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C. R. Mecanique 332 (2004) 725–730

Comptes Rendus Mecanique

An estimate of maximum ground surface motion for non zero surface velocity

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Received 9 March 2004; accepted after revision 6 April 2004

Presented by Jean Salençon

Abstract

The increasing need for probability seismic hazard assessment (PSHA) of critical facilities sometimes leads to unrealistic earthquake scenarios with very high induced ground motions. From a physical standpoint these high motions cannot exist because of the limiting resistance capacity of the soil strata through which the seismic waves travel. A simple analytical model is proposed to bound the maximum ground surface acceleration that any soil deposit can transfer. This model is an extension to non zero ground surface velocity of a previously presented model. ***To cite this article: A. Pecker, C. R. Mecanique 332 (2004).***

Résumé

Estimation du mouvement sismique maximal à la surface du sol pour une celerite d'ondes non nulle en surface. La demande croissante d'études probabilistes de l'aléa sismique pour les installations essentielles pour la sûreté conduit parfois à des scénarios sismiques irréalistes donnant naissance à des mouvements très élevés. D'un point de vue physique ces mouvements ne peuvent exister en raison de la capacité de résistance limitée des couches de sol que traversent les ondes sismiques. Un modèle analytique simple est proposé pour estimer l'accélération maximale en surface que tout profil de sol est susceptible de transmettre. Ce modèle représente une extension au cas d'une célérité d'ondes non nulle en surface d'un modèle présenté précédemment. ***Pour citer cet article : A. Pecker, C. R. Mecanique 332 (2004).***

Keywords: Soils; Earthquake; Wave propagation; Soil strength

Mots-clés : Sols ; Séisme ; Propagation d'ondes ; Résistance du sol

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1631-0721/$ – see front matter
doi:10.1016/j.crme.2004.04.005

Version française abrégée

La demande croissante d'études probabilistes de l'aléa sismique pour les installations essentielles pour la sûreté conduit à considérer des scénarios sismiques présentant des probabilités d'occurrence de l'ordre de 10^{-7} par an donnant parfois naissance à des mouvements très élevés, qui calculés sur la base d'extrapolations de lois d'atténuation statistiques, peuvent atteindre plusieurs g [1]. D'un point de vue physique ces mouvements ne peuvent exister car la capacité de résistance limitée des couches de sol, que traversent les ondes sismiques, limite les mouvements susceptibles d'être transmis. Dans un article précédent [2] un modèle analytique simple a été proposé pour estimer l'accélération maximale en surface qu'un profil de sol surmontant un substratum rocheux est susceptible de transmettre. Ce modèle de type élastoplastique parfait, dont le critère de résistance est de type Mohr–Coulomb, prenait en compte une célérité des ondes variant comme une fonction puissance de la profondeur. Dans le présent article on étend la solution au cas d'une couche de sol pour laquelle la célérité des ondes varie comme une fonction puissance de la profondeur mais présente une valeur non nulle à la surface. Cette configuration permet de traiter, en particulier, le cas des profils de sol argileux surconsolidés, comme illustré sur un exemple pratique.

1. Introduction

The increasing use of Probabilistic Seismic Hazard Assessment (PSHA) for critical facilities leads to the consideration of earthquake scenarios with a probability of occurrence as low as 10^{-7} per year. For such low probabilities the computed ground accelerations, based on extrapolations of statistical attenuation relationships, may reach values as high as a few g, which poses tremendous difficulties for the earthquake resistance design [1]. Obviously such large motions cannot exist because the soil profile, through which the seismic waves travel to reach the ground surface, has a limited resistance capacity and cannot transmit any motion. When failure is reached at any depth within the soil profile, the incident motion is filtered and no motion larger than the motion reached at that stage can be transmitted to the upper strata. The soil acts as a fuse to safely limit the maximum ground surface acceleration. It is therefore necessary to develop an approach that takes into account the soil resistance to derive an upper bound estimate of the ground surface motion.

The approach presented herein is an extension of the previous approach developed in [2], for which is has been shown that, not only reasonable estimates of the maximum peak ground acceleration at the ground surface can be obtained, but predictions are not contradicted by observations. It is based on an analytical solution to the wave equation in an inhomogeneous soil profile: in [2] the stiffness of the soil column was assumed to be zero at the ground surface and to increase with depth; the condition of zero velocity at the ground surface is relaxed in the present paper. Therefore, almost any, reasonably smooth, experimental variation of the shear wave velocity with depth can be accommodated.

2. Equations of motion

Let us consider a soil layer of finite thickness overlying a stiff bedrock, which for the purpose of this study will be considered as a rigid boundary. The soil is assumed to be isotropic elastic, with a shear wave velocity increasing with depth according to some power law:

$$V(z) = V_{\mathrm{S}}\left(\frac{z+d}{d+h}\right)^{p/2} \tag{1}$$

where h is the layer thickness, p a real positive parameter smaller than 2 and V_{S} the shear wave velocity at depth h; d is a strictly positive parameter that can be chosen to fit Eq. (1) to the experimental data. In [2] this parameter was equal to zero giving a zero shear wave velocity at the ground surface. It is convenient to make a change of variable and to define $\zeta = (z+d)/(d+h)$ and $H = d+h$. The wave equation for a plane vertically incident shear wave can be written:

$$\frac{V_S^2}{H^2}\frac{\partial}{\partial\zeta}\left(\zeta^p\frac{\partial u}{\partial\zeta}\right) = \frac{\partial^2 u}{\partial t^2} + \ddot{v}_g(t) \tag{2}$$

where u is the horizontal displacement, relative to the bedrock, and v_g the bedrock displacement motion. The boundary conditions express that the relative displacement at the bedrock interface and the shear stress at the ground surface are equal to 0:

$$\begin{aligned} &\zeta = \frac{d}{H} = \zeta_0, \qquad \tau(\zeta_0) = 0 \Rightarrow \left.\frac{\partial u}{\partial\zeta}\right|_{\zeta=\zeta_0} = 0 \\ &\zeta = 1, \qquad u(1,t) = 0 \end{aligned} \tag{3}$$

Eq. (2) is solved with the mode superposition technique. The modes equation can be written:

$$\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\zeta^p\frac{\mathrm{d}X}{\mathrm{d}\zeta}\right) + \frac{H^2}{V_S^2}\omega^2 X(\zeta) = 0 \tag{4}$$

and its solution is given by $(p < 2)$:

$$X(\zeta) = \zeta^{(1-p)/2}\left[A J_\nu\left(\lambda\zeta^{(2-p)/2}\right) + B Y_\nu\left(\lambda\zeta^{(2-p)/2}\right)\right] \tag{5}$$

where $J_\nu(\cdot)$ and $Y_\nu(\cdot)$ are Bessel's functions of the first and second kind and of order $\nu = (p-1)/(2-p)$; $\lambda = 2\omega H/(2-p)V_S$.

Taking into account the recurrence formula, [3], valid for $J_\nu(\cdot)$ and $Y_\nu(\cdot)$:

$$\mathsf{C}'_\nu(z) = -\mathsf{C}_{\nu+1}(z) + \frac{\nu}{z}\mathsf{C}_\nu(z) \tag{6}$$

The derivative of Eq. (5) can be written:

$$X'(\zeta) = -\frac{2-p}{2}\lambda\zeta^{(1-2p)/2}\left[A J_{\nu+1}\left(\lambda\zeta^{(2-p)/2}\right) + B Y_{\nu+1}\left(\lambda\zeta^{(2-p)/2}\right)\right] \tag{7}$$

Enforcing the boundary conditions, one obtains a linear system of two equations for the unknowns A and B which has a non zero solution provided that:

$$J_\nu(\lambda)Y_{\nu+1}\left(\lambda\zeta_0^{(2-p)/2}\right) - Y_\nu(\lambda)J_{\nu+1}\left(\lambda\zeta_0^{(2-p)/2}\right) = 0 \tag{8}$$

Eq. (8) is the frequency equation with roots ρ_i; the soil column frequencies are given by:

$$f_i = \rho_i\frac{V_S(2-p)}{4\pi H} = \rho_i\frac{V_S(2-p)}{4\pi(h+d)} \tag{9}$$

The roots ρ_i can be computed once and for all as functions of ζ_0; an example, which will serve for illustration below, is given in Fig. 1(a) for $p = 1.0$.

Finally for convenience the mode shapes are normalized to unity at the ground surface $\zeta = \zeta_0$; the final solution for the mode shapes is given, in dimensionless form, by:

$$\begin{aligned} X_i(\zeta) = \frac{\pi}{2}\rho_i\sqrt{\zeta_0}\zeta^{(1-p)/2}\Big[&Y_{(p-1)/(2-p)}\left(\rho_i\zeta^{(2-p)/2}\right)J_{1/(2-p)}\left(\rho_i\zeta_0^{(2-p)/2}\right) \\ &- J_{(p-1)/(2-p)}\left(\rho_i\zeta^{(2-p)/2}\right)Y_{1/(2-p)}\left(\rho_i\zeta_0^{(2-p)/2}\right)\Big] \end{aligned} \tag{10}$$

Following the methodology of [2], the maximum ground surface acceleration due to the contribution of the first N modes is:

$$\ddot{u}_{\max}(z=0) = \left[\sum_{i=1}^{N}\left(\alpha_i S_a(\omega_i,\xi_i)\right)^2\right]^{1/2} \tag{11}$$

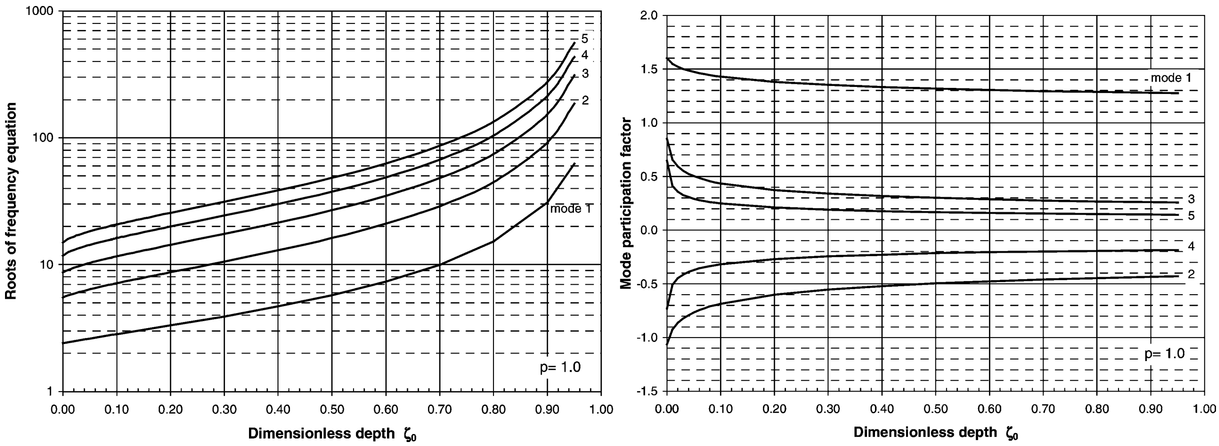

Fig. 1. Roots of the frequency equation; (b) mode participation factors.

Fig. 1. (a) Racines de l'équation aux fréquences propres ; (b) facteurs de participation.

where $S_a(\omega_i, \xi_i)$ is the spectral pseudo acceleration and α_i the modal participation factor:

$$\alpha_i = \frac{\int_{\zeta_0}^{1} X_i(\zeta)\,d\zeta}{\int_{\zeta_0}^{1} X_i^2(\zeta)\,d\zeta} \tag{12}$$

As for the roots, the modal participation factors can be computed once and for all as a function of ζ_0 (Fig. 1(b)).

The shear strain is given, for each mode, by:

$$\gamma_i(z) = \frac{\partial u_i(z)}{\partial z} = \alpha_i \frac{S_a(\omega_i, \xi_i)}{H\omega_i^2} \frac{dX_i(\zeta)}{d\zeta} \tag{13}$$

which can be expressed in dimensionless form by:

$$\chi_i(\zeta) = \frac{\gamma_i(z)\omega_i^2}{S_a(\omega_i, \xi_i)} H = \alpha_i \frac{(2-p)\pi}{4} \rho_i^2 \sqrt{\zeta_0}\,\zeta^{(1-2p)/2} \big[J_{1/(2-p)}\big(\rho_i \zeta^{(2-p)/2}\big) Y_{1/(2-p)}\big(\rho_i \zeta_0^{(2-p)/2}\big) - Y_{1/(2-p)}\big(\rho_i \zeta^{(2-p)/2}\big) J_{1/(2-p)}\big(\rho_i \zeta_0^{(2-p)/2}\big) \big] \tag{14}$$

3. Soil constitutive model

Although the soil behavior is highly non linear from very small strains, as illustrated by the shear stress-shear strain curve in Fig. 2, a simplified elastic-perfectly plastic constitutive relationship is assumed for the soil layer; the shear stress–shear strain curve is defined by two parameters which may depend on the depth z:

- the shear strength $\tau_{max}(z)$;
- the yield engineering shear strain $\gamma_f(z)$ where the engineering shear strain is defined as twice the shear strain.

The shear modulus is then given by $G = \tau_{max}/\gamma_f$.

As soon as, at any depth within the soil profile, the shear strain reaches γ_f, the maximum shear stress that can be transmitted is limited by τ_{max}; the ground surface acceleration cannot therefore exceed the value reached when $\gamma(z) = \gamma_f$.

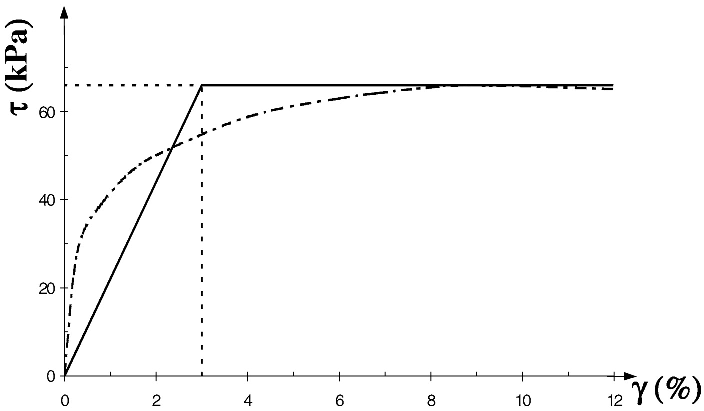

Fig. 2. Soil shear stress-shear strain curve.

Fig. 2. Courbe effort-déformation en cisaillement du sol.

The procedure involves the following steps:

(i) define the input motion at the rock interface by its pseudo acceleration response spectrum S_a^*. This is typically the result of the PSHA;
(ii) compute the eigenfrequencies and mode participation factors from Eqs. (8) and (12);
(iii) plot the shear strain (Eq. (14)) versus depth together with the yield strain γ_f;
(iv) determine the depth z_1 and the scaling factor μ for which $\gamma_f = \mu\gamma(z_1)$;
(v) define $S_a = \mu S_a^*$ the maximum possible pseudo acceleration from which the maximum ground surface acceleration $\ddot{u}_{max}(z = 0)$ is determined (Eq. (11)).

4. Example of application

The procedure outlined above is illustrated with reference to an actual project for which a PSHA and truly non linear site response analyses have led to a ground surface acceleration equal to 4.8 m/s^2 for a 2000 year return period earthquake. The soil profile is composed of 100 m of alluvial deposits underlain by stiffer strata. The undrained shear strength of the top layers has been measured and can be approximated by:

$$S_u(\text{kPa}) = 2.85z(\text{m}) + 25 \tag{15}$$

A typical stress strain curve for the alluvia, measured in laboratory tests on a sample retrieved at 15 m depth, is presented in Fig. 2; it is approximated by the simplified elastic perfectly plastic model with a yield strain, determined as shown, equal to $\gamma_f = 0.03$. The other relevant parameters for the analysis take the following values: $p = 1.0$, $\zeta_0 = 0.08$ and $V_S = 71.9$ m/s. The 'bedrock' motion, as determined from the PSHA, is represented by the pseudo acceleration response spectrum of Fig. 3, scaled for convenience to 10 m/s^2. In reality, the result of the PSHA indicates that for the 2000 year return period earthquake the 'rock' acceleration is equal to 5.0 m/s^2.

The response has been computed with 10 modes to achieve a modal mass greater than 97%. The first three circular frequencies are equal to: $\omega_1 = 0.91$ rd/s, $\omega_2 = 2.26$ rd/s, $\omega_3 = 3.67$ rd/s and the corresponding spectral accelerations for 20% damping, which is relevant for a near failure condition, are equal to $S_{a1}^* = 0.51$ m/s^2, $S_{a2}^* = 3.74$ m/s^2, $S_{a3}^* = 8.46$ m/s^2. The yield strain becomes equal to the induced strain for $\mu \simeq 0.59$ at a critical depth $z_1 = 6$ m below the ground surface. The associated spectral accelerations of the first three modes are: $S_{a1} = \mu S_{a1}^* = 0.30$ m/s^2, $S_{a2} = \mu S_{a2}^* = 2.21$ m/s^2, $S_{a3} = \mu S_{a3}^* = 4.99$ m/s^2 from which the maximum peak ground acceleration (Eq. (11)) is equal to: $\ddot{u}_{max} = 5.6$ m/s^2.

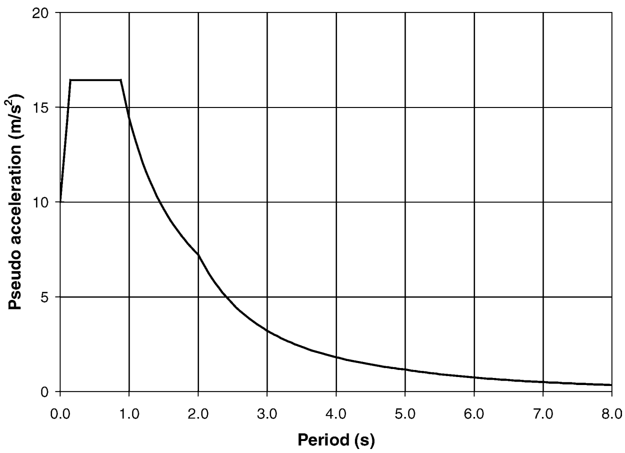

Fig. 3. 20% damped bedrock response spectrum.

Fig. 3. Spectre de réponse au rocher à 20 % d'amortissement.

The approximation of the stress strain curve by the bilinear elastic plastic model involves some arbitrary choice. If instead of 0.03 the yield strain is taken within the range 0.025 to 0.035, which represents reasonable alternatives, the maximum peak ground acceleration varies between 5.4 and 5.8 m/s^2. These values compare favorably with the results of the analyses for the 2000 year return period: for those analyses the PSHA indicated a "bedrock" acceleration of 5 m/s^2 and the non linear site response analyses a surface acceleration of 4.8 m/s^2. The proposed methodology indicates that above a rock ground acceleration of 5.9 m/s^2 the surface motion is limited to 5.4 m/s^2 to 5.8 m/s^2. These maximum values are in good agreement with the empirical relationships derived from field observations, which indicate maximum surface ground accelerations in the range 4.5 m/s^2 to 5.5 m/s^2 for cohesive soils [4].

5. Conclusions

A simple method has been proposed to estimate the maximum ground surface motion that can be observed at the surface of a soil profile whatever the amplitude of the input rock acceleration. The proposed method is robust because the shear strength is a parameter that is routinely measured, rather reliable, and also because the results are not too sensitive to changes in the yield strain. It has been compared to more rigorous numerical site response analyses based on a sophisticated soil constitutive model from which the ground surface acceleration has been found equal to 4.8 m/s^2 for a potential event with a return period of 2000 years. The method indicates that the maximum ground surface acceleration should not exceed 5.4 to 5.8 m/s^2 for a rock motion slightly larger than that associated with the 2000 year return period event. These values are also in good agreement with the empirical relationships derived from field observations.

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