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Reply to “Comments on ‘Large deflection and rotation of Timoshenko beams with frictional end supports under three-point bending’” [C. R. Mecanique 345 (2017), doi:10.1016/j.crme.2017.01.004]

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We would like to thank Professor M. Batista for his interest in our work [1]. In [1], our aim is to present an appropriate method to analyze large deflections and rotations of Timoshenko beams with consideration of frictional force at supports. In this field, little information is available; however, a large number of researches related to large deflection of Euler–Bernoulli beams have been reported. The suggested method consists of two aspects. One is that in the well-known stress–strain constitutive equations used, the strains contain nonlinear trigonometric functions related to the slope angle θ of deflection, rather than to the first leading term of their Taylor series at $\theta = 0$. In our assumptions, we do not require that θ is small enough, which may be arbitrarily large in theory. Thus the constitutive equations are nonlinear. The second is that we adopt the terminal state posterior to deformation to give boundary conditions, rather than the initial state prior to deformation. Just due to the second aspect, the reaction force at supports is no longer vertical, but inclined, which is also essentially different from the classical small deflection treatment, but coincides with large deflection analysis of Euler–Bernoulli beams. It further needs to determine the slope angle of deflection at supports. In small deflection analysis, the reaction force is always assumed vertical and it is not necessary to determine the slope angle of deflection at supports in advance. So we think that it is a new approach to analyze large deflection and rotation of Timoshenko beams. In fact, according to our analysis, the slope angle at the supports can reach over 80 degrees (see Table 1). In [2], although the first leading term is remained and the other remaining terms are neglected when expanding the trigonometric functions as Taylor series at $\theta = 0$, the boundary condition after deformation such as (18) is still adopted. It indicates that the boundary condition related to large deflections is actually applied. In other words, the solution in [2] does not apply for small deflection analysis, but is related to large deflections of Timoshenko beams.

On the other hand, it is not surprising that our solution when neglecting shear deformation cannot reduce to the analytic solution of large deflections on Euler–Bernoulli beams including elliptical functions. The reason is that the starting point of two kinds of large deflections is based on different assumptions. For the latter, the constitutive equation used is related to the bending moment on an element of arc-length. Therefore, large deflections in [1] do not refer to classical large deflections. On the contrary, it provides another feasible but different method to deal with the bending of beams, and the corresponding results should lie between the small deflection and classical large deflection analyses. When comparing with some experimental data available, theoretical predictions based on this approach agree well with experimental data [3]. In addition, it should be mentioned that there are other ways to define large deflections such as using the constitutive

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Table 1
The end slope angle α_0 (in degrees) for frictionless end supports.

ψ	p											
	0.001	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	.95	1
0	0.029	2.868	5.759	8.694	11.703	14.819	18.088	21.578	25.402	29.773	32.304	35.238
	89.977	87.674	85.327	82.934	80.467	77.894	75.167	72.219	68.937	65.108	62.848	60.185
0.1	0.034	3.443	6.914	10.447	14.080	17.863	21.871	26.223	31.154	37.268	41.432	–
	89.977	87.673	85.317	82.900	80.381	77.712	74.817	71.577	67.757	62.753	59.143	–
0.2	0.040	4.017	8.069	12.198	16.452	20.904	25.663	30.935	37.212	46.965	–	–
	89.977	87.674	85.307	82.864	80.291	77.519	74.433	70.829	66.213	58.113	–	–

equations containing von Karman assumptions [4], which is frequently seen particularly for large deflections of plates. Moreover, such solutions of large deflections of beams also cannot reduce to the analytic solution of large deflections on Euler–Bernoulli beams including elliptical functions [5,6]. Therefore, the definition way of large deflections is not unique.

As pointed out in [2], in our analysis we indeed have neglected the contribution of the horizontal displacement on the bending moment since this effect is sufficiently small. This can be seen in the following. If including this effect, one has the following equation

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\xi^2} + p\tan(\alpha_0 - \beta)\, W = -p\xi - p^2\eta\tan(\alpha_0 - \beta)(1-\xi) \tag{1}$$

in place of (31) in [1], where

$$\eta = \frac{I}{AL^2} \tag{2}$$

Taking into account the fact that, for practical cases, I/AL^2 is much less than unity, and p is lower than unity, one may reasonably neglect the contribution of the last term in (1). Based on this reason, we removed the last term of (1) in writing (31) in [1]. Such a treatment can be widely found in the papers on large deflections of classical simply-supported Euler–Bernoulli beams (see, e.g., (3) in [7], (3) in [8], (10) in [9], (1) in [10], etc.). In view of the same cause, we used the relation $\mathrm{d}s\cos\theta = \mathrm{d}x$ in [1], which frequently appears in treating large deflection of Euler–Bernoulli beams [7–10].

Finally, we are grateful to Professor Batista for pointing out some errors in [1]. A negative sign in Eq. (18) and also in the subsequent Eqs. (21) and (24) is missing. Eq. (41) has a little error, in which a superfluous term $p\psi$ is added to the right-hand side of (41). The correct forms of (41) and (44) should read

$$\tan\alpha_0 = \left[p\psi + \frac{1}{\tan(\alpha_0-\beta)}\right]\frac{1}{\cos\sqrt{p\tan(\alpha_0-\beta)}} - \frac{1}{\tan(\alpha_0-\beta)} \tag{3}$$

and

$$\cos\sqrt{ps} = \left[\frac{1+s^2}{(1-\mu s)(1+p\psi s)}\right]^{-1} \tag{4}$$

According to the above resulting equation (4), Table 1 is recalculated in the following. Since $\psi = EI/\left(\kappa GAL^2\right)$ is very small, the influence arising from superfluous $p\psi$ term is quite limited.

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