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Biomimetic flow control

On the scaling of drag reduction by reconfiguration in plants

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ABSTRACT

Slender flexible structures such as plants are deformed by external flow. When the deformation is significant, this results in a reduction of drag. We give a theoretical value of the exponent that characterizes the drag law. This theoretical value is shown to compare well with experimental data on a very large variety of plants. It is found that reconfiguration affects more the local bending stress than the total drag. Moreover, a non-linearity in the bending law does not affect significantly the mechanism.

1. Introduction

There is a large variety of mechanical interactions between plants and the fluid medium where they grow, be it air or water, see for instance [1,2]. Among them, one is of particular interest in the perspective of bio-inspiration: drag reduction by elastic reconfiguration. It has been observed, since the seminal work of Vogel [3,4], that plants or parts of plants experience lower drag than if they were rigid. More specifically, drag has been observed to increase with the flow velocity U not as U^2, as expected for high Reynolds flow, but rather as U^1, and this change of the law for drag has always been associated with significant deformation of the system. Of course simple Reynolds effects have to be ruled out, and the drag reduction is here between a flexible body and a perfectly rigid body. Such a drag reduction has evident biological consequences as it allows organisms to survive permanent stress or rare events [5,2]. In fact by reducing drag at the attachment point the plant will avoid being uprooted or torn from its substrate. Moreover, the reduction of drag induces a general reduction of stress levels in the plant, which allows avoiding local fractures. The passive underlying mechanism, which is the elastic deformation induced by the fluid loading, seems somehow universal.

The level of drag reduction has often been quantified [4,6–8] by a dimensionless number, the Vogel exponent $\mathcal{V}$, such that the power law dependence with the flow velocity reads $U^{2+\mathcal{V}}$. A Vogel number of -1 is a typical value, in the range of flow velocities where plants are deformed.

For the sake of simplicity we assume hereafter that the drag of the body, if it was kept undeformed, would grow like U^2. We implicitly disregard any effect of the Reynolds number in the evolution of drag. For the definition of the Vogel number in the more general case, see [8].

Many experiments exist, using flow from wind tunnels, water flumes or on-site wind or currents, and systems of all kinds: grasses [9], reeds [5], flowers [10], leaves [4,11], tree crowns [12], trees [13], algaes [14], kelp [15], and even submerged trees in water [16], to cite a few. The values of Vogel exponents, in reviews such as [5,17], typically range from -0.2 to -1.2.

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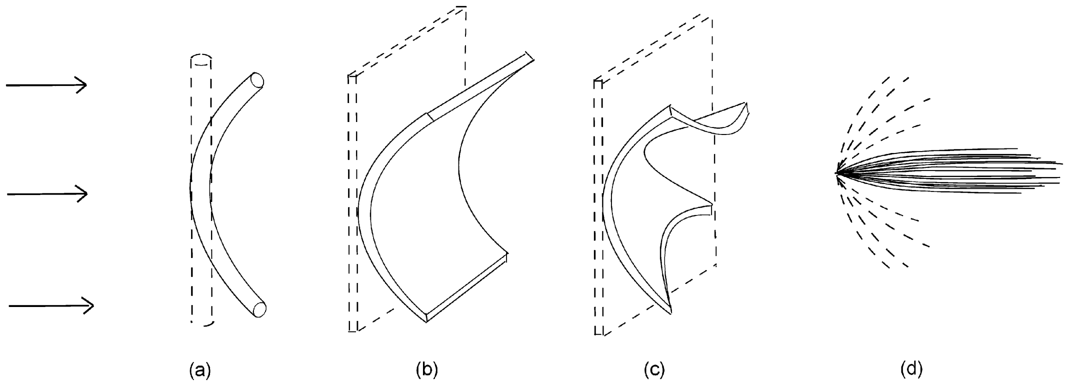

Fig. 1. Reconfiguration of simple systems under flow: (a) a beam in bending induced by cross-flow, (b) a plate in simple bending, (c) a plate in double bending without Gauss curvature, (d) a pack of beams clumped by axial flow.

Considering the ubiquity of the drag reduction mechanism, significant work has been done in recent years to explore the underlying laws of this mechanical interaction between flow and elasticity. Experimental models involving non-biological materials, [6–8,18,19] have been used to quantify the effect of flexibility on this drag reduction. Theoretical approaches that involve simple or advanced modeling of the flow over the flexible body have also been built [6–8,18–20]. A dimensionless parameter has been proposed to scale the deformation of the structure: the Cauchy number, see for instance in [1,8]. It compares the fluid loading with the elastic stiffness, so that reconfiguration only occurs for large values of the Cauchy number. In fact, reconfiguration results in more than a simple change of the slope of the drag law; it has been shown that even for a beam in bending several regions can be distinguished in the drag law, that correspond to distinct values of the Vogel exponent [8]. In the limit of highly deformed structures, simple estimates of the Vogel exponent have recently been proposed and validated for beams, plates or poroelastic media made of beams [8,19]. In this article, we propose an extension of this approach to systems made of beams and plates in simple bending, as are most plants. We also seek to assess the effect of non-linearity in the local bending laws. The corresponding prediction is then compared to recently published experimental results and reviews of existing data.

2. Theoretical value of the Vogel exponent

2.1. Vogel exponent in cross-flow

We consider the total drag F on an elastic system made of plates and beams, meaning that the energy of deformation is mainly dependent on local curvatures. This implies that a stiffness coefficient, B, is defined having the dimensions [$\text{Mass}^1\text{Length}^2\text{Time}^{-2}$] so that the local energy for plates per unit area reads $e \propto B\gamma^2$ where γ is the curvature. For a beam in bending, Fig. 1(a), an additional length d exists, which is such that the integrated stiffness coefficient reads Bd and the energy per unit length is $e \propto Bd\gamma^2$. The length d is typically the diameter of the beam. The local fluid loading is assumed to depend on the dynamic pressure ρU^2 where ρ is the fluid density. This applies to the case of a plate, but reads $\rho U^2 d$ for a beam. Note that drag here is essentially pressure drag.

Following [8], we consider the limit where the system is so deformed that all original length scales, such as its overall size or the typical distance between elements, lose their effect on drag, except the length included in the local stiffness. One may then assume that the drag F will not depend on these length scales, so that for plates $F = F(\rho U^2, B)$ and for beams $F = F(\rho U^2 d, Bd)$. A simple dimensional analysis, see also [8,19], implies that for plates $F \propto (\rho U^2)^{(1/3)} B^{2/3}$ and for beams $F \propto (\rho U^2 d)^{2/3}(Bd)^{1/3}$. In terms of the dependence on velocity, this results in

$$F \propto U^{2/3}, \qquad F \propto U^{4/3} \tag{1}$$

for plates and for beams respectively. The corresponding Vogel exponents are therefore $\mathcal{V} = -4/3$ and $\mathcal{V} = -2/3$. These values are compatible with previous results [6,7]. It must be noted here that the first value implies that the bending of the plate is such that all its original dimensions are lost in terms of drag, as in the crumpling of paper, Fig. 1(c), or the rolling of a disk into a cone [7]. This is a very strong assumption, and the deformation is often such that one of the original length scales, noted Λ, still plays a role, as in the simple bending of a plate, Fig. 1(b). We have then

$$F \propto \left(\rho U^2 \Lambda\right)^{2/3} (B\Lambda)^{1/3} \tag{2}$$

so that the Vogel exponent becomes $\mathcal{V} = -2/3$. The value of $-2/3$ is therefore common for simple bending deformations of beam and plate systems.

2.2. Vogel exponent for packing in axial flow

We consider now the very distinct case of packing of beams under axial flow, Fig. 1(d). This typically applies to packs of macroalgae under flow [21]. If the flow is tangential to the slender structure, drag is essentially friction drag. When the system is strongly deformed, the only length that remains from the original shape is its length, L, and the fluid loading depends on $\rho U^2 L$ at high Reynolds. All cross-flow dimensions have been changed by deformation, so that the drag reads $F(\rho U^2 L, Bd)$. By the same argument as above, we have now

$$F \propto \left(\rho U^2 L\right)^{2/3}(Bd)^{1/3} \quad (3)$$

so that the Vogel exponent is again $\mathcal{V} = -2/3$. In the deformation of a complex natural system, all three configurations, beams in bending, plates in bending, packing of beams in bending, are mixed. Yet, as all lead to a Vogel number of $-2/3$, the combination is expected to lead to this same value.

2.3. The case of a non-linear elastic behavior

The local bending behavior may not be linear. This may be the result of the constitutive law of the material involved, or of local geometrical non-linearities such as ovalizing, see for instance in [15]. Using a simple power law approximation of the stress–strain relation $\sigma \propto \varepsilon^{1/N}$, the local energy of deformation for plates in simple bending, or equivalently for beams, becomes $e \propto B\Lambda\gamma^{(N+1)/N}$ where Λ is the length scale that is not deformed, as in Section 2.2. The stiffness coefficient B has therefore a different dimension, so that drag reads

$$F \propto \left(\rho U^2 \Lambda\right)^{(N+1)/(2N+1)}(B\Lambda)^{N/(2N+1)} \quad (4)$$

The corresponding Vogel number is then simply $\mathcal{V} = -2N/(2N+1)$. When N goes from 1 (linear) to ∞ (perfectly plastic), $\mathcal{V}$ then only varies between $-2/3$ and -1. Even a lower value of the exponent, $N = 1/2$, leads to $\mathcal{V} = -0.5$. We may therefore state that the assumption of linear elastic behavior, made in all theoretical models of this mechanism, is not crucial. This widens the range of application of these models, as the local bending behavior of plant organs is generally not linear in the range of local deformation implied by reconfiguration.

2.4. The Vogel number for moments and stresses

The load parameters of practical importance may not be drag; moments at the base are essential to assess tree uprooting, while local bending stress will control fracture of a leaf or stem. We follow the same approach as above, and the moment is assumed to depend on the same parameters. We have then

$$M \propto \left(\rho U^2 \lambda\right)^{1/3}(B\lambda)^{2/3} \quad (5)$$

where λ is equal either to d (beams), Λ (plates in simple bending) or L and d in axial flow. The corresponding Vogel exponent is $\mathcal{V}_M = -4/3$, defined by $M \propto U^{2+\mathcal{V}_M}$. As the local dimension relating stress to moments is not modified, the stress induced by bending varies in the same way, giving the Vogel exponent for stress $\mathcal{V}_\sigma = -4/3$. For a non-linear elastic behavior we have $\mathcal{V}_M = \mathcal{V}_\sigma = -4N/(2N+1)$. Hence, the reduction of flow-induced moments and stresses by flexibility is much stronger than the reduction in drag.

3. Experimental data

Abundant literature exists on the drag of plants in flow, see for instance in [5] and references therein. In fact, a lot of the tests involve very few data points, so that the value of the Vogel exponent is difficult to assess. The level of reconfiguration, i.e. of deformation, is not always characterized. As a consequence the fit of the drag law by a power law includes several ranges of behavior. This results in an underestimation of the Vogel exponent. We focus here first on some recent experiments where abundant information exists, considering very different types of plants or plant organs. For the sake of clarity, the velocity and drag data have been normalized by the value of the first point where significant reconfiguration was evident. Then, the Vogel exponent is calculated from data above this point. Table 1 summarizes all the corresponding values of Vogel exponents, in comparison with the theoretical values proposed in the preceding section. Note that in these experiments, the Reynolds number typically varies between 10^3 and 10^4 in water and between 10^4 and 10^5 in air, when a reference length of a centimeter is used. The effect of Reynolds number on drag is therefore expected to play a minor role.

Drag on a daffodil (*Narcissus pseudonarcissus*) in air flow was measured by [10]. As the flow velocity is increased, a complex reconfiguration of the flower occurs, involving a full reorientation of the flower in the wind direction, by torsion of the stem. Fig. 2(a) shows a typical drag curve, clearly distinct from the U^2 drag law. The corresponding Vogel exponent is about -0.60.

Fountain Grass (*Pennisetum setaceum*), which is a dense assembly of long leaves, has also been tested [9]. Here, the evolution of drag is remarkably regular, although the geometry of the set of grass leaves is significantly changed in shape.

Table 1
Theoretical and experimental values of the Vogel exponent $\mathcal{V}$.

	Structure	Reference	Vogel exponent $\mathcal{V}$
Theory			
	Beams and plates (linear)	[8,19], present	−0.667
	Beams and plates (non-linear)	Present	[−0.5 −1.]
Experiments			
	Daffodil (*Narcissus pseudonarcissus*)	[10]	−0.60
	Fountain Grass (*Pennisetum setaceum*)	[9]	−0.52
	Poplar (*Populus nigra*) with leaves	[13]	−0.71
	Water parsnip (*Berula erecta*)	[22]	−0.80
	Irish moss (*Chondrus crispus*)	[14]	−0.60
	Aquatic leaves (mean value)	[11]	−0.60
	Aquatic species (41 different, mean value)	[5]	−0.57
	Terrestrial species (12 different, mean value)	[5]	−0.80

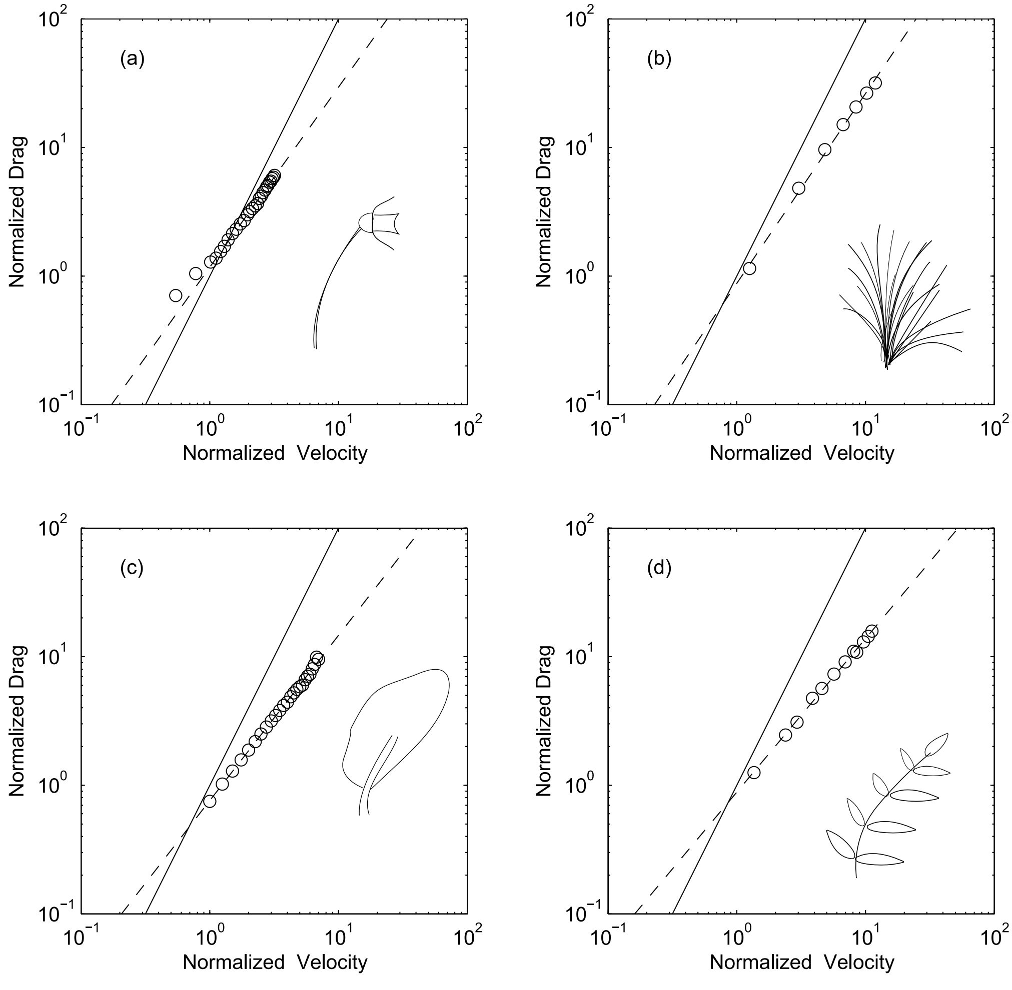

Fig. 2. Experimental data on drag as a function of the free stream velocity. All values of velocity and of drag are arbitrarily normalized for the sake of clarity, as only the power law dependence is considered. (—) Drag of a rigid body, $F \propto U^2$. (- -) Power law fit of the experimental data, $F \propto U^{2+\mathcal{V}}$ where $\mathcal{V}$ is the Vogel exponent. (a) Daffodil [10], $\mathcal{V} = -0.60$. (b) Fountain Grass [9], $\mathcal{V} = -0.52$. (c) Poplar with leaves [13], $\mathcal{V} = -0.71$. (d) Water parsnip [22], $\mathcal{V} = -0.80$.

Drag data are shown in Fig. 2(b). The corresponding Vogel exponent is about −0.52 [9]. Note that the large number of individual grass leaves probably plays a role in smoothing the drag curve. A comparison with the data of [19] is of interest, particularly on the role of porosity, but falls outside the scope of this article.

Although drag has been measured on parts of trees in wind tunnels [12], full trees have not often been considered, because of the difficulty of outdoor measurements. The recent work by [13] shows data on several poplars (*Populus nigra*), in winter (no leaves) and in spring (with leaves), under a large range of wind velocity. Drag data on a poplar with leaves are shown in Fig. 2(c). The corresponding Vogel exponent is about −0.71 for this particular tree. Without leaves it is slightly different (−0.81).

A series of aquatic plants have been tested in [22], and the Vogel exponents have been measured. We show here data on *Berula erecta*, Fig. 2(d). The corresponding Vogel exponent is about −0.80 for this particular plant, but many individuals have been tested. Remarkably, it was found that plants growing in very different levels of mean velocity did not differ in terms of Vogel exponent. Other data on a small algae [14] and on a series of leaves of various aquatic plants are also reported, Table 1.

Finally, we show in Table 1 the mean values of Vogel numbers reported by [5], obtained from the literature on 49 different aquatic species and 12 terrestrial species. The mean value is slightly different for aquatic species (−0.57) and for terrestrial species (−0.80); this difference might not be related to biological factors but to the smaller number of points generally obtained in water flume experiments.

4. Discussion and conclusions

The comparison between theory and experiments in Table 1 shows good agreement; all values of the Vogel exponent fall near the theoretical value of −2/3 for systems ranging from a single leaf of an aquatic plant to a full poplar.

The Vogel exponent $\mathcal{V}$ is a simple measure of the effect of drag reduction by reconfiguration. From a theoretical point of view, only in the limit of very strong deformation of the structure does such a simple law apply. Yet, as can be seen in the experimental data on many different systems, drag does depend on velocity with a modified power law, $F \propto U^{2+\mathcal{V}}$ over a large range of velocities. For particular systems such as a single beam or a rectangular plate, the reference value of $\mathcal{V} = -2/3$ was obtained previously [8,19]. Here, we have shown that this value is related to the loss of one characteristic length in the system, during reconfiguration. If one length is lost, be it for beams, plates or packs of beams in axial flown the Vogel exponent is always −2/3. If two lengths are lost, as in the crumpling of paper, it becomes −4/3. The observations of deformation of the plant systems that we have mentioned in this paper show that simple bending dominates. The Vogel number is therefore expected to be −2/3 in most cases. The possible local non-linearities do not modify significantly the value of the Vogel exponent. This may be related to the issue of loss of length scale effect, mentioned above, which is not affected directly by this non-linearity. Similarly, as shown in [19], porosity does not play a major effect. Considering all this it seems therefore reasonable to state that the Vogel exponent can be expected to be near −2/3, and therefore that drag will grow as $F \propto U^{4/3}$.

This may be used to understand an apparent paradox mentioned above: in a work on aquatic plants, [22], it was noted that plants that grew in a stressing environment differed in geometry, being are shorter, but that their Vogel exponent was similar. One would expect that a stressing environment would induce a growth that favors reconfiguration; this is not the case, because the Vogel exponent is only related to the topology of the deformation of the system, which is simple bending for all plant components.

Another important result of the dimensional analysis presented above is that the dependence of moments and bending stresses with the flow velocity are very different from those of drag. Moments are much more affected by flexibility because of the combined effect of load reduction and reduction of distances; they grow as $U^{2/3}$, as do stresses. In a plate system with double bending, stress would theoretically become independent of the flow velocity.

Drag reduction by reconfiguration seems to be a very robust and efficient mechanism. Plants have relied on it to bear large flow loading, and one may expect fruitful application in the design of slender flexible man-made structures.

Finally, the concept of drag reduction by reconfiguration may be extended to in-elastic behavior, such as brittle fracture. Assuming that flow-induced stress results in local fractures, it has been shown recently that flow-induced pruning is an alternative strategy in branched systems to reduce drag [23].

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