Available online at www.sciencedirect.com

C. R. Physique 5 (2004) 881–892

COMPTES RENDUS PHYSIQUE

http://france.elsevier.com/direct/COMREN/

Fundamental metrology/Métrologie fondamentale

Towards a new kilogram definition based on a fundamental constant

Wolfgang Schwitz *, Beat Jeckelmann, Philippe Richard

Swiss Federal Office of Metrology and Accreditation (METAS), Lindenweg 50, 3003 Bern-Wabern, Switzerland

Available online 5 November 2004

Presented by Guy Laval

Abstract

The further development of the International System of Units and the redefinition of the mass unit based on a fundamental constant is a priority task of the metrology community. Two main strategies are pursued today, counting atoms and relating mechanical to electrical power. In this article the actual status of the kilogram and the different proposed methods are reviewed. ***To cite this article: W. Schwitz et al., C. R. Physique 5 (2004).***

Résumé

Vers une définition du kilogramme basée sur une constante fondamentale. Un des plus grand défis actuels de la communauté de la métrologie consiste à poursuivre le développement du Système International d'unités et à redéfinir l'unité de masse à partir d'une constante fondamentale. Deux stratégies principales sont suivies actuellement, le comptage d'atomes et la réalisation de l'équivalence des puissances mécanique et électrique. Cet article présente la situation actuelle du kilogramme ainsi que les différentes méthodes proposées à ce jour. ***Pour citer cet article : W. Schwitz et al., C. R. Physique 5 (2004).***

Keywords: Kilogram; Planck constant; Avogadro; Watt balance

Mots-clés : Kilogramme ; Constante de Planck ; Avogadro ; Balance de watt

---

1. Introduction

Mass and the corresponding definition and realisation of units has been of importance throughout human history. Like length or time, mass is a very familiar quantity in daily life as well as in science, technology and industry. The measurement of mass or weighing was and will always be a dominant activity in manufacturing processes and in trade of goods. As can be seen from the many related regulations, the unit of mass and its applications were always of political and economic relevance. The unit of mass *kilogram* was originally derived from one cubic decimetre of water at its maximum density and was realised for the

---

* Corresponding author.

E-mail addresses: wolfgang.schwitz@metas.ch (W. Schwitz), beat.jeckelmann@metas.ch (B. Jeckelmann), philippe.richard@metas.ch (P. Richard).

doi:10.1016/j.crhy.2004.05.005

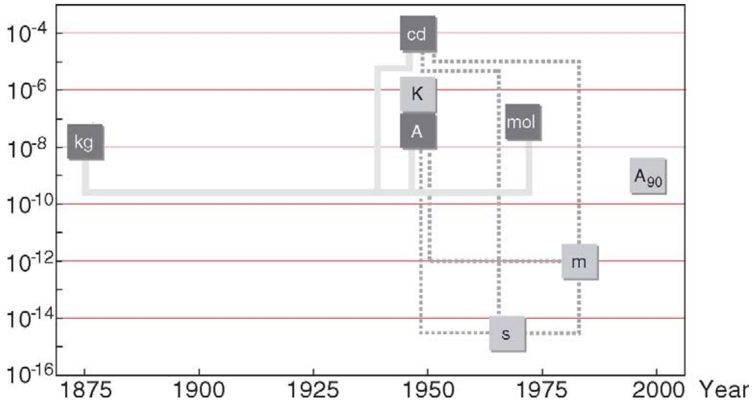


Fig. 1. The kilogram and its relation to other base units with the year of adoption by the General Conference of Weights and Measures and today's accuracies of realisation. A_{90} is the representation of the ampere realised in terms of the Josephson and quantum Hall effect and the conventional values for $K_{\text{J-90}}$ and $R_{\text{K-90}}$ (see Section 2.1). For the further development of the SI it is crucial to eliminate the dependency (gray lines) on the *International Prototype Kilogram* $\mathfrak{K}$, to define the unit of mass based on a fundamental constant and to allow kg-realisations with relative uncertainties $\leqslant 10^{-8}$.

first time in the 1790s in France [1] as a platinum cylinder standard, known as the *Kilogramme des Archives* [2]. Under the Metre Convention of 1875 it was decided that the new kilogram definition had to be consistent with the existing one. After a long development, fabrication and evaluation process for an adequately stable platinum–iridium standard, the *International Prototype Kilogram* ($\mathfrak{K}$) was deposited in 1889 at the Bureau International des Poids et Mesures (BIPM) in Sèvres near Paris. Unlike the other units in the Système International d'unités (SI) [3], the first international artefact definition for the unit of mass is still in use nowadays, but has been quite intensively debated and questioned during the past two decades. There are good reasons for this debate, mainly the annoying and not fully understood drifts [4] between $\mathfrak{K}$, its six official copies and the national prototype copies, and the dependency on the only remaining base unit defined by an artefact. Clearly, a material object like a mass standard does unavoidably have an exchange with the environment across its surface, and therefore, its mass is subject to small and not easily predictable changes. In view of the accuracies and consistencies within the SI needed these days, it is obvious that the *International Prototype Kilogram* of 1889 may not hold much longer as the definition for the mass unit. As shown in Fig. 1, the possible drift of $\mathfrak{K}$ not only affects the mass unit but three other base units as well.

In the present situation we may well recall James Clerk Maxwell's visionary statement of 1870 [5]: *"If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wave-length, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules"*. During the past decades, base and other important units have been related to atomic or fundamental constants, in the very spirit of Maxwell's vision, with one exception, the mass unit. In the present state of research and technology, the mass unit clearly needs a new definition, based on a fundamental constant, preferably in a way as to allow various realisation methods [6,7]. Furthermore, consistency is required with the present definition and value of the mass unit within the SI, that is with $\mathfrak{K}$. Therefore, $\mathfrak{K}$ has to be measured and monitored by such new realisation methods with a relative accuracy better than the suspected drifts, i.e. at the level of 1 part in 10^8. In the following review we summarise the actual status of the unit of mass kilogram with regard to its role in the SI and the problems which have arisen from the possible drifts of the $\mathfrak{K}$. We discuss the two groups of methods potentially able to monitor $\mathfrak{K}$, namely counting atoms of known mass and electromechanical methods based on equivalence of electrical and mechanical power. We discuss the Avogadro, ion accumulation, watt balance as well as the voltage balance and superconducting levitation methods. A discussion of possible new kilogram definitions and an outlook on their implementation and realisation concludes the review.

2. Present status of the kilogram

The present definition of the unit of mass in the SI: *"The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram"* [3] is as old as the *International Prototype Kilogram* $\mathfrak{K}$ itself (see Fig. 1). This artefact is made of a Pt/Ir-alloy (90% Pt and 10% Ir) and has a cylindrical shape (height $\backsimeq$ diametre $\backsimeq$ 39 mm). A large majority of the member states of the Metre Convention possesses a copy of $\mathfrak{K}$, often referred to as the national prototype of the kilogram.

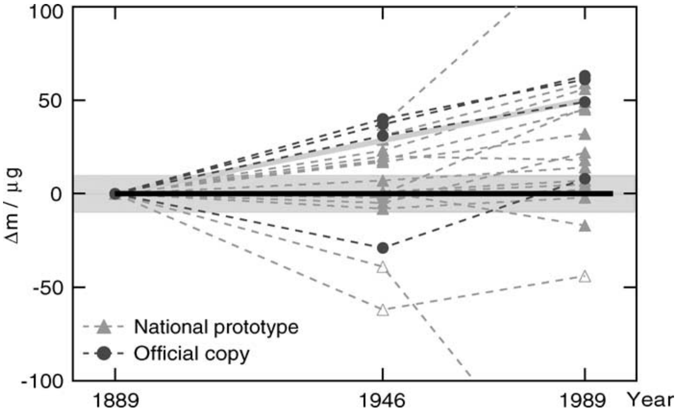


Fig. 2. Relative change in mass of four out of the six official copies and seventeen national prototypes with respect to the mass of the *International Prototype Kilogram* $\mathfrak{K}$ [4]. The black horizontal line is for the assumed constant value of $\mathfrak{K}$ according to the definition. The solid gray line is the average mass drift (50 µg in 100 years) of the national prototypes. The gray band of ±10 µg represent the uncertainty to be reached to consider a new definition of the kilogram. Finally the open triangles are for national prototypes with low mass stability.

National prototypes are directly used for the dissemination of the mass unit in each country. The initial mass determination of the first 40 prototypes using $\mathfrak{K}$ as reference was finished in 1889 with a standard uncertainty of 3 µg.[1]

2.1. Role of the kilogram in the SI

One of the major disadvantages of the definition of the unit of mass is that the amount of material constituting $\mathfrak{K}$ is subject to changes in time. Long-term observation of the relative mass drift between the international prototype and its copies [8,4] indicate that the long-term variation of the kilogram could be as much as 5 parts in 10^9 per year (see Fig. 2).

A drifting mass unit also influences the electrical units, since they are linked to the kilogram through the ampere definition (see Fig. 1). In 1990, international electrical reference standards based on the Josephson and the quantum Hall effects were introduced [9] by defining conventional values $K_{\text{J-90}}$ and $R_{\text{K-90}}$ for the Josephson constant and the von Klitzing constant respectively. All electrical quantities can be measured in terms of these two conventional values. As a consequence, the worldwide uniformity and consistency of electrical measurements has improved by almost two orders of magnitude. However, due to the uncertainty of the values of the constants K_J and R_K of several parts in 10^7, results expressed in the 1990 'practical system' of electrical units may differ from the results in the SI by this amount. Moreover, the difference may change with time because of the possible drift of $\mathfrak{K}$.

This inconsistency is not yet a problem in most practical applications. However, to prepare the SI for the future needs of science and technology, a replacement of the kilogram based on a fundamental constant is needed. There is general agreement among metrologists that a replacement of the present kilogram definition should be considered when the experiments relating mass and fundamental constants reach a relative uncertainty of $\leqslant 1 \times 10^{-8}$.

2.2. Results of periodic verifications of national prototypes

Since the official sanction of $\mathfrak{K}$ in 1889 and the distribution of the national prototypes, only three comparisons were undertaken altogether.

First Periodic Verification (1899–1911). An initial stability check was performed already ten years after the distribution of the national prototypes. Two out of the 25 verified prototypes had changed by as much as 50 µg. The changes of the others were found insignificant in comparison to the uncertainty of measurement of 10 µg.

Second Periodic Verification (1947–1954). Some comparisons between $\mathfrak{K}$ and its six official copies demonstrated the necessity to develop a procedure to reproducibly clean the prototypes. The BIPM cleaning and washing procedure for the Pt/Ir prototypes was developed during the Second World War [10]. Before the second periodic verification, all prototypes were cleaned and washed. Four prototypes out of the first 40 gained more than 30 µg since the first verification. Unfortunately the effect of cleaning and washing was not studied and the uncertainty of measurement was not reported.

---

[1] It should be noted that throughout this paper, the term uncertainty refers either to standard uncertainty or relative standard uncertainty.

Third Periodic Verification (1988–1992). During this verification, the effect of cleaning and washing was studied in detail. It was possible to determine the short-term and long-term re-contamination rate of the prototypes. Based on these studies, the definition of the kilogram was completed the following way: *"The kilogram is equal to the mass of the international prototype kilogram immediately after cleaning and washing using the BIPM method"*. The detailed results of the third periodic verification, which included a total of 52 Pt/Ir-prototypes, were described by Girard [4]. The standard uncertainty for the mass of each national prototype was 2.3 µg. The third periodic verification confirmed that the mass of the national prototypes and the six official copies tends to increase over time with respect to the mass of $\mathfrak{K}$. More detail can be found in a recent comprehensive review of the SI unit of mass by Davis [11].

Fig. 2 shows the relative mass change of 23 prototypes with respect to the mass of $\mathfrak{K}$. Up to now no explanation was found for this average relative increase in mass of the official copies and the national prototypes was found. The doubt will remain until a success in the on-going experiments described below is achieved.

3. Counting atoms

Among the approaches to define and realise a unit of mass that is no longer based on an artefact, the idea of an atomic mass standard is probably the most intuitive one. This is quite in line with Maxwell's [5] idea more than 130 years ago, where he suggested to seeking molecular quantities in order to get 'permanent standards' of measurement. The underlying concept of atomic mass dates back to times even before Maxwell. But only in 1971 the General Conference on Weights and Measures officially defined the atomic mass unit m_u as $1/12$ of the ^{12}C nuclide's mass. Today's accepted value of m_u is the result of the 1998 CODATA [12] least-squares adjustment with a relative uncertainty of 7.9×10^{-8}, which in turn is mainly caused by the uncertainty of the Planck constant measurement with the watt balance at the National Institute of Standards and Technology (NIST) in the US [13] in 1998 (see also Section 5). In this line of thought, the kilogram would be defined as a fixed number of 'elementary masses'. For the practical realisation, the mass of the chosen atom or particle has to be known in terms of the kilogram, and in addition, the number of atoms in a macroscopic body has to be measured. Two approaches are pursued today. The first involves the determination of the number of atoms in a large silicon crystal by measuring the unit cell volume, the macroscopic density and the isotopic composition. In the second approach, a beam of ionised atoms is collected and weighed. The number of ions is derived from the electric current leaving the collector.

3.1. The Avogadro constant

As a consequence of the progress made in the semiconductor industry, almost perfect Si-crystals with a mass up to several kilograms can be grown today. The crystal structure allows the counting of atoms in a macroscopic volume. The Si-crystal has a cubic lattice structure with eight atoms per unit cell. From the ratio between the volume V of a macroscopic Si-crystal and the volume of the unit cell a^3, the total number of atoms is deduced as

$$N = 8\frac{V}{a^3} = \frac{m}{m(\mathrm{Si})}, \tag{1}$$

where m is the mass of the crystal and $m(\mathrm{Si})$ the mass of a single Si-atom. The latter can be expressed as the ratio of the molar mass of Si, $M(\mathrm{Si})$, and the Avogadro constant N_A which determines the number of Si-atoms in a mol. The Avogadro constant establishes the link between microscopic and macroscopic masses, and we may relate this constant and the mass of the crystal as follows:

$$N_\mathrm{A} = 8\frac{V}{a^3} \cdot \frac{M(\mathrm{Si})}{m}. \tag{2}$$

The experiment involves, thus, the measurement of the following quantities: the lattice spacing of a silicon crystal, the macroscopic density of the crystal and the mean molar mass of the Si derived from the isotopic composition of the Si in the crystal.

Many metrology institutes are involved in the Avogadro project. A comprehensive review on the work carried out so far was recently published by Becker [14,15].

An x-ray interferometer is used to measure the side length of a unit cell. The principle of the technique is shown in Fig. 3(a). Three thin silicon plates fabricated from the same crystal are mounted in such a way that the rows of atoms are well oriented with respect to each other. The x-ray beams reaching the surface of the third plate form a standing wave pattern reproducing the lattice period. If the third plate is moved perpendicularly through the beam, a sinusoidal intensity modulation is detected by the x-ray detector. The movement of the third plate is measured with an optical interferometer calibrated in terms of the metre. Measuring the path length of the third plate and counting simultaneously the x-ray intensity maxima yields an average length of the unit cell.

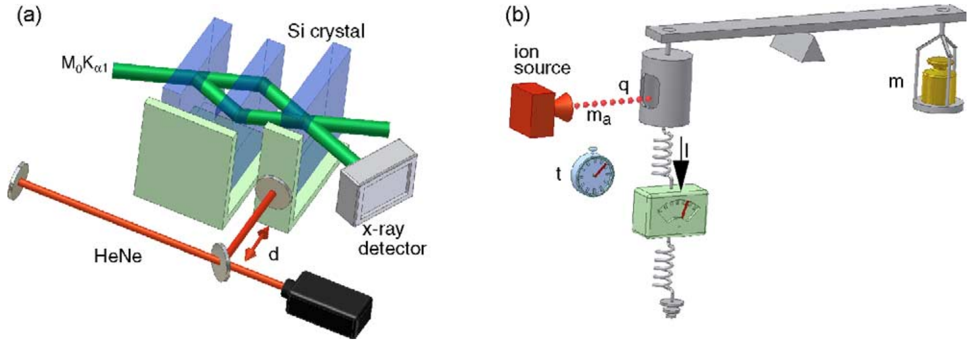


Fig. 3. (a) Schematic set-up of a combined x-ray and optical interferometer (see [14]). (b) Experimental concept of determining a mass based on atomic mass unit by ion accumulation. m_a/q and m/Q are the mass-to-charge ratio of the single ion and the total of accumulated ions respectively, I is the electric current measured over the time t (see [16]).

Due to the influence of impurity atoms, Si self-point defects and the actual isotopic composition, the cell dimension varies slightly from one crystal to another. Extensive studies [17] have been carried out to quantify the different influence factors. At present, the lattice constant can be determined with a relative uncertainty below one part in 10^8.

As we have already seen, the number n of Si-atoms per unit cell in a crystal without defects is eight. Due to impurity atoms on regular lattice sites, impurities on interstitial sites, Si-vacancies and Si-self-interstitials, n differs slightly from an integer: $n = 8(1 + \delta_n)$. In high quality silicon, the correction term δ_n is in the order of 1×10^{-8}.

Unfortunately, natural silicon used to grow the crystals consists of three isotopes ^{28}Si, ^{29}Si and ^{30}Si. The mean molar mass is thus given by:

$$M(\mathrm{Si}) = \sum f({}^i\mathrm{Si}) M({}^i\mathrm{Si}), \qquad \sum f({}^i\mathrm{Si}) \equiv 1, \tag{3}$$

where the $f({}^i\mathrm{Si})$ are the isotope abundance factors. The relative masses of the three isotopes and, thus, the molar mass values $M({}^i\mathrm{Si})$ are known to better than 1 part in 10^8. The difficult part is the determination of the isotope fractions. In natural silicon the values are approximately: $f({}^{28}\mathrm{Si}) \simeq 0.922$, $f({}^{29}\mathrm{Si}) \simeq 0.047$ and $f({}^{30}\mathrm{Si}) \simeq 0.031$. Up to now only two institutes, the NIST [18] and the Institute for Reference Materials and Measurements (IRMM) of the European Commission in Belgium [19] have published isotopic abundance results based on gas mass spectrometry. The uncertainty contribution to the Avogadro experiment from this source amounts about to 2 parts in 10^7. As an alternative, thermal neutron capture spectroscopy was proposed [20] and may bring further improvement.

To determine the volume and mass of the crystal whose unit cell dimension has been measured, nearly perfect silicon spheres are formed by Leistner and Giardini at the CSIRO-NML in Australia [21]. Using conventional grinding and lapping techniques, spheres with a diameter of about 94 mm and a mass close to 1 kg are made. For the best spheres, the deviation from ideal roundness is of the order of 30 nm only. The volume of such a sphere is determined by measuring a set of diameters with a laser interferometer [22,23]. These measurements are affected and limited by the structure of the surface layers formed by silicon oxides. The surface contaminants (oxides, water vapor, hydrocarbons) have also to be accounted for in the calculation of the total mass. Analytical methods such as optical ellipsometry, IR- and x-ray spectroscopy or low-energy electron diffraction are used to characterise the surface layers.

Finally, the mass of the silicon sphere is determined using a mass comparator. If the measurement is performed in air, buoyancy corrections have to be applied. In vacuum, the effect of adsorbed water molecules on the surface of the Si-sphere has to be studied in detail.

At present, the Avogadro experiment has a relative uncertainty of 3.4×10^{-7} [24]. All the different techniques contributing to this result are close to the state-of-the-art in their respective fields and it seems difficult to further reduce the uncertainty, at least with natural silicon. Therefore, for a further improvement, a project making use of 99.99% ^{28}Si is on-going. In this experiment, the anticipated relative uncertainty of the molar mass is expected to be $\leqslant 3 \times 10^{-8}$, i.e. an order of magnitude below what is now achieved with natural silicon [15].

3.2. *The ion accumulation*

Already in the 1960s research was undertaken to accumulate and count ions by an electrochemical method: electrolysis. The quantity of interest in this case is the Faraday constant, F, which determines the number of moles of a substance X that the passage of the amount of electric charge $Q = I \cdot t$ will deposit on an electrode or dissolve from it during electrolysis. The Faraday constant may be expressed as (see e.g. [12]):

$$F = \frac{M(X)}{z \cdot m(X)} \cdot I \cdot t, \tag{4}$$

where $M(X)$ is the molar mass of the entity X, z denotes the charge number, and $m(X)$ is the mass change of the electrode. The most accurate measurement of F by an electrochemical method was carried out at NIST using a silver dissolution coulometer. In 1980, the experiment achieved a relative uncertainty of 1.3×10^{-6} [27]. By that time, however, the method had reached basic limitations and was, therefore, abandoned.

The idea of counting atoms of known or defined mass by means of ion accumulation in vacuum was proposed in the early 1990s by Gläser (Physikalisch-Technische Bundesanstalt (PTB)) [25]. Because the classic way of counting atoms up to a weighable mass with an electronic counter would need millions of years, the author proposed to use an ion beam and measure its electric current I over the accumulation time, that is, the total electric charge Q. As the ratio between the accumulated mass and its total charge m/Q is the same as for a single ion m_a/q, a measurement of q/Q immediately leads to m/m_a and therefore to number of accumulated ions N.

As shown in Fig. 3(b), the total electric charge Q carried by the ion beam (typically ^{197}Au or ^{209}Bi) is determined by measuring its electric current I over the accumulation time t. In the PTB experiment [16,26] – the only one of its kind so far – the current I is determined by the voltage drop over a resistance in the current path. The voltage is traceable to the Josephson voltage standard and the resistance to the quantised Hall standard. This leads to a simple ratio between the ion mass m_a and the total collected mass m

$$m_a/m = 2z\Big/\left(n_1 n_2 \int f \, \mathrm{d}t\right), \tag{5}$$

where z is the charge state q/e of the single ion, n_1 the quantum Hall plateau number and n_2 the Josephson step number. The frequency f and the time t are measured based on the same atomic clock and, therefore, an eventual offset from the SI second is irrelevant. Despite the simplicity of the idea, the PTB experiment is quite demanding according to their recent report [26]. First results with a $^{197}\mathrm{Au}^{1+}$ beam were obtained with a relative uncertainty of 1.5 % for the mass of the gold atom and the deduced value for m_u is in agreement with that published in [12]. The experiment has proved to be working in principle, but is still at an early stage with regard to the projected relative uncertainty of $\leqslant 1 \times 10^{-8}$. A significant reduction of the uncertainty has been envisaged by improving the ion beam current and optics, the mass comparator, detection of foreign and lost particles, etc.

4. Electromechanical methods

The second approach towards a new definition of the kilogram is to use electrical or electromechanical methods to relate the unit of mass to fundamental constants at a macroscopic scale. In this case, the link is established by comparison of electrical and mechanical power. The gravitational force acting on a test mass is compensated by a Lorenz force in the watt balance experiment, by the force acting on a diamagnetic body in the superconducting levitation experiment and by an electrostatic force in the voltage balance experiment. In all three experiments, the electrical parameters are directly measured in terms of the quantised Hall resistance and the Josephson voltage standard, and are, thus, related to the Planck constant.

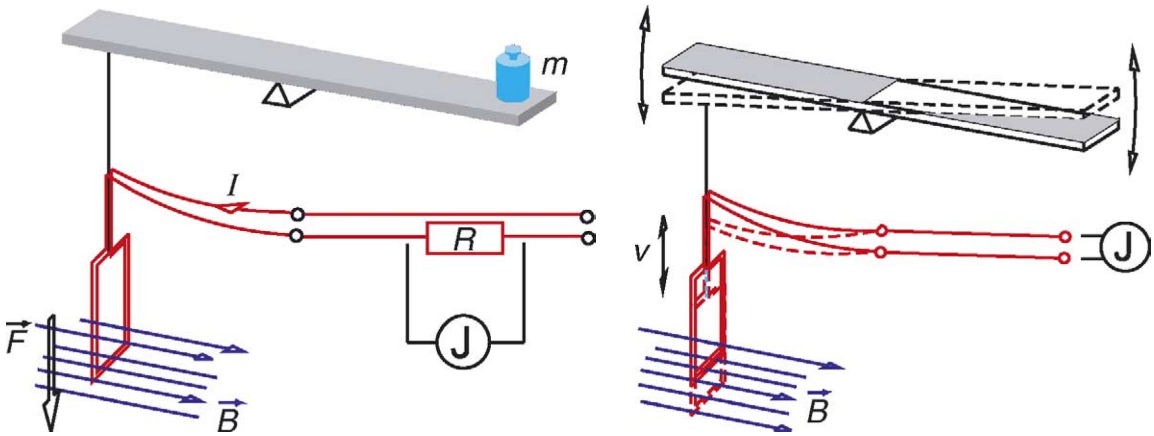
4.1. The watt balance

The concept of the moving coil watt balance was proposed almost 30 years ago by Kibble [28]. A comprehensive review on the existing watt balance experiments was published in 2003 by Eichenberger et al. [29]. The experiment is performed in two parts within the same experimental set-up (see Fig. 4). The first part is a static force compensation where a coil is suspended from one arm of a balance. The coil is immersed in a stationary horizontal magnetic field of flux density B. The current I in the coil exerts a force on the conductor given by

$$\vec{F} = I \cdot \oint \overrightarrow{\mathrm{d}l} \times \vec{B}, \tag{6}$$

where l is the conductor length of the coil. The vertical component F_z of this force is balanced against the weight of the test mass m and we have $F_z = mg$ where g is the local acceleration due to gravity. In the second part of the experiment, the coil is moved at a constant velocity v in the vertical direction through the magnetic field and the voltage U induced across the coil is measured, being

$$U = \oint (\vec{v} \times \vec{B}) \cdot \overrightarrow{\mathrm{d}l} = -\oint (\overrightarrow{\mathrm{d}l} \times \vec{B}) \cdot \vec{v}. \tag{7}$$



(a) (b)

Fig. 4. Principle of the watt balance experiment. (a) Static mode: the electromagnetic force (F) acting on the current carrying coil (I) is balanced against the weight (mg) of the test mass. (b) Dynamic mode: the coil is moved at constant velocity (v) in the vertical direction through the magnetic field (B) and the induced voltage measured in comparison with a Josephson voltage standard (J).

In case of a strictly vertical movement of the coil, the integrals $\oint(\overrightarrow{\mathrm{d}l} \times \vec{B})$ are the same in Eqs. (6) and (7) at the location of the weighing. The elimination of these terms then leads to

$$U \cdot I = m \cdot g \cdot v. \tag{8}$$

The experiment thus allows the virtual comparison of the watt realised electrically (left-hand side of the equation) to the watt realised mechanically. The voltage U can be measured against a Josephson voltage standard. This is most conveniently done using a programmable Josephson voltage standard [30].

Using the expressions of the Josephson frequency, the quantum number of the voltage step, the current, the voltage drop across a resistance calibrated against a quantum Hall resistance standard, and the quantised Hall resistance, Eq. (8) can be rewritten as

$$m = \mathcal{C}\,\frac{f_{\mathrm{J}} \cdot f'_{\mathrm{J}}}{g \cdot v} \cdot h, \tag{9}$$

where $\mathcal{C}$ represents different calibration constants, f_{J} the Josephson frequency measured during the dynamic phase and f'_{J} the Josephson frequency measured during the static phase and h the Planck constant. The watt balance, thus, allows us to express the test mass in terms of the metre, the second and the Planck constant. One of the major advantages of the experiment is that neither the geometry of the coil nor the flux density produced by the source magnet have to be known. Moreover, only virtual electrical and mechanical energy are related. This means that in contrast to the superconducting magnetic levitation described below, real energy dissipation does not enter into the basic equation of the experiment.

The velocity signal comes from a carefully designed interferometer [31] and either this signal or the induced voltage can be used in a regulation loop to control the motion. Since the sign of the induced voltage is reversed when the direction of the motion is inverted, voltage offsets in the electrical circuit can be removed when up and down measurement results are averaged.

In the static mode of the experiment, a current I flowing in the coil generates a Lorentz force to balance the mechanical force F produced by the test mass in the gravitational field. In practice, the balance is underloaded by half of the value of the test mass. A first weighing with a current producing the force needed to balance the system without test mass is followed by a second one, where the sign of the current is reversed and the test mass placed on the balance. These currents are controlled to keep the balance at the equilibrium position and the values are measured with the help of a standard resistor, periodically calibrated against the quantum Hall resistance standard, and a voltage reference.

The mass m of the test mass is determined in air by the classical methods of mass metrology using a mass comparator. The test mass is then directly traceable to national prototypes of the kilogram. As watt balances are operated under vacuum, the mass of the test mass should be also known under vacuum. The immediate advantage of the vacuum measurement is the suppression of the air buoyancy correction. The main disadvantage is the discontinuity of the mass scale between air and vacuum due to the physical and chemical adsorbed layers at the surface of the test mass. Different methods to experimentally overcome the discontinuity of the mass scale between air and vacuum were proposed by Kibble and Robinson [32,33].

Finally, an accurate determination of the absolute value of the gravitational acceleration g next to the experiment and synchronously to the static mode measurement is required to get the expression of the mechanical force $F = m \cdot g$.

The first moving coil apparatus was developed at the National Physical Laboratory (NPL) in the UK, based on Kibble's proposal of 1976 [28]. The final result of the initial set-up with a relative standard uncertainty of 2×10^{-7} was published in 1990 [34]. An improved apparatus, presented the same year reached a short term reproducibility in the order of 1 part in 10^8 [35], and a new result for the Planck constant may be expected in the near future. The second watt balance was built at the NIST. The first results published in 1989 [36,37] had an relative standard uncertainty of 1.3×10^{-6}. Further improvements [38,39] led to the result reported in 1998 with a relative standard uncertainty of 9×10^{-8} [13]. Further improvements are ongoing. The Swiss Federal Office of Metrology and Accreditation (METAS), started to build the third watt balance experiment in 1997 [40]. The new design features implemented in the METAS watt balance consist mainly in a clear separation between the static and dynamic phase of the experiment and a drastic size reduction using a 100 g mass. The METAS apparatus is fully assembled and the testing and evaluation phase has started [41]. First results can be expected within the coming years. The fourth project was initiated in 2000 at the Bureau National de Métrologie (BNM) in France. The French group plans to have an operational instrument in a new laboratory by the end of 2006. Finally, in 2002 the BIPM decided to join the club with the proposal of a cryogenic watt balance.

4.2. Other electrical methods

4.2.1. The superconducting magnetic levitation

This experiment makes use of the force acting on a body with diamagnetic properties in a non-uniform magnetic field. The idea was first brought up by Sullivan and Frederich [42] as a possibility to realise the ampere. When a superconductor in the Meissner state is introduced into the field of a coil with decreasing magnetic flux Φ in the vertical direction, a stable levitation of the body can be obtained (see Fig. 5(a)). The energy equation of the system can then be written as

$$I \cdot U \cdot \mathrm{d}t = I \cdot \mathrm{d}\Phi = \mathrm{d}A + \mathrm{d}W, \tag{10}$$

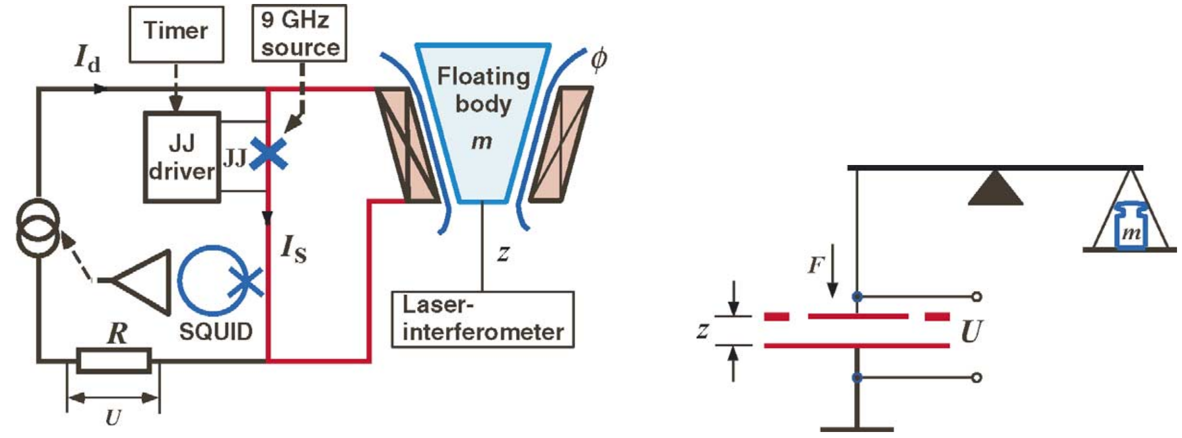
where A is the work done by the field to increase the gravitational energy of the body and W the magnetic field energy. If the levitated body has ideal diamagnetic properties and the coil circuit is superconducting as well, the energy terms are given by

$$A = m \cdot g \cdot z, \quad W = \frac{1}{2}\Phi \cdot I. \tag{11}$$

Considering two equilibrium positions with heights z_l and z_h, where the subscript h and l denote high and low position respectively, the energy difference takes the form

$$\int_{\Phi_l}^{\Phi_h} I \cdot \mathrm{d}\Phi = \frac{1}{2}(\Phi_h I_h - \Phi_l I_l) + mg(z_h - z_l). \tag{12}$$

In practice the experiment can be realised as shown in Fig. 5(a). The superconducting coil is driven by a supply circuit which is controlled by a SQUID ammeter so that the drive current I_d corresponds to the coil current I_s. The drive current is determined



(a) (b)

Fig. 5. (a) Principle of the superconducting magnetic levitation. A superconducting body is floating in a magnetic flux, produced by a superconducting coil; (b) principle of the voltage balance.

by the voltage drop across a resistance standard calibrated in terms of the quantised Hall resistance. When the Josephson junction in the coil circuit is biased on the first step for a time interval t, the flux in the coil is increased by $\Delta\Phi = f_J t \Phi_0$, where f_J is the Josephson frequency, and $\Phi_0 = h/2e$ the magnetic flux quantum. If $\Delta\Phi$ is large enough, the superconducting body of mass m is levitated and reaches the equilibrium position z_l which is measured by laser interferometry. By repeating the process, a series of equilibrium positions can be obtained where Eq. (12) describes the energy between any two positions. Since the flux change can be expressed in units of Φ_0 and the coil current can be measured using the Josephson and the quantum Hall effect, the experiment relates the mass of the floating body to the Planck constant.

The experiment has some major metrological difficulties to overcome. The most important problems are: all unwanted energy expenditure, e.g. due to horizontal force components on the trajectory of the levitated object or distortion of the object under the force of levitation, has to be avoided. The floating body has to be a perfect diamagnet and its mass has to be known in low temperature environment.

The approach of the superconducting magnetic levitation has been pursued at the National Research Laboratory of Metrology (NRLM, now NMIJ) in Japan [43,44] and the D. I. Mendeleyev Research Institute of Metrology (VNIIM) in Russia [45]. The NRLM group has reached a resolution of 1 part in 10^6 in its experiment [46]. A new set-up which should reduce some of the systematic errors was proposed in 2001 [47]. In the same year, a design study for a magnetic levitation system was presented by the Centre for Metrology and Accreditation (MIKES) in Finland [48]. The MIKES group is developing a cryogenic calorimeter to measure the energy losses due to the non-ideal diamagnetic properties of the levitated body.

4.2.2. The voltage balance

The principle of this approach is illustrated in Fig. 5(b). The electrostatic force acting between the plates of capacitance C is compared with the weight mg of the test mass m, where g is the gravitational acceleration. The movable plate of the capacitor is suspended from the balance. In the equilibrium position of the balance, the forces are connected by the relationship

$$m \cdot g = \frac{1}{2} U^2 \frac{\partial C}{\partial z}, \tag{13}$$

where U is the voltage across the capacitor and $\partial C/\partial z$ the capacitance gradient in the vertical direction. The measurement of the voltage is performed against a Josephson voltage standard and the capacitance change is expressed in terms of the quantised Hall resistance. In this way, a link between the test mass and the Planck constant is established. In a typical set-up (see [49] for a review), the voltage needed is around 10 kV and the test mass is a few grams.

Using this approach, Funck and Sienknecht from the PTB [50] reached a relative standard uncertainty of 6.3×10^{-7} in the determination of h [12]. The experiment was also carried out at the University of Zagreb [51]. A relative uncertainty of 3.5×10^{-7} in the determination of the volt was obtained in 1987–1988 [9]. Subsequently, several systematic errors were found by Bego et al. in the set-up which led to improvements [51] and the proposition for a new 100 kV voltage balance [52]. To our knowledge, however, this work is not carried on, at least at the moment.

With the present techniques, the voltage balance approach does not promise to reach an uncertainty below 1 part in 10^7. The main problems are the high voltage required in the experiment, the voltage and frequency dependence of the capacitance and its mechanical imperfections.

5. New definition of the kilogram and conclusions

Mass is an important physical property and the selection of the mass as a base quantity in the SI is quite logical. However, when it comes to a definition and realisation of the base unit of mass, present and future requirements on the stability and reproducibility in the SI are clearly beyond the capabilities of an artefact like the *International Prototype Kilogram* $\mathfrak{K}$. Therefore, a new definition based on a fundamental constant is urgently needed.

The experiments described in this paper establish a link between the kilogram and fundamental constants. The approach based on counting atoms could lead to a definition of the form [53]: "*The kilogram is the mass of* $5.018\ldots \times 10^{25}$ *free* ^{12}C *atoms at rest in their ground state.*" In this definition, the numerical value is derived from $m_u = M_0/N_A$, where m_u is the atomic mass unit and M_0 denotes the molar mass constant which has the value 10^{-3} kg mol^{-1} by definition. The numerical value is, thus, $(N_A/12) \cdot 10^3$ mol.

In the second line of experiments comparing electrical and mechanical power, a link between the kilogram and the Planck constant is established. As the Planck constant plays a unique role among the fundamental constants, both as quantum of action and as a factor of proportionality in many equations, it would be a natural choice to fix the value of h and to link the kilogram to this value using experiments like the watt balance. According to propositions of Taylor and Mohr [53], the new definition of the kilogram could read as follows: "*The kilogram is the mass of a body at rest whose equivalent energy equals the energy of a collection of photons whose frequencies sum to* 135 639 274 × 10^{42} Hz", or in other words: "*One kilogram is a mass such*

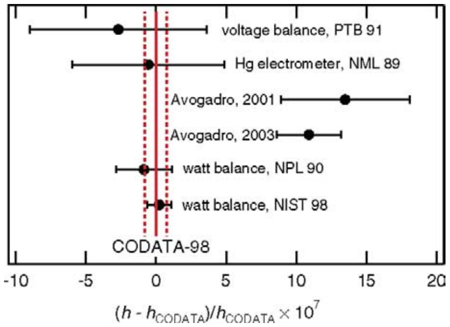


Fig. 6. Experimental values for the Planck constant.

that the Plank constant h is exactly 6.626 068 76 × 10^{-34} Js". This definition is based on the well-known Einstein relation $E = mc^2$, where c is the speed of light fixed with the definition of the metre, and the relation $E = h\nu$ valid for the energy of photons. Note that it is also possible to use a definition based on a fixed value of the Planck constant for the Avogadro route. The Planck and Avogadro constants are related by

$$h = \frac{cA_r(e)M_0\alpha^2}{2R_\infty N_\mathrm{A}}, \tag{14}$$

where $A_r(e)$ is the relative atomic mass of the electron, α is the fine structure constant, and R_∞ is the Rydberg constant. The combined uncertainty of this group of constants is below 1 part in 10^8. The value of the molar mass constant is $M_0 = 10^{-3}$ kg mol^{-1} exactly.

An overview on the present status of the experiments aiming at a new definition of the kilogram can be gained by looking at the published results for the Planck constant. In Fig. 6, all results with a relative standard uncertainty below 1×10^{-6} are shown. With the exception of the results deduced from the Avogadro experiments, the values are taken from [12]. Due to improvements in the analysis, they may, in some cases, differ from the data first published by the experimenters. The values labeled 'Avogadro 2001' and 'Avogadro 2003' are determined from the latest published values of the Avogadro constant [54,55,24].

The relative differences between h calculated using Eq. (14) and the CODATA value are $(1.3 \pm 0.5) \times 10^{-6}$ and $(1.1 \pm 0.3) \times 10^{-6}$ respectively. This may point to an unresolved systematic error in one of the experiments.

The research work outlined in this review, undertaken during the past two decades to replace the artefact definition of the base unit kilogram, is impressive, both in extent and in variety. While the options for a new definition based on a fundamental constant are already clear today, the various experiments are at quite different stage of development and none has achieved the necessary accuracy level so far. From this point of view the Avogadro and the watt balance experiments are more advanced than the other routes and important results may be expected within a couple of years. But a new definition may only be considered when experimental agreement is reached at the suspected relative uncertainty level of the *International Prototype Kilogram* $\mathfrak{K}$, that is at 10^{-8}. This may be the case towards the end of the decade. Furthermore, the new definition should preferably not refer to a particular atom, but to a fundamental constant only, such as the mass of an elementary particle or the Planck constant h.

References

[1] W. Smeaton, The foundation of the metric system in France in the 1790s, Platinum Metals Rev. 44 (3) (2000) 125–134.
[2] H. Moreau, Le système métrique, Chiron, Paris.
[3] The International System of Units, Bureau international des poids et mesures, BIPM, 1998.
[4] G. Girard, The third periodic verification of national prototypes of the kilogram (1988–1992), Metrologia 31 (1994) 317–336.
[5] J. Maxwell, Report of the British Association for the Advancement of Science 40 (1870) 215.
[6] B.N. Taylor, The possible role of the fundamental constants in replacing the kilogram, IEEE Trans. Instrum. Meas. 40 (2) (1991) 86–91.
[7] V. Kose, B.R.L. Siebert, W. Wöger, General principles for the definition of the base units in the SI, Metrologia 40 (2003) 146–153.
[8] T.J. Quinn, The kilogram: the present state of our knowledge, IEEE Trans. Instrum. Meas. 40 (2) (1991) 81–85.
[9] B.N. Taylor, T.J. Witt, New international electric reference standards based on the Josephson and quantum Hall effects, Metrologia 26 (1989) 47–62.
[10] G. Girard, The washing and cleaning of kilogram prototypes at the BIPM, BIPM report.
[11] R. Davis, The SI unit of mass, Metrologia 40 (2003) 299–305.
[12] P.J. Mohr, B.N. Taylor, CODATA recommended values of the fundamental physical constants: 1998, Rev. Mod. Phys. 72 (2) (2000) 351–495.

[13] E.R. Williams, R.L. Steiner, D.B. Newell, P.T. Olsen, Accurate measurement of the Planck constant, Phys. Rev. Lett. 81 (12) (1998) 2404–2407.

[14] P. Becker, History and progress in the accurate determination of the Avogadro constant, Rep. Prog. Phys. 64 (2001) 1945–2008.

[15] P. Becker, Tracing the definition of the kilogram to the Avogadro constant using a solicon single crystal, Metrologia 40 (2003) 366–375.

[16] M. Gläser, Tracing the atomic mass unit to the kilogram by ion accumulation, Metrologia 40 (2003) 376–386.

[17] J. Martin, U. Kuetgens, J. Stümpel, P. Becker, The silicon lattice parameter – an invariant quantity of nature, Metrologia 35 (1998) 811–817.

[18] I.L. Barnes, L.J. Moore, L.A. Machlan, T.J. Murphy, W.R. Shields, Absolute isotopic abundance ratios and the atomic weight of a reference sample of silicon, J. Res. Nat. Bur. Stand. A 79 (1975) 727–735.

[19] P. de Bièvre, G. Lenaers, T.J. Murphy, H.S. Peiser, S. Valkiers, The chemical preparation and characterization of specimens for absolute measurements of the molar mass of an element, exemplified by silicon, for redeterminations of the Avogadro constant, Metrologia 32 (1995) 103–110.

[20] S. Röttger, A. Paul, U. Keyser, Prompt (n,gamma)-spectrometry for the isotopic analysis of silicon crystals for the Avogadro project, IEEE Trans. Instrum. Meas. 46 (1997) 560–562.

[21] A.J. Leistner, W.J. Giardini, Fabrication and sphericity measurements of single-crystal silicon spheres, Metrologia 31 (1994) 231–243.

[22] R.A. Nicolaus, G. Bönsch, A novel interferometer for dimensional measurement of a silicon sphere, IEEE Trans. Instrum. Meas. 46 (2) (1997) 563–565.

[23] K. Fujii, Present state of the solid and liquid density standards, Metrologia 41 (2004) S1–S15.

[24] P. Becker, H. Bettin, H.-U. Danzenbrink, M. Gläser, U. Kuetgens, A. Nicolaus, D. Schiel, P. de Bièvre, S. Valkiers, P. Taylor, Determination of the Avogadro constant via the silicon route, Metrologia 40 (2003) 271–287.

[25] M. Gläser, Proposal for a novel method of precisely determining the atom mass unit by accumulation of ions, Rev. Sci. Instrum. 62 (1991) 2493–2494.

[26] P. Becker, M. Gläser, Avogadro constant and ion accumulation: steps towards a redifinition of the SI unit of mass, Meas. Sci. Technol. 14 (2003) 1249–1258.

[27] V.E. Bower, R.S. Davis, The electrochemical equivalent of pure silver – a value of the Faraday constant, J. Res. Nat. Bur. Stand. 85 (3) (1980) 175–191.

[28] B.P. Kibble, A measurement of the gyromagnetic ratio of the proton by the strong field method, in: J.H. Sanders, A.H. Wapstra (Eds.), in: Atomic Masses and Fundamental Constants, vol. 5, Plenum Press, New York, 1976, pp. 545–551.

[29] A. Eichenberger, B. Jeckelmann, P. Richard, Tracing Planck's constant to the kilogram by electromechanical methods, Metrologia 40 (2003) 356–365.

[30] C. Burroughs, S.P. Benz, T.E. Harvey, C.A. Hamilton, 1 Volt DC programmable Josephson voltage standard, IEEE Trans. Appl. Supercond. 9 (2) (June 1999) 4145–4149.

[31] A. Courteville, Y. Salvadé, R. Dändliker, High-precision velocimetry: optimization of a Fabry–Perot interferometer, Appl. Opt. 39 (10) (2000) 1521–1526.

[32] B.P. Kibble, Comparing a mass in vacuum with another in air by conventional weighing, Metrologia 27 (1990) 157–158.

[33] I.A. Robinson, Comparing in-air and in-vacuum mass standards without buoyancy corrections via in-vacuum weighing, Metrologia 27 (1990) 159.

[34] B.P. Kibble, I.A. Robinson, J.H. Belliss, A realization of the SI watt by the NPL moving-coil balance, Metrologia 27 (1990) 173–192.

[35] I.A. Robinson, B.P. Kibble, Progress in relating the kilogram to Planck's constant with the NPL watt balance, in: Conference on Precision Electromagnetic Measurements, CPEM, Conference Digest, 2002, pp. 574–575.

[36] P.T. Olsen, R.E. Elmquist, W.D. Philips, E.R. Williams, G.R. Jones, V.E. Bower, A measurement of the NBS electrical watt in SI units, IEEE Trans. Instrum. Meas. 38 (2) (1989) 238–244.

[37] M.E. Cage, R.F. Dziuba, R.E. Elmquist, B.F. Field, G.R. Jones, P.T. Olsen, W.D. Phillips, J.Q. Shields, R.L. Steiner, B.N. Taylor, E.R. Williams, NBS determination of the fine-structure constant, and of the quantized Hall resistance and Josephson frequency-to-voltage quotient in SI units, IEEE Trans. Instrum. Meas. 38 (2) (1989) 284–289.

[38] P.T. Olsen, W.L. Tew, E.R. Williams, R.E. Elmquist, H. Sasaki, Monitoring the mass standard via the comparison of mechanical to electrical power, IEEE Trans. Instrum. Meas. 40 (2) (1991) 115–120.

[39] R.L. Steiner, A.G. Gillespie, K. Fujii, E.R. Williams, D.B. Newell, A. Picard, G.N. Stenbakken, P.T. Olsen, The NIST watt balance: progress toward the monitoring of the kilogram, IEEE Trans. Instrum. Meas. 46 (2) (1997) 601–604.

[40] W. Beer, B. Jeanneret, B. Jeckelmann, P. Richard, A. Courteville, Y. Salvadé, R. Dändliker, A proposal for a new moving-coil experiment, IEEE Trans. Instrum. Meas. 48 (2) (1999) 192–195.

[41] W. Beer, A.L. Eichenberger, B. Jeanneret, B. Jeckelmann, A.R. Pourzand, P. Richard, J.P. Schwarz, Status of the METAS watt balance experiment, IEEE Trans. Instrum. Meas. 52 (2) (2003) 626–630.

[42] D.B. Sullivan, N.V. Frederich, Can superconductivity contribute to the determination of the absolute ampere, IEEE Trans. Magnetics 13 (1977) 396–399.

[43] F. Shiota, K. Hara, A study of a superconducting magnetic levitation system for an absolute determination of the magnetic flux quantum, IEEE Trans. Instrum. Meas. IM-36 (2) (1987) 271–274.

[44] F. Shiota, Y. Miki, A. Namba, Y. Nezu, Y. Sakamoto, T. Morokuma, K. Hara, Absolute determination of the magnetic flux quantum using superconducting magnetic levitation, IEEE Trans. Instrum. Meas. 44 (2) (1995) 583–586.

[45] E.T. Frantsuz, Y.D. Gorchakov, V.M. Khavinson, Measurements of the magnetic flux quantum, Planck constant, and elementary charge at VNIIM, IEEE Trans. Instrum. Meas. 41 (4) (1992) 482–485.

[46] F. Shiota, Y. Miki, Y. Fujii, T. Morokuma, Y. Nezu, Evaluation of equilibrium trajectory of superconducting magnetic levitation system for the future kg unit of mass, IEEE Trans. Instrum. Meas. 49 (5) (2000) 1117–1121.
[47] Y. Fujii, Y. Miki, F. Shiota, T. Morokuma, Mechanism for levitated superconductor experiment, IEEE Trans. Instrum. Meas. 50 (2) (2001) 580–582.
[48] K. Riski, P. Heikkinen, H. Kajastie, J. Manninen, H. Rossi, K. Nummila, E. Frantsuz, V. Khavinson, Design of a superconductiong magnetic levitation system, in: Proceedings IMEKO TC3 2001, 2001, pp. 239–246.
[49] V. Bego, Determination of the volt by means of voltage balances, Metrologia 25 (1988) 127–133.
[50] T. Funck, V. Sienknecht, Determination of the volt with the improved PTB voltage balance, IEEE Trans. Instrum. Meas. 40 (2) (1991) 158–161.
[51] V. Bego, J. Butorac, K. Poljančić, Voltage balance for replacing the kilogram, IEEE Trans. Instrum. Meas. 44 (2) (1995) 579–582.
[52] V. Bego, J. Butorac, D. Ilić, Realization of the kilogram by measuring at 100 kV with the voltage balance ETF, IEEE Trans. Instrum. Meas. 48 (2) (1999) 212–215.
[53] B.N. Taylor, P.J. Mohr, On the redefinition of the kilogram, Metrologia 36 (1999) 63–64.
[54] P. de Bièvre, S. Valkiers, R. Kessel, P.D.P. Taylor, P. Becker, H. Bettin, A. Peuto, S. Pettorruso, K. Fujii, A. Waseda, M. Tanaka, R.D. Deslattes, H.S. Peiser, M.J. Kenny, A reassessment of the molar volume of silicon and the Avogadro constant, IEEE Trans. Instrum. Meas. 50 (2) (2001) 593–597.
[55] P. Becker, The molar volume of single-crystal silicon, Metrologia 38 (2001) 85–86.