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Hydrodynamic formation of stable anisotropic structures in Couette flow of dilute suspension

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Abstract

The problem of rotary motion of rigid axially symmetric elongated particles in the Couette flow of dilute suspension with anisotropic carrier fluid is solved. It is shown that the stable stationary solutions of the dynamical set of ordinary differential equations describing the particles rotary motion are possible in the case of forming the stationary anisotropy in the carrier fluid of the suspension. It allows us to detect the stationary orientation of suspended particles and formation of stable anisotropic liquid-crystalline structures in the considered suspension under the action of hydrodynamic forces. The study of rheological properties of such a structured suspension shows that it behaves as a viscoelastic quasi-Newtonian anisotropic liquid medium. ***To cite this article: E.Yu. Taran et al., C. R. Mecanique 332 (2004).***

Résumé

Formation hydrodynamique pour structures anisotropiques stables dans un flux de Couette d'une suspension diluée. On résoud le problème sur les mouvement rotatoire de particules rigides axiallement symétriques allongées dans le flux de Couette de la suspension diluée à liquide porteur anisotropique. Il est montré que dans le cas de la création d'une anisotropie stationnaire dans le liquide porteur de la suspension des solutions stables stationnaires du système dynamique des équations différentielles ordinaires qui décrivent un movement rotatiore d'une particule sont possibles. Cela nous a permis de révéler une orientation stationnaire des particules pondérées et la formation dans la suspension considérée des structures anisotropiques mésomorphes sous une action des forces hydrodynamiques. L'examen des propriétés rhéologiques d'une telle suspension structurisée a montré qu'elle se comporte un milieu liquide anisotropique visqueux et élastique quasi-newtonien. ***Pour citer cet article : E.Yu. Taran et al., C. R. Mecanique 332 (2004).***

Keywords: Rheology; Dilute suspension; Anisotropic carrier fluid; Couette flow

Mots-clés : Rhéologie ; Suspension diluée ; Liquide porteur anisotropique ; Flux de Couette

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On résoud le problème sur les mouvement rotatoire de particules rigides axiallement symétriques allongées dans le flux de Couette de la suspension diluée à liquide porteur anisotropique. Comme un modèle rhéologique du liquide porteur de la suspension et du modèle hydrodynamique des particules pondérées nous utilisons le liquide anisotropique d'Ericksen (1), (2) [1] et un haltére triaxe symétrique [2]. Pour l'examen du mouvement rotatoire des particules pondérées l'équation définissante vectorienne pour leur vitesse angulaire (4), obtenue en [3] se transforme en un système normal des équations différentielles ordinaires relativement au composants de vecteur unitaire n_i définissant l'orientation des particules pondérées. Cela nous permet de ramener le problème spatial sur le mouvement rotatoire des particules pondérées dans la flux de Couette (3) de la suspension à la solution et à l'examen du comportement de qualité du système dynamique des équations différentielles ordinaires (5) dans son plan à phase On_1n_2. L'obtention d'une 0 solution du système (5), la preuve de sa stabilité à l'aide d'une méthode de Liapounov à des conditions (7), ainsi que la solution numérique du système (5) et l'examen de ses trajectoires de phase montrent que dans le flux de Couette de la suspension considérée une structure mésomorphe anisotropique à deux directions préferentielles mutuellement perpendiculaires est formée. La première direction se forme par une orientation uniforme stationnaire des particules pondérées (6) dans la direction, perpendiculaire au plan de glissement du flux de Couette, et la seconde direction se forme dans le plan de glissement par une orientation stationaire du directeur unitaire u_i par une anisotropie du liquide porteur de la suspension. La solution numérique du système (5) et l'examen des ses trajectoires de phase montrent aussi que dans les conditions (8) toutes les particules pondérées dans leur rotation sous une action des forces hydrodynamiques se trouvent finalement dans le plan de glissement du flux de Couette. Après cela elles continuent à tourner periodiquement dans le plan de glissement à une vitesse angulaire (9) si leur relation axielle $\tilde{p}$ est plus petite qu'une certaine valeur critique $\tilde{p}^*$. Dans le cas de $\tilde{p} \geqslant \tilde{p}^*$ de telles particules sont suspendues dans une certaine position angulaire dans le plan de glissement qui ne dépend pas de la vitesse de glissement K. L'angle entre les particules suspendues stationairement et le directeur u_i orienté stationnairement dans le plan de glissement diminue avec une augmentation de relation axiale $\tilde{p}$ et/ou une augmentation du parametre Δ qui charactérise l'anisotropie du liquide porteur. La recherche des propriétés visqueuses et élastiques des systèmes mésomorphes anisotropiques biaxiaux formés dans le flux stationnaire de Couette de la suspension à l'aide de l'équation rhéologique obtenue en [3] montre qu'une telle suspension se comporte comme un milieu liquide anisotropique visqueux et élastique quasi-newtonien.

1. Introduction

The flow is associated with an irreversible deformation of flowing medium and a change of its structure. Nevertheless, in this paper, we detect formation of stable biaxial anisotropic structures of liquid-crystalline type under the action of hydrodynamic forces in a steady-state Couette flow of dilute suspension of rigid axially symmetric particles with an anisotropic carrier fluid. As a rheological model of the suspension carrier fluid we use the Ericksen anisotropic fluid [1] determined by rheological equations

$$t_{ij} = -p\delta_{ij} + 2\mu d_{ij} + (\mu_1 + \mu_2 d_{km} u_k u_m) u_i u_j + 2\mu_3 (d_{jk} u_k u_i + d_{ik} u_k u_j) \tag{1}$$

$$\dot{u}_i = \omega_{ik} u_k + \lambda (d_{ik} u_k - d_{km} u_k u_m u_i) \tag{2}$$

where t_{ij} is the stress tensor; d_{ij} is the deformation rate tensor, $d_{ij} = (1/2)(v_{i,j} + v_{j,i})$; ω_{ij} is the velocity vortex tensor, $\omega_{ij} = (1/2)(v_{i,j} - v_{j,i})$; u_i is the unit director defining anisotropy in each point of fluid; p is the isotropic pressure; λ and μ's are rheological constants; the dot over u_i denotes a local time derivation; δ_{ij} is the Kronecker delta.

As a hydrodynamic model of suspended particles possessing zero buoyancy, we use a symmetric triaxial dumbbell [2] with mutually perpendicular axes $L_1,\ L_2,\ L_3$ $(L_1 \geqslant L_2 = L_3)$ that are intersecting in one point and are divided into equal parts by it. For investigating the rotary dynamics of suspended particles in gradient flows of the considered suspension and its rheological properties, constitutive equations obtained in [3] are applied.

An investigation of dynamics of suspended particles in gradient flows of suspensions is the necessary stage in the construction of their rheological models and in the study of the suspensions rheological properties. Thus, the study of rotational motion of spherical and ellipsoidal suspended particles by Einstein [4] and Jeffery [5] in gradient flows of corresponding dilute suspensions with the Newtonian carrier fluid allowed them to derive the expressions for the apparent viscosity of such suspensions.

2. Structuring induced by flow in dilute suspension with anisotropic carrier fluid

Dilute suspension of triaxial dumbbells with the Ericksen anisotropic carrier fluid [3] is used in the paper as a model suspension to investigate the influence of the stable stationary anisotropy of the suspension carrier fluid on the dynamics of rigid axially symmetric suspended particles under the action of hydrodynamic forces arising in the Couette flow

$$v_1 = 0, \quad v_2 = Kx_1, \quad v_3 = 0 \quad (K = const) \tag{3}$$

of the suspension and also on the rheological properties of the suspension as a whole.

According to [1], the above mentioned stable stationary anisotropy in the Couette flow of the Ericksen anisotropic fluid (1), (2) is formed by the stable stationary orientation $u_1 = \cos\beta,\ u_2 = \sin\beta,\ u_3 = 0$ of its unit director u_i in the plane of shear Ox_1x_2 of the Couette flow, which arise at the condition $|\lambda| \geqslant 1$. Here, β is the angle between the axis Ox_1 and director u_i defined by the equation $\mathrm{ctg}^2\beta = (\lambda - 1)/(\lambda + 1)$.

The equation which is used in the paper to investigate the rotary dynamics of suspended particles

$$\begin{aligned}&\tilde{p}(-\xi_{ij}N_j + \xi_{kl}N_k n_l n_i - \xi_{jl}d_{jk}n_l n_k n_i + \xi_{il}d_{lj}n_j) - \xi_{jl}(N_i n_j n_l + d_{ij}n_l - d_{jk}n_l n_k n_i)\\ &\quad + \varepsilon_{ijk}\xi_{kl}n_j n_l \varepsilon_{nms}(\xi_{sf}N_n n_m n_f - \xi_{ls}d_{ml}n_n)(\xi_{\alpha\alpha} - \xi_{\alpha\beta}n_\alpha n_\beta)^{-1} = 0\end{aligned} \tag{4}$$

was obtained in [3] as a result of the vector multiplication by vector n_i of the equation of rotary motion of suspended dumbbell particles under the action of hydrodynamic forces in arbitrary gradient flows of the considered suspension. In (4), $\tilde{p}$ is the axial ratio of triaxial dumbbells, $\tilde{p} = L_1/L_2$; $N_i = \dot{n}_i - \omega_{ik}n_k$, where n_i is the unit vector characterizing orientation of the main axis L_1 of the triaxial dumbbell in Cartesian coordinate system $x_1,\ x_2,\ x_3$ used in the paper; ξ_{ik} is the friction drag tensor of beads placed at the ends of dumbbell axes $L_1,\ L_2,\ L_3$, $\xi_{ik} = \zeta_\perp\delta_{ik} + (\zeta_\parallel - \zeta_\perp)u_i u_k$, where $\zeta_\parallel$ and $\zeta_\perp$ are the longitudinal and transverse (relatively to the director u_i of the Ericksen anisotropic fluid) frictional coefficients of the dumbbell beads respectively; ε_{ijk} is the Levi-Civita tensor.

The solution of the vector equation (4) with respect to $\dot{n}_i$ allows us to transform it into the normal set of ordinary differential equations with respect to the components n_i $(i = 1, 2, 3)$ of the vector n_i. Due to the relation $n_i n_i = 1$, we have $n_3 = \pm\sqrt{1 - n_1^2 - n_2^2}$. Therefore, only two equations of such a set suffice for investigating the rotary dynamics of suspended particles. The non-uniqueness in calculations of n_3 does not influence on results, because the vectors n_i and $-n_i$ define identically the angular position of the main axis L_1 of the triaxial dumbbell. It allows us to reduce the spatial problem of rotary motion of suspended particles in the Couette flow (3) of suspension to the solution and investigation of qualitative behavior of dynamical set of two differential equations

$$\begin{aligned}\begin{pmatrix}\dot{n}_1\\ \dot{n}_2\end{pmatrix} &= \frac{K}{2}\begin{pmatrix}-n_2\\ n_1\end{pmatrix} + \frac{K}{2(\tilde{p}^2 + 1 + (\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2)}\cdot\biggl\{(\tilde{p}^2 - 1)\begin{pmatrix}n_2\\ n_1\end{pmatrix}\\ &\quad - (\Delta - 1)(n_1\cos\beta + n_2\sin\beta)\begin{pmatrix}\sin\beta\\ \cos\beta\end{pmatrix} + \frac{1}{1 + \tilde{p}^2\Delta - (\tilde{p}^2 - 1)(\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2}\\ &\quad \times\bigl[-\tilde{p}^2(\Delta - 1)^2(n_1\cos\beta + n_2\sin\beta)^2\sin 2\beta\end{aligned}$$

$$
\begin{aligned}
&+ (\Delta - 1)(n_1\cos\beta + n_2\sin\beta)(n_1\sin\beta + n_2\cos\beta)\\
&\times \big(1 - 2\tilde p^2 + \tilde p^2\Delta - (\tilde p^2 - 1)(\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2\big)\\
&- \big(\tilde p^2 - 1\big)\big(1 + \tilde p^2\Delta + (\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2\big)2n_1n_2\big]\cdot\begin{pmatrix} n_1\\ n_2\end{pmatrix}\\
&+ \frac{(\Delta - 1)\tilde p^2}{1 + \tilde p^2\Delta - (\tilde p^2 - 1)(\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2}\cdot\big[(\Delta - 1)(n_1\cos\beta + n_2\sin\beta)\sin 2\beta\\
&+ \big(\tilde p^2 - 1\big)(n_1\cos\beta + n_2\sin\beta)2n_1n_2 + 2(n_1\sin\beta + n_2\cos\beta)\big]\cdot\begin{pmatrix}\cos\beta\\ \sin\beta\end{pmatrix}\\
&+ \frac{(\Delta - 1)^2(n_1\cos\beta + n_2\sin\beta)(1 - n_1^2 - n_2^2)}{(1 + \tilde p^2)(\Delta + 1) - (\tilde p^2 - 1)(\Delta - 1)(n_1\cos\beta + n_2\sin\beta)^2}\\
&\times \big[\big(\tilde p^2 - 1\big)(n_1\cos\beta + n_2\sin\beta)(n_1\cos\beta - n_2\sin\beta) + \big(\tilde p^2 + 1\big)\cos 2\beta\big]\cdot\begin{pmatrix}-\sin\beta\\ \cos\beta\end{pmatrix}\Bigg\}
\end{aligned}
\tag{5}
$$

in its phase plane On_1n_2. In (5), $\Delta = \zeta_\parallel/\zeta_\perp$.

The set of Eqs. (5) has the evident stationary solution $n_1 = 0,\ n_2 = 0$. The stationary angular position

$$n_1 = 0, \qquad n_2 = 0, \qquad n_3 = \pm 1 \tag{6}$$

of suspended particles which is perpendicular to the plane of shear of the Couette flow (3) corresponds to this solution. At the conditions

$$\{\lambda \leqslant -1;\ \Delta > 1\} \qquad \text{or} \qquad \{\lambda \geqslant 1;\ \Delta < 1\} \tag{7}$$

such a position of suspended particles is stable. This result comes out from our investigation of the stability of the position of equilibrium (0, 0) of the set of Eqs. (5) in its phase plane On_1n_2. The employment of the first method of Lyapunov [6] shows that the point (0, 0) is the stable focus of the set (5). In addition, the numerical solution of the set (5) at the conditions (7) together with investigating its phase trajectories demonstrate as well that all suspended particles randomly oriented before the flow of the suspension turn under hydrodynamic forces until position (6).

By this is meant that the stable biaxial anisotropic liquid-crystalline structure with two mutually perpendicular preferred directions is formed in the Couette flow (3) of the considered suspension at the conditions (7). The first direction is formed in the shear plane by stationary orientation of the unit director u_i of the Ericksen anisotropic carrier fluid (1), (2) and the second direction is formed by uniform stationary orientation of suspended particles in the direction perpendicular to the shear plane.

Numerical solution of the set (5) and investigating its phase trajectories at the conditions

$$\{\lambda \leqslant -1;\ \Delta < 1\} \qquad \text{or} \qquad \{\lambda \geqslant 1;\ \Delta > 1\} \tag{8}$$

show that all suspended particles randomly oriented before the flow of the suspension eventually get to the plane of shear Ox_1x_2 of the Couette flow (3) in their rotation under the action of hydrodynamic forces. After that, they continue to rotate periodically in the plane of shear with angular velocity

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{(K/2)(1 + \tilde\lambda\cos 2\varphi) - K(\Delta - 1)[\tilde\lambda\cos\varphi\sin\beta\sin(\varphi - \beta) - (\sin^2\beta)/(\tilde p^2 + 1)]}{1 + ((\Delta - 1)/2)[1 + \tilde\lambda\cos 2(\varphi - \beta)]} \tag{9}$$

if their axial ratio $\tilde p$ is less than the certain critical value $\tilde p^\star$ depending on parameters λ and Δ, which characterize the stationary anisotropy of the carrier fluid. In (9), φ is the angle between the axis Ox_1 and the main axis L_1 of suspended dumbbell particle; $\tilde\lambda = (\tilde p^2 - 1)/(\tilde p^2 + 1)$. In the case of the Newtonian carrier fluid ($\Delta = 1$), the angular velocity (9) of triaxial dumbbell coincides with that

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{K}{2}(1 + \tilde\lambda\cos 2\varphi)$$

of ellipsoidal suspended particles with the axial ratio $\tilde{p}$ obtained by Jeffery [5], and at $\tilde{p} = 1$ $(L_1 = L_2 = L_3)$ it coincides with angular velocity $\mathrm{d}\varphi/\mathrm{d}t = K/2$ obtained by Einstein [4] for suspended spherical particles in the Couette flow (3) of the corresponding dilute suspensions with the Newtonian carrier fluid.

Contrary to the suspensions with the Newtonian carrier fluid, the rotary motion of elongated triaxial dumbbells $(\tilde{p} > 1)$ in the Couette flow (3) of the suspension with anisotropic carrier fluid is stopped if $\tilde{p} \geqslant \tilde{p}^{\star}$. In this case, such particles hover in the certain angular position under the action of hydrodynamic forces. Their stationary orientation does not depend of the shear rate K, but it depends on parameters $\tilde{p},\ \lambda,\ \Delta$. The angle between the preferred direction of biaxial liquid-crystalline structure formed in the suspension at the conditions (8) by uniformly oriented suspended particles and stationary oriented director u_i of the anisotropic carrier fluid in the plane of shear is decreased while increasing axial ratio $\tilde{p}$ and/or parameter Δ which characterizes the anisotropy of the carrier fluid.

3. Rheological properties of structured dilute suspension

Rheological equation of the dilute suspension with anisotropic carrier fluid considered in the present paper was obtained in [3] within the framework of the structure-phenomenological approach. In the case of arising the stable biaxial anisotropic liquid-crystalline structures in gradient flows of the suspension, its rheological equation takes the form

$$T_{ij} = t_{ij}^0 + \frac{n_0 L_2^2}{2}\big\{\xi_{ik}\big(d_{kj} + (\tilde{p}^2 - 1)d_{kn}n_n n_j + \tilde{p}^2\omega_{kn}n_k n_j - \omega_{jn}n_k n_n\big) + \varepsilon_{jnk}n_n\xi_{ki}\varepsilon_{lms}(\omega_{lq}n_q n_m\xi_{sf}n_f + d_{mf}n_l\xi_{fs})(\xi_{\alpha\alpha} - \xi_{\alpha\beta}n_\alpha n_\beta)^{-1}\big\} \tag{10}$$

where T_{ij} is the stress in the suspension; t_{ij}^0 is the stress in the Ericksen anisotropic fluid (1), (2) at $\mu_1 = 0$; n_0 is the number of suspended particles per unit volume of the suspension. With the constraint $\mu_1 = 0$, the Ericksen anisotropic fluid behaves as a non-Binghamian liquid medium [1] when $t_{ij} = -p\delta_{ij}$ in the absence of fluid flow $v_i \equiv 0$.

The employment of Eq. (10) shows that the suspension with uniformly stationary oriented particles behaves as a viscoelastic anisotropic liquid medium. The effective shear viscosity μ_a of such a suspension obtained for the Couette flow (3) at the conditions (7) is defined by the expression

$$\mu_a \equiv \frac{T_{xy} + T_{yx}}{2K} = \mu + \mu_2\frac{\lambda^2 - 1}{4\lambda^2} + \mu_3 + n_0 L_2^2\zeta_\perp\frac{\lambda^2(\Delta + 1)^2 - (\Delta - 1)^2}{8\lambda^2(\Delta + 1)} \tag{11}$$

From (11) it follows that the considered suspension behaves as a quasi-Newtonian anisotropic liquid medium. Its effective viscosity μ_a does not depend on the shear rate K, but depends on the biaxial anisotropy arising in suspension, that is, on the stationary orientations of director u_i and suspended particles. The elastic properties of the suspension is characterized by manifestation of the Weissenberg effect, that is, by non-zero differences of normal stresses $\sigma_1 = T_{yy} - T_{zz}$ and $\sigma_2 = T_{xx} - T_{zz}$

$$\sigma_1 = K\frac{\sqrt{\lambda^2 - 1}}{\lambda}\left(\mu_2\frac{\lambda + 1}{4\lambda} + \mu_3 + n_0 L_2^2\zeta_\perp\frac{\Delta - 1}{8\lambda}\left(\lambda + \frac{\Delta - 1}{\Delta + 1}\right)\right)$$

$$\sigma_2 = K\frac{\sqrt{\lambda^2 - 1}}{\lambda}\left(\mu_2\frac{\lambda - 1}{4\lambda} + \mu_3 + n_0 L_2^2\zeta_\perp\frac{\Delta - 1}{8\lambda}\left(\lambda - \frac{\Delta - 1}{\Delta + 1}\right)\right)$$

in the steady-state Couette flow (3) of the suspension.

4. Conclusions

A comparison between results obtained in the present paper with results obtained by Einstein [4] and Jeffery [5] shows that the dilute suspension of rigid particles with anisotropic carrier fluid can behave radically differently in gradient flows than corresponding dilute suspensions with the Newtonian carrier fluid. Formation of stable stationary anisotropy in the carrier fluid in the Couette flow of dilute suspension so changes rotational dynamics of rigid axially symmetric elongated suspended particles that the stable stationary biaxial structures are set up in the suspension under the action of hydrodynamic forces.

Eqs. (1), (2) of the Ericksen anisotropic fluid may be used for rheological modelling of liquid-crystalline media of nematic type [7]. Suspensions formed on the base of such liquid-crystalline media may appear, for example, during the manufacturing of composite materials through the reinforcement of liquid-crystalline matrices with elongated undeformable uniformly oriented particles. In order that to use the results obtained in the present paper as a theoretical model of forming of such composite materials, it should be verify experimentally above all the possibility of hydrodynamic forming of stable biaxial anisotropic structures in pre-composite suspensions and also obtain numerical values of λ, μ's, $\zeta_{\|}$ and $\zeta_{\perp}$ characterizing the real pre-composite suspension.

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