

Specific behaviour of thermosolutal convection induced in a vertical porous medium in the case of a separation coefficient identical to the ratio of buoyancy forces

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Abstract

Thermosolutal natural convection induced in a vertical porous layer heated and salted with uniform fluxes is studied analytically and numerically. The study is focused on a specific case where the separation coefficient is identical to the ratio of buoyancy forces. Analytical results, describing both pseudo-conductive and boundary layer regimes, are discussed. Specific behaviour, corresponding to this particular situation, is presented. *To cite this article: M. Er-Raki et al., C. R. Mecanique 336 (2008).*

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Résumé

Comportements spécifiques de la convection thermosolutale induite dans une cavité poreuse verticale dans le cas où le coefficient de séparation est identique au rapport des forces de volume. La convection thermosolutale induite dans une couche poreuse verticale, soumise à des flux de chaleur et de masse uniformes, est étudiée analytiquement et numériquement. L'étude est focalisée sur un cas spécifique pour lequel le coefficient de séparation est identique au rapport des forces de poussées thermique et solutale. Des résultats analytiques, décrivant les régimes pseudo-conductif et de couche limite, sont discutés. Des comportements spécifiques, correspondant à cette situation particulière, sont discutés. *Pour citer cet article : M. Er-Raki et al., C. R. Mecanique 336 (2008).*

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Mots-clés: Milieux poreux; Effet Soret; Convection thermosolutale; Dynamique des fluides; Étude analytique et numérique

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L'étude du phénomène de la thermodiffusion (effet Soret) et son influence sur le comportement des écoulements induits, par double diffusion, dans des milieux poreux ont suscité l'intérêt des chercheurs spécialisés dans ce domaine depuis plusieurs décennies. Cet intérêt est justifié par l'importance pratique du phénomène dans divers domaines (industriel, naturel et environnemental).

La revue bibliographique révèle que la plupart des études expérimentales et théoriques se rapportant à ce phénomène sont relatives à des cavités rectangulaires confinant un milieu poreux ou purement fluide. Des difficultés inhérentes aux études expérimentales sur la thermodiffusion ont été évoquées dans un article récent par Platten [1]. Parmi les études expérimentales qui ont touché différents aspects en présence de la thermodiffusion, on peut citer comme exemples les travaux Rehberg et Ahlers [2] (phénomènes de bifurcations), Rosanne et al. [3] (effet de la thermodiffusion sur le transfert de masse) et Platten et Costesèque [4] (mesure du coefficient de Soret dans des milieux poreux et fluides). Sur le plan théorique, l'effet de la thermodiffusion sur la convection naturelle thermosolutale induite dans des milieux poreux ou dans des milieux fluides [5–9] a été pris en compte dans plusieurs études récentes. Dans ces études, les auteurs ont présenté des résultats très variés portant sur l'impact de l'effet Soret sur les transferts thermiques et massiques, les différents types de bifurcations, la multiplicité des solutions, le phénomène d'hystérésis, . . .

Le présent travail porte sur une approche analytique et numérique de l'effet combiné du phénomène de la double diffusion et de la thermodiffusion dans une couche poreuse verticale de hauteur H' et de largeur L' dont les faces verticales sont soumises à des flux de chaleur, q' , et de masse, j' (Fig. 1). Le milieu poreux est supposé homogène, isotrope et saturé par un fluide binaire incompressible et obéissant à l'approximation de Boussinesq.

Le modèle mathématique de Darcy décrivant ce problème est donné par les Éqs. (1) à (5). Les conditions aux limites associées sont données par l'Éq. (6).

Les variables adimensionnelles ζ , ψ , T , S , u et v apparaissant dans ces équations représentent respectivement la vorticit , la fonction de courant, la temp rature, la concentration et les composantes horizontale et verticale de la vitesse. Les param tres R_T , Le et S_P repr sentent le nombre de Rayleigh–Darcy thermique, le nombre de Lewis et le param tre de Soret, respectivement.

Dans cette  tude, on s'int resse en particulier   l' tude de la dynamique des  coulements et l'analyse des comportements thermiques et massiques observ s pour $S_P = 1$. Cette situation correspond physiquement   un coefficient de s paration identique au rapport des forces de volume thermiques et solutales.

Les transferts thermiques et massiques sont repr sent s respectivement par les nombres adimensionnels de Nusselt, Nu , et de Sherwood, Sh .

La solution analytique approch e du probl me, d velopp e pour $S_P = 1$, est bas e sur le concept de l' coulement parall le. Ce concept, souvent valide pour des conditions de flux impos s si l'effet de bord est n gligeable ($A \gg 1$), permet les simplifications donn es par l' q. (7) et conduit   une solution analytique offrant la possibilit  d'une analyse tr s d taill e du probl me  tudi .

Des r sultats correspondant aux r gimes pseudo-conductif et de couche limite sont pr sent s dans cette  tude. Les expressions analytiques d crivant ces deux r gimes peuvent  tre d duites directement de celles de l' coulement parall le respectivement pour des valeurs faibles ( qs. (19)   (23)) et importantes ( qs. (24)   (29)) de Ω . Il est   noter que, en r gime de couche limite, Ω est proportionnel   R_T^m ($m \in \mathbb{R}^+$) et il est en m me temps inversement proportionnel   δ ( paisseur de la couche limite).

Dans ce travail, on pr sente  galement une partie num rique qui a servi pour la validation des r sultats analytiques. La m thode num rique utilis e est bas e sur une technique des diff rences finies int grant les  quations r gissantes dans leur int gralit  i.e. ; sans introduire les approximations de l' coulement parall le. Les r sultats pr sent s ont  t  obtenus avec un maillage de 81×201 (maillage permettant de retrouver les r sultats de la solution analytique avec un bon accord) et un rapport de forme $A = 8$.

La discussion portera sur l'impact de quelques param tres de contr le sur les caract ristiques thermiques et massiques du probl me. Ainsi, sur la Fig. 2 on illustre les  volutions de Ω , Nu et Sh en fonction du nombre de Rayleigh thermique, R_T , pour $Le = 10$, $S_P = 1$ et $N = -3$. Cette figure montre que, pour des valeurs suffisamment faibles de R_T , le transfert de chaleur est domin  par un r gime o  les gradients de temp rature sont nuls dans la direction verticale et la concentration est uniforme au sein de la cavit  ; il en r sulte $Nu \approx 1$ et $Sh \rightarrow \infty$. La Fig. 2 montre aussi une bonne concordance entre les r sultats de l' coulement parall le et ceux correspondant aux r gimes pseudo-conductif

(pour $\Omega \ll 1$) et de couche limite (pour $\Omega \gg 1$), atteints respectivement pour des valeurs de R_T suffisamment faibles et suffisamment élevées.

Les variations de l'épaisseur de la couche limite, δ , avec R_T peuvent être également déduites de la Fig. 2 ($\delta \propto \Omega^{-1}$). Comme on pourrait s'y attendre, l'effet de R_T est caractérisé par une réduction de l'épaisseur de la couche limite à mesure que ce paramètre augmente.

Les variations de Ω , Nu et Sh , avec le rapport des forces de volume, N ($N > 0$), sont présentées sur la Fig. 3 pour $R_T = 100$, $S_P = 1$ et $Le = 10$. Comme montré sur cette figure, les grandeurs Ω , Nu et Sh restent pratiquement insensibles à l'augmentation de N jusqu'à l'atteinte d'une certaine valeur de ce paramètre à partir de laquelle un changement de tendance est observé; il est caractérisé par une diminution drastique de Ω et une augmentation de Nu et Sh (augmentation moins importante pour Nu) avec N . On peut ainsi retenir que la solution s'écarte du régime de couche limite (δ devient important) pour des valeurs suffisamment élevées de N . Par contre, pour des valeurs suffisamment faibles de N , un accord est observé entre les résultats de l'écoulement parallèle et celui de type couche limite ($\Omega \gg 1$). Il est à noter que les évolutions de Sh et Nu avec N sont caractérisées par des comportements asymptotiques dont les limites sont atteintes à des valeurs très élevées de N .

Une comparaison des profils de température et de concentration est illustrée sur les Figs. 4(a) ($S_P = 0,5$) et (b) ($S_P = 1$) pour $R_T = 10^3$, $Le = 10$ et $N = 3$. Sur la Fig. 4(a), correspondant à $S_P = 0,5$, un comportement de type couche limite est observé dans les profils de T et S . Le comportement change cependant lorsque le flux de masse imposé (j') est compensé par l'effet Soret ($S_P = 1$). En effet, la Fig. 4(b) montre bien que le comportement de couche limite est observé pour la température mais il est absent pour la concentration. Pour cette valeur particulière de S_P , le comportement de couche limite restera toujours absent pour le profil de concentration et ce, indépendamment des valeurs attribuées aux autres paramètres de contrôle.

1. Introduction

The Soret effect on thermal and solutal natural convection developed in fluid and porous media has received growing attention in the last decades. This interest is justified by the practical importance of this phenomenon in several natural, environmental and industrial processes.

The literature review shows that, most of the previous works (theoretical and experimental works) related to the Soret effect have considered rectangular geometries submitted to various thermal and/or solutal boundary conditions. In spite of the numerous difficulties inherent to the experimental aspect of the thermodiffusion as explained in a recent paper [1] about different techniques used to measure the Soret coefficient, authors had recourse to the experimental methods to approach some aspects of the thermodiffusion. Hence, bifurcations phenomena in a horizontal porous layer of normal-fluid ^3He - ^4He mixture heated from below were studied experimentally by Rehberg and Ahlers [2]. Thermo-diffusion in a solution of sodium chloride contained in compact clay was studied experimentally by Rosanne et al. [3] and a comparative study of the Soret coefficient measured in a free fluid and a porous medium was conducted by Platten and Costesèque [4]. Theoretically, different aspects relative to the coupling between thermodiffusion and natural convection have been considered in previous studies. The critical conditions corresponding to the onset of convection were determined by several authors for both horizontal [5–7] and vertical [8,9] rectangular enclosures. The importance of such category of problems, for which the stability analysis is possible, remains in their attractive aspects; characterized by the richness of the behaviours susceptible to be encountered (multiplicity of solutions, nature of bifurcations, reversal gradient of concentration and many other behaviours).

The object of the present note consists of studying analytically and numerically the coupling between thermosolutal natural convection and the Soret effect in a vertical porous cavity. Attention is mainly focused on the particular situation where the Soret parameter, defined as the ratio of the separation coefficient to the buoyancy ratio, is equal to unity. An analytical solution is proposed to describe the pseudo-conductive and the boundary layer regimes.

2. Statement of the problem

The geometry under consideration is sketched in Fig. 1. It corresponds to an isotropic, homogeneous and saturated vertical porous layer of height H' and width L' such that $A = H'/L' \gg 1$. The vertical walls of the layer are submitted to uniform fluxes of heat, q' , and mass, j' , while its horizontal walls are considered adiabatic and impermeable to

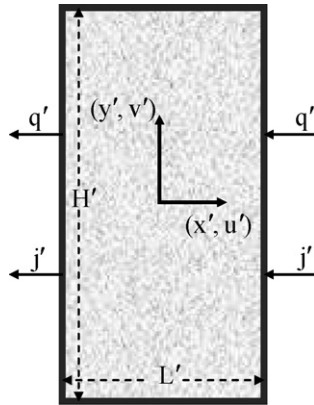


Fig. 1. Schematic of the physical system.
 Fig. 1. Schéma de la configuration étudiée.

mass transfer. Using the Darcy model and the Boussinesq approximation and assuming constant physical properties, the non-dimensional Partial Differential Equations (PDE) governing this problem are:

$$\eta \frac{\partial \zeta}{\partial t} + \zeta = R_T \left(\frac{\partial T}{\partial x} + N \frac{\partial S}{\partial x} \right) \tag{1}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T \tag{2}$$

$$\varepsilon \frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{Le} (\nabla^2 S + S_P \nabla^2 T) \tag{3}$$

$$\nabla^2 \psi = -\zeta \tag{4}$$

$$u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

The associated boundary conditions are as follows:

$$\left. \begin{aligned} x = \pm 1/2, \quad \psi = 0, \quad \frac{\partial T}{\partial x} = 1, \quad \frac{\partial S}{\partial x} = 1 - S_P \\ y = \pm A/2, \quad \psi = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial S}{\partial y} = 0 \end{aligned} \right\} \tag{6}$$

where ζ , ψ , T , S , u and v are the dimensionless vorticity, stream function, temperature, concentration, horizontal and vertical components of the velocity, respectively.

The governing parameters appear explicitly in the set of governing equations; they are the Soret parameter, $S_P = S'_0 D_T \Delta T' / (D \Delta S')$, the thermal Rayleigh–Darcy number, $R_T = g \beta_T K q' L'^2 / (\lambda \alpha \nu)$, the Lewis number, $Le = \alpha / D$ and the buoyancy ratio, $N = \beta_S \Delta S' / \beta_T \Delta T'$.

The main concern here deals with an analytical extension of the parallel flow analysis to a specific case where the buoyancy ratio is identical to the separation coefficient. A standard numerical method based on a finite difference technique was used to solve numerically the full governing equations. It is to underline that the numerical solution is used here only with the objective to validate the approximate analytical results. The grid used is uniform by zone; finer in the boundary layer regions. The numerical results reported here were obtained with a grid of 81×201 and an aspect ratio $A = 8$.

3. Results and discussion

3.1. Approximate analytical solution

The present problem can be simplified by assuming the parallel flow approximation, valid in the central part of the cavity for sufficiently large aspect ratios of the enclosure ($A \gg 1$). The validity of this hypothesis yields the following simplifications:

$$\psi(x, y) = \psi(x); \quad T(x, y) = C_T y + \theta_T(x); \quad S(x, y) = C_S y + \theta_S(x) \quad (7)$$

where C_T and C_S are respectively unknown temperature and concentration gradients in the y -direction.

Using the simplifications allowed by the parallel flow conditions with $S_P = 1$, the set of PDE describing the problem reduce to the following set of ordinary equations:

$$\frac{d^2\psi}{dx^2} = -R_T \frac{d}{dx}(\theta_T + N\theta_S) \quad (8)$$

$$\frac{d^2\theta_T}{dx^2} = -C_T \frac{d\psi}{dx} \quad (9)$$

$$\frac{d^2\theta_S}{dx^2} = (C_T - Le C_S) \frac{d\psi}{dx} \quad (10)$$

The corresponding boundary conditions on the active walls can be rewritten as:

$$\psi = 0, \quad \frac{d\theta_T}{dx} = 1, \quad \frac{d\theta_S}{dx} = 0 \quad \text{for } x = \pm 1/2 \quad (11)$$

By solving the resulting system (8)–(10), with the boundary conditions (11), a parallel flow solution is obtained as follows:

$$\psi(x) = k \left[1 - \frac{\cosh(\Omega x)}{\cosh(\Omega/2)} \right] \quad (12)$$

$$T(x, y) = C_T y + (1 - kC_T)x + kC_T \frac{\sinh(\Omega x)}{\Omega \cosh(\Omega/2)} \quad (13)$$

$$S(x, y) = C_S y + k(C_T - Le C_S)x - k(C_T - Le C_S) \frac{\sinh(\Omega x)}{\Omega \cosh(\Omega/2)} \quad (14)$$

with

$$\Omega = \sqrt{R_T [C_T(1 - N) + N Le C_S]} \quad \text{and} \quad k = \frac{R_T}{\Omega^2} \quad (15)$$

The energy balances at each horizontal section of the enclosure, lead to the following expressions for the parameters C_T and C_S :

$$C_T = \frac{A_1}{1 + A_0} \quad \text{and} \quad C_S = \frac{(Le A_0 - 1)}{(Le^2 A_0 + 1)} C_T \quad (16)$$

where

$$A_0 = k^2 \left(\frac{3}{2} - \frac{3 \tanh(\Omega/2)}{\Omega} - \frac{\tanh^2(\Omega/2)}{2} \right), \quad A_1 = k \left(1 - \frac{2 \tanh(\Omega/2)}{\Omega} \right) \quad (17)$$

The analytical expressions of Nusselt and Sherwood numbers are deduced as:

$$Nu = \frac{1}{T(\frac{1}{2}, 0) - T(-\frac{1}{2}, 0)} = \frac{1}{1 - C_T A_1} \quad \text{and} \quad Sh = \frac{1}{S(\frac{1}{2}, 0) - S(-\frac{1}{2}, 0)} = \frac{1}{(C_T - Le C_S) A_1} \quad (18)$$

3.2. Limiting cases

In the following subsections, the analytical solutions of the pseudo-conductive and boundary layer regimes are deduced from the parallel flow solution by introducing appropriate simplifications.

3.2.1. The pseudo-conductive regime

Such a regime can be observed at sufficiently small values of R_T (Ω close to zero). For this situation, the approximate expressions that characterize fluid flow and heat and mass transfers within the cavity can be written as follows:

$$\psi(x) = \frac{R_T}{2} \left(-x^2 + \frac{1}{4} \right) \tag{19}$$

$$T(x, y) = C_T y + x + \frac{R_T C_T}{6} \left(x^3 - \frac{3}{4} x \right) \tag{20}$$

$$S(x, y) = C_S y - \frac{R_T (C_T - Le C_S)}{6} \left(x^3 - \frac{3}{4} x \right) \tag{21}$$

The expressions of Nu and Sh , corresponding to this regime are:

$$Nu = \frac{12}{12 - R_T C_T}; \quad Sh = \frac{12}{R_T (C_T - Le C_S)} \tag{22}$$

where C_T and C_S are given by:

$$C_T = \frac{10 R_T}{120 + R_T^2}; \quad C_S = \frac{(Le R_T^2 - 120)}{(Le^2 R_T^2 + 120)} C_T \tag{23}$$

3.2.2. The boundary layer regime

This regime is reached at large values of Ω ($\Omega \gg 1$) where the latter parameter is inversely proportional to the thickness δ of the vertical boundary layer. The simplified expressions of ψ , T and S , corresponding to the boundary layer regime can be given under the following forms:

$$\psi(x) = k \left(1 - e^{\Omega(\lambda x - \frac{1}{2})} \right) \tag{24}$$

$$T(x, y) = C_T y + (1 - k C_T) x + \lambda k C_T \frac{e^{\Omega(\lambda x - \frac{1}{2})}}{\Omega} \tag{25}$$

$$S(x, y) = C_S y + k(C_T - Le C_S) x - \lambda k(C_T - Le C_S) \frac{e^{\Omega(\lambda x - \frac{1}{2})}}{\Omega} \tag{26}$$

where $\lambda = 1$ (-1) for the positive (negative) values of x .

The expressions of the parameters C_T and C_S become:

$$C_T = \frac{k(1 - \frac{2}{\Omega})}{1 + G(1 - \frac{3}{\Omega})}, \quad C_S = \left(\frac{Le G(1 - \frac{3}{\Omega}) - 1}{Le^2 G(1 - \frac{3}{\Omega}) + 1} \right) C_T \tag{27}$$

with

$$G = k^2 \tag{28}$$

The Nusselt and Sherwood numbers are reduced to:

$$Nu = \frac{1}{1 - k C_T (1 - \frac{2}{\Omega})}, \quad Sh = \frac{1}{k(C_T - Le C_S) (1 - \frac{2}{\Omega})} \tag{29}$$

By combining Eqs. (15), (27) and (28), one obtains the following equation for G :

$$\xi_1 G^2 - \xi_2 \Omega G - \Omega^2 = 0 \tag{30}$$

with

$$\begin{cases} \xi_1 = Le^2 (\Omega - 3) \\ \xi_2 = (Le^2 + N Le + N) (\Omega - 3) + (N Le + N - 1) \end{cases} \tag{31}$$

The only root of Eq. (30) satisfying Eq. (28) is given by:

$$G = \frac{\Omega}{2} \left(\frac{\xi_2 + \sqrt{\xi_2^2 + 4 \xi_1}}{\xi_1} \right) \tag{32}$$

The approximate expressions of Eq. (32) are given hereafter at sufficiently large values of Ω in the cases of opposing and aiding thermal and solutal buoyancy forces.

3.2.2.1. *Boundary layer regime under opposing buoyancy forces effect ($N < 0$).* It corresponds to counteracting solutal and thermal buoyancy forces (due to the imposed fluxes). For this situation, the expression of the parameter G can be approximated by:

$$G \cong \frac{Le^2 + N Le + N}{Le^2} \Omega \quad \text{for } N > \frac{-Le^2}{1 + Le} \tag{33}$$

and

$$G \cong \frac{-1}{Le^2 + N Le + N} \quad \text{for } N < \frac{-Le^2}{1 + Le} \tag{34}$$

It can be deduced from Eqs. (33) and (34) that two different boundary layer behaviours are possible for $N < 0$. It is to underline that the existence of two different behaviours is proved in the case of opposing buoyancy ($N < 0$) even for $S_P \neq 1$.

At sufficiently large values of R_T , the expressions of Ω deduced from Eqs. (33) and (34), are respectively given as:

$$\Omega \cong R_T^{2/5} Le^{2/5} (Le^2 + N Le + N)^{-1/5} \tag{35}$$

$$\Omega \cong R_T^{1/2} [-(Le^2 + N Le + N)]^{1/4} \tag{36}$$

3.2.2.2. *Boundary layer regime under aiding buoyancy forces effect ($N > 0$).* For this case, it can be easily demonstrated that only the expression of Ω given by Eq. (35) is possible for any positive value of N . This means that, for $S_P = 1$, the boundary layer regime under aiding buoyancy forces effect, is characterized by the existence of a unique behaviour for which Ω varies as $R_T^{2/5}$. For $S_P \neq 1$, it was reported [10] that, the two boundary layer behaviours, given by Eqs. (35) and (36), are possible even for positive values of N .

3.3. Flow characteristics and heat and mass transfers for $S_P = 1$

In this section, the evolutions of Ω , Nu and Sh versus R_T and N are discussed. Hence, in Fig. 2, these variations are illustrated versus R_T for $Le = 10$, $S_P = 1$ and $N = -3$. It can be seen that the analytical parallel flow solution, represented by solid lines, is in good agreement with the numerical results depicted by full circles. As predicted by Eq. (22), this figure shows that, when R_T is small enough, the heat transfer is dominated by the pure diffusive regime

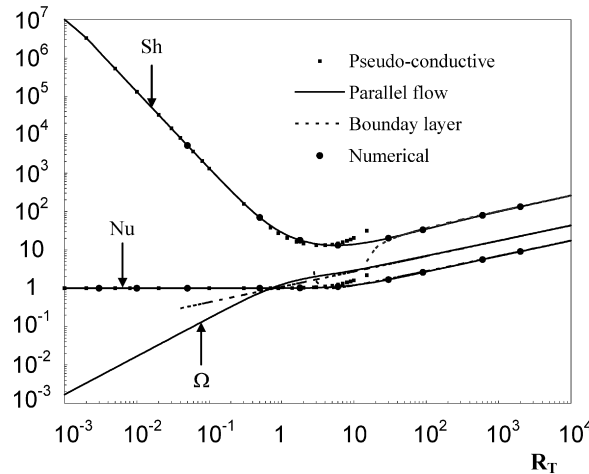


Fig. 2. Effect of R_T on Ω , Nu and Sh for $Le = 10$, $N = -3$, $S_P = 1$ and $A = 8$.
 Fig. 2. Effet de R_T sur Ω , Nu et Sh pour $Le = 10$, $N = -3$, $S_P = 1$ et $A = 8$.

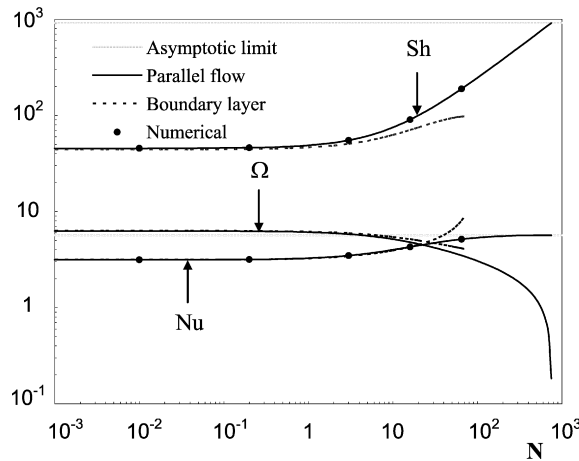


Fig. 3. Variations of Ω , Nu and Sh with N for $R_T = 100$, $Le = 10$, $S_P = 1$ and $A = 8$.

Fig. 3. Variations de Ω , Nu et Sh avec N pour $R_T = 100$, $Le = 10$, $S_P = 1$ et $A = 8$.

($Nu \cong 1$) whereas the mass transfer is characterized by the equality of concentrations on the vertical walls ($Sh \rightarrow \infty$). In accordance with the analytical discussion of the limiting cases, Fig. 2 shows also a good agreement between the results based on the parallel flow assumption and those corresponding to the pseudo conductive and boundary layer regimes at sufficiently small and large values of R_T , respectively.

The evolution of the boundary layer thickness, δ , with R_T can be deduced from that of Ω ($\delta \propto \Omega^{-1}$), given by Eq. (35) for which δ varies as $R_T^{-2/5}$.

The variations of Ω , Nu and Sh with N ($N > 0$) are illustrated in Fig. 3 for $R_T = 100$, $S_P = 1$ and $Le = 10$. A good agreement is observed also between the results obtained by the approximate analytical solution and those obtained numerically. All the variations are characterized first by a negligible effect of N on the evolutions of Ω , Nu and Sh , as long as the parameter N remains lower than some threshold ($N \sim 1$) above which a change in the trends is observed. In fact, above this limit, the variations of Ω are characterized by a drastic decrease (increase of δ), those of Sh and Nu by an increase (important in the case of Sh and limited in the case of Nu) towards asymptotic limits reached at high values of N (not yet reached in the figure). It is seen from Fig. 3 that the boundary layer approximation is valid in the case of heat transfer driven flow (relatively small values of N) while the boundary layer behaviour disappears at high values of N (case of solute transfer driven flow).

As mentioned before, it is reported in a previous study [10] that, for $S_P \neq 1$, the horizontal profiles of both temperature and concentration exhibit boundary layer behaviours which are characterized by horizontal gradients of T and S close to zero in the core region of the cavity and important in a very thin layer adjacent to the vertical boundaries. By comparing the different profiles of T and S illustrated in Figs. 4(a) (with $S_P = 0.5$) and (b) (with $S_P = 1$), it can be seen that, for $S_P = 0.5$, both T and S profiles exhibit boundary layer behaviours while the situation is different in the case of $S_P = 1$. In fact, Fig. 4(b) shows that no boundary layer behaviour is observed in the concentration profile for $S_P = 1$.

4. Conclusions

The Soret effect on fluid flow and heat and mass transfers, induced by double-diffusive natural convection in a vertical porous enclosure, subject to horizontal gradients of temperature and concentration, is studied analytically and numerically. The Soret effect is represented here by the specific value, $S_P = 1$, corresponding to the case where the separation coefficient is equal to the ratio of buoyancy forces. Results corresponding to the pseudo-conductive ($R_T \ll 1$) and boundary layer ($R_T \gg 1$) regimes are presented and validated numerically and against those of the parallel flow solution. The disappearance of the boundary layer regime at sufficiently large values of N is proved. For this particular situation, where the external mass flux is compensated by the Soret effect, the boundary layer regime remains absent in the case of concentration independently of the governing parameters.

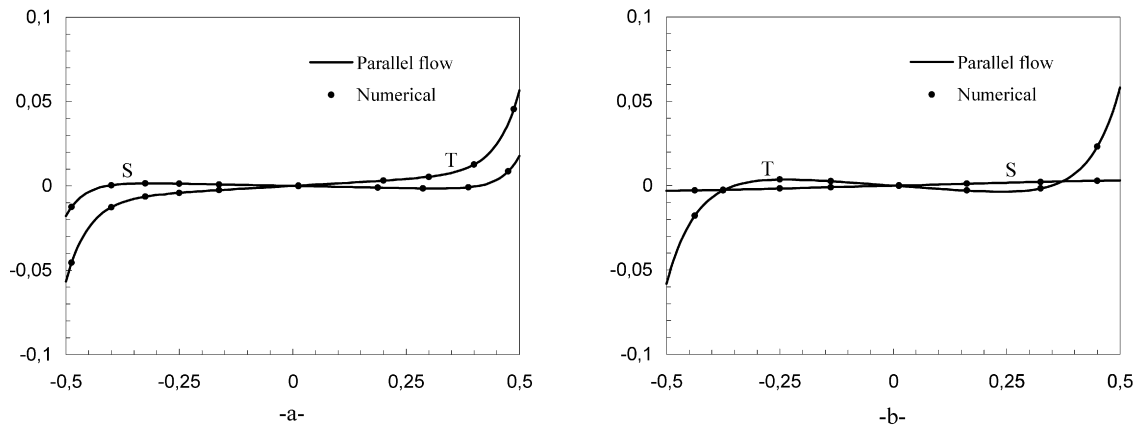


Fig. 4. Horizontal profiles of T and S at mid-height of the enclosure for $A = 8$, $R_T = 10^3$, $Le = 10$, $N = 3$: (a) $S_P = 0.5$ and (b) $S_P = 1$.

Fig. 4. Profils horizontaux de T et S à mi-hauteur de la cavité pour $A = 8$, $R_T = 10^3$, $Le = 10$, $N = 3$: (a) $S_P = 0,5$ et (b) $S_P = 1$.

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