Entanglement of unstable atoms: modifications of the emission properties

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Abstract

We analyse the influence of entanglement on the emission properties of atoms. To this end, first, we propose a scheme for the preparation of a pair of entangled Helium atoms, one in the ortho and the other in the para spin configuration. We discuss a realistic scenario for this process, based in the double ionization of He by intense laser fields. These states are used to analyse disentanglement and the role of entanglement in the spontaneous emission from the pair. In particular, we show that the decaying rate of an entangled atom is different from that in a product state, modifying the temporal emission distribution and lifetime of the atoms.

1 Introduction

Entanglement has become a central issue in quantum theory. Many aspects of the subject have been already deeply analysed as, for instance, questions about its experimental generation and verification [1, 2, 3], the existence of good measures for mixtures [4], its application in quantum information processing and foundational tests [1], and many others.

More recently, several authors have studied the behavior of entanglement during the process of spontaneous emission. Diósi [5] and Dood *et al* [6] showed that entanglement is destroyed by the same decoherence mechanisms that degrade quantum interference. In a fundamental paper [7] the disentanglement dynamics originated by the spontaneous emission process was analyzed (see also [8, 9] and references therein). In that approach it was demonstrated that, in the general case, entanglement decays at least as fast as the sum of decaying

rates of the individual states. Also the entanglement existent between an atom and a spontaneously emitted photon has been evaluated [10]. Using the Schmidt decomposition [11], that entanglement can be estimated in a quantitative way. The same approach can be extended to other decaying systems, even when there is interaction between the products of the process [12]. In a related context it has been proposed in Ref. [13] to use squeezed fields to preserve entanglement against spontaneous emissions.

In this paper we want to consider a less known aspect of the problem, the influence of entanglement on the emission properties of unstable atoms. This aspect is complementary to those studied in Refs. [7, 10]. We analyze the problem by considering a system particularly well suited for this purpose, an entangled state of excited ortho- and para-Helium atoms. Recently, it has been suggested the possibility of preparing metastable superpositions of ortho- and para-Helium states [14]. We shall begin this paper addressing the problem of state preparation, and we shall discuss the interaction with intense laser filed as a possible scenario in which this entanglement appears naturally.

Then we will proceed to study radiative properties by calculating the decay rates of the entangled pair in terms of those of unentangled atoms. Three main conclusions will be obtained. First, we shall derive the decaying rate of the entangled state using a simple general method that does not resort to the master equation of the system (as done in Ref. [7]). Later, and as the most important result of the paper, we show that the decaying rates of entangled and free unstable atoms are different. The amplitude of this effect is measured by expectation values of the non-trivial part of the evolution operator. Finally, being the decaying rates different, we can expect the temporal emission distributions and lifetimes also to be different. We confirm this behavior by calculating the emission distributions in entangled and product states. The results presented here aim to provide a deeper understanding of some fundamental properties of entangled unstable states, that are in principle accessible to experimental scrutiny.

We must remark the resemblances and connections of our work with other related problems that have already been addressed in the literature. In the field of quantum information, the authors in [15, 16] have analyzed the decay of the density matrix coherences of one or several qubits, used as quantum registers, when they are coupled with a bath. The density matrix coherences measure the degree of superposition between the two states of the qubit. When the qubit interacts with the bath the state of the complete system, qubit plus bath, becomes entangled. The decaying rates of the coherences show a strong dependence on the characteristics of the entangled state and the bath. The connection of this behaviour with our main result is clear. At variance with these papers, we shall not consider the decaying of a superposition state but that of an excited one. On the other hand, the resemblances of our approach with the work in [17, 18] (and references therein), where the authors have considered the modifications of the emission properties associated with the presence of

boundaries, will be addressed in the Discussion section.

2   Preparation of the state

In Ref. [14] it was discussed how to prepare a superposition of ortho- and para-Helium states. That proposal can easily be extended to the generation of entangled states. Suppose we have two Helium ions He^+ with their electrons in the $|\uparrow\rangle$ state (in a predetermined axis, for instance the z-one): $|He^+(\uparrow\rangle\rangle_L|He^+(\uparrow)\rangle_R$. We also assume that both ions are placed at distant regions denoted by L and R. Next, we prepare a pair of electrons in the entangled state

$$|e_L e_R\rangle = \frac{1}{\sqrt{2}}|e(\uparrow)\rangle_L|e(\downarrow)\rangle_R + \frac{1}{\sqrt{2}}|e(\downarrow)\rangle_L|e(\uparrow)\rangle_R \tag{1}$$

When there is capture, the interaction between electrons and ions is given by

$$|He^+(\uparrow)\rangle|e(\uparrow)\rangle \to |He_{or}\rangle \tag{2}$$

and

$$|He^+(\uparrow)\rangle|e(\downarrow)\rangle \to |He_{pa}\rangle \tag{3}$$

Therefore, because of the linearity of quantum theory we have

$$|He^+(\uparrow)\rangle_L|He^+(\uparrow)\rangle_R|e_L e_R\rangle \to$$
$$\frac{1}{\sqrt{2}}|He_{or}\rangle_L|He_{pa}\rangle_R + \frac{1}{\sqrt{2}}|He_{pa}\rangle_L|He_{or}\rangle_R \tag{4}$$

This is a maximally entangled state of otho- and para-Helium.

In order to actually implement this scheme one must be cautious with the energies of the incident electrons. We want to prepare metastable states. The simplest choice is the $1s2s$ states of both configurations, ortho- and para-Helium: we must prepare state (4) with $He_{pa}(1s2s)$ and $He_{or}(1s2s)$. The difference of energy between the lower state of the ortho- $(1s2s)$ and the first excited state of the para-Helium $(1s2s)$ is small and it seems possible to fulfill that condition.

From now on we shall only refer to these states, using the notation

$$
\begin{aligned}
|\Psi\rangle &= \frac{1}{\sqrt{2}}|He_{or}(1s2s)\rangle_L|He_{pa}(1s2s)\rangle_R + \frac{1}{\sqrt{2}}|He_{pa}(1s2s)\rangle_L|He_{or}(1s2s)\rangle_R \\
&= \frac{1}{\sqrt{2}}|or\rangle_L|pa\rangle_R + \frac{1}{\sqrt{2}}|pa\rangle_L|or\rangle_R
\end{aligned} \tag{5}
$$

In the next section we discuss two ways to actually implement the above scheme.

3 Implementation of the scheme

Interestingly, a situation similar to the above procedure takes place during the interaction of strong laser radiation with a cloud of He atoms. It is known that, at intensities near saturation, the most probable path for the double ionization of He is the so-called sequential process [19]. In this case, the two electrons are ionized at each of the field maxima of a laser cycle. Since the field interaction is not relativistic, the ionized electron pair retains the same spin configuration as the He ground state. In addition, as the field reverses its sign during the laser cycle, the electrons are emitted to opposite sides of the parent atom, configuring a spatially separated entangled pair. On the other hand, the single ionization of He occurs typically with a higher rate than the double. Therefore, the ionized pair has higher probability to encounter single ionized atoms than double ionized. As a consequence, there is a better chance to recombine in the form of entangled pairs of neutral He than of He^+. Half of these neutral recombinations will produce He entangled pairs of the form given in Eq. (4). The final yield depends on the particular cross section of the recombination process. However note that, as we are considering low-energy metastable states in our discussion, their fraction will increase as other (unwanted) configurations will decay via spontaneous processes.

4 Lifetimes of entangled states

Our starting point is Eq. (5). The states $1s2s$ of ortho- and para-Helium atoms are metastable. Therefore, after some time they will decay emitting photons. As the energies of the photons emitted by ortho and para atoms are (slightly) different, we can in principle know if the emission event took place for a para- or ortho-atom and, consequently, if the companion atom rests in an ortho- or para-state. Moreover, since the two particles are spatially well separated, the events to emit in L or to emit in R are distinguishable. We can know where and by what type of state was the photon emitted. The two alternatives in Eq. (5) become distinguishable and there is no longer a superposition of two-particle states. The pure two-particle state (5) disentangles to a mixture of states

$$|g\rangle_i |or\rangle_j \ , |g\rangle_i |pa\rangle_j \tag{6}$$

with $i, j = L, R$, $i \neq j$ and $|g\rangle$ denoting the ground state, i. e., the para-Helium $1s1s$ state at which the two $1s2s$ states decay.

We can ask now how fast this process is or, equivalently, by the lifetime of the entangled state (5). This property can be easily evaluated using the standard techniques for the calculation of lifetimes of unstable atoms. The lifetime of (5), denoted as $\tilde{\tau}$, is given by $\tilde{\tau} = 1/\tilde{\Gamma}$ with $\tilde{\Gamma}$ the decaying rate of $|\Psi\rangle$. From now on in order to avoid any confusion between the decaying rates and lifetimes of entangled and non-entangled

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states, those corresponding to the first case will be denoted by a tilde.

The decaying can occur at two different locations,

$$|\Psi\rangle \rightarrow |g\rangle_L |h\rangle_R \tag{7}$$

at the left side, and

$$|\Psi\rangle \rightarrow |h\rangle_L |g\rangle_R \tag{8}$$

at the right side. $|h\rangle$ can have two different values at each side, $h = or, pa$.

Since we add probabilities and not amplitudes (because, as signaled before, the alternatives of emission at L or at R are distinguishable) we have that the total decaying rate is

$$\tilde{\Gamma} = \tilde{\Gamma}_L + \tilde{\Gamma}_R \tag{9}$$

where $\tilde{\Gamma}_L$ and $\tilde{\Gamma}_R$ denote the decaying rates of the two channels associated with the two above equations.

Let us evaluate $\tilde{\Gamma}_i$, $i = L, R$. Taking into account again that the emissions by atoms ortho and para are distinguishable and, consequently, we must add probabilities instead of amplitudes we have that the probability of the decaying process per unit time in the interval (t_0, t) is

$$\tilde{\Gamma}_i(t, t_0) = \frac{1}{t - t_0} \sum_{|h\rangle_j} |_j\langle h|_i\langle g|\hat{U}(t, t_0)|\Psi\rangle|^2 \tag{10}$$

with $h = or, pa$ and $i, j = L, R; i \neq j$.

When we take the limit of $t - t_0$ very small we obtain the instantaneous value of the decaying rate. This justifies the use of the same notation, $\tilde{\Gamma}$, for the instantaneous decaying rates and for this probability per unit time. $\hat{U}(t, t_0)$ is the evolution operator between times t and t_0. It is given by $\exp(-i\hat{\mathcal{H}}(t - t_0))$ with $\hat{\mathcal{H}}$ the Hamiltonian operator of the system. We can decompose the Hamiltonian in the form $\hat{\mathcal{H}} = \hat{\mathcal{H}}_L + \hat{\mathcal{H}}_R$ because there is not interaction between the well separated parts L and R (note that $\hat{\mathcal{H}}_L = \hat{\mathcal{H}}_R$ except by the fact that they act in different spatial regions). Therefore, since $[\hat{\mathcal{H}}_L, \hat{\mathcal{H}}_R] = 0$ the evolution operator factorizes as $\hat{U} = \hat{U}_L \hat{U}_R$.

Now, we are in position to evaluate the disentangling rate per unit time in the interval (t_0, t):

$$\tilde{\Gamma}(t, t_0) = \tilde{\Gamma}_L(t, t_0) + \tilde{\Gamma}_R(t, t_0) =$$
$$\tilde{\Gamma}_{or}^L(t, t_0) + \tilde{\Gamma}_{pa}^L(t, t_0) + \tilde{\Gamma}_{or}^R(t, t_0) + \tilde{\Gamma}_{pa}^R(t, t_0) =$$
$$\tilde{\Gamma}_{or}(t, t_0) + \tilde{\Gamma}_{pa}(t, t_0) \tag{11}$$

Finally, for the lifetime we have

$$\tilde{\tau} \approx \frac{1}{\tilde{\Gamma}_{or} + \tilde{\Gamma}_{pa}} \tag{12}$$

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Equation (12) shows that the lifetime of the entangled state is always shorter than those of the component states. In our case, it is slightly shorter than that of the $1s2s$ state of the para-Helium. This result agrees with that obtained in [7], where it was shown that entanglement decays at least as fast as the sum of the separate rates.

The result can easily be understood in terms of the probabilities associated with the two decay channels. As the two channels represent distinguishable alternatives the probability of disentanglement is the sum of the two decay probabilities and, consequently, larger than any of them.

5 Decaying rates of atoms in entangled states

In this section we analyze the relation between entangled and unentangled decaying rates. Let us see, for instance, the case $h = or$ in Eq. (10), which corresponds to the evolution $|\Psi\rangle \to |g\rangle_i |or\rangle_j$. The probability amplitude of this process in the interval (t_0, t) is

$$
\begin{aligned}
{}_j\langle or|{}_i\langle g|\hat{U}(t,t_0)|\Psi\rangle = {} & \frac{1}{\sqrt{2}}\, {}_j\langle or|\hat{U}_j(t,t_0)|pa\rangle_j \, {}_i\langle g|\hat{U}_i(t,t_0)|or\rangle_i \\
& + \frac{1}{\sqrt{2}}\, {}_j\langle or|\hat{U}_j(t,t_0)|or\rangle_j \, {}_i\langle g|\hat{U}_i(t,t_0)|pa\rangle_i = \\
& \frac{1}{\sqrt{2}}\mathcal{M}^j_{op}\mathcal{M}^i_{go} + \frac{1}{\sqrt{2}}\mathcal{M}^j_{oo}\mathcal{M}^i_{gp}
\end{aligned}
\tag{13}
$$

where the notation for the matrix elements $\mathcal{M}$ is obvious.

It must be noted the presence of the term $\mathcal{M}^j_{op}\mathcal{M}^i_{go}$. Physically, it represents the emission by an atom ortho at i (leaving the atomic state $|g\rangle_i$) and the transition $|pa\rangle \to |or\rangle$ at j (giving rise to the presence of an unstable atom ortho at j). This term contributes to the total decaying rate of the channel $|\Psi\rangle \to |g\rangle_i |or\rangle_j$, but not to the emission rate of photons emitted from the para configuration.

The probability of this transition is extremely small and, consequently, we shall neglect this contribution here. With this approximation the only relevant contribution to the disentangling channel $|\Psi\rangle \to |g\rangle_i |or\rangle_j$ is $\mathcal{M}^j_{oo}\mathcal{M}^i_{gp}$. The physical meaning of these matrix elements is clear. $|\mathcal{M}^i_{gp}|^2/(t-t_0)$ gives the probability of decaying of a free unstable para-type atom (located at i) per unit time in the interval (t_0, t), $\Gamma^i_{pa}(t, t_0)$. As discussed before, in the limit of $t - t_0$ very small it gives the instantaneous decaying rate of free unstable para-type atoms at side i, $\Gamma^i_{pa}(t_0)$. It also equals the emission rate of photons emitted from the para configuration at i.

The other matrix element, $\mathcal{M}^j_{oo}$, can be expressed in the well-known way

$$
\mathcal{M}^j_{oo} = {}_j\langle or|1 + \hat{W}_j(t,t_0)|or\rangle_j = 1 + W^{or}_j(t,t_0)
\tag{14}
$$

The operator $\hat{W}_j$ contains all the terms in the expansion of the evolution operator depending on the Hamiltonian, i. e., it represents the non-trivial terms. On the other hand, $W_j^{or}(t, t_0) = {}_j\langle or|\hat{W}_j(t, t_0)|or\rangle_j$ is the expectation value of the operator $\hat{W}_j(t, t_0)$ in the ortho state.

Thus, the squared modulus of $\mathcal{M}_{oo}^j \mathcal{M}_{gp}^i / \sqrt{2}$ divided by $t - t_0$ gives the probability of disentanglement by emission of a photon of type para at side i. Moreover, it equals the emission rate by an unstable para-atom in an entangled state at side i. Taking into account this equality, we use the same notation for the disentanglement and decaying for unstable entangled atom rates:

$$\tilde{\Gamma}_{pa}^i(t, t_0) = \frac{1}{2}\Gamma_{pa}^i(t, t_0)(1 + |W_j^{or}(t, t_0)|^2 + 2Re(W_j^{or}(t, t_0))) \qquad (15)$$

The total decaying rate per unit time of atoms of type para in an entangled state is

$$\tilde{\Gamma}_{pa}(t, t_0) = \tilde{\Gamma}_{pa}^L(t, t_0) + \tilde{\Gamma}_{pa}^R(t, t_0) = \Gamma_{pa}(t, t_0)(1 + |W^{or}(t, t_0)|^2 + 2Re(W^{or}(t, t_0))) \qquad (16)$$

In the last term we have dropped all the indexes referring to the location of the atom $(j = L, R)$ because all the variables are equal on both sides due to the symmetry of the problem. A similar result can be obtained for the ortho case with obvious modifications.

We have obtained (in the limit of $t - t_0$ very small) the decaying rates of atoms in entangled states. This expression clearly shows that the decaying rate of an atom in an entangled state differs from that in a product state due to the presence of W^h, $h = or, pa$.

As we shall see in next section this property has interesting physical implications. The importance of these differences is measured in the para case by

$$\frac{\tilde{\Gamma}_{pa}(t - t_0) - \Gamma_{pa}(t - t_0)}{\Gamma_{pa}(t - t_0)} = |W^{or}(t, t_0)|^2 + 2Re(W^{or}(t, t_0)) \qquad (17)$$

The disentangling rate per unit time in the interval (t_0, t) can be expressed in terms of the unentangled rates as:

$$\tilde{\Gamma}(t, t_0) = \Gamma_{or}(t, t_0) + \Gamma_{pa}(t, t_0) + \Lambda(t, t_0) \qquad (18)$$

with

$$\Lambda(t, t_0) = |W^{or}(t, t_0)|^2 + |W^{pa}(t, t_0)|^2 + 2Re(W^{or}(t, t_0) + W^{pa}(t, t_0)) \qquad (19)$$

In the case that the term Λ is very small (using the limit $t \to t_0$ and assuming a stationary situation where the decaying rates are time independent) we have

$$\tilde{\tau} \approx \frac{1}{\Gamma_{or} + \Gamma_{pa}} \qquad (20)$$

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6 Distributions of emitted photons. Lifetimes

As discussed in the previous section the decaying rates of entangled atoms differ from those of free unstable ones. Then one can expect the temporal distributions of emitted photons and lifetimes of unstable atoms do not be equal in both cases. We confirm this point by an explicit calculation of the time-dependent distributions.

If we denote by n and n_h respectively the number of pairs of entangled particles and non-entangled unstable atoms (the unstable atoms produced in the disentangling process) of type h, taking the usual approach of an exponential decay, their rules of change are given by

$$\frac{dn}{dt} = -\tilde{\Gamma} n \tag{21}$$

and

$$\frac{dn_h}{dt} = \tilde{\Gamma}_H n - \Gamma_h n_h \tag{22}$$

where $h, H = or, pa$ and $H \neq h$. Along this section we use the instantaneous emission rates and assume, by simplicity, that they are independent of time. The first equation says that the number of entangled states only varies because of disentanglement. The rule for n_h is different. We have a source term $\tilde{\Gamma}_H n$ that gives the number of entangled pairs that generate an unstable atom of type h (the emitted photon in the disentangling process is of type H). On the other hand, we have the usual decay term, $\Gamma_h n_h$. The above equations can easily be solved. Taking the initial conditions $n(0) = n_0$ (with n_0 the initial number of entangled pairs) and $n_h(0) = 0$ (initially all the atoms are entangled) we have

$$n(t) = n_0 e^{-\tilde{\Gamma} t} \tag{23}$$

and

$$n_h(t) = n_0 \frac{\tilde{\Gamma}_H}{\tilde{\Gamma} - \Gamma_h} (e^{-\Gamma_h t} - e^{-\tilde{\Gamma} t}) \tag{24}$$

To solve the second equation we have taken for the particular solution $\propto e^{-\tilde{\Gamma} t}$ and for the general of the homogeneous equation $\propto e^{-\Gamma_h t}$.

These expressions are to be compared with that obtained for product states, which is derived from

$$\frac{dn_h^p}{dt} = -\Gamma_h n_h^p \tag{25}$$

with solution

$$n_h^p(t) = n_0 e^{-\Gamma_h t} \tag{26}$$

where we have denoted all the variables related to the product state by the superscript p.

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Clearly, $n_h(t)$ and $n_h^p(t)$ are, in general, different. The most interesting manifestation of these different behaviours is, probably, the fact that the lifetimes are not equal. In effect, the lifetime of a free (or in a product state) unstable atom of type h, τ_h, is defined by the relation

$$n_h^p(\tau_h) = n_0 e^{-1} \tag{27}$$

This definition can be extended to the case in which the atoms are initially entangled. In this case, however, one must be cautious because the unstable atoms of type h (before of decaying to the stable state) can be in an entangled or in a product state. The number of atoms in both types of states must be added. Then the definition of the lifetime of the atom of type h when initially is in an entangled state, $\tilde{\tau}_h$, is

$$n(\tilde{\tau}_h) + n_h(\tilde{\tau}_h) = n_0 e^{-1} \tag{28}$$

because the number of unstable atoms of type h in an entangled state at time t equals the number of entangled pairs at that time, $n(t)$.

Then $\tilde{\tau}_h$ is given by the solution of the equation

$$\left(1 - \frac{\tilde{\Gamma}_H}{\tilde{\Gamma} - \Gamma_h}\right) e^{-\tilde{\Gamma}\tilde{\tau}_h} + \frac{\tilde{\Gamma}_H}{\tilde{\Gamma} - \Gamma_h} e^{-\Gamma_h \tilde{\tau}_h} = e^{-1} \tag{29}$$

From an operative point of view it is not easy to determine experimentally the number of atoms of each type existent at a given time. It is much easier to measure the number of photons of each type emitted at that time, and from this data to obtain the number of unstable atoms. We denote by $N_h(t)$ the number of photons of type h emitted at time t. It is given by

$$\frac{dN_h}{dt} = \tilde{\Gamma}_h n + \Gamma_h n_h \tag{30}$$

Using the expressions for $n(t)$ and $n_h(t)$ and the initial condition $N_h(0) = 0$ (in this step we must use $\tilde{\Gamma} = \tilde{\Gamma}_h + \tilde{\Gamma}_H$) we obtain

$$N_h(t) = n_0 + \frac{n_0}{\tilde{\Gamma}}\left(\frac{\Gamma_h \tilde{\Gamma}_H}{\tilde{\Gamma} - \Gamma_h} - \tilde{\Gamma}_h\right) e^{-\tilde{\Gamma}t} + n_0 \frac{\tilde{\Gamma}_H}{\Gamma_h - \tilde{\Gamma}} e^{-\Gamma_h t} \tag{31}$$

This expression differs from the distribution of emitted photons by an atom of type h when it is in a product state, which is

$$N_h^p(t) = n_0(1 - e^{-\Gamma_h t}) \tag{32}$$

The number of unstable atoms of type h at any time can be deduced from the easily measurable distributions of emitted photons when the temporal distribution is measured on a large number of single repetitions of the experiment

(instead of a large number of pairs prepared at the same time in the same zone). In effect, then for each pair we can determine if each photon has been emitted before than the other (causing the disentangling process) or after it (the emission is caused by a non-entangled unstable atom). We denote the distribution of the first type of photons by $N_h^{(1)}(t)$ and that of the second ones by $N_h^{(2)}(t)$. Then $n_h(t) = N_H^{(1)}(t) - N_h^{(2)}(t)$. We can also determine $n(t)$ using the relation $n(t) = n_0 - N_H^{(1)}(t) - N_h^{(1)}(t)$ In contrast, when all the pairs are at the same time in the same zone (for instance, a cloud of initially entangled pairs) one cannot determine if the photon is of type (1) or (2) and it is not possible to deduce the distributions $n(t)$ and $n_h(t)$.

It must be signaled as a byproduct that the above emission distributions could be used to determine without additional measurements if a state is entangled or not. This type of characterization of entanglement is usually denoted as a direct measurement of entanglement [2]. A similar proposal can be found in Ref. [20], where the authors use the correlations function of the emitted light for detecting entanglement in a system comprising coupled excitonic qubits.

7 Discussion

We have shown in this paper that it is, in principle, possible to prepare metastable entangled states of atoms. Although we have restricted our considerations to the Helium the same scheme can be extended to metastable states of other atoms. For states for which the transition between them can be neglected (as for ortho- and para-Helium), the extension is immediate. When that condition is not fulfilled the matrix element representing the transition must be taken into account. In future work we plan to discuss how to actually implement the scheme proposed here.

The proposed technique allows to analyze the properties of unstable atoms in entangled states. Three principal results have been obtained in this paper: (i) We have derived the disentanglement rate in a simple and intuitive way that does not resort to more involved techniques as the master equation used in Ref. [7]. (ii) We have shown that the decaying rates of the atoms are different for entangled and product states. (iii) As a consequence of (ii), the temporal distributions of emitted photons and lifetimes of unstable atoms are modified by the presence of entanglement.

The results (ii) and (iii) provide a complementary view to the relation between entanglement and spontaneous emission presented in [7, 8, 9, 10] (and point (i) here). In effect, in these references (and the evaluation of $\tilde{\tau}$ here) one studies how the entanglement is generated [10] or destroyed [7] by spontaneous emission. The results presented here are complementary to the above ones, showing that entanglement is not only a passive element, but also an active one that can modify the emission properties.

The modifications of the emission properties and lifetimes due to the presence of entanglement provide a second example of modifications of the properties of unstable atoms. In [18] it was shown how the spontaneous emission rate is altered when placed inside a conducting wedge. Physically, this effect can be explained because the non-trivial boundary conditions affect the interaction between atoms and the quantum electromagnetic field. In our example the modifications are associated with the entanglement of the atom with another atom. The wavefunction, with which we evaluate all the physical properties of the system, is different from that of a free system.

Finally, it is worthy to examine our results from the point of view of the reduced density matrix. It is remarkable the fact that, according to our analysis, we have properties of the individual system that are modified by the presence of entanglement. It is sometimes vaguely stated that only joint properties of the two-particle system are sensitive to the presence of entanglement. The reduced (or single-particle) density matrix of any of the atoms for state (5) has the form $\rho_h = Tr_H(|\Psi\rangle\langle\Psi|) = \frac{1}{2}|h\rangle_R\langle h| + \frac{1}{2}|h\rangle_L\langle h|$, which is equivalent to a mixture of states $|h\rangle_R$ and $|h\rangle_L$. The expected value of any observable $\hat{A}$ associated with the system is $\langle\hat{A}\rangle = Tr(\rho_h\hat{A})$. Thus, the expectation value of any observable for the entangled state and the mixture are equal and, consequently, one cannot use the values of $\langle\hat{A}\rangle$ to distinguish between both types of states. Nevertheless, the decaying rates, lifetimes and emission distributions are not expectation values of observables and are not constrained by the above restriction.

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