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String theory and fundamental forces/Théorie des cordes et forces fondamentales

Particles and strings in six-dimensional (2, 0) theory

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Abstract

In 1995, we learned of the rather surprising existence of a completely new class of *quantum theories in six space–time dimensions with* $(2,0)$ *superconformal symmetry*. Some important reasons to study these theories are: (i) Finding the right conceptual framework to define them is a very challenging problem, that will probably take a long time to solve. It is likely to involve new interesting mathematical structures with connections in particular to algebra and geometry. (ii) They give rise to certain Yang–Mills theories with maximally extended supersymmetry upon compactification on a two-torus. This may be a way to find an S-dual formulation of these lower dimensional theories. (iii) They arise within string/M-theory as decoupled subsectors localized on certain space–time impurities such as branes or singularities. (This is in fact how these theories were first discovered (see Witten, hep-th/9507121).) This may provide an opportunity to study aspects of these higher dimensional theories without having to deal with the conceptual subtleties of quantum gravity. ***To cite this article: M. Henningson, C. R. Physique 5 (2004).***

Résumé

Particules et cordes dans les théories (2, 0) en dimension six. En 1995, l'existence plutôt surprenante d'une classe complètement nouvelle de *théories quantiques en dimensions six avec symétrie superconforme* $(2,0)$ a été découverte. Quelques-unes des raisons pour les étudier sont : (i) Trouver le cadre conceptuel correct pour les définir est un challenge qui prendra probablement beaucoup de temps. Il est probable que ce cadre introduira de nouvelles structures mathématiques ayant en particulier des liens avec l'algèbre et la géométrie. (ii) Elles donnent lieu à des théories de Yang–Mills avec une supersymétrie étendue maximale après compactification sur un tore bidimensionnel. Cela pourrait permettre de trouver une formulation S-duale de ces théories en dimension inférieure. (iii) Elles interviennent en théorie M et cordes à travers un sous-secteur découplé localisé sur certaines impuretés de l'espace-temps, comme des branes ou singularités. (C'est en fait ainsi que ces théories ont à l'origine été découvertes (voir Witten, hep-th/9507121).) Ceci pourrait être l'occasion pour étudier des aspects de ces théories en dimension supérieure, sans avoir à faire avec les subtilités conceptuelles de la gravité quantique. ***Pour citer cet article : M. Henningson, C. R. Physique 5 (2004).***

In this article, I will report on some work on this problem performed in collaboration with P. Arvidsson and E. Flink. More details and references can be found in our recent publications [1,2].

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1. Tensor multiplet particles ...

The $(2,0)$ supersymmetry algebra in six dimensions comprises the $SO(5,1)$ Lorentz algebra and an $SO(5)_R$ internal R-symmetry algebra. Its best known representation is the so called tensor multiplet of massless space–time fields:

field	$SO(5,1)$	$SO(5)_R$
ϕ	scalar	vector
ψ	anti-chiral spinor	spinor
$h = db$	self-dual three-form	scalar

Second quantization of these fields yields a Fock space of massless particle states.

These particles transform as scalars, spinors, and self-dual tensors under the $SO(4) \subset SO(5,1)$ little group of rotations in the directions transverse to the spatial momentum of the particles.

Dimensional reduction of these fields gives rise to a massless vector multiplet of maximally extended supersymmetry. This is a first indication of the relationship between the six-dimensional $(2,0)$ theories and lower-dimensional Yang–Mills theories.

One would now like to construct an interacting theory including these particles. A major problem is then that the two-form field b does not naturally couple to a particle. So it appears difficult to construct a 'non-Abelian' version of the tensor multiplet theory.

2. ... and self-dual strings

However, the two-form field b can be coupled to a self-dual string in a natural way.

The presence of such strings in the theory is in fact indicated already by the $(2,0)$ supersymmetry algebra, if we extended it with 'central charges' Z so that the anti-commutator of the supercharges Q reads

$$\{Q, Q\} = P + Z.$$

(Here P is the six-momentum.) The quantum numbers of the generators are given by

generator	$SO(5,1)$	$SO(5)_R$
Q	chiral spinor	spinor
P	vector	scalar
Z	vector	vector

(The operator Z is not truly central, since it transforms non-trivially under both the $SO(5,1)$ Lorentz group and the $SO(5)$ R-symmetry group.)

A straight string element in the spatial direction V coupled to a tensor multiplet ϕ, ψ, h gives rise to such a central charge Z of the form $Z = \langle\phi\rangle \otimes V$. Unitarity imposes the bound $|P| \geqslant |Z|$ that limits the tension (i.e. the mass per unit length) of the string. We will be interested in the BPS-case where this bound is saturated so that the tension of the string is given by $\langle\phi\rangle$.

A configuration with a straight and static such string is invariant under half of the supersymmetries. Acting with the remaining spontaneously broken supersymmetry generators, we may build up a multipet of string states transforming as scalars, spinors, and vectors under the little group $SO(4) \subset SO(5,1)$ of transverse rotations.

Dimensional reduction along the string direction of this multiplet gives rise to the particles of a massive vector multiplet of maximally extended supersymmetry. This is another indication that (spontaneously broken) non-Abelian gauge symmetry may appear after compactification.

3. The Lagrangian formulation

Continuous symmetries spontaneously broken by a string gives rise to Goldstone fields on its two-dimensional world-sheet $\Sigma \subset \mathbb{R}^{1,5}$:

field	$SO(4) \subset SO(5,1)$
$X^{\perp}$ (non-chiral)	vector
Θ^{+} (left-moving)	chiral spinor
Θ^{-} (right-moving)	anti-chiral spinor

The dynamics of the space–time fields ϕ, ψ, and $h = db$ and the world-sheet fields $X^{\perp}$, Θ^{+}, and Θ^{-} is governed by the unique action

$$S = \frac{1}{4\pi\lambda^2} \int_{\mathbb{R}^{1,5}} \mathrm{d}^6 x (\partial\phi\partial\phi + \bar{\psi}\partial\!\!\!/\psi + h^2) + \int_{\Sigma} \mathrm{d}^2\sigma \sqrt{\phi\phi} (DX^{\perp}DX^{\perp} + \overline{\Theta}^{+}\not{D}\Theta^{+} + \overline{\Theta}^{-}\not{D}\Theta^{-}) + e\int_{\Sigma} b + \cdots,$$

where h fulfills the modified Bianchi identity

$$dh = 2\pi q \delta_{\Sigma}$$

appropriate in the presence of a magnetically charged string. Here δ_{Σ} denotes the Poincaré dual four-form of the string world-sheet Σ, and

$\lambda =$ coupling constant,

$e =$ string electric charge,

$q =$ string magnetic charge.

Decoupling of the anti self-dual part of h (which is not part of the tensor multiplet) requires that $e = \frac{1}{\lambda^2} q$.

4. The coupling constant and the charges

The quantum theory of a chiral two-form b (i.e. with self-dual field strength $h = db$) can only be defined for a rational value of the coupling constant λ^2.

The correct value appears to be $\lambda^2 = 2$. (This follows from consistency requirements when a five-brane is embedded into eleven-dimensional M-theory, or alternatively by considering the equal-time commutation relations of Wilson–'t Hooft surface observables for two linked surfaces in five spatial dimensions.) The relationship between electric and magnetic charges is thus $e = \frac{1}{2} q$.

The topological class of the 'wave function' of a system of two strings with electric-magnetic charges (e, q) and (e', q') is determined by the quantity

$$e \cdot q' + q \cdot e' \in \mathbb{Z}.$$

This is analogous to the familiar case of dyonic particles in four space–time dimensions. A crucial difference, however, is the relative sign between the two terms. The plus sign means that also for two identical strings we get a non-zero result. So it appears that even to define the quantum theory of a single dyonic string might be a subtle problem...

Reinstating that $e = \frac{1}{2} q$ and $e' = \frac{1}{2} q'$, as required for the decoupling of anti self-dual part of h, we get the condition

$$q \cdot q' \in \mathbb{Z}$$

for all magnetic string charges q and q'.

5. The normal bundle anomaly

Diffeomorphisms of $\mathbb{R}^{1,5}$ that leave Σ invariant appear as $SO(4)$ gauge transformations on the normal bundle $N = (T\Sigma)^{\perp}$. The theory must respect this symmetry.

However, because of the modified Bianchi identity $dh = 2\pi q \delta_{\Sigma}$, the electric coupling

$$e\int_{\Sigma} b = e \int_{\mathbb{R}^{1,5}} b \wedge \delta_{\Sigma}$$

classically suffers from an anomaly inflow. This is described by descent on the four-form

$$I^{\text{class}} = q \cdot e \delta_{\Sigma}|_{\Sigma} = q \cdot e \chi(N).$$

Here $\chi(N)$ denotes the Euler class of N, and the last equality follows from the Thom isomorphism between the cohomology of Σ and cohomology with compact vertical support on the total space of N. (As usual when applying the descent procedure, one should actually consider a two-parameter family of world-sheets rather than a single Σ.)

The chiral and anti-chiral fermions Θ^+ and Θ^- give a further one-loop contribution to the anomaly:

$$I^{\text{quant}} = -\chi(N).$$

All other terms in the action are non-anomalous, so consistency of the theory requires that I^{class} and I^{quant} cancel. Reinstating that $e = \frac{1}{2}q$, we thus find that

$$q \cdot q = 2$$

for all magnetic string charges q.

6. The ADE-classification

The general solution to the conditions $q \cdot q' \in \mathbb{Z}$ and $q \cdot q = 2$ are labeled by the series $A_{1,2,\ldots}$, $D_{4,5,\ldots}$, or $E_{6,7,8}$.

This is in one-to-one correspondence with the classification of simple singularities of hyper-Kähler four-folds $K \simeq \mathbb{C}^2/\Gamma$ for $\Gamma =$ discrete subgroup of $SU(2)$.

Indeed, type IIB string theory on $\mathbb{R}^{1,5} \times K$ gives a realization of $(2,0)$ theory.

More mysteriously, it also indicates that a compactified $(2,0)$ theory may most straightforwardly only give rise to Yang–Mills theories with maximally extended supersymmetry and a simply laced gauge group (i.e. $SU(r+1)$, $SO(2r)$, or $E_{6,7,8}$).

In the Coulomb phase of such a Yang–Mills theory:

– Massless vector multiplets associated with Cartan generators originate from massless tensor multiplets.
– Massive vector multiplets associated with root generators originate from tensile self-dual strings.

Of course, we usually think of massive vector multiplets as originating from massless vector multiplets that have acquired a mass through spontaneous breaking of the gauge symmetry and the Higgs phenomenon. However, there is no sign of any 'non-Abelian' symmetry relating tensor multiplet particles and self-dual strings in six dimensions.

7. Scattering processes

Consider an infinitely extended, approximately straight and static 'bare' string. It is always surrounded by a non-vanishing configuration of the space–time tensor multiplet fields ϕ, ψ, and h. It is therefore natural to try to change variables and instead work with a 'dressed' string, which includes this surrounding self-field.

This viewpoint would be useful when considering e.g. the scattering of tensor multiplet quanta off a string. Indeed, because of the $\sqrt{\phi\phi}$ string tension, tensor multiplet quanta of energy E then couple to string wave quanta with strength $E/(\sqrt{\phi\phi})^{1/2}$. For $E^2 \ll \sqrt{\phi\phi}$, the tensor multiplets are thus described by a free space–time field theory, and the string waves are described by some decoupled world-sheet conformal field theory. Interactions could then be described perturbatively in the small dimensionless parameter $E/(\sqrt{\phi\phi})^{1/2}$.

But a first, crucial question is: What is the two-dimensional conformal field theory that describes low-energy excitations on the string world-sheet?

8. The decoupled world-sheet theory

The degenerate ground states of the string are a representation of the Clifford algebra of the Θ^+ and Θ^- zero modes.

Above these ground states, there are four bosonic ($X^\perp$) and four fermionic (Θ^+ and Θ^-) polarizations of string wave excitations. But because of the electro-magnetic interaction, the bosonic string waves are not free.

To investigate the interactions between these excitations, we replace $X^\perp$ with 'spherical' variables $r \in \mathbb{R}$ and $g \in S^3 \simeq SU(2)$, modulo $(r, g) \sim (-r, -g)$. The normal bundle structure group acts as $g \mapsto ugv^{-1}$ for $(u, v) \in SU(2) \times SU(2) \simeq \text{Spin}(4)$.

The 'radial' coordinate r is a free boson, but we suggest that g is governed by a 'level one' $SU(2)$ Wess–Zumino–Witten model. The reason is that this reproduces the classical bosonic normal bundle anomaly described previously. As before, this anomaly is cancelled by the fermionic quantum contribution.

The WZW kinetic term of g implies that the effective space–time geometry probed by the string wave fluctuations is blown up at the locus of the string.

9. Conclusion ...

The six-dimensional $(2,0)$ theories exhibit a tightly constrained structure, apparent already in the effective low-energy description in terms of particles and strings. Our most important results are that

– The classical theory is anomalous and thus inconsistent.
– Supersymmetry is essential for anomaly cancellation.
– There is an ADE-classification of consistent models.
– The low-energy string world-sheet theory is a non-trivial conformal field theory.

10. ... and outlook

Here are two aspects of the theory that we are currently investigating:

– Investigations of scattering processes, particularly in a compactified situation, may give insights into the six-dimensional origins of (spontaneously broken) non-Abelian gauge symmetry in certain lower-dimensional Yang–Mills theories.
– Particles and strings are useful at a generic point in the moduli space with non-vanishing $\langle\phi\rangle >$. A major challenge is to instead use the full superconformal symmetry of the theory. But we do not yet know the correct framework for such a construction.

Acknowledgements

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