncing models are under pressure as they naturally predict either exactly scale-invariant scalar perturbations, or

[3] At low energy, the four-dimensional effective action of the ekpyrotic model is equivalent to a modified version of the pre-big-bang model.

[4] Moduli fields are fields that appear after compactification. They are identifiable with the (*a priori* unfixed) sizes and shapes of cycles in the higher dimensional Calabi–Yau manifold which the theory lives on.

even slightly blue spectra. Reddening the spectrum often demands new components that may contribute in a non-negligible way and produce unobserved isocurvature modes. Furthermore, the bounce itself, involving either NEC violating fields or positive spatial curvature, might induce large non-Gaussianities [21]. Present-day bouncing models scarcely agree with all existing observational constraints, but it must be noted that neither do most inflationary models [4]. More work is needed to reach definite conclusions, perhaps along the lines of purely quantum models?

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References

[1] V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, Cambridge, UK, 2005.
[2] P. Peter, J.-P. Uzan, Primordial Cosmology, Oxford Graduate Texts, Oxford University Press, Oxford, UK, 2009.
[3] M. Lemoine, J. Martin, P. Peter, Inflationary Cosmology, Lect. Notes Phys., vol. 738, 2008.
[4] J. Martin, C. Ringeval, V. Vennin, Encyclopædia inflationaris, Phys. Dark Universe 5–6 (2014) 75.
[5] P. Ade, et al., Planck 2013 results. XXII. Constraints on inflation, Astron. Astrophys. 571 (2014) A22.
[6] P. Ade, et al., Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. 571 (2014) A16.
[7] D. Battefeld, P. Peter, A critical review of classical bouncing cosmologies, Phys. Rep. 571 (2014) 1–66.
[8] S. Hawking, G. Ellis, The Large Scale Structure of Space-Time, Cambridge University Press, Cambridge, UK, 1973.
[9] A. Borde, A.H. Guth, A. Vilenkin, Inflationary space-times are incomplete in past directions, Phys. Rev. Lett. 90 (2003) 151301.
[10] J. Martin, P. Peter, Parametric amplification of metric fluctuations through a bouncing phase, Phys. Rev. D 68 (2003) 103517.
[11] F.T. Falciano, M. Lilley, P. Peter, A classical bounce: constraints and consequences, Phys. Rev. D 77 (2008) 083513.
[12] M. Lilley, L. Lorenz, S. Clesse, Observational signatures of a non-singular bouncing cosmology, J. Cosmol. Astropart. Phys. 1106 (2011) 004, arXiv:1104.3494 [gr-qc].
[13] P. Peter, N. Pinto-Neto, Cosmology without inflation, Phys. Rev. D 78 (2008) 063506.
[14] J. Martin, Inflation and precision cosmology, Braz. J. Phys. 34 (2004) 1307–1321.
[15] S.D. Vitenti, N. Pinto-Neto, Large adiabatic scalar perturbations in a regular bouncing universe, Phys. Rev. D 85 (2012) 023524.
[16] N. Pinto-Neto, S. Vitenti, Comments on "Growth of covariant perturbations in the contracting phase of a bouncing Universe" by A. Kumar, Phys. Rev. D 89 (2014) 028301.
[17] V. Belinsky, I. Khalatnikov, E. Lifshitz, Oscillatory approach to a singular point in the relativistic cosmology, Adv. Phys. 19 (1970) 525–573.
[18] J. Khoury, B.A. Ovrut, P.J. Steinhardt, N. Turok, The ekpyrotic universe: colliding branes and the origin of the hot Big Bang, Phys. Rev. D 64 (2001) 123522.
[19] P. Peter, N. Pinto-Neto, Has the Universe always expanded?, Phys. Rev. D 65 (Jan. 2002) 023513.
[20] P. Peter, N. Pinto-Neto, D.A. Gonzalez, Adiabatic and entropy perturbations propagation in a bouncing universe, J. Cosmol. Astropart. Phys. 0312 (2003) 003.
[21] X. Gao, M. Lilley, P. Peter, Non-gaussianity excess problem in classical bouncing cosmologies, Phys. Rev. D 91 (2015) 023516.
[22] P. Peter, N. Pinto-Neto, Primordial perturbations in a nonsingular bouncing universe model, Phys. Rev. D 66 (Sept. 2002) 063509.
[23] N. Arkani-Hamed, H.-C. Cheng, M.A. Luty, S. Mukohyama, Ghost condensation and a consistent infrared modification of gravity, J. High Energy Phys. 0405 (2004) 074.
[24] L.R. Abramo, P. Peter, K-bounce, J. Cosmol. Astropart. Phys. 009 (Sept. 2007) 001.
[25] E.I. Buchbinder, J. Khoury, B.A. Ovrut, On the initial conditions in new ekpyrotic cosmology, J. High Energy Phys. 0711 (2007) 076.
[26] C. Lin, R.H. Brandenberger, L. Perreault Levasseur, A matter bounce by means of ghost condensation, J. Cosmol. Astropart. Phys. 1104 (2011) 019.
[27] R. Kallosh, J.U. Kang, A.D. Linde, V. Mukhanov, The new ekpyrotic ghost, J. Cosmol. Astropart. Phys. 0804 (2008) 018.
[28] B. Xue, P.J. Steinhardt, Unstable growth of curvature perturbation in non-singular bouncing cosmologies, Phys. Rev. Lett. 105 (2010) 261301.
[29] B. Xue, P.J. Steinhardt, Evolution of curvature and anisotropy near a nonsingular bounce, Phys. Rev. D 84 (2011) 083520.
[30] C. Deffayet, D.A. Steer, A formal introduction to Horndeski and Galileon theories and their generalizations, Class. Quantum Gravity 30 (2013) 214006.
[31] P. Creminelli, K. Hinterbichler, J. Khoury, A. Nicolis, E. Trincherini, Subluminal Galilean genesis, J. High Energy Phys. 1302 (2013) 006.
[32] D.A. Easson, I. Sawicki, A. Vikman, When matter matters, J. Cosmol. Astropart. Phys. 1307 (2013) 014.
[33] M. Gasperini, G. Veneziano, The pre-Big Bang scenario in string cosmology, Phys. Rep. 373 (Jan. 2003) 1–2.
[34] J.C. Fabris, R.G. Furtado, P. Peter, N. Pinto-Neto, Regular cosmological bouncing solutions in low energy effective action from string theories, Phys. Rev. D 67 (June 2003) 124003.
[35] S. Tsujikawa, Construction of nonsingular cosmological solutions in string theories, Class. Quantum Gravity 20 (2003) 1991–2014.
[36] J. Khoury, B.A. Ovrut, P.J. Steinhardt, N. Turok, Ekpyrotic universe: colliding branes and the origin of the hot Big Bang, Phys. Rev. D 64 (Dec. 2001) 123522.
[37] S. Kachru, L. McAllister, Bouncing brane cosmologies from warped string compactifications, J. High Energy Phys. 03 (2003) 018.
[38] D. Easson, R. Gregory, G. Tasinato, I. Zavala, Cycling in the throat, J. High Energy Phys. 04 (2007) 026.
[39] P. Peter, E.J.C. Pinho, N. Pinto-Neto, Noninflationary model with scale invariant cosmological perturbations, Phys. Rev. D 75 (Jan. 2007) 023516.