Available online at www.sciencedirect.com
SCIENCE DIRECT®
C. R. Mecanique 333 (2005) 459–466
COMPTES RENDUS MECANIQUE
http://france.elsevier.com/direct/CRAS2B/

Derivation of a well-posed and multidimensional drift-flux model for boiling flows

Olivier Grégoire *, Matthieu Martin

CEA Saclay, 91191 Gif sur Yvette cedex, France

Received 18 January 2005; accepted after revision 12 May 2005

Presented by Sébastien Candel

Abstract

In this Note, we derive a multidimensional drift-flux model for boiling flows. Within this framework, the distribution parameter is no longer a scalar but a tensor that might account for the medium anisotropy and the flow regime. A new model for the drift-velocity vector is also derived. It intrinsically takes into account the effect of the friction pressure loss on the buoyancy force. On the other hand, we show that most drift-flux models might exhibit a singularity for large void fraction. In order to avoid this singularity, a remedy based on a simplified three field approach is proposed. ***To cite this article: O. Grégoire, M. Martin, C. R. Mecanique 333 (2005).***

Résumé

Dérivation d'un modèle d'écart de vitesse bien posé et multidimensionnel pour les écoulements bouillants. Dans cette Note, nous établissons un modèle d'écart de vitesse multidimensionnel pour les écoulements bouillants. Dans ce contexte, le paramètre de distribution n'est plus un scalaire mais un tenseur pouvant rendre compte du caractère anisotrope du milieu et de la nature de l'écoulement. Un nouveau modèle pour le vecteur de vitesse de dérive est également établi. Il prend intrinsèquement en compte l'impact des pertes de charge par frottement sur la force de flottabilité. Par ailleurs, nous montrons que, pour de forts taux de vide, la plupart des modèles d'écart de vitesse peuvent présenter une singularité. Une méthode, basée sur une analyse multi-champ simplifiée, est proposée qui permet de s'affranchir de cette singularité. ***Pour citer cet article : O. Grégoire, M. Martin, C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Boiling two-phase flows; Drift-flux model; Multidimensional; Void fraction

Mots-clés : Mécanique des fluides numérique ; Écoulements diphasiques bouillants ; Modèle d'écart de vitesse ; Multidimensionnel ; Taux de vide

* Corresponding author.
E-mail addresses: olivier.gregoire@cea.fr (O. Grégoire), matthieu.martin@cea.fr (M. Martin).

1631-0721/$ – see front matter
doi:10.1016/j.crme.2005.05.002

Version française abrégée

Nous nous intéressons au calcul des écoulements bouillants à haute pression rencontrés dans les sytèmes hydrauliques de grande taille comme les échangeurs de chaleur ou encore les cœurs des réacteurs nucléaires. De par leur grande taille, ces écoulements complexes ne sont pas accessibles dans leur ensemble au calcul détaillé. Il faut donc les représenter au travers d'une approche homogénéisée, moyennée en espace. Nous adoptons ici la moyenne volumique développée pour les milieux poreux [8].

Le caractère diphasique des écoulements étudiés pose le problème de la modélisation des déséquilibres cinématiques et thermiques entre les phases. Dans cet article, nous nous intéressons à la modélisation aux échelles macroscopiques de l'écart de vitesse entre les phases vapeur (indice v) et liquide (indice $_l$), défini par $\boldsymbol{U}_r = \boldsymbol{U}_v - \boldsymbol{U}_l$, dans les écoulements bouillants. Les modèles d'écart de vitesse (ou modèles de « dérive ») les plus employés dérivent généralement du modèle présenté la première fois par Zuber et Findlay [2] et rigoureusement dérivé par la suite par Ishii et al. [4,3]. Ils sont en principe établis dans un contexte unidimensionnel et reposent sur deux idées de base : la vitesse vapeur résulte de la superposition d'une vitesse d'entraînement et d'une vitesse supplémentaire, appelée vitesse de dérive (V_G) ; la vitesse de dérive traduit l'équilibre entre la force de flottabilité et le frottement interfacial. La vitesse d'entraînement rend compte des déformations des profils de vitesse et de taux de vide au travers du paramètre de distribution : $c_0 = \langle \alpha j \rangle_{\mathsf{f}} / (AJ)$, où $\langle \cdot \rangle_{\mathsf{f}}$ désigne la moyenne spatiale (voir Éq. (1)), j est la vitesse du centre de volume du mélange, J sa moyenne volumique, α est le taux de vide et A sa moyenne volumique. La vitesse vapeur moyenne est alors définie par : $U_v \equiv c_0 J + V_G$. Nous proposons dans cette note d'étendre la formulation de ce modèle au contexte multidimensionnel selon l'Éq. (4). Le paramètre de distribution n'est alors plus un scalaire mais un tenseur, noté $\mathbb{C}_0$, qui peut rendre compte de l'anisotropie du milieu et du régime d'écoulement. Dans cette Note, on supposera que $\mathbb{C}_0$ est diagonalisable.

On souhaite par ailleurs mieux rendre compte de l'adhérence des bulles en parois pour les écoulements en ébullition nucléée. Pour ce faire, nous proposons une adaptation de la correction d'adhérence originale [4] en distinguant la vapeur produite en paroi (à faible vitesse) de celle produite au sein de l'écoulement principal (approximativement à la vitesse du liquide). On propose également de prendre en compte l'impact des pertes de charge par frottement sur la force de flottabilité et donc sur la vitesse de dérive. Pour ce faire, nous substituons la force de flottabilité généralisée aux pertes de charge par frottement à la flottablité induite par le seul gradient de pression hydrostatique. Là encore la dérivation se place dans le cadre multidimensionnel.

Malgré ces améliorations, le modèle résultant (comme la plupart des modèles de la littérature) présente une singularité aux forts taux de vide. Nous mettons en évidence cette singularité et introduisons un taux de vide seuil (A_s) au delà duquel cette singularité est susceptible d'apparaître. Une approche basée sur une analyse simplifiée à trois champs (un champ pour le cœur de vapeur et deux pour le film liquide pariétal et les goutelettes disséminées dans la vapeur) est proposée pour $A > A_s$. Finalement, nous fermons ce modèle en imposant la continuité du module de la vitesse relative au voisinage de A_s.

Selon que l'on utilise un modèle simple de mélange avec trois ou quatre équations de bilan ou encore un modèle plus complet à six équations pour caractériser l'écoulement diphasique, ce modèle d'écart de vitesse pourra être utilisé pour calculer directement les vitesses vapeur et liquide ou encore pour recaler dynamiquement le coefficient de frottement interfacial [1].

1. Introduction

We are interested in calculating large scale hydraulic systems like heat exchangers or nuclear reactor cores. Such systems frequently exhibit two-phase flow regimes. The size of systems under study does not allow detailed calculations. Therefore a space-homogenized approach is needed and media we deal with might be seen as a strongly anisotropic, ordered and periodic macro-porous media. For the case of light water nuclear reactors, the medium is mainly built with fuel rod bundles. The two-phase aspect of such flows leads to non-equilibrium between phases: thermal non-equilibrium during the phase change and kinematic non-equilibrium. Drift-flux models address the issue of predicting the relative velocity between phases. They are mostly used within the so-called three and four-equation models for which balance equations for the overall two-phase mixture are derived. However, even more complex models such as the six-equation model (in a spatially averaged formulation), might require a drift-flux model to dynamically recalculate the macroscopic interfacial drag coefficient [1].

Drift-flux models are generally derived within a one-dimensional framework [2,3]. They are assumed accurate when the motions of both phases are strongly coupled and when the pressure gradient is dominated by gravity. Various improvements of the original model have been proposed. Among these, one can cite the important work of Ishii and colleagues [4,3,5,6] who propose a rigorous derivation method and various correlations for a broad range of flow conditions. In this Note, we focus on the multidimensional modeling of the kinematic non-equilibrium between phases for large scale vessels with high pressure drops. We introduce a drift-flux model for multidimensional flows that accounts for the supplementary buoyancy force induced by the frictional pressure gradient. We thus propose a complementary approach that notably extends to multidimensional purposes the recent work of Hibiki and Ishii [6]. However, whatever the model for the distribution parameter is, provided that it exhibits values greater than one, we also show that drift-flux models may exhibit a singularity for large void fractions. In order to avoids this singularity, we also introduce a model dedicated to large void fraction cases.

The guideline of the paper is the following. In Section 2, we present the drift-flux model for multidimensional applications. A new model for the distribution parameter is presented in Section 3. The derivation of the drift velocity is addressed in Section 4. Finally, it is shown in Section 5 that for large void fractions, this drift-flux model features a singularity. An original method is then proposed to overcome this difficulty.

2. Multidimensional formulation for the drift-flux model

We introduce the spatial average over a representative elementary volume (REV) in order to derive a macroscopic model of the flow enclosed within the solid matrix. This approach is analogous to that used to deal with porous media. We assume that the REV is appropriate to the geometrical characteristics of the media under study following [8] recommendations. Let us call ΔV the volume of the REV and ΔV_{f} the volume of fluid embedded within ΔV. We define the spatial average

$$\langle\zeta\rangle_{\mathsf{f}}(\mathbf{x},t) \equiv \frac{1}{\Delta V_{\mathsf{f}}(\mathbf{x})} \int\limits_{\Delta V_{\mathsf{f}}(\mathbf{x})} \zeta(\mathbf{y},t)\,\mathrm{d}V_{\mathbf{y}} \tag{1}$$

Any quantity ζ splits into: $\zeta \equiv \langle\zeta\rangle_{\mathsf{f}} + \delta\zeta$ where $\delta\zeta$ is called the deviation of ζ from its spatial average $\langle\zeta\rangle_{\mathsf{f}}$. The porosity of the medium is given by: $\phi \equiv \Delta V_{\mathsf{f}}/\Delta V$. If variation length scales of the macroscopic quantities are large with respect to the filter size (i.e. REV length scale) then the spatial average can be assumed idempotent [8].

The quantities of interest in this Note are: α, $\rho_{v(l)}$, $c = \alpha\rho_v/\rho$, $\boldsymbol{u}_{v(l)}$, $\rho\boldsymbol{u} = \alpha\rho_v\boldsymbol{u}_v + (1-\alpha)\rho_l\boldsymbol{u}_l$, $\boldsymbol{j} = \alpha\boldsymbol{u}_v + (1-\alpha)\boldsymbol{u}_l$ that denote respectively the void fraction, the vapor (resp. liquid) density, the vapor mass fraction, the vapor (resp. liquid) velocity, the mass flux and the volumetric flux. Since flows under study feature two-phases, it is relevant to introduce the void fraction weighted space average and its symmetrical, say the liquid fraction weighted space average. Flows under study are weakly compressible and we assume that the density for each component is nearly constant at the REV scale: $\langle\rho_v\rangle_{\mathsf{f}} \simeq \rho_v$, $\langle\rho_l\rangle_{\mathsf{f}} \simeq \rho_l$ and $\langle\rho\rangle_{\mathsf{f}} = \langle\alpha\rangle_{\mathsf{f}}\rho_v + (1-\langle\alpha\rangle_{\mathsf{f}})\rho_l$. However, due to the mixing between vapor and liquid, large density variations of the mixture are encountered. Hence, we also introduce the density-weighted spatial average. For the sake of simplicity, we shall use the following notations for averaged quantities:

$$\begin{aligned} &A \equiv \langle\alpha\rangle_{\mathsf{f}}, \quad C \equiv \langle\alpha\rangle_{\mathsf{f}}\rho_v/\langle\rho\rangle_{\mathsf{f}}, \quad \boldsymbol{U}_v \equiv \langle\alpha\boldsymbol{u}_v\rangle_{\mathsf{f}}/A, \quad \boldsymbol{U}_l \equiv \big\langle(1-\alpha)\boldsymbol{u}_l\big\rangle_{\mathsf{f}}/(1-A) \\ &\boldsymbol{U}_r \equiv \boldsymbol{U}_v - \boldsymbol{U}_l, \quad \boldsymbol{U} \equiv A\rho_v\boldsymbol{U}_v + (1-A)\rho_l\boldsymbol{U}_l \quad \text{and} \quad \boldsymbol{J} \equiv A\boldsymbol{U}_v + (1-A)\boldsymbol{U}_l \end{aligned} \tag{2}$$

Let us note that $\boldsymbol{J} = \langle \boldsymbol{j} \rangle_{\mathsf{f}}$ and $\langle \rho \rangle_{\mathsf{f}} \boldsymbol{U} = \langle \rho \boldsymbol{u} \rangle_{\mathsf{f}}$. On the contrary, $\boldsymbol{U}_r$ is not the spatial average of the local relative velocity, i.e. $\boldsymbol{U}_r \neq \langle u_v - u_l \rangle_{\mathsf{f}}$. From the above definitions, we deduce the following relations

$$\begin{aligned} &\boldsymbol{U}_v = \boldsymbol{J} + (1 - A)\boldsymbol{U}_r, \qquad \boldsymbol{U}_l = \boldsymbol{J} - A\boldsymbol{U}_r \\ &\boldsymbol{U}_v = \boldsymbol{U} + (1 - C)\boldsymbol{U}_r, \quad \boldsymbol{U}_l = \boldsymbol{U} - C\boldsymbol{U}_r \quad \text{and} \quad \boldsymbol{J} = \boldsymbol{U} + (A - C)\boldsymbol{U}_r \end{aligned} \tag{3}$$

The basic idea of the drift-flux model is that the dispersed phase velocity results from the addition of the mixture mean motion and the buoyancy induced velocity, hereafter called the drift velocity, $\boldsymbol{V}_G$. Since the pioneering work of Zuber and Findlay [2], it is customary to write the mean vapor velocity with respect to the volumetric flux $\boldsymbol{J}$ and the distribution parameter C_0. The distribution parameter stands for the spatial correlation: $C_0 = \langle \alpha j \rangle_{\mathsf{f}} / (AJ)$ and consequently accounts at the macroscopic scale for the spatial heterogeneities of the flow embedded in the REV. In particular, it accounts for the boundary layers contributions. Hence, the overall contribution $C_0 \boldsymbol{J}$ might be perceived as the entrainment of vapor by the mean motion of the mixture. Drift-flux models are generally derived within a one-dimensional framework where the relative velocity simply reads $U_v = c_0 J + V_G$. For multidimensional cases, we introduce the tensor of the distribution parameters $\mathbb{C}_0$. Within this framework, vapor velocity might be written:

$$\boldsymbol{U}_v \equiv \mathbb{C}_0 \boldsymbol{J} + \boldsymbol{V}_G \tag{4}$$

For the sake of simplicity, we shall assume that $\mathbb{C}_0$ is diagonalizable and that its eigen vectors constitute an orthogonal basis. Nevertheless, this assumption is fully achieved if $\mathbb{C}_0$ is real and symmetric. Furthermore, flows we deal with, more or less follow the direction of the solid matrix. Hence, as a first approximation, we assume that frame defined by the eigen basis of $\mathbb{C}_0$ matches the solid matrix frame. Within this framework, Eq. (4) written along the ith direction simply reads: $U_{v_i} = c_{0_i} J_i + V_{G_i}$.

3. Model for the distribution parameter $\mathbb{C}_0$

In the original work of Ishii [4], the one-dimensional distribution parameter is expressed

$$C_0 = \left[C_\infty + (1 - C_\infty)\sqrt{\rho_v/\rho_l}\,\right] \times \underbrace{\left[1 - \exp(-\zeta A)\right]}_{\text{bubbles adherence at walls}} \tag{5}$$

where C_∞ and ζ ($\gg 1$) are phenomenological constants. The tendency for bubbles to stick to walls in sub-cooled nucleate boiling flows is taken into account by the exponential contribution. In such flows, void fraction is low and the exponential contribution reduces C_0. Indeed, vapor velocity tends to $\boldsymbol{V}_G$ when the void fraction tends towards 0. For high void fraction values, the adherence contribution simply vanishes. In this paper, we propose to clearly discriminate the motion of vapor produced by nucleation processes from the motion of vapor generated at interfaces that already exist. Within the sub-cooled boiling regime, the bulk flow temperature is significantly smaller than saturation temperature and nucleation mass exchange dominates. Bubbles appear at nucleation sites at wall. They may slide along the wall, grow or even detach. Detached bubbles might condensate into bulk flow or reattach downstream. Since within this regime, bubbles stay within wall vicinity, their velocity remain small compared with the bulk velocity. This phenomenology is modeled by the so-called ‘adherence’ contribution. In contrast, within the developed nucleate boiling and the saturated boiling regimes, extensive vapor generation occurs that is both due to nucleation, $\langle \Gamma_v^N \rangle_{\mathsf{f}}$ and vaporization, $\langle \Gamma_v^G \rangle_{\mathsf{f}}$. Bubbles rapidly detach from active nucleation sites and progressively migrate inside the entire flow. During this, they undergo condensation, dilatation, fragmentation or coalescence. Consequently, vapor inclusions are no more restricted to walls vicinity, they achieve much higher velocities. Following these remarks, we propose

$$\mathbb{C}_0 = \left[\mathbb{C}_\infty + (1 - \mathbb{C}_\infty)\sqrt{\rho_v/\rho_l}\,\right] \times \left[1 - \exp(-\zeta A) \times \left\langle \Gamma_v^W \right\rangle_{\mathsf{f}} / \langle \Gamma_v \rangle_{\mathsf{f}}\right] \tag{6}$$

where $\langle\Gamma_v\rangle_{\mathsf{f}} \equiv \langle\Gamma_v^N\rangle_{\mathsf{f}} + \langle\Gamma_v^G\rangle_{\mathsf{f}}$ is the total rate of vapor generation. For sub-cooled nucleate boiling flows: $\langle\Gamma_v^N\rangle_{\mathsf{f}}/\langle\Gamma_v\rangle_{\mathsf{f}} \to 1$ while for fully developed boiling flows, we have: $\langle\Gamma_v^N\rangle_{\mathsf{f}}/\langle\Gamma_v\rangle_{\mathsf{f}} \to 0$. The diagonal tensor $\mathbb{C}_\infty$ for the coefficients involved in the distribution parameter correlation is a property of the medium and of the flow regime. Hence, it might vary according to space and to the flow conditions.

4. Derivation of the drift velocity V_G

The drift velocity is defined as the velocity of the dispersed phase with respect to the volume center of the mixture. It is tightly linked to buoyancy. In order to derive a vectorial model for the drift velocity, we enhance the 1D method developed by Ishii and Zuber [3]. In what follows, the flow under study is called the 'original flow'. We also consider two stereotype flows:

(i) the steady and homogeneous flow without shear that exhibits the same flow conditions (temperature, pressure gradient, void fraction) as the original flow at the location of interest; this situation shall be referred to as the 'homogeneous' flow;
(ii) the motionless infinite flow that shows only one single bubble with the same Sauter and drag radii and the same pressure gradient as the original flow; all quantities related to this second flow will be denoted ∞ and this flow shall be referred to as the 'single particule' flow.

Furthermore, we assume: a pressure equilibrium between liquid and vapor: $P_v = P_l = P$, that the liquid phase is continuous and vapor phase is dispersed. The basic balance equations are given by the so-called '6 Eqs. model' [3,1]. Under steadiness and homogeneity assumptions, spatially averaged balance equations for the continuity of vapor and liquid simply read: $\langle\Gamma_v\rangle_{\mathsf{f}} = 0$. Since the space average of the rate of vapor generation $\langle\Gamma_v\rangle_{\mathsf{f}}$ is zero, we also neglect momentum exchange due to phase change. Hence, the spatially averaged equations of vapor and liquid momentum can be expressed

$$\langle \boldsymbol{M}_v''\rangle_{\mathsf{f}} + A\boldsymbol{\tau}_w^v - A\nabla P + \rho_v A\boldsymbol{g} = 0 \tag{7}$$

$$-\langle \boldsymbol{M}_v''\rangle_{\mathsf{f}} + (1-A)\boldsymbol{\tau}_w^l - (1-A)\nabla P + \rho_l(1-A)\boldsymbol{g} = 0 \tag{8}$$

where $\boldsymbol{g}$ is gravity, $\langle \boldsymbol{M}_v''\rangle_{\mathsf{f}}$ is the momentum exchange and $\boldsymbol{\tau}_w^{v,l}$ stands respectively for wall friction for vapor and liquid. Major contributions to momentum exchanges are: the drag force, the added mass force, the lift force and the turbulent dispersion force. Under the steadiness and homogeneity assumptions, the last three contributions vanish. Only drag remains: $\boldsymbol{M}_v'' = \boldsymbol{M}_D$. Adding Eqs. (7) and (8), we get the expression of the pressure gradient that accounts for the hydrostatic equilibrium

$$\nabla P = \rho\boldsymbol{g} + A\boldsymbol{\tau}_w^v + (1-A)\boldsymbol{\tau}_w^l \tag{9}$$

Introducing Eq. (9) in (7) and taking into account the definition: $\rho \equiv A\rho_v + (1-A)\rho_l$, we get:

$$\langle \boldsymbol{M}_D\rangle_{\mathsf{f}} = A(1-A)\big[(\rho_l-\rho_v)\boldsymbol{g} + \boldsymbol{\tau}_w^l - \boldsymbol{\tau}_w^v\big] \tag{10}$$

Eq. (10) simply expresses the equilibrium between drag and buoyancy forces. From a macroscopic point of view, the friction pressure loss can be seen as a supplementary volumetric force. In that way, it adds to the hydrostatic (gravity induced) pressure gradient to yield the overall pressure gradient. The volumetric buoyancy force involved in (10) is expressed:

$$\boldsymbol{M}_B \equiv (1-A)\big[(\rho_l-\rho_v)\boldsymbol{g} + \boldsymbol{\tau}_w^l - \boldsymbol{\tau}_w^v\big] \tag{11}$$

Following [5,1], we postulate the macroscopic form of the drag force:

$$\langle \boldsymbol{M}_D\rangle_{\mathsf{f}} \equiv -C_D\rho_l\frac{\langle a^I\rangle_{\mathsf{f}} R_{sm}}{8R_D}\|\boldsymbol{U}_r\|\boldsymbol{U}_r \tag{12}$$

where C_D denotes the drag coefficient, a^I is the volumetric density of interfacial area between phases, R_{sm} stands for the mean Sauter radius and R_D is the mean drag radius. By definition, we have for dispersed two-phase flows: $a^I \equiv 3\alpha/r_{sm}$. We approximate $\langle a^I\rangle_{\mathsf{f}} \simeq 3A/R_{sm}$, where $R_{sm} = \langle r_{sm}\rangle_{\mathsf{f}}$ and we get

$$\langle \boldsymbol{M}_D\rangle_{\mathsf{f}} = A\boldsymbol{M}_B \simeq -\frac{3C_D\rho_l A}{8R_D}\|\boldsymbol{U}_r\|\boldsymbol{U}_r \tag{13}$$

Hence, Eq. (13) yields the expression for $\boldsymbol{U}_r$:

$$\boldsymbol{U}_r\|\boldsymbol{U}_r\| = -\frac{8R_D}{3\rho_l C_D}\boldsymbol{M}_B \quad \text{and} \quad \|\boldsymbol{U}_r\|^2 = \frac{8R_D}{3\rho_l C_D}\|\boldsymbol{M}_B\| \tag{14}$$

We now consider the second stereotype flow with a single inclusion. This inclusion has the same characteristics as the population of bubbles of the homogeneous flow, e.g. the same Sauter and drag radii. Furthermore, since, this second stereotype flow is infinite, void fraction is zero. Hence, we have $\rho_{l_\infty} = \rho_l$, $R_{D_\infty} = R_D$, $R_{sm_\infty} = R_{sm}$ and $\nabla P_\infty = \nabla P$. Within this framework, Eq. (14) becomes

$$\boldsymbol{U}_{r_\infty}\|\boldsymbol{U}_{r_\infty}\| = -\frac{8R_D}{3\rho_l C_{D_\infty}}\boldsymbol{M}_{B_\infty} \quad \text{and} \quad \|\boldsymbol{U}_{r_\infty}\|^2 = \frac{8R_D}{3\rho_l C_{D_\infty}}\|\boldsymbol{M}_{B_\infty}\| \tag{15}$$

In order to derive an expression for $\boldsymbol{M}_{B_\infty}$, we introduce $\boldsymbol{g}_\infty$, the volumetric force field that induces the same pressure gradient for both stereotype flows. For the second stereotype flow, Eq. (9) is greatly simplified into

$$\boldsymbol{0} = -\nabla P + \rho_l\boldsymbol{g}_\infty \quad \Longleftrightarrow \quad \rho_l\boldsymbol{g}_\infty = \nabla P = \rho\boldsymbol{g} + A\boldsymbol{\tau}_w^v + (1-A)\boldsymbol{\tau}_w^l \tag{16}$$

Therefore, we get from definition (11): $\boldsymbol{M}_{B_\infty} = \lim_{A_\infty\to 0, \boldsymbol{U}_{l_\infty}\to 0, \boldsymbol{g}\to\boldsymbol{g}_\infty}\boldsymbol{M}_B = (\rho_l - \rho_v)\boldsymbol{g}_\infty$. As it appears in [3,6], many correlations for the interfacial drag coefficient C_D exist. Each correlation is tightly connected to a specific flow regime. We focus in this Note on high pressure two-phase flows such as those encountered during accidental events in nuclear reactor cores. For such flows, it is inferred [7] that the drag coefficient for distorted-bubbly-flows is valid. This remark even applies to high void fraction situations where two-phase flows may look like foam rather than annular flows [7]. For a single particule submitted to the generalized buoyancy force $\boldsymbol{M}_{B_\infty}$, the interfacial drag coefficient can be expressed

$$C_{D_\infty} = 4R_D/3 \times \sqrt{\|\boldsymbol{M}_{B_\infty}\|/\sigma} \tag{17}$$

Let us recall that σ is the surface tension and that $\sqrt{\sigma/\|\boldsymbol{M}_{B_\infty}\|}$ is the Laplace length scale for the overall pressure gradient of the original flow. From Eqs. (15) and (17), we derive the modulus of the maximal ascending velocity for one single particule submitted to the pressure gradient of the original flow

$$\|\boldsymbol{U}_{r_\infty}\| = \sqrt{2}\big(\sigma\|\boldsymbol{M}_{B_\infty}\|\rho_l^{-2}\big)^{1/4} \tag{18}$$

From Eqs. (14) and (15), we also derive the average ascending velocity for the population of bubbles of the homogeneous flow submitted to the same pressure gradient

$$\boldsymbol{U}_r = -\|\boldsymbol{U}_{r_\infty}\|\sqrt{C_{D_\infty}/C_D} \times \boldsymbol{M}_B/\sqrt{\|\boldsymbol{M}_B\| \times \|\boldsymbol{M}_{B_\infty}\|} \tag{19}$$

Since for the homogeneous flow under study, the distribution parameter value is 1, we have: $\boldsymbol{U}_v = \boldsymbol{J} + \boldsymbol{V}_G$. This yields: $\boldsymbol{V}_G = (1-A)\boldsymbol{U}_r$. For the multi-particulate configuration, Ishii and Zuber [3] propose: $C_D = C_{D_\infty}(1-A)^{-\theta}$ with $\theta = 1/2, 1$ or $3/2$ according to the ratio of the dynamical viscosities: $\mu_v/\mu_l \ll 1, \simeq 1$ or $\gg 1$ respectively. Taking into account this latter correlation in Eqs. (19) and (18), we finally get the vectorial expression for the drift velocity

$$\boldsymbol{V}_G = -\sqrt{2}\big(\sigma\|\boldsymbol{M}_{B_\infty}\|\rho_l^{-2}\big)^{1/4}(1-A)^{(1+\theta/2)} \times \boldsymbol{M}_B/\sqrt{\|\boldsymbol{M}_B\| \times \|\boldsymbol{M}_{B_\infty}\|} \tag{20}$$

5. Ill-posedness of the distribution parameter for large void fraction and remedy

Since, $\mathbb{C}_0$ is diagonal, we deduce from Eqs. (2), (3) and (5), that along the ith direction

$$U_{r_i} = \big[(C_{0_i} - 1)J_i + V_{G_i}\big]/(1 - A) \tag{21}$$

$$= \big[(C_{0_i} - 1)U_i + V_{G_i}\big]/\big[1 - C + C_{0_i}(C - A)\big] \tag{22}$$

Obviously $\rho_v \ll \rho_l$ and since $C = A/[(A + (1 - A)\rho_l/\rho_v]$, we have $C \leqslant A$. Hence, for the ith component of $\mathbb{C}_0$ greater than one, we can show that the denominator (hereafter denoted D) of (22) cancels for

$$A = 1/\big[C_{0_i}(1 - \rho_v/\rho_l)\big] \quad \text{or} \quad A = 1 \tag{23}$$

For $C_{\infty_i} > 1$, we have from definition (5): $C_{0_i} < C_{\infty_i}$. Hence, it is easy to show from (23) that D_i will be positive if $A < 1/c_{\infty_i}$. For multidimensional cases we define the threshold void fraction value: $A_M \equiv \min_{i=1,2,3}(1/C_{\infty_i})$. A direct way to avoid the cancellation of D_i is to define a well-posed expression for the distribution parameter as in [6]. However, this method leads to rather complicated formulations for the distribution parameter and may not be adapted for multidimensional configurations. In this approach, a specific correlation is devoted to each flow regime (e.g. bubbly, churn and annular flows) [3] and the lack of continuity at transitions between various flow regimes for the relative velocity still remains an issue. On the other hand, we have empirically noticed that correlations (5) and (20) provide satisfactory results for high pressure flows ($P > 80$ bars for water-steam flows) even for high void fractions (typically $A = 0.8$). This result corroborates experimental observations performed by François [7] that indicate a strong coupling between phases even for high void fractions. For very high void fractions (typically $A > 0.9$), we infer that such flows achieve the annular regime. Hence, the method presented in Section 4 to derive the drift-velocity may not apply anymore. In the sequel to this Note, we propose a simple method to avoid the singularity. We assume that within the annular regime, the liquid phase splits into a continuous film stuck to walls and a field of burst droplets that move at the vapor velocity. Let us call β the fraction of liquid constituting the film, $\boldsymbol{U}_{\mathtt{f}}$ its velocity and $\boldsymbol{U}_d$ the droplet field velocity. The overall flow-rate then becomes

$$\boldsymbol{G} \equiv \rho\boldsymbol{U} \equiv A\rho_v\boldsymbol{U}_v + (1 - A)\rho_l\big[\beta\boldsymbol{U}_{\mathtt{f}} + (1 - \beta)\boldsymbol{U}_d\big] \tag{24}$$

We now consider the very simple approximation: $\boldsymbol{U}_{\mathtt{f}} = 0$ and $\boldsymbol{U}_d = \boldsymbol{U}_v$. We then deduce the expression for the relative velocity within this annular flow

$$\boldsymbol{U}_r^a = \beta\boldsymbol{U}_v \quad \text{or in an other way} \quad \boldsymbol{U}_r^a = \beta\boldsymbol{U}/\big[1 - \beta(1 - C)\big] \tag{25}$$

The model relies on the following basic ideas:

- introduce a threshold void fraction A_s such as $A_s \leqslant A_M$;
- for $A \leqslant A_s$, the drift-flux model presented in § 2 fits over ; this relative velocity is denoted $\boldsymbol{U}_r^M$;
- for $A > A_s$, use $\boldsymbol{U}_r = \boldsymbol{U}_r^a$.

In order to ensure an approximate continuity of the relative velocity at the void fraction threshold, we impose $\|\boldsymbol{U}_r^M\| = \|\boldsymbol{U}_r^a\|$ for $A = A_s$. We also assume that, at the same location, departure from colinearity between $\boldsymbol{U}_r^M$ and $\boldsymbol{U}_r^a$ is small. Since A_s is supposed to be large, properties of both phases shall be taken at the saturation point for boiling flows: $\rho_v = \rho_v^{\mathrm{sat}}$, $\rho_l = \rho_l^{\mathrm{sat}}$. Hence, we also have: $\rho_s = A_s\rho_v^{\mathrm{sat}} + (1 - A_s)\rho_l^{\mathrm{sat}}$, $\boldsymbol{U}_s = \boldsymbol{U}(A_s) = \boldsymbol{G}/\rho_s$, $C_s = A_s\rho_v^{\mathrm{sat}}/\rho_s$, $\boldsymbol{c}_{0_s} = \boldsymbol{c}_0(A_s, \rho_v^{\mathrm{sat}}, \rho_l^{\mathrm{sat}})$ and $\boldsymbol{V}_{\boldsymbol{G}_s} = \boldsymbol{V}_{\boldsymbol{G}}(A_s, \rho_v^{\mathrm{sat}}, \rho_l^{\mathrm{sat}})$. Finally, we assume that β is constant for the whole domain: $A_s \leqslant A < 1$. According to that latter assumption we have in particular: $\beta = \beta(A_s)$. Thus, we deduce from (25)

$$\beta\big\|\boldsymbol{U}_v(A_s)\big\| = \big\|\boldsymbol{U}_r^a(A_s)\big\| = \big\|\boldsymbol{U}_r^M(A_s)\big\| \tag{26}$$

Table 1
Summary of the drift-flux model and its remedy for the singularity occurring for large void fractions

Void fraction	Assumptions	Modeling
$A < A_s$	The drift-flux model presented in Section 2 (Eq. (4)), Section 3 (Eq. (6)) and Section 4 (Eq. (20)) fits over.	$\boldsymbol{U}_r = \boldsymbol{U}_r^M$
$A = A_s$	Parameter β is constant for $A \geqslant A_s$. Continuity of $\\|\boldsymbol{U_r}\\|$ and quasi-colinearity of $\boldsymbol{U_r}$ and $\boldsymbol{U}$ nearby A_s.	In particular $\beta = \beta(A_s)$. $\beta = \beta(A_s) = \frac{\\|\boldsymbol{U}_r^M(A_s)\\|}{\\|\boldsymbol{U}_s\\|+(1-C_s)\\|\boldsymbol{U}_r^M\\|}$
$A > A_s$	The flow is assumed to be annular and at the saturation point. Three fields are considered: a continuous vapor core, a continuous liquid film at walls and a field of droplets dispersed within the vapor core. We assume: $\boldsymbol{U}_d = \boldsymbol{U}_v$: dispersed droplets carried out like a passive scalar by the vapor flow; $\boldsymbol{U}_\mathtt{f} = \boldsymbol{0}$: liquid film stuck to walls.	Three velocities are calculated: $\boldsymbol{U}_v$, $\boldsymbol{U}_\mathtt{f}$ and $\boldsymbol{U}_d$. The distribution parameter (β) among the continuous and dispersed liquid phase is introduced. We have: $\boldsymbol{U}_r = \boldsymbol{U}_r^a = \beta \boldsymbol{U}_v$

According that $\boldsymbol{U}_r^M$ and $\boldsymbol{U}_r^a$ are assumed quasi-colinear and have the same direction, we deduce from Eqs. (3) and (26) the expression for β

$$\beta = \frac{\|\boldsymbol{U}_r^M(A_s)\|}{\|\boldsymbol{U}_s\| + (1 - C_s)\|\boldsymbol{U}_r^M(A_s)\|} \tag{27}$$

The model developed to calculate the relative velocity for large void fraction is summarized in Table 1.

6. Conclusions

This Note proposes a drift-flux model usable for multidimensional simulations. It is based on a tensorial extension of the original model of Ishii [4]. The so-called distribution parameter is now a tensor. Each space direction has its own distribution parameter that is a property of the medium and of the flow regime. We also propose an improvement of the adherence correction that better accounts for nucleate boiling flows: vapor generated at walls is now treated separately from vapor generated in the bulk flow. The derivation of the drift-flux velocity is also presented. This velocity takes into account the overall pressure gradient and not only the hydrostatic pressure gradient. It thus accounts for friction losses of pressure in the buoyancy force expression. However, it is shown that, when the distribution parameter is greater than one (which is the most general case) and for large void fraction, drift-flux models generally exhibit a singularity. A remedy to this singularity is proposed that is based on a simplified three-field analysis. The calibration of phenomenological coefficients and the validation of the model will be presented in a forthcoming paper.

References

[1] D. Bestion, The physical closure laws in the CATHARE code, Nuclear Engrg. Design 124 (1990).
[2] N. Zuber, J.A. Findlay, Average volumetric concentration in two-phase flow, J. Heat Transfer 87 (1965).
[3] M. Ishii, N. Zuber, Drag coefficient and relative velocity in bubbly, droplet or particulate flows, AIChE J. 25 (1979).
[4] M. Ishii, One dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes, ANL-77-47, 1977.
[5] T. Hibiki, M. Ishii, Distribution parameter and drift velocity of drift-flux model in bubbly flow, Int. J. Heat Mass Transfer 45 (2002).
[6] T. Hibiki, M. Ishii, One dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes, Int. J. Heat Mass Transfer 46 (2003).
[7] F. François, Étude et caractérisation des écoulements diphasiques dans un réacteur à eau sous pression embarqué en similitude Fréon, Thèse de Doctorat, INPG Grenoble, 2001.
[8] M. Quintard, S. Whitaker, Transport in ordered and disordered porous media II: generalized volume averaging, Transport Porous Med. 14 (1994) 179.