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Motion of a porous sphere in a spherical container

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Abstract

The creeping motion of a porous sphere at the instant it passes the center of a spherical container has been investigated. The Brinkman's model for the flow inside the porous sphere and the Stokes equation for the flow in the spherical container were used to study the motion. The stream function (and thus the velocity) and pressure (both for the flow inside the porous sphere and inside the spherical container) are calculated. The drag force experienced by the porous spherical particle and wall correction factor is determined. ***To cite this article: D. Srinivasacharya, C. R. Mecanique 333 (2005).***

Résumé

Mouvement d'une sphère poreuse à l'intérieur d'un récipient sphérique. Nous considérons le mouvement rampant d'une sphère poreuse à l'instant où elle traverse le centre d'un récipient sphérique. L'étude du mouvement a été faite en utilisant le modèle de Brinkman d'un écoulement à l'intérieur d'une sphère poreuse et l'équation de Stokes décrivant l'écoulement dans un récipient sphérique. La fonction de courant (et ainsi la vitesse) et la pression (pour l'écoulement à l'intérieur de la sphère poreuse et à l'intérieur du récipient sphérique) Ont été calculées. La force de résistance au mouvement éprouvée par une particule poreuse sphérique et le facteur de correction dû à l'effet de la paroi sont aussi determinés. ***Pour citer cet article : D. Srinivasacharya, C. R. Mecanique 333 (2005).***

Keywords: Porous media; Porous sphere; Spherical container; Brinkman model; Drag; Wall correction factor

Mots-clés : Milieux poreux ; Sphère poreuse ; Récipient sphérique ; Modèle de Brinkman ; Traînée ; Facteur de correction d'une parois

1. Introduction

The problems of the motion of a particle at the instant it passes the centre of the spherical container serves as a model of interaction in multi-particle systems. This class of problems is important because it provides some

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information on wall effects. A number of researchers have considered the creeping motion of rigid or fluid sphere in a spherical or spheroidal container. Cunnigham [1] and Williams [2], independently, considered the motion of a solid sphere in a spherical container. Haberman and Sayre [3] have made an analogous study for the motion of an inner Newtonian fluid sphere. Ramkissoon and Rahaman investigated the motion of inner non-Newtonian (Reiner–Revlin) fluid sphere in a spherical container [4] and a solid spherical particle in a spheroidal container [5].

It is well known that problems of fluid flow around porous particles are encountered in many important engineering applications such as flows of fluids through porous beds (fixed or fluidized), sedimentation of fine particulate suspensions, modeling of polymer macromolecule coils in solvent, catalytic reactions where porous pallets are used, folc settling process etc. A survey of literature regarding the fluid flows past and within porous bodies indicates that while abundant information is available for flows in an infinite expanse of fluid, very little information is available for flows in enclosures.

In this Note, we consider the creeping motion of a porous sphere in a spherical container. We have used the Brinkman's model for the flow inside the porous particle and Stokes model for the flow within the spherical container. The stream function and the pressure for both the flows inside porous particle and within the spherical container are calculated. The drag experienced by the porous particle is calculated and its variation is studied numerically.

2. Formulation of the problem

Consider a porous spherical particle of radius a passing the center of a spherical vessel of radius b containing an incompressible Newtonian viscous fluid. This is equivalent to the inner particle at rest while the outer spherical container moves with a constant velocity U in the negative Z direction. We assume that the flow within the spherical container is Stokesian, and Brinkman's law [6] governs the flow inside the porous spherical particle.

The equations of motion for the region within the spherical container are

$$\operatorname{div} \vec{q}^{\,(1)} = 0 \tag{1}$$

$$\operatorname{grad} p^{(1)} + \mu \operatorname{curl} \operatorname{curl} \vec{q}^{\,(1)} = 0 \tag{2}$$

where $\vec{q}^{\,(1)}$ is the velocity, μ is the coefficient of viscosity and $p^{(1)}$ is the pressure.

For the region inside the porous sphere, the equations of motion are:

$$\operatorname{div} \vec{q}^{\,(2)} = 0 \tag{3}$$

$$\operatorname{grad} p^{(2)} + \frac{\mu}{k} \vec{q}^{\,(2)} + \mu \operatorname{curl} \operatorname{curl} \vec{q}^{\,(2)} = 0 \tag{4}$$

where $\vec{q}^{\,(2)}$ is the velocity, $p^{(2)}$ is the pressure, μ is viscosity and k is the permeability of the porous medium.

Let (r, θ, ϕ) denote a spherical polar co-ordinate system with the origin at the centre of the sphere of radius a and diameter coinciding with the line of motion of the inner sphere as the initial line. Since the flow of the fluid is in the meridian plane and the flow is axially symmetric, all the physical quantities are independent of ϕ. Hence, we assume that

$$\vec{q}^{\,(i)} = \left[u^{(i)}(r,\theta)\vec{e}_r + v^{(i)}(r,\theta)\vec{e}_\theta\right], \quad i = 1, 2 \tag{5}$$

where $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$ are unit base vectors and $h_1 = 1$, $h_2 = r$ and $h_3 = r \sin\theta$ are the corresponding scale factors in the spherical polar coordinate system.

3. Solution of the problem

Introducing the stream functions $\psi^{(i)}(r,\theta),\ i=1,2$, through

$$u^{(i)} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta},\quad v^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r},\quad i=1,2 \tag{6}$$

in Eqs. (1)–(4) and eliminating the pressure from the resulting equations, we get the following dimensionless equations for $\psi^{(i)},\ i=1,2$,

$$E^4\psi^{(1)} = 0 \tag{7}$$

and

$$E^2(E^2-\alpha^2)\psi^{(2)} = 0 \tag{8}$$

where $\alpha^2 = a^2/k$ and E^2 denotes the Stokes stream function operator given by

$$E^2 = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}\right] \tag{9}$$

The boundary conditions are:

– The normal velocity component is continuous at the boundary of the sphere i.e.,

$$u^{(1)}(r,\theta) = u^{(2)}(r,\theta)\quad \text{on } r=a \tag{10}$$

– The tangential velocity component is continuous at the boundary of the sphere i.e.,

$$v^{(1)}(r,\theta) = v^{(2)}(r,\theta)\quad \text{on } r=a \tag{11}$$

– Continuity of tangential stresses components at the boundary of the sphere i.e.,

$$\tau_{r\theta}^{(1)}(r,\theta) = \tau_{r\theta}^{(2)}(r,\theta)\quad \text{on } r=a \tag{12}$$

– Continuity of the pressure distributions at the boundary of the sphere i.e.,

$$p^{(1)}(r,\theta) = p^{(2)}(r,\theta)\quad \text{on } r=a \tag{13}$$

On the outer sphere, the condition of impenetrability leads to

$$u^{(1)}(r,\theta) = U\cos\theta\quad\text{and}\quad v^{(1)}(r,\theta) = -U\sin\theta\quad \text{on } r=b \tag{14}$$

and the condition that velocity and pressure must have no singularities anywhere in the flow field.

The boundary conditions (10)–(14) in terms of the stream function are

$$\psi^{(1)}(r,\theta) = \frac{1}{2}r^2\sin^2\theta,\quad \psi_r^{(1)}(r,\theta) = r\sin^2\theta\quad \text{on } r=1/\eta \tag{15}$$

and

$$\begin{aligned}&\psi^{(1)}(r,\theta) = \psi^{(2)}(r,\theta),\quad \psi_r^{(1)}(r,\theta) = \psi_r^{(2)}(r,\theta)\\ &\psi_{rr}^{(1)}(r,\theta) = \psi_{rr}^{(2)}(r,\theta),\quad p^{(1)}(r,\theta) = p^{(2)}(r,\theta)\quad \text{on } r=1\end{aligned} \tag{16}$$

where $\eta = a/b$.

The solutions of (7) and (8), which are nonsingular everywhere in the flow region, are

$$\psi^{(1)} = \sum_{n=2}^{\infty}\left[A_n r^n + B_n r^{1-n} + C_n r^{n+2} + D_n r^{3-n}\right]\vartheta_n(\zeta) \tag{17}$$

and

$$\psi^{(2)} = \sum_{n=2}^{\infty} \left[E_n r^n + F_n \sqrt{r} I_{n-1/2}(\alpha r) \right] \vartheta_n(\zeta) \tag{18}$$

where A_n, B_n, C_n, D_n, E_n, and F_n, are arbitrary constants, $I_{n-1/2}(\alpha r)$ denote the modified Bessel function of the first kind and $\vartheta_n(\zeta)$ is the Gegenbauer function of the first and second kind of order n and degree $-1/2$.

Using (17) in (2) and integrating the resulting equations, we get the pressure distribution $p^{(1)}$ within the spherical container as

$$p^{(1)} = -\sum_{n=2}^{\infty} \left[\frac{(4n+2)r^{n-1}}{n-1} C_n - \frac{6-4n}{n} D_n r^{-n} \right] P_{n-1}(\zeta) \tag{19}$$

Similarly, the pressure distribution $p^{(2)}$ within the porous sphere is given by

$$p^{(2)} = \alpha^2 \sum_{n=2}^{\infty} E_n \frac{r^{n-1}}{n-1} P_{n-1}(\zeta) \tag{20}$$

Using the boundary conditions (15) and (16), the constants appearing in the solutions of the problem are seen to be

$$\begin{aligned} A_2 = \big\{ &\left[(-9\eta^5 + 5\eta^3 + 4)\alpha^5 + (-126\eta^5 + 30\eta^3 + 6)\alpha^3 - 270\eta^5\alpha \right] \cosh(\alpha) \\ &+ 3\left[90\eta^4 + 5\alpha^4(3\eta^2 - 1)\eta^3 + 2\alpha^2(36\eta^5 - 5\eta^3 - 1) \right] \sinh(\alpha) \big\} / X_1 \end{aligned} \tag{21}$$

$$B_2 = -2\alpha^2 \left\{ \alpha\left[15\eta^3 + (\eta^3 - 1)\alpha^2 - 6 \right] \cosh(\alpha) + 3\left[-5\eta^3 + \alpha^2(1 - 2\eta^3) + 2 \right] \sinh(\alpha) \right\} / X_1 \tag{22}$$

$$C_2 = 3\alpha^2\eta^3 \left\{ \alpha\left[6\eta^2 + (\eta^2 - 1)\alpha^2 \right] \cosh(\alpha) + \left[\alpha^2(1 - 3\eta^2) - 6\eta^2 \right] \sinh(\alpha) \right\} / X_1 \tag{23}$$

$$D_2 = 6\alpha^2 \left\{ \alpha\left[15\eta^5 + (\eta^5 - 1)\alpha^2 \right] \cosh(\alpha) + \left[\alpha^2(1 - 6\eta^3) - 15\eta^5 \right] \sinh(\alpha) \right\} / X_1 \tag{24}$$

$$E_2 = -6\left\{ \alpha\left[45\eta^5 + (6\eta^5 - 5\eta^3 - 1)\alpha^2 \right] \cosh(\alpha) + \left[\alpha^2(1 + 5\eta^3 - 21\eta^5) - 45\eta^5 \right] \sinh(\alpha) \right\} / X_1 \tag{25}$$

$$F_2 = -3\sqrt{2\pi}\alpha^{7/2}[3\eta^5 - 5\eta^3 + 2] / X_1 \tag{26}$$

where

$$\begin{aligned} X_1 = \alpha\big[&\left\{ -270\eta^5 + \alpha^4(\eta - 1)^4(4\eta^2 + 7\eta + 4) + 6\alpha^2(10\eta^6 - 21\eta^5 + 10\eta^3 + 1) \right\} \cosh(\alpha) \\ &- 3\left\{ -90\eta^5 + \alpha^4(\eta - 1)^3(8\eta^2 + 9\eta + 3)\eta + 2\alpha^2(10\eta^6 - 36\eta^5 + 10\eta^3 + 1) \right\} \sinh(\alpha) \big] \end{aligned} \tag{27}$$

and

$$A_n = B_n = C_n = D_n = E_n = F_n = 0 \quad \text{for } n \geqslant 3 \tag{28}$$

4. Drag on the body and wall effects

The drag experienced by the inner spherical particle is given by [7]

$$F = \mu\pi \int_0^{\pi} \varpi^3 \frac{\partial}{\partial r} \left(\frac{E^2\psi}{\varpi^2} \right) r \, \mathrm{d}\theta \tag{29}$$

where $\varpi = r\sin\theta$. On carrying out the integration, it is found to be

$$Drag = 24\mu\pi a U \alpha^2 \left\{ \alpha\left[15\eta^5 + (\eta^5 - 1)\alpha^2 \right] \cosh(\alpha) + \left[\alpha^2(1 - 6\eta^3) - 15\eta^5 \right] \sinh(\alpha) \right\} / X_1 \tag{30}$$

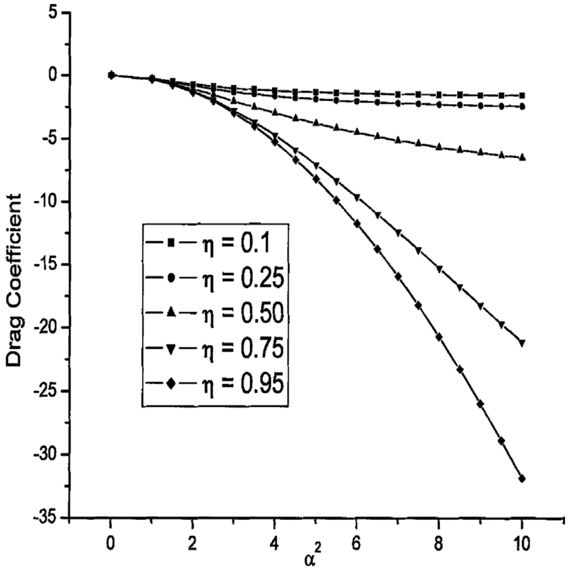

Fig. 1. The variation of drag coefficient with α^2 for various values of η.

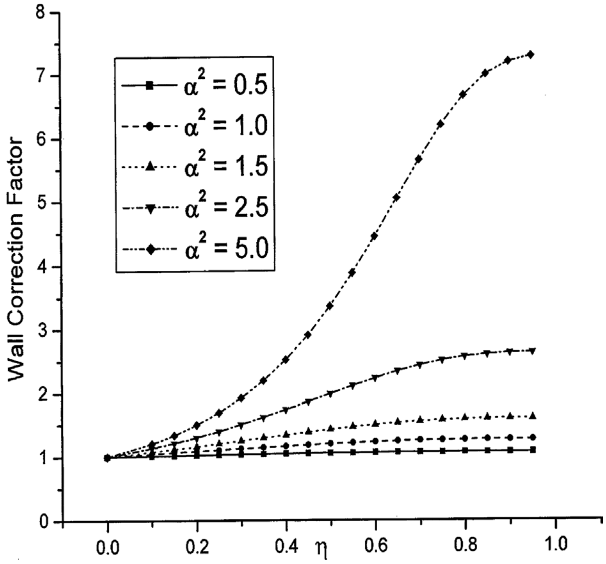

Fig. 2. The variation of wall correction factor with η for various values of α^2.

The variation of drag coefficient $D_N = Drag/(24\mu\pi a U)$ with respect to α^2 for various values of η is shown in Fig. 1. It can be observed from the figure that, as the inner porous particle size (η) increases, the drag coefficient decreases and this is expected. The drag coefficient is decreasing as the permeability parameter α^2 is increasing.

As $b \to \infty$, we get the drag on a porous sphere in the case of streaming in an unbounded medium,

$$\frac{12\pi\mu U a\alpha^2\{-\alpha\cosh(\alpha)+\sinh(\alpha)\}}{\alpha(3+2\alpha^2)\cosh(\alpha)-3\sinh(\alpha)} \tag{31}$$

which agrees with the drag on the porous sphere case derived Brinkman [6], Neale et al. [8].

The wall correction factor W_c is defined as the ratio of the actual drag experienced by the particle in the enclosure and the drag on a particle in an infinite expanse of fluid. With the aid of Eqs. (30) and (31) this becomes

$$\begin{aligned} W_c = {} & 2\{\alpha[15\eta^5+(\eta^5-1)\alpha^2]\cosh(\alpha)+[\alpha^2(1-6\eta^3)-15\eta^5]\sinh(\alpha)\} \\ & \times\{\alpha(3+2\alpha^2)\cosh(\alpha)-3\sinh(\alpha)\}/\{-\alpha\cosh(\alpha)+\sinh(\alpha)\}X_1 \end{aligned} \tag{32}$$

The variation of W_c against η for various values of α^2 is shown in Fig. 2. It can be observed from the figure that, as η increases, the wall correction factor increases. The wall correction factor is increasing as the permeability parameter α^2 is increasing.

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