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Characterization of the flow for a single fluid in an excavation damaged zone

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Abstract

The aim of this Note is to quantify the change of characteristics of the media of an Excavated Damaged Zone (EDZ) affected by several fractures. For this, we consider Darcy flow through matrix blocks and fractures with permeability of order $\varepsilon^2\delta^\theta$ and 1 respectively. ε is the size of a typical porous block, δ representing the relative size of the fracture and θ is a parameter characterising the permeability ratio. We derive the global behavior from the limit as ε and δ tend to zero. The resulting homogenized equation is of dual-porosity type for $\theta = 2$, but it is a simple-porosity model with effective coefficients for $\theta > 2$, and there is no flow at the macroscopic level when $0 < \theta < 2$. ***To cite this article: B. Amaziane et al., C. R. Mecanique 332 (2004).***

Résumé

Caractérisation de l'écoulement d'un fluide monophasique en zone endommagée. Le but de cette Note est de quantifier les changements dans les paramètres de l'écoulement au sein d'un milieu poreux lorsque celui-ci est endommagé par l'apparition de fissures en grand nombre. Pour cela, on considère l'écoulement d'un fluide régi par la loi de Darcy avec une perméabilité de l'ordre de $\varepsilon^2\delta^\theta$ dans les matrices et d'ordre un dans le réseau de fissures. Pour décrire les diverses situations nous avons caractérisé respectivement par ε, δ et θ la taille des blocs, l'épaisseur relative des fractures et le rapport des perméabilités. On étudie alors le comportement asymptotique de ce problème lorsque ε et δ tendent vers zéro. On montre que le problème homogénéisé est un modèle à double porosité pour $\theta = 2$, un modèle à simple porosité avec des coefficients effectifs lorsque $\theta > 2$ mais qu'il n'y a pas d'écoulement pour le modèle globalement équivalent avec $0 < \theta < 2$. ***Pour citer cet article : B. Amaziane et al., C. R. Mecanique 332 (2004).***

Keywords: Porous media; EDZ; Double porosity model

Mots-clés : Milieu poreux ; EDZ ; Milieu poreux fracturé ; Modèle de double porosité

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1. Introduction

We assume a geological situation where originally the host rock was a low permeability porous rock, and under tectonic stresses, these rocks became fissured. The resulting porous system is then made of two regions having very different properties (see, e.g., [1,2]). The region constituted by the fissure set represents a small part of the total volume, but it is highly permeable, whereas, on the contrary, the porous block has a very high storage fluid capacity with a low permeability. Therefore, the large-scale description will have to incorporate the two different flow mechanisms. For some permeability ratios and some fissure width, the large-scale description is achieved by introducing the so-called double porosity model. It was introduced first for describing the global behavior of fractured porous media by Barenblatt et al. [1] and it is since used in a wide range of engineering specialities related to geohydrology, petroleum reservoir engineering, civil engineering or soil science. More recently, fractured rock domains corresponding to the so-called Excavation Damaged Zone (EDZ) received an increasing attention in connection with the behavior of geological isolation of radioactive waste after the drilling of the wells or shafts.

The usual double porosity model assumes that the width of the fractures containing highly permeable porous media is of the same order as the block size. The related homogenization problem was studied in [3], and was then revisited in the mathematical literature by many other authors (see, e.g., [4–8], and [9]). The main goal of this Note is to describe all the different situations, according to the different type and size of fissuring in the EDZ, by means of two parameters, the relative size of the fissures, δ, and the permeability ratio. An example is given at the end. In order to have explicit formulae and a better understanding of the physics, we will assume the fissure set to be described by a thin, periodic network of intersecting plane fractures. The periodicity assumption was used in order to get more 'tractable' physical models, but the situation where the fractures are randomly spaced will lead qualitatively to similar results, as soon as the associated random field is ergodic and stationary. We assume that the domain Ω is made of a set of porous blocks called $M^{\varepsilon,\delta}$ with permeability of order $\varepsilon^2\delta^\theta$, where ε is the normalized microscopic length scale of a typical porous block ($0<\varepsilon\ll 1$), δ is the relative fracture width and $\theta>0$ is a parameter characterizing the permeability ratio; these porous blocks are surrounded by a system of connected and thin fissures $F^{\varepsilon,\delta}$ with permeability of order 1. The fissure thickness is much smaller than the size of the blocks and, therefore, the measure of the set $F^{\varepsilon,\delta}=\Omega\setminus\overline{M^{\varepsilon,\delta}}$ tends to zero as $\varepsilon\to 0$ and $\delta\to 0$. This model is described by the following linear parabolic equation:

$$\Phi^{\varepsilon,\delta}(x)\partial_t u^{\varepsilon,\delta}-\operatorname{div}\bigl(a^{\varepsilon,\delta}(x)\nabla u^{\varepsilon,\delta}\bigr)=f^{\varepsilon,\delta}(x) \tag{1}$$

where $a^{\varepsilon,\delta}$ is the permeability of the medium, $\Phi^{\varepsilon,\delta}$ is the porosity of the medium, and $f^{\varepsilon,\delta}$ is the source density.

In this Note, in order to describe the various situations occurring in the EDZ, we study the homogenization problem for (1) when ε and δ tend to zero. In the case $\theta=2$, the macroscopic model is nonlocal in time, which corresponds to a memory phenomenon. In contrast with the usual double porosity model we obtain, as in [10], an additional linear source term. For $\theta>2$, the macroscopic behavior is given back by a single-porosity model. When $0<\theta<2$, the solution of (1) converges in a suitable topology to zero, which means that in this case there is no flow at the macroscopic level; the flow is trapped. Similar questions, with different parameters and different scope, were considered in [5,7] and [10]. In [10], this type of microstructure, but with a fixed relative fissure size $\delta=\varepsilon^{\alpha/2}$, $\alpha>0$, was modeled with only one parameter ε. In [7] the simpler case $\theta=0$ was considered.

2. Problem statement and the main result

We consider the following initial boundary value problem for the function $u^{\varepsilon,\delta}:\Omega_T\to\mathbb{R}$:

$$\begin{cases}\Phi^{\varepsilon,\delta}(x)\partial_t u^{\varepsilon,\delta}-\operatorname{div}\bigl(a^{\varepsilon,\delta}(x)\nabla u^{\varepsilon,\delta}\bigr)=f^{\varepsilon,\delta}(x) & \text{in } \Omega_T\\ u^{\varepsilon,\delta}(t,x)=0 & \text{on } (0,T)\times\partial\Omega\\ u^{\varepsilon,\delta}(0,x)=0 & \text{in } \Omega\end{cases} \tag{2}$$

where Ω is a bounded domain in $\mathbb{R}^3$ with a smooth boundary $\partial\Omega$, $T > 0$ is given, and Ω_T denotes the cylinder $\Omega_T = (0, T) \times \Omega$. Let $\mathcal{Y} = (0, 1)^3$ be a periodic cell. We assume that $\mathcal{M}^\delta$ is an open cube centered at the point $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ with length equal to $1 - \delta$ and we reproduce $\mathcal{M}^\delta$ by periodicity, obtaining a periodic open set M^δ in $\mathbb{R}^3$. We denote by F^δ the periodic set $F^\delta = \mathbb{R}^3 \setminus \overline{M^\delta}$, which is obtained from the set $\mathcal{F}^\delta = \mathcal{Y} \setminus \overline{\mathcal{M}^\delta}$.

The functions $a^{\varepsilon,\delta} : \mathbb{R}^3 \to \mathbb{R}$ and $\Phi^{\varepsilon,\delta} : \mathbb{R}^3 \to \mathbb{R}$ are defined by

$$a^{\varepsilon,\delta}(x) = a\varepsilon^2\delta^\theta \cdot \mathbf{1}^\delta_M\left(\frac{x}{\varepsilon}\right) + 1 \cdot \mathbf{1}^\delta_F\left(\frac{x}{\varepsilon}\right), \qquad \Phi^{\varepsilon,\delta}(x) = \Phi_M \cdot \mathbf{1}^\delta_M\left(\frac{x}{\varepsilon}\right) + \Phi_F \cdot \mathbf{1}^\delta_F\left(\frac{x}{\varepsilon}\right) \tag{3}$$

with $0 < a, \theta, \Phi_M, \Phi_F < +\infty$, where $\mathbf{1}^\delta_F(\frac{x}{\varepsilon})$ and $\mathbf{1}^\delta_M(\frac{x}{\varepsilon})$ denote the characteristic periodic functions of the sets F^δ and M^δ defined in the following way: $\mathbf{1}^\delta_M(\frac{x}{\varepsilon})$ is the periodic function of period $\varepsilon\mathcal{Y}$ which takes the value 1 in the set $M^{\varepsilon,\delta}$, union of the sets obtained from $\varepsilon\mathcal{M}^\delta$ by translations of vectors $\varepsilon\sum_{i=1}^n k_i\vec{e}_i$, where $k_i \in \mathbb{Z}$ and $\vec{e}_i$, $1 \leqslant i \leqslant 3$, is the canonical basis of $\mathbb{R}^3$, and which takes the value 0 in the set $F^{\varepsilon,\delta}$, complementary in $\mathbb{R}^3$ of this union. In other words, $\mathbf{1}^\delta_M(\frac{x}{\varepsilon})$ is the characteristic function of the set $M^{\varepsilon,\delta}$, while $\mathbf{1}^\delta_F(\frac{x}{\varepsilon})$ is the characteristic function of $F^{\varepsilon,\delta}$. The function $a^{\varepsilon,\delta}$ therefore takes the value 1 on the set $F^{\varepsilon,\delta}$ which is of asymtotically small measure (of order δ), while it takes small values $a\varepsilon^2\delta^\theta$ on the set $M^{\varepsilon,\delta}$ (where ε and δ take small values, as we will assume below).

Notice that the measure of $\mathcal{F}^\delta$, is calculated as follows:

$$\left|\mathcal{F}^\delta\right| = 1 - (1 - \delta)^3 = 3\delta - 3\delta^2 + \delta^3 \sim 3\delta \tag{4}$$

as $\delta \to 0$. Then the measure of the set $F^{\varepsilon,\delta}$ is such that $\lim_{\delta\to 0}\lim_{\varepsilon\to 0}|F^{\varepsilon,\delta}| = 0$.

We make the following assumption on the source term $f^{\varepsilon,\delta} : \Omega \to \mathbb{R}$:

(H1) $f^{\varepsilon,\delta} \in L^2(\Omega)$ such that $f^{\varepsilon,\delta}(x) \equiv 0$ for $x \in M^{\varepsilon,\delta}$;
(H2) $f^{\varepsilon,\delta}$ L_δ-converges to a function $f \in L^2(\Omega)$, according to Definition 2.1, below.

It is already known (see, e.g., [11]) that for any $\varepsilon, \delta > 0$ there exists a unique solution $u^{\varepsilon,\delta}$ of problem (2) belonging to $C([0, T]; H^1(\Omega))$ and that the ε and δ limits are commuting [6]. We then choose to study the asymptotic behavior of the solutions $u^{\varepsilon,\delta}$ first as $\varepsilon \to 0$ and then as $\delta \to 0$.

Due to the vanishing measure of the fissure, we should define the convergence of sequences according to the singularity of the fissure measure. For this, inspired by [12,6,13] and [14] we define:

Definition 2.1. A sequence $\{v^{\varepsilon,\delta}\} \subset L^2(F^{\varepsilon,\delta})$ is said to L_δ-converge to a function $v \in L^2(\Omega)$ if

$$\lim_{\delta\to 0}\lim_{\varepsilon\to 0}\frac{1}{|F^{\varepsilon,\delta}|}\left\|v^{\varepsilon,\delta} - v\right\|^2_{L^2(F^{\varepsilon,\delta})} = 0$$

Models corresponding to the various situations are given by the following convergence results:

Theorem 2.2. *Let $\theta = 2$ in* (3)*, then for any $t \in]0, T[$, under assumptions* (H1), (H2), *$u^{\varepsilon,\delta}$ solution of* (2) *L_δ-converges to u^* solution of a global model with an additional linear source term and the fracture porosity as effective porosity*:

$$\begin{cases} \Phi_F u^*_t - \dfrac{2}{3}\Delta u^* = f(x) - \dfrac{2\sqrt{a\Phi_M}}{\sqrt{\pi}}\displaystyle\int_0^t \dfrac{u^*_t(x,\tau)}{\sqrt{t-\tau}}\,\mathrm{d}\tau & \text{in } \Omega_T \\ u^*(t,x) = 0 & \text{on } (0,T)\times\partial\Omega \\ u^*(0,x) = 0 & \text{in } \Omega \end{cases} \tag{5}$$

Theorem 2.3. *If $\theta > 2$ in* (3)*, then for any $t \in]0, T[$, under assumptions* (H1), (H2)*, $u^{\varepsilon,\delta}$ solution of* (2) *L_δ-converges to u^* solution of a simple porosity model with effective constant porosity and permeability*:

$$\begin{cases} \Phi_F u_t^* - \dfrac{2}{3}\Delta u^* = f(x) & \text{in } \Omega_T \\ u^*(t,x) = 0 & \text{on } (0,T)\times\partial\Omega \\ u^*(0,x) = 0 & \text{in } \Omega \end{cases} \tag{6}$$

Theorem 2.4. *If $\theta < 2$ in* (3)*, then for any $t \in]0, T[$, under assumptions* (H1), (H2)*, $u^{\varepsilon,\delta}$ solution of* (2) *L_δ-converges to* 0*. Thus, there is no flow through the effective porous medium at the global scale.*

3. Sketch of the proof of Theorem 2.2

Knowing from [6] that the limit as $(\varepsilon,\delta) \to 0$ does not depend on the order, Theorem 2.2 will be proved in two main steps. On the first step fixing δ we pass to the limit as $\varepsilon \to 0$; we obtain then a boundary value problem considered in the whole domain Ω but with coefficients depending on the parameter δ. On the second step we pass to the limit as $\delta \to 0$ and obtain, finally, the macroscopic model (5).

Step 1. Let us first fix δ.

In the following, for Ω^ε any subdomain of the domain Ω, we will say that the the sequence $\{v^\varepsilon\} \subset L^2(\Omega^\varepsilon)$ converges in the space $L^2(\Omega^\varepsilon)$ to a function $v \in L^2(\Omega)$ if $\lim_{\varepsilon\to 0} \|v^\varepsilon - v\|_{L^2(\Omega^\varepsilon)} = 0$ (see, e.g., [15], p. 14, or [4], p. 1247).

The convergence of the homogenization process of problem (2) for fixed $\delta > 0$ when $\varepsilon \to 0$ will be given by Theorem 3.1 (see, e.g., [4]), and for this, we consider $u_\lambda^{\varepsilon,\delta} : \Omega \to$ the Laplace transform of $u^{\varepsilon,\delta}$ solution of Eq. (2) and study then the corresponding initial boundary value problem:

$$\begin{cases} \Phi^{\varepsilon,\delta}(x) u_\lambda^{\varepsilon,\delta} - \operatorname{div}\big(a^{\varepsilon,\delta}(x)\nabla u_\lambda^{\varepsilon,\delta}\big) = \lambda^{-1} f^{\varepsilon,\delta}(x) & \text{in } \Omega \\ u_\lambda^{\varepsilon,\delta}(x) = 0 & \text{on } \partial\Omega \end{cases} \tag{7}$$

$\lambda > 0$. It is known that, for any $\varepsilon > 0$, there exists a unique solution $u_\lambda^{\varepsilon,\delta} \in H_0^1(\Omega)$ of problem (7). The asymptotic behavior of the functions $u_\lambda^{\varepsilon,\delta}$ as $\varepsilon \to 0$ is given by the following theorem.

Theorem 3.1. *Let the conditions of Theorem* 2.2 *be fulfilled. Then $u_\lambda^{\varepsilon,\delta}$ solution of* (7)*, converges in the space $L^2(F^{\varepsilon,\delta})$ to $u_\lambda^\delta = u_\lambda^\delta(x)$ solution of*

$$\begin{cases} \lambda\big|\mathcal{F}^\delta\big|\Phi_F u_\lambda^\delta - \operatorname{div}_x\big(A^\delta \nabla_x u_\lambda^\delta\big) + b_\lambda^\delta u_\lambda^\delta = \lambda^{-1}\big|\mathcal{F}^\delta\big| f(x) & \text{in } \Omega \\ u_\lambda^\delta(x) = 0 & \text{on } \partial\Omega \\ \lambda \Phi_M U_\lambda^\delta - a\delta^2 \Delta_y U_\lambda^\delta = 0 & \text{in } \mathcal{M}^\delta \\ U_\lambda^\delta(y) = 1 & \text{on } \partial\mathcal{M}^\delta \end{cases} \tag{8}$$

where $A^\delta = \{a_{ij}^\delta\}$ is the homogenized permeability tensor defined by:

$$a_{ij}^\delta = \int_{\mathcal{F}^\delta} (\vec{e}_i + \nabla_y w_i)\cdot(\vec{e}_j + \nabla_y w_j)\,\mathrm{d}y \tag{9}$$

w_i being the unique solution in $H_\#^1(\mathcal{F}^\delta)\setminus\mathbb{R}$ of

$$\begin{cases} -\Delta w_i = 0 \quad \text{in } \mathcal{F}^\delta \\ (\vec{e}_i + \nabla_y w_i)\cdot\vec{\nu} = 0 \quad \text{on } \partial\mathcal{M}^\delta \\ y \to w_i(x,y) \quad \mathcal{Y}\text{-periodic} \end{cases} \tag{10}$$

and where b_λ^δ is a positive constant given by:

$$b_\lambda^\delta = \lambda \Phi_M \int_{\mathcal{M}^\delta} U_\lambda^\delta(y)\,\mathrm{d}y \tag{11}$$

Remark 1. Theorem 3.1 can be reformulated as follows. Let $\tilde{u}_\lambda^{\varepsilon,\delta}$ be an extension of $u_\lambda^{\varepsilon,\delta}$ from the set $F^{\varepsilon,\delta}$ to Ω which exists (see [16]). Then we have that under conditions of Theorem 2.2 the sequence $\{\tilde{u}_\lambda^{\varepsilon,\delta}\}$ converges in $L^2(\Omega)$ to u_λ^δ solution of (8).

Step 2. Now we pass to the limit as δ tends to 0. First we study the asymptotic behavior of b_λ^δ in (11) as $\delta \to 0$. Applying the ideas of [10] one can show that

$$b_\lambda^\delta = 6\delta\sqrt{a\lambda\Phi_M}\cdot\bigl(1+\mathrm{o}(1)\bigr), \quad \text{as } \delta \to 0 \tag{12}$$

Following now the arguments of [15] and using (4) one easily shows that the sequence $\{u_\lambda^\delta\}$ converges in $L^2(\Omega)$ to a function u_λ^*, the solution of

$$\begin{cases} \lambda\Phi_F u_\lambda^* - \frac{2}{3}\Delta u_\lambda^* + 2\sqrt{a\lambda\Phi_M}\,u_\lambda^* = \lambda^{-1} f(x) & \text{in } \Omega \\ u_\lambda^*(x) = 0 & \text{on } \partial\Omega \end{cases} \tag{13}$$

Let us show now that $u_\lambda^{\varepsilon,\delta}$, solution of (7), L_δ-converge to u_λ^*, solution of (13). In fact, we have

$$\frac{1}{|F^{\varepsilon,\delta}|}\bigl\|u_\lambda^{\varepsilon,\delta} - u_\lambda^*\bigr\|^2_{L^2(F^{\varepsilon,\delta})} \leqslant C\bigl(\bigl\|\tilde{u}_\lambda^{\varepsilon,\delta} - u_\lambda^\delta\bigr\|^2_{L^2(\Omega)} + \bigl\|u_\lambda^\delta - u_\lambda^*\bigr\|^2_{L^2(\Omega)}\bigr) \tag{14}$$

where $\tilde{u}_\lambda^{\varepsilon,\delta}$ is the extension of $u_\lambda^{\varepsilon,\delta}$ from the set $F^{\varepsilon,\delta}$ to Ω defined in Remark 1 and C is a constant independent of ε, δ. Now the L_δ-convergence of $u_\lambda^{\varepsilon,\delta}$ to u_λ^* easy follows from Remark 1 and the strong convergence in $L^2(\Omega)$ of the sequence $\{u_\lambda^\delta\}$ to u_λ^*.

Therefore, the following result is proved.

Theorem 3.2. *Let $\theta = 2$. Then under assumptions* (H1), (H2) *and* (3), *$u_\lambda^{\varepsilon,\delta}$ solution of* (7) *L_δ-converges to u_λ^* solution of* (13).

Now we are in position to complete the proof of Theorem 2.2. Using the arguments similar to ones from [17], we prove that the inverse Laplace transform of u_λ^*, denoted u^*, is solution of (5) and we are in position to deduce the statement of Theorem 2.2 from Theorem 3.2 . This completes the proof of Theorem 2.2.

4. Sketch of the proofs of Theorems 2.3 and 2.4

The proofs of Theorems 2.2 and 2.3 are exactly the same, except that b_λ^δ/δ tends to 0 when $\theta > 2$ and $\delta \to 0$.

The idea of the proof of Theorem 2.4 is the following. The term b_λ^δ/δ tends to $+\infty$ when $\theta < 2$ and $\delta \to 0$. Since the energy of the system is finite, $u_\lambda^{\varepsilon,\delta}$ solution of (7) should L_δ-converge to 0. Then the proof of Theorem 2.4 may be completed in a similar way as the proof of Theorem 2.2.

5. Conclusion

A typical example of the situation described by the above theory is the following. We start with an initially non-damaged porous media, with no source or sink inside the considered zone, with effective permeability of order ε^4, i.e., permeability and porosity of order $\varepsilon^k = \varepsilon^{12}$ and $\varepsilon^m = \varepsilon^8$, respectively. Assume, the damaging of this zone corresponds to a net of 10 connected fissures per square meter ($\varepsilon = 1/10$) each of which has thickness of order $1/100 = \varepsilon^2$, with effective permeability of order ($\varepsilon^\alpha \delta^{-\theta} = \varepsilon$) and with a source term of flow inside the fissure net of order ε. The above theorems applied to this situation give three different behaviors, according to the order of the fissure net porosity δ^θ. If the fissure porosity is of order ε^5 ($\theta = 2.5$), then, according to Theorem 2.3, there will be some flow through the damaged zone, which then behaves effectively like a porous media with effective permeability of order $\mathrm{O}(1)$, exactly $= (2/3)/\phi_f$, and with source flow of density ε. On the other hand, if the fissure porosity is of order ε^3 ($\theta = 1.5$), then, according to Theorem 2.4, there will be no flow through the damaged zone, which then behaves effectively like the non-damaged porous media. In between ($\theta = 2$), according to Theorem 2.2, the damaged zone will then behave effectively according to a simplified 'double porosity' model as in (5).

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