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Modelling of unsteady 2D cavity flows using the Logvinovich independence principle

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Abstract

A simple model for two-dimensional cavity flows is presented. It is based upon the Logvinovich independence principle. Each section of the cavity is assumed to behave independently of the neighbouring ones. The equation of evolution of the cavity interface is derived. It mainly takes into account an added mass effect and is similar to the well-known Rayleigh–Plesset equation relative to spherical bubbles. The dynamics of the 2D cavity is controlled by the pressure difference between infinity and the cavity. The model proves to be in good agreement with Tulin's solution for a steady cavity flow and easily applicable to unsteady cavity flows. ***To cite this article: C. Pellone et al., C. R. Mecanique 332 (2004).***

Résumé

Modélisation des écoulements cavitants bidimensionnels basée sur le principe d'indépendance de Logvinovich. Une modélisation simple des écoulements cavitants bidimensionnels est proposée. Elle est basée sur le principe d'indépendance de Logvinovich qui suppose que chaque section de cavité se comporte indépendamment des voisines. L'équation d'évolution de l'interface est présentée dans cette Note. Elle prend essentiellement en compte un effet de masse ajoutée et est comparable à l'équation de Rayleigh–Plesset qui régit l'évolution d'une bulle sphérique. La dynamique d'une cavité bidimensionnelle est contrôlée par la différence entre la pression de cavité et la pression à l'infini. Le modèle est en bon accord avec la solution de Tulin pour un écoulement supercavitant stationnaire et est facilement applicable à une configuration instationnaire. ***Pour citer cet article : C. Pellone et al., C. R. Mecanique 332 (2004).***

Keywords: Computational fluid mechanics; Two-phase flows; Cavitation

Mots-clés : Mécanique des fluides numérique ; Écoulements diphasiques ; Cavitation

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1. Introduction

This paper addresses the modelling of two-dimensional unsteady cavity flows. Cavity flows are characterized by the coexistence of two phases, liquid and vapour. The latter can be due to cavitation or to the injection of non-condensable gas into the liquid flow, as it is the case for ventilated supercavitating flows.

The two phases are supposed to be separated by a well-defined interface. It is assumed that there is no intimate mixing between them in the form of small bubbles, for instance, or at least that such a zone is limited in space. This is especially the case for supercavitation, which is actually characterized by a vapour cavity of usually large size with respect to the cavitator. The interface of such a supercavity is clearly defined except in the vicinity of cavity closure where the cavity breaks up into smaller scale vapour structures. Fig. 1 shows a typical view of a natural supercavity generated by a two-dimensional circular cylinder. Such cavity flows can be found around high-speed supercavitating torpedoes as an example.

Several techniques are available for the modelling of cavity flows. Apart from analytical or quasi-analytical methods applicable to simple and often linearized configurations [1], we can mention the boundary element method which is particularly powerful [2]. Other methods based on the resolution of Navier–Stokes equations are also available with various types of cavitation models (see e.g. [3–5]).

In this paper, a different model based on the Logvinovich independence principle is proposed. This principle has been widely used in Russia and Ukraine for the modelling of axisymmetric cavity flows ([6–9], see also [1]). In axisymmetric configurations, a logarithmic singularity appears as the radial distance from the axis tends to infinity. It is then necessary to limit the computational domain to an artificial maximum radius. In the two-dimensional case, this singularity is expected to be stronger and a special procedure, presented in this paper, has to be developed to overcome the difficulty.

The Logvinovich independence principle consists in assuming that each cross-section of the cavity evolves independently of the neighbouring ones and that its evolution is mainly controlled by the pressure difference between the cavity and the pressure at infinity or in other words by the cavitation number σ (cf. Eq. (6) for a definition of σ). In comparison with Navier–Stokes techniques that are very time-consuming, one of the main advantages of this technique is its simplicity, in particular for unsteady cavity flows. It does not take into account viscous effects, which is generally not a serious limitation for supercavity flows since they are mainly governed by inertia. The model presented below also requires that the slenderness of the cavity is small enough. This is generally true for supercavitation since the cavity length increases more rapidly than its thickness as the cavitation number is decreased (cf. Eq. (10)).

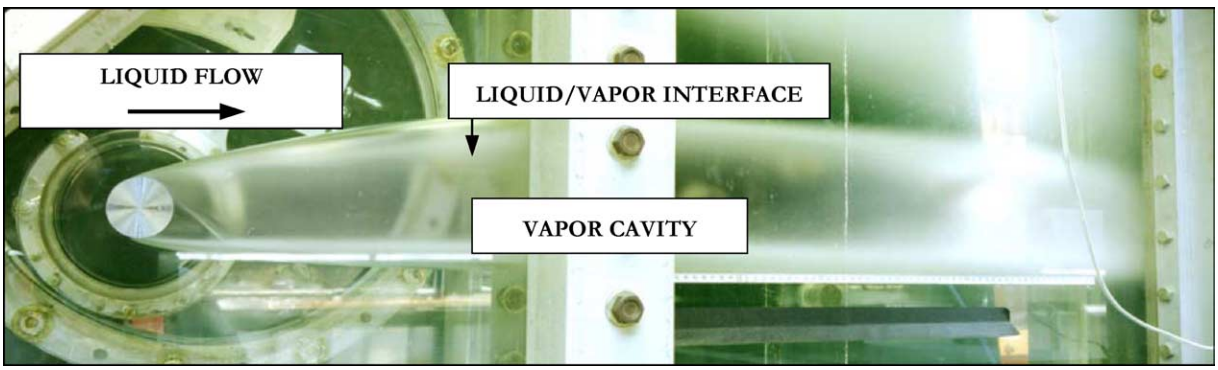

Fig. 1. Supercavity flow around a two-dimensional circular cylinder in the LEGI hydrodynamic tunnel. The dissymmetry between the upper and lower part of the cavity is due to the free surface. (Cylinder diameter: 5 cm, flow height in undisturbed conditions: 40 cm, flow velocity: 12 m/s, cavitation number: 0.05.)

2. Equation of evolution of the cavity interface

Consider the flow around a two-dimensional cavitator and its cavity in an infinite liquid medium as schematically shown on Fig. 2. For simplicity, we consider a symmetric configuration with respect to the plane $y = 0$. The slender body approximation consists in assuming that the velocity does not differ significantly from the velocity at infinity U_∞. In other words, the two components u and v of the flow velocity shown in Fig. 2 are supposed to be everywhere small with respect to U_∞.

The v-component on the cavity interface (denoted v_{c}) is deduced from the kinematic condition on the interface. This condition infers that, if a fluid particle is on the interface at a given time, it will remain on it at any subsequent time until it reaches the closure region where it will separate from the cavity. Hence, v_{c} is given by:

$$v_{\mathrm{c}} = \frac{\partial y_{\mathrm{c}}}{\partial t} + U_\infty \frac{\partial y_{\mathrm{c}}}{\partial x} \tag{1}$$

where $y = y_{\mathrm{c}}(x, t)$ is the equation of the cavity interface at time t.

Furthermore, the v-component vanishes at infinity where the flow is uniform. Then, it is expected to decrease when moving away from the cavity. For a spherical bubble, the radial velocity behaves like $1/r^2$ [1]. In the axisymmetric case, the continuity equation shows that it decreases as $1/r$. By integration, this leads to the logarithmic singularity for the pressure already mentioned. For the two-dimensional case, we assume that the v-component behaves like a given power $1/y^n$ of the distance y to the plane of symmetry. It will be shown later that exponent n depends upon the cavitation number σ and approaches 1 when σ approaches 0. Nevertheless, n is supposed to be always greater than 1 so that no singular behaviour is expected. The v-component is then assumed to be given by:

$$v(x, y, t) = v_{\mathrm{c}} \left[\frac{y_{\mathrm{c}}}{y}\right]^n = \left[\frac{\partial y_{\mathrm{c}}}{\partial t} + U_\infty \frac{\partial y_{\mathrm{c}}}{\partial x}\right]\left[\frac{y_{\mathrm{c}}}{y}\right]^n \tag{2}$$

In the slender body approximation and for an inviscid fluid, the momentum balance in the y-direction writes:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{3}$$

where p is the pressure, ρ the liquid density and $\mathrm{d}/\mathrm{d}t$ the transport derivative given by:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x} \tag{4}$$

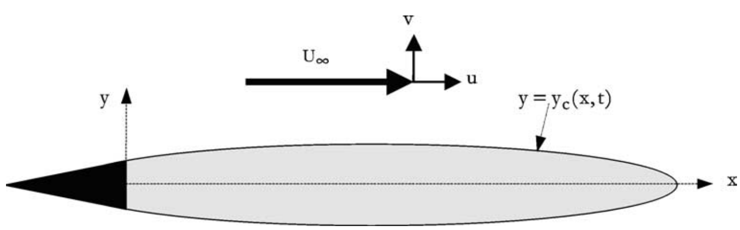

Fig. 2. Schematic view of a cavity flow and general notations.

[1] r is the distance to the bubble centre in the spherical case or to the cavity axis in the axisymmetric case.

The equation of evolution of the cavity interface is obtained by introducing expression (2) for the v-component in the momentum balance (3) and integrating from the cavity interface y_c where the pressure is the cavity pressure p_c to infinity where p is denoted p_∞. We finally obtain the following equation:

$$\boxed{y_c \frac{d^2 y_c}{dt^2} + n\left(\frac{dy_c}{dt}\right)^2 = -(n-1)U_\infty^2 \frac{\sigma}{2}} \tag{5}$$

In this equation d/dt is still the transport derivative (4) and σ is the usual cavitation number defined by:

$$\sigma = \frac{p_\infty - p_c}{(1/2)\rho U_\infty^2} \tag{6}$$

Eq. (5) is similar to the Rayleigh–Plesset equation[2] which allows the computation of the time evolution of a spherical bubble when submitted to a given pressure difference $p_\infty - p_c$. It suggests to follow any cross section of the 2D cavity in a Lagrangian way, as it is advected downstream at flow velocity U_∞. During its downstream movement, the temporal evolution of the cross-sectional area is given by Eq. (5). Initial conditions have to be specified at the instant of shedding of the cross section by the cavitator. They concern the initial size of the cross-section and its derivative which are connected respectively to the size of the cavitator and the slope of the wall at cavity detachment.

Eq. (5) expresses the Logvinovich principle according to which the temporal evolution of a given cross section of the cavity is independent of the neighbouring ones. The main effect that is taken into account in Eq. (5) is an added mass effect connected to the inertia of the surrounding liquid.

3. Discussion

At this step, the exponent n is still unknown. In order to determine it, the steady case is examined. The shape of the cavity is given by the steady version of Eq. (5):

$$\frac{d^2(y_c^2)}{dx^2} = -(n-1)\sigma \tag{7}$$

To get Eq. (7), we assumed n close to unity in the left-hand side of Eq. (5) only. It is checked below that this is a good approximation. This equation has the following elliptic solution for the shape of the cavity interface:

$$\frac{y_c^2}{(n-1)(\sigma/2)(\ell^2/4)} + \frac{(x-\ell/2)^2}{\ell^2/4} = 1 \tag{8}$$

where ℓ is the cavity length. This solution is such that the cavity thickness is zero at both ends $x = 0$ and $x = \ell$. The cavity slenderness is:

$$\delta = \sqrt{(n-1)\frac{\sigma}{2}} \tag{9}$$

In order to determine n, we compare the present solution to the classical one obtained by Tulin [10] for the cavity flow about a symmetrical body in an infinite flow field (see also [1]). According to Tulin's linearized solution, the cavity is also elliptic and its slenderness is:

$$\delta = \frac{\sigma/2}{1+\sigma/2} \tag{10}$$

[2] The left-hand side of the classical Rayleigh–Plesset equation would correspond to $n = 3/2$.

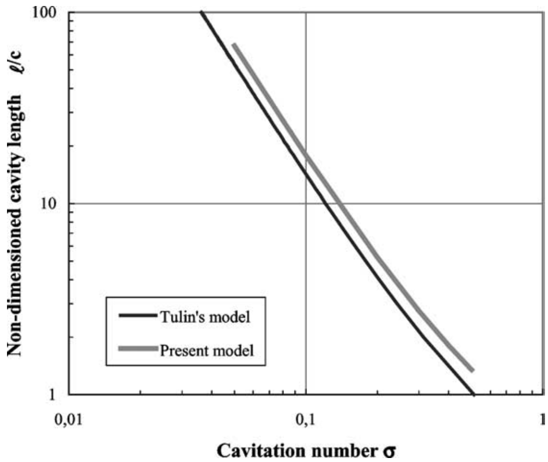

Fig. 3. Comparison of the present model with Tulin's solution for the evolution of the cavity length with the cavitation number. Case of a symmetrical wedge of chord length c and half vertex angle 8 degrees in an infinite flow field. The cavity length is non-dimensioned by the wedge chord length.

The identification of Eqs. (9) and (10) allows the determination of exponent n:

$$n = 1 + \frac{\sigma/2}{(1+\sigma/2)^2} \tag{11}$$

Exponent n is then always greater than unity and approaches unity when σ tends to zero.

Finally, the equation of evolution of the steady cavity is:

$$\frac{\mathrm{d}^2(y_c^2)}{\mathrm{d}x^2} = -2\left[\frac{\sigma/2}{1+\sigma/2}\right]^2 \tag{12}$$

The solution of Eq. (12) in terms of the evolution of the cavity length with the cavitation number is compared to Tulin's solution in Fig. 3 in the particular case of a wedge. The present solution is in good agreement with Tulin's original one. The systematic difference which is observed between the two is not so critical since it is well known that, from an experimental viewpoint, cavity closure is affected by large fluctuations due to cavity unsteadiness and that the cavity length is then not defined very accurately in practice.

It has been checked that, if the exact value of n given by Eq. (11) is taken instead of unity in the left-hand side of Eq. (12), this makes no significant difference in the range of cavitation numbers considered in Fig. 3.

In the unsteady case, the temporal evolution of the cavity is governed by Eq. (5). The principle of the simulation of an unsteady cavity is then the following. At each time step, a new 'slice' of cavity is shed by the cavitator. Each slice is advected downstream independently and the evolution of its cross section is governed by Eq. (5). At each time step, the whole cavity is reconstructed by piling up the different slices of cavity shed at previous time steps. At time steps $1, 2, \ldots, k$, the cavity is then made of $1, 2, \ldots, k$ slices respectively.

Unsteadiness can have several origins. It can be caused by variations of pressure or cavitation number in Eq. (5). Another source of unsteadiness may be due to the movement of the cavitator or to the possible deformation of its walls at constant cavitation number. If so, from a mathematical viewpoint, the unsteady behaviour of the cavity originates in the initial conditions, at the instant of shedding of each cross section, and not in Eq. (5) itself. Unsteadiness can also be due to pressure oscillations inside the cavity, as observed for ventilated pulsating supercavities. The present model is able to account for these various sources of unsteadiness.

A difficulty arises as for the choice of exponent n in the unsteady case. It has been shown that n is a function of the cavitation number for the steady case. It seems then reasonable to use the same determination of n (Eq. (11)) in the unsteady case. However, this is not fully justified and needs to be further validated against experimental results.

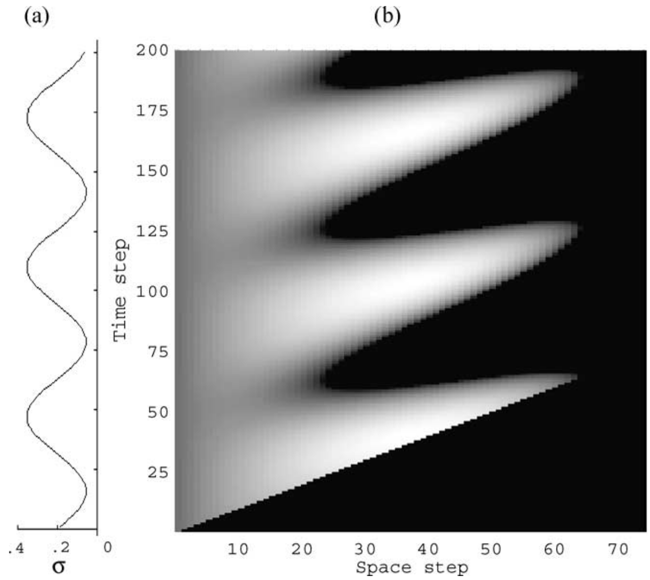

Fig. 4. Unsteady behaviour of the cavity past a symmetrical wedge in the case of sinusoidal oscillations of the cavitation number (chord length: $c = 1$, half vertex angle: 8 degrees, time step: $\Delta t = 0.1$, flow velocity: $U_\infty = 1$, space step: $\Delta x = U_\infty \Delta t = 0.1$). Variation of the applied cavitation number σ with time t: $\sigma = 0.2 - 0.15 \sin t$. Top view of the cavity at different time steps between 0 and 200, coloured by cavity thickness (maximum thickness in white). The origin of space steps corresponds to the basis of the wedge from which the cavity detaches.

An example of solution is given in Fig. 4 for an oscillatory cavitation number in the case of the symmetrical wedge already considered in Fig. 3. Fig. 4(b) presents a collection of top views of the cavity as a function of the distance from the wedge basis at different times. The grey level is representative of the cavity thickness. The linear part at the bottom of diagram 4(b) corresponds to the starting stage and more precisely to the advection of the very first cross section of the cavity shed at the first time step.

Here, the oscillation frequency has been chosen so that it is comparable to the advection time ℓ/U_∞ based on the maximum cavity length. Significant unsteady effects are then expected. This can be seen by the phase difference between the oscillations in pressure and cavity length. When the cavitation number is minimum, the cavity length is not maximum and, moreover, the maximum cavity length is much smaller than that which would be estimated from Fig. 3 assuming a quasi-steady behaviour.

It can be also observed on Fig. 4 that at some instants (typically around time step 125), the cavity is made up of two separate parts. This is characteristic of the development of an undulation of the cavity interface which leads to a local pinching of the cavity and the subsequent separation of part of the cavity. The model is then capable of predicting the break-off of the cavity and the production of a large scale vapour structure which collapses downstream.

The present model appears to have the capabilities of predicting the steady and unsteady behaviours of two-dimensional cavity flows. It is based upon an Eq. (5) which can be considered as a 2D version of the Rayleigh–Plesset equation and which can easily be solved numerically. The model appears to be intermediate between analytical techniques which are limited to simple and usually steady configurations and more sophisticated and time-consuming techniques as the resolution of Navier–Stokes equation together with a cavitation model.

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