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Laminar forced convection with viscous dissipation in a concentric annular duct

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Abstract

Forced convection heat transfer in fully developed flows of viscous dissipating fluids in concentric annular ducts is analyzed analytically. Special attention has been paid to the effect of the viscous dissipation. Two different cases of the thermal boundary conditions are considered: uniform heat flux at the outer wall and adiabatic inner wall (Case A) and uniform heat flux at the inner wall and adiabatic outer wall (Case B). Solutions for the velocity and temperature distributions and the Nusselt number are obtained for different values of the aspect ratio and the Brinkman number. The present analytical results for the case without the viscous dissipation effect are compared with those available in the literature and an excellent agreement is observed. ***To cite this article: M. Avcı, O. Aydın, C. R. Mecanique 334 (2006).***

Résumé

Convection laminaire forcée avec dissipation visqueuse dans un conduit annulaire concentrique. On analyse de manière analytique le transfert de chaleur par convection forcée dans des écoulements pleinement établis à l'intérieur des conduits annulaires concentriques. On considère deux cas de conditions aux limites : un flux de chaleur uniforme à travers la paroi extérieure avec une paroi intérieure adiabatique (Cas A) ou bien un flux de chaleur uniforme à travers la paroi intérieurs avec la paroi extérieure adiabatique (Cas B). Les solutions pour les distributions de la vitesse et de la chaleur, ainsi que le nombre de Nusselt ont été trouvées, correspondant aux valeurs différentes du rapport d'aspect et du nombre de Brinkman. Les résultats trouvés ici en l'absence d'effet de dissipation sont comparés avec ceux que l'on trouve dans la littérature, avec un excellent accord entre les deux. ***Pour citer cet article : M. Avcı, O. Aydın, C. R. Mecanique 334 (2006).***

Keywords: Computational fluid mechanics; Annular flow; Aspect ratio; Viscous dissipation; Brinkman number

Mots-clés : Mécanique des fluides numérique ; Écoulement annulaire ; Rapport d'aspect ; Dissipation visqueuse ; Nombre de Brinkman

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Nomenclature

Br	Brinkman number, Eq. (8)	
c_p	specific heat at constant pressure	
k	thermal conductivity	W/mK
Nu	Nusselt number	
P	pressure	N/m^2
Pr	Prandtl number	
r	radial coordinate	m
r^*	aspect ratio for the annuli $= r_i/r_o$	
r_i	inner radius of the annuli	
r_m	the radius where the maximum velocity occurs	m
r_m^*	dimensionless form of r_m, Eq. (2)	
r_o	outer radius of the annuli	
R	dimensionless radial coordinate	
q_w	wall heat flux	W/m^2
T	temperature	K
u	velocity	m/s
z	axial coordinate	

Greek symbols

α	thermal diffusivity	m^2/s
μ	dynamic viscosity	Pa s
ρ	density	kg/m^3
υ	kinematic viscosity	m^2/s
θ	dimensionless temperature, Eq. (6)	

Subscripts

m	mean
w	wall

1. Introduction

The concentric annular pipe flow is of considerable industrial importance, as it is frequently encountered in heat exchangers, gas-cooled nuclear reactors, gas turbines, extruders, and oil/gas drilling wells. Forced convection in annular ducts has been investigated extensively in the literature [1,2]. Manglik and Fang [3] studied effect of eccentricity and thermal boundary conditions on laminar fully developed flow.

Viscous dissipation plays a role like an internal heat generation source in the energy transfer, which, in the following, effects temperature distributions and, in turn, heat transfer rates. This heat source is caused by the shearing of fluid layers. The merit of the effect of the viscous dissipation depends on whether the duct wall is hot or cold. A review of the existing literature regarding the effect of the viscous dissipation is given by Aydın [4,5]. He studied the effects of the viscous dissipation on the thermal transport for both hydrodynamically and thermally developed flow [4] and hydrodynamically developed but thermally developing flow [5]. In a recent study, Aydın and Avcı [6] analytically examined laminar heat convection in a Poiseuille flow of a Newtonian fluid with constant properties by taking the viscous dissipation into account.

In the present study we aimed at analytically investigating the effect of viscous dissipation on steady state laminar heat transfer in the concentric annular pipe flow. The effect of the Brinkman number and the aspect ratio of the annular geometry on the temperature profile and, in the following, the Nusselt number are determined for two different configurations of the thermal boundary conditions.

2. Analysis

Consider steady, hydrodynamically and thermally fully developed, laminar flow of an incompressible in a concentric annular duct. The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature. The axial heat conduction in the fluid and in the wall is neglected. The fully developed velocity profile for concentric annular duct is given by [2]:

$$\frac{u}{u_m} = 2\left(\frac{1-(r/r_o)^2 + 2r_m^{*2}\ln(r/r_o)}{1+r^{*2}-2r_m^{*2}}\right) \tag{1}$$

Here r_m^* designates the dimensionless radius where the maximum velocity occurs $(\partial u/\partial r = 0)$. It is given by

$$r_m^* = \left(\frac{1-r^{*2}}{2\ln(1/r^*)}\right)^{1/2} \tag{2}$$

The conservation of energy including the effect of the viscous dissipation can be written as follows:

$$u\frac{\partial T}{\partial z}=\frac{\upsilon}{Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\mu}{\rho c_p}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2 \tag{3}$$

where the second term in the right hand side is the viscous dissipation term.

The constant heat flux at wall is assumed, which states that

$$k\frac{\partial T}{\partial r}\bigg|_{r=r_o}=q_w \tag{4}$$

where q_w is positive when its direction is to the fluid (the hot wall), otherwise it is negative (the cold wall).

For the uniform wall heat flux case, the first term in the left-side of Eq. (3) is

$$\frac{\partial T}{\partial z}=\frac{\mathrm{d}T_w}{\mathrm{d}z} \tag{5}$$

By introducing the following non-dimensional quantities:

$$U=\frac{u}{u_m},\quad R=\frac{r}{r_o},\quad \theta=\frac{T-T_w}{\frac{q_w r_o}{k}} \tag{6}$$

Eq. (3) can be written as

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left(R\frac{\mathrm{d}\theta}{\mathrm{d}R}\right)=a\left[\frac{2-2R^2+4r_m^{*2}\ln R}{1+r^{*2}-2r_m^{*2}}\right]-4Br\left[\frac{-2R+(2r_m^{*2}/R)}{1+r^{*2}-2r_m^{*2}}\right]^2 \tag{7}$$

where $a=\frac{u_m k r_o Pr}{q_w \upsilon}\frac{\mathrm{d}T_w}{\mathrm{d}z}$ and Br is the Brinkman number given as:

$$Br=\frac{\mu u_m^2}{r_o q_w} \tag{8}$$

Two different forms of the thermal boundary conditions are applied, which are shown in Fig. 1: uniform heat flux at the outer wall and adiabatic inner wall (Case A), uniform heat flux at the inner wall and adiabatic outer wall (Case B). In the following these two different cases are examined separately:

For the Case A, the thermal boundary conditions in the dimensionless form are written as:

$$\theta=0,\quad \frac{\partial\theta}{\partial R}\bigg|_{R=1}=1\quad \text{at } R=1,\qquad \frac{\partial\theta}{\partial R}\bigg|_{R=r^*}=0\quad \text{at } R=r^* \tag{9}$$

The solution of Eq. (7) and a under the thermal boundary conditions given above in Eq. (9) is obtained as, respectively,

$$a=\frac{-1/r^*-r^*+2r_m^{*2}/r^*+4Br\frac{(r^{*3}-1/r^*-4r_m^{*2}(r^*-1/r^*)+4r_m^{*4}\ln r^*/r^*)}{(1+r^{*2}-2r_m^{*2})}}{r^*-1/2r^*-r^{*3}/2+r_m^{*2}(1/r^*-r^*+2r^*\ln r^*)} \tag{10}$$

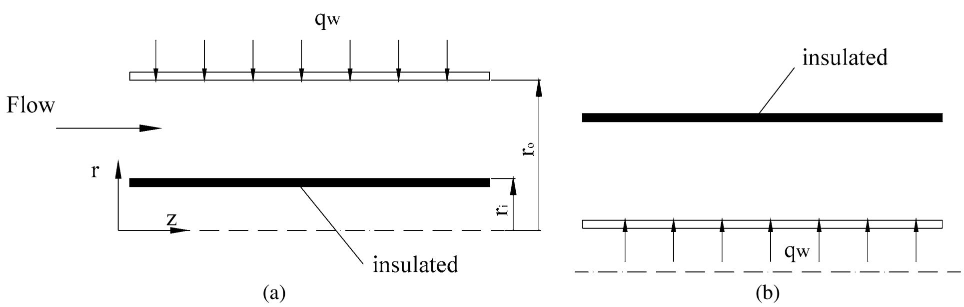

Fig. 1. Schematic diagram of flow domain (a) Case A, (b) Case B.

Fig. 1. Représentation schématique du domaine d'écoulement : (a) Cas A, (b) Cas B.

$$\theta(R)=\frac{a[-3/8+R^2/2-R^4/8-\ln R/2+r_m^{*2}(1-R^2+R^2\ln R+\ln R)]}{[1+r^{*2}-2r_m^{*2}]} - Br\frac{[-1+R^4-4\ln R-16r_m^{*2}(-1/2+R^2/2-\ln R)+8r_m^{*4}(\ln R)^2]}{[1+r^{*2}-2r_m^{*2}]^2}+\ln R \tag{11}$$

In the Case B, the thermal boundary conditions in the dimensionless form are given as:

$$\theta=0,\quad \left.\frac{\partial\theta}{\partial R}\right|_{R=r^*}=-1\quad \text{at } R=r^*,\qquad \left.\frac{\partial\theta}{\partial R}\right|_{R=1}=0\quad \text{at } R=1 \tag{12}$$

Similarly, the solution of Eq. (7) and a under the conditions of Eq. (12) gives

$$a=\frac{-1-r^{*2}+2r_m^{*2}+\frac{4Br(r^{*3}-1/r^*-4r_m^{*2}(r^*-1/r^*)+4r_m^{*4}\ln r^*/r^*)}{[1+r^{*2}-2r_m^{*2}]}}{r^*-1/2r^*-r^{*3}/2+r_m^{*2}(1/r^*-r^*+2r^*\ln r^*)} \tag{13}$$

$$\theta(R)=\frac{a[\frac{R^2-r^{*2}}{2}-\frac{R^4-r^{*4}}{8}-\frac{\ln R-\ln r^*}{2}+r_m^{*2}(R^2(\ln R-1)+r^{*2}(1-\ln r^*)+\ln R-\ln r^*)]}{1+r^{*2}-2r_m^{*2}} - Br\frac{[R^4-r^{*4}-4(\ln R-\ln r^*)-16r_m^{*2}(\frac{R^2-r^{*2}}{2}-\ln R+\ln r^*)+8r_m^{*4}((\ln R)^2-(\ln r^*)^2)]}{[1+r^{*2}-2r_m^{*2}]^2} \tag{14}$$

In the fully developed flow, it is usual to utilize the mean fluid temperature, θ_m, rather than the center line temperature when defining the Nusselt number. This mean or bulk temperature is given by:

$$\theta_m=\frac{\int U(R)\theta(R)\,\mathrm{d}A}{\int U(R)\,\mathrm{d}A} \tag{15}$$

The forced convective heat transfer coefficient is given as follows:

$$h=\frac{q_w}{T_w-T_m} \tag{16}$$

which is obtained from the Nusselt number that is defined as:

$$Nu=\frac{q_w 2(r_o-r_i)}{(T_w-T_m)k}=\frac{q_w 2r_o(1-r^*)}{(T_w-T_m)k}=-\frac{2}{\theta_m}(1-r^*) \tag{17}$$

After performing necessary substitutions, the Nusselt numbers for Cases A and B are obtained, respectively as:

$$\begin{aligned}Nu=&\big(288(r^*-1)(1-2r_m^{*2}+r^{*2})^2\big((r^{*2}-1)(-1+2r_m^{*2}+r^{*2})-4r_m^{*2}r^{*2}\ln(r^*)\big)\big)\\&\Big/\Big((r^{*2}-1)\big(a(-1+2r_m^{*2}-r^{*2})\big(21+57r^{*2}-51r^{*4}+9r^{*6}+108r_m^{*4}(1+r^{*2})\\&+4r_m^{*2}(-25-52r^{*2}+29r^{*4})\big)+4\big(9(1-2r_m^{*2}+r^{*2})^2(-3+8r_m^{*2}r^{*2})\\&+2Br\big(-39+432r_m^{*6}-3r^{*2}-3r^{*4}+9r^{*6}-54r_m^{*4}(9+r^{*2})+r_m^{*2}(232+124r^{*2}-92r^{*2})\big)\big)\big)\\&+24r^{*2}\ln r^*\big(-6(1-2r_m^{*2}+r^{*2})^2(-2+4r_m^{*2}+r^{*2})-8Br\big(36r_m^{*6}+3(r^{*2}-2)\\&-9r_m^{*4}(r^{*2}+4)+r_m^{*2}(33-12r^{*2}+r^{*4})\big)+a(-1+2r_m^{*2}-r^{*2})\big(6-3r^{*2}\\&+r_m^{*2}\big(-15+6\big(-1+6(-1+3r_m^{*2})r^{*2}+5r^{*4}\big)\big)+12r_m^{*2}\ln r^*\big(-a(-1+2r_m^{*2}-r^{*2})\\&\times\big(-1+r_m^{*2}(r^{*2}+2)\big)+2\big((1-2r_m^{*2}+r^{*2})^2+2Br\big(2+r_m^{*2}(-10+6r_m^{*2}+r^{*2})\big)\big)\\&-16Br\ln r^* r_m^{*4}\big)\big)\big)\Big)\end{aligned} \tag{18}$$

$$\begin{aligned}Nu=&\big(288(r^*-1)(1-2r_m^{*2}+r^{*2})^2\big((r^{*2}-1)(-1+2r_m^{*2}+r^{*2})-4r_m^{*2}r^{*2}\ln(r^*)\big)\big)\\&\Big/\Big((r^{*2}-1)\big(a(-1+2r_m^{*2}-r^{*2})(-75+69r^{*2}-39r^{*4}+9r^{*6}+36r_m^{*4}(-11+5r^{*2})\\&+32r_m^{*2}(11-7r^{*2}+2r^{*4}))+8Br\big(-432r_m^{*6}+r_m^{*4}(774-234r^{*2})\end{aligned}$$

$$
\begin{aligned}
&-4r_m^{*2}(103-41r^{*2}+4r^{*4})+3(19-5r^{*2}-5r^{*4}+3r^{*6})\big)\big)+24\ln r^*\big(-8Br\big(-3+36r_m^{*6}r^{*2}\\
&+3r_m^{*4}(-8-12r^{*2}+5r^{*4})-2r_m^{*2}(-9-3r^{*2}+r^{*6})\big)+a(-1+2r_m^{*2}-r^{*2})\\
&\times\big(-3+6r_m^{*4}(-2-4r^{*2}+3r^{*4})+4r_m^{*2}(3+3r^{*2}-3r^{*4}+r^{*6})\big)\\
&-12r_m^{*4}\big(-ar^{*4}+(1-2r_m^{*2}+r^{*2})+4Br\big(-1+r_m^{*2}(2+4r^{*2})\big)\big)\ln r^*\big)\Big)
\end{aligned}
\tag{19}
$$

One limiting condition of the Case A, when r^* goes to 0 is a circular duct with a uniform heat flux. For this case, Eq. (18) leads to

$$\lim_{r^*\to 0} Nu = \frac{48}{11+24Br}$$

which has been already shown by Aydin [4]. Moreover, for the case without the viscous dissipation ($Br = 0$), the above equation leads to the usual value, $Nu = 4.364$. The other limit of the Case A, when r^* goes to 1, is the parallel plates geometry. For this case, when exclude the effect of the viscous dissipation ($Br = 0$), Eq. (18) leads to

$$\lim_{x\to 1} Nu = \frac{70}{13}$$

which is the value for heat transfer between two parallel plates, the upper one with constant heat flux and the adiabatic lower one [1].

3. Results and discussion

Here, we study the forced convection flow in the concentric cylindrical annular duct including the effect of viscous dissipation. The problem is steady, laminar, and hydrodynamically and thermally fully developed. Two different cases of the thermal boundary conditions are considered: uniform heat flux at the outer wall and adiabatic inner wall (Case A) and uniform heat flux at the inner wall and adiabatic outer wall (Case B).

Above, we have already validated our analysis by checking our results for the limiting cases of the aspect ratio. In order to give a more credit to the validity of our results, we compare them with those of [1] and [3] neglecting the viscous dissipation effect. As seen from Table 1, an excellent agreement is obtained.

Fig. 2(a) and (b) illustrate the variation of the Nusselt number with the aspect ratio of the annulus, r^* for different values of the Brinkman number at Cases A and B, respectively (see also Tables 2 and 3). The effect of the viscous dissipation expressed by the Brinkman number can be best explained in terms of the energy balance. For positive values of Br, i.e., the positive heat fluxes at the wall, Nu decreases with an increase at Br. This is due to decreasing temperature differences between the wall and the bulk fluid or the decreasing temperature gradient at the wall, as stated earlier. And, as expected, negative Br values representing the negative heat flux values at the wall will increase the Nusselt number as a result of increasing temperature gradient at the wall.

For each case, at $Br = -0.1$, we observe a singularity around $r^* = 0.8$ in the $Nu - r^*$ behavior. We attribute these singularities to the energy balance between the heat flux supplied by the wall and the viscous dissipation or shear heat. At the singular point, the heat supplied by the wall is balanced by the shear heat. And, beyond this point, the heat transfer changes its direction.

Table 1
Nusselt number values for different values r^* at $Br = 0.0$

Tableau 1
Nombre de Nusselt pour plusieurs valeurs de r^* à $Br = 0,0$

r^*	Case A		Case B		
	Present	Ref. [1]	Present	Ref. [1]	Ref. [3]
0	4.36364	4.36364	∞	∞	–
0.2	4.88259	4.88259	8.49892	8.49892	8.4373
0.4	4.97917	4.97917	6.58330	6.58330	–
0.6	5.09922	5.09922	5.91171	5.91171	–
0.8	5.23654	5.23654	5.57849	5.57849	5.5832
1.0	5.38462	5.38462	5.38462	5.38462	–

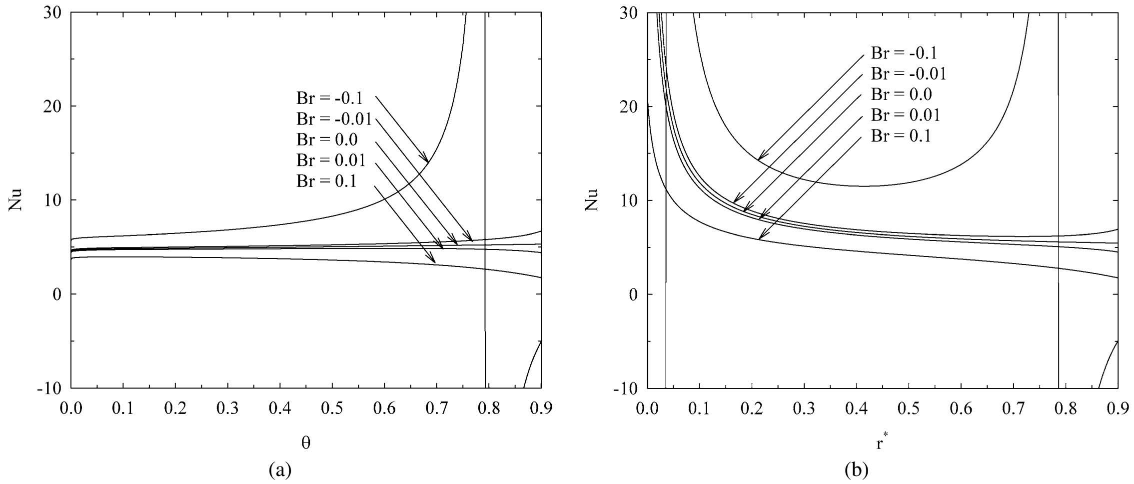

Fig. 2. The variation of the Nusselt number with the r^* at different values of *Br* (a) Case A, (b) Case B.

Fig. 2. Variation du nombre de Nusselt avec r^* correspondant aux valeurs différentes de *Br* (a) Cas A, (b) Cas B.

Table 2
Nusselt number values for different values r^* with *Br* (Case A)

Tableau 2
Nombre de Nusselt pour plusieurs valeurs de r^* avec *Br* (Cas A)

r^*	Br				
	−0.10	−0.01	0.00	0.01	0.10
0	5.58140	4.46097	4.36364	4.27046	3.58209
0.2	6.45162	5.00429	4.88259	4.76667	3.92743
0.4	7.36427	5.14583	4.97917	4.82296	3.76105
0.6	10.06916	5.36397	5.09922	4.85937	3.41409
0.8	−491.8396	5.82527	5.23654	4.75589	2.60441

Table 3
Nusselt number values for different values r^* with *Br* (Case B)

Tableau 3
Nombre de Nusselt pour plusieurs valeurs de r^* avec *Br* (Cas B)

r^*	Br				
	−0.10	−0.01	0.00	0.01	0.10
0	−20.0000	−200.000	∞	200.000	20.0000
0.2	14.73789	8.87461	8.49892	8.15375	5.97116
0.4	11.51366	6.87782	6.58330	6.31297	4.60945
0.6	13.81972	6.27053	5.91171	5.59174	3.76009
0.8	−72.78537	6.25156	5.57849	5.03626	2.68630

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