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Mean velocity profiles of fully-developed turbulent flows near smooth walls

Distribution de la vitesse moyenne près de la paroi lisse dans un écoulement turbulent

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A B S T R A C T

The work proposes an indirect turbulence model to represent the mean streamwise velocity profile of fully-developed turbulent channel flows near smooth walls. The proposed turbulence model highlights that the parameters of the velocity distribution are functions of the friction Reynolds number. It is also shown that the proposed expression for the velocity distribution is in line with the principles of dimensional analysis; it allows one to satisfy the imposed boundary conditions; in the regions close to the wall it allows to reproduce (with good agreement) the velocity profiles available in literature obtained through a Direct Numerical Simulation (DNS).

R É S U M É

Un modèle de turbulence a été développé pour représenter la distribution de la vitesse moyenne locale dans les écoulements uniformes turbulents. Le modèle proposé permet de décrire la distribution de la vitesse moyenne (en fonction du nombre de Reynolds) près de la paroi lisse d'un canal à section rectangulaire très large. Il est démontré que l'expression proposée est conforme aux principes de l'analyse dimensionnelle; elle permet de satisfaire les conditions aux limites imposées au problème ; elle permet de reproduire, dans les zones près de la paroi, les distributions de la vitesse disponibles dans la littérature et obtenues par intégration numérique directe des équations de Navier–Stokes (DNS).

1. Introduction

The impossibility for the logarithmic law and for the power law to represent the mean velocity profiles in every point of a generic cross section of a fully-developed turbulent wall-bounded flow [1,2] has led some authors to propose, in the regions close to the wall, velocity distribution laws based on experimental considerations, on semi-empirical theories or on indirect turbulence models [3–5].

An exhaustive study of the near-wall behaviour of turbulent wall-bounded flows is quoted in Buschmann et al. [6]. In Poggi et al. [7] there is an experimental technique which allows to obtain detailed information on turbulent quantities in the regions close to the wall.

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Using the equation adopted by van Driest [8] – concerning the mixing length – and the study method based on the eddy viscosity, Absi [9] has recently deduced the near smooth wall mean velocity profile from the integration of the momentum equation. With reference to fully-developed turbulent flows in smooth channels having a very wide rectangular cross section, Absi's method allows to determine the mean streamwise velocity profile u^+ in the region close to the wall defined by $0 \leqslant y^+ \leqslant 20$, having indicated, as usual, with u^+ and y^+ the non-dimensional quantities referred to the distance from the wall y, to the kinematic viscosity of the fluid υ and to the friction velocity u_τ, this latter defined by the relation $u_\tau = \sqrt{\tau_0/\rho}$, where τ_0 is the wall-shear stress and ρ the fluid density.

A different criterion to define the mean streamwise velocity profile in the domain $0 \leqslant y^+ \leqslant 20$ has been proposed by Bucci et al. [10]. The criterion, defined for fully-developed turbulent flows in smooth pipes, is based on the possibility offered by the electrolytic tank to realise an electric field distribution formally superimposable to the required velocity profile. Starting from these results, this note proposes an indirect turbulence model to represent the mean streamwise velocity profile of fully-developed turbulent channel flows near smooth walls. The proposed turbulence model highlights that the parameters of the velocity distribution are functions of the friction Reynolds number. It is also shown that the proposed expression for the velocity distribution is in line with the principles of dimensional analysis; it allows to satisfy the boundary conditions imposed to the problem; it allows to reproduce, in the regions near the wall defined by $0 \leqslant y^+ \leqslant 20$, the velocity profiles available in literature obtained through a Direct Numerical Simulation of Navier–Stokes equations (DNS).

2. Velocity profile near smooth walls

In Bucci et al. [10] it has been shown that it is possible to implement, in an electrolytic tank with a circular shape, an electric field distribution combining two electric field distributions. In detail, the first of them, ΔV_1, is obtained with a circular thin lamina with a diameter D made up of leading material with constant conductivity σ, run through by a uniform current with density j, for which:

$$\Delta V_1 = \frac{j}{4\sigma}\left(\frac{D^2}{4} - r^2\right) = C_1\left(1 - 4\frac{r^2}{D^2}\right) \tag{1}$$

where r measures the distance from the axis; the second distribution of electric field ΔV_2 is obtained applying the potential difference ΔV_0 between the condenser armours of a coaxial cylindrical condenser with internal diameter d and with external diameter D:

$$\Delta V_2 = \Delta V_0 \frac{\ln\frac{2r}{D}}{\ln\frac{d}{D}} = C_2 \ln\frac{2r}{D} \tag{2}$$

By superimposing ΔV_1 and ΔV_2, the resultant electric field distribution is:

$$\Delta V = \Delta V_1 + \Delta V_2 = C_1\left(1 - 4\frac{r^2}{D^2}\right) + C_2 \ln\frac{2r}{D} \tag{3}$$

The experimental installation used and the method to take over data are described in Bucci et al. [10] that can be consulted for every details.

For the purposes proposed by this note it is sufficient to remember that it is possible to generate an electric field distribution completely superimposable to the mean velocity profiles of fully-developed turbulent pipe flows in the regions near the smooth walls.

The results obtained by Bucci et al. [10] lead to assign to the mean streamwise velocity profile the expression:

$$u^+ = \beta_1\left(1 - 4\frac{r^2}{D^2}\right) + \beta_2 \ln\frac{2r}{D} \tag{4}$$

being: $u^+ = \frac{\bar{u}}{u_\tau}$ the non-dimensional velocity, defined by the relation between the mean streamwise velocity $\bar{u}$ and the friction velocity u_τ; r the distance from the axis of the pipe with diameter D; β_1 and β_2 two functions of friction Reynolds number $Re_\tau = \frac{u_\tau D}{\upsilon}$.

Considering the relation:

$$\frac{2r}{D} = 1 - 2\frac{y^+}{Re_\tau} \tag{5}$$

Eq. (4) can be expressed as a function of the non-dimensional distance from the wall y^+:

$$u^+ = \lambda_1 y^+\left(1 - \frac{y^+}{Re_\tau}\right) + \lambda_2 \ln\left(1 - \frac{2y^+}{Re_\tau}\right) \tag{6}$$

where

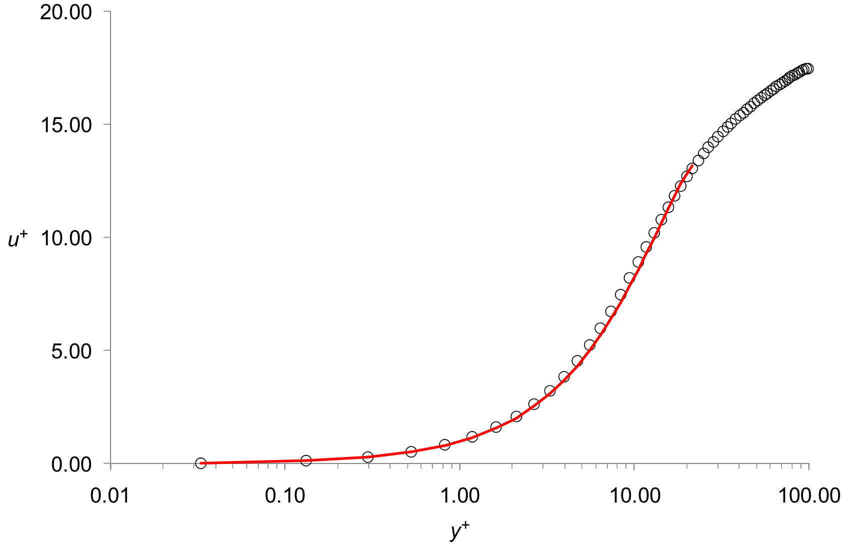

Fig. 1. $Re_\tau = 110$; ∘ DNS data [12,13]; — model results.

$$y^+ = \frac{y u_\tau}{\upsilon} \tag{7}$$

$$\lambda_1 = 4\frac{\beta_1}{Re_\tau} \tag{8}$$

$$\lambda_2 = \beta_2 \tag{9}$$

The analysis of the results obtained by Bucci et al. [10] leads to identify in the domain $0 \leqslant y^+ \leqslant 20$ the validity field of Eq. (6).

Eq. (6) finds its theoretical grounding in the following indirect turbulence model.

The formal analogy between the velocity distribution and the electric field potential distribution – obtained by the combination of two independent electric field distributions – allows one to give to viscous shear stress distribution τ^+_{vis} the expression:

$$\tau^+_{vis} = \frac{du^+}{dy^+} = \lambda_1\left(1 - \frac{2y^+}{Re_\tau}\right) - 2\frac{\lambda_2}{Re_\tau - 2y^+} \tag{10}$$

Based on the Reynolds averaged Navier–Stokes equations, according to which:

$$\tau^+_{tot} = \tau^+_{vis} + \tau^+_{tur} \tag{11}$$

the total shear stress τ^+_{tot} and the turbulent shear stress τ^+_{tur} are expressed as:

$$\tau^+_{tot} = \lambda_1\left(1 - \frac{2y^+}{Re_\tau}\right) \tag{12}$$

$$\tau^+_{tur} = 2\frac{\lambda_2}{Re_\tau - 2y^+} \tag{13}$$

Eq. (13) is in line with the complete similarity hypothesis [11]. In fact, introduced as a quantity representative of the turbulent stresses τ_{tur} the function $d\bar{u}_{tur}/dr$, being $\bar{u}_{tur}$ the velocity distribution which exclusively considers the effects connected to the components of turbulent fluctuation, the complete similarity hypothesis allows to put:

$$\tau_{tur} = \frac{r}{u_{c\text{-}tur}}\frac{d\bar{u}_{tur}}{dr} = k \tag{14}$$

where the velocity $u_{c\text{-}tur}$ is introduced to characterise the purely turbulent flow, and $k = constant$.

Setting:

$$\tau^+_{tur} = \frac{1}{u_\tau}\frac{d\bar{u}_{turb}}{dy^+} \tag{15}$$

Eq. (14) provides:

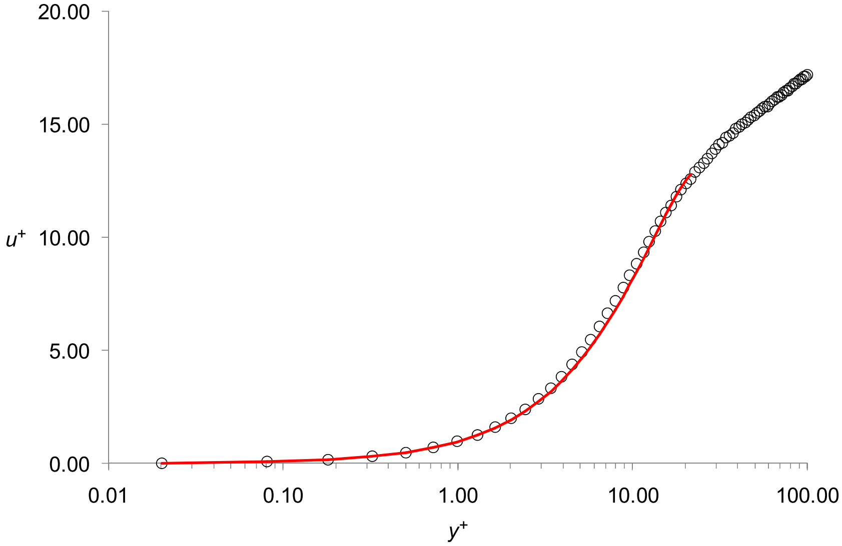

Fig. 2. $Re_\tau = 150$; ○ DNS data [12,13]; — model results.

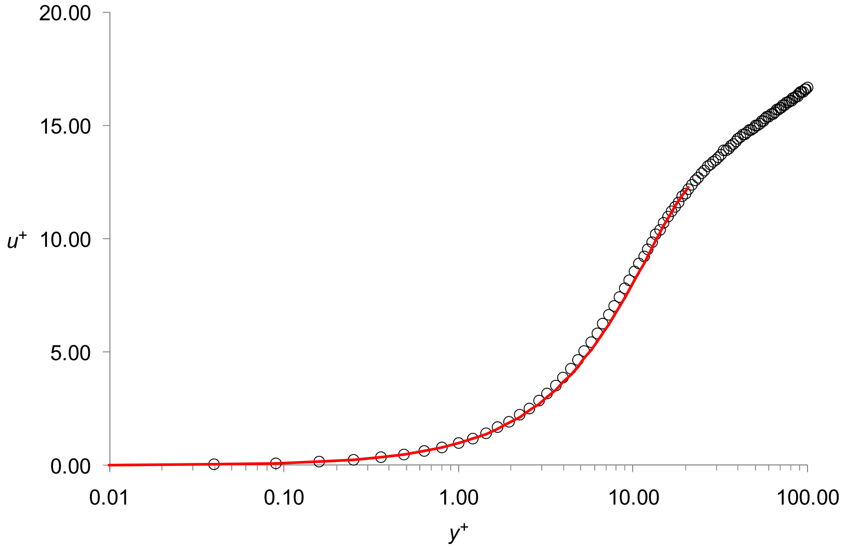

Fig. 3. $Re_\tau = 300$; ○ DNS data [12,13]; — model results.

$$\tau^+_{tur} = \lambda_2 \frac{1}{Re_\tau - 2y^+} \tag{16}$$

with:

$$\lambda_2 = -k\frac{u_{c\text{-}tur}}{u_\tau} = f(Re_\tau) \tag{17}$$

The functions λ_1 and λ_2 are connected to the relation:

$$\lambda_2 = \frac{1}{2}(\lambda_1 - 1)\,Re_\tau \tag{18}$$

that is obtained from the boundary condition:

$$\left.\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right|_{y^+=0} = 1 \tag{19}$$

The line of reasoning followed allows one to highlight that the parameters of the velocity distribution are functions of the friction Reynolds number. It is also shown that Eq. (6), besides being in line with the principles of dimensional analysis, allows one to respect the boundary conditions associated to the problem considered.

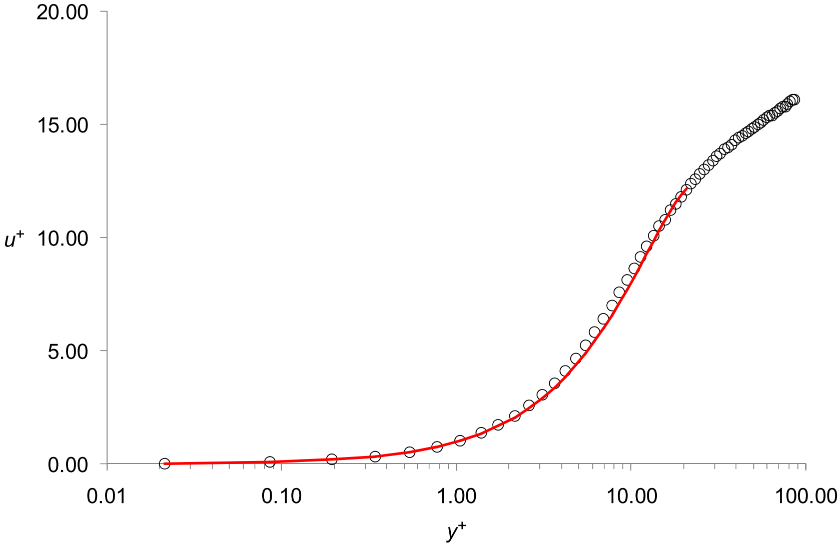

Fig. 4. $Re_\tau = 650$; ∘ DNS data [12,13]; — model results.

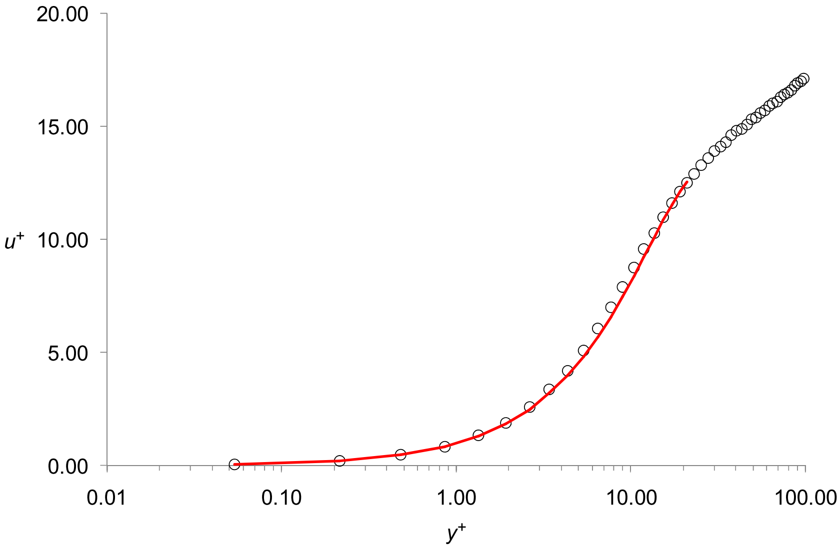

Fig. 5. $Re_\tau = 180$; ∘ DNS data [14]; — model results.

The above-mentioned considerations can be extended also to the case of fully-developed turbulent flows in smooth channels having a very wide rectangular cross section. Substituted Eq. (4) with the equation:

$$u^+ = \omega_1\left(1 - \frac{\delta^2}{h^2}\right) + \omega_2 \ln\frac{\delta}{h} \tag{20}$$

being h the channel half-width, $\delta = h - y$, ω_1 and ω_2 two functions of friction Reynolds number $Re_\tau = \frac{u_\tau h}{\upsilon}$ and considered the relation:

$$\frac{\delta}{h} = 1 - \frac{y^+}{Re_\tau} \tag{21}$$

Eq. (6) is substituted by the equation:

$$u^+ = \varphi_1 y^+\left(1 - \frac{y^+}{2\,Re_\tau}\right) + \varphi_2 \ln\left(1 - \frac{y^+}{2\,Re_\tau}\right) \tag{22}$$

having put:

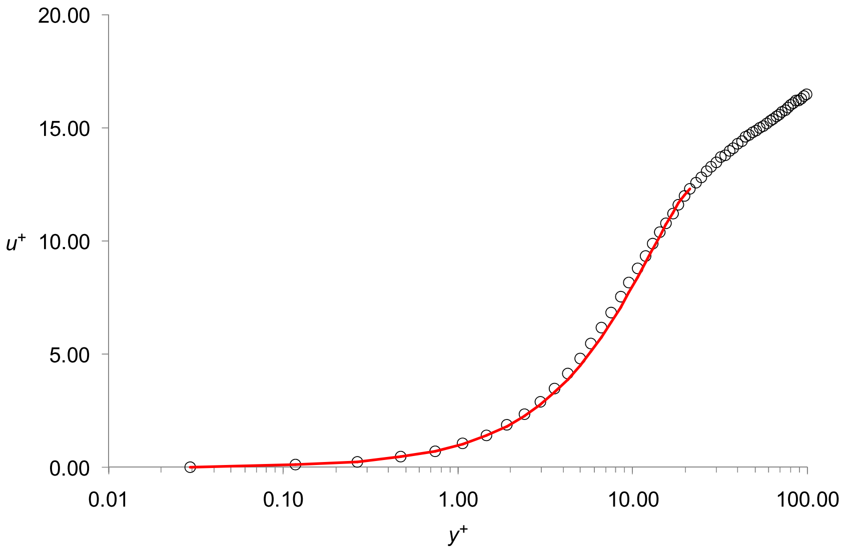

Fig. 6. $Re_\tau = 395$; ○ DNS data [14]; — model results.

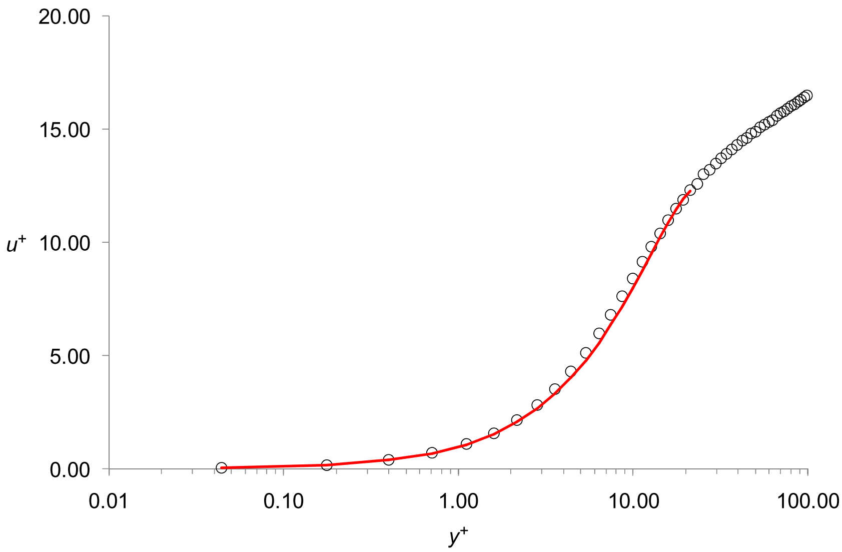

Fig. 7. $Re_\tau = 590$; ○ DNS data [14]; — model results.

$$\varphi_1 = 2\frac{\omega_1}{Re_\tau} \tag{23}$$

$$\varphi_2 = \omega_2 \tag{24}$$

Finally, the respect of the boundary condition expressed by Eq. (19) involves that:

$$\varphi_2 = (\varphi_1 - 1)\,Re_\tau \tag{25}$$

Accordingly to the approach adopted by Bucci et al. [10], the domain $0 \leqslant y^+ \leqslant 20$ defines the validity field of Eq. (22).

3. Model calibration and results analysis

The line of reasoning followed leads back the given problem, concerning the definition of mean streamwise velocity profiles in the domain $0 \leqslant y^+ \leqslant 20$, to search of the connection existing between the function φ_1 and friction Reynolds number Re_τ. For this purpose it has been useful to have recourse to the velocity profiles available in literature deduced with the numerical method DNS [12,13]. With reference to the fully-developed turbulent channel flows, on the basis of the numerical results deduced with the DNS for $Re_\tau = 110$; 150; 300; 650, it is proposed to give to Eq. (23) the expression:

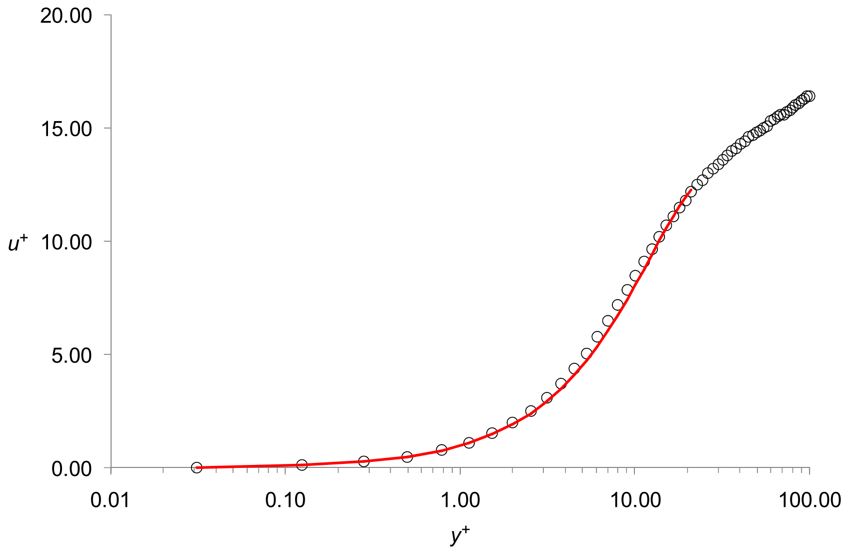

Fig. 8. $Re_\tau = 950$; ∘ DNS data [15]; — model results.

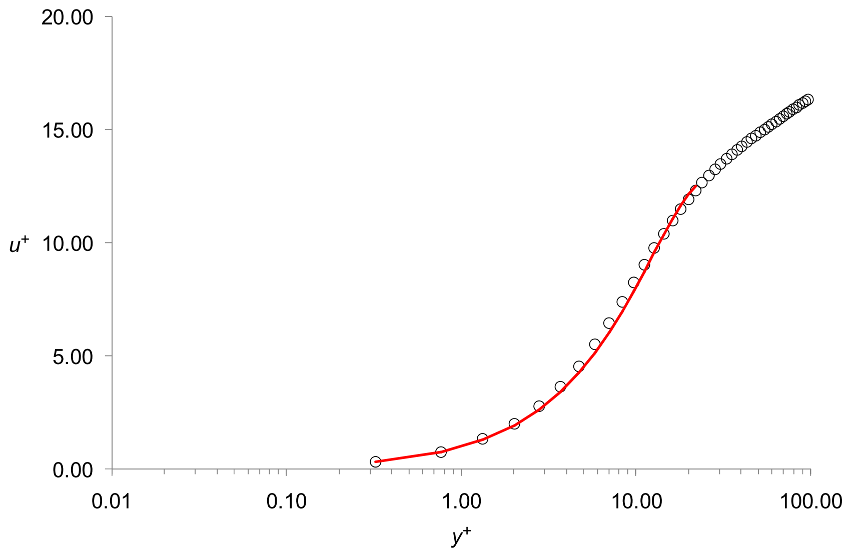

Fig. 9. $Re_\tau = 2003$; ∘ DNS data [16]; — model results.

$$\varphi_1 = 0.025\, Re_\tau^{0.969} \tag{26}$$

The comparison between the DNS data and the results obtained with the application of Eq. (22), completed with Eqs. (25) and (26) is reported in Figs. 1, 2, 3 and 4.

The robustness of the proposed model is verified using the numerical results deduced with the DNS for $Re_\tau = 180$; 395; 590; 950; 2003 [14–16]. The comparison between the DNS data and the results obtained with the application of Eq. (22), completed with Eqs. (25) and (26) is reported in Figs. 5, 6, 7, 8 and 9.

The model results show good agreement with DNS data: for the examined cases, the percentage error was less than 7%.

In the buffer region, the velocity distribution values calculated with the proposed expression are smaller than those ones obtained with DNS. The presence of this systematic error is closely connected to the adopted indirect turbulence model and, in particular, to the complete similarity hypothesis introduced to characterise the turbulent shear stress: this hypothesis, therefore, finds partial confirmation on the basis of the obtained results.

4. Conclusions

This short paper may be regarded as a direct continuation of Bucci et al.'s work [10]: in this work it has been proposed a criterion for the deduction of the mean streamwise velocity profile in turbulent pipe flows in the region near the smooth walls. In the same work it has been shown that the expression proposed for the velocity profile is formally similar to that defining an electric field distribution generated in an electrolytic tank. The validity of the approach has been verified proceeding to the comparison between the electric field values collected in the electrolytic tank and the experimental data of the mean streamwise velocity known in Literature.

After showing that the expression proposed by Bucci et al. [10] finds its justification in theoretical basis and not only in the best agreement to the experimental data, this note proposes an expression to represent the mean streamwise velocity profile in turbulent channel flows near smooth walls. Moreover, it is shown that the expression proposed, besides satisfying the boundary conditions imposed to the problem, allows to reproduce with good agreement, in the regions defined by $0 \leqslant y^+ \leqslant 20$, the velocity profiles available in literature obtained through a Direct Numerical Simulation of Navier–Stokes equations (DNS).

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