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On the accuracy of analytical methods for turbulent flows near smooth walls

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ABSTRACT

This Note presents two methods for mean streamwise velocity profiles of fully-developed turbulent pipe and channel flows near smooth walls. The first is the classical approach where the mean streamwise velocity is obtained by solving the momentum equation with an eddy viscosity formulation [R. Absi, A simple eddy viscosity formulation for turbulent boundary layers near smooth walls, C. R. Mecanique 337 (2009) 158–165]. The second approach presents a formulation of the velocity profile based on an analogy with an electric field distribution [C. Di Nucci, E. Fiorucci, Mean velocity profiles of fully-developed turbulent flows near smooth walls, C. R. Mecanique 339 (2011) 388–395] and a formulation for the turbulent shear stress. However, this formulation for the turbulent shear stress shows a weakness. A corrected formulation is presented. Comparisons with DNS data show that the classical approach with the eddy viscosity formulation provides more accurate profiles for both turbulent shear stress and velocity gradient.

1. Introduction

Upon appropriate normalization, fully-developed pipe and channel flows can be regarded as the same in the near-wall region. Monty et al. (2009) [1] found that the inner-scaled mean velocity is identical in the region $y < 0.25\delta$, where y is the distance from the wall and δ is either the channel half-height, pipe radius or boundary layer thickness. The properties of shear and normal Reynolds stresses very close to the wall of turbulent channel/pipe flows and boundary layers were investigated by Buschmann et al. (2009) [2] from direct numerical simulations (DNS) and physical experiments data.

For turbulent channel and pipe flows the total shear stress is given by [3–6]

$$\tau_{tot}^{+} = \frac{\mathrm{d}U^{+}}{\mathrm{d}y^{+}} + \tau_{tur}^{+} = \left(1 - \frac{y^{+}}{\mathrm{Re}_{\tau c}}\right) = \left(1 - \frac{2y^{+}}{\mathrm{Re}_{\tau p}}\right) \tag{1}$$

where y^{+} is the non-dimensional distance from the wall (non-dimensionalized by the wall friction velocity u_τ and the kinematic viscosity ν, $y^{+} = yu_\tau/\nu$) and U^{+} the non-dimensional mean streamwise velocity ($U^{+} = U/u_\tau$), $\mathrm{Re}_{\tau c} = u_\tau h/\nu$ and $\mathrm{Re}_{\tau p} = u_\tau D/\nu$ are respectively friction Reynolds numbers for channels and pipes with the channel half-width h and the pipe diameter D (parameters with subscripts "c" and "p" are respectively for channels and pipes).

Two analytical methods for the determination of mean streamwise velocity $U^{+}(y^{+})$ profiles of fully-developed turbulent pipe and channel flows near smooth walls are presented. The first is the classical approach where the mean streamwise velocity is obtained by solving the momentum equation with an eddy viscosity formulation [7]. The second approach

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presents a formulation of the velocity profile based on an analogy with the electric field distribution generated in an electrolytic tank [8]. Based on this velocity profile, a formulation for the turbulent shear stress τ^+_{tur} was presented [8]. However, the related total shear stress is not in accordance with the classical profile (Eq. (1)) and becomes valid only for a condition which involves a contradiction indicating that $\tau^+_{tur} = 0$ and therefore that the flow is laminar and not turbulent. The aim of this Note is to present: (1) a solution for this shortcoming, (2) a comparison between these two methods.

2. The classical approach with an eddy viscosity formulation

An ordinary differential equation for the velocity distribution is obtained from Eq. (1), for the case of a channel flow, as

$$\frac{\mathrm{d}U^+}{\mathrm{d}y^+} = \frac{1}{1+\nu_t^+}\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) \tag{2}$$

where $\nu_t^+ = \nu_t/\nu$. Resolution of Eq. (2) needs the dimensionless eddy viscosity ν_t^+ which is given by [7]

$$\nu_t^+ = \kappa C_\nu B^{0.5} y^{+2} e^{-y^+/(2A_k^+)}\left(1 - e^{-y^+/A_l^+}\right) \tag{3}$$

where κ is the Kármán constant (≈ 0.4), C_ν, $A_k^+ = 8$, $A_l^+ = 26$ are coefficients and B is a Re_τ-dependent parameter given by $B(\mathrm{Re}_{\tau c}) = 0.0164 \ln(\mathrm{Re}_{\tau c}) + 0.0334$ [7]. This analytical eddy viscosity formulation is based on a near-wall analytical solution for the turbulent kinetic energy [9] and the van Driest mixing length equation [10].

Predicted mean streamwise velocity profiles (obtained from Eqs. (2) and (3)) were assessed by DNS data of fully-developed turbulent channel flows [7]. Comparisons with DNS data [11] show good agreement for $y^+ \leqslant 20$ and that for $\mathrm{Re}_\tau < 395$, the coefficient C_ν is Re_τ-dependent. However, for $\mathrm{Re}_\tau \geqslant 395$ the coefficient of proportionality C_ν in the eddy viscosity equation is independent of Re_τ and equal to 0.3 [7].

3. A formulation based on an electric field distribution

3.1. Mean streamwise velocities near smooth walls

Bucci et al. [12] have shown that it is possible to implement in an electrolytic tank an electric field distribution completely superimposable to the mean velocity profiles of fully-developed turbulent pipe flows in the regions near the smooth walls. Starting from this result, Di Nucci and Fiorucci [8] have proposed to assign to the mean streamwise velocity profiles the expressions:

$$U_p^+ = \lambda_1 y^+\left(1 - \frac{y^+}{\mathrm{Re}_{\tau p}}\right) + \lambda_2 \ln\left(1 - \frac{2y^+}{\mathrm{Re}_{\tau p}}\right) \tag{4}$$

and

$$U_c^+ = \varphi_1 y^+\left(1 - \frac{y^+}{2\,\mathrm{Re}_{\tau c}}\right) + \varphi_2 \ln\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) \tag{5}$$

where λ_1, λ_2, φ_1 and φ_2 are functions of friction Reynolds numbers [8].

Comparisons between mean streamwise velocity profiles obtained by Eq. (5) and DNS data [11] for turbulent channel flows show good agreement for $y^+ \leqslant 20$ [8].

3.2. Turbulent shear stress near smooth walls

3.2.1. Former formulation

The derivative of Eq. (4) gives

$$\frac{\mathrm{d}U_p^+}{\mathrm{d}y^+} = \lambda_1\left(1 - \frac{2y^+}{\mathrm{Re}_{\tau p}}\right) - 2\frac{\lambda_2}{\mathrm{Re}_{\tau p} - 2y^+} \tag{6}$$

The boundary condition $\frac{\mathrm{d}U^+}{\mathrm{d}y^+} = 1$ at $y^+ = 0$ allows to write a relation between λ_1 and λ_2 as

$$\lambda_2 = \frac{1}{2}(\lambda_1 - 1)\,\mathrm{Re}_{\tau p} \tag{7}$$

From Eq. (6) and by analogy with Eq. (1), the total and turbulent shear stresses were expressed respectively by [8]

$$\tau^+_{tot} = \lambda_1\left(1 - \frac{2y^+}{\mathrm{Re}_{\tau p}}\right) \tag{8}$$

and

$$\tau^+_{tur\,p} = 2\frac{\lambda_2}{\mathrm{Re}_{\tau p} - 2y^+} \tag{9}$$

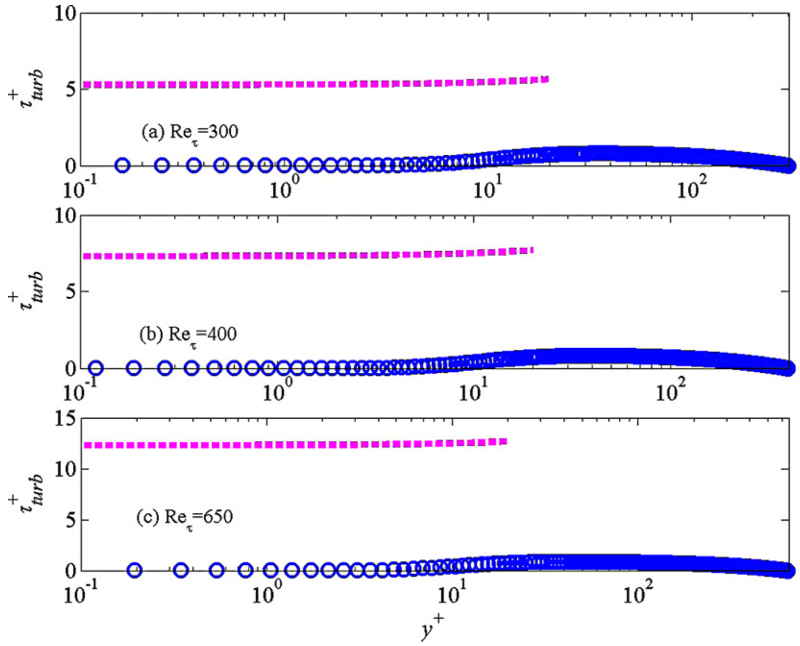

Fig. 1. Turbulent shear stress for turbulent channel flows, curve: former formulation of the second method (Eq. (11)); symbols: DNS data of Iwamoto et al. [11] for different friction Reynolds numbers.

For channel flows, Eq. (6) and Eq. (9) become, respectively:

$$\frac{\mathrm{d}U_c^+}{\mathrm{d}y^+} = \varphi_1\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) - \frac{\varphi_2}{\mathrm{Re}_{\tau c} - y^+} \tag{10}$$

and

$$\tau^+_{tur\,c} = \frac{\varphi_2}{\mathrm{Re}_{\tau c} - y^+} \tag{11}$$

Fig. 1 shows that the turbulent shear stress obtained by Eq. (11) is unable to allow adequate prediction of the DNS data. Functions φ_1 and φ_2 are given by $\varphi_1 = 0.025\,\mathrm{Re}_{\tau c}^{0.969}$ and $\varphi_2 = (\varphi_1 - 1)\,\mathrm{Re}_{\tau c}$ [8].

In this formulation, Eq. (8) is different from Eq. (1). It becomes valid and reverts to Eq. (1) only for $\lambda_1 = 1$. This condition involves from Eq. (7) that $\lambda_2 = 0$ and therefore (from Eq. (9)) that $\tau^+_{tur} = 0$ which indicates that the flow is laminar and not turbulent. Moreover, the turbulent shear stress (Eqs. (9) and (11)) does not respect the boundary condition $\tau^+_{tur} = 0$ at $y^+ = 0$: therefore, these equations must be corrected.

3.2.2. Corrected formulation

From Eqs. (1) and (6), τ^+_{tur} is given by

$$\tau^+_{tur\,p} = (1 - \lambda_1)\left(1 - \frac{2y^+}{\mathrm{Re}_{\tau p}}\right) + \frac{2\lambda_2}{\mathrm{Re}_{\tau p} - 2y^+} \tag{12}$$

or by using Eq. (7)

$$\tau^+_{tur\,p} = 8\lambda_2 y^+ \frac{\mathrm{Re}_{\tau p} - y^+}{\mathrm{Re}^2_{\tau p}(\mathrm{Re}_{\tau p} - 2y^+)} \tag{13}$$

Eq. (13) presents the interest that it contains only one parameter λ_2. Moreover, Eq. (13) allows us to verify the boundary condition $\tau^+_{tur} = 0$ at $y^+ = 0$.

The former formulation of turbulent shear stress given by Eqs. (9) and (11) is therefore not suitable. For pipe flows, Eq. (9) should be replaced by Eq. (12) or Eq. (13).

4. Comparisons and discussions

In order to assess both methods, we present comparisons for turbulent shear stress and velocity gradient. The validation is obtained by comparison with the DNS data of fully-developed turbulent channel flows of Iwamoto et al. [11].

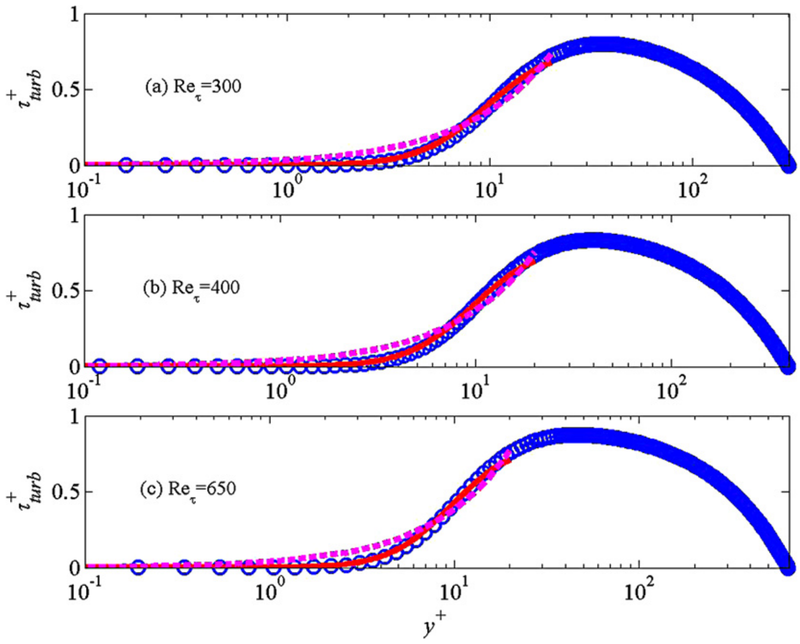

Fig. 2. Turbulent shear stress profiles near smooth walls for turbulent channel flows; dashed lines: corrected formulation of the second method (Eq. (15)); solid line: classical approach (Eq. (16)); symbols: DNS data.

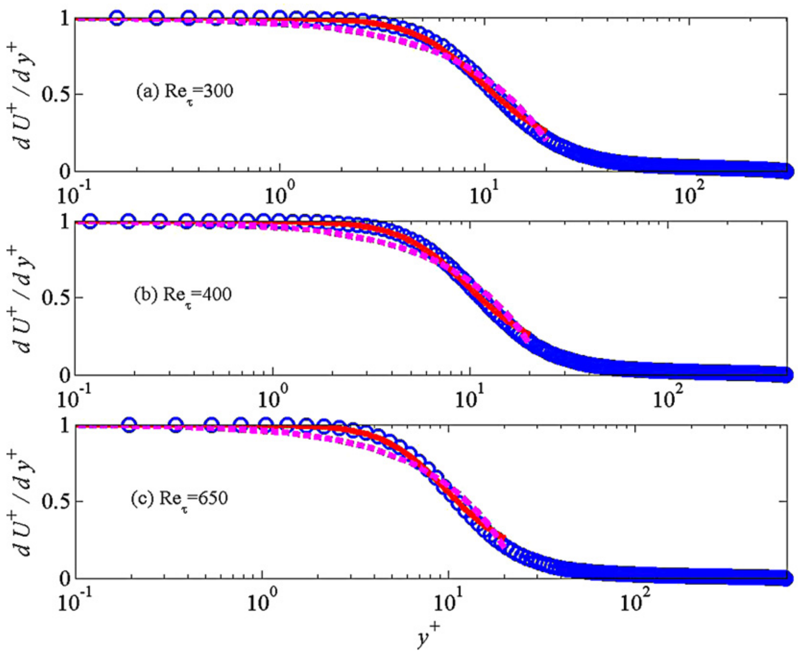

Fig. 3. Velocity gradient profiles near smooth walls for turbulent channel flows; dashed lines: Eq. (14); solid line: classical approach (Eqs. (2) and (3)); symbols: DNS data.

For channel flow, we write the velocity gradient and the turbulent shear stress of the second method as:
The derivative of Eq. (4) gives

$$\frac{\mathrm{d}U_c^+}{\mathrm{d}y^+} = \varphi_1\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) - \frac{\varphi_2}{\mathrm{Re}_{\tau c} - y^+} \tag{14}$$

From Eqs. (1) and (14), the turbulent shear stress is given for channel flows by

$$\tau^+_{tur\,c} = (1 - \varphi_1)\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) + \frac{\varphi_2}{\mathrm{Re}_{\tau c} - y^+} \tag{15}$$

For the classical approach, the turbulent shear stress is obtained from Eqs. (1) and (2) as

$$\tau^+_{tur\,c} = \frac{\nu_t^+}{1+\nu_t^+}\left(1 - \frac{y^+}{\mathrm{Re}_{\tau c}}\right) \tag{16}$$

Fig. 2 shows that the corrected formulation (Eq. (15)) improves the former one and allows a good description of the turbulent shear stress. However, comparisons with DNS data show that the classical approach (Eq. (16)) provides more accurate profiles.

Comparisons of velocity gradient obtained by the two methods and DNS data (Fig. 3) show that the classical approach (Eqs. (2) and (3)) provides more accurate results.

5. Conclusions

In this Note, two methods for mean streamwise velocity profiles of fully-developed turbulent pipe and channel flows near smooth walls are presented. The first is the classical approach where the mean streamwise velocity is obtained from the momentum equation with an eddy viscosity formulation. The second approach presents a formulation of the velocity profile based on an analogy with the electric field distribution generated in an electrolytic tank. We presented a corrected formulation for the turbulent shear stress which improves the former one. Comparisons with DNS data show that the classical approach with the eddy viscosity formulation provides more accurate profiles for both turbulent shear stress and velocity gradient.

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