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Disturbance rejection-based robust control for micropositioning of piezoelectric actuators

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ABSTRACT

Precise control of piezoelectric actuators used in micropositioning applications is strongly under the effect of internal and external disturbances. Undesired external forces, unmodelled dynamics, parameter uncertainties, time variation of parameters and hysteresis are some sources of disturbances. These effects not only degrade the performance efficiency, but also may lead to closed-loop instability. Several works have investigated the positioning accuracy for constant and slow time-varying disturbances. The main concern is controlling performance and also the presence of time-varying perturbations. Considering unknown source and magnitude of disturbances, the estimation of the existing disturbances would be inevitable. In this paper, a compound disturbance observer-based robust control is developed to achieve precise positioning in the presence of time-varying disturbances. In addition, a modified disturbance observer is proposed to remedy the effect of switching behaviour in the case of slow time variations. A modified Prandtl–Ishlinskii (PI) operator and its inverse are utilized for both identification and real-time compensation of the hysteresis effect. Experimental results depict that the proposed approach achieves precise micropositioning in the presence of estimated disturbances.

1. Introduction

Internal and external disturbances are challenging issues in the control of dynamic systems, especially in micromanipulation applications. Disturbances include joint frictions, actuator perturbations, unmodelled dynamics, parameter uncertainties, undesired external forces and nonlinear behaviours such as hysteresis. As a result, the performance can strongly be degraded and may even lead to closed-loop instability. This problem is of serious concern in piezoelectric actuators, in particular in micromanipulation processes.

To deal with this issue, several robust control approaches such as sliding mode, adaptive and observer-based control have been presented for piezoelectric actuators [1–3]. However it has been assumed that the system deals with a known bounded disturbance which should be small enough. The assumptions are conservative for mechanical systems, especially when disturbance includes dynamic uncertainties. To eliminate the mentioned deficiencies, online disturbance estimation

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and feedforward compensation have been utilized as an alternative solution [4–7]. In such a case, the necessity of known boundedness assumption can be ignored.

Two control approaches have been generally considered, i.e. *Adaptive Perturbation Estimation Based Control* and *Disturbance Observer-Based Control* (*DOBC*). In the first approach, the perturbation could be adaptively estimated and compensated based on the closed-loop positioning error [1]. An adaptive perturbation estimation based on computed torque control for nonlinear systems has been presented in [4]. The proposed method guarantees asymptotic convergence for only slow varying disturbances. In addition, the convergence of perturbation to the real value could not be generally achieved. It would be achievable in special cases such as persistent excitation. In another works of the authors, a robust control combined with adaptive perturbation estimation has been developed for micropositioning actuators [5]. The major drawback has been the assumption of the separation of external force and perturbation. This issue is one of the points to be considered in this paper.

In the second approach, a disturbance observer coupled with different control approaches has been proposed [6–12]. In this method, the controller includes two separate parts: first, a conventional controller under the assumption that there is no disturbance or the disturbance is exactly known; second, the disturbance observer for the disturbance feedforward compensation [6,7]. Disturbance observers have many advantages as increasing the robustness of the closed-loop system [6]. It can be also utilized as an external force estimator in sensorless manipulations [8].

Several DOBC approaches have been proposed based on the linear system theories [9,10]. But the methods could not be effectively implemented for nonlinear systems. As a result, nonlinear disturbance observer-based control (NDOBC) has been of interest in the last decade. Robust motion control and output control based on a disturbance observer have been proposed for nonlinear systems [11,12]. The proposed observer does not have any analytical convergence. In addition, the low-frequency performance causes bounded tracking error in high-frequency applications. One of the most popular nonlinear disturbance observers with the analytical convergence has been introduced by Chen et al. for a two-link revolute manipulator [13]. The observer has also been improved for the general revolute manipulators [14]. Mohammadi et al. [15] has recently revised the observer for a general structure of manipulators. In this structure, the asymptotic convergence for slow time-varying disturbances has been deduced. In the case of fast time-varying disturbances, the convergence error would be ultimately bounded.

Based on the aforementioned disturbance observer, different approaches such as sliding mode and computed torque controls have been proposed for uncertain nonlinear systems [16,17]. A modified disturbance observer-based feedback linearization has also been investigated for a teleoperation system [18]. In all the mentioned methods, the perfect convergence is derived for slow time-varying and constant disturbances. But the major challenge is the case of time-varying disturbances. The controller performance for time-varying disturbances has been investigated for special disturbances such as linear exogenous systems [19]. An integral based sliding mode control has also been proposed for time-varying disturbances [20]. Conservative assumptions such as boundedness of both first and second time derivatives of disturbances have been taken in this case. However, improper control gains may make a limit cycle in the closed-loop system.

In this paper, a compound disturbance observer-based robust control has been designed for time-varying disturbances. The necessity condition for the asymptotic convergence is boundedness of the disturbance variation. This point has been a common assumption in dynamic systems [7,15,20]. Based on the approach used by other researchers, a modified observer has also been developed to eliminate the switching behaviour of the controller for slow time-varying disturbances [18]. The proposed approach has been implemented for a precise micropositioning piezoelectric actuator. Therefore, a generalized Prandtl–Ishlinskii model and its inverse are utilized for the identification and online feedforward compensation of hysteresis. This leads to the elimination of the actuator's hysteretic behaviour. PI estimation errors, probably unmodelled dynamics and undesired external forces are the major parts of the disturbances. The experimental results demonstrate that the PI model has achieved hysteresis identification and compensation. In addition, the proposed controller tracks the desired trajectories precisely in the presence of disturbances.

2. General nonlinear dynamic modelling and control design

In this section, the control approaches will be designed for a general nth-order nonlinear dynamic system. Absolutely, the proposed approaches can be also utilized for linear dynamic systems. A general nonlinear dynamic system is represented as follows:

$$\boldsymbol{X}^{(n)} = \boldsymbol{F}\big(\boldsymbol{X}, \dot{\boldsymbol{X}}, \ldots, \boldsymbol{X}^{(n-1)}\big) + \boldsymbol{B}\big(\boldsymbol{X}, \dot{\boldsymbol{X}}, \ldots, \boldsymbol{X}^{(n-1)}\big)\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{d}_{\text{ext}}(\boldsymbol{t}) \tag{1}$$

where $\boldsymbol{X} = [x_1, x_2, \ldots, x_m]$ is the vector of m generalized coordinates. $\boldsymbol{X}^{(i)} = [x_1^{(i)}, x_2^{(i)}, \ldots, x_m^{(i)}]$ is the ith time derivative of the generalized coordinates. $\boldsymbol{u}(\boldsymbol{t})$ is the input and $\boldsymbol{d}_{\text{ext}}$ includes external disturbances such as joint frictions, actuator perturbations and undesired external forces.

Due to parametric uncertainties, unmodelled dynamics and identification errors, the dynamic is described as:

$$\begin{aligned}\boldsymbol{X}^{(n)} &= \boldsymbol{f} + \tilde{\boldsymbol{f}} + (\boldsymbol{b} + \tilde{\boldsymbol{b}})\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{d}_{\text{ext}}(\boldsymbol{t}) \\ &= \boldsymbol{f} + \boldsymbol{b}\boldsymbol{u}(\boldsymbol{t}) + \tilde{\boldsymbol{f}} + \tilde{\boldsymbol{b}}\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{d}_{\text{ext}}(\boldsymbol{t})\end{aligned} \tag{2}$$

$\boldsymbol{f}, \boldsymbol{b}$ are the known parts and $\tilde{\boldsymbol{f}}, \tilde{\boldsymbol{b}}$ are the unknown parts of $\boldsymbol{F}$ and $\boldsymbol{B}$, respectively [4]. The unknown part can be considered as the internal disturbances $\boldsymbol{d}_{\text{int}}(\boldsymbol{t}) = \tilde{\boldsymbol{f}} + \tilde{\boldsymbol{b}}\boldsymbol{u}(\boldsymbol{t})$. With regards to dynamic model (2), total uncertainties could be represented as a disturbance term $\boldsymbol{d}(\boldsymbol{t})$ where $\boldsymbol{d}(\boldsymbol{t}) = \boldsymbol{d}_{\text{int}}(\boldsymbol{t}) + \boldsymbol{d}_{\text{ext}}(\boldsymbol{t})$. As a result, the dynamic model of the system would be simplified to Eq. (3).

$$\boldsymbol{X}^{(n)} = \boldsymbol{f} + \boldsymbol{b}\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{d}(\boldsymbol{t}) \tag{3}$$

The objective is the accurate trajectory tracking of generalized coordinates regarding the existence of all disturbances. Designing a robust control structure could be a solution for this problem. In this case, the boundedness of $\boldsymbol{d}(\boldsymbol{t})$ is the necessary condition. Considering the unknown parts of dynamic matrices, the boundedness of $\boldsymbol{d}(\boldsymbol{t})$ is a wrong assumption. Therefore, the disturbance estimation can be an alternative solution. The controller includes two separate steps. First, a robust controller will be designed with the assumption that no disturbance exists or it is completely known. Second, an observer will be proposed for the estimation and feedforward compensation of the disturbance. Finally, the asymptotic stability of the closed-loop system should be investigated.

To design a robust control structure, the sliding surface $\boldsymbol{S}$ is represented as:

$$\begin{aligned} &\boldsymbol{S} = \boldsymbol{e}^{(n-1)} + \boldsymbol{K}_{(\boldsymbol{n}-2)}\boldsymbol{e}^{(n-2)} + \cdots + \boldsymbol{K}_1\dot{\boldsymbol{e}} + \boldsymbol{e} \\ &\boldsymbol{e} = \boldsymbol{X} - \boldsymbol{X}_{\text{d}} \\ &\boldsymbol{e}^{(i)} = \boldsymbol{X}^{(i)} - \boldsymbol{X}_{\text{d}}^{(i)} \end{aligned} \tag{4}$$

where $\boldsymbol{X}_{\mathbf{d}}$ is defined as the desired trajectory vector and $\boldsymbol{K}_{(\boldsymbol{i})}$ is an $n \times n$ diagonal positive definite constant matrix so that $\boldsymbol{S}$ would be Hurwitz.

Theorem 1. *Considering the general dynamic model (3), the control input (5):*

$$\boldsymbol{u}(\boldsymbol{t}) = \boldsymbol{b}^{-1}\left(-\boldsymbol{f} - \boldsymbol{d}(\boldsymbol{t}) + \boldsymbol{X}_{\text{d}}^{(n)} - \left(\boldsymbol{K}_{(\boldsymbol{n}-2)}\boldsymbol{e}^{(n-1)} + \cdots + \boldsymbol{K}_1\ddot{\boldsymbol{e}} + \dot{\boldsymbol{e}}\right) - \boldsymbol{\eta}_1\boldsymbol{S} - \boldsymbol{\eta}_2\,\text{sgn}(\boldsymbol{S})\right) \tag{5}$$

could guarantee the asymptotic stability of the closed-loop dynamic system regarding the existence of all disturbances. $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are diagonal positive definite constant control gain matrices.

Proof. Substituting the control input (5) in the dynamic model (3) could result the closed-loop dynamic system as follows:

$$\dot{\boldsymbol{S}} + \boldsymbol{\eta}_1\boldsymbol{S} + \boldsymbol{\eta}_2\,\text{sgn}(\boldsymbol{S}) = 0 \tag{6}$$

The positive definite Lyapunov function candidate $\boldsymbol{V} = \frac{1}{2}\boldsymbol{S}^{\text{T}}\boldsymbol{S}$ would be considered. Substituting the closed-loop dynamic in the time derivative of the Lyapunov function leads to:

$$\dot{\boldsymbol{V}} = \boldsymbol{S}^{\text{T}}\dot{\boldsymbol{S}} = -\boldsymbol{S}^{\text{T}}\boldsymbol{\eta}_1\boldsymbol{S} - \boldsymbol{S}^{\text{T}}\boldsymbol{\eta}_2\,\text{sgn}(\boldsymbol{S}) \tag{7}$$

With regards to compensation of all coupling terms and diagonal constant positive definite property of gain matrices $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$, Eq. (7) can be expressed as follows:

$$\dot{\boldsymbol{V}} = \boldsymbol{S}^{\text{T}}\dot{\boldsymbol{S}} = -\boldsymbol{S}^{\text{T}}\boldsymbol{\eta}_1\boldsymbol{S} - \boldsymbol{S}^{\text{T}}\boldsymbol{\eta}_2\,\text{sgn}(\boldsymbol{S}) = \sum_{i=1}^{m}\left(-\boldsymbol{\eta}_{1i}\boldsymbol{S}_{\boldsymbol{i}}^2 - \boldsymbol{\eta}_{2i}|\boldsymbol{S}_{\boldsymbol{i}}|\right) < 0 \tag{8}$$

Negative definiteness of $\dot{V}$ guarantees the asymptotic stability of closed-loop dynamic system, even with the existence of all disturbances. As a result, $\dot{V}$ would tend to zero as $t \to \infty$.

$$\dot{V} < 0 \quad \to \quad (\boldsymbol{S} \to 0) \quad \to \quad (\boldsymbol{e} \to 0) \quad \to \quad (\boldsymbol{X} \to \boldsymbol{X}_{\text{d}}) \qquad \square$$

The proposed control structure could properly stabilize the dynamic system. The main problem would be the measurement of unknown disturbances. The unknown inheritage of disturbances is a challenging problem that could be released by an appropriate disturbance observer. As a result, the estimated disturbance could be replaced in the control structure.

2.1. Disturbance observer structure

The proposed observer [13] is utilized as a disturbance observer. The disturbance estimation approach could be represented as:

$$\dot{\hat{\boldsymbol{d}}}(\boldsymbol{t}) = -L\hat{\boldsymbol{d}}(t) + L\left(\boldsymbol{X}^{(n)} - \boldsymbol{f} - \boldsymbol{b}\boldsymbol{u}(\boldsymbol{t})\right) \tag{9}$$

where $\hat{\boldsymbol{d}}(t)$ is the estimated disturbance and L is the positive definite observer gain matrix. The estimation error is defined as $\tilde{\boldsymbol{d}}(t) = \boldsymbol{d}(t) - \hat{\boldsymbol{d}}(t)$. Differentiating both sides and importing the observer structure (9), the closed-loop dynamic (10) would be achieved.

$$\dot{\tilde{\boldsymbol d}}(t)=\dot{\boldsymbol d}(t)-\dot{\hat{\boldsymbol d}}(t)=\dot{\boldsymbol d}(t)-L\hat{\boldsymbol d}(t)+L\big(\boldsymbol X^{(\boldsymbol n)}-\boldsymbol f-\boldsymbol{bu}(\boldsymbol t)\big)$$
$$\dot{\tilde{\boldsymbol d}}(t)+L\tilde{\boldsymbol d}(t)=\dot{\boldsymbol d}(t) \tag{10}$$

$\dot{\boldsymbol d}(t)$ is the time variation of disturbance. With regards the positive definiteness of L, the closed-loop dynamic (10) can be considered as a stable system with the input $\dot{\boldsymbol d}(t)$. It could be assumed that the time variation of the disturbance would be bounded as $\|\dot{\boldsymbol d}(t)\|\leqslant\bar d$, where $\bar d$ is a positive constant. Disturbances are the combination of both high-frequency and low-frequency signals. After filtering by a properly designed filter, the time variation can be approximated to be bounded. This assumption is a conventional point mentioned in different Refs. [7,15,20].

To investigate the closed-loop stability, a positive definite Lyapunov function $V(t)=\frac{1}{2}\tilde{\boldsymbol d}^{\mathrm T}\tilde{\boldsymbol d}$ is considered. The time derivative would be as follows:

$$\dot V(t)=\tilde{\boldsymbol d}^{\mathrm T}\dot{\tilde{\boldsymbol d}}=-\tilde{\boldsymbol d}^{\mathrm T}L\tilde{\boldsymbol d}+\tilde{\boldsymbol d}^{\mathrm T}\dot{\boldsymbol d} \tag{11}$$

Based on the quadratic property of $\lambda_{\min}(L)\|\tilde{\boldsymbol d}\|^2\leqslant\tilde{\boldsymbol d}^{\mathrm T}L\tilde{\boldsymbol d}\leqslant\lambda_{\max}(L)\|\tilde{\boldsymbol d}\|^2$, the ultimately boundedness of the closed-loop could be deduced:

$$\dot V(t)=-\tilde{\boldsymbol d}^{\mathrm T}L\tilde{\boldsymbol d}+\tilde{\boldsymbol d}^{\mathrm T}\dot{\boldsymbol d}\leqslant-\lambda_{\min}(L)\|\tilde{\boldsymbol d}\|^2+\|\tilde{\boldsymbol d}\|\|\dot{\boldsymbol d}\|\leqslant 0\quad\text{if }\|\tilde{\boldsymbol d}\|\geqslant\|\dot{\boldsymbol d}\|/\lambda_{\min}(L) \tag{12}$$

As a result, the disturbance estimation error would be bounded as $\|\tilde{\boldsymbol d}\|\leqslant\|\dot{\boldsymbol d}\|/\lambda_{\min}(L)\leqslant\bar d/\lambda_{\min}(L)$.

Remark 1. In the case of slow time-varying disturbances, i.e. $\dot{\boldsymbol d}(t)\approx 0$, the proposed disturbance observer approach would generate an asymptotic closed-loop dynamic $\dot{\tilde{\boldsymbol d}}(t)+L\tilde{\boldsymbol d}(t)=0$. As a result, the asymptotic convergence of the estimated disturbance to the real value could be deduced as $\tilde{\boldsymbol d}(t)\to 0$.

Remark 2. The existence of a high-order differentiation of states in the disturbance observer's structure creates a challenging concern due to the lack of appropriate sensors. This problem will appear as an acceleration measurement issue in conventional second-order dynamic systems. An alternative solution to remedy this problem will be discussed later.

2.2. Disturbance observer-based robust control for time-varying disturbances

Considering the observed disturbances, the disturbance estimation error would be bounded. Therefore, a robust control approach could be designed for precise trajectory tracking.

Theorem 2. *Considering the general dynamic model (3), the proposed control input and disturbance observer dynamics (13):*

$$\boldsymbol u(\boldsymbol t)=\boldsymbol b^{-1}\big(-\boldsymbol f-\hat{\boldsymbol d}(\boldsymbol t)+\boldsymbol X_{\mathrm d}^{(n)}-\big(\boldsymbol K_{(\boldsymbol n-2)}\boldsymbol e^{(n-1)}+\cdots+\boldsymbol K_1\ddot{\boldsymbol e}+\dot{\boldsymbol e}\big)-\boldsymbol\eta_1\boldsymbol S-\boldsymbol\eta_2\operatorname{sgn}(\boldsymbol S)\big)$$
$$\dot{\hat{\boldsymbol d}}(\boldsymbol t)=-L\hat{\boldsymbol d}(t)+L\big(\boldsymbol X^{(n)}-\boldsymbol f-\boldsymbol{bu}(\boldsymbol t)\big) \tag{13}$$

could guarantee the asymptotical stability of the closed-loop dynamic system regarding the existence of all disturbances.

Proof. Substituting the control input (13) in the dynamic model (3) could result in the closed-loop dynamic system as follows:

$$\dot{\boldsymbol S}+\boldsymbol\eta_1\boldsymbol S+\boldsymbol\eta_2\operatorname{sgn}(\boldsymbol S)=\tilde{\boldsymbol d} \tag{14}$$

where $\tilde{\boldsymbol d}$ is the disturbance estimation error. The positive definite Lyapunov function candidate $\boldsymbol V=\frac{1}{2}\boldsymbol S^{\mathrm T}\boldsymbol S$ would be considered. Substituting the closed-loop dynamic in the time derivative of the Lyapunov function, one has:

$$\dot{\boldsymbol V}=\boldsymbol S^{\mathrm T}\dot{\boldsymbol S}=-\boldsymbol S^{\mathrm T}\boldsymbol\eta_1\boldsymbol S-\boldsymbol S^{\mathrm T}\boldsymbol\eta_2\operatorname{sgn}(\boldsymbol S)+\boldsymbol S^{\mathrm T}\tilde{\boldsymbol d}=\sum_{i=1}^{m}\big(-\eta_{1i}\boldsymbol S_{\boldsymbol i}^2-\eta_{2i}|\boldsymbol S_{\boldsymbol i}|+\boldsymbol S_{\boldsymbol i}\tilde{\boldsymbol d}_{\boldsymbol i}\big)$$
$$\leqslant\sum_{i=1}^{m}\big(-\eta_{1i}\boldsymbol S_{\boldsymbol i}^2-\eta_{2i}|\boldsymbol S_{\boldsymbol i}|+|\boldsymbol S_{\boldsymbol i}||\tilde{\boldsymbol d}_{\boldsymbol i}|\big) \tag{15}$$

Based on the disturbance observer analysis in the last section, the boundedness of the estimation error $\|\tilde{\boldsymbol d}\|\leqslant\|\dot{\boldsymbol d}\|/\lambda_{\min}(L)\leqslant\bar d/\lambda_{\min}(L)$ has been achieved. As a result, it can be considered that any element of the estimation error vector is also bounded, i.e. $\|\tilde{\boldsymbol d}\|\leqslant\bar d/\lambda_{\min}(L)\to|\tilde{\boldsymbol d}_{\boldsymbol i}|\leqslant\bar d/\lambda_{\min}(L)$. Therefore,

$$\dot{\boldsymbol V}\leqslant\sum_{i=1}^{m}\big(-\eta_{1i}\boldsymbol S_{\boldsymbol i}^2-\eta_{2i}|\boldsymbol S_{\boldsymbol i}|+|\boldsymbol S_{\boldsymbol i}||\tilde{\boldsymbol d}_{\boldsymbol i}|\big)\leqslant\sum_{i=1}^{m}\big(-\eta_{1i}\boldsymbol S_{\boldsymbol i}^2-\eta_{2i}|\boldsymbol S_{\boldsymbol i}|+|\boldsymbol S_{\boldsymbol i}|\bar d/\lambda_{\min}(L)\big) \tag{16}$$

Assuming $\boldsymbol{\eta}_{2i} = \boldsymbol{\eta}'_{2i} + \bar{d}/\lambda_{\min}(L)$

$$\dot{\boldsymbol{V}} \leqslant \sum_{i=1}^{m} \left(-\boldsymbol{\eta}_{1i}\boldsymbol{S}_{\boldsymbol{i}}^2 - \boldsymbol{\eta}'_{2i}|\boldsymbol{S}_{\boldsymbol{i}}|\right) \leqslant 0 \tag{17}$$

where $\boldsymbol{\eta}'_{2i}$ is a positive constant. Negative definiteness of $\dot{\boldsymbol{V}}$ guarantees the asymptotical stability of the closed-loop dynamic system regarding the existence of time-varying disturbances. As a result, $\dot{V} < 0 \rightarrow (\boldsymbol{S} \rightarrow 0) \rightarrow (\boldsymbol{e} \rightarrow 0) \rightarrow (\boldsymbol{X} \rightarrow \boldsymbol{X}_{\mathrm{d}})$. □

2.3. Modified disturbance observer-based control for slow time-varying disturbances

It has been demonstrated in the last section that the proposed control structure (13) could precisely track the desired trajectory in the presence of time-varying disturbances. It is obvious that the asymptotic convergence can also be achieved for slow time-varying disturbances. But in this section, it will be shown that the necessity of Sgn(.) function can be eliminated by modifying the observer's structure for the case of slow time-varying disturbances. Therefore, the controller performance can be considerably improved in this case. Based on [18], by importing a feedback of tracking error in the observer's structure, a modified disturbance observer and control input would be proposed for precise trajectory tracking for slow time-varying external disturbances.

Theorem 3. *Considering the general dynamic model (3), the modified disturbance observer is proposed as*:

$$\dot{\hat{\boldsymbol{d}}}(\boldsymbol{t}) = -L\hat{\boldsymbol{d}}(t) + L\left(\boldsymbol{X}^{(n)} - \boldsymbol{f} + \boldsymbol{bu}(\boldsymbol{t})\right) + \boldsymbol{S} \tag{18}$$

where the sliding surface $\boldsymbol{S}$ *is coupled with the observer structure. Utilizing the proposed observer, the control input (19)*:

$$\boldsymbol{u}(\boldsymbol{t}) = \boldsymbol{b}^{-1}\left(-\boldsymbol{f} - \hat{\boldsymbol{d}}(\boldsymbol{t}) + \boldsymbol{X}_{\mathrm{d}}^{(n)} - \left(\boldsymbol{K}_{(\boldsymbol{n}-2)}\boldsymbol{e}^{(n-1)} + \cdots + \boldsymbol{K}_1\ddot{\boldsymbol{e}} + \dot{\boldsymbol{e}}\right) - \boldsymbol{\eta}_1\boldsymbol{S}\right) \tag{19}$$

could guarantee the asymptotically stability of closed-loop dynamic system regarding the existence of slow time-varying disturbances.

Proof. Considering the modified observer structure, the observer's closed-loop dynamic is derived as:

$$\dot{\tilde{\boldsymbol{d}}}(t) + L\tilde{\boldsymbol{d}}(t) = -\boldsymbol{S} \tag{20}$$

Substitution of control input (19) into the dynamic model (3) could result in the closed-loop dynamic system as follows:

$$\dot{\boldsymbol{S}} + \boldsymbol{\eta}_1\boldsymbol{S} = \tilde{\boldsymbol{d}} \tag{21}$$

where $\tilde{\boldsymbol{d}}$ is the disturbance estimation error. The positive definite Lyapunov function candidate $\boldsymbol{V} = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S} + \frac{1}{2}\tilde{\boldsymbol{d}}^{\mathrm{T}}\tilde{\boldsymbol{d}}$ would be considered. By substituting the closed-loop dynamics (20) and (21) in the time derivative of the Lyapunov function:

$$\dot{\boldsymbol{V}} = \boldsymbol{S}^{\mathrm{T}}\dot{\boldsymbol{S}} + \tilde{\boldsymbol{d}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{d}}} = -\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\eta}_1\boldsymbol{S} + \boldsymbol{S}^{\mathrm{T}}\tilde{\boldsymbol{d}} - \tilde{\boldsymbol{d}}^{\mathrm{T}}L\tilde{\boldsymbol{d}} - \tilde{\boldsymbol{d}}^{\mathrm{T}}\boldsymbol{S} = -\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\eta}_1\boldsymbol{S} - \tilde{\boldsymbol{d}}^{\mathrm{T}}L\tilde{\boldsymbol{d}} < 0 \tag{22}$$

The negative definiteness of $\dot{V}$ results in the fact that $\dot{\boldsymbol{V}}$ would tend to zero as $t \rightarrow \infty$.

$$\dot{V} < 0 \quad \rightarrow \quad (\boldsymbol{S} \rightarrow 0), (\tilde{\boldsymbol{d}} \rightarrow 0) \quad \rightarrow \quad (\boldsymbol{X} \rightarrow \boldsymbol{X}_{\mathrm{d}}), (\hat{\boldsymbol{d}} \rightarrow \boldsymbol{d})$$

Therefore, precise trajectory tacking and perfect disturbance estimation could be achieved. □

3. Dynamic modelling of micropositioning piezoelectric actuators

A second-order dynamics has been utilized for piezoelectric actuators. The model was divided into two parts. The first part is a second-order linear dynamics that refers to the mass–spring–damper system. The second part describes the nonlinear portion of the dynamics, i.e. the hysteresis nonlinearity effect [21]. Fig. 1 shows the linear second-order dynamics of a piezoelectric actuator that would be added by hysteresis nonlinearity effect into the input.

The governing equation in free motion is represented as (23):

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = H_{\mathrm{F}}\big(v(t)\big) \tag{23}$$

where $x(t)$ and $v(t)$ represent the actuator's displacement and input voltage. m, c and k denote mass, damping and stiffness impedances, respectively. $H_{\mathrm{F}}(v(t))$ expresses the hysteretic relation between the input voltage and the excitation force.

In many cases, there is an undesired external contact force, as shown in Fig. 2. The purpose is the perfect tracking in the presence of undesired forces and also disturbances.

Considering uncertainties, hysteresis estimation error, probably unmodelled dynamics and undesired external forces, a perturbation term $d(t)$ would be added to the dynamic model:

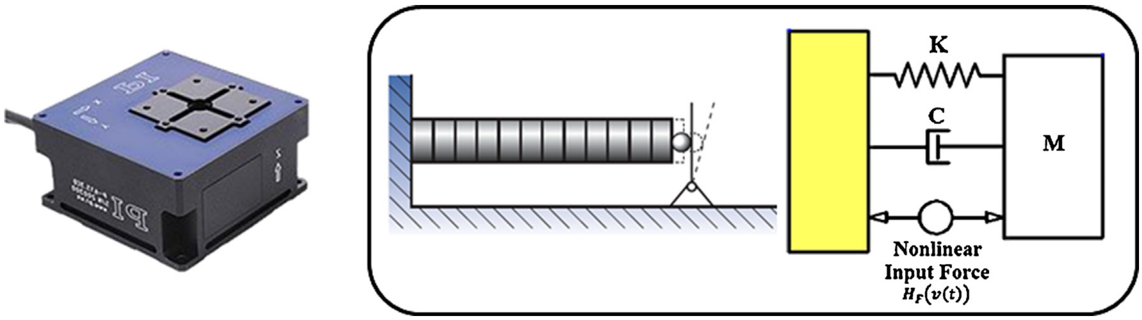

Fig. 1. Second-order nonlinear dynamic model of piezoelectric actuators.

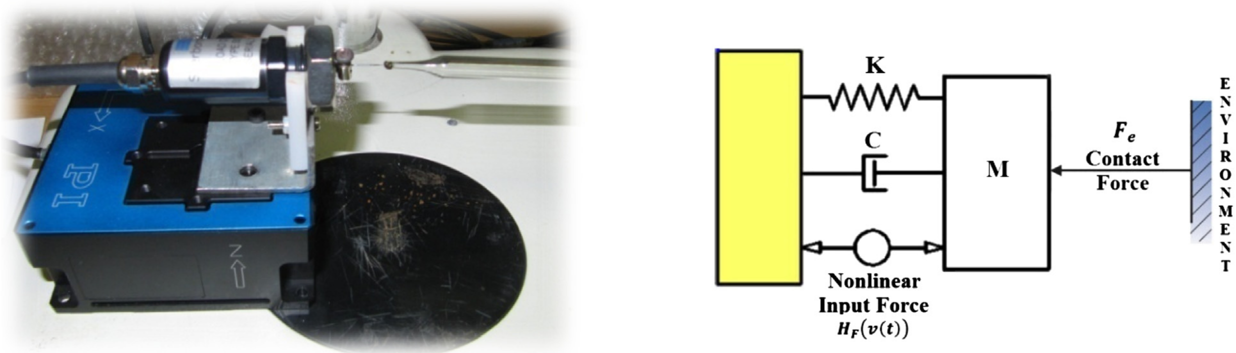

Fig. 2. Modelling of the piezoelectric contact with an environment.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = H_{\mathrm{F}}\big(v(t)\big) - d(t) \tag{24}$$

The main concern resides in the identification of the nonlinear hysteresis behaviour $H_{\mathrm{F}}(v(t))$. But this identification requires the force sensors for actuation force measurement. Therefore, an alternative solution has been utilized.

With regards to the rate independent property of hysteresis behaviour, in the low-frequency operation, the inertia and damping effects could be neglected for hysteresis identification. Hence, (23) can be transformed to (25) for the free motion case.

$$x(t) = \frac{1}{k} H_{\mathrm{F}}\big(v(t)\big) = H_x\big(v(t)\big) \tag{25}$$

This equation describes the input voltage and actuator displacement relative to each other. By this facilitation, the constraint of identification of the voltage–force nonlinear hysteretic relation is not necessary. By the identification of the hysteresis mapping between the input voltage and the actuator displacement ($H_x(v(t))$), it is scaled up with a factor k to obtain $H_{\mathrm{F}}(v(t))$. Finally, the dynamic model of the system is as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = kH_x\big(v(t)\big) - d(t) \tag{26}$$

3.1. Generalized Prandtl–Ishlinskii (PI) hysteresis model

A generalized Prandtl–Ishlinskii model is used for both hysteresis identification and compensation. The most important advantage of this model is its simplicity and the fact that its inverse could be calculated analytically. In addition, this approach could be utilized in open-loop systems where the feedback of signals is not accessible. The rate-independent backlash operator is the primary operator in the PI hysteresis model, as shown in Fig. 3.

This operator could be defined as follows:

$$\begin{aligned} &y(t) = w_{\mathrm{h}} H_r[x, y_0](t) \\ &H_r = max\big[x(t) - r, \min\big\{x(t) + r, y(t - T)\big\}\big] \\ &y(0) = max\big[x(0) - r, \min\big\{x(0) + r, y_0\big\}\big] \end{aligned} \tag{27}$$

where x is the control input, y is the actuator response, r is the control input threshold value or the magnitude of backlash, and T is the sampling period. The initial consistency condition $y(0)$ is usually but not necessarily initialized to zero.

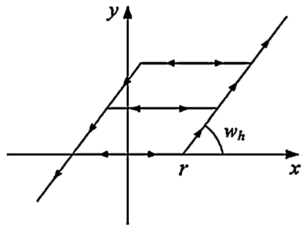

Fig. 3. Backlash operator with threshold r and weighting value w_h.

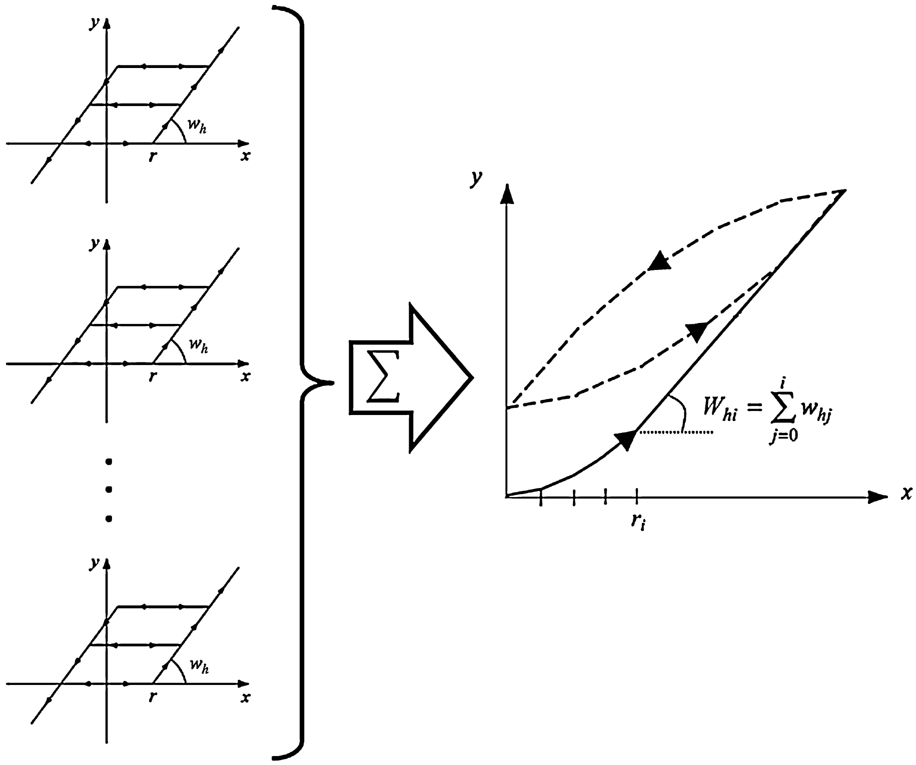

Fig. 4. Summation of backlash operators.

The Prandtl–Ishlinskii method expresses that the hysteresis loop could be identified as the summation of a number of backlash operators with different thresholds (r) and weights (w_h).

$$y(t) = \sum_{i=0}^{n} w_h^{T_i} H_r^i[x, y_0](t) \tag{28}$$

The summation of backlash operators is shown in Fig. 4.

The key idea of an inverse feedforward compensation of hysteresis is to cascade the inverse hysteresis operator H_x^{-1} with the actual hysteresis. As a result, an identity mapping between the desired actuator output $x_d(t)$ and the actuator response $x(t)$ would be obtained. The structure of the inverse feedforward hysteresis compensation is shown in Fig. 5.

As shown in Fig. 5, the inverse PI operator H_x^{-1}, uses $x_d(t)$ as an input and transforms it into a control input $v = H^{-1}(x_d)$, which produces $x(t)$ in the hysteretic system that closely tracks $x_d(t)$.

As a result, the feedforward positioning of the actuator for low frequency trajectories is achieved as follows:

$$x(t) = H_x\big[H_x^{-1}\big[x_d(t)\big]\big] \tag{29}$$

3.2. Observer-based robust control design for a piezoelectric actuator

Utilizing the inverse PI hysteresis model and choosing the input voltage as:

$$v = H_x^{-1}\left(\frac{1}{k}u(t)\right) \tag{30}$$

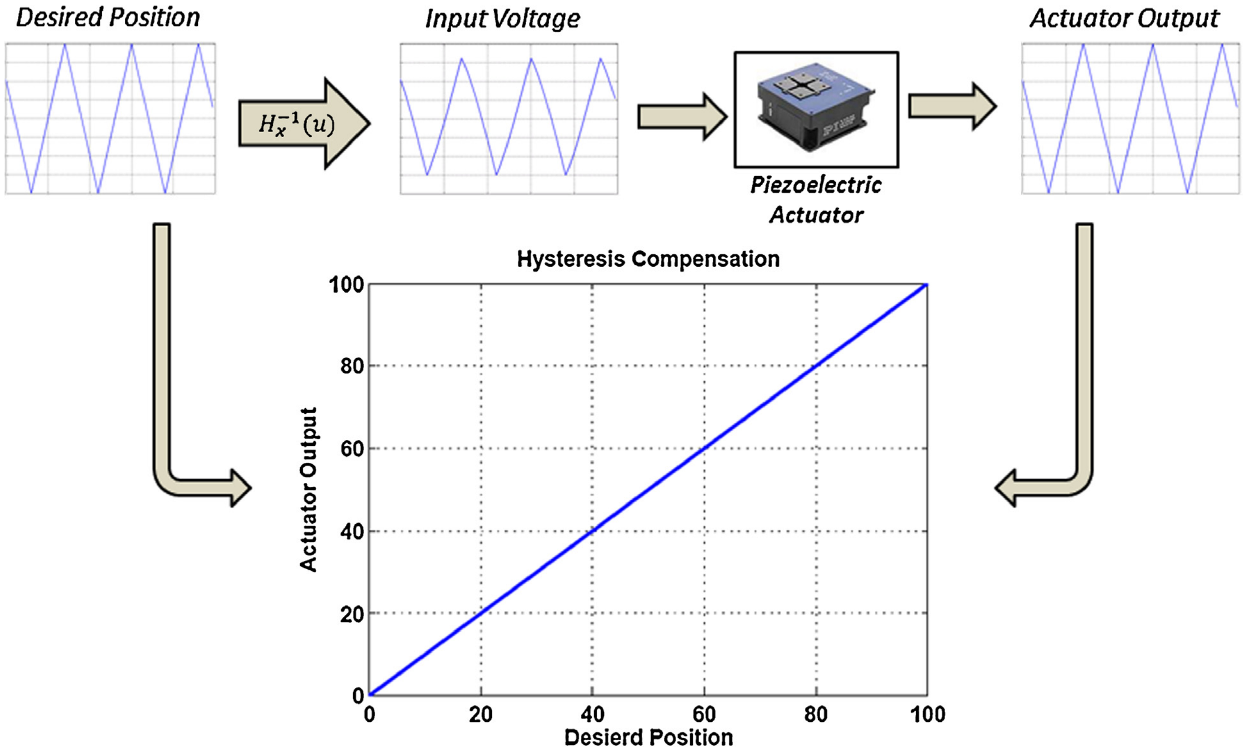

Fig. 5. Inverse feedforward compensation of hysteretic effect.

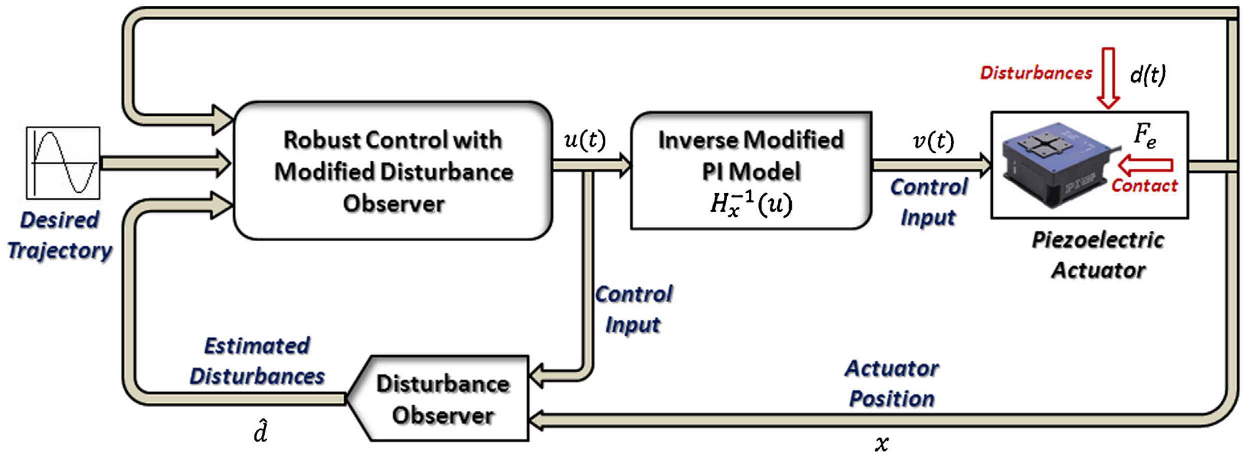

Fig. 6. The block diagram of the closed-loop system.

it appears that the dynamics of actuator would be linearized as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = u(t) - d(t) \tag{31}$$

where u would be the control input. Eq. (31) implies the linear structure of the actuator due to the use of the inverse PI model. The total disturbance $d(t)$ would contain model uncertainties such as hysteresis estimation error, unmodelled dynamics and external forces.

The objective is to get an accurate trajectory tracking of the actuator position. Tracking errors and sliding surfaces are defined as:

$$\begin{aligned} S &= \dot{\tilde{x}} + \lambda\tilde{x} \\ \tilde{x} &= x - x_{\mathrm{d}} \\ \dot{\tilde{x}} &= \dot{x} - \dot{x}_{\mathrm{d}} \end{aligned} \tag{32}$$

where λ is a positive constant.

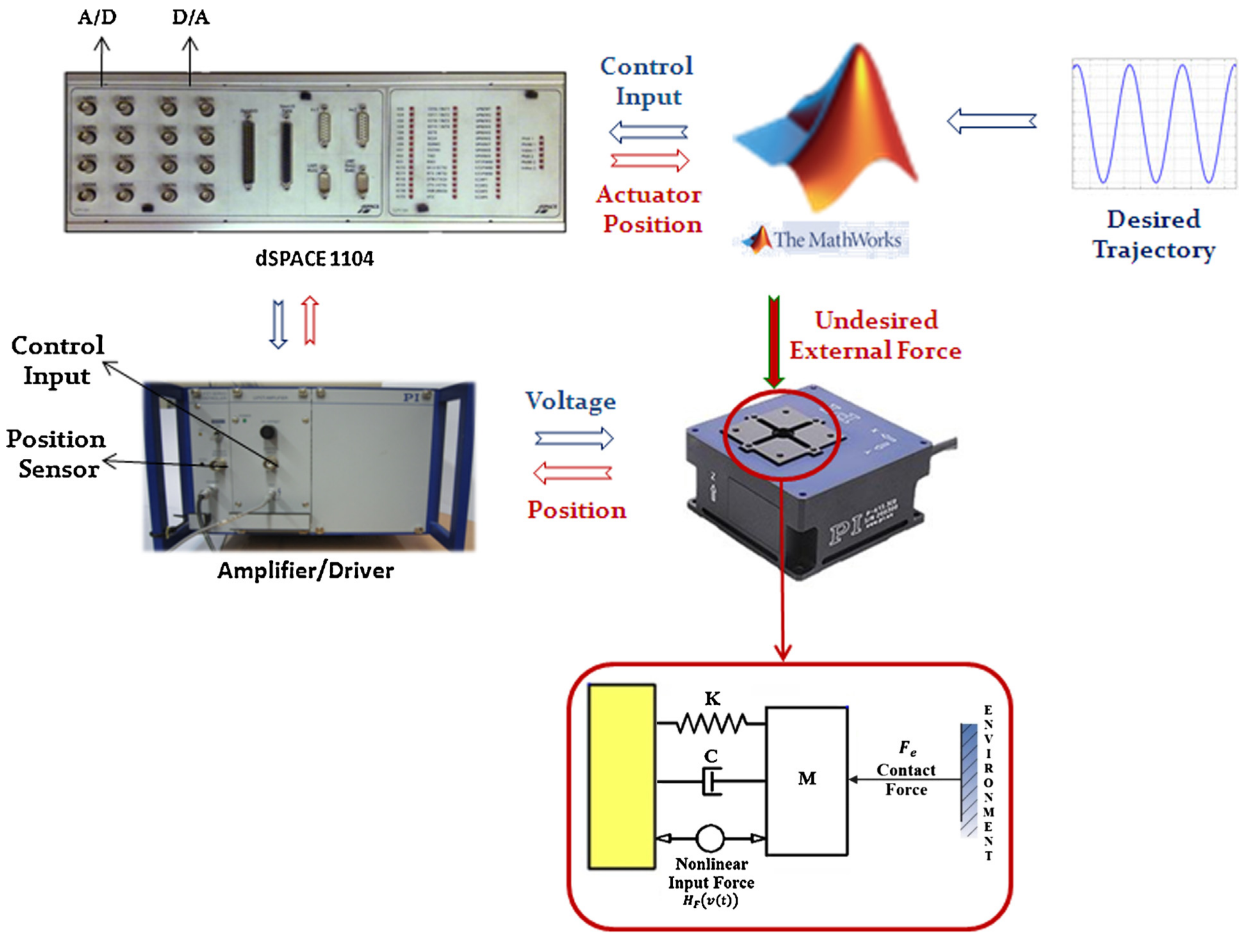

Fig. 7. The experimental setup.

The observer-based robust control (33) is proposed for precise micropositioning in the presence of constant and time-varying disturbances.

$$
\begin{aligned}
&v = H_x^{-1}\left\{\frac{1}{k}\left[\hat{d} + m(\ddot{x}_{\mathrm{d}} - \lambda\dot{\tilde{x}}) - c\dot{x} - kx - \eta_1 S\right]\right\} \\
&\dot{\hat{d}} = -L\hat{d} + L\big(u - m\ddot{x}(t) + c\dot{x}(t) + kx(t)\big) + S
\end{aligned}
\tag{33}
$$

η_1, η_2 are positive coefficients. As mentioned in Remark 2, the proposed observer includes the acceleration term. To overcome this problem, a new auxiliary parameter z would be defined as $z = \hat{d} - p$, where parameter p should be determined [13]. By differentiating both sides and substituting the observer structure, a new dynamic could be achieved for the observer as follows:

$$
\begin{aligned}
&\dot{z} = \dot{\hat{d}} - \dot{p} \\
&\dot{z} = -L\hat{d} + L\big(u - m\ddot{x}(t) + c\dot{x}(t) + kx(t)\big) + S - \dot{p} \\
&\dot{z} = -L(z + p) + L\big(u - m\ddot{x}(t) + c\dot{x}(t) + kx(t)\big) + S - \dot{p}
\end{aligned}
\tag{34}
$$

If parameter p were designed as $\dot{p} = -Lm\ddot{x}(t)$, the new observer dynamics is derived as:

$$
\dot{z} = -Lz + L[u + p - c\dot{x} - kx] + S \tag{35}
$$

Therefore, by properly defining parameter p, the acceleration measurement would be rectified. As a result, the observer structure is modified as (36).

$$
\begin{aligned}
&\hat{d} = z + p \\
&\dot{z} = -Lz + L[u + p - c\dot{x} - kx] + S
\end{aligned}
$$

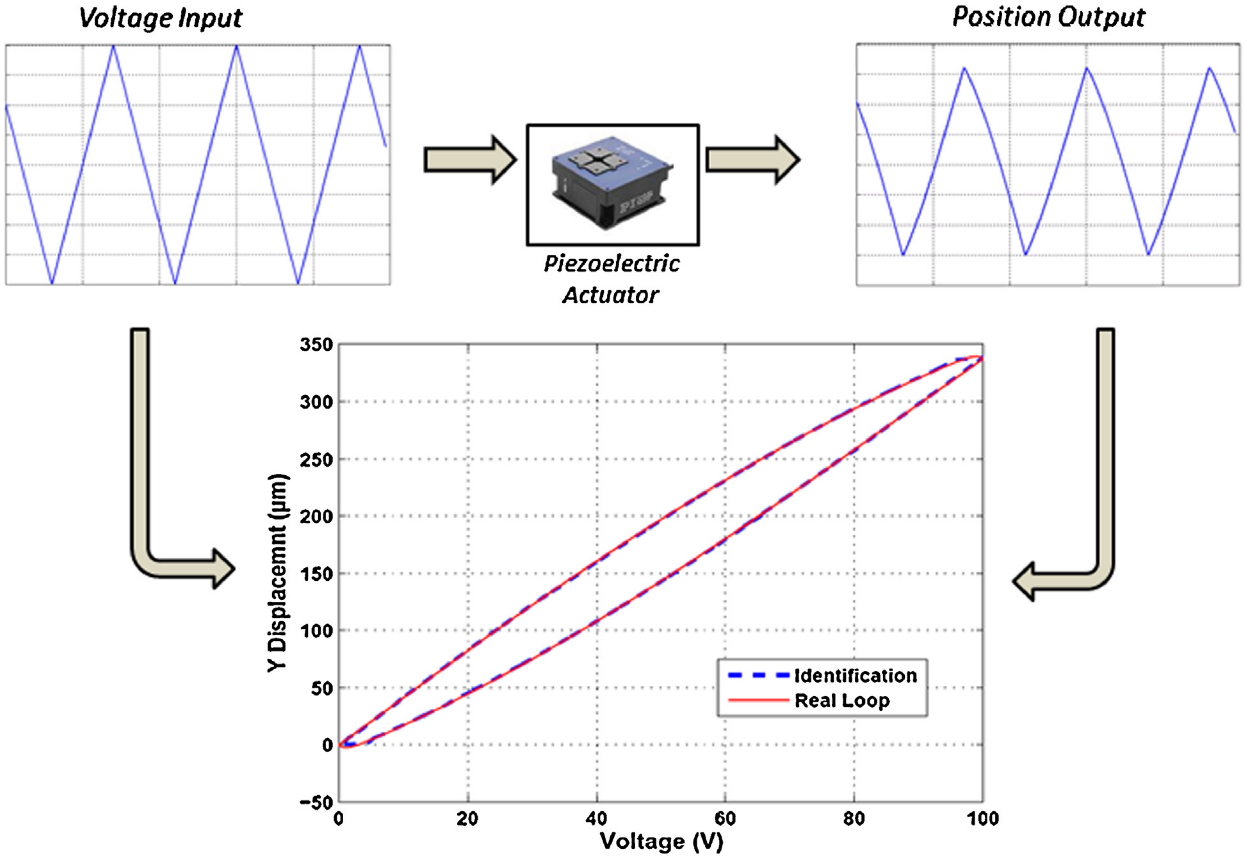

Fig. 8. The triangular quasi-static input voltage for hysteresis estimation.

$$p = -Lm\dot{x} \tag{36}$$

Differentiating $\hat{d} = z + p$ and importing the $\dot{p}$ and $\dot{z}$ dynamics, it is clear that the closed-loop $\dot{\tilde{d}} + L\tilde{d} = -S$ could be achieved in the same way as the original observer.

Fig. 6 shows the overall block diagram of the proposed disturbance observer based robust control for piezoelectric actuators.

4. Experimental results

The proposed controller was implemented experimentally. The experimental setup is shown in Fig. 7. The setup contains a P-615 NanoCube piezoelectric actuator with a 420-mm maximum displacement in the X and Y directions. A DS1104 dSPACE data acquisition and a controller board are used for data capturing. Analogue-to-digital inputs (A/D) have 12 and 16 bits. The sampling frequency is considered to be 1 kHz.

Matlab/Simulink software is utilized for implementation of the control approach. The capacitive sensors had been installed inside the actuator by the manufacturer. Based on the catalogue data, the open-loop resolution and the repeatability are 1 nm and 7.5 nm, respectively.

4.1. Hysteresis estimation and compensation

The modified PI model could estimate the actuator's hysteresis behaviour. To eliminate the actuator dynamic behavior, a low-frequency quasi-static input is utilized for hysteresis identification, as shown in Fig. 8. As a result, the precise hysteresis identification could be achieved.

4.2. Controller performance without disturbance observer structure

For the first case, a conventional robust controller has been implemented for trajectory tracking. The perturbation caused by the dynamic modelling and identification errors exist in this case. The desired position is set as $x_d(t) = 150 + 120\sin 3t$. Fig. 9 depicts the precise trajectory tracking and the tracking error, respectively.

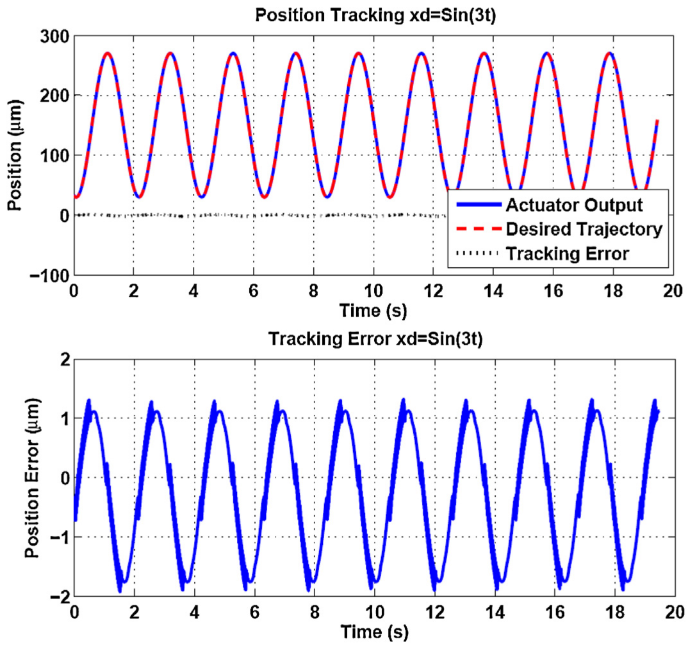

Fig. 9. Trajectory tracking and tracking error for a harmonic trajectory.

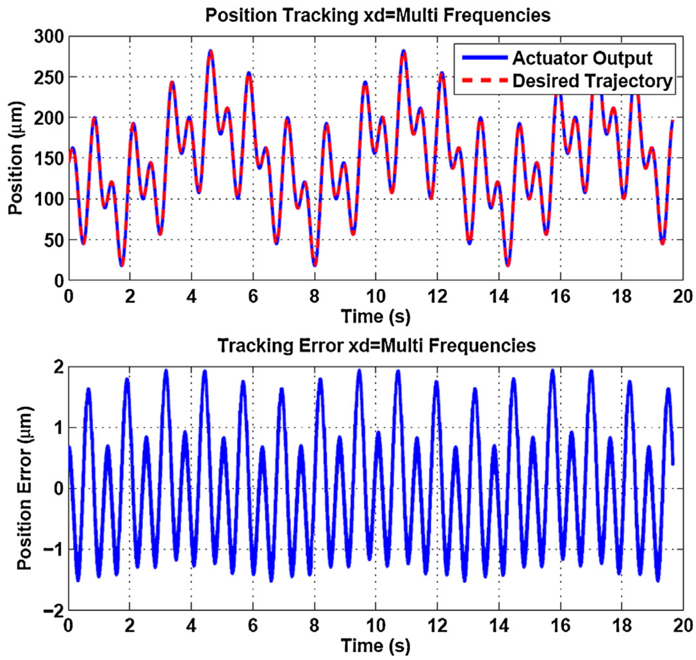

Fig. 10. (Color online.) Multi-frequency trajectory tracking.

Evaluating the controller performance for a multi-frequency trajectory, $x_d(t) = 150 + 48(\sin t + \sin 5t + \sin 10t)$ is utilized as the desired trajectory. Fig. 10 shows that the controller precisely tracks the multi-frequency trajectories in the presence of perturbations.

The control input is shown in Fig. 11.

The controller, without the observer's structure, could track precisely multi-frequency trajectories in the presence of only small perturbations and disturbances. In other words, the controller's performance would be degraded for large disturbances such as external forces. As a result, the observer's structure should be activated in such cases.

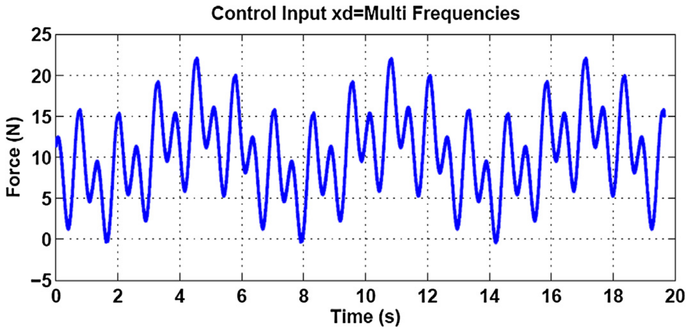

Fig. 11. Control input for multi-frequency trajectory tracking.

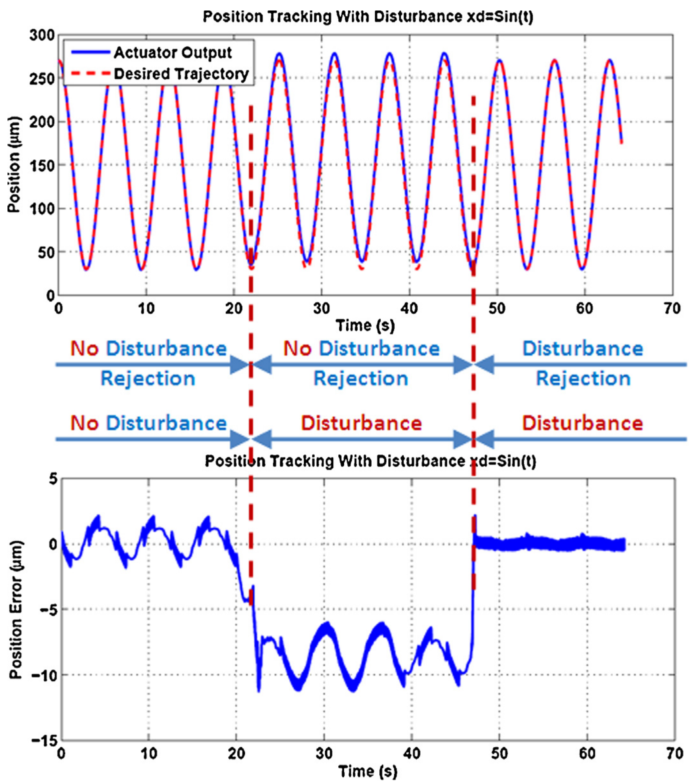

Fig. 12. (Color online.) Controller performance with and without the disturbance observer.

4.3. Controller's performance with the disturbance observer structure

The controller's performance should be evaluated in the presence of large disturbances such as external forces. Therefore, an external force is exerted to the actuator at the second 23. Such a disturbance clearly degrades the controller's performance, as shown in Fig. 12. To elliminate this problem, the disturbance observer is activated at second 47.

Tracking is appropriately improved by the proposed controller. The disturbance observer could precisely estimate perturbations and compensate the tracking error efficiently.

Finally, the proposed controller has been implemented on the experimental setup. In such a case, the disturbance rejection is activated during the whole time. The external force is exerted in the time space between 25 and 50 s. Fig. 13 shows that the tracking is accurately done in the presence of disturbances.

The used controller gains are displayed in Table 1.

5. Conclusion

Piezoelectric actuators are widely used for precise micropositioning. The positioning accuracy is effectively degraded in the presence of disturbances. Considering the disturbance magnitude in the controller could improve the performance.

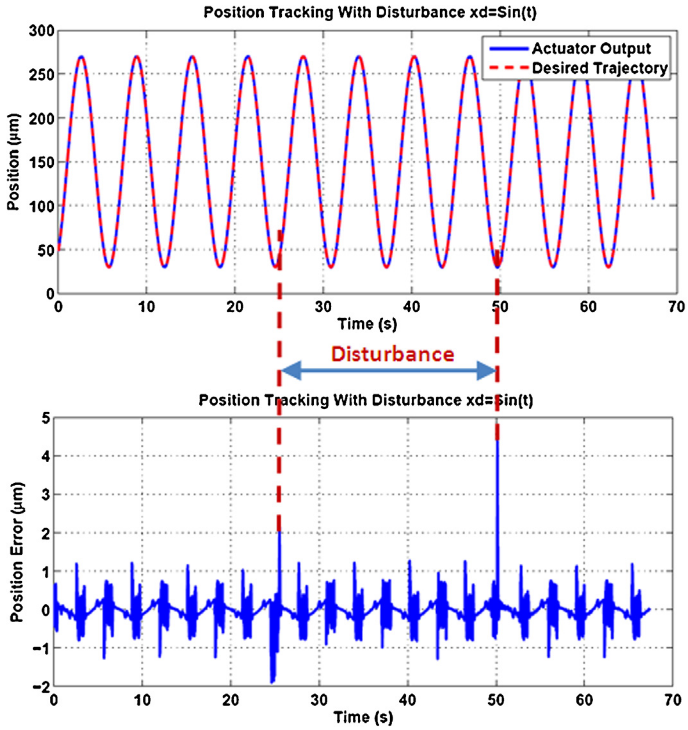

Fig. 13. (Color online.) The proposed controller performance in the presence of disturbance.

Table 1
Controller parameters.

Parameter	Value	Parameter	Value
m	2	λ	150
c	3000	η_1	1100
k	70 000	L	20

Due to the unknown source of disturbances, it should be precisely observed. The modified Prandtl–Ishlinskii (PI) model was utilized for both hysteresis identification and feedforward compensation. Therefore, the nonlinear dynamics of the actuator is transformed into a linear model. A disturbance observer coupled with a robust controller is proposed for precisely trajectory tracking in the presence of disturbances. The controller is designed for constant and time-varying disturbances. In addition, the switching behaviour of the controller has been elliminated by proposing a modified observer structure for slow time varying disturbances. Experimental results validate the controller performance without disturbance observer for small perturbations. By exerting large disturbances such as external forces, the observer could appropriately estimate and compensate disturbances. Trajectory tracking could be precisely implemented by the proposed control approach in the presence of disturbances.

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