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Singular layers for transmission problems in thin shallow shell theory: Rigid junction case

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ABSTRACT

In this Note we study two-dimensional transmission problems for the linear Koiter's model of an elastic multi-structure composed of two thin shallow shells. This work enters in the framework of singular perturbation of problems depending on a small parameter ε. The formal limit problem fails to give a solution satisfying all boundary and transmission conditions; it gives only the outer solution. Both in the case of regular or singular loadings, we derive a limit problem which allows us to determine the inner solution explicitly.

RÉSUMÉ

Dans cette Note on s'intéresse à des problèmes de transmission bidimensionnels pour le modèle linéaire de Koiter d'une multi-structure composée de deux coques peu profondes élastiques minces. Ce travail rentre dans le cadre des problèmes de perturbation singulière dépendant d'un petit paramètre ε. Le problème limite formel ne donne pas une solution vérifiant toutes les conditions au bord et de transmission ; il donne seulement la solution « extérieure ». Pour des chargements singuliers ou réguliers nous identifions le problème limite qui nous permet de déterminer la solution « intérieure » explicitement.

Version française abrégée

Dans cette Note, on étudie un problème de transmission bidimensionnel, il s'agit d'un problème simplifié du modèle linéaire de coque peu profonde de Koiter avec la condition de jonction rigide (voir [1, I.7 et III.3]). Ce travail rentre dans le cadre des problèmes de perturbation singulière dépendant d'un petit paramètre ε. On considère deux coques peu profondes minces C et $\tilde{C}$ de même épaisseur ε, de surface moyenne S et $\tilde{S}$. La surface moyenne est elliptique totalement encastrée présentant une arête Γ. Les coefficients d'élasticité peuvent être différents dans chaque partie de la multi-structure, mais ils sont de l'ordre O(1).

Soient ω et $\tilde{\omega}$ deux domaines plans, ϕ et $\tilde{\phi}$ deux bijections telles que :

$$\omega = (0,\pi)_{x_1} \times (0,1)_{x_2}, \qquad \tilde{\omega} = (0,\pi)_{x_1} \times (0,1)_{\tilde{x}_2}, \qquad \Sigma = \{(x_1,0,0) \mid 0 < x_1 < \pi\} \tag{1}$$

$$\Omega = \omega \cup \Sigma \cup \tilde{\omega}, \qquad S = \phi(\omega), \qquad \tilde{S} = \tilde{\phi}(\tilde{\omega}), \qquad \Gamma = \phi(\Sigma) = \tilde{\phi}(\Sigma), \qquad \Gamma_0 = \partial\Omega \cap \partial\omega \quad \text{et} \quad \tilde{\Gamma}_0 = \partial\Omega \cap \partial\tilde{\omega} \tag{2}$$

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D'après [1, III.3] la formulation variationnelle du problème de jonction rigide est donnée par (3). Pour ε fixé l'existence et l'unicité est une conséquence directe du lemme de Lax–Milgram. Le problème limite formel i.e. $\varepsilon = 0$ est mal posé. Les conditions au bord et de transmission ne peuvent être satisfaites, une couche limite ordinaire sur le bord et une couche limite de transmission sur Σ apparaissent. Dans le cas des chargements réguliers (resp. singuliers), notre méthode consiste à effectuer le changement de variables et d'inconnues (7) et (8) (resp. (7) et (17)). On obtient alors un nouveau problème (10) (resp. (18)) dont la solution du problème limite formel est une approximation valable au voisinage de la zone de transmission.

1. Introduction

This Note is devoted to transmission problems for the linear Koiter's model of an elastic multi-structure composed of two thin shallow shells for the rigid junction case [1, I.7 and III.3]. We consider the case of two elliptic surfaces such that the common middle surface is stiff and piecewise elliptic with a clamped fold (see [2]). We recall that the surfaces (subjected to the corresponding kinematic boundary conditions) which do not admit pure bending are called stiff [2]. The elasticity coefficients can be different in each part of the junction but they have the same order O(1). This work is in the general framework of singular perturbation of problems depending on a small parameter ε. The formal limit problem does not give a conventional approximate solution, boundary layers and singular layers along the edge appear. We only deal here with a rigorous treatment of the singular layers arising along the edge. We use the convention of summation of repeated indices, which run from 1 to 2 when they are Greek. The notation $(.)^{\{c\}}$ means that the property holds true for both $\omega, \tilde{\omega}, u, \tilde{u}, x, \tilde{x}, \ldots$, i.e., on both shells.

2. The model problem and its formal limit

Let us consider two thin elliptic shallow shells C and $\tilde{C}$ with the same thickness ε, whose middle surfaces S and $\tilde{S}$ intersect along a common boundary Γ. Namely we assume that the domains are given by (1)–(2), i.e., $S^{\{c\}}$ is the image through the bijection $\phi^{\{c\}}$ of the bounded open subset $\omega^{\{c\}}$ of the plane of variables $x_1^{\{c\}}, x_2^{\{c\}}$ which represent the local coordinates in the basis:

$$a_\alpha^{\{c\}} = \partial_\alpha^{\{c\}} \phi^{\{c\}} = \frac{\partial \phi^{\{c\}}}{\partial x_\alpha^{\{c\}}}, \quad \alpha = 1, 2 \quad \text{and} \quad a_3^{\{c\}} = \left|a_1^{\{c\}} \times a_2^{\{c\}}\right|^{-1}\left(a_1^{\{c\}} \times a_2^{\{c\}}\right)$$

We suppose that the common middle surface is clamped along the whole boundary.

We consider a simplified problem, when the coefficients of the second fundamental form of the middle surface $S^{\{c\}}$ satisfy $b_{11}^{\{c\}} = b_{22}^{\{c\}} = 1$, $b_{12}^{\{c\}} = 0$ and the elasticity coefficients $A^{\alpha\beta\gamma\lambda,\{c\}}$ form the unit matrix:

$$A^{\alpha\beta\gamma\lambda,\{c\}}\gamma_{\gamma\lambda}\left(v^{\{c\}}\right) = T^{\alpha\beta,\{c\}}\left(v^{\{c\}}\right) = \gamma_{\alpha\beta}\left(v^{\{c\}}\right) = \frac{1}{2}\left(\partial_\alpha^{\{c\}} v_\beta^{\{c\}} + \partial_\beta^{\{c\}} v_\alpha^{\{c\}}\right) - b_{\alpha\beta}^{\{c\}} v_3^{\{c\}}$$

$$\frac{A^{\alpha\beta\gamma\lambda,\{c\}}}{12}\varrho_{\gamma\lambda}\left(v^{\{c\}}\right) = M^{\alpha\beta,\{c\}}\left(v^{\{c\}}\right) = \partial_{\alpha\beta}^{\{c\}} v_3^{\{c\}}$$

According to [1, III.3], the variational formulation of the problem is:

$$\text{Find } \left(u^\varepsilon, \tilde{u}^\varepsilon\right) \in H \quad \text{s.t.} \quad a\left(u^\varepsilon, v\right) + \varepsilon^2 b\left(u^\varepsilon, v\right) + \tilde{a}\left(\tilde{u}^\varepsilon, \tilde{v}\right) + \varepsilon^2 \tilde{b}\left(\tilde{u}^\varepsilon, \tilde{v}\right) = \langle f, v\rangle + \langle \tilde{f}, \tilde{v}\rangle, \quad \forall (v, \tilde{v}) \in H \tag{3}$$

where

$$H = \left\{(v, \tilde{v}) \in V \times \tilde{V} \;\middle|\; \left\{v = \tilde{v} \text{ and } \frac{\partial v_3}{\partial x_2} = \frac{\partial \tilde{v}_3}{\partial \tilde{x}_2}\right\} \text{ on } \Sigma\right\}$$

$$V^{\{c\}} = \left\{v^{\{c\}} \in H^1\left(\omega^{\{c\}}\right)^2 \times H^2\left(\omega^{\{c\}}\right) \;\middle|\; v^{\{c\}}_{|\Gamma_0^{\{c\}}} = \left(\frac{\partial v_3^{\{c\}}}{\partial n^{\{c\}}}\right)_{|\Gamma_0^{\{c\}}} = 0\right\}$$

$$a^{\{c\}}(u, v) = \int_{\omega^{\{c\}}} \gamma_{\alpha\beta}(u)\gamma_{\alpha\beta}(v)\,\mathrm{d}x^{\{c\}}, \qquad b^{\{c\}}(u, v) = \int_{\omega^{\{c\}}} \varrho_{\alpha\beta}(u)\varrho_{\alpha\beta}(v)\,\mathrm{d}x^{\{c\}}$$

For a fixed ε the existence and uniqueness of problem (3) are ensured by the Lax–Milgram lemma. This solution satisfies the transmission system (of total order 16):

$$\begin{cases} -\partial_\alpha T^{\alpha\beta}(u^{\varepsilon,\{c\}}) = f^{\beta,\{c\}} \\ -b_{\alpha\beta}^{\{c\}} T^{\alpha\beta}(u^{\varepsilon,\{c\}}) + \varepsilon^2 \partial_{\alpha\beta\lambda\mu}^{4,\{c\}} u_3^{\varepsilon,\{c\}} = f^{3,\{c\}} \end{cases} \quad \text{in } \omega^{\{c\}} \tag{4}$$

with the Dirichlet boundary conditions: $u_\alpha^\varepsilon = u_3^\varepsilon = \frac{\partial u_3^\varepsilon}{\partial n} = \tilde{u}_\alpha^\varepsilon = \tilde{u}_3^\varepsilon = \frac{\partial \tilde{u}_3^\varepsilon}{\partial \tilde{n}} = 0$ on $\partial\Omega$, the rigid junction conditions and the action–reaction principle giving (eight) transmission conditions:

$$\begin{cases} \tilde{u}_2^\varepsilon - u_2^\varepsilon \cos(\theta) + u_3^\varepsilon \sin(\theta) = 0 \\ \tilde{u}_3^\varepsilon - u_2^\varepsilon \sin(\theta) - u_3^\varepsilon \cos(\theta) = 0 \\ u_1^\varepsilon = \tilde{u}_1^\varepsilon \\ \dfrac{\partial u_3^\varepsilon}{\partial x_2} = \dfrac{\partial \tilde{u}_3^\varepsilon}{\partial \tilde{x}_2} \end{cases} \quad \text{and} \quad \begin{cases} T^{12} = \tilde{T}^{12} \\ T^{22} = 0 \quad \text{and} \quad \tilde{T}^{22} = 0 \quad \text{on } \Sigma \\ M^{22} = \tilde{M}^{22} \end{cases} \tag{5}$$

when θ is the angle between the two shells along the edge Σ.

The formal limit problem i.e. $\varepsilon = 0$ ("membrane problem") is:

$$\begin{cases} -\partial_\alpha T^{\alpha\beta}(u^{\{c\}}) = f^{\beta,\{c\}} \\ -b_{\alpha\beta} T^{\alpha\beta}(u^{\{c\}}) = f^{3,\{c\}} \quad \text{in } \omega^{\{c\}} \\ u^{\{c\}}_{\alpha|\Gamma_0^{\{c\}}} = 0 \end{cases} \tag{6}$$

with four transmission conditions on Σ:

$$u_1 = \tilde{u}_1, \qquad T^{12} = \tilde{T}^{12}, \qquad T^{22} = \tilde{T}^{22} = 0 \quad \text{on } \Sigma$$

Proposition 2.1. *The formal limit problem is not elliptic in the Agmon, Douglis and Nirenberg sense, as the transmission conditions do not satisfy the Shapiro–Lopatinski condition.*

Proof. See for instance [3, VII.5]. □

Remark 1. The previous proposition asserts that even if $f^{i,\{c\}}$ are smooth, the formal limit problem is ill posed. Any solution $(u, \tilde{u})$ does not satisfy the transmission conditions at Σ and the boundary conditions on $\partial\Omega$. Therefore, we may expect that $(u^\varepsilon, \tilde{u}^\varepsilon)$ will develop boundary layers (transmission layers) along Σ and (standard) boundary layers along $\partial\Omega$.

3. The case of regular loadings

In this section we suppose that $\theta = \frac{\pi}{2}$, the loadings $f^{3,\{c\}}$ are smooth and $f^{\alpha,\{c\}} = 0$. We perform a dilatation of the coordinate normal to the layer

$$y_1 = x_1, \qquad y_2^{\{c\}} = \frac{x_2^{\{c\}}}{\eta(\varepsilon)} \quad \text{for} \quad \left(x_1^{\{c\}}, x_2^{\{c\}}\right) \in \omega^{\{c\}} \tag{7}$$

The domain $\omega^{\{c\}}$ then becomes $B_\eta^{\{c\}} =]0, \pi[_{y_1} \times]0, \frac{1}{\eta}[_{y_2^{\{c\}}}$.

For the consistency of forthcoming developments we shall choose $\eta = \sqrt{\varepsilon}$ as is usual in elliptic shell layers. We shall further make the change of unknowns

$$u_\alpha^\varepsilon(x) = \eta u_\alpha^\eta(y), \qquad u_3^\varepsilon(x) = u_3^\eta(y) \quad \text{and} \quad \tilde{u}_\alpha^\varepsilon(x) = \eta \tilde{u}_\alpha^\eta(\tilde{y}), \qquad \tilde{u}_3^\varepsilon(x) = \tilde{u}_3^\eta(\tilde{y}) \tag{8}$$

The test functions are chosen to satisfy the kinematic transmission conditions

$$v(y_1, 0) = \tilde{v}(y_1, 0), \qquad \partial_2 v_3(y_1, 0) = \tilde{\partial}_2 \tilde{v}_3(y_1, 0) \tag{9}$$

Let us also define the expressions introduced in [4]:

$$\ell_0^{\{c\}}\left(v^{\{c\}}\right) = \left[\ell_0^1, \ell_0^2, \ell_0^3, \ell_0^4\right]^{\{c\}} = \left[-v_3, \frac{1}{2}\partial_2 v_1, \partial_2 v_2 - v_3, \partial_2^2 v_3\right]^{\{c\}}$$

$$\ell_1^{\{c\}}(u) = \left[\ell_1^1, \ell_1^2, \ell_1^3\right]^{\{c\}} = \left[\partial_1 u_1, \frac{1}{2}\partial_1 u_2, \partial_1 \partial_2 u_3\right]^{\{c\}}$$

If $f^{\{c\}} = (0, 0, f_3^{\{c\}})$, the variational formulation (3) becomes

$$\text{Find } \left(u^\eta, \tilde{u}^\eta\right) \in H_\eta \quad \text{s.t. } a^\eta\left(u^\eta, v\right) + \tilde{a}^\eta\left(\tilde{u}^\eta, \tilde{v}\right) = \int_{B_\eta} f_3 v_3 \, dy_1 \, dy_2 + \int_{\tilde{B}_\eta} f_3 \tilde{v}_3 \, dy_1 \, d\tilde{y}_2, \quad \forall (v, \tilde{v}) \in H_\eta \tag{10}$$

where

$$H_\eta = \{(v,\tilde v)\in V_\eta\times\tilde V_\eta \text{ fulfilling (9)}\}$$

$$V_\eta^{\{c\}} = \left\{ v^{\{c\}}\in H^1\big(B_\eta^{\{c\}}\big)^2\times H^2\big(B_\eta^{\{c\}}\big) \;\middle|\; v^{\{c\}}_{|\Gamma_0^{\eta,c}} = \left(\frac{\partial v_3^{\{c\}}}{\partial n}\right)_{|\Gamma_0^{\eta,c}} = 0\right\}$$

$$\Gamma_0^\eta = \partial B_\eta \setminus \{y_2=0\} \quad \text{and} \quad \tilde\Gamma_0^\eta = \partial \tilde B_\eta \setminus \{\tilde y_2=0\}$$

$$a^{\eta,\{c\}}(u^\eta,v) = a_{00}^{\{c\}}(u^\eta,v) + \eta\big(a_{01}^{\{c\}}(u^\eta,v)+a_{10}^{\{c\}}(u^\eta,v)\big) + \eta^2 a_{11}^{\{c\}}(u^\eta,v) + \eta^4 a_{22}^{\{c\}}(u^\eta,v) \tag{11}$$

$$a_{00}^{\{c\}}(u,v) = \int_{B_0^{\{c\}}} \big[\ell_0^1(u)\ell_0^1(v)+\ell_0^2(u)\ell_0^2(v)+\ell_0^3(u)\ell_0^3(u)+\ell_0^4(u)\ell_0^4(v)\big]\,dy^{\{c\}}$$

$$a_{01}^{\{c\}}(u,v) = \int_{B_0^{\{c\}}} \big[\ell_0^1(u)\ell_1^1(v)+2\ell_0^2(u)\ell_1^2(v)\big]\,dy^{\{c\}}, \qquad a_{10}^{\{c\}}(u,v) = \int_{B_0^{\{c\}}} \big[\ell_1^1(u)\ell_0^1(v)+2\ell_1^2(u)\ell_0^2(v)\big]\,dy^{\{c\}}$$

$$a_{11}^{\{c\}}(u,v) = \int_{B_0^{\{c\}}} \big[\ell_1^1(u)\ell_1^1(u)+2\ell_1^2(u)\ell_1^2(v)+\ell_1^3(u)\ell_1^3(v)\big]\,dy^{\{c\}}, \qquad a_{22}^{\{c\}}(u,v) = \int_{B_0^{\{c\}}} \partial_1^2 u_3\partial_1^2 v_3\,dy^{\{c\}}$$

Since we may extend the elements of $V^{\eta,\{c\}}$ by zero for $y_2^{\{c\}}\geqslant \frac{1}{\eta}$, we have the inclusion $H_\eta\subset H_{\eta'}$ for $\eta>\eta'$. This extension by zero is also used in the definition of $a_{00}^{\{c\}}$, $a_{01}^{\{c\}}$, $a_{10}^{\{c\}}$, $a_{11}^{\{c\}}$ and $a_{22}^{\{c\}}$.

3.1. A priori estimates

The equivalence between (3) and (10) allows us to deduce estimates for the solution $(u^\eta,\tilde u^\eta)$.

Lemma 3.1. *For η fixed, if $f_3\in L^2(B_0)$, $\tilde f_3\in L^2(\tilde B_0)$ and $(u^\eta,\tilde u^\eta)$ is the solution of (10) then*

$$\|u_3^\eta\|_{L^2((0,\pi)_{y_1};H^2(0,\infty))} + \|\tilde u_3^\eta\|_{L^2((0,\pi)_{y_1};H^2(0,\infty))} + \eta^2\|\partial_1^2 u_3^\eta\|_{L^2(B_0)} + \eta^2\|\partial_1^2\tilde u_3^\eta\|_{L^2(\tilde B_0)} \leqslant C$$
$$\|\ell_0(u^\eta)\|_{(L^2(B_0))^4} + \|\tilde\ell_0(\tilde u^\eta)\|_{(L^2(\tilde B_0))^4} + \eta\|\ell_1(u^\eta)\|_{(L^2(B_0))^3} + \eta\|\tilde\ell_1(\tilde u^\eta)\|_{(L^2(\tilde B_0))^3} \leqslant C$$

for some C independent of η.

3.2. Limit problem and convergence theorem

In order to construct the energy space for the limit problem, let

$$E_0^{\{c\}} = L^2\big(B_0^{\{c\}}\big)^4 \quad \text{and} \quad E_1^{\{c\}} = L^2\big(B_0^{\{c\}}\big)^4 \tag{12}$$

We may consider the space $\bigcup_{\eta\searrow 0} H_\eta$ with the norm:

$$\|(v,\tilde v)\|_{V_0\times\tilde V_0} := \big(\|\ell_0(v)\|_{E_0}^2 + \|\ell_1(v)\|_{E_1}^2 + \|\ell_0(\tilde v)\|_{\tilde E_0}^2 + \|\ell_1(\tilde v)\|_{\tilde E_1}^2\big)^{\frac12} \tag{13}$$

Let H_0 be the completion of $\bigcup_{\eta\searrow 0} H_\eta$ with this norm.

Lemma 3.2. *The quantity $\|v\|_{\bar H_0} = (\|\ell_0(v)\|_{E_0}^{\frac12} + \|\tilde\ell_0(\tilde v)\|_{\tilde E_0}^{\frac12})^{\frac12}$ is a norm on H_0.*

Hence we define the larger space $\bar H_0$ as the completion of H_0 with respect to the norm $\|\cdot\|_{\bar H_0}$. On the basis of the a priori estimates obtained in Lemma 3.1 we have:

Theorem 3.3. *Let $(u^\eta,\tilde u^\eta)$ be the solution of (10), then $(u^\eta,\tilde u^\eta)\to(u,\tilde u)$ weakly in $\bar H_0$ where $(u,\tilde u)\in\bar H_0$ is the solution of:*

$$a_{00}(u,v)+\tilde a_{00}(\tilde u,\tilde v) = \int_{B_0} f_3 v_3\,dy_1\,dy_2 + \int_{\tilde B_0}\tilde f_3\tilde v_3\,dy_1\,d\tilde y_2, \quad \forall (v,\tilde v)\in\bar H_0 \tag{14}$$

The corresponding limit problem in the inner variables y takes the following form. Choosing appropriate test functions in (14), we obtain the partial differential equations

$$\begin{cases} -\partial_2\mathring{\gamma}_{12}(u) - \partial_2\mathring{\gamma}_{22}(u) = 0 \\ -\partial_2\mathring{\gamma}_{12}(u) = 0 \\ -b_{\alpha\beta}\mathring{\gamma}_{\alpha\beta}(u) + \partial_2^4 u_3 = f_3 \end{cases} \text{in } B_0, \quad \text{and} \quad \begin{cases} -\tilde{\partial}_2\mathring{\gamma}_{12}(\tilde{u}) - \tilde{\partial}_2\mathring{\gamma}_{22}(\tilde{u}) = 0 \\ -\tilde{\partial}_2\mathring{\gamma}_{12}(\tilde{u}) = 0 \\ -\tilde{b}_{\alpha\beta}\mathring{\gamma}_{\alpha\beta}(\tilde{u}) + \tilde{\partial}_2^4\tilde{u}_3 = \tilde{f}_3 \end{cases} \text{in } \tilde{B}_0 \tag{15}$$

when $\mathring{\gamma}_{11}(v^{\{c\}}) = -v_3^{\{c\}}$, $\mathring{\gamma}_{12}(v^{\{c\}}) = \frac{1}{2}\partial_2^{\{c\}} v_1^{\{c\}}$ and $\mathring{\gamma}_{22}(v^{\{c\}}) = \partial_2^{\{c\}} v_2^{\{c\}} - v_3^{\{c\}}$, with the transmission conditions on Σ:

$$\begin{cases} u_3 = 0 \\ \tilde{u}_3 = 0 \\ \partial_2 u_3 = \partial_2\tilde{u}_3 \end{cases} \quad \text{and} \quad \begin{cases} \partial_2^3 u_3 = \tilde{\partial}_2^3\tilde{u}_3 = 0 \\ \partial_2^2 u_3 = \tilde{\partial}_2^2\tilde{u}_3 = 0 \\ \partial_2 u_1 = \tilde{\partial}_2\tilde{u}_1 = 0 \\ \partial_2 u_2 - u_3 = \tilde{\partial}_2\tilde{u}_2 - \tilde{u}_3 = 0 \end{cases} \tag{16}$$

Remark 2. The conditions (16) satisfied by the solution of the limit problem is the formal limit of conditions (5) (in the inner variables y) as $\varepsilon \to 0$.

4. The case of a singular loading

Let us now consider the loading $f(x_1, x_2) = (0, 0, F(x_1)\delta(x_2))$, where F is a given function in $L^2(0,\pi)$ and δ denotes the Dirac mass. We use the same change of variables as in (7) with the following change of unknowns:

$$u_\alpha^\varepsilon(x) = u_\alpha^\eta(y), \qquad u_3^\varepsilon(x) = \eta^{-1}u_3^\eta(y) \quad \text{and} \quad \tilde{u}_\alpha^\varepsilon(x) = \tilde{u}_\alpha^\eta(y), \qquad \tilde{u}_3^\varepsilon(x) = \eta^{-1}\tilde{u}_3^\eta(y) \tag{17}$$

and obtain the new problem: Find $(u^\eta, \tilde{u}^\eta) \in H_\eta$ solution of (a^η and $\tilde{a}^\eta$ being given by (11))

$$a^\eta\big(u^\eta, v\big) + \tilde{a}^\eta\big(\tilde{u}^\eta, \tilde{v}\big) = \int_0^\pi F(y_1)v_3(y_1, 0)\,\mathrm{d}y_1 + \eta\int_{\tilde{B}_\eta} \tilde{f}_3\tilde{v}_3(y_1, \tilde{y}_2)\,\mathrm{d}\tilde{y}, \quad \forall(v, \tilde{v}) \in H_\eta \tag{18}$$

If $F \in L^2(0,\pi)$ and $\tilde{f}_3 \in L^2(\tilde{B}_0)$ we obtain the same a priori estimates as in Lemma 3.1 and the following limit problem:

Theorem 4.1. *Let $(u^\eta, \tilde{u}^\eta)$ be the solution of (18), then $(u^\eta, \tilde{u}^\eta) \to (u, \tilde{u})$ weakly in $\bar{H}_0$ where $(u, \tilde{u}) \in \bar{H}_0$ is the solution of*

$$a_{00}(u, v) + \tilde{a}_{00}(\tilde{u}, \tilde{v}) = \int_0^\pi F(y_1)v_3(y_1, 0)\,\mathrm{d}y_1, \quad \forall(v, \tilde{v}) \in \bar{H}_0 \tag{19}$$

5. Concluding remarks

We have shown that for the case of transmission problems (the rigid junction case) of two thin shallow shells, even if the loadings are smooth, the formal limit problem (membrane) is ill posed, singular layers appearing along the edge. A dilatation of the coordinate normal to the layer gives a new problem with a well posed variational limit problem, that gives the inner solution satisfying all the transmission conditions as $\varepsilon \to 0$. The use of the different scaling (8) and (17) shows that we must take in account the singularities of the loadings. The immediate extension, apart from more general shell problem, is to consider a general shell without assumptions on the geometry of the middle surface. Another interesting extension is to consider very different elasticity coefficients on each part of the multi-structure with a fold of arbitrary angle, which does not seem to be entirely straightforward, the elastic junction model seems to be more consistent.

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