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Interaction of electromagnetic fields with the environment/Interaction du champ électromagnétique avec l'environnement

Some numerical models to compute electromagnetic antenna–structure interactions

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Abstract

The problem of the radiation pattern from sources in the presence of perfectly conducting objects is of great interest in the design of transmitting or receiving antennas on structures. The optimal antenna location is particularly hard to find because of the many costly tests which must be managed. A reliable and fast electromagnetic code, able to model antenna/structure couplings, can allow engineers to reduce this cost considerably. In this article, we present two different numerical approaches to solve this problem, according to the kind of antenna, and its location. In the first case, the antenna is in the vicinity of the structure and is only known by its radiation pattern in free-space. A new far to near field transformation is used to compute the incident field induced by the antenna on the structure. In the second case, the antenna is integrated into the structure. The technique consists in a physical and geometrical modeling of the antenna, using a Finite Element Method. In both cases, a Boundary Element Method coupled with a Fast Multipole Method, is used because of its suitability to cope with propagation outside the obstacle. Some numerical examples are given to illustrate the accuracy and efficiency of these two approaches. ***To cite this article: N. Zerbib et al., C. R. Physique 6 (2005).***

Résumé

Modèles numériques pour le calcul des intéractions electromagnétiques antenne–structure. La prévision du diagramme de rayonnement de sources en présence d'objets parfaitement conducteurs est un problème de grand intérêt dans la conception d'antenne pour les télecommunications. La détermination de la position optimale d'une antenne est un problème difficile à résoudre à cause du nombre de tests coûteux qui doivent être réalisés. La simulation numérique par un code fiable et rapide capable de modéliser les intéractions électromagnétiques antenne/structure permet de réduire considérablement le nombre d'essais à mettre en œuvre pour résoudre ce type de problème. Dans cet article, nous présentons deux approches différentes pour résoudre ce problème en fonction du type et de la position de l'antenne. Dans le premier cas, l'antenne est placée à proximité de la structure et est connue uniquement par son diagramme de rayonnement en espace libre. Une nouvelle transformation champ lointain – champ proche est alors utilisée pour calculer le champ incident induit par l'antenne sur la structure. Dans le second cas, l'antenne fait partie de la structure. La technique consiste alors à modéliser physiquement et géométriquement l'antenne en utilisant une méthode d'Eléments Finis. Dans les deux cas, une méthode d'Equations Intégrales, couplée à une Méthode Multipôle rapide, est employée pour la propagation des ondes électromagnétiques en dehors de l'obstacle. Des exemples numériques sont présentés afin d'illustrer la précision et la robustesse de ces méthodes. ***Pour citer cet article : N. Zerbib et al., C. R. Physique 6 (2005).***

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Mots-clés : Intéractions antenne–structure ; Méthode d'Eléments Finis

1. Introduction

Usually, antennas are characterized by their radiation pattern in free space. This characterization is performed by measurement of the radiated electric field, received or transmitted by the stand alone antenna. When the antenna is put on a structure, the radiation pattern is perturbed by the electromagnetic interaction with the surrounding structure. It can induce a performance loss of the communication system. To ensure a good link budget of the transmission, we need to analyze these perturbations. The direct measurement of the perturbed radiation pattern on site, or on a mock-up, is complex and expensive. Numerical methods provide an efficient tool to foresee and analyze the antenna/structure interaction. Usable numerical methods for antenna/structure interaction analysis can be classified in two families:

– *Asymptotic methods* (*geometrical or optic theories*): The size of the structure must be large in comparison of the wavelength λ. These methods are well suited for 'high frequency' problems if the geometry does not have small details.
– *Exact methods* (*e.g., Method of Moment, noted MoM*): They provide a high accuracy but are limited by the size of the problem (few λ).

In this article, we focus our attention only on the exact method. We are interested in computation of antennas put on a perfectly metallic structure (e.g., a satellite). The purpose is to compute the radiation pattern of the antenna, with the surrounding metallic structure. We have distinguished two configurations we solve with different numerical techniques:

– *Case* 1: The antenna is in the vicinity of the structure. The antenna is close to the structure without contact. Moreover, the antenna is known, with its far-field radiation pattern. A typical case is a quadrifilar antenna.
– *Case* 2: The antenna is integrated into the structure. This is the case of patch antennas, for example.

In both approaches, the electromagnetic problem on the structure (called external problem) is described with a Boundary Element Method (BEM). Let us briefly recall the Stratton–Chu relations between the electromagnetic field (E, H) and the equivalent currents (J, M):

$$\begin{cases} E(x) = E^{\mathrm{inc}}(x) + \mathrm{i}kZTJ(x) + KM(x), & x \in \Omega^{\mathrm{ext}} \\ H(x) = H^{\mathrm{inc}}(x) - KJ(x) + \mathrm{i}kZ^{-1}TM(x), & x \in \Omega^{\mathrm{ext}} \end{cases} \tag{1}$$

where Ω^{ext} is the surrounding medium assumed to be the free-space and k denotes the wave number, Z the free-space intrinsic impedance. The respective potentials are given by

$$\begin{cases} TJ(x) = \displaystyle\int_\Gamma G(x,y)J(y)\,\mathrm{d}\Gamma(y) + \frac{1}{k^2}\int_\Gamma \underset{x}{\mathrm{grad}}\, G(x,y)\,\underset{\Gamma}{\mathrm{div}}\, J(y)\,\mathrm{d}\Gamma(y) \\ K\phi(x) = \displaystyle\int_\Gamma \underset{y}{\mathrm{grad}}\, G(x,y) \times J(y)\,\mathrm{d}\Gamma(y) \end{cases} \tag{2}$$

$G(x, y) = \frac{\exp(\mathrm{i}k|x-y|)}{4\pi|x-y|}$ is the Green kernel and div_Γ is the surfacic divergence. The equivalent currents are linked to the electromagnetic field by

$$J = n \times H, \qquad M = -n \times E \tag{3}$$

where n is the unit normal to Γ. This representation of the electromagnetic field allows us to reduce the scattering problem to the determination of the currents J. The boundary condition of the perfectly conducting property of the scatterer amounts to an integral equation which can be solved by means of the Method of Moments (MoM). After meshing the structure, the linear system obtained is solved by an iterative solver coupled with a Fast Multipole Method (FMM).

The difference between the two approaches (Cases 1 and 2) occurs in the treatment of the antenna (called the internal problem). In Case 1, we use a new far to near field transformation to compute the incident field on the structure induced by the antenna. The far field radiation pattern of the antenna is explicitly used. This technique uses a FMM based algorithm. In Case 2, the internal problem is solved with a Finite Element Method (FEM) coupled with a MoM method for the external domain. In the following, we describe these two methods in more details and then numerical examples are given.

2. Case 1: The far to near transformation and MoM

In Case 1, remember that the antenna description should be the far field radiation pattern, issued from measurement or simulation. The method used to solve this case is based on a two step approach. In the first step, we use a new far to near field transformation in order to compute the incident electric field over the structure. In a second step, we solve the integral equation by using an iterative solver coupled with a FMM algorithm. In this way, the size of problem usually solved by the MoM could be extended and we are able to consider large structures.

2.1. The first step: The far to near field transformation

The antenna (the source) is described by its far field radiation pattern. It could be generated independently by measurement or computation. The problem consists in evaluating the electrical field $E_a(x_s)$ radiated by the antenna on each point of the structure noted by x_s. Usually we use an exponential approximation (based on physical optics) of the Green kernel:

$$E_a(x_s) \approx E_a^{\infty}(r)\frac{\mathrm{e}^{jkr}}{|r|}$$

with $E_a^{\infty}(r)$ the antenna far field along the r direction. This approximation is easy to compute and widely used within the electromagnetism community, but it is still only valid if the distance antenna-satellite is larger than a few $lambda$. It is the main restriction for our application. The new approach we used is more involved. It is based on a multipole expansion of the Green kernel [1]. We consider only a perfectly metallic structure. Therefore, the antenna radiated field $E_a(x_s)$ is related to the antenna current by equations (EFIE, MFIE or CFIE) described in [2] as (1) where the magnetic current equals to 0. Using the Gegenbauer's theorem for the Green's kernel and usual multipole techniques [3,4,1], we can write the near field expression in the form

$$E_a(x_s) \simeq \frac{k}{4\pi}\int_{S^2} E_a^{\infty}(\hat{s})T^L(\hat{s}, D)\mathrm{e}^{+\mathrm{i}k(x_s - c_{B_{x_s}})\cdot\hat{s}}\,\mathrm{d}\sigma(\hat{s}) \tag{4}$$

with

$$T^L(\hat{s}, D) = -i\sum_{\ell=0}^{L} i^{\ell}(2\ell+1)h_{\ell}^{(1)}\big(k|D|\big)P_{\ell}(\widehat{D}\cdot\hat{s})$$

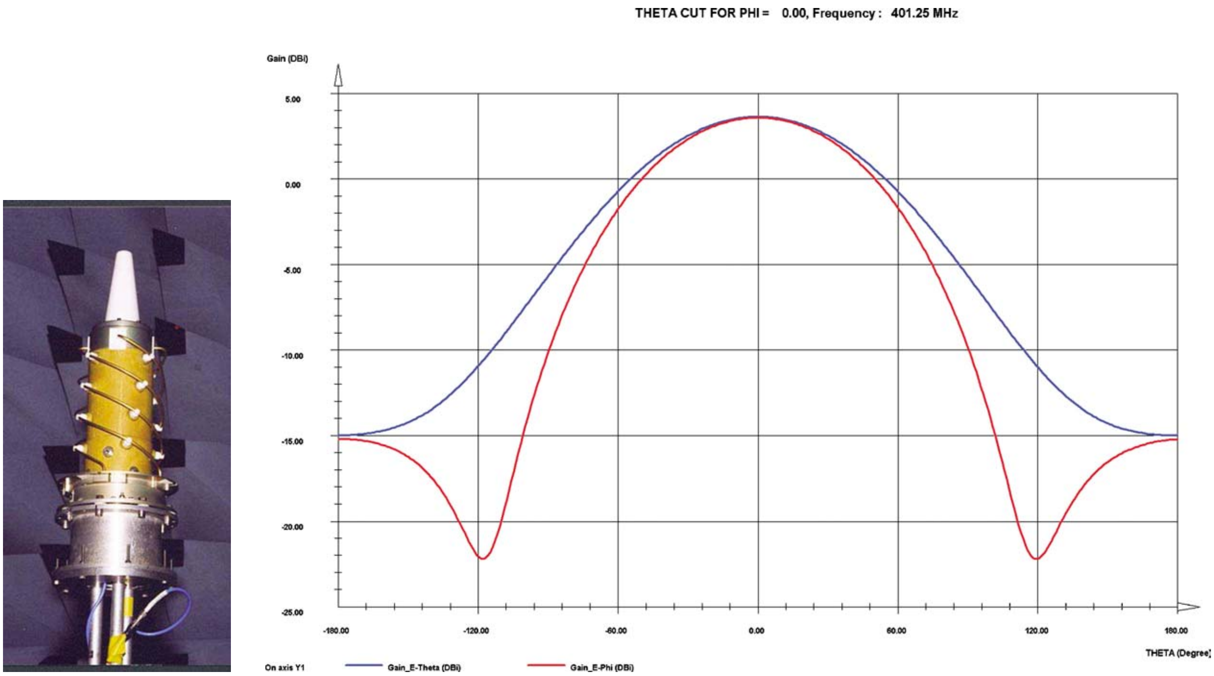
where S^2 is the 3D unit sphere. $P_\ell(x)$ is the Legendre polynomial of order ℓ, $h_\ell^{(1)}(u)$ is the spherical Hankel function of order ℓ. D is the distance between two boxes in the same level and the parameter L determines the number of terms in the Gegenbauer series. This equation means that the near field can be retrieved at each point x_s with the knowledge of the far field pattern in all directions. To calculate the far field in any direction, we extrapolate the measured data with a spherical harmonic expansion (see [1] for more details).

2.2. The second step: Integral equations coupled with a FMM

The currents induced by the antenna on the structure are computed by an Integral Equation defined on the satellite. Due to the large dimension of this structure, we use an iterative solver coupled with a FMM algorithm. The main advantage of the FMM is the capability of extending the application domain of exact methods up to one million of degrees of freedom. It allows us to deal with realistic size of satellite. The FMM algorithm is described, for instance in [4,3].

2.3. Results for Case 1

This method, described above, has been used to solve the problem of a quadrifilar antenna (DORIS) on a satellite as illustrated on Fig. 1. The stand alone antenna is described by its far field pattern (E_θ, E_ϕ) (see Fig. 1). The frequency is fixed to 401.25 MHz. The distance between the antenna and the satellite is about 0.5λ. The near to far field transformation algorithm has been applied in order to compute the incident electric field on the whole satellite. Then, the EFIE integral equation has been solved in 16 GMRES iterations using a 8-level FMM. On Fig. 2, we have drawn the two components of the radiation pattern of the perturbed antenna. For comparison, we give the radiation pattern of the antenna + satellite fully computed with MoM. We can see a good agreement between the two results. Many numerical examples have been computed in order to evaluate the performance of this approach. It gives very accurate results if some requirements are enforced. The minimum distance between the antenna and the structure must be larger than a quarter of the wavelength, otherwise some overflows in the FMM scheme



(a) (b)

Fig. 1. Description of the Doris antenna. (a) Configuration. (b) Radiation pattern.

may occur. Moreover, an other distance criteria imposes a minimum distance, depending on the number of spherical harmonic coefficients used to interpolate the far field pattern. In practice, we obtain a minimal distance of about third of the wavelength. This distance is much smaller than the distance usually used in the classical approach.

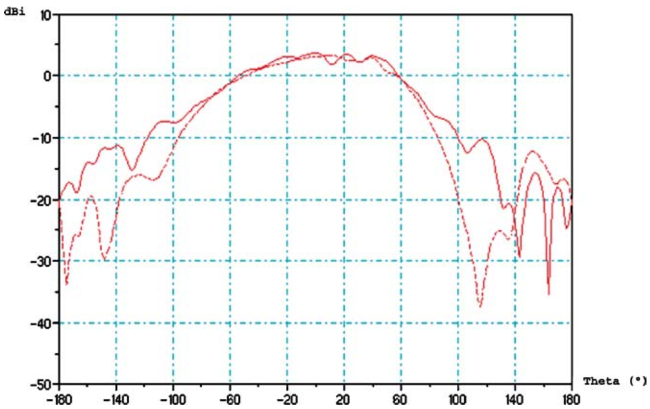
3. Case 2: The FE method coupled with the MoM

In this section, we present another method to compute the electromagnetic antenna–structure interactions when the antenna is integrated into the structure. Unlike the previous procedure, it consists in a physical and geometrical modeling of the antenna. To do it, a Finite Element Method (FEM) is used for its flexibility to deal with varying dielectric properties of materials. For the external problem, a Boundary Element Method is used for its suitability to cope with the propagation outside the obstacle. The algorithm used to solve this case is also based on a two step procedure.

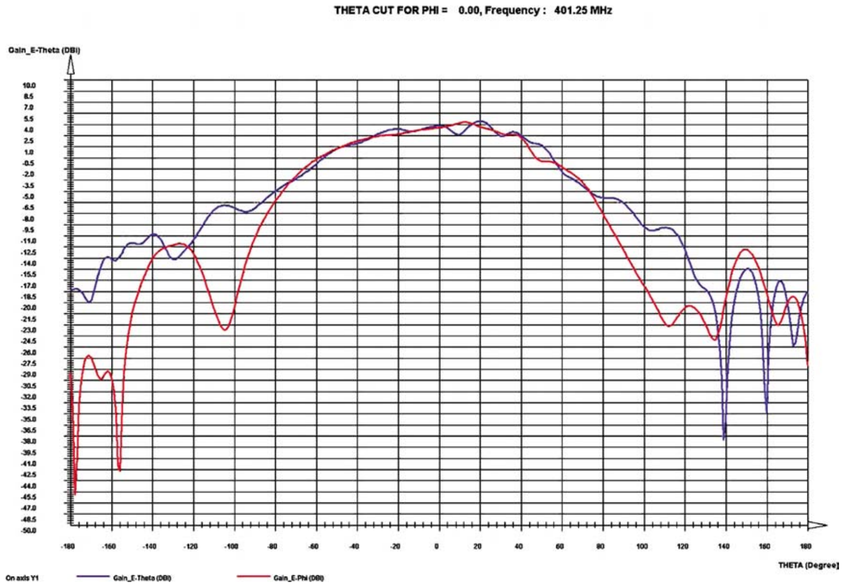
3.1. The first step: Formulation of the antenna fields by the FEM

Consider an arbitrarily-shaped antenna filled by an heterogeneous dielectric where (ε_r, μ_r) are respectively the relative electric permittivity and the relative magnetic permeability. We denote the antenna domain by Ω_A and the dielectric interface separating the interior from the exterior of the antenna by Σ. For a electric source $\mathbf{J_A}$ inside the antenna, the variational formulation of the Maxwell's equations verified by the electromagnetic field in Ω_A is given by

$$\frac{1}{\mathrm{i}kZ_0}\int_{\Omega_A}\left(\frac{1}{\mu_r}\nabla\times\mathbf{E}.\nabla\times\mathbf{E}'-k^2\varepsilon_r\mathbf{E}.\mathbf{E}'\right)\mathrm{d}\Omega_A-\mathrm{i}k\int_{\Sigma}T\mathbf{M}(y).\mathbf{M}'(x)\,\mathrm{d}\Sigma(x)$$
$$+\int_{\Sigma}K\mathbf{J}(y).\mathbf{M}'(x)\,\mathrm{d}\Sigma(x)+\frac{1}{2}\int_{\Sigma}\mathbf{n}\times\mathbf{J}(y).\mathbf{M}'(x)\,\mathrm{d}\Sigma(x)=-\int_{\Omega_A}\mathbf{J_A}.\mathbf{E}'\,\mathrm{d}\Omega_A \tag{5}$$



(a)



(b)

Fig. 2. (E_θ, E_ϕ) components of the radiation pattern of the perturbed antenna. (a) Our method. (b) MoM analysis.

3.2. The second step: Formulation of the exterior fields by the BEM

In order to avoid spurious modes which may render meaningless the numerical solving, specially for the coupled FE–BE approach, a Combined Field Integral Equation (CFIE) must be used for the BE part [2]. This CFIE is obtained by a suitable combination of the electric field tested by Rao–Wilton–Glisson (*RWG*) functions and the magnetic field tested by $n \times RWG$ functions (*RWG* functions turned by $\pi/2$ around the normal n of the surface). However, when the magnetic currents are also involved in the integral representation of the electromagnetic field, the CFIE requires the composition of the $n\times$ operator with the metallic EFIE integral operator T, which is not easy to compute.

To evaluate this composition, it is not obvious that we can move the gradient into the test function because $n \times RWG$ is not a continuous function on the surface of the obstacle. However, if we consider separately two triangles of the mesh, we can use a part integration formula and apply the Gauss divergence theorem [5]. Due to the fact that the surface divergence of $n \times RWG$ function vanishes, the original outer integral over the triangle is reduced to a line integral over the edges of this triangle. The charge densities relative to the divergence of $n \times RWG$ functions are no longer the usual integrable functions but rather Dirac distributions concentrated along the edges of the mesh. Then, the stability of the numerical scheme becomes highly questionable.

In this article, we present an accurate and robust method to overcome the above difficulties. It has been inspired by [6] essentially. It consists in using of a mixed BE method whose algorithm is performed in two steps. For magnetic currents $\mathbf{M}$, we first join an auxiliary unknown X to the formulation to evaluate the image of the currents $\mathbf{M}$ by the integral operator T, $\mathbf{X} = T\mathbf{M}$.

This new equation is translated variationally by $\int_\Sigma \mathbf{X}.\mathbf{X}'\,\mathrm{d}S = \int_\Sigma T\mathbf{M}.\mathbf{X}'\,\mathrm{d}S$ where both unknown $\mathbf{X}$ and test functions $\mathbf{X}'$ are expressed on a unusual finite element space RWG_{aug}.

This space RWG_{aug} is built in order to solve the difficult problem of mass lumping for edge finite elements. In a second step, the contribution of the $\mathbf{n}\times$ operator, given by $\int_\Sigma \mathbf{n}\times\mathbf{X}.\mathbf{J}'\,\mathrm{d}S$, is added. A judicious treatment enables us to add the contribution of $\mathbf{n}\times\mathbf{X}$ at the level of the assembly process and so to remove the auxiliary unknown from the final linear system to solve.

3.3. Resolution of the global system

The global system to be solved resulting from the FE–BE coupling is given by

$$\begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22}+B_1 & B_2 \\ 0 & B_3 & D \end{pmatrix}\begin{pmatrix} \mathbf{E} \\ \mathbf{M} \\ \mathbf{J} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \tag{6}$$

where $(A_{i,j})_{i=1,2}^{j=1,2}$ represents the FE sparse blocks, whereas $(B_i)_{i=1,2,3}$ and D represents the BE dense blocks. Hence, the global form of this matrix prevents the use of the available libraries on parallel platforms which can deal with only one type of matrix, either sparse or dense, simultaneously.

As the obstacle involves a (relatively) small dielectric part compared to the large-sized metallic object, we propose a method to solve this system in two steps. First, a sparse matrix parallel multifrontal library [7] is used to remove the FE degrees of freedom $\mathbf{E}$ that do not correspond to an equivalent magnetic current $\mathbf{M}$ from the equations. At this step, we are led to solve the following dense system

$$\begin{pmatrix} C & B_2 \\ B_3 & D \end{pmatrix}\begin{pmatrix} \mathbf{M} \\ \mathbf{J} \end{pmatrix} = \begin{pmatrix} \widetilde{F}_2 \\ F_3 \end{pmatrix} \tag{7}$$

where $C = B_1 + (A_{22} - A_{21}A_{11}^{-1}A_{12})$ and $\widetilde{F}_2 = F_2 - A_{21}A_{11}^{-1}F_1$. The well-known parallel library SCALAPACK is used to perform the LU decomposition of C relative to the coupling of the magnetic currents with themselves. Secondly, we are led to solve the system

$$\big(D - B_3C^{-1}B_2\big)\mathbf{J} = \widetilde{F}_3 \tag{8}$$

where $\widetilde{F}_3 = F_3 - B_3C^{-1}\widetilde{F}_2$ and J are the degrees of freedom of the equivalent electric currents. For large-sized metallic structures, the resolution of the BE part (8) can be done by means of the FMM only which in return requires a good preconditioning for the global system to be solved by a Krylov method. The block D corresponds to the impedance matrix obtained for the CFIE formulation when the dielectric part is also assumed a perfectly conducting metal. The system (8) is then solved by the GMRES algorithm [8] using as preconditioner a Sparse Approximate Inverse of D only. Since the extent of the magnetic currents $\mathbf{M}$ is assumed to be (relatively) small, the contribution of $B_3C^{-1}B_2$ is considered like a low rank perturbation of the main block D. Each iteration is performed using an evaluation of the matrix-vector products DJ by the FMM and $B_3C^{-1}B_2J$ from two matrix-vector products and a forward and a backward substitutions.

3.4. Results for Case 2

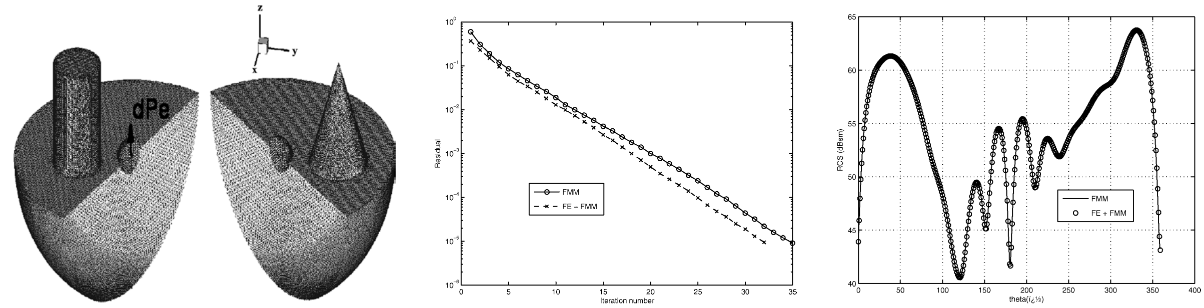
A numerical example is presented to demonstrate the efficiency of our approach. The scatterer is depicted in Fig. 3. It consists in a dielectric sphere ($\varepsilon_r = 1,\ \mu_r = 1$) of radius $R_1 = 0.25$ cm integrated into a half metallic sphere of radius $R_2 = 2$ cm close to a 1.5 cm high metallic cylinder of radius 0.4 cm and a 1.5 cm high metallic cone of radius 0.5 cm. The source is an electric dipole located at the center of the dielectric sphere with a z-polarization and the wavenumber is $k = 12\ \mathrm{m}^{-1}$. The mesh density is about **12** points per wavelength.

We study the convergence behavior based on the number of preconditioned iterations. We compare the behavior of the complete system

$$(D - B_3C^{-1}B_2)J = \widetilde{F}_3 \tag{9}$$

with the simpler system obtained when the dielectric is supposed not to exist

$$DJ = F_3 \tag{10}$$



(a) (b) (c)

Fig. 3. Comparisons between the two FMM solvers. (a) Antenna integrated in large metallic structure. (b) The convergence behaviour. (c) The bistatic RCS.

In Fig. 3, we observe that the number of iterations required for the GMRES algorithm for (9) remains approximately the same as for (10) whose coefficients matrix is D only. The convergence for (9) is reached after 32 iterations whereas 35 iterations are needed for (10). The comparison of the bistatic RCS with a mesh density about 8 points per wavelength is given in Fig. 3. The agreement between both solvers is remarkably good.

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