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Wheel/rail contact model for rail vehicle dynamics

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ABSTRACT

An advanced method using progressive concept of correspondence is applied to create new wheel/rail model based on the virtual penetration theory. The geometry and mechanism of contact are solved simultaneously because of the independency in a defined correspondence. This model takes the penetrated profiles of wheel and rail and also associated creeps as its inputs, and produces driving contact forces as its outputs. The advantage of model is that there is no need to pre-tabulate rigid contact situation. This method allows calculating flexible, non-elliptical, multiple contact patches during integration of the model.

1. Introduction

The contact problem is a combination of geometrical and elastic based difficulty. Each of them has particular complexity. In the first problem, the geometry of contact, the main attention is on finding the contact angle between two penetrated surfaces. To do this, various methods, such as solving a set of algebraic equations and nodal search, are presented up to now [1–3]. The application of interpolation methods is very common in them and this leads to difference in the details. In the majority of the cases, the main effort is on decreasing the time of solution with a sufficient accuracy.

The other problem, the mechanism of contact, is a complicated problem with several influential factors. The new investigations prove that the Hertzian assumptions for the contact lead to unpromising results [4]. Particularly for contact surfaces of wheel and rail, the radius curvature changes considerably along the profile. This causes that the new methods have been emerged having a base of theory of Hertz such as the sets of ellipses and virtual penetration. Piotrowski and Chollet get a comprehensive overview about them [4]. Moreover the wheel/rail problem is tied to the rolling contact problem and met with a stick-slip phenomenon at contact patch. Although Kalker [5] solves this problem based on the half-space assumption for three dimension bodies, but it is too time consuming for the simulation of railroad vehicle/track systems. Consequently the simplified methods develop for modeling creep forces. Most of them are designed based on Kalker linear theory which with the Hertzian assumption in its base. Up to now, most of literatures divide the geometrical and elastic based difficulties. Some of them focus only on solving the geometry of contact and finding geometrical information such as the rigid contact pair, contact angle, penetration distance and etc. [1–3]. Unfortunately some other methods concern just about developing new approaches based on Hertz and Kalker linear theories and they express more complicated formulas [6,7]. They supposed that the undeformed surfaces of two bodies, touching in the geometrical point of contact, and then they are shifted towards each other. But in practical points of view, in each step of simulation of railroad vehicle/track systems with flexible contact, the direction of penetration is unknown. For instance, Ayasse and Chollet [6] recommend to tabular the geometrical values of each strip in a preprocessing step for implementation their algorithm in a dynamic model. However pre-calculated

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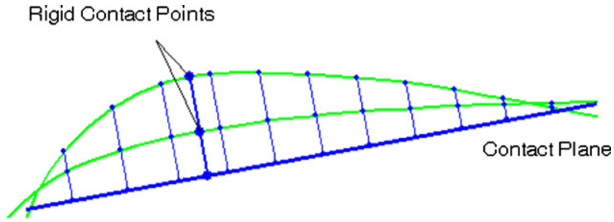

Fig. 1. Projecting on Contact Plane method.

tables does not require a very fast or completely reliable method of calculation, it has pre-calculation cost for only one wheel and rail profiles and doesn't have capacity to modeling system with variable profile.

Now, based on above description, some questions will be raised:

1. Which parameters cause a separation between geometrical and elastic procedure?
2. Why most of literatures divide their model to geometrical and elastic process?
3. Is it possible finding a method to destroy this separation?
4. Is it feasible to solve these two problems simultaneously?

The authors believe that some basic rules must be re-examined to answer these questions. The principal efforts of this article are on developing of the concept of correspondence in contact zone, and employ it to present a new wheel/rail contact model.

During last decades, only two correspondences, projecting on contact plane and nearest point, are commonly used and there is no new development. In a current work, the new correspondence based on isosceles triangle will be presented. Moreover it will be demonstrated that only by adopting a preceding methods to the new presented correspondence, we can present a new wheel/rail model. In this model the new control based method will become combined with STRIPES [6] in the new environment of correspondence bases. This model finds the contact angle and the contacts forces simultaneously. In addition it has all of STRIPES method properties.

2. The strategy of solution

The contact problem is a mutually situation which requires a solution. In this situation the surfaces of two bodies faced to each other such as embattle of two armies. Notwithstanding these two bodies are independent, in fact a typical logical connection makes a relation between them. This relation is distinctive correspondence between two surfaces. The writers believe that depiction of this correspondence is a key for simplification of contact problem in multibody dynamics applications. For instance when we have rigid contact, it is possible to make correspondence by projecting the points of two surfaces to the contact plane which be tangent to the surfaces at contact point. However, in cases of flexible contact that the surfaces are penetrated towards each other, formation of the correspondence is more complex. In the most literatures, two specimens of corresponding points are commonly applied for penetrated surfaces. The first one is a rigid contact pair that line segment between them is perpendicular to the tangents of the surfaces. The second one is intersectional pair which has a same global position. For the other points near the contact zone, there is no exact defined correspondence until now. There are two methods presented up to the present time and a new method will be introduced in this article. They are as below:

2.1. Projecting on contact plane

In this method, first the location of rigid contact pair must be determined for understanding the contact angle and contact plane. Then the projection of points on the contact plane characterizes the correspondence between two surfaces. In a widespread manner, this method is used for contacts theories and models [4,6–8]. When the undeformed surfaces of two bodies, touching in the geometrical point of contact, and then they are shifted towards each other, Projecting on Contact Plane method is usable. As can be seen in Fig. 1, from a practical point of view, in dynamic simulations this method involves with three sequential problems:

1. Finding the rigid contact pair and contact angle.
2. Projecting the points of one surface to the contact plane.
3. Inverse projecting of projected points to another surface.

In above procedure there, the first and third ones don't have a simple solution. This method makes one-to-one correspondence between two surfaces. In another view, it makes a dependent correspondence. It means that there isn't possible to find independently corresponding point from other information such as contact angle. The contact zone is determined by the coordinates which are attached to contact plane.

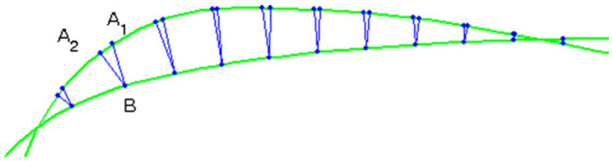

Fig. 2. Nearest Points method.

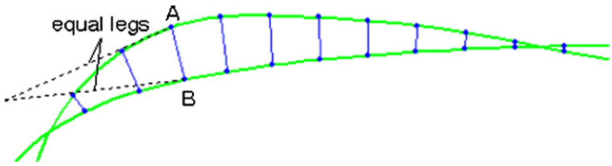

Fig. 3. Isosceles Pairs method.

2.2. Nearest points

This method expresses that the nearest point on the opposite surface is corresponding point. However it satisfies rigid contact and intersectional pairs but it isn't one-to-one correspondence and just unilateral. For instance, in Fig. 2, point B is the nearest point to point A_1, but the nearest point to B at opposite surface is A_2. As we know for the smoothing surface, the line segment between one point outside the surface and nearest point on the surface is normal to surface. But if this line segment is also normal to opposite surface, then these two points is rigid contact pair. However this correspondence is unilateral, it can help us to find rigid contact pair and some algorithms work base on this rule [1,2]. Other important advantage of this method is that the corresponding point of each point is independent from others. In this method the contact zone is determined by the parameter which represents the arc length of one curve.

2.3. Isosceles pairs

This method state that two points on opposite surfaces are corresponding when crossing point of normal (or tangent for 2D) lines of them makes an isosceles triangle which the crossing point is a head of triangle. According to Fig. 3 in this method, the angles between normals (or tangent for 2D) and a line segment are equal. This independent rule satisfies rigid contact and intersectional pairs and it is possible to prove that this relation is locally one-to-one. In comparison with previous methods, this method is independent correspondence and in addition it is one-to-one. The contact zone is determined by parameters which represent the arc lengths of the curve that is traversed by middle point of corresponding points.

After introducing with three different methods, we must select one method or combined method for modeling the contact. It seems that one-to-one corresponding is more logical, because the contact problem has symmetry connection in its nature.

Also when the correspondence is dependent, it means that the procedure is more time consuming. In this article, these two parameters force us to concentrate on Isosceles Pairs method which has one-to-one property and independence together.

Now the question is that how can we apply this method in railway vehicle dynamics. First we should find a fast procedure to make correspondence between facing surfaces and then estimate normal and tangents forces. In most cases the slop of contact changes continuously along boundary. This means that the correspondence of contact is also continuous and this is the key of problem. If two corresponding points move with controlled speeds along the surface, then all of corresponding points will be swept. The control object is holding the angle difference at zero. So there is no need to use searching algorithms or solving a set of algebraic equations and this subject improve the robustness and quickness of procedure.

Now the problem is dealing with control base difficulty. The time and accuracy limitations increase the importance of control procedure. However a lot of resources of control are available but it is preferred to come back to the innate behavioral patterns of contact geometry. In the next section we will discuss about motion on two curves in details. After geometrical solution, all of the discrete corresponding points can be applied for analysis of contact forces and this is one of the benefits of a stated method.

3. Wheel/rail contact model

3.1. The geometry of profiles

The smoothing cubic spline representation is used for measured cross-section of wheel and rail. In these functions, the vertical position (y) of profile coordinate system depends only on the lateral position (x). Also the first and second derivatives can be obtained directly from spline functions. Clearly there isn't necessary that profile's coordinate is parallel to global coordinates. The global position is routinely obtained by the transformation matrix and global position vector of

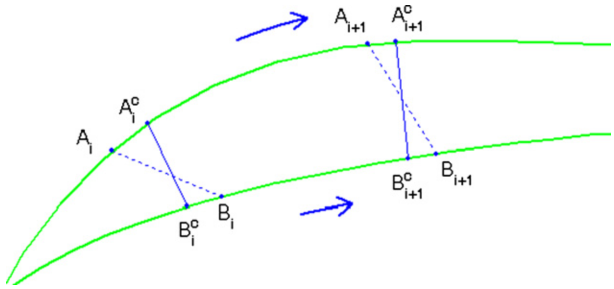

Fig. 4. Traveling procedures.

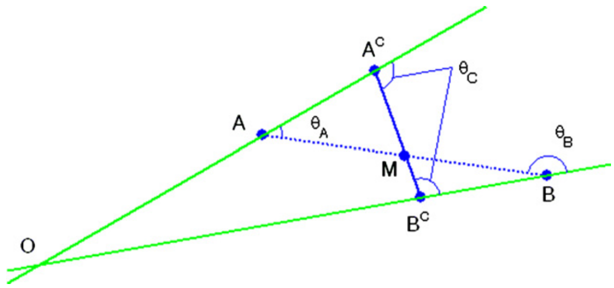

Fig. 5. Correction mechanism.

profile origin. In this article there is supposed that in each dynamic step, all of the situation information is function of (x), such as location, cant, radius and other.

3.2. Traveling on curves

During traveling on curves two procedures are successively repeated:

1. *Stepping:* This means that both points must traverse same arc length on curves. Later in next stage, the traveling will be corrected, so there isn't necessary to estimate arc length exactly. Knowing the cant at current position is enough for estimation. In traveling usually there is needed to pass intersectional pair, and it is conceivable that the paired points encounter with them and this may confuse the motion. Therefore the Crossing Distance (*CD*) will be defined to predict remained arc length to junction.

$$CD = \frac{\overline{AB}}{2\cos(\theta_C)} \tag{1}$$

   In this stage, selected arc length must have at least 20% deviation from *CD*. Also *CD* helps us to step with a variable arc stepping. If the traveling is out of penetrated zone, there is not necessary to move accurately. So it is preferred to increase step arc.
2. *Correction:* In this stage, the leading point must be pulled back and the lagging point must be pushed forward. As can be seen in Figs. 4 and 5, points of A and B are produced from stepping stage and A^c and B^c are the corrected points. To do this, the curves are marked as two crossing lines which are tangent to the points. $\Delta\theta = \theta_A - \theta_B$ presents the objective error of traveling, wherein:

$$\theta_A = \mathrm{Arc}\cos\left(\frac{\overrightarrow{AB} \cdot \overrightarrow{T_A}}{|\overrightarrow{AB}| \cdot |\overrightarrow{T_A}|}\right) \tag{2}$$

$$\theta_B = \mathrm{Arc}\cos\left(\frac{\overrightarrow{BA} \cdot \overrightarrow{T_B}}{|\overrightarrow{AB}| \cdot |\overrightarrow{T_B}|}\right) \tag{3}$$

   and clearly:

$$\theta_C = (\theta_A + \theta_B)/2 \tag{4}$$

   M is the middle point of $\overline{AB}$. So it is possible to show that:

$$\overline{AA^c} = \overline{BB^c} = \frac{\sin(\Delta\theta/2)}{\sin(\theta_C)} \times \frac{\overline{AB}}{2} \tag{5}$$

   therefore arc length error is obtained.

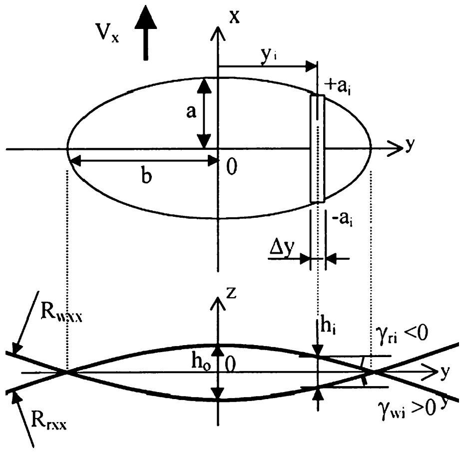

Fig. 6. Local parameters in the contact area [6].

3.3. Rigid contact pair and contact angle

As stated, Rigid contact pair is one of the corresponding points. In its situation the value of θ_C is equal to 90°. Two different approaches can be used to find its accurate position of it. The first approach controls arc length stepping until achieve desired tolerance. The second one is moving with small steppings inside the penetrated zone, and after traversing the whole faced profiles, rigid contact pair will be found.

3.4. Calculation of contact forces

Piotrowski and Chollet [4] give a comprehensive overview of the methods used in vehicle system dynamics that including multi-point contact simultaneously. They divide methods to two groups. The first one which is called as set of ellipses has superposition idea in its nature. The other methods are based on so-called virtual penetration. The assumption for this method is the semi-elliptical normal stress distribution in the rolling direction. Each of them has specific preferences. The consideration of this article is on Stripes method [6]. This method attempts to estimate the non-elliptical contact patches with a discrete extension of the Hertz theory. The Stripes method approximately solves the normal contact problem. In order to solve the tangential problem, the FASTSIM algorithm is adapted to Stripes. Quest tried to assess it by exact results of FEM and BEM methods in his PhD thesis. In his presented paper [8], he compares Stripes results, in terms of force versus displacement law, contact shape, and pressure distribution inside the contact patch, with ABAQUS FEM solver on three test cases of a curvature change, a conforming contact, and an overlapping of contact patches. In addition the tangent stress distribution creep forces are compared with Contact results.

In the presented paper, all of the formulation demonstrated by Stripes method is used without any conversions and just the definition of strip is converted from projecting correspondence to isosceles correspondence. First the STRIPES method formulations will be presented in a condensed form and then the effect of correspondence conversions will be discussed.

3.4.1. The Stripes method

In general form, the Stripes method estimates stresses at each strip after compensation of the shape ratio error. This compensation is done for adaptation of the shape ratio of this method with Hertz theory in constant curvature condition. If the compensation is done on the A curvature which corresponds to the rolling direction, then the following simple expressions is obtained as [6]:

$$A_0^C = \frac{n_0^2}{m_0^2} B_0 \tag{6}$$

According to Fig. 6, participation depth (h_i) and participation length (a_i) of each strip is determined based on rigid contact penetration (δ_0):

$$h_0 = \varepsilon_0 \delta_0 \tag{7}$$

$$\varepsilon_0 = \frac{n_0^2 B_0}{r_0(A_0 + B_0)} \tag{8}$$

$$h_i = \max\big(0, \delta_0(1 - \varepsilon_0) - z_i\big) \tag{9}$$

$$a_i = \sqrt{\frac{h_i}{A_i^C}} \tag{10}$$

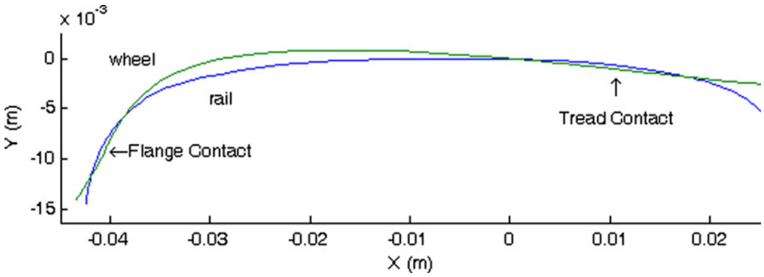

Fig. 7. Wheel/rail sample situation with flange and tread contacts.

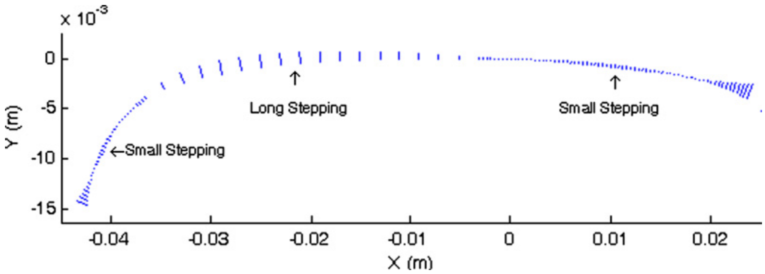

Fig. 8. Discretization of the contact area by isosceles base strips: case 1.

The subscript i denotes strip number and 0 denotes rigid contact strip, where n and r stand for Hertz parameters which are tabulated according to the A/B. The normal and shear stresses are expressed as below:

$$\sigma_{zz,i} = \frac{1}{\pi}\frac{1}{n_i r_i \varepsilon_i}\frac{E}{1-\upsilon^2}\sqrt{1-\left(\frac{x}{a_i}\right)^2}\frac{h_i}{a_i} \tag{11}$$

$$\sigma_{zx,i} = -\frac{3}{8}Gc_{11,i}v_{x,i}\left(1-\frac{x}{a_i}\right)\frac{a_i}{a_0} \tag{12}$$

$$\sigma_{zx,i} = -\left(\frac{3}{8}Gc_{22,i}v_{y,i} + \frac{2}{\pi}\sqrt{\frac{n_i}{m_i}}Gc_{23,i}\varphi_i(a_i+x)\right)\left(1-\frac{x}{a_i}\right)\frac{a_i}{a_0} \tag{13}$$

$$N_i = \frac{1}{2}\frac{1}{n_i r_i \varepsilon_i}\frac{E}{1-\upsilon^2}h_i\delta y_i \tag{14}$$

3.4.2. Effect of correspondence conversions

The strips in isosceles correspondence in comparison with Stripes method have a little rotation. It is assumed that the midpoint of isosceles strip is on the plane of projecting strip. By linear approximation, it is possible to show that:

$$z_i^{is} \approx z_i/\cos(\alpha) \tag{15}$$

As can be seen in Eq. (8) the value of h_i^{is} is larger than h_i. Also because the coordinate of isosceles correspondence moves with the mid point of strip, the differential distance of isosceles correspondence (δy^{is}) is larger than δy. By the same approximation we have:

$$\delta y_i^{is} \approx \delta y_i/\cos(\alpha) \tag{16}$$

therefore

$$z_i^{is}\delta y_i^{is} \approx z_i \delta y_i \tag{17}$$

Hence in Eq. (14), the normal force is approximately constant.

4. Simulation results

In this section, we demonstrate a sample situation of wheel/rail contact with tread and flange contact, as shown in Fig. 7. UIC60 and S1002 profiles are selected in this examination. Figs. 8 and 9 show the motion of corresponding pairs between two faced surfaces while passing two penetrated zones for two cases. In case 1, arc length stepping is 0.5 and in case 2, it is 0.1 mm. Clearly out of penetrated zone, it is increased to improve calculation speed. As it can be seen in Fig. 10, the error

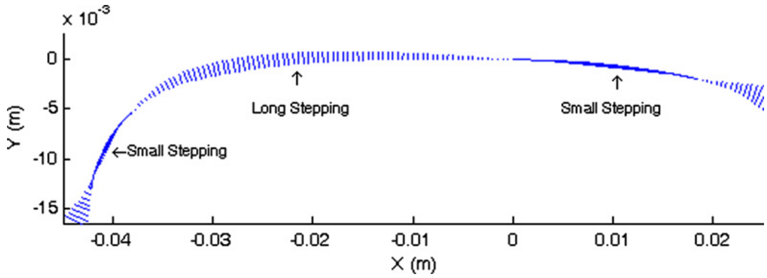

Fig. 9. Discretization of the contact area by isosceles base strips: case 2.

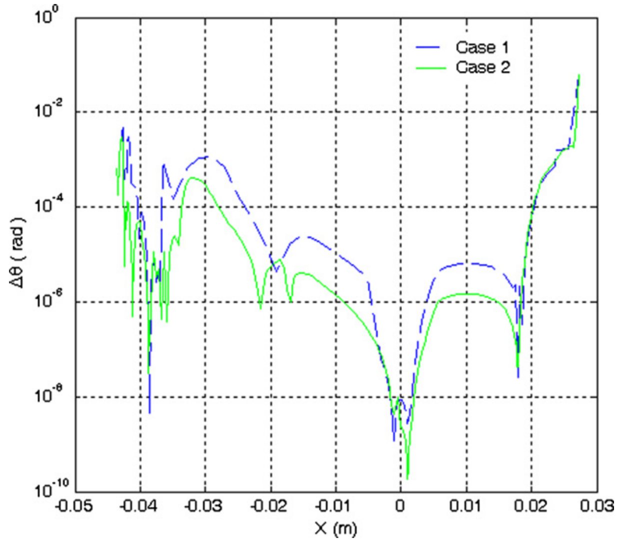

Fig. 10. Error of traveling.

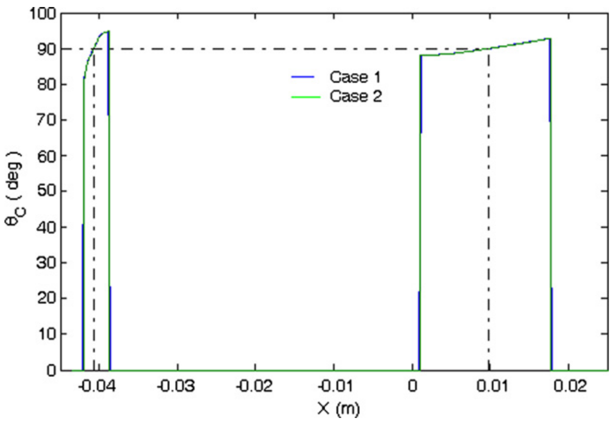

Fig. 11. θ_C of the contact area by isosceles base strips.

of traveling is decreased by reducing arc stepping length from 0.5 to 0.1 mm. Near the tread contact patch, this error is less than 1e−5, and also near the flange contact patch, it doesn't exceed 1e−3. The cause of this increase near the flange is the quick growth of slope at curvatures. After traveling on curves, the value of θ_C is plotted for penetrated zone, as is illustrated Fig. 11.

Knowing the fact that at Rigid contact pair the value of θ_C is equal to 90°, the location of Rigid contact pair is determined, based on that, the contact angle is determined. Fig. 12 shows determination of contact angle in flange and tread contact zones. In Tables 1 and 2, the results of wheel/rail contact model are listed. The error of contact angle and contact position are estimated based on exact geometrical solution. Also the error of normal force is relative to fine solution of Stripes method.

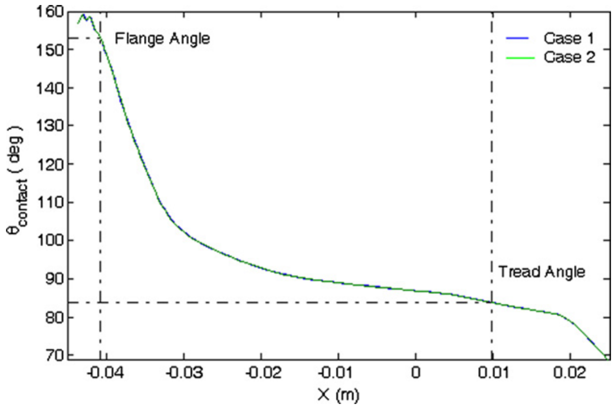

Fig. 12. Contact angles in flange and tread contact zones.

Table 1
The results of wheel/rail contact model: case 1.

	Value	Error (%)
Tread contact		
Contact location (mm)	9.723	0.23
Contact angle (deg)	83.92	0.17
Normal force (kN)	205.8	0.12
Flange contact		
Contact location (mm)	−40.94	0.29
Contact angle (deg)	153.54	0.33
Normal force (kN)	66.4	3.26

Table 2
The results of wheel/rail contact model: case 2.

	Value	Error (%)
Tread contact		
Contact location (mm)	9.757	0.03
Contact angle (deg)	83.08	0.03
Normal force (kN)	206.1	0.02
Flange contact		
Contact location (mm)	−40.83	0.02
Contact angle (deg)	153.09	0.04
Normal force (kN)	65.3	0.85

5. Conclusion

The concept of geometrical correspondence in the contact zone is developed in this paper. The new correspondence based on isosceles triangle is illustrated. In comparison with previous methods, this method offers independent correspondence which is in addition one-to-one. A new wheel/rail model was developed based on new method. In this model the new control based method was combined with Stripes method in the new correspondence-bases environment. This model determines the contact angle and the contacts forces simultaneously, while all Stripes method properties are retained.

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