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## Introduction of acoustical diffraction in the radiative transfer method

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### Abstract

This Note presents an original approach to include diffraction in the radiative transfer method when applied to acoustics. This approach leads to a better spatial description of the acoustical energy. An energetic diffraction coefficient and some diffraction sources are introduced to model the diffraction phenomena. The amplitudes of these sources are determined by solving a linear system of equations resulting from the power balance between all acoustical sources. The approach is applied on bidimensional examples and gives good results except at geometrical boundaries. *To cite this article: E. Reboul et al., C. R. Mecanique 332 (2004).*

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### Résumé

**Introduction de la diffraction acoustique dans la méthode du transfert radiatif.** Cette Note présente une méthode originale permettant d'inclure la diffraction acoustique dans la méthode du transfert radiatif. Cette nouvelle approche conduit à une meilleure description de la répartition de l'énergie acoustique. Elle repose sur l'introduction d'un coefficient de diffraction énergétique, et de sources diffractantes pour modéliser le phénomène de diffraction. L'amplitude de ces sources est déterminée en résolvant un système linéaire d'équations qui traduit le bilan d'énergie entre toutes les sources acoustiques. Cette méthode est appliquée à des problèmes bi-dimensionnels, et fournit de bons résultats excepté aux frontières géométriques. *Pour citer cet article : E. Reboul et al., C. R. Mecanique 332 (2004).*

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*Mots-clés :* Acoustique ; Ondes ; Vibrations ; Théorie Géométrique de la Diffraction (TGD) ; Diffraction ; Transfert radiatif ; Méthode énergétique ; Hautes fréquences

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### Version française abrégée

La méthode du transfert radiatif de l'énergie repose sur une approche énergétique appliquée à l'étude des vibrations structurales et à l'acoustique dans le domaine des hautes fréquences. Concernant les vibrations structurales, elle se présente comme une amélioration de l'analyse statistique de l'énergie (SEA) en permettant de prédire la répartition spatiale de l'énergie. Dans la mesure où l'hypothèse de champ diffus n'est pas nécessaire, elle est particulièrement adaptée à l'étude des systèmes fortement amortis à hautes fréquences. Appliquée à l'acoustique, cette méthode est parfois appelée méthode de radiosité [3], et est utilisée pour le calcul des durées de réverbération en acoustique des salles. Notons que la méthode du transfert radiatif est basée sur l'hypothèse de décorrélation des ondes, ce qui ne permet pas de reproduire les phénomènes d'interférences. Cette méthode a été appliquée au cas de la réflexion diffuse [5], et étendue aux cas de la réflexion partiellement diffuse [6], et de la réflexion spéculaire [7,8]. Cependant, seuls les termes d'acoustique géométrique sont pris en compte, ainsi les effets de diffraction sont négligés bien que ceux-ci conduisent à une redistribution spatiale de l'énergie. L'objectif de cette étude est donc d'inclure la diffraction acoustique dans la méthode du transfert radiatif de l'énergie acoustique, afin de décrire plus précisément la répartition de l'énergie. L'étude est menée dans une configuration bi-dimensionnelle.

L'approche repose sur l'introduction d'un coefficient de diffraction énergétique. On définit l'intensité diffractée spécifique comme la puissance diffractée par unité d'angle solide. Le coefficient de diffraction énergétique se présente comme le rapport entre l'intensité diffractée spécifique et l'intensité incidente dans la direction d'incidence – Éqs. (1)–(6). En adoptant une approche onde plane, le coefficient de diffraction énergétique peut donc s'écrire comme le carré du module du coefficient de diffraction introduit par la Théorie Géométrique de la Diffraction (TGD).

La méthode du transfert radiatif est appliquée au calcul du champ acoustique dans deux exemples : la diffraction d'une onde plane par un dièdre, et la diffraction d'une onde cylindrique par un obstacle rectangulaire. D'après le principe de superposition linéaire, l'intensité acoustique peut s'écrire comme la somme de trois contributions : l'intensité directe, l'intensité réfléchie et l'intensité diffractée. L'intensité directe est définie par l'Éq. (7). L'intensité réfléchie est décrite par l'Éq. (8), obtenue en appliquant la méthode des sources images. Le champ diffracté s'écrit comme la somme des contributions issues de sources de diffraction d'amplitude  $\sigma_i$  (Éq. (9)) et situées aux points de diffraction  $\mathbf{p}_i$ . Leur amplitude dépend de la direction de diffraction et est calculée en réalisant un bilan de puissance en chaque point de diffraction  $\mathbf{p}_i$ . L'intensité spécifique diffractée en  $\mathbf{p}_i$  est donnée par l'Éq. (10). Pour les deux exemples, aucune onde n'est réfléchie en direction des points de diffraction. Par conséquent, en présence d'un seul point de diffraction comme dans le cas de la diffraction par un dièdre, l'intensité incidente au point de diffraction n'est due qu'au champ direct dans la direction d'incidence en ce point. L'amplitude de la source de diffraction est donnée par l'Éq. (11). Dans le cas de la diffraction par un obstacle rectangulaire, l'intensité incidente en un point de diffraction  $\mathbf{p}_i$  est la somme de l'intensité directe, mais aussi de l'intensité issue des trois autres points de diffraction qui diffractent dans la direction de  $\mathbf{p}_i$ . L'Éq. (12) conduit alors à quatre relations – Éqs. (13)–(16), qui, appliquées à des directions de diffraction particulières – Éqs. (17)–(20), aboutissent à un système linéaire d'ordre 8. Ce système est résolu par la méthode de Gauss. L'amplitude de toutes les sources acoustiques étant maintenant connue, le champ acoustique total est obtenu en sommant toutes les contributions (Éqs. (21)).

Les Figs. 1(a) et 2(a) présentent les cartes de niveau de pression en dB obtenues pour les deux exemples étudiés à l'aide de la méthode du transfert radiatif. Les Figs. 1(b) et 2(b) présentent un tracé linéaire de la densité d'énergie acoustique le long d'une ligne de contour : ce tracé est issu du précédent calcul et d'un calcul de référence obtenu à l'aide des formules analytiques de la TGD dans le cadre du dièdre, et en appliquant la méthode des éléments de frontière (BEM) implantée dans le logiciel Sysnoise<sup>®</sup> dans le cadre de l'obstacle rectangulaire. Les interférences ne sont pas visibles sur les cartes de pression obtenues en appliquant la méthode de l'étude en raison de l'hypothèse de décorrélation des ondes (somme des contributions énergétiques). La comparaison des résultats montre une bonne concordance entre la méthode du transfert radiatif et la TGD ou la BEM en dehors des frontières géométriques. Dans ces zones, le champ ne peut être décrit en termes de rayons et donc la présente méthode,

tout comme la TGD, ne sont plus valables. Pour conclure, la méthode du transfert radiatif est particulièrement intéressante pour étudier la diffraction multiple car elle permet de prendre en compte simultanément tous les ordres de diffraction.

## 1. Introduction

The radiative transfer method is based on an integral energy approach well-suited for high frequency analyses in structural vibrations and acoustics. In the field of structural vibrations, this method allows to predict the spatial distribution of energy inside subsystems, and then appears to be an improvement of the Statistical Energy Analysis (SEA). It was applied to dynamics of multi-plate structures [1], radiation of plates [2], or sound transmission through panels. As the diffuse field assumption is not required, this method is particularly well-suited for highly damped systems at high frequency. It is a main advantage having regard to SEA. In the field of acoustics, the radiative transfer method is sometimes called the radiosity method [3] and was applied in room acoustics to predict reverberation times beyond the validity of Sabine's theory. The method is basically equivalent to the ray-tracing method [4]. It relies on the assumption that all acoustical sources, whether they are actual sources or secondary sources introduced by the method, are uncorrelated. This assumption leads to neglect interference effects: results can be interpreted as band frequency averaged results. Miles [5] applied this method to enclosures with diffuse reflecting boundaries, and the method was extended by Kuttruff [6] for partially diffuse reflection, by Le Bot [7] and by Franzoni et al. [8] for specular reflection. In these studies, only geometrical terms are taken into account and diffraction effects are neglected. However the diffraction of sound waves can bring acoustical energy into shadow zones that are prevented from direct incidence.

This Note presents a way to include diffraction into the radiative transfer method for getting a better spatial description of the energy distribution. In Section 2, an energy coefficient for diffraction is proposed; in Section 3, diffraction sources are calculated in the frame of the radiative transfer method to study multiple diffraction. In Section 4, two bidimensional examples are analysed to show the efficiency of the method.

## 2. Calculation of the energetic diffraction coefficient

The problem is considered to be bidimensional. Consider an incoming plane pressure wave in the direction  $\mathbf{u}$ , and impinging on a structure with a diffracting point. The incident wave has amplitude  $p_0$  at the diffraction point, frequency  $\omega$  and wavenumber  $k$ . The intensity in the incident direction  $\mathbf{u}$  can thus be expressed as:

$$I_{\text{inc}} = \mathbf{I}_{\text{inc}} \cdot \mathbf{u} = \frac{|p_0|^2}{2\rho c} \quad (1)$$

where  $c$  is the sound velocity and  $\rho$  is the fluid density. The incident wave is diffracted in the direction  $\mathbf{v}$ : according to the Geometrical Theory of Diffraction (GTD) [9], the diffracted wave can be described at any point  $\mathbf{r}$  as:

$$p_{\text{diff}} = p_0 d_v(\mathbf{u}, \mathbf{v}) \frac{e^{ikr}}{\sqrt{r}} \quad (2)$$

with [10]:

$$d_v(\mathbf{u}, \mathbf{v}) = \frac{e^{i\pi/4}}{\sqrt{2\pi k}} v \sin(v\pi) \left( \frac{1}{\cos(v\pi) - \cos(v(\phi + \phi_0))} + \frac{1}{\cos(v\pi) - \cos(v(\phi - \phi_0))} \right) \quad (3)$$

$d_v(\mathbf{u}, \mathbf{v})$  is the diffraction coefficient introduced by the GTD due to the source in the direction  $\mathbf{u}$  and the observation point in the direction  $\mathbf{v}$ .  $\phi$  and  $\phi_0$  are source and observation angular positions and are defined to the right side of

the wedge.  $d_\nu$  depends on the inner angle of the diffracting wedge  $2\Omega$  by means of the wedge index  $\nu$  defined as  $\nu = \pi/(2\pi - 2\Omega)$ . Thus, the diffracted intensity in the direction  $\mathbf{v}$  can be written as:

$$I_{\text{diff}} = \mathbf{I}_{\text{diff}} \cdot \mathbf{v} = \frac{|p_0|^2}{2\rho c} |d_\nu(\mathbf{u}, \mathbf{v})|^2 \quad (4)$$

Let us introduce the diffracted specific intensity defined as the diffracted power per unit solid angle:

$$P_{\text{diff}} = \frac{|p_0|^2}{2\rho c} |d_\nu(\mathbf{u}, \mathbf{v})|^2 \quad (5)$$

It can be expressed in terms of the intensity in the incident direction:

$$P_{\text{diff}} = D_\nu(\mathbf{u}, \mathbf{v}) I_{\text{inc}} \quad (6)$$

where  $D_\nu(\mathbf{v}, \mathbf{u}) = |d_\nu(\mathbf{u}, \mathbf{v})|^2$  is the energetic diffraction coefficient which is the square of the diffraction coefficient introduced by the GTD.

### 3. Energy formulation

The radiative transfer method is here applied to determine the whole acoustical field in presence of a diffracting structure. The study is carried out on two examples: the diffraction of a plane wave around a wedge with only one diffracting point (Fig. 1), and the diffraction of a cylindrical wave at edges of a rectangular obstacle with four diffracting points (Fig. 2). According to the superposition principle valid at high frequency, the acoustical intensity  $\mathbf{I}$  can be considered as the superposition of three contributions: the direct intensity  $\mathbf{I}_{\text{dir}}$ , the reflected intensity  $\mathbf{I}_{\text{refl}}$ , and the diffracted intensity  $\mathbf{I}_{\text{diff}}$ .

The direct intensity coming from the primary source  $\mathbf{s}$  of amplitude  $\rho_0$  can be written at point  $\mathbf{r}$  as:

$$\mathbf{I}_{\text{dir}} = \begin{cases} \rho_0 e^{-m\mathbf{r}\cdot\mathbf{u}} \mathbf{u} = \rho_0 \mathbf{u} & \text{for an incident plane wave in the direction } \mathbf{u} \\ \rho_0 \mathbf{H}(\mathbf{s}, \mathbf{r}) & \text{for an incident cylindrical wave} \end{cases} \quad (7)$$

where  $m$  is the atmospheric attenuation factor taken in this study equal to 0,  $\mathbf{H}(\mathbf{s}, \mathbf{r}) = (e^{-ms}/(2\pi s)) \mathbf{u}_{\text{sr}} = (1/(2\pi s)) \mathbf{u}_{\text{sr}}$  is the kernel function for cylindrical waves where  $\mathbf{u}_{\text{sr}} = (\mathbf{r} - \mathbf{s})/s$  is the source-receiver unit vector, and  $s = |\mathbf{r} - \mathbf{s}|$  is the source-receiver distance.

The image-source method is used to evaluate the reflected intensity. The reflected field can be written as the direct field in the following way:

$$\mathbf{I}_{\text{refl}} = \begin{cases} \rho_0 \mathbf{u}' & \text{for a plane wave reflected in the direction } \mathbf{u}' \\ \rho_0 \mathbf{H}(\mathbf{s}', \mathbf{r}) & \text{for a reflected cylindrical wave} \end{cases} \quad (8)$$

where  $\mathbf{s}'$  is the symmetric of the primary source  $\mathbf{s}$  with respect to the reflection plan.

The diffracted field is described by diffraction sources of amplitude  $\sigma_i$  which are located at diffraction points  $\mathbf{p}_i$ . The amplitudes depend on the diffraction direction  $\mathbf{u}_{\mathbf{p}_i\mathbf{r}} = (\mathbf{r} - \mathbf{p}_i)/|\mathbf{r} - \mathbf{p}_i|$ . The diffracted intensity can be written by means of the kernel function  $\mathbf{H}$  for cylindrical waves:

$$\mathbf{I}_{\text{diff}} = \sum_i \sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}}) \mathbf{H}(\mathbf{p}_i, \mathbf{r}) \quad (9)$$

Amplitudes of diffraction sources are calculated by applying the power balance at each point of diffraction. The emitted specific intensity  $P_{d,i}$  is related to the intensity  $I_{\text{inc},i}$  in the incident direction  $\mathbf{u}_{\text{inc},i}$  by means of Eq. (6).  $P_{d,i}$  is the diffracted power per unit solid angle  $d\phi$ . The diffracted power is the flux of energy inside the solid angle  $d\phi$  from the diffraction source of amplitude  $\sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}})$ . The expression for the emitted specific intensity is the following:

$$P_{d,i} = \lim_{\varepsilon \rightarrow 0} \sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}}) \frac{e^{-m\varepsilon}}{2\pi\varepsilon} \varepsilon d\phi \times \frac{1}{d\phi} = \frac{\sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}})}{2\pi} \quad (10)$$

The incident intensity results from the superposition of the direct, the reflected and the diffracted intensity taken in the incident direction. For the two examples, the reflected waves never impinge upon diffraction points, so the reflected field is not taken into account in the calculation of the incident intensity. In the case of the wedge where there is only one diffraction point  $\mathbf{p}$  at the top of the wedge, the intensity  $I_{inc}$  is thus simply the direct intensity at this point taken in the incident direction. As a consequence, Eq. (6), giving the amplitude of the diffraction source  $\sigma(\mathbf{u}_{\mathbf{p}\mathbf{r}})$ , can simply be written in the following way:

$$\frac{\sigma(\mathbf{u}_{\mathbf{p}\mathbf{r}})}{2\pi} = \rho_0 D_v(\mathbf{u}, \mathbf{u}_{\mathbf{p}\mathbf{r}}) \tag{11}$$

In the case of diffraction around a rectangular obstacle, four diffraction sources  $\sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}})$  ( $i \in \{1, 2, 3, 4\}$ ) have to be introduced at each corner of the obstacle. The incident intensity at each diffraction point  $\mathbf{p}_i$  is then the sum of the contributions coming from the primary source, and from other diffraction points  $\mathbf{p}_j$  which diffract in the direction of point  $\mathbf{p}_i$ . Thus the power balance at point of diffraction  $\mathbf{p}_i$  leads to the following equation:

$$\frac{\sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}})}{2\pi} = \rho_0 D_v(\mathbf{u}_{\mathbf{s}\mathbf{p}_i}, \mathbf{u}_{\mathbf{p}_i\mathbf{r}}) H(\mathbf{s}, \mathbf{p}_i) + \sum_{j \neq i} D_v(\mathbf{u}_{\mathbf{p}_j\mathbf{p}_i}, \mathbf{u}_{\mathbf{p}_i\mathbf{r}}) \sigma_j(\mathbf{u}_{\mathbf{p}_j\mathbf{p}_i}) H(\mathbf{p}_j, \mathbf{p}_i) \tag{12}$$

This equation illustrates the energy transfer from all source points to the diffraction point  $i$ . Applying this equation to all points of diffraction, it leads to a linear matrix equation which is solved numerically by Gaussian elimination to give the amplitude of all diffraction sources. For example, in the case of the rectangular obstacle, the system is the following:

$$\begin{aligned} \frac{\sigma_1(\mathbf{u}_{\mathbf{p}_1\mathbf{r}})}{2\pi} &= \rho_0 D_v(\mathbf{u}_{\mathbf{s}\mathbf{p}_1}, \mathbf{u}_{\mathbf{p}_1\mathbf{r}}) H(\mathbf{s}, \mathbf{p}_1) + D_v(\mathbf{u}_{\mathbf{p}_2\mathbf{p}_1}, \mathbf{u}_{\mathbf{p}_1\mathbf{r}}) \sigma_2(\mathbf{u}_{\mathbf{p}_2\mathbf{p}_1}) H(\mathbf{p}_2, \mathbf{p}_1) \\ &\quad + D_v(\mathbf{u}_{\mathbf{p}_3\mathbf{p}_1}, \mathbf{u}_{\mathbf{p}_1\mathbf{r}}) \sigma_3(\mathbf{u}_{\mathbf{p}_3\mathbf{p}_1}) H(\mathbf{p}_3, \mathbf{p}_1) \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\sigma_2(\mathbf{u}_{\mathbf{p}_2\mathbf{r}})}{2\pi} &= \rho_0 D_v(\mathbf{u}_{\mathbf{s}\mathbf{p}_2}, \mathbf{u}_{\mathbf{p}_2\mathbf{r}}) H(\mathbf{s}, \mathbf{p}_2) + D_v(\mathbf{u}_{\mathbf{p}_1\mathbf{p}_2}, \mathbf{u}_{\mathbf{p}_2\mathbf{r}}) \sigma_1(\mathbf{u}_{\mathbf{p}_1\mathbf{p}_2}) H(\mathbf{p}_1, \mathbf{p}_2) \\ &\quad + D_v(\mathbf{u}_{\mathbf{p}_4\mathbf{p}_2}, \mathbf{u}_{\mathbf{p}_2\mathbf{r}}) \sigma_4(\mathbf{u}_{\mathbf{p}_4\mathbf{p}_2}) H(\mathbf{p}_4, \mathbf{p}_2) \end{aligned} \tag{14}$$

$$\frac{\sigma_3(\mathbf{u}_{\mathbf{p}_3\mathbf{r}})}{2\pi} = D_v(\mathbf{u}_{\mathbf{p}_1\mathbf{p}_3}, \mathbf{u}_{\mathbf{p}_3\mathbf{r}}) \sigma_1(\mathbf{u}_{\mathbf{p}_1\mathbf{p}_3}) H(\mathbf{p}_1, \mathbf{p}_3) + D_v(\mathbf{u}_{\mathbf{p}_4\mathbf{p}_3}, \mathbf{u}_{\mathbf{p}_3\mathbf{r}}) \sigma_4(\mathbf{u}_{\mathbf{p}_4\mathbf{p}_3}) H(\mathbf{p}_4, \mathbf{p}_3) \tag{15}$$

$$\frac{\sigma_4(\mathbf{u}_{\mathbf{p}_4\mathbf{r}})}{2\pi} = D_v(\mathbf{u}_{\mathbf{p}_3\mathbf{p}_4}, \mathbf{u}_{\mathbf{p}_4\mathbf{r}}) \sigma_3(\mathbf{u}_{\mathbf{p}_3\mathbf{p}_4}) H(\mathbf{p}_3, \mathbf{p}_4) + D_v(\mathbf{u}_{\mathbf{p}_2\mathbf{p}_4}, \mathbf{u}_{\mathbf{p}_4\mathbf{r}}) \sigma_2(\mathbf{u}_{\mathbf{p}_2\mathbf{p}_4}) H(\mathbf{p}_2, \mathbf{p}_4) \tag{16}$$

$\mathbf{p}_1, \mathbf{p}_3$  are the top left-hand and right-hand corners, and  $\mathbf{p}_2, \mathbf{p}_4$  are the bottom left-hand and right-hand corners. These equations are thus applied for the particular cases where:

$$\mathbf{u}_{\mathbf{p}_1\mathbf{r}} = \mathbf{u}_{\mathbf{p}_1\mathbf{p}_3} \text{ and } \mathbf{u}_{\mathbf{p}_1\mathbf{p}_2} \tag{17}$$

$$\mathbf{u}_{\mathbf{p}_2\mathbf{r}} = \mathbf{u}_{\mathbf{p}_2\mathbf{p}_1} \text{ and } \mathbf{u}_{\mathbf{p}_2\mathbf{p}_4} \tag{18}$$

$$\mathbf{u}_{\mathbf{p}_3\mathbf{r}} = \mathbf{u}_{\mathbf{p}_3\mathbf{p}_1} \text{ and } \mathbf{u}_{\mathbf{p}_3\mathbf{p}_4} \tag{19}$$

$$\mathbf{u}_{\mathbf{p}_4\mathbf{r}} = \mathbf{u}_{\mathbf{p}_4\mathbf{p}_3} \text{ and } \mathbf{u}_{\mathbf{p}_4\mathbf{p}_2} \tag{20}$$

leading to a linear system of order 8. After knowing all sources amplitudes, the whole acoustical field can be written by summing all source contributions in the following way:

$$\mathbf{I} = \begin{cases} \rho_0 \mathbf{u} + \rho_0 \mathbf{u}' + \sigma(\mathbf{u}_{\mathbf{p}\mathbf{r}}) \mathbf{H}(\mathbf{p}, \mathbf{r}) \\ \text{for incident plane wave on a wedge} \\ \rho_0 \mathbf{H}(\mathbf{s}, \mathbf{r}) + \rho_0 \mathbf{H}(\mathbf{s}', \mathbf{r}) + \sum_{i=1}^4 \sigma_i(\mathbf{u}_{\mathbf{p}_i\mathbf{r}}) \mathbf{H}(\mathbf{p}_i, \mathbf{r}) \\ \text{for incident cylindrical wave on a rectangular obstacle} \end{cases} \tag{21}$$

where each source contribution is taken into account only if the receiver point  $\mathbf{r}$  is in the radiation area of this source (for example, points behind the rectangular barrier are not in the radiation area of the direct source).

#### 4. Numerical results

The wedge inner angle is  $2\Omega = \frac{\pi}{4}$ . The plane wave is impinging upon the wedge with an angle  $\phi_0 = 267.5^\circ$  compared to the right side of the wedge. The atmospheric attenuation factor is  $m = 0$ , and the frequency is  $f = 2500$  Hz for both examples. Figs. 1(a) and 2(a) show the map of the sound pressure level  $L_p$  in dB calculated with the radiative transfer method around the wedge and around the barrier. Figs. 1(b) and 2(b) compare the energy density level obtained with the method and with reference calculations along a contour line plotted in Figs. 1(a) and 2(a). The distance along the contour line  $s$  is taken equal to 0 at the left-bottom point of the line. The reference calculation is performed using the asymptotic formulation given by the GTD in the case of the wedge, and using the Boundary Element Method (BEM) implemented in the software Sysnoise<sup>®</sup> in the case of the barrier. In this last case, it is integrated over a third-octave band centered on  $f = 2500$  Hz. Interferences are not visible on both sound pressure maps obtained with the method because of the high frequency uncorrelation assumption. The comparison shows a good agreement between the method and the GTD as well as with the BEM, except at geometrical boundaries which mark the transition between the presence or the absence of some geometrical rays. These boundaries result in white linear areas on both SPL maps, and divergent peaks of the energy density on curves obtained with the GTD or with the radiative transfer method. Indeed, at these boundaries the diffraction coefficient and so the energy diffraction coefficient tends to infinity: the acoustical field cannot be divided into geometrical and diffraction parts so the radiative transfer method as well as the GTD cannot be applied. The radiative transfer method is particularly interesting for the study of multiple diffraction as all orders of diffraction are taken into account in the calculation, what would require to work with infinite series applying the GTD: for example, in the case of the rectangular barrier, the study of diffraction by means of the GTD would involve an infinite number of rays between the source and a receiver point while the radiative transfer method only involve four diffraction sources.

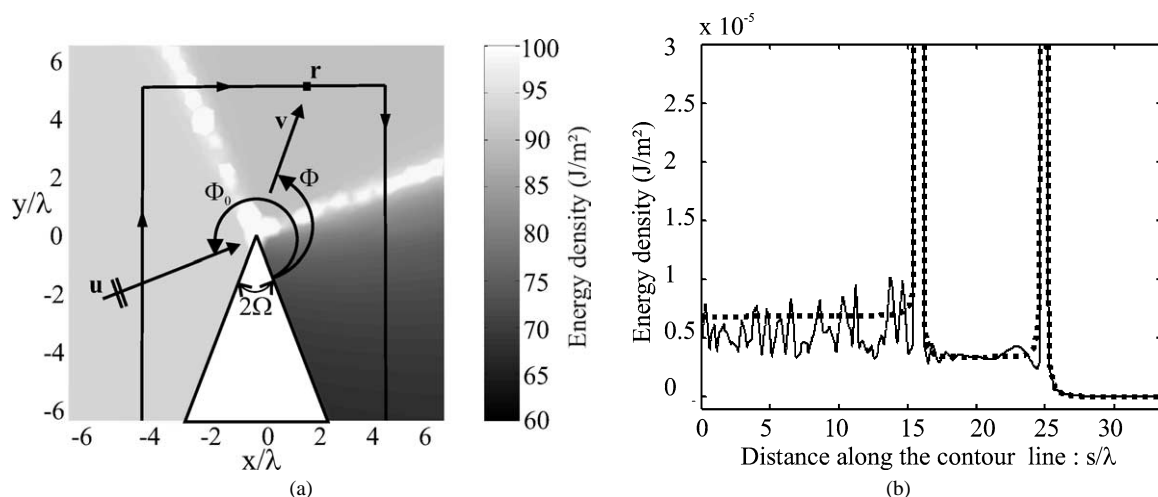


Fig. 1. Diffraction of a plane wave around a wedge: (a) map of the sound pressure level  $L_p$  in dB; (b) comparison of the energy density ( $\text{J}/\text{m}^2$ ) along the contour line estimated with the radiative transfer method (dot line) and with the GTD (solid line).

Fig. 1. Diffraction d'une onde plane sur un dièdre : (a) carte du niveau de pression  $L_p$  en dB ; (b) densité d'énergie ( $\text{J}/\text{m}^2$ ) le long de la ligne de contour estimée à l'aide de la méthode du transfert radiatif (trait pointillé) et de la TGD (trait plein).

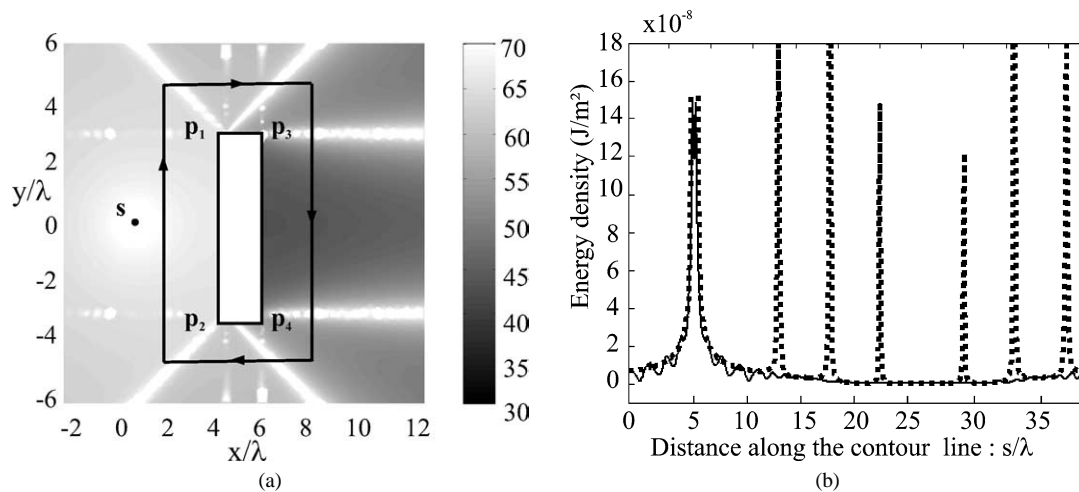


Fig. 2. Diffraction around a rectangular barrier: (a) map of the sound pressure level  $L_p$  in dB; (b) energy density ( $\text{J}/\text{m}^2$ ) along the contour line estimated with the radiative transfer method (dot line) and with the BEM (solid line).

Fig. 2. Diffraction autour d'une barrière rectangulaire : (a) carte du niveau de pression  $L_p$  en dB ; (b) densité d'énergie ( $\text{J}/\text{m}^2$ ) le long de la ligne de contour estimée à l'aide de la méthode du transfert radiatif (trait pointillé) et de la méthode BEM (trait plein).

## 5. Conclusion

The radiative transfer method has been extended here to the study of acoustical diffraction. This leads to a better description of the acoustical field around a structure as diffraction is a current phenomenon. The method has proved to give good results compared to reference calculations except at geometrical boundaries where it is not suitable. The study discussed here was on bidimensional examples; ongoing work of the authors consists in adapting it for three-dimensional geometries.

## References

- [1] A. Le Bot, Energy transfer for high frequencies in built-up structures, *J. Sound Vib.* 250 (2) (2002) 247–275.
- [2] V. Cotoni, A. Le Bot, Radiation of plane structures at high frequency using an energy method, *Int. J. Acoust. Vib.* 6 (4) (2001) 209–214.
- [3] H. Kuttruff, Energetic sound propagation in rooms, *Acta Acustica united with Acustica* 83 (1997) 622–628.
- [4] A. Le Bot, A. Bocquillet, Comparison of an integral equation on energy and the ray-tracing technique in room acoustics, *J. Acoust. Soc. Am.* 108 (4) (2000) 1732–1740.
- [5] R.N. Miles, Sound field in a rectangular enclosure with diffusely reflecting boundaries, *J. Sound Vib.* 92 (2) (1984) 203–226.
- [6] H. Kuttruff, Stationary propagation of sound energy in flat enclosures with partially diffuse surface reflection, *Acta Acustica united with Acustica* 86 (2000) 1028–1033.
- [7] A. Le Bot, A functional equation for the specular reflection of rays, *J. Acoust. Soc. Am.* 112 (4) (2002) 1276–1287.
- [8] L.P. Franzoni, D.B. Bliss, J.W. Rouse, An acoustic boundary element method based on energy and intensity variables for prediction of high-frequency broadband sound fields, *J. Acoust. Soc. Am.* 110 (6) (2001) 3071–3080.
- [9] J.B. Keller, Geometrical theory of diffraction, *J. Opt. Soc. Am.* 52 (2) (1962).
- [10] A.D. Pierce, *Acoustics – An Introduction to its Physical Principles and Applications*, second printing, Acoustical Society of America, American Institute of Physics, New York, 1991.