

# Influence of capillary effects on strength of non-saturated porous media

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Received 16 June 2005; accepted after revision 8 November 2005

Available online 9 December 2005

Presented by Pierre Suquet

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## Abstract

The mechanical behavior of a partially saturated porous medium is addressed by means of a micro-to-macro reasoning. First, an estimate of the quadratic average over the solid phase of the equivalent shear strain is proposed. The latter is used in the framework of a nonlinear homogenization technique ('modified secant' method) in order to model the nonlinear poroelastic behavior in partially saturated conditions. The determination of the macroscopic strength criterion is then considered. Finally, the influence of membrane tension effects on strength is investigated. *To cite this article: L. Dormieux et al., C. R. Mecanique 334 (2006).*

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## Résumé

**Influence des effets capillaires sur la résistance d'un milieu poreux non saturé.** On met en œuvre une démarche de changement d'échelle pour étudier le comportement mécanique d'un milieu poreux non saturé. On propose d'abord une estimation de la moyenne quadratique sur la phase solide, supposée élastique linéaire, de la déformation déviatorique équivalente. Celle-ci est mise à profit ensuite pour la modélisation du comportement poroélastique non linéaire dans le cadre d'une méthode d'homogénéisation non linéaire de type « sécante modifiée ». Pour finir, on présente une détermination approchée du critère de rupture macroscopique et on étudie l'influence des tensions de membrane sur ce critère. *Pour citer cet article : L. Dormieux et al., C. R. Mecanique 334 (2006).*

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*Keywords:* Porous media; Nonlinear homogenization; Strength; Unsaturated

*Mots-clés:* Milieu poreux; Homogénéisation non linéaire; Résistance; Non saturé

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## 1. Introduction

The representative elementary volume  $\Omega$  of a porous medium is made up of a homogeneous solid matrix  $\Omega^S$  and of a pore space  $\Omega^P$ . The latter is divided into a liquid domain  $\Omega^\ell$  at pressure  $p_\ell$  and a gaseous domain  $\Omega^g$

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at pressure  $p_g$ . We introduce the capillary pressure  $p_c = p_g - p_\ell$ , and the mean pressure in the whole pore space:  $\bar{p} = (1 - S_r)p_g + S_r p_\ell = p_g - S_r p_c$ , where  $S_r$  is the saturation ratio (volume fraction of the liquid domain in the pore space). In Sections 2, 3 and 4, we assume that surface tension effects can be neglected. This means that capillary effects are only taken into account through the difference between  $p_\ell$  and  $p_g$ . In contrast, Section 5 is devoted to the influence of these surface tensions.

The microscopic position vector is  $\underline{z}$ . The mean values of a field  $a(\underline{z})$  over the solid, porous, liquid or gaseous domain are denoted by  $\langle a \rangle_s$ ,  $\langle a \rangle_p$ ,  $\langle a \rangle_\ell$  and  $\langle a \rangle_g$  respectively. The mean value over the whole r.e.v. is denoted by  $\langle a \rangle$ . The porosity and the volume fractions of the liquid and the gaseous phases are denoted by  $\varphi$ ,  $\varphi^\ell$  and  $\varphi^g$ .

The micromechanical point of view considers the r.e.v. as a structure. The microscopic strain and stress tensors are denoted by  $\boldsymbol{\varepsilon}$  and  $\boldsymbol{\sigma}$ . The applied loading is characterized by the pressures  $p_g$  and  $p_\ell$ , and the macroscopic strain tensor  $\boldsymbol{E}$ , in the framework of uniform strain boundary conditions on the microscopic displacement field  $\underline{\xi}$  ( $\underline{\xi}(\underline{z}) = \boldsymbol{E} \cdot \underline{z}$ ,  $\underline{z} \in \partial\Omega$ ). The macroscopic stress tensor is then defined as  $\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma} \rangle$ .

Given a second order tensor  $\boldsymbol{a}$ , we introduce the intensity of the isotropic part  $a_m = \text{tr} \boldsymbol{a} / 3$ , the deviatoric part  $\boldsymbol{a}^{\text{dev}} = \boldsymbol{a} - a_m \mathbf{1}$ , and the intensity of the deviatoric part  $a_d = \sqrt{\boldsymbol{a}^{\text{dev}} : \boldsymbol{a}^{\text{dev}} / 2}$ . The second and fourth order identity tensors are respectively denoted by  $\mathbf{1}$  and  $\mathbb{I}$ . We shall also refer to  $\mathbb{J}$  and  $\mathbb{K}$  defined as  $\mathbb{J} = \mathbf{1} \otimes \mathbf{1} / 3$  and  $\mathbb{K} = \mathbb{I} - \mathbb{J}$ .

## 2. Average strain level in a linear elastic solid phase

In this section, the solid phase is supposed to be isotropic linear elastic (tensor of elastic moduli  $\mathbb{C}^s$ , bulk and shear moduli  $k^s$  and  $\mu^s$ ). The macroscopic state equation can be established as [1]:

$$\boldsymbol{\Sigma} + \bar{p} \boldsymbol{B} = \mathbb{C}^{\text{hom}} : \boldsymbol{E} \quad (1)$$

$\mathbb{C}^{\text{hom}}$  is the macroscopic elastic tensor in drained conditions ( $p_\ell = p_g = 0$ ), and  $\boldsymbol{B}$  is the Biot tensor. The Bishop effective stress appears on the left-hand side.

As an estimate of the deviatoric strain level in the solid phase, the quadratic average  $\langle \varepsilon_d^2 \rangle_s$  is considered. The starting point is the density  $\Psi^*$  of potential energy of the solid phase per unit volume of porous medium. The potential energy of the solid in the r.e.v. is  $|\Omega| \Psi^*$  and comprises the term of elastic energy and the work of the fluid pressures:

$$|\Omega| \Psi^* = \frac{1}{2} \int_{\Omega^s} \boldsymbol{\varepsilon} : \mathbb{C}^s : \boldsymbol{\varepsilon} dV_z - p_\ell \int_{\Omega^\ell} \text{tr} \boldsymbol{\varepsilon} dV_z - p_g \int_{\Omega^g} \text{tr} \boldsymbol{\varepsilon} dV_z \quad (2)$$

and differentiating with respect to  $\mu^s$ , we obtain:

$$|\Omega| \frac{\partial \Psi^*}{\partial \mu^s} = \int_{\Omega^s} \boldsymbol{\varepsilon}^{\text{dev}} : \boldsymbol{\varepsilon}^{\text{dev}} dV_z + \int_{\Omega^s} \frac{\partial \boldsymbol{\varepsilon}}{\partial \mu^s} : \mathbb{C}^s : \boldsymbol{\varepsilon} dV_z + \int_{\Omega^g} -p_g \mathbf{1} : \frac{\partial \boldsymbol{\varepsilon}}{\partial \mu^s} dV_z + \int_{\Omega^\ell} -p_\ell \mathbf{1} : \frac{\partial \boldsymbol{\varepsilon}}{\partial \mu^s} dV_z \quad (3)$$

Observing that the microscopic stress respectively reads  $\mathbb{C}^s : \boldsymbol{\varepsilon}$  in  $\Omega^s$ ,  $-p_g \mathbf{1}$  in  $\Omega^g$ , and  $-p_\ell \mathbf{1}$  in  $\Omega^\ell$ , Hill's lemma yields:

$$\frac{\partial \Psi^*}{\partial \mu^s} = 2(1 - \varphi) \langle \varepsilon_d^2 \rangle_s + \left\langle \boldsymbol{\sigma} : \frac{\partial \boldsymbol{\varepsilon}}{\partial \mu^s} \right\rangle = 2(1 - \varphi) \langle \varepsilon_d^2 \rangle_s \quad (4)$$

This remark and Hill's lemma applied to (2) allow one to obtain:

$$\Psi^* = \frac{1}{2} (\boldsymbol{\Sigma} : \boldsymbol{E} - p_\ell \varphi^\ell \mathbf{1} : \langle \boldsymbol{\varepsilon} \rangle_\ell - p_g \varphi^g \mathbf{1} : \langle \boldsymbol{\varepsilon} \rangle_g) \quad (5)$$

In order to take advantage of (4), it appears that we need estimates for the average volume strains in both the liquid and gas domains. The latter can be derived from the solution of the Eshelby problem for prestressed inclusions (see for instance [2]). From now on, assuming an isotropic distribution of the solid and pore phases, the pores are represented by spherical inclusions in the solid matrix. Accordingly, the average strain in  $\Omega^\alpha$  ( $\alpha = \ell, g$ ) is estimated as the uniform strain that is induced in a spherical pore saturated by a fluid at the pressure  $p_\alpha$ , embedded in an infinite solid medium subjected to uniform strain boundary conditions at infinity:  $\underline{\xi} \rightarrow \boldsymbol{E}_0 \cdot \underline{z}$  when  $|\underline{z}| \rightarrow \infty$  (see Fig. 1).  $\boldsymbol{E}_0$  represents the average strain  $\langle \boldsymbol{\varepsilon} \rangle_s$  in the solid phase:

$$\langle \boldsymbol{\varepsilon} \rangle_\ell = (\mathbb{I} - \mathbb{S})^{-1} : (\boldsymbol{E}_0 + p_\ell \mathbb{P} : \mathbf{1}); \quad \langle \boldsymbol{\varepsilon} \rangle_g = (\mathbb{I} - \mathbb{S})^{-1} : (\boldsymbol{E}_0 + p_g \mathbb{P} : \mathbf{1}); \quad \langle \boldsymbol{\varepsilon} \rangle_s = \boldsymbol{E}_0 \quad (6)$$

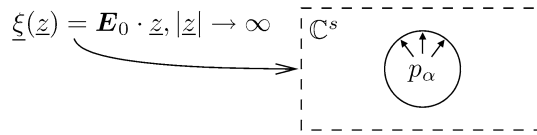


Fig. 1. The generalized Eshelby problem.

Fig. 1. Le problème d'Eshelby généralisé.

where  $\mathbb{S}$  is the Eshelby tensor of a spherical inclusion in the solid matrix and  $\mathbb{P} = \mathbb{S} : \mathbb{C}^{s-1}$ . We now take advantage of the average rule  $\mathbf{E} = \langle \boldsymbol{\varepsilon} \rangle$  that provides  $\mathbf{E}_0$  as a function of  $\mathbf{E}$ ,  $p_\ell$  and  $p_g$ , and we introduce this expression of  $\mathbf{E}_0$  into (6):

$$\langle \boldsymbol{\varepsilon} \rangle_\alpha = p_\alpha (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1} + (\mathbb{I} - (1 - \varphi)\mathbb{S})^{-1} : (\mathbf{E} - \varphi \bar{p} (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1}) \tag{7}$$

with  $\alpha = \ell$  or  $g$ . Returning to (5), it appears that we have to estimate the total work of the fluid pressures. Introducing  $\bar{p}^2 = (1 - S_r)p_g^2 + S_r p_\ell^2$  and using (7), we obtain<sup>1</sup>:

$$p_\alpha \varphi^\alpha \mathbf{1} : \langle \boldsymbol{\varepsilon} \rangle_\alpha = \bar{p} \mathbf{B} : (\mathbf{E} - \varphi \bar{p} (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1}) + \varphi \bar{p}^2 \mathbf{1} : (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1} \tag{8}$$

where we have used the Mori–Tanaka estimate  $\mathbf{B} = \varphi \mathbf{1} : (\mathbb{I} - (1 - \varphi)\mathbb{S})^{-1}$  of the Biot tensor (to be consistent with (6)). After some algebra, the above quantity can also be rearranged into:

$$p_\alpha \varphi^\alpha \mathbf{1} : \langle \boldsymbol{\varepsilon} \rangle_\alpha = \bar{p} \mathbf{B} : \mathbf{E} + \frac{\bar{p}^2}{N} + \varphi (\bar{p}^2 - \bar{p}^2) \mathbf{1} : (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1} \tag{9}$$

where  $N$  is the Biot modulus ( $1/N = (\mathbf{B} - \varphi \mathbf{1}) : \mathbb{C}^{s-1} : \mathbf{1}$ ). The last step consists in introducing the state equation (1) and (9) into (5). This yields the following estimate of the potential energy in unsaturated conditions:

$$\psi^* = \frac{1}{2} \mathbf{E} : \mathbb{C}^{\text{hom}} : \mathbf{E} - \bar{p} \mathbf{B} : \mathbf{E} - \frac{\bar{p}^2}{2N} - \frac{\varphi}{2} (\bar{p}^2 - \bar{p}^2) \mathbf{1} : (\mathbb{I} - \mathbb{S})^{-1} : \mathbb{P} : \mathbf{1} \tag{10}$$

Interestingly, it is readily seen that  $\bar{p}^2 - \bar{p}^2 = S_r(1 - S_r)p_c^2$ . Considering spherical pores and recalling the specific expression of the Eshelby tensor for a spherical inclusion, the above expression reduces to:

$$\psi^* = \frac{1}{2} \mathbf{E} : \mathbb{C}^{\text{hom}} : \mathbf{E} - \bar{p} \mathbf{B} : \mathbf{E} - \frac{\bar{p}^2}{2N} - \frac{3\varphi}{8\mu^s} S_r(1 - S_r)p_c^2 \tag{11}$$

Eq. (11) appears as the extension to unsaturated conditions of the expression of the potential energy of the solid phase derived classically in the saturated case: the homogeneous pore pressure  $P$  is replaced by the average  $\bar{p}$ . Still, from an energy point of view, the last term in the above equation shows that the unsaturated conditions are not equivalent to saturated conditions with  $P = \bar{p}$ . We now return to (4) and we take advantage of the macroscopic isotropy. From Biot relations, we have [3]:

$$\mathbf{B} = \left(1 - \frac{k^{\text{hom}}}{k^s}\right) \mathbf{1}; \quad \frac{\partial \mathbf{B}}{\partial \mu^s} = -\frac{1}{k^s} \frac{\partial k^{\text{hom}}}{\partial \mu^s} \mathbf{1}; \quad \frac{\partial 1/N}{\partial \mu^s} = -\frac{1}{k^{s2}} \frac{\partial k^{\text{hom}}}{\partial \mu^s} \tag{12}$$

Using (4), (11), (12) and the state equation (1), we derive an expression of  $\langle \varepsilon_d^2 \rangle_s$  as a function of  $\boldsymbol{\Sigma}$ ,  $\bar{p}$  and  $p_c$ :

$$4(1 - \varphi) \langle \varepsilon_d^2 \rangle_s = -\frac{\partial 1/k^{\text{hom}}}{\partial \mu^s} (\Sigma_m + \bar{p})^2 - \frac{\partial 1/\mu^{\text{hom}}}{\partial \mu^s} \Sigma_d^2 + \frac{3\varphi}{4\mu^{s2}} S_r(1 - S_r)p_c^2 \tag{13}$$

For practical use of (13), we have to select a homogenization scheme providing estimates of the drained macroscopic elastic moduli  $k^{\text{hom}}$  and  $\mu^{\text{hom}}$ :

$$k^{\text{hom}} = K(k^s, \mu^s, \varphi); \quad \mu^{\text{hom}} = M(k^s, \mu^s, \varphi) \tag{14}$$

The Mori–Tanaka scheme is adopted in the sequel.

<sup>1</sup> Summation over repeated subscript is assumed.

### 3. Nonlinear poroelastic behavior

We now address the nonlinear elastic behavior of the solid phase in isothermal conditions. We further assume that the state equation of the solid phase can be put in the form:

$$\boldsymbol{\sigma} = \mathbb{C}^s(\varepsilon_d) : \boldsymbol{\varepsilon} \quad \text{with } \mathbb{C}^s(\varepsilon_d) = 3k^s \mathbb{J} + 2\mu^s(\varepsilon_d) \mathbb{K} \quad (15)$$

$\mathbb{C}^s(\varepsilon_d)$  is classically referred to as secant stiffness tensor. This nonlinearity only affects the shear modulus. The nonlinear macroscopic poroelastic behavior can be estimated by means of the modified secant method [4]. The idea is to approximate the tensor  $\mathbb{C}^s(\varepsilon_d(\underline{z}))$  in every point  $\underline{z}$  of  $\Omega^s$  by a uniform value  $\mathbb{C}^s(\varepsilon_d^{\text{ef}})$  where  $\varepsilon_d^{\text{ef}}$  is the so-called effective deviatoric strain: it represents an estimate of the deviatoric strain level in the solid phase. We use here the quadratic average introduced in Section 2:

$$\forall \underline{z} \in \Omega^s, \quad \mathbb{C}^s(\varepsilon_d(\underline{z})) \approx \mathbb{C}^s(\varepsilon_d^{\text{ef}}) \quad \text{with } \varepsilon_d^{\text{ef}} = \sqrt{\langle \varepsilon_d^2 \rangle_s} \quad (16)$$

The macroscopic state equation then has the same structure as the one obtained in the linear case (1):

$$\boldsymbol{\Sigma} + \bar{p} \mathbf{B}(\varepsilon_d^{\text{ef}}) = \mathbb{C}^{\text{hom}}(\varepsilon_d^{\text{ef}}) : \mathbf{E} \quad (17)$$

The effective strain is the solution  $\varepsilon_d^{\text{ef}}(\boldsymbol{\Sigma} + \bar{p} \mathbf{1}, p_c, S_r)$  of the nonlinear problem (13) and (14) with  $\mu^s = \mu^s(\varepsilon_d^{\text{ef}})$ .

### 4. Strength in partially saturated conditions

We now investigate the influence of partially saturated conditions on the strength of a porous medium.

We consider the case of a solid matrix of the von Mises type (strength criterion  $f^s(\boldsymbol{\sigma}) = \sigma_d - k \leq 0$ ). We use the results obtained in Sections 2 and 3 to estimate the domain  $G^{\text{hom}}$  of admissible macroscopic stresses  $\boldsymbol{\Sigma}$ . We introduce a fictitious elastic solid whose behavior is nonlinear and described by (15). The shear modulus  $\mu^s(\varepsilon_d)$  is chosen so that the microscopic stress  $\boldsymbol{\sigma}$  tends asymptotically towards the boundary of the domain of microscopic admissible stresses ( $f^s(\boldsymbol{\sigma}) = 0$ ) when the strain  $\varepsilon_d$  is large enough (we symbolically note  $\varepsilon_d \rightarrow \infty$ ):

$$f^s\left(\lim_{\varepsilon_d \rightarrow \infty} \mathbb{C}^s(\varepsilon_d) : \boldsymbol{\varepsilon}\right) = 0 \quad (18)$$

In the von Mises case, we can take  $k^s$  as a constant and  $\mu^s \approx k/(2\varepsilon_d)$ .

Keeping  $p_\ell$  and  $p_c$  constant (the saturation ratio  $S_r$  is then also constant), we look for the stress states reached asymptotically on radial macroscopic strain paths  $\lambda \rightarrow \mathbf{E} = \lambda \mathbf{e}$ , where  $\mathbf{e}$  represents the direction of the strain path in the space of macroscopic strains. The macroscopic stress asymptotically developed on this strain path is located on the boundary of  $G^{\text{hom}}$ :  $\lim_{|\mathbf{E}| \rightarrow \infty} \boldsymbol{\Sigma} \in \partial G^{\text{hom}}$ . When the local deviatoric strain  $\varepsilon_d$  is large enough, note that  $\mu^s/k^s \ll 1$ , so the fictitious solid has an incompressible behavior. In that case, the Mori–Tanaka scheme (14) becomes simpler:

$$k^{\text{hom}} = \frac{4(1-\varphi)}{3\varphi} \mu^s(\varepsilon_d^{\text{ef}}); \quad \mu^{\text{hom}} = \frac{1-\varphi}{1+2\varphi/3} \mu^s(\varepsilon_d^{\text{ef}}) \quad (19)$$

Combining (13), (19) and recalling that  $\mu^s(\varepsilon_d) \approx k/(2\varepsilon_d)$  in the domain of large strains, it is found that the macroscopic stress state asymptotically lies on an elliptic curve of the  $(\Sigma_m, \Sigma_d)$  plane:

$$\frac{3\varphi}{4(1-\varphi)^2} (\Sigma_m + \bar{p})^2 + \frac{1+2\varphi/3}{(1-\varphi)^2} \Sigma_d^2 = k^2 - \frac{3\varphi S_r(1-S_r)}{4(1-\varphi)} p_c^2 \quad (20)$$

Strictly speaking, the strength predicted by the above criterion is not controlled by the effective stress  $\boldsymbol{\Sigma} + \bar{p} \mathbf{1}$  since the capillary pressure  $p_c$  also appears on the right-hand side. The unsaturated strength domain differs from the saturated one at a uniform pressure  $\bar{p}$ . This can be intuitively related to the extra amount of deviatoric stress created by the difference of pressure in neighboring pores. This can be taken into account in replacing the shear strength  $k$  of the solid by the corrected strength  $k'$  given by:

$$k' = k \sqrt{1 - \frac{3\varphi S_r(1-S_r)}{4(1-\varphi)} \frac{p_c^2}{k^2}} \quad (21)$$

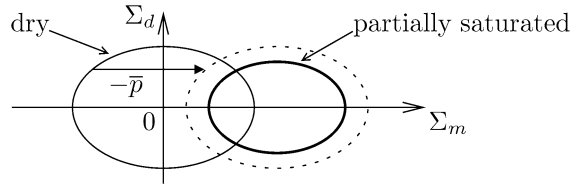


Fig. 2. Macroscopic strength criterion in the case of a von Mises solid matrix.

Fig. 2. Critère de résistance macroscopique dans le cas d'une matrice solide de von Mises.

From a geometrical point of view, let us start from the strength criterion in the dry case, that corresponds to the ellipse centered at the origin in Fig. 2. The strength criterion corresponding to the saturated state with fluid pressure  $\bar{p}$  is obtained by a translation to the right, since we expect that  $\bar{p} < 0$  (suction). The strength criterion for the unsaturated state is obtained from the previous ellipse by the homothety with ratio  $k'/k < 1$ .

### 5. Influence of membrane tension effects on strength

Capillary effects are responsible for membrane stresses developing in the interfaces  $I^{\alpha\beta}$  between the phases ( $\alpha, \beta \in \{\ell, s, g\}$ ). In a 3-dimensional description, the latter can be translated in terms of Cauchy stresses of the form  $\gamma^{\alpha\beta} \mathbf{1}_T(\underline{z}) \delta_{\alpha\beta}(\underline{z})$  where  $\gamma^{\alpha\beta}$  is the surface tension,  $\mathbf{1}_T$  is the identity tensor of the plane  $T$  tangent to  $I^{\alpha\beta}$ , and  $\delta_{\alpha\beta}$  is the Dirac distribution associated to the surface  $I^{\alpha\beta}$  [5].

We again restrict to connected spherical pores and we refer to a partition of the pore space into gas-filled pores and liquid-filled pores according to the pore radius  $r$ . More precisely, the pore is gas-filled (resp., liquid-filled) if  $r > r^*$  (resp.,  $r < r^*$ ). In other words,  $r^*$  is the critical radius separating liquid- and gas-filled pores. Neglecting hysteretic phenomena, it can be regarded as a function of  $p_c$ . The distribution of pore size is characterized by the density  $\alpha(r)$  ( $\alpha(r) dr$  is the volume fraction of the pores with radius in the interval  $[r, r + dr]$ ). The contribution of the liquid-gas interface is neglected.

Due to surface tension in the interfaces, the surface force applied on the solid at the solid-fluid boundary  $I^{\alpha s}$  is not equal to the pore pressure  $p_\alpha$ . The latter must be replaced by  $p' = p_\alpha - 2\gamma^{\alpha s}/r$ . Note that the corrected fluid pressure  $p'$  is not uniform, neither in  $\Omega^\ell$  nor in  $\Omega^g$ . Still, the developments of the previous sections can be easily adapted. The definition of the potential energy  $\Psi^*$  can be generalized in the form:

$$|\Omega| \Psi^* = \frac{1}{2} \int_{\Omega^s} \boldsymbol{\varepsilon} : \mathbb{C} : \boldsymbol{\varepsilon} dV_z - \int_{\Omega^\ell \cup \Omega^g} p'(\underline{z}) \text{tr} \boldsymbol{\varepsilon} dV_z \tag{22}$$

Despite the heterogeneity of  $p'(\underline{z})$ , it is readily seen that (4) remains valid. However,  $\bar{p}$  and  $\overline{p^2}$  in (10) must be replaced by

$$\bar{p}' = \int_0^\infty \alpha(r) p'(r) dr; \quad \overline{p'^2} = \int_0^\infty \alpha(r) p'^2(r) dr \tag{23}$$

Instead of (20), the equation of the macroscopic limit states in the  $(\Sigma_m, \Sigma_d)$ -plane now reads:

$$\frac{3\varphi}{4(1-\varphi)^2} (\Sigma_m + \bar{p}')^2 + \frac{1+2\varphi/3}{(1-\varphi)^2} \Sigma_d^2 = k^2 - \frac{3\varphi}{4(1-\varphi)} (\overline{p'^2} - \bar{p}'^2) \tag{24}$$

For practical implementation, the density function  $\alpha(r)$  is required. The latter is derived from a classical procedure [6] which is briefly recalled. The starting point is the capillary pressure curve  $p_c = p_c(S_r)$ . In the sequel, the expression [7]

$$p_c(S_r) = M(S_r^{-1/m} - 1)^{1-m} \tag{25}$$

with  $0 < m < 1$ , is adopted.  $M$  controls the magnitude of the capillary pressure induced by drying. A constant ratio  $\chi$  between pore access radius and pore radius is assumed. For a perfectly wetting fluid, the Laplace law yields the capillary pressure in the form  $p_c = 2(\gamma^{sg} - \gamma^{s\ell})/(\chi r^*)$ . Combining this equation with (25) allows to consider the

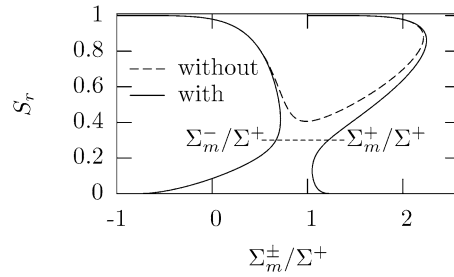


Fig. 3. Isotropic strength disregarding (without) or taking into account (with) membrane tensions;  $\varphi = 0.3$ ,  $\chi = 0.1$ ,  $m = 0.8$ ,  $M/k = 3.1$ ,  $\gamma^{s\ell} = 0$ .

Fig. 3. Résistance en traction/compression isotrope avec ou sans effet de membrane :  $\varphi = 0.3$ ,  $\chi = 0.1$ ,  $m = 0.8$ ,  $M/k = 3.1$ ,  $\gamma^{s\ell} = 0$ .

critical radius as a function  $r^*(S_r)$  of the saturation ratio. According to the definitions of  $\alpha(r)$  and  $r^*$ , the saturation ratio can be expressed as

$$S_r = \int_0^{r^*} \alpha(r) dr \quad (26)$$

Finally, differentiating (26) with respect to  $S_r$  yields the density function:

$$\alpha(r^*) = \frac{1}{dr^*/dS_r} \quad (27)$$

We now investigate the evolution of the strength during a drying experiment ( $\dot{S}_r < 0$ ,  $p_g = 0$ ), starting from the saturated state ( $S_r = 1$ ,  $p_\ell = 0$ ). As shown by (24), the domain of admissible stresses is completely characterized by the minimum ( $\Sigma_m^-$ ) and maximum ( $\Sigma_m^+$ ) admissible mean stresses. In the saturated state, note that  $|\Sigma_m^\pm| = \Sigma^+$ . In the model including membrane stresses effects,  $\Sigma_m^-$  and  $\Sigma_m^+$  are plotted against  $S_r$  at Fig. 3 respectively on the left and right solid curves. When membrane effects are disregarded,  $\Sigma_m^-$  and  $\Sigma_m^+$  are the intercepts of the dashed curve with the horizontal line  $y = S_r$ . This illustrates the fact that the model without surface tension effects significantly underestimates the material strength. For the considered set of parameters, this model predicts that the material strength vanishes for the critical saturation ratio  $S_r^{cr} = 0.4$ . In contrast, when surface tension effects are taken into account, the domain of admissible stresses is never empty, whatever the value of  $S_r$ .

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