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Flow stability analysis and excitation using pulsating jets

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Abstract

Classical flow stability applied to transition from laminar to turbulent flow may also describe the behavior of vorticity fluctuations created by a pulsating jet placed along a solid boundary. A numerical laminar flow experiment involving a pulsating jet placed along the surface of a duct with flow separation downstream, resulted in eliminating most part of the separated flow region. Applying the same approach to a turbulent flow, it was possible to develop a turbulent stability flow formulation and apply successfully turbulent pulsating jet flow separation control. ***To cite this article: D. Skamnakis, K. Papailiou, C. R. Mecanique 333 (2005).***

Résumé

Analyse de stabilité de l'écoulement et excitation par jets pulsés. La théorie de stabilité classique de l'écoulement laminaire, appliquée à la région de transition d'une couche limite, peut décrire aussi le comportement des fluctuations de vorticité créées par un jet pulsé, placé le long d'une paroi solide. Une expérience numérique sur l'écoulement laminaire dans un conduit avec décollement a confirmé la vérité de cette thèse. Cette analyse théorique s'est développée pour le cas d'un jet pulsé turbulent qui a été placé dans une configuration analogue. L'expérience numérique réalisée sur ce cas turbulent a produit des résultats qui ont confirmés les expériences réalisées dans une soufflerie subsonique. ***Pour citer cet article : D. Skamnakis, K. Papailiou, C. R. Mecanique 333 (2005).***

Keywords: Control; Flow; Pulsating; Synthetic; Jet; Stability

Mots-clés : Automatique ; Écoulement ; Jet ; Stabilité

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1. Introduction

Pulsating jets placed at a certain position along a solid wall limiting a turbulent flow, create vortices (vorticity fluctuations), which, under certain conditions, may increase in strength as they travel downstream. If, in this case, these vortices (considered as separate flow entities) remain at a close distance to the solid wall, they provide kinetic energy to the boundary layer region. If, also, the boundary layer is separated, it may reattach, in case it receives an adequate amount of kinetic energy. Hence the importance attributed to this phenomenon. Exchange of kinetic energy between the flow and the traveling vortex takes place, when the jet frequency is found in a certain range suggesting that, for this transfer to take place, a flow instability is exited. Of course, the level of the jet kinetic energy (pulsation amplitude) is an important parameter, but the needed level of pulsating jet energy is low, if one takes into account that in order to succeed in reattaching a separated flow, the energy required externally is one or two orders of magnitude lower than that, which would be required, if the same results were to be obtained using continuous suction. This phenomenon has been known for quite a few years now [1–3], but the information was obtained exclusively from experiment, until very recently, when unsteady calculations made their appearance. Two recent reviews on this subject are presented in [4,5]. They are describing the progress made in the understanding of the physics and the materialization of the techniques available for obtaining practical results. Essentially, they describe the experimental work performed until quite recently. Nevertheless efforts to simulate these unsteady flows, appear continuously during the last years [6,7]. What is still unclear is the nature of the flow instabilities, underlying the physical phenomena examined. This will be the object of the present work. Understanding will be helped by examining, first, the laminar case.

2. Stability analysis for laminar flow

The known stability of laminar flow is best described by the (well-established) Tollmien–Schlichting's theory, which can be found in some detail in [8], which has been related to transition from laminar to turbulent flow, resulting from the amplification of existing velocity fluctuations in the boundary layer. Corresponding linearized stability equation is the well known Orr–Sommerfeld equation.

The same stability analysis, may be applied to the vorticity equation, considering vorticity fluctuations, which are the appropriate perturbations related to the present development. The vorticity equation reads:

$$\vartheta_t \omega + u \cdot \nabla \omega = \nu \Delta \omega \tag{1}$$

Assuming the corresponding vorticity base (2D parallel) flow $\Omega = \vartheta_y U(y)$ and a fluctuation of it $\omega' = -\Delta \Psi$ through a stream function $\Psi(x, t, t) = \Phi(y) \mathrm{e}^{\mathrm{i}ax+bt}$, as it is expected, the same stability (Orr–Sommerfeld) equation will hold. *Consequently, the same results, which were obtained for the velocity fluctuations behavior, are valid also for the vorticity fluctuations, with the small difference that the amplitude function Φ becomes for the elementary vorticity fluctuation equal to*:

$$\Phi_\omega = a^2 \Phi - \Phi'' \tag{2}$$

The stability analysis described above provides information related to the location examined and the velocity profile associated to it.

3. Numerical experiment for laminar flow and the flow control problem

The described stability analysis has been applied, as already pointed out in the Introduction, to velocity fluctuations and was verified by experiment (Schubauer and Skramstad [9]). Until now, the stability analysis of vorticity

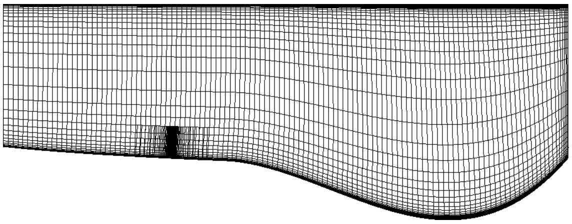

Fig. 1. Case study 1: grid used.

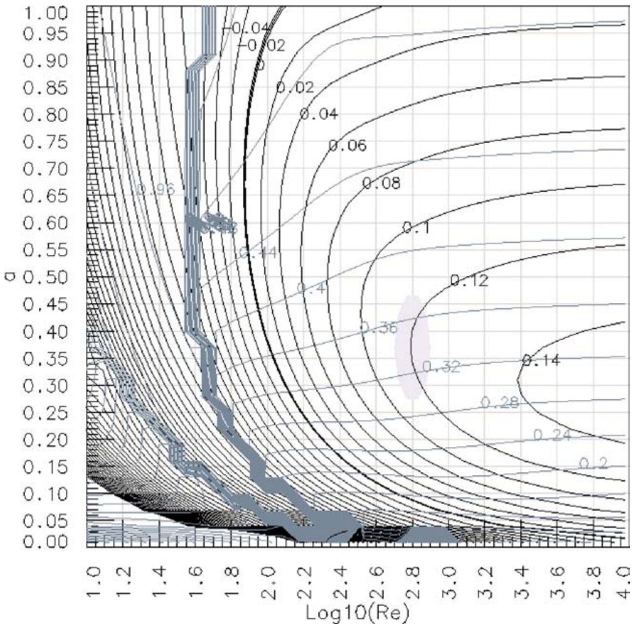

Fig. 2. Linear stability analysis results for 'Hartree' profile with $H_{12} = 3.67$. Contours of gain and velocity of perturbation are presented.

fluctuations and their amplification have not yet been considered and has not been related to the flow control work referred to in the introduction.

In this section, a numerical experiment will be set up, intended to provide information concerning separation control through vorticity fluctuations rather than transition to turbulent flow due to velocity fluctuations. An internal flow case was selected of a duct presented in Fig. 1, along with the grid, used for the computation of the flow.

A point was selected prior but very close to separation ($Re_{\delta 1} = 700$), for which $H_{12} = 3.67$ (when separation, according to the Hartree profile family used, occurs at $H_{12} = 4.023$). For this velocity profile, stability analysis calculations were performed and the results are presented in Fig. 2.

On this figure the amplification factor ci and the wave traveling velocity cr are also present. It can be noticed, that for log 10, highest amplification is achieved for a wave number $a = 0.34$, while at the same time the wave traveling velocity is $cr = 0.36$. A pulsating (synthetic in this case) jet was placed at the position indicated in Fig. 2, with a jet maximum velocity, which was varying, in order to study its influence on the separation region reduction. The frequency used corresponded to the wavelength.

In Fig. 3 left, the starting state of the flow is presented, when no control has not yet been applied. An extended separated flow region is evident. When the pulsation starts its motion, then, as this can be seen in Fig. 3 right,

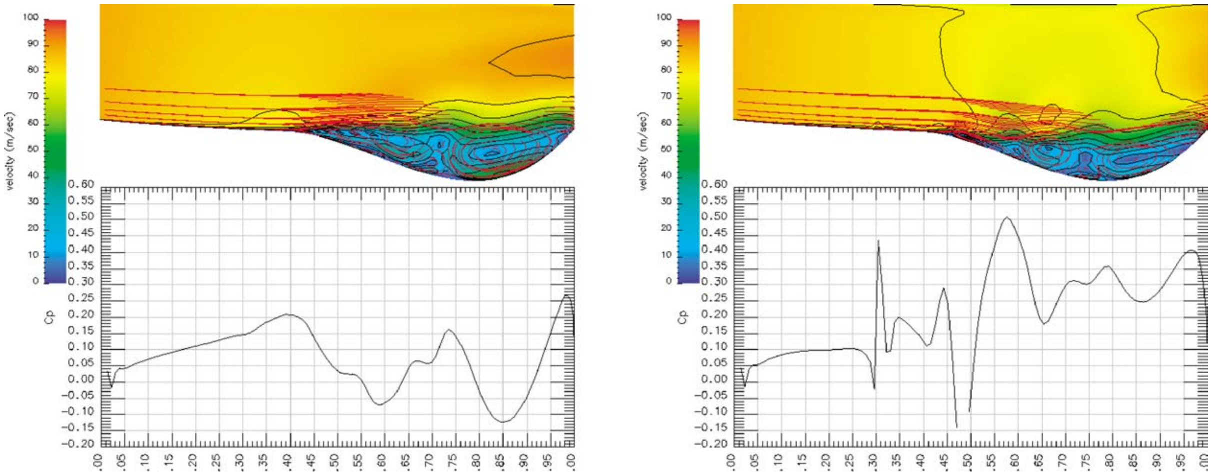

Fig. 3. Flow field simulation. Left: no control. Right: flow control applied.

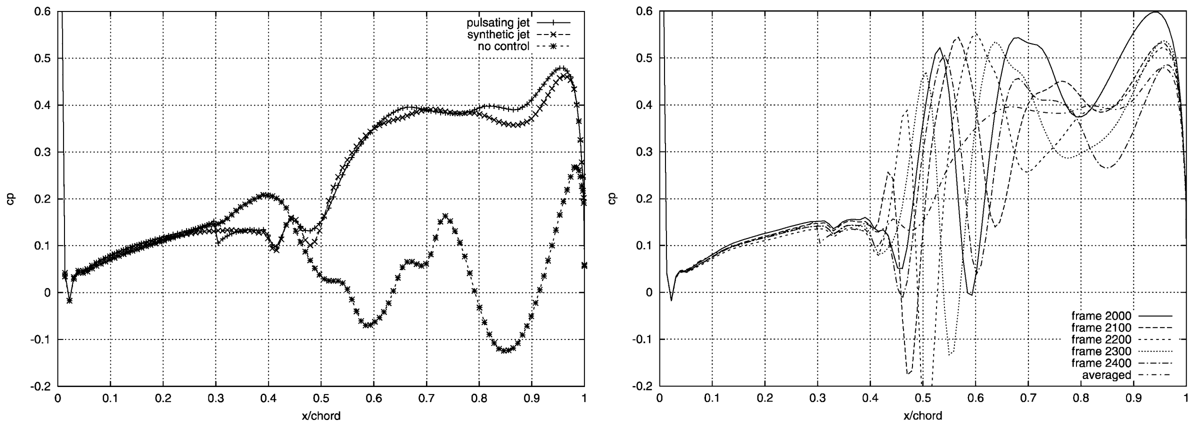

Fig. 4. *cp* distribution, left: averaged for a synthetic and a pulsating jet, right: instantaneous and mean *cp* distribution ($cm = 0.01\%$) used.

vortices are being formed at the pulsating jet level and start rolling downstream. From the corresponding pressure distribution, it is evident that the pressure inside the separation region is increasing and, when a cycle has been completed, the pressure has been increased considerably, indicating that the flow separation has been reduced. From the presented instantaneous flow pictures, it can be seen that the produced vortices, which are rolling downstream and increase in size very quickly, are staying quite close to the solid wall, succeeding to reenergetize the existing dead water region.

Fig. 4 provides time mean distributions of static pressure along the wall, without and with control. The obtained pressure rise gain with flow control is quite evident, although no appreciable difference can be observed for the pulsating cases presented. In addition, instantaneous static pressure distributions are presented in Fig. 4, along with the mean static pressure ones. It becomes obvious that the phenomenon is highly unsteady.

The effort used for the flow control (effectiveness in a way) is next examined through the momentum coefficient, defined as:

$$cm = \frac{\textit{injected momentum}}{\textit{chord} \times \textit{free sream dynamic pressure}} = \frac{(\rho_{\text{jet}} U_{\text{jet}})(h U_{\text{jet}})}{\textit{chord} * \frac{1}{2}\rho U_\infty^2} \tag{3}$$

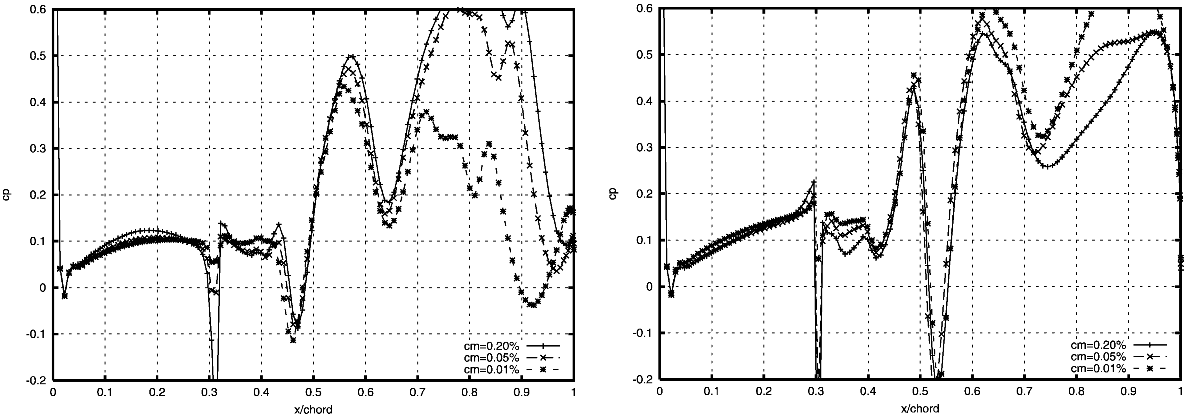

Fig. 5. *cp* distribution for various *cm*. Left figure concerns averaged pressure coefficient values at an early (in time) stage of the excitation. Right figure concerns averaged pressure coefficient values at a later (in time) stage of the excitation. Appreciable differences exist at an early stage, as the low cm pulsating jet needs more time to achieve its maximum effectiveness.

In Fig. 5 the computation results for three cases, where synthetic jets were placed at the same position, pulsating with the same frequency but possessing different amplitudes (different *cm* values), are presented. The averaging has been performed during the initial stages of the computation. It can be seen that with a very low kinetic energy supply, the obtained results are very nearly the same, although it takes more time to the lower *cm* pulsating jets to become as effective.

4. Stability analysis for turbulent flow

The manipulation performed on the laminar Navier–Stokes equations is quite clear, in view of the fact that these equations have a specific unquestionable form particularly concerning the diffusion terms. For the turbulent case, questions may arise as to the turbulent model, which is used, in view of the fact that, as this was pointed out very early [8], diffusion plays an important role in the stability characteristics of the flow. However, experiment suggests, as this was already stated, that instabilities exist and are excited for the turbulent case, as well. It was decided, then, to investigate whether the approach used in the laminar case, could have its equivalent in the turbulent case. Observing that the inner wall layers were playing an important part in helping the created vortex structure to acquire rapidly strength as it rolls downstream, it was decided to use the Boussinesq approximation, which describes relatively well the inner layers.

A similar to the laminar case approach resulted in developing an *Orr–Sommerfeld-like* equation for turbulent flows, which can be expressed ($\psi(x,y,t) \to \phi(y)\mathrm{e}^{\mathbf{i}ax+bt}$):

$$\begin{aligned}&\left(U-\mathbf{i}\frac{b}{a}\right)(\phi''-a^2\phi)-U''\phi\\&\quad=-\mathbf{i}\frac{\nu_{\mathrm{eff}}}{a}(\phi''''-2a^2\phi''+a^4\phi)\underbrace{-\mathbf{i}\frac{1}{a}(a^2\phi+\phi'')\nu''_{\mathrm{eff}}-\mathbf{i}\frac{1}{a}(-a^2\phi'+\phi''')\nu'_{\mathrm{eff}}}_{\text{additional terms due to turbulence}}\end{aligned} \tag{4}$$

It can be seen that this Orr–Sommerfeld equation differs from the one valid for laminar flow only in the expression of the diffusion terms, which depend gravely upon the turbulence modeling adopted. It has to be stressed that $\nu_{\mathrm{eff}} = \nu_{\mathrm{eff}}(y)$ and that use of the correct distribution in computations is quite important for obtaining correct results.

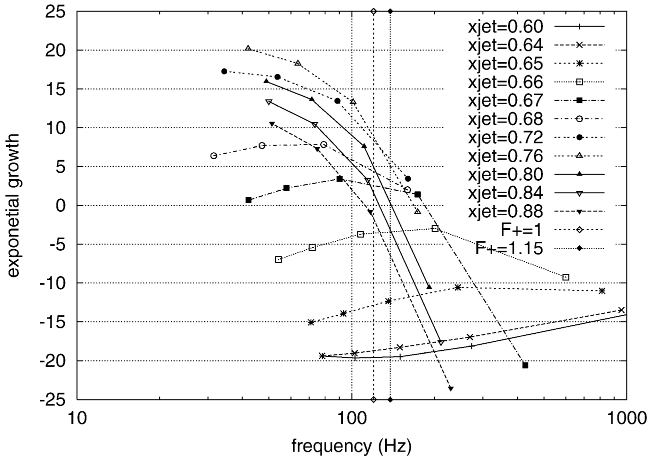

Fig. 6. Stability analysis results for various wall positions of the duct flow experiment of [9]. The experimentally used jet frequency ($F+ = 1.15$), the suggested optimal jet frequency by Wygnanski [4], and the one suggested by the stability analysis performed ($f = 100$ Hz) appear, also, on this figure.

5. Numerical experiment for turbulent flow

A turbulent experiment was considered, concerning as previously a duct, which possesses a hump, producing an airfoil-like pressure distribution along with the lower duct surface [10]. A severe adverse pressure gradient at around 60% of the hump chord, relaxed further downstream, creates a separation bubble. The flow is turbulent throughout the duct. A synthetic jet is placed at 64% of the hump chord and operates slightly upstream of the position, where the separation region starts.

Numerical solutions of Eq. (4) were obtained for eleven wall positions, indicated in Fig. 6 by 'xjet' (relative to chord wall positions). Wave lengths from 0.05 to 0.65 m were examined for a Reynolds number of 7×10^6 (1.3 m chord).

In order to avoid dumping the vortex generated by the pulsating jet, the value of its frequency should normally be appropriate and capable to destabilize the flow. Although the performed analysis provides information only for the considered position, it is clear that, if the vortex frequency is associated with amplification, the vortex will increase its strength as it travels downstream, provided that it rests near the solid wall, where the boundary layer develops. To this end, it is important to have in mind that not only a single but a range of frequencies destabilize the flow (at different amplification rates), ensuring amplification of the vorticity disturbances.

With these remarks, the stability analysis, which results for the specific experiment will be examined. Fig. 6 provides stability analysis results for the eleven positions along the hump mentioned above, that is the disturbance exponent growth for a given frequency, given the profile of each position and the flow Reynolds number. Additionally, the two reduced frequencies $F+ = 1$ (1̃20 Hz) and 1.15 (1̃40 Hz) are indicated, the first one advocated by Greenblatt and Wygnanski [4], to be the most efficient one found experimentally for active control, and the second one used during the experiment. It can be seen, in the first place that the first four slot positions do not possess the ability to amplify disturbances, whatever the frequency. Particularly the one associated with the slot position (xjet = 0.64) is the second one of the previously mentioned four positions. Consequently, no amplification is expected for the created pulsating vorticity structure at this position, for the considered range of frequencies. It can be seen, though, that as the vortical structure travels at a short distance downstream (successive positions xjet = 0.65, 0.66, 0.67) with the same frequency, which is changing very slowly, it will traverse the neutral stability line (zero exponent growth) and will enter the unstable region. From there on, the flow will become more unstable and the vortex structure will gain energy.

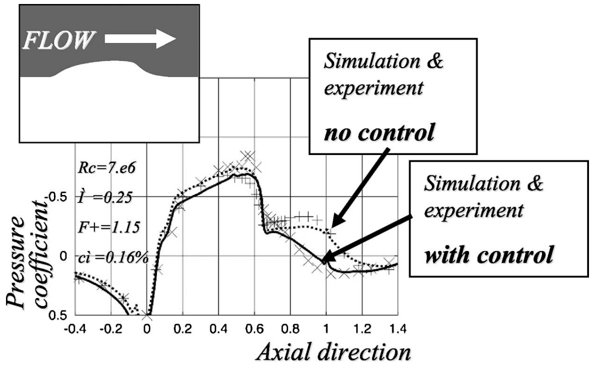

Fig. 7. Comparison between theoretical and experimental results for the duct flow of [9].

It must be stressed that between position 0.66 and 0.67, where the neutral stability line is over passed, the maximum growth is obtained for a frequency of the range of the one that has be selected experimentally. The lower frequency of that range (100 Hz) is more advantageous than the one, which has been experimentally selected, because positive exponent growth rates are achieved quickly and are combined with large exponent growth levels further downstream. However, the experimentally chosen frequency is close to the one the authors advocate being best. On the other hand, experiments performed with $F+ = 0.5$ ($\sim$60 Hz) have successfully eliminated separation, something which has to be taken into account in the judgment pronounced above.

Finally, the more unstable frequency for the vortex structure considered will tend to decrease, considering a downstream position, something, which will enhance instability, as the exponent growth will become positively larger. If the chosen frequency will be to low, it will be cut out by upstream positions. This discussion provides proof of the validity of the stability analysis proposed, but it also provides guidance for choosing the pulsating jet frequency and position for efficient control of both laminar and turbulent flows.

An unsteady computation performed for the experiment of reference [10], in steady and synthetic jet conditions, using a two equation URANS, is presented in Fig. 7, in order to complete the information available for this work. This calculation (Navier–Stokes solver with a two-equation turbulence model) is added here, in order to support the choice of the turbulence model used for performing the turbulent stability calculations. From the discussion held above, it appears that the nature of the instability for turbulent flows is identical with that of laminar ones. However, improvements in turbulence modeling could provide more correct expressions for the diffusion terms and more accurate solutions, something which cannot be checked with the available experimental data.

6. Conclusions

Experiment has indicated that flow separation can be reduced or even eliminated by positioning a pulsating jet at an appropriate position, with a frequency, which can excite existing flow instabilities. Then, the created vortex structure is traveling downstream, growing in intensity on the expense of the flow kinetic energy and resulting in eliminating the existing separation region, if the pulsation amplitude is sufficient. The externally supplied kinetic energy for obtaining this result was found experimentally to be very low, as the bulk of the required energy was provided by the external to the boundary layer flow.

The authors considered first a laminar boundary layer observing that the Tollmien–Schlichting stability analysis is also applicable to vorticity fluctuations. A numerical experiment performed for laminar flow in a duct, confirmed that the frequency indicated by the Orr–Sommerfeld equation was the proper on for excitation and flow control of the duct laminar separation, which was eliminated, when an observing appropriate synthetic jet was applied. Further, a stability equation corresponding to the laminar Orr–Sommerfeld equation was developed for turbulent flow, based on the Boussinesq assumption, which was very similar to its laminar counterpart but with additional viscous (diffusion) terms.

A control experiment for turbulent flow, performed in a duct was considered and a stability analysis was performed. It was found that the stability analysis produce a best frequency value to be used for the pulsating jet, which was quite close to the one selected experimentally. Computations performed, using a turbulent flow Navier–Stokes solver, this time, demonstrated that the experiment could be reproduced sufficiently close by computation. The turbulence model implemented in the Navier–Stokes solver used for the computation (a two-equation model) was close to the one used for the turbulent stability analysis. Although for turbulent flow, the representation of diffusion terms could still be questioned, the form of the stability equation developed and the computational results seem to confirm the hypothesis that the nature of stability in both laminar and turbulent flows, involving vorticity fluctuations, is identical with the one identified by Ralley in the late 1800s, which is well known to-day under the name of Tollmien–Schlichting stability.

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