C. R. Mecanique 342 (2014) 258–262

Contents lists available at ScienceDirect

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An asymptotic model of a multimaterial with a thin piezoeletric interphase

Un modèle asymptotique d'un multimatériau avec une interphase piézoélectrique mince

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A R T I C L E I N F O

Article history:
Received 19 December 2013
Accepted after revision 7 February 2014
Available online 18 March 2014

Keywords:
Asymptotic analysis
Piezoelectric materials
Interfaces

Mots-clés :
Analyse asymptotique
Matériaux piézoélectriques
Interfaces

A B S T R A C T

We study the electromechanical behavior of a multimaterial constituted by a linear piezoelectric transversely isotropic plate-like body with high rigidity embedded between two generic three-dimensional piezoelectric bodies by means of the asymptotic expansion method. After defining a small real dimensionless parameter ε, which will tend to zero, we characterize the limit model and the associated limit problem. Moreover, we identify the non-classical electromechanical transmission conditions between the two three-dimensional bodies.

R É S U M É

On étudie le comportement électromécanique d'un multimatériau constitué d'un corps de type plaque piézoélectrique isotrope transverse de grande rigidité inséré entre deux corps 3D piézoélectriques génériques à travers la méthode des développements asymptotiques. Après avoir défini un petit paramètre adimensionnel ε qui tend vers zéro, on caractérise le modèle limite et son problème limite associé. De plus, on identifie les conditions de transmission éléctroméchaniques non classiques entre les deux corps 3D.

Version française abrégée

Depuis quelques années, les structures qui utilisent les matériaux intelligents, comme les piézoélectriques, avec des fonctions d'actionneur et capteur, ont acquis une importance considérable. Les matériaux piézoélectriques jouent un rôle de plus en plus important dans les nouvelles technologies, grâce à leur capacité à convertir l'énergie mécanique en énergie électrique et vice-versa. Les structures intelligentes sont conçues pour identifier, réagir et s'adapter aux changements de l'environnement de travail. Les matériaux piézoélectriques peuvent être intégrés directement dans la structure même, sous

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http://dx.doi.org/10.1016/j.crme.2014.02.003
1631-0721/

la forme de plaques piézoélectriques, en créant des assemblages complexes constitués d'une alternance de couches minces ayant des propriétés électromécaniques très différentes.

L'étude d'une couche mince élastique insérée entre deux matériaux élastiques et possédant des propriétés d'un ordre de grandeur différent de celles de ces deux matériaux s'est largement développée à la suite des travaux de Bessoud et al. [1], Chapelle–Ferent [3] et Lebon–Rizzoni [7]. Les méthodes asymptotiques ont été aussi appliquées dans le cas piézoélectrique d'une série de conducteurs minces immergés dans une matrice piézoélectrique (voir [6]). Dans le présent travail, on considère le problème du comportement électromécanique dans le cas statique d'un corps de type plaque mince piézoélectrique inseré entre deux corps tridimensionnels piézoélectriques génériques.

Plus précisément, dans l'espace euclidien tridimensionnel identifié par $\mathbb{R}^3$ muni du repère cartésien $\mathbf{e}_i$, soient Ω^+ et Ω^- deux ouverts disjoints à bords réguliers $\partial\Omega^+$ et $\partial\Omega^-$. Soit $\omega := \{\partial\Omega^+ \cap \partial\Omega^-\}^\circ$ un domaine de $\mathbb{R}^2$ de mesure bidimensionnelle non nulle et soit $\tilde{x} = (x_\alpha)$ un point générique de ω. La couche intermédiaire est insérée en déplaçant Ω^+ (respectivement Ω^-) suivant $\mathbf{e}_3$ (respectivement $-\mathbf{e}_3$), d'une quantité $\varepsilon h > 0$, où ε est un petit paramètre réel positif. Soient $\Omega^{\pm,\varepsilon} := \{x^\varepsilon := x \pm \varepsilon h \mathbf{e}_3;\ x \in \Omega^\pm\}$, $\Omega^{m,\varepsilon} := \omega \times (-\varepsilon h, \varepsilon h)$ et $\Omega^\varepsilon := \Omega^{+,\varepsilon} \cup \Omega^{m,\varepsilon} \cup \Omega^{-,\varepsilon}$. De plus, on définit avec $S^{\pm,\varepsilon} := \omega \times \{\pm\varepsilon h\} = \Omega^{\pm,\varepsilon} \cap \Omega^{m,\varepsilon}$, les faces supérieure et inférieure du domaine de type plaque, $\Gamma^{\pm,\varepsilon} := \partial\Omega^{\pm,\varepsilon}/S^{\pm,\varepsilon}$, et $\Gamma^{m,\varepsilon}_{lat} := \partial\omega \times (-\varepsilon h, \varepsilon h)$, sa surface latérale. Soient $(\Gamma^\varepsilon_{mD}, \Gamma^\varepsilon_{mN})$ et $(\Gamma^\varepsilon_{eD}, \Gamma^\varepsilon_{eN})$ deux partitions de $\partial\Omega^\varepsilon := \Gamma^{\pm,\varepsilon} \cup \Gamma^{m,\varepsilon}_{lat}$, avec Γ^ε_{mD} et Γ^ε_{eD} de mesure de Lebesgue strictement positive. Le multimatériau est fixé sur Γ^ε_{mD} et soumis à un potentiel électrique connu $\varphi^\varepsilon = \hat{\varphi}^\varepsilon$ sur Γ^ε_{eD}. On applique des forces de surface g^ε_i sur Γ^ε_{mN} et une densité de charges électriques d^ε sur Γ^ε_{eN}. L'assemblage est soumis à des forces de volume f^ε_i et à des forces électriques F^ε qui agissent dans $\Omega^{\pm,\varepsilon}$. On suppose ainsi, sans perdre de généralité, que $\Omega^{m,\varepsilon}$ et $\Gamma^{m,\varepsilon}_{\text{lat}}$ ne sont pas chargés. Les corps $\Omega^{\pm,\varepsilon}$ sont constitués d'un matériau piézoélectrique générique, alors que $\Omega^{m,\varepsilon}$ est constitué d'un matériau piézoélectrique isotrope transverse par rapport à $\mathbf{e}_3$. Le problème physique sur le domaine variable Ω^ε s'écrit sous la forme variationnelle (1).

Afin d'étudier le problème limite, on transforme l'ouvert variable en un ouvert fixe, grâce au changement de variables usuel (voir [4]). Le problème variationnel mis à l'échelle présente une structure polynomiale par rapport à ε ; on peut donc supposer le développement asymptotique de la solution suivant : $\mathbf{u}(\varepsilon) = \mathbf{u}^0 + \varepsilon^2\mathbf{u}^2 + \varepsilon^4\mathbf{u}^4 + \cdots$ et $\varphi(\varepsilon) = \varphi^0 + \varepsilon^2\varphi^2 + \varepsilon^4\varphi^4 + \cdots$. Les termes principaux, $\mathbf{u}^0$ et φ^0 dans Ω^m, des développements asymptotiques vérifient les relations : $u^{m,0}_\alpha(\tilde{x}, x_3) = \bar{u}^0_\alpha(\tilde{x}) - x_3\partial_\alpha w(\tilde{x})$, $u^{m,0}_3 = w(\tilde{x})$, $\varphi^{m,0}(\tilde{x}, x_3) = \sum_{k=0}^2 \phi^k(\tilde{x})x_3^k$. Ceci implique que le champ de déplacement limite dans la couche de type plaque satisfait les hypothèses cinématiques de Kirchhoff–Love et que le potentiel électrique limite est une fonction quadratique de x_3 dépendant des valeurs des potentiels électriques à l'interface avec les corps Ω^+ et Ω^-. L'état électromécanique limite défini par $\mathbf{u}^0$ et φ^0 vérifie le problème (2). L'énergie de surface associée à cette formulation s'écrit sous la forme :

$$\mathcal{J}(\bar{\mathbf{u}}^0, w^0, \varphi^0) = h\int_\omega \left\{2\mu e_{\alpha\beta}(\bar{\mathbf{u}}^0)e_{\alpha\beta}(\bar{\mathbf{u}}^0) + Be_{\sigma\sigma}(\bar{\mathbf{u}}^0)e_{\tau\tau}(\bar{\mathbf{u}}^0) + \frac{D}{4h^2}[\![\varphi^0]\!]^2\right\} \mathrm{d}\tilde{x} + \frac{h^3}{3}\int_\omega \left\{2\mu\partial_{\alpha\beta}w^0\partial_{\alpha\beta}w^0 + \left(\frac{BD + C^2}{D}\right)\Delta w^0 \Delta w^0\right\} \mathrm{d}\tilde{x}$$

qui représente l'énergie d'interface bidimensionnelle définie sur le plan moyen de l'inclusion ω ; elle est associée à une énergie de surface d'une plaque de Kirchhoff–Love piézoélectrique.

Grâce à la méthode des développements asymptotiques formels, on remplace l'énergie piézoélectrique de la couche intermédiaire de type plaque par une énergie de surface particulière, qui génère un problème de transmission non classique entre les corps Ω^+ et Ω^-. Les conditions de transmission obtenues (3) se distinguent en conditions de transmission mécaniques et conditions de transmission électriques : les premières peuvent être considérées comme une généralisation piézoélectrique des conditions obtenues dans [1] ; les secondes sont des conditions non homogènes de type Robin, typiques dans les problèmes électromagnétiques.

1. Introduction

The technology of smart structures provides a new degree of design flexibility for advanced composite structural members. The key to the technology is the ability to allow the structure to sense and react in a desired fashion, improving its performances for what concerns structural vibrations, acoustic signature, and aerodynamic stability. The new concept of adaptive structure requires, for instance, the use of piezoelectric sensors and actuators for controlling the mechanical behavior of structural systems. Piezoelectric materials may be integrated into a host structure to change its shape and to enhance its mechanical properties with different configurations: for instance, a piezoelectric transducer can be embedded into the structure to be controlled or it can be glued on it, as in the case of piezo-patches. Moreover, the same piezoelectric actuators are often obtained by alternating different thin layers of material with highly contrasted electromechanical properties. This generates different types of complex multimaterial assemblies, in which each phase interacts with the others. The successful application of the asymptotic methods to obtain a mathematical justification of the most used models of plates in elasticity [4] and piezoelectricity [8,9] has stimulated the research toward a rational simplification of the modeling of complex structures obtained joining elements of different dimensions and/or materials of highly contrasted properties.

The direct solution to such problems by a standard finite element method is too expensive from a computational point of view and the presence of strong contrasts in the geometry and mechanical properties causes numerical instabilities. That is why specific asymptotic expansions are used and allow one to replace the original problem by a set of problems in which the thin layer, for instance, is substituted by a two-dimensional surface. The thin inclusion of a third material between two other ones when the rigidity properties of the inclusion are highly contrasted with respect to those of the surrounding materials has been deeply investigated in the case of linear elasticity, see [1,3,7] and, also, in the case of thin conductor plates embedded into a piezoelectric matrix [5].

In this work, we consider a particular piezoelectric multimaterial, constituted by two generic three-dimensional piezoelectric bodies separated by a thin piezoelectric transversely isotropic plate-like body with high rigidity. By defining a small real parameter ε, associated with the thickness and the electromechanical properties of the middle layer, we perform an asymptotic analysis by letting ε tend to zero, following the approach by P.G. Ciarlet [4]. Then we characterize the limit model and its associated limit problem. In the simplified model, the intermediate plate-like body "disappears", and it is replaced by a specific electromechanical surface energy defined over the middle plane of the plate. This surface energy then results in ad hoc transmission conditions between the two piezoelectric bodies in terms of the jump of stresses, electric displacements, and electric potentials.

2. Piezoelectric interphase: asymptotic behavior

Let us consider a three-dimensional Euclidean space identified by $\mathbb{R}^3$ and such that the three vectors $\mathbf{e}_i$ form an orthonormal basis. Let Ω^+ and Ω^- be two disjoint open domains with smooth boundaries $\partial\Omega^+$ and $\partial\Omega^-$. Let $\omega := \{\partial\Omega^+ \cap \partial\Omega^-\}^\circ$ be the interior of the common part of the boundaries, which is assumed to be a non-empty domain in $\mathbb{R}^2$ having a positive two-dimensional measure. We consider the assembly constituted by two solids bonded together by an intermediate thin plate-like body $\Omega^{m,\varepsilon}$ of thickness $2h^\varepsilon$, where $0 < \varepsilon < 1$ is a dimensionless small real parameter which will tend to zero. We suppose that the thickness h^ε of the middle layer depends linearly on ε, so that $h^\varepsilon = \varepsilon h$. More precisely, we denote respectively with $\Omega^{\pm,\varepsilon} := \{x^\varepsilon := x \pm \varepsilon h\mathbf{e}_3;\ \ x \in \Omega^\pm\}$, the translation of Ω^+ (resp. Ω^-) along the direction $\mathbf{e}_3$ (resp. $-\mathbf{e}_3$) of the quantity εh, with $\Omega^{m,\varepsilon} := \omega \times (-\varepsilon h, \varepsilon h)$, the intermediate plate-like domain, and with $\Omega^\varepsilon := \Omega^{+,\varepsilon} \cup \Omega^{m,\varepsilon} \cup \Omega^{-,\varepsilon}$, the reference configuration of the assembly. Moreover, we define with $S^{\pm,\varepsilon} := \omega \times \{\pm\varepsilon h\} = \Omega^{\pm,\varepsilon} \cap \Omega^{m,\varepsilon}$ the upper and lower faces of the intermediate plate-like domain, $\Gamma^{\pm,\varepsilon} := \partial\Omega^{\pm,\varepsilon}/S^{\pm,\varepsilon}$, and $\Gamma^{m,\varepsilon}_{\text{lat}} := \partial\omega \times (-\varepsilon h, \varepsilon h)$, its lateral surface. Let $(\Gamma^\varepsilon_{mD}, \Gamma^\varepsilon_{mN})$ and $(\Gamma^\varepsilon_{eD}, \Gamma^\varepsilon_{eN})$ be two suitable partitions of $\partial\Omega^\varepsilon := \Gamma^{\pm,\varepsilon} \cup \Gamma^{m,\varepsilon}_{\text{lat}}$, with both Γ^ε_{mD} and Γ^ε_{eD} of strictly positive Lebesgue measure. The multimaterial is, on one hand, clamped along Γ^ε_{mD} and at an electric potential $\varphi^\varepsilon = \hat{\varphi}^\varepsilon$ on Γ^ε_{eD} and, on the other hand, subjected to surface forces g^ε_i on Γ^ε_{mN} and to a surface density of electric charges d^ε on Γ^ε_{eN}. The assembly is also subjected to body forces f^ε_i and electrical loadings F^ε acting in $\Omega^{\pm,\varepsilon}$. We suppose, without loss of generality, that $\Omega^{m,\varepsilon}$ and $\Gamma^{m,\varepsilon}_{\text{lat}}$ are both free of mechanical and electrical charges. We finally assume that $\Omega^{\pm,\varepsilon}$ are constituted by two homogeneous linearly piezoelectric materials, while $\Omega^{m,\varepsilon}$ is made by a homogeneous linearly piezoelectric material, transversely isotropic with respect to $\mathbf{e}_3$.

The electromechanical state at the equilibrium is determined by the pair $s^\varepsilon := (\mathbf{u}^\varepsilon, \varphi^\varepsilon)$. The physical variational problem defined over the variable domain Ω^ε reads as follows:

$$\begin{cases} \text{Find } s^\varepsilon \in V^\varepsilon(\Omega^\varepsilon) \text{ such that} \\ A^{-,\varepsilon}(s^\varepsilon, r^\varepsilon) + A^{+,\varepsilon}(s^\varepsilon, r^\varepsilon) + A^{m,\varepsilon}(s^\varepsilon, r^\varepsilon) = L^\varepsilon(r^\varepsilon) \quad \text{for all } r^\varepsilon \in V^\varepsilon(\Omega^\varepsilon) \end{cases} \tag{1}$$

where $V^\varepsilon(\Omega^\varepsilon) := \{r^\varepsilon := (\mathbf{v}^\varepsilon, \psi^\varepsilon) \in H^1(\Omega^\varepsilon; \mathbb{R}^3) \times H^1(\Omega^\varepsilon);\ \mathbf{v}^\varepsilon = \mathbf{0} \text{ on } \Gamma^\varepsilon_{mD},\ \psi^\varepsilon = \hat{\varphi}^\varepsilon \text{ on } \Gamma^\varepsilon_{eD}\}$. The bilinear forms $A^{\pm,\varepsilon}$ and $A^{m,\varepsilon}$ are defined by

$$A^{\pm,m,\varepsilon}(s^\varepsilon, r^\varepsilon) := \int_{\Omega^{\pm,m,\varepsilon}} \{A^{\pm,m,\varepsilon}_{ijk\ell} e^\varepsilon_{k\ell}(\mathbf{u}^\varepsilon) e^\varepsilon_{ij}(\mathbf{v}^\varepsilon) + H^{\pm,m,\varepsilon}_{ij} E^\varepsilon_j(\varphi^\varepsilon) E^\varepsilon_i(\psi^\varepsilon) + P^{\pm,m,\varepsilon}_{ihk}\big(E^\varepsilon_i(\psi^\varepsilon) e^\varepsilon_{hk}(\mathbf{u}^\varepsilon) - E^\varepsilon_i(\varphi^\varepsilon) e^\varepsilon_{hk}(\mathbf{v}^\varepsilon)\big)\}\, \mathrm{d}x^\varepsilon$$

The functional $L(\cdot)$ is the linear application associated with the applied electromechanical charges.

Let us suppose that the components of the elasticity tensor, dielectric tensor and coupling tensor of the intermediate plate-like body $\Omega^{m,\varepsilon}$ satisfy, respectively, $A^{m,\varepsilon}_{ijk\ell} := \frac{1}{\varepsilon} A^m_{ijk\ell}$, $H^{m,\varepsilon}_{ij} := \frac{1}{\varepsilon} H^m_{ij}$, $P^{m,\varepsilon}_{ijk} := \frac{1}{\varepsilon} P^m_{ijk}$, while the elastic and electric constants of $\Omega^{\pm,\varepsilon}$ are independent of ε.

In order to study the asymptotic behavior of the solution of problem (1) when ε tends to zero, we apply the usual change of variables (see [4]), which transforms the problem posed on a variable domain Ω^ε onto a problem over fixed domain $\Omega := \Omega^\pm \cup \Omega^m$, where $\Omega^m := \omega \times (-h, h)$. The rescaled problem takes the following form, with $V(\Omega) := \{r := (\mathbf{v}, \psi) \in H^1(\Omega; \mathbb{R}^3) \times H^1(\Omega);\ \mathbf{v} = \mathbf{0} \text{ on } \Gamma_{mD},\ \psi = \hat{\varphi} \text{ on } \Gamma_{eD}\}$:

$$\begin{cases} \text{Find } s(\varepsilon) \in V(\Omega) \text{ such that} \\ A^-(s(\varepsilon), r) + A^+(s(\varepsilon), r) + A^m(s(\varepsilon), r) = L(r) \quad \text{for all } r \in V(\Omega) \end{cases}$$

where $A^m(s(\varepsilon), r) := \frac{1}{\varepsilon^4} a^m_{-4}(s(\varepsilon), r) + \frac{1}{\varepsilon^2} a^m_{-2}(s(\varepsilon), r) + a^m_0(s(\varepsilon), r) + \varepsilon^2 a^m_2(s(\varepsilon), r)$, with

$$a^m_{-4}\big(s(\varepsilon),r\big) := \int_{\Omega^m} \tau_1 e_{33}\big(\mathbf{u}(\varepsilon)\big)e_{33}(\mathbf{v})\,\mathrm{d}x$$

$$a^m_{-2}\big(s(\varepsilon),r\big) := \int_{\Omega^m} \big(\tau_2 e_{33}\big(\mathbf{u}(\varepsilon)\big)e_{\sigma\sigma}(\mathbf{v}) + \big(\tau_2 e_{\sigma\sigma}\big(\mathbf{u}(\varepsilon)\big) + \delta_3\partial_3\varphi(\varepsilon)\big)e_{33}(\mathbf{v}) + 2\eta e_{\alpha 3}\big(\mathbf{u}(\varepsilon)\big)e_{\alpha 3}(\mathbf{v}) - \delta_3 e_{33}\big(\mathbf{u}(\varepsilon)\big)\partial_3\psi\big)\,\mathrm{d}x$$

$$a^m_{0}\big(s(\varepsilon),r\big) := \int_{\Omega^m} \big(2\mu e_{\alpha\beta}\big(\mathbf{u}(\varepsilon)\big)e_{\alpha\beta}(\mathbf{v}) + \big(\lambda e_{\sigma\sigma}\big(\mathbf{u}(\varepsilon)\big) + \delta_1\partial_3\varphi(\varepsilon)\big)e_{\tau\tau}(\mathbf{v}) + \delta_2\partial_\alpha\varphi(\varepsilon)e_{\alpha 3}(\mathbf{v}) - 2\delta_2 e_{\alpha 3}\big(\mathbf{u}(\varepsilon)\big)\partial_\alpha\psi + \big(\gamma_2\partial_3\varphi(\varepsilon) - \delta_1 e_{\sigma\sigma}\big(\mathbf{u}(\varepsilon)\big)\big)\partial_3\psi\big)\,\mathrm{d}x$$

$$a^m_{2}\big(s(\varepsilon),r\big) := \int_{\Omega^m} \gamma_1\partial_\alpha\varphi(\varepsilon)\partial_\alpha\psi\,\mathrm{d}x$$

In the sequel, only if necessary, we will note, respectively, with $\mathbf{v}^\pm$ and $\mathbf{v}^m$ the restrictions of functions $\mathbf{v}$ to $\Omega^\pm$ and Ω^m.

We look for the following asymptotic expansion of the unknown $s(\varepsilon) = s^0 + \varepsilon^2 s^2 + \varepsilon^4 s^4 + \cdots$. Hence, $\mathbf{u}(\varepsilon) = \mathbf{u}^0 + \varepsilon^2\mathbf{u}^2 + \varepsilon^4\mathbf{u}^4 + \cdots$ and $\varphi(\varepsilon) = \varphi^0 + \varepsilon^2\varphi^2 + \varepsilon^4\varphi^4 + \cdots$. The leading term $s^0 = (\mathbf{u}^0, \varphi^0)$ of the asymptotic expansion in Ω^m satisfies the following relations:

$$\begin{cases} u^{m,0}_\alpha(\tilde{x}, x_3) = \bar{u}^0_\alpha(\tilde{x}) - x_3\partial_3 w(\tilde{x}), \quad u^{m,0}_3 = w(\tilde{x}) \\ \varphi^{m,0}(\tilde{x}, x_3) = \sum_{k=0}^2 \phi^k(\tilde{x})x_3^k \end{cases}$$

where

$$\phi^0 = \frac{\varphi^{+,0} + \varphi^{-,0}}{2} + \frac{Ch^2}{2D}\Delta\bar{u}^0_3, \qquad \phi^1 = \frac{\varphi^{+,0} - \varphi^{-,0}}{2h}, \qquad \phi^2 = -\frac{C}{2D}\Delta\bar{u}^0_3$$

The limit displacement field $\mathbf{u}^{m,0}$ belongs to $V_{KL} := \{\mathbf{v} \in H^1(\Omega^m; \mathbb{R}^3);\ e_{i3}(\mathbf{v}) = 0\}$ and it verifies the Kirchhoff–Love kinematical assumptions. The limit electric potential $\varphi^{m,0}$ is a second-order polynomial function of x_3 depending on the values of $\varphi^{\pm,0} := \varphi^0|_{S^\pm} = \varphi^0(\tilde{x}, \pm h)$ which denote the restrictions of φ^0 on the s between Ω^+ and Ω^-; it belongs to the space $\Psi := \{\psi \in L^2(\Omega^m);\ \partial_3\psi \in L^2(\Omega^m)\} = H^1(-h, h; L^2(\omega))$.

The previous characterization of the electric potential $\varphi^{m,0}$ is a rigorous justification of the a priori assumptions conjectured by M. Bernadou and C. Haenel in [2]. We can also notice that the regularity of the electric potential only depends on the regularities of $\varphi^{\pm,0}$ and of $\bar{u}^0_3$.

The limit electromechanical state $s^0 = (\mathbf{u}^0, \varphi^0)$ is the solution of the following limit problem:

$$\begin{cases} \text{Find } s^0 \in \mathcal{V} \text{ such that} \\ A^-\big(s^0, r\big) + A^+\big(s^0, r\big) + A^m_{KL}\big(s^0, r\big) = L(r) \quad \text{for all } r \in \mathcal{V} \end{cases} \tag{2}$$

where $\mathcal{V} := \{r := (\mathbf{v}, \psi) \in H^1(\Omega/\Omega^m; \mathbb{R}^3) \times H^1(\Omega/\Omega^m);\ r^m \in V_{KL} \times \Psi,\ \mathbf{v} = \mathbf{0} \text{ on } \Gamma_{mD},\ \psi = \hat{\varphi} \text{ on } \Gamma_{eD}\}$, and

$$A^m_{KL}\big(s^0, r\big) := 2h\int_\omega \left\{2\mu e_{\alpha\beta}\big(\bar{\mathbf{u}}^0\big)e_{\alpha\beta}(\bar{\mathbf{v}}) + \left(Be_{\sigma\sigma}\big(\bar{\mathbf{u}}^0\big) + C\frac{[\![\varphi^0]\!]}{2h}\right)e_{\tau\tau}(\bar{\mathbf{v}}) + \left(-Ce_{\sigma\sigma}\big(\bar{\mathbf{u}}^0\big) + D\frac{[\![\varphi^0]\!]}{2h}\right)\frac{[\![\psi]\!]}{2h}\right\}\mathrm{d}\tilde{x} + \frac{2h^3}{3}\int_\omega \left\{2\mu\partial_{\alpha\beta}w^0\partial_{\alpha\beta}v_3 + \left(\frac{BD + C^2}{D}\right)\Delta w^0\Delta v_3\right\}\mathrm{d}\tilde{x}$$

is the bilinear form associated with the Kirchhoff–Love transversely isotropic piezoelectric plate energy defined over the middle plane ω of the plate. $]\!]f[\![:= f^+ - f^-$ denotes the jump function at the interface ω between Ω^+ and Ω^-.

3. The electromechanical interface problem

The aim of this section is to derive a coupled electromechanical interface problem between the two piezoelectric bodies Ω^+ and Ω^- with some ad hoc transmission conditions at the interface ω. By virtue of the asymptotic methods, we replace the three-dimensional electromechanical energy of the intermediate piezoelectric layer with a specific two-dimensional surface energy defined over the middle plane of the plate. This surface energy generates non-classical transmission conditions between the two three-dimensional bodies. By rewriting problem (2) in its differential form after an integration by parts, we obtain:

electrostatic problems in $\Omega^\pm$

$$\begin{cases} \partial_i D_i^\pm(\mathbf{u}^0,\varphi^0) = F & \text{in } \Omega^\pm \\ D_i^\pm(\mathbf{u}^0,\varphi^0) n_i = d & \text{on } \Gamma_{eN} \\ \varphi^0 = \hat{\varphi} & \text{on } \Gamma_{eD} \end{cases}$$

elasticity problems in $\Omega^\pm$

$$\begin{cases} -\partial_j \sigma_{ij}^\pm(\mathbf{u}^0,\varphi^0) = f_i & \text{in } \Omega^\pm \\ \sigma_{ij}^\pm(\mathbf{u}^0,\varphi^0) n_j = g_i & \text{on } \Gamma_{mN} \\ \mathbf{u}^0 = \mathbf{0} & \text{on } \Gamma_{mD} \end{cases}$$

where $D_i^\pm(\mathbf{u}^0,\varphi^0) = P_{ijk}^\pm e_{jk}(\mathbf{u}^0) + H_{ij}^\pm E_j(\varphi^0)$ and $\sigma_{ij}^\pm(\mathbf{u}^0,\varphi^0) := A_{ijk\ell}^\pm e_{k\ell}(\mathbf{u}^0) - P_{ijk}^\pm E_k(\varphi^0)$ denote, respectively, the electric displacement field and the Cauchy stress tensor on $\Omega^\pm$;

transmission conditions on ω

$$\begin{cases} [\![\sigma_{\alpha 3}]\!] - C\,[\![\partial_\alpha \varphi^0]\!] = \partial_\beta n_{\alpha\beta}(\boldsymbol{\eta}^0) & \text{on } \omega \\ [\![\sigma_{33}]\!] = \partial_{\alpha\beta} m_{\alpha\beta}(\boldsymbol{\eta}^0) & \text{on } \omega \\ [\![D_3(\mathbf{u}^0,\varphi^0)]\!] = 0 & \text{on } \omega \\ [\![\mathbf{u}^0]\!] = \mathbf{0} & \text{on } \omega \end{cases} \tag{3}$$

where $\boldsymbol{\eta}^0 := \mathbf{u}^0|_\omega$ is the restriction of $\mathbf{u}^0$ on ω, $n_{\alpha\beta}(\boldsymbol{\eta}^0) := 2h(2\mu e_{\alpha\beta}(\boldsymbol{\eta}^0) + B e_{\sigma\sigma}(\boldsymbol{\eta}^0)\delta_{\alpha\beta})$ and $m_{\alpha\beta}(\boldsymbol{\eta}^0) := -\frac{2h^3}{3}(2\mu\partial_{\alpha\beta}\eta_3^0 + (C-B)\Delta\eta_3^0\delta_{\alpha\beta})$ represent, respectively, the components of the membrane stress tensor and the components of the moment tensor.

Remark. The previous electromechanical interface problem can be considered as a generalization in the case of piezoelectric assemblies of the transmission problem obtained in [1] for thin elastic inclusions with high rigidity. The particular jump conditions at the interface yield to a non-standard transmission problem which can be solved by an adapted Neumann–Neumann domain decomposition algorithm [6].

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