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MCAO for very large telescopes/OAMC pour les très grands télescopes

Introduction to Multi-Conjugate Adaptive Optics systems

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Abstract

The paper describes the basic concepts underlying the design of Multi-Conjugate Adaptive Optics systems. The paper takes into account mainly three papers. The first paper, published in 1988, is due to J. Beckers and it introduces the currently used multi-conjugation concept and quantifies the increase in FoV achieved applying this concept with respect to the single conjugate adaptive system. The second paper considered is due to M. Tallon and R. Foy. This paper, published in 1990, provides a quantitative method to estimate the 3D map of the atmospheric phase perturbation. This is the needed information to properly control the multi-conjugate system introduced by J. Beckers. Finally a third paper, published in 2000 and due to R. Ragazzoni et al. is considered, where the Layer Oriented scheme is described opening the multi-conjugate adaptive system field to natural guide stars. These three papers stimulated a lot of other contributions from several people and some of them are briefly discussed in the paper. ***To cite this article: S. Esposito, C. R. Physique 6 (2005).***

Résumé

Introduction aux systèmes d'optique adaptative multi-conjuguée. Cet article présente les notions de base qui gouvernent le dimensionnement des systèmes d'optique adaptative multi-conjuguée [OAMC]. La synthèse présentée ici reprend principalement les résultats de trois articles. Le premier, publié en 1988 par J. Beckers, introduit le concept de multi-conjugaison and évalue le gain attendu en champ de vue par rapport à l'optique adaptative classique. Le deuxième article fondateur est celui de M. Tallon and R. Foy. Ce travail, publié en 1990, propose une méthode pour estimer quantitativement la carte 3D des perturbations de phase atmosphériques. Cette information est essentielle pour commander les systèmes multi-conjugués introduits par J. Beckers. Un troisième article majeur, publié en 2000 par R. Ragazzoni et al., décrit le principe de l'analyse de front d'onde dite « Layer Oriented ». Cette technique permet d'ouvrir le domaine de l'OAMC vers l'utilisation d'étoiles naturelles pour l'analyse de front d'onde. Ces trois articles ont bien sûr stimulé un grand nombre d'autres contributions émanant de divers instituts, l'essentiel de ces travaux est discuté dans le texte qui suit. ***Pour citer cet article : S. Esposito, C. R. Physique 6 (2005).***

1. Introduction

The aim of this article is to give a short review of the basic concepts underlying the design of Multi-Conjugate Adaptive Optics (MCAO) systems for astronomical applications. Such systems have been proposed in literature to overcome a strong limitation of the classical Adaptive Optics (AO) systems, namely, the small size of the isoplanatic patch. In a conventional AO system the scientific target and the reference star are usually different astronomical objects located in different sky positions. In this condition the propagation paths of reference and target wavefronts through the turbulent atmosphere are different. This situation generates different aberrations in the above mentioned wavefronts. On top of that the difference between the reference and scientific wavefront perturbations is not constant in the telescope FoV. It depends on the angular distance between the

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reference source and the scientific target. The largest angular distance that still allows a good quality correction, the so called isoplanatic angle, was introduced by Fried in 1982 [1]. The usual values for good telescope sites are about some arcsecs in the V band and some tens of arcsecs in K band. These relatively small values gave rise to the search of new techniques able to obtain large corrected FoV. This search lead J. Beckers in 1988 [2] (JB88) to propose, as a new solution to increase the isoplanatic patch, a Multi-Conjugate Adaptive Optics system defined as "*an AO system that uses an array of adaptive mirrors each one placed at a conjugate of a different atmospheric layer*". In Becker's paper this system was supposed to use laser generated reference star (after Foy and Labeyrie [3]), so overcoming another very strong AO system limitation, i.e., the limited sky coverage. The JB88 paper highlighted the gain in isoplanatic patch dimension obtained correcting with multiple mirrors the atmospheric phase perturbations. However, to operate such a multi-conjugate system the knowledge of the phase perturbations due to each atmospheric layer conjugated to the system mirrors is required. A three-dimensional wavefront sensing process or, as called later, a tomographic wavefront sensor, has to be part of a working MCAO system. This problem was pointed out in JB88 paper and a possible solution was outlined. However a computing scheme for the wavefront sensing process was not given. The basic concept for the wavefront sensing process in MCAO was stated by M. Tallon and R. Foy (1990) [4]. This paper had, as primary goal, to demonstrate that the wavefront measurements obtained using multiple laser generated reference sources, each one acting as a reference source for a wavefront sensor, allow one "*to retrieve the three-dimensional map of the phase aberration through the atmosphere*". At the same time this method allows solving or reducing the so called cone effect [5] introduced by the use of reference sources at a finite distance from the telescope. This cone effect reduction was achieved using the three-dimensional wavefront measurements to reconstruct the wavefront aberration undergone by the scientific object. The concepts presented in these two articles have been elaborated by many authors in various directions, generating many improvements in the MCAO system theory, together with completely new ideas. To mention only a few names, the work of Johnston and Welsh [6], Sasiela [7], Ellerbroeck and Rigaut [8,9], investigated intensely the MCAO system performance as a whole, while work due to Fusco et al. [10] addressed especially the field of the phase reconstruction algorithm and noise propagation associated with tomographic wavefront sensing. On the other hand, an important idea in MCAO system design was contributed by Ragazzoni et al. (2000) (RR00) with the introduction of the so called 'Layer-oriented Multi-reference wavefront sensing' scheme [11]. This concept was proposed to obtain a MCAO system design based on natural guide star and scalable to Extremely Large Telescopes (ELT). The starting point to trigger the idea is to optically co-add light from different reference stars and use this overall intensity for wavefront sensing. The authors evaluated the "*amount of photons coming from natural sources in the sky*" and obtained an average equivalent brightness between $V = 15.1$ and $V = 13.1$ according to Galactic latitude. These numbers are the basis for the use of the Layer Oriented technique. From the year 2000 the MCAO system theory developed much further, and in parallel; several real systems has been designed and are currently under realization and test. Among these we mention the Gemini South LGS MCAO [12], a star oriented MCAO system using laser generated guide star, MAD the ESO project for an MCAO system demonstrator featuring a star oriented and a layer-oriented tomographic wavefront sensor [13], and Nirvana [14] a high order MCAO system for the interferometric observing mode of LBT. The realization and test of these systems will surely contribute much to the design of MCAO systems for the next generation of Extremely Large Telescopes.

2. MCAO: starting the field

The well-known limitation of single reference astronomical AO systems is the reduced size of the corrected FoV. As reported in the introduction, the numerical values are some arcsecs and some tens of arcsecs in the visible and NIR, respectively. Increasing the size of the corrected FoV was the aim of Beckers, who in 1988 proposed the use of MCAO [2]. The concept in Beckers' proposal is to use more than one mirror to correct the atmospheric perturbations. In order to efficiently do this, each of the correcting mirrors has to be optically conjugated to a certain turbulent layer. The proposed solution can be easily understood by considering the angular anisoplanatism effect of a single atmospheric layer. Our discussion in this article will take care mainly of the angular anisoplanatism effect, because this is the main effect that MCAO systems want to reduce. Let us consider the single layer case as reported in Fig. 1(a). We assume a single turbulence layer infinitely thin and a deformable mirror conjugated to the telescope pupil of diameter D. In Fig. 1, as is usually the case, the reference source for the AO system is placed in a different direction with respect to the scientific object. The angular anisoplanatism error arises as a combination of two facts: firstly the correction is estimated looking along a different line of sight with respect to the scientific object line of sight; and secondly the wavefront correction is applied in a plane that is not the plane of the atmospheric disturbance or a plane optically conjugate to it. The combination of these two effects creates a shifting error between the atmospheric phase perturbation and the applied correction. Considering the notation introduced in Fig. 1, this shift is readily computed in the case of a single layer infinitely thin (Fig. 1(a)) and is θH. In the configuration of Fig. 1(a), Fried showed [1] that the Angular Anisoplanatism wavefront Error (AAE) variance is given by

$$\sigma^2 = (\theta/\theta_0)^{5/3} \quad [\mathrm{rad}^2] \tag{1}$$

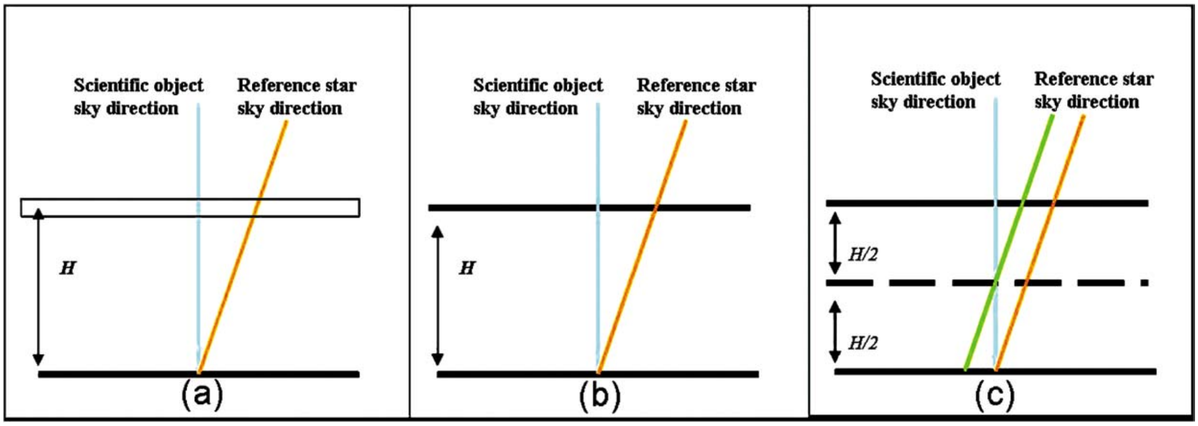

Fig. 1. A sketch of the various geometry and DM conjugation used in the paper for angular anisoplanatism computation: (a) ground conjugate with infinitely thin layer at altitude H over the telescope pupil, (b) ground conjugate with extended layer of thickness H, (c) same case but with DM conjugate to mid thickness.

where θ and θ_0 are the reference to scientific object angular distance and the so called isoplanatic angle, respectively. For a single, infinitely thin, turbulent layer the expression for θ_0 is the following:

$$\theta_0 = 0.31 r_0/H \tag{2}$$

where

$$r_0 = A\left[\int_0^\infty C_n^2(h)\,\mathrm{d}h\right]^{-3/5} \tag{3}$$

is the Fried parameter [15], H is the layer altitude over the telescope and $C_n^2(h)$ is the so called refraction index fluctuation structure constant measuring the turbulence strength at the altitude h. It is now easy to see that the error variance quantified by Eq. (1) is almost proportional to the square (5/3) of the normalized beam displacement $\theta H/0.31 r_0$. If the computed correction is applied directly at the layer altitude, or equivalently the DM is placed in a plane conjugate to that altitude, the beam displacement is zero and the AAE can be made zero. It is interesting to note that the considerations above do not refer to the telescope diameter and so hold for any D (note that Eq. (1) is valid for $D/r0 \gg 1$, a condition usually met in astronomical AO). We report now the formula of the AAE in the case of a thick layer. Again, according to Fried, the AAE for a continuous turbulence distribution surrounding the telescope aperture (a thick layer) is given by Eq. (1), where the layer altitude H is substituted by an effective layer height given by

$$\overline{H} = \left[\frac{\int_0^\infty C_n^2(h) h^{6/5}\,\mathrm{d}h}{\int_0^\infty C_n^2(h)\,\mathrm{d}h}\right]^{5/6} \tag{4}$$

A sketch of this second case is reported in Fig. 1(b). The effective height of an uniform distribution of $C_n^2(h)$ results $(5/11)^{5/6} H = 0.52H$. In the following we will consider this effective height as equal to $0.5H$ for simplicity. So, even in this case, the AAE error is related to an effective shifting error given by $\theta\overline{H}$. It is easy to see, considering what was stated before for the thin layer, that this average beam displacement can be reduced conjugating the DM to the midpoint of the thick layer as reported in Fig. 1(c). Applying Eqs. (2) and (4) to the two sublayers placed below and above the DM, we can compute the overall θ_0 of this layer in the DM conjugated configuration of Fig. 1(c). The expression we find for the $\theta_{0\,\mathrm{sub}}$ of each sub-layer is achieved considering the new layer width of $H/2$ and is given by

$$\theta_{0\,\mathrm{sub}} = \frac{0.31 A (C_n^2 H/2)^{-3/5}}{0.5H/2} = \theta_0 2^{8/5} \tag{5}$$

Using the above results and considering that the AAE of the complete layer is the sum of the AAE of the two sub-layers, we find

$$\sigma^2 = 2\left(\frac{\theta}{\theta_{0\,\mathrm{sub}}}\right)^{5/3} = 2\left(\frac{\theta}{\theta_0 2^{8/5}}\right)^{5/3} = \left(\frac{\theta}{2\theta_0}\right)^{5/3} \tag{6}$$

This reduction of the AAE by approximately a factor of four arises for two reasons: firstly we sum the squares of two beam displacements being half of the thick layer with ground conjugate case previously discussed; secondly we normalize these displacements to the sub-layer r_0 roughly twice the single layer r_0. In other words in the single layer case a proper conjugation of the DM can increase the θ_0 value by a factor two. This approach was applied by Beckers to the case of N DMs in an uniformly distributed atmospheric turbulence. In this case he supposed placing N equally spaced DMs in the portion of the atmosphere where C_n^2 has significant values. The results achieved under this assumption can be applied to the case of a layered atmosphere with similar or better results. Following the derivation made for the AAE of a thick single layer with conjugated DM, we know that the θ_0 value for the ith atmospheric layer with conjugated DM, indicated in the following as θ_{0i-dm}, is twice the standard θ_{0i}. Moreover, the single ith thick layer θ_{0i} value with no DM conjugation is obtained using Eqs. (2) and (4) so that

$$\theta_{0i-dm} = 2\cdot\theta_{0i} = 2\times 0.31A(C_n^2 H/N)^{-3/5}\big/\big(H/(2N)\big) = 2\cdot\theta_0 N^{8/5} \tag{7}$$

Now the overall AEE of the N layers with conjugated DMs can be achieved summing in quadrature the AAE of the various layers so that we find

$$\sigma^2 = \sum_{i=1}^{N}\left(\frac{\theta}{\theta_{0i-dm}}\right)^{5/3} = \sum_{i=1}^{N}\left(\frac{\theta}{2\theta_0 N^{8/5}}\right)^{5/3} = \left(\frac{\theta}{2\theta_0 N}\right)^{5/3} \tag{8}$$

The achieved result show that the use of N DMs conjugated to different altitudes in a homogeneous atmosphere increase the isoplanatic FoV by a factor $2N$ with respect to the case of a single DM conjugated to ground. It is interesting to note that the r_{0i} values of each turbulent layer are clearly larger than the overall r_0 of the whole atmosphere so that the DM sampling on each layer can be decreased with respect to the sampling used in the case of a single DM system. Let us consider placing one actuator per r_{0i} so that the actuator pitch on the ith layer is given by $(d_{\mathrm{act}})_i = r_{0i}$. Considering again N layers and a telescope of diameter D we can obtain an estimate of the overall number of actuators required in an MCAO system using N DMs. This estimate is readily found to be

$$N_{\mathrm{act}} = \sum_{i=1}^{N}\left(\frac{D}{(d_{\mathrm{act}})_i}\right)^2 = \sum_{i=1}^{N}\left(\frac{D}{r_{0i}}\right)^2 = \left(\frac{D}{r_0}\right)^2 N^{-1/5} \tag{9}$$

From this formula we find that an MCAO system has roughly the same number of actuators as a single mirror AO system. However, this configuration has a larger fitting error [16] than the single DM system. To show this, we consider the MCAO system DMs fitting error. This term is the one that directly sums up to the angular anisoplanatism error estimate above. Summing the fitting error formula in quadrature we find:

$$\sigma_{\mathrm{fit}}^2 = \sum_{i=1}^{N}\big((d_{\mathrm{act}})_i/r_{0i}\big)^{5/3} = N \tag{10}$$

So, a sampling of one actuator per r_{0i} gives a fitting error N times larger than the fitting error of a single DM conjugated to ground and having one actuator per overall r_0. So if the fitting error has to remain the same of the single mirror case the sampling on the ith DM has to be increased. In particular, we find directly from the above equation that

$$(d_{\mathrm{act}})_i = r_{0i}/N^{3/5} \tag{11}$$

The increase of the corrected FoV and the relatively small penalty on the number of total actuators given by last equation, made MCAO very attractive, so that this paper of J. Beckers started a completely new field.

3. Getting the 3D phase perturbation

In all the above we have implicitly supposed that we are able to measure the wavefront aberrations introduced by each of the N turbulent layers conjugated to each DMs. A way to do this was outlined in JB88 [2], but to have a quantitative way to disentangle the contributions of the various layers we have to consider a second paper due to M. Tallon and R. Foy (TF90) [4]. This paper, published in 1990, proposed a quantitative way to retrieve the 3D map of the atmospheric perturbations using several laser guide stars. Moreover, the proposed method was devised in order to solve the cone effect problem [5] that arises when artificial reference stars generated at a finite distance from the telescope are used for wavefront sensing. The proposed concept is sketched in Fig. 2, adapted from the original paper. The basics assumptions used in the paper are:

- the turbulence is modelled using a finite number of thin turbulent layers. Two of these layers are represented in Fig. 2;

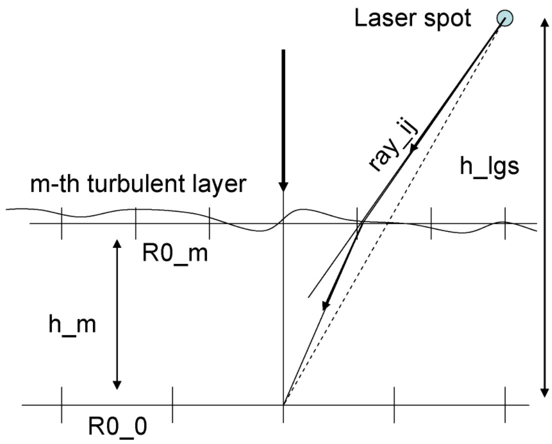

Fig. 2. A schematic picture of the wavefront sensing arrangement proposed by Tallon and Foy to recover the 3D phase perturbation map and the cone effect. Note here the different sampling steps in the two represented layers.

- the approximation of geometrical propagation is made. Rays propagate along straight lines between the layers and no interference effects are taken into account;
- phase distortion in the telescope pupil are measured using a Shack–Hartmann sensor with an appropriate sampling;
- all rays, leaving the laser source and reaching the sensor subapertures, do not depart significantly from their unperturbed path. In other words, the ray propagation directions experienced very small changes crossing each turbulent layer. A consequence of this is that the rays do not exit from the cone defined by the laser guide star as a vertex and the SH subaperture as base.

In this hypotheses we see that the phase aberration seen in a given sensor subaperture is the sum of the phase perturbations encountered on the various layers by the ray travelling from a given reference source to the center of the mentioned subaperture. Considering all the N_{sub} subapertures of the SH sensor the authors wrote a system of $2N_{\mathrm{sub}}$ equations having as unknowns the phase perturbation of the M layers considered. The system of equations is extended by considering that we are using N_{star} reference stars. The total number of $2N_{\mathrm{sub}}N_{\mathrm{star}}$ equations thus results, allowing for the retrieval of the same number of phase perturbations located on the M layers. From a purely mathematical point of view the problem is well stated when

$$2N_{\mathrm{sub}}N_{\mathrm{star}} \geqslant N_{\mathrm{phase}} \tag{12}$$

where N_{phase} is the overall number of phase perturbations that we want to determine in the various turbulent layers. However, there are some geometrical consideration to make in order to arrange the stars and the sensors subapertures in such a way that the system can be properly written down. The sampling patches in the turbulent layers can be of the order of the layer r_0 that is larger, as showed in the previous section, than the overall r_0. Each of the sampling areas so defined has to be crossed by at least one ray. To achieve this, the sensor subapertures have to be smaller than the smaller projection of the layer sampling areas. This requirement can be translated in formulas so that we obtain for the minimum subaperture size

$$d_{\mathrm{sub}} \leqslant r_{0k}\frac{H}{H-h_k} \tag{13}$$

where h_k and r_{0k} are the kth layer altitude and r_0 values, respectively. The authors presented some LGS configurations that allowed them to properly sample the 3D phase map as needed. They proposed two configurations with 3 and 4 stars, respectively. The three star configuration is reported in Fig. 3, adapted from the TF90 [4] paper. In this configuration the three stars are placed on the vertex of an equilateral triangle. Referring to Fig. 3 the largest fully corrected FoV is determined by the diameter Dn of disk inscribed in the area covered by the overposition of the three pupil footprints at the highest layer. The angular diameter of the corrected FoV indicated as θ_{MCAO} is given by

$$\theta_{\mathrm{MCAO}} = \frac{Dn - D}{h_n} \tag{14}$$

where D is the telescope pupil diameter and h_n is the height of the highest layer. The geometrical condition for non-zero corrected FoV, i.e., shaded surface larger than telescope pupil projection, can be imposed in this three sources case, so that TF90 obtains

$$h_n/H \leqslant 1 - \frac{\sqrt{3}}{2} \tag{15}$$

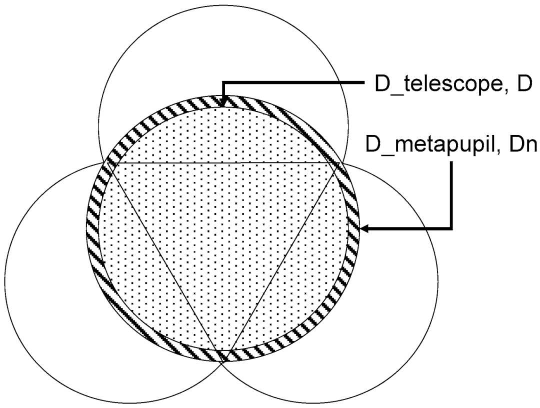

Fig. 3. The proposed arrangement of three star and the main elements used to find the multi-conjugate FoV. The picture shows the three footprint of the LGS on the highest turbulent layer, the achieved FoV that is the larger disk inscribed in the LGS footprints and the telescope pupil projection.

The above condition translates in a maximum altitude for the highest sampled turbulent layer. Considering an 8 m telescope and three sodium laser guide stars focused at 90 km altitude in the atmosphere we find for the maximum h_n a value of 12 km. Forcing the sampling of this 12 km layer, in the considered geometrical arrangement, will give an MCAO FoV equal to zero. Assuming a value of 9 km for the highest DM conjugation altitude the considered paper reports a corrected FoV of 7 arcsec while the sources have to be placed 48 arcsec off axis. The achieved corrected FoV is relatively small but it has to be considered that this is the completely corrected FoV. The partially corrected FoV where part of the telescope pupil is not included in the meta-pupil for some sky direction is larger than the mentioned 7 arcsec value. Finally, the authors developed a similar discussion for a four reference star case (arranged in a square geometry). In this case the authors obtained a FoV of 50 arcsec for the same highest layer altitude of 9 km placing the reference star 1.23 arcmin off axis. The paper thus reported a quantitative way to obtain a wavefront reconstruction matrix for an MCAO system and showed that the FoV can be increased using three or better four reference stars. The methods outlined in this paper triggered the extensive work on MCAO sketched in the last section of this article.

4. The Layer Oriented scheme: a new approach

As stated before, the MCAO systems are based on the use of more than one deformable mirror, each one conjugated to a different altitude and on a wavefront sensing scheme able to retrieve the three-dimensional phase perturbation due to the atmosphere. In particular, the WFS has to be able to identify the phase perturbations ascribed to a particular layer conjugate to a particular DM of the AO system. The TF90 paper gets the 3D perturbation looking along different line of sights to several laser guide stars. The same scheme is not easily applied to NGS mainly because of the magnitude required of the reference star constellation made up of three or four quite bright stars arranged in a favorable manner. An alternative approach was introduced by Ragazzoni et al. in 2000 [11]. This new approach is called 'layer oriented' and is based on the idea to co-add the light of different natural guide stars to perform the WFS operation. To estimate the viability of such an arrangement the authors computed, using the Bahall and Soneira model of the Galaxy star distribution, the total photon flux received from stars brighter then 20th mag and fainter than 10th. The considered reference stars have to be located in a field of view that can be imaged by practical optics and was set to some arcminutes. This total flux turns out to be in the range of 14–12 equivalent V mag, going from the North Galactic Pole to the galactic plane. So, this integrated photon flux is comparable with the photon flux usually needed by the single star AO system to operate properly. Let us consider now what is the concept of a wavefront sensor that works co-adding the light of several guide star with the aid of Fig. 4. Let us assume that we have N stars in the technical FoV, each one focused on a pyramid (three stars are represented in Fig. 4 for clarity). Each pyramid will split the received star light in four beams reaching the re-imaging optics. The re-imaging optics will create four images of the telescope exit pupil in correspondence with each one of the three pyramids. This situation is reported in Fig. 4, where only two pupils per pyramid are reported for clarity. It is now important to note that because the exit pupil of the telescope is a single object, all the three set of four pupil images will be imaged overimposed. This is the situation for the two pupil images of Fig. 4. The exit pupil images will be found in the focal plane of the re-imaging optics. The turbulence located on the telescope entrance pupil will be imaged

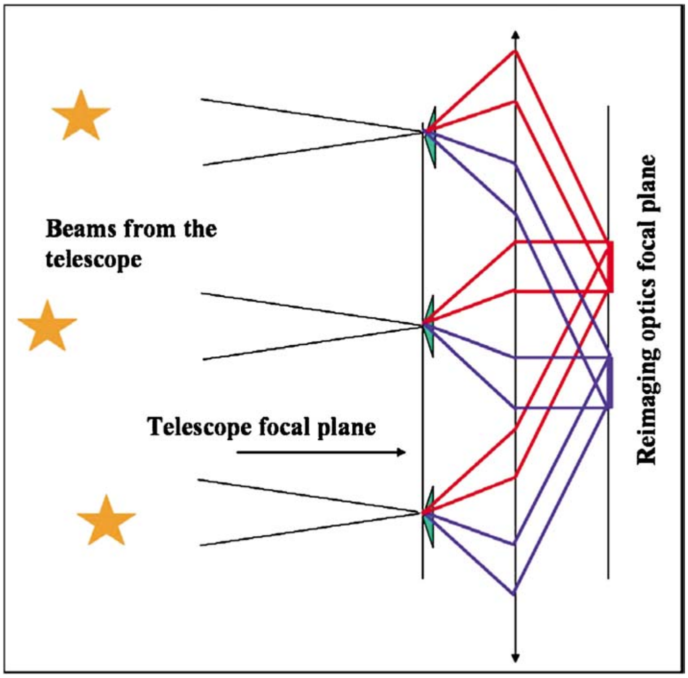

Fig. 4. A sketch of the optical layout of a layer oriented WFS based on pyramid sensors. Only two pupils are represented of the four produced by the pyramid sensor for clarity. Note that in the Layer Oriented scheme the number of pyramid has to be equal to the number of reference stars.

there while other layers will be imaged closer to the re-imaging lens. Quantitatively, the image of a layer placed at altitude H above the telescope is created a distance z from the re-imaging lens focal plane given by

$$z = H(f_{\text{reim}}/f_{\text{tel}})^2 \tag{16}$$

where f_{reim} and f_{tel} are the focal length of the re-imaging optics and the telescope, respectively. Placing two detectors at distances 0 and z allows us to sense the atmospheric perturbation of those two particular layers. To see why and to what extent this is true let us consider all the rays from the considered reference stars that pass through a given point on the analyzed turbulent layer, labelled layer lth. This point is re-imaged by the telescope and re-imaging optics in the corresponding position on the detector conjugated to the considered lth layer. We assume here a linear relationship between sensor signals and wavefront derivatives so that we can write for the signal due to a single reference star [17]

$$(S_x)_i = \frac{\Delta I_i}{I_i} \propto (\partial w/\partial x)_i \tag{17}$$

where ΔI_i is the light unbalance in the CCD pixels (four pixels in the pyramid case) used to compute the WFS signal and I_i is the ith reference star intensity in the considered pixel group. Because we co-add the light from all the reference stars we have that the overall signal S_x, measured in the considered point, is given by the following expression

$$S_x = \frac{\sum_{i=1}^{N_{\text{star}}} \Delta I_i}{\sum_{i=1}^{N_{\text{star}}} I_i} = \frac{\sum_{i=1}^{N_{\text{star}}} (S_x)_i I_i}{\sum_{i=1}^{N_{\text{star}}} I_i} = \frac{\sum_{i=1}^{N_{\text{star}}} (\partial w/\partial x)_i I_i}{\sum_{i=1}^{N_{\text{star}}} I_i} \tag{18}$$

where $(\partial w/\partial x)_i$ is the overall X wavefront derivative experienced by the ray arriving from the ith reference star to the considered sampling point placed on the lth layer. X and Y define here a standard bi-dimensional coordinate system placed in the pupil image plane. The above wavefront derivative can be written as

$$(\partial w/\partial x)_i = (\partial w/\partial x)_l + \overline{(\partial w/\partial x)_i} \tag{19}$$

where the first term is the contribution of the analyzed layer to the WFS signal. This contribution is the same for all the considered reference sources. The second term represent the atmospheric perturbation due to all the remaining layers on the ray starting from the ith source and crossing the analyzed point at layer l. Substituting this formula in Eq. (18) we find

$$S_x = (\partial w/\partial x)_l + \frac{\sum_{i=1}^{N} \overline{(\partial w/\partial x)}_i I_k}{\sum_{ki=1}^{N} I_i} \tag{20}$$

This formula tell us that the signal of such a wavefront sensor co-adding the light of several stars on the same pupil images is made up of two terms. A first is exactly the quantity that we want to measure $(\partial w/\partial x)_l$ and a second which can be considered as noise. This last term is the average of all the remaining wavefront derivatives cumulated by each one of the rays passing from the considered point in the analyzed layer l. This last term can be small when a large number of stars is considered and so different aberrations are averaged out or when wavefront aberrations contain only high spatial frequencies. For this reason this term should be small when the system is properly working in a closed loop so that the perturbations placed out from the analyzed layer are small in amplitude and contain mainly wavefront residuals with high spatial frequencies. These residuals cancel out easily, having a low spatial correlation, in the averaging process making up the second term. Finally we note that the averaging process of the noise term can be less effective if the reference stars have quite different magnitudes as seen from Eq. (20). A qualitative discussion of the layer oriented bootstrap phase in the simple case of two turbulent layers and two deformable mirrors is given below referring to Fig. 5. This figure reports in columns 1, 2 and 3, as a function of the iterations, the shape of the two DMs conjugated to the turbulent layers, the wavefront perturbation at the two layers after the DMs correction and the signals of the two wavefront sensors conjugated to the two considered layers, respectively. Moreover, we assume for simplicity in the following that the WFSs used sense directly the wavefront phase. At first the two DMs deformations are null (Fig. 5, column 1). We start with two different aberrations on the two atmospheric layers 1 and 2 (Fig. 5, column 2). Then each of the WFSs perform a sensing step. We assume here that the LO WFS is working with many stars all located inside a cone of angular radius α. The effect of the deformation in the layer 2 is to introduce a negative signal on the wavefront measurement of layer 1. The amplitude of this coupling signal is reduced with respect to the effective value of the perturbation because the layer 2 is 'out of focus' for WFS 1. In fact the coupling signal is spread on layer 1 on an area larger that the area of the original perturbation on layer 2. The spreading area is approximately given by $\alpha\delta h$ where δh is the distance between the two considered layers. The same considerations apply to the coupling of layer 2 to layer 1. The results of the sensing step is reported in column 3 for the two analyzed layers. After a first sensing step we pass to apply the sensed shapes to upgrade the system DMs surface. The new situation is reported in the second row of Fig. 5. The new DMs shape is reported together with the initial atmospheric perturbation for that layer (dashed curve) for comparison. The corrected wavefront (atm + dm) shows the bootstrap phase and the coupling effect. Layer 1 is close to being flat, because the layer 2 perturbation produced a small coupling effect. Layer 2 is not well corrected and is showing a small bump due to coupling with the layer 1 having a larger initial perturbation. In the next iteration layer 2 is recovered too. This is because the low spatial frequencies in the layer 1 at the second step are corrected and so the coupling effect of the corrected layer 1 on layer 2 becomes negligible. This brief description shows that the layer

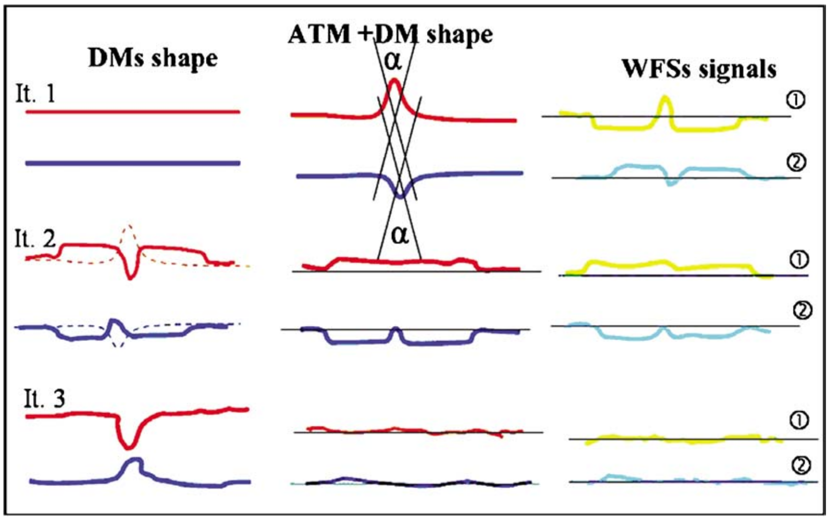

Fig. 5. A sketch of the bootstrap and coupling effects in the closed loop operation of the layer oriented wavefront sensing scheme. For simplicity the WFSs are supposed to measure the wavefront phase directly.

oriented scheme relies on a bootstrap phase, aimed at reducing the layers optical coupling, to enter a good correction regime. This wavefront sensing scheme has some differences with respect to the approach described in TF90. The first is that the signal from a detector can be used directly to drive the DM conjugated to the analyzed layer. This means that the MCAO system control matrix is split in smaller matrices, two in the considered cases. Moreover, the use of LGSs that is strongly needed for the approach proposed by TF90 is to some extent solved [18] because this method co-add the light from several NGSs to reach a sufficient number of detected photons per integration time on the WFSs. Finally we note that adding a DM and so a sensed layer in a layer oriented WFS reduces the number of photons in all the used WFSs. This is not the case for the TF90 method.

5. MCAO control matrix and real systems

The elements introduced so far enable us to state some short considerations about one of the critical elements of an MCAO system, namely the control or reconstruction matrix. The TF90 paper makes clear that the dimension of the reconstruction matrix depends on the number of DMs N_{DM} and their actuators N_{act}, the number of reference stars N_{star} and the number of WFS subapertures N_{sub}. Quantitatively, this single control matrix has dimensions $N_{\mathrm{act}} \cdot N_{\mathrm{DM}} \times N_{\mathrm{star}} \cdot 2 \cdot N_{\mathrm{sub}}$. This matrix can be quite large for an 8 m telescope. In the case of 4 reference stars, 3 DMs with 16×16 actuators and 4 wavefront sensors with 15×15 subapertures we find a matrix dimension of 768×1800. The numerical simulation and optimization of such a matrix has been the subject of several papers using different approaches [6–10,19]. Other papers have analyzed the limits of the wavefront reconstruction accuracy [20,21]. Some other investigation about system modes has been conducted [22]. The matrix presented in Tallon and Foy work is a so called zonal matrix. Another approach was described by Ragazzoni et al. [23] where a modal matrix is considered. An experiment was set up to demonstrate that a reconstruction matrix able to combine the signals from three WFSs obtaining a good wavefront estimate in a certain angular direction [24]. The experimental results showed that the tomographic reconstruction was three times better than the simple average of the three WFSs signal. Coming back to the matrix dimension it is useful to analyze it for the case of a layer oriented system. In this case, as was already pointed out, the control matrix is divided in a number of matrices equal to the number of used DMs. The dimension of the single matrix is insensitive to the number of reference stars and is given by $N_{\mathrm{act}} \times 2N_{\mathrm{subap}}$. This approach has two advantages in terms of computing power. Firstly, the control matrix is split into more than one matrix, allowing a parallelized computation. Secondly, each of these sub matrices has the typical dimension of a single reference star AO system control matrix. A comparison between LO and classical MCAO is a well-known subject and is beyond the scope of the present article. It is important here to report that an on-sky experiment called Multi-Conjugate Adaptive Demonstrator (MAD) [13] has been set-up by a European collaboration to test both approaches to MCAO at VLT. This experiment will be performed using NGS and should tell us much about MCAO system behavior pros and cons. To have an LGS MCAO system working, we need to wait the Gemini system [12] that seems the first one that should be on-line. A comparison of the results coming out from these two activities should give us many elements for the design of the next generation of MCAO systems to be developed to achieve the ultimate performance of the Extremely Large Telescopes presently under study.

6. Conclusions

The article has given a short review of the basic concepts and methods used in the field of multi-conjugate AO system. The field was opened by a well-known paper due to J. Beckers that demonstrated the powerful effect of performing the adaptive correction using more than one DM, each one properly conjugated to a certain turbulent layer. A side product of Beckers work is that even a single mirror system can work with an improved FoV (a reduced angular anisoplanatism error) if the single mirror is conjugated to a proper altitude. However, multiple mirror correction assumes that the system is able to single out the aberrations due to a certain layer. A method for this was suggested in the J. Beckers paper [2], but a quantitative solution was not given. This second but fundamental step towards MCAO systems is due to Tallon and Foy that presented in 1990 a wavefront sensing arrangement to retrieve the 3D map of the phase perturbation and solve the cone effect problem at the same time [4]. The concepts stated in these two papers put in motion the work of many other people, and the MCAO field started to grow faster and faster. In the year 2000 another wavefront sensing method, using natural guide star was presented by R. Ragazzoni called layer oriented approach [11]. This sensing method takes advantage of summing optically the light from several reference stars so partly overcoming the usual difficulty of finding a suitable reference star or, even worst, a suitable constellation of reference stars as required in MCAO. Until now, the wavefront sensing schemes available remain these two, and the discussion about using laser guide star or natural guide star is not resolved. Both approaches are pursued and we should see in a while the results of the LGS based MCAO system of Gemini together with the results of the NGS layer oriented demonstrator called MAD to be installed at the VLT. The comparison of the performance of these two system will provide, I think, new elements and new ideas on how to proceed in MCAO system development.

References

[1] D.L. Fried, Anisoplanatism in adaptive optics, J. Opt. Soc. Am. 72 (1982) 52–61.

[2] J.M. Beckers, Increasing the size of the isoplanatic patch within multiconjugate adaptive optics, in: M.-H. Ulrich (Ed.), Proceedings of European Southern Observatory Conference and Workshop on Very Large Telescopes and Their Instrumentation, in: ESO Conference and Workshop Proceedings, vol. 30, European Southern Observatory, Garching, Germany, 1988, pp. 693–703.

[3] R. Foy, A. Labeyrie, Feasibility of adaptive telescopes using laser probe, Astron. Astrophys. 152 (1985) 129–131.

[4] M. Tallon, R. Foy, Adaptive telescope with laser probe: isoplanatism and cone effect, Astron. Astrophys. 235 (1990) 549–557.

[5] D.L. Fried, J.F. Belsher, Analysis of fundamental limits to artificial-guide-star adaptive-optics-system performance for astronomical imaging, J. Opt. Soc. Am. A 11 (1994) 277–287.

[6] D.C. Johnston, B.M. Welsh, Analysis of multiconjugate adaptive optics, J. Opt. Soc. Am. A 11 (1994) 394–408.

[7] R.J. Sasiela, Wave-front correction by one or more synthetic beacons, J. Opt. Soc. Am. A 11 (1994) 379–393.

[8] B.L. Ellerbroek, First order performance evaluation of adaptive optics systems for atmospheric turbulence compensation in extended field of view astronomical telescopes, J. Opt. Soc. Am. A 11 (1994) 783–805.

[9] B.L. Ellerbroek, F. Rigaut, Methods for correcting tilt anisoplanatism in laser-guide-star-based multi-conjugate adaptive optics, J. Opt. Soc. Am. A 18 (2001) 2539–2547.

[10] T. Fusco, J.M. Conan, G. Rousset, L.M. Mugnier, V. Michau, Optimal wave-front reconstruction strategies for multiconjugate adaptive optics, J. Opt. Soc. Am. A 18 (2001) 2527–2538.

[11] R. Ragazzoni, Adaptive optics for 100 m class telescopes: New challenges require new solutions, in: P. Wizinowich (Ed.), Adaptive Optical Systems Technology, in: Proc. SPIE, vol. 4007, 2000, pp. 1076–1087.

[12] B. Ellerbroek, F. Rigaut, B.J. Bauman, C. Boyer, S.L. Browne, R.A. Buchroeder, J.W. Catone, P. Clark, C. d'Orgeville, D.T. Gavel, G. Herriot, M.R. Hunten, E. James, E.J. Kibblewhite, I.T. McKinnie, J.T. Murray, D. Rabaud, L.K. Saddlemyer, J. Sebag, J. Stillburn, J.M. Telle, J.P. Veran, Multiconjugate adaptive optics for Gemini-South, in: P. Wizinowich, D. Bonaccini (Eds.), Adaptive Optical System Technologies II, in: Proc. SPIE, vol. 4839, 2003, pp. 55–66.

[13] E. Marchetti, R. Brast, B. Delabre, R. Donaldson, E. Fedrigo, C. Frank, N. Hubin, J. Kolb, M. Le Louarn, J.-L. Lizon, S. Oberti, R. Reiss, J. Santos, S. Tordo, R. Ragazzoni, C. Arcidiacono, A. Baruffolo, E. Diolaiti, J. Farinato, E. Vernet-Viard, MAD status report, in: D. Bonaccini Calia, B.L. Ellerbroek, R. Ragazzoni (Eds.), Advancements in Adaptive Optics, in: Proc. SPIE, vol. 5490, 2004, pp. 236–247.

[14] T. Herbst, R. Ragazzoni, A. Eckart, G. Weigelt, The LINC-NIRVANA interferometric imager for the Large Binocular Telescope, in: A.F.M. Moorwood, I. Masanori (Eds.), in: Ground-Based Instrumentation for Astronomy, in: Proc. SPIE, vol. 5492, 2004, pp. 1045–1052.

[15] D.L. Fried, Optical resolution through a randomly inhomogeneous medium for very long and very short exposures, J. Opt. Soc. Am. 56 (1966) 1372–1379.

[16] R. Hudgin, Wave-front compensation error due to finite corrector-element size, J. Opt. Soc. Am. 67 (1977) 393–396.

[17] R. Ragazzoni, Pupil plane wavefront sensing with an oscillating prism, J. Modern Opt. 43 (1996) 289.

[18] C. Arcidiacono, E. Diolaitib, R. Ragazzoni, J. Farinato, E. Vernet, Sky coverage for Layer Oriented MCAO: a detailed analytical and numerical study, in: D. Bonaccini Calia, B.L. Ellerbroek, R. Ragazzoni (Eds.), Advancements in Adaptive Optics, in: Proc. SPIE, 5490, 2004, pp. 563–573.

[19] R. Flicker, B.L. Ellerbroek, F.J. Rigaut, Comparison of multiconjugate adaptive optics configurations and control algorithms for the Gemini-South 8 m telescope, in: P. Wizinowich (Ed.), Adaptive Optical Systems Technology, in: Proc. SPIE, vol. 4007, 2000, pp. 1032–1043.

[20] A. Tokovinin, E. Viard, Limiting precision of tomographic phase, J. Opt. Soc. Am. A 18 (2001) 873–882.

[21] M. Lloyd-Hart, N.M. Milton, Fundamental limits on isoplanatic correction with multiconjugate adaptive optics, J. Opt. Soc. Am. A 20 (2003) 1949–1957.

[22] M. le Louarn, M. Tallon, Analysis of modes and behavior of a multiconjugate adaptive optics system, J. Opt. Soc. Am. A 19 (2002) 912–925.

[23] R. Ragazzoni, E. Marchetti, F. Rigaut, Modal tomography for adaptive optics, Astron. Astrophys. 342 (1999) L53–L56.

[24] R. Ragazzoni, E. Marchetti, G. Valente, Adaptive-optics corrections available for the whole sky, Nature 403 (2000) 54–56.