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Averaged model of a cross hydrodynamic–mechanic process in a double porosity medium

Mikhail Panfilov [a,*], Remy Marmier [b], Laurent Jeannin [b]

[a] *Laboratoire environnement, géomécanique & ouvrages, École nationale supérieure de géologie – INPL, 54501 Vandoeuvre-lès-Nancy, France*
[b] *Institut français du pétrole, 1 and 4, avenue de Bois-Préau, 92852 Rueil-Malmaison cedex, France*

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Abstract

For the single-phase flow of a compressible liquid in a deformable double porosity medium, the closed homogenized model is obtained with a total splitting between various cross mechanic–hydrodynamic effects. The transfer between matrix and fractures is completed by the peristaltic effect and the effect of flow arising due to shearing strain. In the equation of deformations, a new stress appears being generated by the cross effects and matrix relaxation. ***To cite this article: M. Panfilov et al., C. R. Mecanique 334 (2006).***

Résumé

Modèle homogénéisé d'un processus hydrodynamique–mécanique couplé en milieu de double porosité. Un modèle homogénéisé de l'écoulement d'un fluide monophasique compressible en milieu déformable double porosité, où les effets couplés mécaniques–hydrodynamiques sont totalement séparés, est obtenu. Le transfert entre la matrice et les fractures est complété par un effet péristaltique et par un écoulement du aux déformations de cisaillement. De nouvelles contraintes apparaissent suite aux effets couplés et à la relaxation de la matrice. ***Pour citer cet article : M. Panfilov et al., C. R. Mecanique 334 (2006).***

Keywords: Porous media; Double porosity; Deformation; Transfer; Peristaltic; Darcy; Homogenization; High heterogeneity; Cross effect

Mots-clés : Milieux poreux ; Double porosité ; Déformation ; Transfert ; Péristaltique ; Darcy ; Homogénéisation ; Forte hétérogénéité ; Effet couplé

Version française abrégée

Le modèle macroscopique d'écoulement en milieu de double porosité est connu dans le cas d'un milieu non déformable. Il possède un terme d'échange provoqué par la différence de pression de matrice et de fracture. Dans le cas d'un milieu déformable le modèle macroscopique n'a été obtenu qu'à l'aide de certaines approximations (équivalence de pressions en matrice et fractures) qui supprimaient la double porosité. La difficulté essentielle d'obtention

* Corresponding author.
E-mail addresses: mikhail.panfilov@ensg.inpl-nancy.fr (M. Panfilov), remy.marmier@ifp.fr (R. Marmier).

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du modèle macroscopique est liée à l'inséparabilité des variables d'écoulement et de déformation à l'échelle du problème cellulaire, ce qui amène à l'apparition du comportement macroscopique fortement non local. Dans la présente note nous avons obtenu le modèle macroscopique pour le cas d'un milieu déformable, en conservant la double porosité et la différence de pressions. Le problème original d'écoulement-déformation est décrit par (1)–(3). Le modèle homogénéisé a la forme (5)–(7), avec des paramètres effectifs définis à l'aide de (8) et de problèmes cellulaires (9).

L'approche est basée sur le remplacement de la forte non localité (une mémoire longue) par une non localité faible (une mémoire courte) qui permet de décomposer tous les phénomènes non-locaux sous forme d'une superposition. Ceci a été atteint à l'aide d'un choix entre les paramètres ω et ε définissant le rapport de perméabilités et l'échelle de l'hétérogénéité respectivement. La structure des développements asymptotiques a la forme (4).

Dans le modèle obtenu, le terme d'échange entre la matrice et les fractures contient deux nouveaux effets : un effet péristaltique qui signifie l'apparition d'un écoulement en fractures à cause de la compression/détente volumique de la matrice, tandis que le deuxième effet décrit l'apparition de l'écoulement à cause de la déformation de cisaillement. A son tour, l'équation de déformation montre l'apparition de nouvelles contraintes dues aux effets couplés.

1. Introduction

The flow of a low compressible liquid in an elastic deformable double porosity medium is examined in terms of the homogenized behaviour. We apply the two-scale asymptotic homogenization method. The basic properties of a double porosity medium are a small value of the parameter ω which is the ratio between the matrix and fracture permeabilities, and a small value of the parameter ε which is the dimensionless heterogeneity scale (a fast oscillating heterogeneity). The averaged models of flow in a non-deformable medium are well known [1,2]. All they display a difference between the fracture and matrix pressures, a delay in matrix behaviour and a transfer process generated by the pressure difference.

For a double porosity medium, it is usually assumed that $\omega \sim \varepsilon^2$, which leads to a non-stationarity of the cell-problem and, as a consequence, to arising of a transfer term in the macroscopic model which is described by an integro-differential operator meaning the appearance of a strong non-locality (a long-term memory) in the system behaviour. In a non-deformable medium this non-local operator can be successfully obtained, analyzed and simulated, which is not the case of a deformable medium that occurs to be almost non-homogenizable due to high correlations between the cross effects at the micro-scale. A typical cross effect has a twice nature: the displacement in rocks can be caused by the pressure gradient (fluid flow), whereas the fluid pressure can be perturbed by a variation in time of the porosity which is related to the divergence of the rock displacement vector. Due to this the displacement is a space convolution with the pressure gradient, while the pressure is a space and time convolution with the divergence of the displacement. The macroscopic model is then expected to represent a coupled system of the integro-differential equations with implicitly defined space and time kernels. Such a system is useless being more complicated than the original microscale model.

The effective mathematical description of such a combined strongly correlated cross process can be obtained only in some particular cases. Due to this in various papers devoted to the examined system some approximations are always introduced in order to obtain constructive results. Basically such an approximation consists in assuming that the matrix and fracture pressures are equivalent [4], or the zero approximation for pressure does not depend on the fast variable [3]. Both these hypotheses remove immediately any double porosity.

In the present Note we applied the approach developed in Panfilov [5] where both the true double porosity model (with two different pressures) was obtained and the non-locality was reduced to a short-term memory. This case is characterized by a softer ratio between ω and ε: $\varepsilon^2 < \omega \ll 1$. Due to this, we have succeeded in converting the nonlocal operators into the differential relaxation operators of high order and to split all the cross effects from one other. The obtained macroscale model is a true double porosity model with two pressures and exchange terms which represent a generalization of the Barenblatt's and Warren–Root's term. We show that various exchange effects are characterized by a number of various relaxation times. All the effective parameters including the relaxation times are determined in a closed form through the solutions of the cell problems. The obtained cross terms have a clear physical origin which is discussed in the final part of the Note.

2. Problem formulation

Let us consider the one-phase weakly compressible flow in a linearly deformable medium consisted of highly permeable fractures (medium $\mathcal{F}$) submerged into a tight porous matrix (medium $\mathcal{M}$). The fracture network is connected. Each of the sub-domains Ω^α, $\alpha = \mathcal{F}, \mathcal{M}$, is characterized by the rock permeability, K^α, and the porosity ϕ^α. The fluid viscosity is assumed to be constant. The medium elastic deformations are described by Lamé's coefficients, whereas the Biot parameter is assumed to be equal to one in both sub-domains within the framework of this paper. The last assumption means that the solid matter of rocks in non-deformable.

The medium heterogeneity is periodic. The ratio between the period size and the overall domain size is a small parameter ε. We assume also that the medium is highly heterogeneous with respect to permeability, so that the matrix and fracture mean permeabilities, $K^{\mathcal{F}0}$ and $K^{\mathcal{M}0}$, are very different from one other: $\omega = K^{\mathcal{M}0}/K^{\mathcal{F}0} \ll 1$.

Let $\mathbf{x} = (x_1, x_2, x_3)$ be the space coordinates, τ the time, p the fluid pressure, $\mathbf{u}$ the elastic rock displacement vector. The fluid flow is then described by the system of mass balance equations, in which the flow velocity obeys the Darcy law:

$$\beta^{\mathcal{F}} \frac{\partial p}{\partial \tau} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_i^{\mathcal{F}}}{\partial \tau}\right) = \frac{\partial}{\partial x_i}\left(a^{\mathcal{F}} \frac{\partial p}{\partial x_i}\right) \tag{1a}$$

$$\beta^{\mathcal{M}} \frac{\partial p}{\partial \tau} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_i^{\mathcal{M}}}{\partial \tau}\right) = \omega \frac{\partial}{\partial x_i}\left(a^{\mathcal{M}} \frac{\partial p}{\partial x_i}\right) \tag{1b}$$

while the saturated rock deformations are described by the equilibrium between all the acting stresses (for $\alpha = \mathcal{F}, \mathcal{M}$):

$$\frac{\partial}{\partial x_i}\left(\mu^\alpha \left(\frac{\partial u_k^\alpha}{\partial x_i} + \frac{\partial u_i^\alpha}{\partial x_k}\right) + \nu^\alpha \frac{\partial u_j^\alpha}{\partial x_j}\delta_{ik} - æ p \delta_{ik}\right) = 0, \quad k = 1, 2, 3 \tag{2}$$

The conditions at the matrix-fracture interface Γ imply the continuity for the normal stresses, the fluid pressure, the normal fluid flow rate and the normal displacement:

$$\left[\left(\mu\left(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\right) + \nu \frac{\partial u_j}{\partial x_j}\delta_{ik} - æ p \delta_{ik}\right) n_i\right]_\Gamma = 0, \quad [p]_\Gamma = 0 \tag{3a}$$

$$\omega a^{\mathcal{M}} \frac{\partial p}{\partial x_i} n_i \bigg|_{\Gamma^{\mathcal{M}}} = a^{\mathcal{F}} \frac{\partial p}{\partial x_i} n_i \bigg|_{\Gamma^{\mathcal{F}}}, \quad [u_n]_\Gamma = 0 \tag{3b}$$

where n is the exterior normal vector to the interface Γ, ν^i and μ^i are the Lamé coefficients of the medium rocks (dimensionless); $a^{\mathcal{F}} = K^{\mathcal{F}}/K^{\mathcal{F}0}$ and $\omega a^{\mathcal{M}} = K^{\mathcal{M}}/K^{\mathcal{F}0}$ are the fluid mobility (permeability divided to viscosity); $\beta = \phi C_f$ where ϕ is the porosity, while C_f is the fluid compressibility; $æ = P^0 L/N^0 U^0$ is the ratio between the normal strain in a saturated medium and in a dry medium.

System (1)–(3) is formulated in dimensionless form. The following characteristic scales were selected for variables with dimension: N^0: the mean value between all the Lamé coefficients; B^0: the mean compressibility between $\beta^{\mathcal{F}}$ and $\beta^{\mathcal{M}}$; P^0: the maximum pressure drop along the overall medium; $K^{\mathcal{F}0}$: the mean permeability in sub-domain $\mathcal{F}$; L: the domain size; $T = B^0 L^2/K^{\mathcal{F}0}$: the characteristic time of elastic perturbation propagation along the distance L through medium $\mathcal{F}$; $U^0 = P^0 B^0 L$: the characteristic elastic deformation caused by the pressure drop P^0.

3. Two-scale asymptotic expansions

In the classic version of the double porosity media parameter ω is usually equal to ε^2 which leads to a difference between the matrix and the fracture pressures of order one. This leads in turn to arising of an integro-differential operator in the macroscale model which is responsible for the transfer between matrix and fracture media [5]. In the case of a deformable medium, the ratio $\omega \sim \varepsilon^2$ does not permit to separate the fast and slow variables both at the scale of the cell problem and of the averaged equation [3]. So the obtained model can not be examined as really homogenized.

As shown in [5], the effective averaged model with two various pressures in matrix and fractures can be obtained if parameter ω is small but larger that ε^2. In this case the difference between pressures will be lower than one, but still significant. We select the ratio between ω and ε in the following way: $\varepsilon^2 \ll \omega \ll 1$. More strictly, the range between parameters is the following: $\varepsilon^2 < \omega < \varepsilon < \varepsilon^2/\omega \ll 1$. The asymptotic expansions for Eqs. (1)–(3) take the form:

$$p(x,y,\tau) = p_0(x,\tau) + \begin{cases} \dfrac{\varepsilon^2}{\omega} p_{01}^{\mathcal{M}}(x,y,\tau) + \varepsilon p_{10}^{\mathcal{M}}(x,y,\tau) + \omega\cdots, & y \in Y^{\mathcal{M}} \\ \dfrac{\varepsilon^2}{\omega} p_{01}^{\mathcal{F}}(x,\tau) + \varepsilon p_{10}^{\mathcal{F}}(x,y,\tau) + \varepsilon\dfrac{\varepsilon^2}{\omega} p_{11}^{\mathcal{F}}(x,y,\tau) + \cdots, & y \in Y^{\mathcal{F}} \end{cases} \tag{4a}$$

$$\mathbf{u}(x,y,\tau) = \mathbf{u}_0(x,\tau) + \frac{\varepsilon^2}{\omega}\mathbf{u}_{01}(x,\tau) + \varepsilon\mathbf{u}_{10}(x,y,\tau) + \cdots, \quad y \in Y^{\mathcal{F}}, Y^{\mathcal{M}} \tag{4b}$$

The next steps of the constructive two-scale homogenization technique are as follows: (i) 'y' is examined as a new independent variable; (ii) all the original functions and differential operators formulated in terms of x, τ are replaced by their two-scale extended versions in terms of x, τ, y ($\partial/\partial x \to \partial/\partial x + \varepsilon^{-1}\partial/\partial y$); (iii) the solution to the new two-scale problems is searched in the space of y-periodic functions; (iv) Eqs. (4) are substituted into the two-scale formulations of Eqs. (1)–(3); (v) applying the regular perturbation technique, the obtained flow equations are decomposed into an infinite system which determines the successive approximations; (vi) the solvability of the obtained problems imposes some integral conditions on the coefficients of series (4), according to the Fredholm alternative; (vii) the macroscopic model follows from the solvability condition applied to the zero terms of the asymptotic series.

4. Macroscopic model

For the macroscale pressures in fractures and matrix, $P^{\mathcal{F}}$ and $P^{\mathcal{M}}$, and the displacement U we obtain the following result with keeping the terms up to ε^2/ω:

$$\langle\beta\rangle\frac{\partial P^{\mathcal{F}}}{\partial\tau} + \frac{\partial^2 U_j}{\partial\tau\partial x_j} - \frac{\partial}{\partial x_i}\left(\widehat{K}_{ik}\frac{\partial P^{\mathcal{F}}}{\partial x_k}\right) = -\tau^{\varphi}\frac{\partial^2 P^{\mathcal{F}}}{\partial\tau^2} - \tau^{\theta}_{mn}\frac{\partial^2\widehat{G}_{mn}}{\partial\tau^2} - \tau^{\sigma}\frac{\partial^3 U_j}{\partial\tau^2\partial x_j} \tag{5a}$$

$$P^{\mathcal{M}} = P^{\mathcal{F}} + \tau^p\frac{\partial P^{\mathcal{F}}}{\partial\tau} + \tau^G_{mn}\frac{\partial\widehat{G}_{mn}}{\partial\tau} + \tau^u\frac{\partial^2 U_j}{\partial\tau\partial x_j} \tag{5b}$$

$$\begin{aligned} &\frac{\partial}{\partial x_i}\left(\hat{\mu}_{mnki}\left(\frac{\partial U_m}{\partial x_n} + \frac{\partial U_n}{\partial x_m}\right) + \hat{\nu}_{ki}\frac{\partial U_j}{\partial x_j} - \text{æ} P^{\mathcal{F}}\delta_{ik}\right) \\ &\quad = -\bar{\tau}^{\varphi}_{ki}\frac{\partial^2 P^{\mathcal{F}}}{\partial\tau\partial x_i} + \bar{\tau}^{\theta}_{mnki}\frac{\partial^2\widehat{G}_{mn}}{\partial\tau\partial x_i} + \bar{\tau}^{\sigma}_{ki}\frac{\partial^3 U_j}{\partial\tau\partial x_j\partial x_k}, \quad k = 1,2,3 \end{aligned} \tag{5c}$$

where $\widehat{G}_{mn} \equiv \dfrac{\partial U_m}{\partial x_n} + \dfrac{\partial U_n}{\partial x_m}$

The effective permeability is determined as:

$$\widehat{K}_{ik} = \left\langle a^{\mathcal{F}}(y)\left(\frac{\partial\psi_k^{\mathcal{F}}}{\partial y_i} + \delta_{ik}\right)\right\rangle_{\mathcal{F}} \tag{6}$$

The effective Lamé coefficients are defined as in a dry medium:

$$\hat{\mu}_{mnki} = \left\langle\mu\left(\frac{\partial\xi_{kmn}}{\partial y_i} + \frac{\partial\xi_{imn}}{\partial y_k}\right) + \nu\frac{\partial\xi_{jmn}}{\partial y_j}\delta_{ik} + \mu\delta_{km}\delta_{ik}\right\rangle \tag{7a}$$

$$\hat{\nu}_{ki} = \left\langle\mu\left(\frac{\partial\zeta_k}{\partial y_i} + \frac{\partial\zeta_i}{\partial y_k}\right) + \nu\frac{\partial\zeta_j}{\partial y_j}\delta_{ik} + \nu\delta_{ik}\right\rangle \tag{7b}$$

Nine relaxation tensors caused by the double porosity have the form:

$$\tau^p = \frac{\varepsilon^2}{\omega}\langle\varphi\rangle_{\mathcal{M}}, \qquad \tau^G_{mn} = \frac{\varepsilon^2}{\omega}\langle\theta_{mn}\rangle_{\mathcal{M}}, \qquad \tau^u = \frac{\varepsilon^2}{\omega}\langle\sigma\rangle_{\mathcal{M}} \tag{8a}$$

$$\tau^{\varphi} = \frac{\varepsilon^2}{\omega}\langle\beta\varphi\rangle_{\mathcal{M}}, \qquad \tau^{\theta}_{mn} = \frac{\varepsilon^2}{\omega}\langle\beta\theta_{mn}\rangle_{\mathcal{M}}, \qquad \tau^{\sigma} = \frac{\varepsilon^2}{\omega}\langle\beta\sigma\rangle_{\mathcal{M}} \tag{8b}$$

$$\bar{\tau}^{p}_{ki} = \frac{\varepsilon^2}{\omega}\left\langle \mu^{\mathcal{M}}\left(\frac{\partial\bar{\varphi}_k}{\partial y_i} + \frac{\partial\bar{\varphi}_i}{\partial y_k}\right) + \nu^{\mathcal{M}}\frac{\partial\bar{\varphi}_j}{\partial y_j}\delta_{ik} - \varphi\delta_{ik}\right\rangle_{\mathcal{M}} \tag{8c}$$

$$\bar{\tau}^{\theta}_{mnki} = \frac{\varepsilon^2}{\omega}\left\langle \mu^{\mathcal{M}}\left(\frac{\partial\bar{\theta}_{kmn}}{\partial y_i} + \frac{\partial\bar{\theta}_{imn}}{\partial y_k}\right) + \nu^{\mathcal{M}}\frac{\partial\bar{\theta}_{jmn}}{\partial y_j}\delta_{ik} - \theta_{mn}\delta_{ik}\right\rangle_{\mathcal{M}} \tag{8d}$$

$$\bar{\tau}^{\sigma}_{ki} = \frac{\varepsilon^2}{\omega}\left\langle \mu^{\mathcal{M}}\left(\frac{\partial\bar{\sigma}_i}{\partial y_k} + \frac{\partial\bar{\sigma}_k}{\partial y_i}\right) + \nu^{\mathcal{M}}\frac{\partial\bar{\sigma}_j}{\partial y_j}\delta_{ik} - \sigma\delta_{ik}\right\rangle_{\mathcal{M}} \tag{8e}$$

The difference between the macroscale model and the original microscale equations (1) is of order of $\varepsilon^2 + \varepsilon^3\omega$ which tends to zero, as $\varepsilon, \omega \to 0$.

5. Cell problems

The effective parameters are defined through intermediary functions $\psi_k(y)$, $\varphi(y)$, $\theta_{km}(y)$, $\sigma(y)$, $\xi_{kmn}(y)$, $\zeta_k(y)$, $\bar{\varphi}_k(y)$, $\bar{\theta}_{kmn}(y)$, $\bar{\sigma}_k(y)$ $(k, m, n = 1, 2, 3)$, which are the solutions to the following series of cell problems:

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(\mu\left(\dfrac{\partial\xi_{kmn}}{\partial y_i} + \dfrac{\partial\xi_{imn}}{\partial y_k}\right) + \nu\dfrac{\partial\xi_{jmn}}{\partial y_j}\delta_{ik} + \mu\delta_{km}\delta_{in}\right) = 0, \quad y \in Y \\ \langle\xi_{kmn}\rangle = 0, \quad \xi_{kmn} \text{ is 1-periodic} \end{cases} \tag{9a}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(\mu\left(\dfrac{\partial\zeta_k}{\partial y_i} + \dfrac{\partial\zeta_i}{\partial y_k}\right) + \nu\dfrac{\partial\zeta_j}{\partial y_j}\delta_{ik} + \nu\delta_{ik}\right) = 0, \quad y \in Y \\ \langle\zeta_k\rangle = 0, \quad \zeta_k \text{ is 1-periodic} \end{cases} \tag{9b}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(a^{\mathcal{M}}\dfrac{\partial\varphi}{\partial y_i}\right) = \beta^{\mathcal{M}}, \quad y \in Y^{\mathcal{M}} \\ \varphi|_{\Gamma} = 0 \end{cases} \tag{9c}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(a^{\mathcal{M}}\dfrac{\partial\theta_{mn}}{\partial y_i}\right) = \dfrac{\partial\xi_{jmn}}{\partial y_j}, \quad y \in Y^{\mathcal{M}} \\ \theta_{mn}|_{\Gamma} = 0 \end{cases} \tag{9d}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(a^{\mathcal{M}}\dfrac{\partial\sigma}{\partial y_i}\right) = 1 + \dfrac{\partial\zeta_j}{\partial y_j}, \quad y \in Y^{\mathcal{M}} \\ \sigma|_{\Gamma} = 0 \end{cases} \tag{9e}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(\mu\left(\dfrac{\partial\bar{\theta}_{kmn}}{\partial y_i} + \dfrac{\partial\bar{\theta}_{imn}}{\partial y_k}\right) + \nu\dfrac{\partial\theta_{jmn}}{\partial y_j}\delta_{ik} - æ\theta_{mn}\delta_{ik}\right) = 0, \quad y \in Y^{\mathcal{M}} \\ \bar{\theta}_{kmn}|_{\Gamma} = 0 \end{cases} \tag{9f}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(\mu\left(\dfrac{\partial\bar{\varphi}_k}{\partial y_i} + \dfrac{\partial\bar{\varphi}_i}{\partial y_k}\right) + \nu\dfrac{\partial\bar{\varphi}_j}{\partial y_j}\delta_{ik} - æ\varphi\delta_{ik}\right) = 0, \quad y \in Y^{\mathcal{M}} \\ \bar{\varphi}_k|_{\Gamma} = 0 \end{cases} \tag{9g}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(\mu\left(\dfrac{\partial\bar{\sigma}_k}{\partial y_i} + \dfrac{\partial\bar{\sigma}_i}{\partial y_k}\right) + \nu\dfrac{\partial\bar{\sigma}_j}{\partial y_j}\delta_{ik} - æ\sigma\delta_{ik}\right) = 0, \quad y \in Y^{\mathcal{M}} \\ \bar{\sigma}_k|_{\Gamma} = 0 \end{cases} \tag{9h}$$

$$\begin{cases} \dfrac{\partial}{\partial y_i}\left(a^{\mathcal{F}}\dfrac{\partial\psi_k}{\partial y_i}\right) = -\dfrac{\partial a^{\mathcal{F}}}{\partial y_k}, \quad y \in Y^{\mathcal{F}} \\ \langle\psi_k\rangle_{\mathcal{F}} = 0, \quad \psi_k \text{ is 1-periodic} \end{cases} \tag{9i}$$

Due to the Fredholm alternative for an elliptic equation, each problem (9a), (9b) and (9i) has a unique periodic solution defined up to an additive constant which is strictly determined in our case by the condition on the average value. Each system (9c), (9d), (9e), (9f), (9g) and (9h) has a unique solution as a Dirichlet problem for the Laplace-like equation in a bounded domain $Y^{\mathcal{M}}$ with piece-wise regular boundary.

6. Analysis of the macroscopic model

System (4.1) shows the existence of two different macroscopic pressures, $P^{\mathcal{F}}$ and $P^{\mathcal{M}}$, which is a typical property of a double porosity medium resulting only from the high difference between the matrix and fracture permeabilities. At the same time, the macroscopic displacement **U** is defined in a uniform way for matrix and fractures.

Eq. (5a) can be presented with introducing the transfer function between matrix and fracture, q, for which we can obtain an equivalent formulation:

$$\begin{aligned}
&\langle\beta\rangle_{\mathcal{F}}\frac{\partial P^{\mathcal{F}}}{\partial\tau}+\frac{\partial^2 U_j}{\partial\tau\,\partial x_j}-\frac{\partial}{\partial x_i}\left(\widehat{K}_{ik}\frac{\partial P^{\mathcal{F}}}{\partial x_k}\right)=q\\
&q\frac{\tau^p}{\langle\beta\rangle_{\mathcal{M}}}=\left(P^{\mathcal{M}}-P^{\mathcal{F}}\right)+\left(\tau^u+\tau^G_{mm}\right)\frac{\partial^2 U_m}{\partial\tau\,\partial x_m}+\underbrace{\left(\tau^G_{mn}+\tau^G_{nm}\right)}_{m\neq n}\frac{\partial^2 U_m}{\partial\tau\,\partial x_n}
\end{aligned}\qquad(10)$$

with a summation over indexes m and n in all the terms.

The first term is the classic transfer caused by a pressure difference. The second term is a cross deformation-flow effect which implies that a volumetric matrix compaction–extension causes a flow in the fractures. This is a typical peristaltic effect observed in flexible tubes (blood artery, intestine, rubber hose), when an elastic expansion-dilatation of tube walls produces liquid flow inside the tube. The third term represents a flow in fracture caused by shearing strains in the matrix.

The equation of stresses, (7), can be presented in the following form more convenient to be explained:

$$\begin{aligned}
&\frac{\partial}{\partial x_i}\left(\Sigma^{(0)}_{ik}+\Sigma^{(1)}_{ik}\right)=0,\quad i,k=1,2,3\\
&\Sigma^{(0)}_{ik}\equiv\hat{\mu}_{mnki}\left(\frac{\partial U_m}{\partial x_n}+\frac{\partial U_n}{\partial x_m}\right)+\hat{\nu}_{ki}\frac{\partial U_j}{\partial x_j}-æP^{\mathcal{F}}\delta_{ik},\\
&\Sigma^{(1)}_{ik}\equiv\bar{\tau}^{\varphi}_{ki}\frac{\partial P^{\mathcal{F}}}{\partial\tau}-\bar{\tau}^{\theta}_{mnki}\frac{\partial\widehat{G}_{mn}}{\partial\tau}-\bar{\tau}^{\sigma}_{ki}\frac{\partial^2 U_j}{\partial\tau\,\partial x_j}
\end{aligned}$$

The stress $\Sigma^{(0)}_{ik}$ may be examined as the classic averaged result which corresponds to a moderately heterogeneous medium without double porosity. The new stress $\Sigma^{(1)}_{ik}$ is caused by the double porosity, i.e., by the exceeding pressure in matrix, which provokes additional deformations.

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