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Extension of the Kida law in turbulence

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Abstract

We extend the validity range of Kida's log-stable law of stability index $\alpha = 1.65$ and intermittency parameter $\mu = 0.2$ to a new range of Reynolds number. This law describes intermittencies in fully developed turbulent flows or more precisely the p.d.f. of turbulence dissipation. Former measurements of the hyper-flatness factors of order 4, 5, 6 of turbulent velocity increments, coming from both experimental works and numerical simulations are used. We show that the power-law variation of these hyper-flatness factors with Taylor scale based Reynolds numbers Re_λ can be fitted, for Re_λ ranging from 35 to 750, by a log-stable law of stability index $\alpha = 1.65$ and intermittency parameter $\mu = 0.21$. ***To cite this article: N. Rimbert, O. Séro-Guillaume, C. R. Mecanique 331 (2003).***

Résumé

Extension de la loi de Kida en turbulence. On étend le domaine de validité de la loi de Kida d'indice de stabilité $\alpha = 1{,}65$ et de paramètre d'intermittence $\mu = 0{,}2$ à une nouvelle gamme de nombre de Reynolds. Cette loi décrit les intermittences en turbulence pleinement développée ou plus précisément la distribution de densité de probabilité de la dissipation de la turbulence. On utilise les résultats des mesures des coefficients d'hyper-aplatissement d'ordre 4, 5 et 6 des incréments de vitesse turbulente issues de précédentes études expérimentales et numériques. Nous montrons que la variation en loi de puissance de ces coefficients avec le nombre de Reynolds construit sur la micro-échelle de Taylor λ peut être ajustée pour Re_λ compris entre 35 et 750 à l'aide d'une loi log-stable d'indice de stabilité $\alpha = 1{,}65$ et de paramètre d'intermittence $\mu = 0{,}21$. ***Pour citer cet article : N. Rimbert, O. Séro-Guillaume, C. R. Mecanique 331 (2003).***

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Mots-clés : Turbulence ; Intermittence ; Théorie de la cascade ; Distribution log-Lévy

Version française abrégée

L'objet de cette Note est de montrer comment la loi log-stable de Kida d'indice de stabilité $\alpha = 1{,}65$ et de paramètre d'intermittence $\mu = 0{,}21$ [1] permet de rendre compte d'un certain nombre de résultats expérimentaux

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et numériques récents sur l'étude des intermittences en turbulence pleinement développée. Cette loi est une généralisation de la loi log-normale obtenue théoriquement par Kolmogorov en atomisation [4] et appliquée en turbulence [2,3]. Parmi les travaux antérieurs sur cette loi, on peut citer comme nous l'avons déjà fait, ceux de Kida [1,9], puis ceux de Novikov [5] qui la généralisent aux lois log-infiniment divisible et de nombreux autres encore, comme Castaing et al. [6] ou Schertzer et al. [7]. Il est difficile de distinguer les différents modèles proposé en utilisant les moments jusqu'à l'ordre 10 et [7] constitue une intéressante comparaison entre les lois log-stables et log-Poisson. [7] discute aussi du problème important de la divergence des moments statistiques d'ordre supérieur à α.

Les lois stables forment un cas particulier des lois infiniment divisibles. Les résultats de Kida ont été obtenus à partir des expériences d'Anselmet et al. [8] pour un nombre de Reynolds Re_λ construit sur la micro-échelle de Taylor λ, $Re_\lambda = 852$. Ces mêmes résultats ont ensuite été directement comparés [9] aux mesures de la p.d.f. de la dissipation de la turbulence atmosphériques obtenues par Stewart et al. [10] pour une gamme de Re_λ situés aux alentours de 1000. Nous allons étendre la gamme des nombres de Reynolds où cette loi semble s'appliquer en utilisant les résultats de simulations numériques directes de la turbulence homogène isotrope obtenus par Kerr [11] et les résultats expérimentaux de Belin et al. [12] obtenus à partir de l'écoulement d'hélium dans un cylindre situé entre deux disques contra-rotatifs. Les premiers correspondant à une gamme de nombres de Reynolds Re_λ compris entre 35 et 200 et les seconds à une gamme de nombres de Reynolds Re_λ compris entre 200 et 750.

La Fig. 1 représente l'évolution en fonction du nombre de Reynolds des coefficients d'hyper-aplatissement des incréments de vitesse définis par l'Éq. (1) :

$$H_p(r) = \frac{G_p(r)}{(G_2(r))^{p/2}} = \frac{\langle |u(x+r) - u(x)|^p \rangle}{\langle |u(x+r) - u(x)|^2 \rangle^{p/2}} \tag{1}$$

pour $\eta \lesssim r \lesssim 10\eta$, intervalle où leur valeur est sensiblement constante (η représente l'échelle de Kolmogorov). Après ajustement, on obtient alors les corrélations (2), (3) et (4) entre les H_p pour $p = 2, 3, 4$ et le nombre de Reynolds Re_λ.

$$H_4 = 0{,}95 Re_\lambda^{0,376} \tag{2}$$

$$H_5 = 1{,}07 Re_\lambda^{0,642} \tag{3}$$

$$H_6 = 0{,}99 Re_\lambda^{0,989} \tag{4}$$

Ainsi que les corrélations (10) et (11) pour les coefficients d'hyper-aplatissement entre eux :

$$H_4 = 0{,}96 H_6^{0,38} \tag{10}$$

$$H_5 = 1{,}08 H_6^{0,65} \tag{11}$$

On suppose que la dissipation est répartie suivant une loi log-stable d'indice de stabilité α, de paramètre d'asymétrie $\beta = -1$, et de paramètre d'échelle σ. On peut alors écrire les coefficients d'hyper-aplatissement sous la forme (13) en utilisant l'argument de Kolmogorov [3] reliant, à haut Reynolds, les incréments de vitesse $|u(x+r) - u(x)|$, à ε_r la dissipation de la turbulence moyennée sur une sphère de rayon r par :

$$\left|u(x+r) - u(x)\right| \sim \varepsilon_r^{1/3} r^{1/3} \tag{12}$$

On a pour une loi log-stable complètement asymétrique [12] :

$$\left\langle \varepsilon_r^q \right\rangle = \exp\left[q m_{\ln \varepsilon_r} - \frac{\sigma^\alpha}{\cos(\pi\alpha/2)} q^\alpha \right], \quad q > 0$$

On peut alors calculer les coefficients d'hyper-aplatissement H_p et les exposants ξ_p via :

$$H_p = \frac{\langle \varepsilon_r^{p/3} \rangle}{\langle \varepsilon_r^{2/3} \rangle^{p/2}} = \exp(\xi_p) \tag{13}$$

L'indice de stabilité α peut être obtenu à partir de la mesure du rapport des ξ_p comme indiqué par les relations (15) et (16). Le résultat de Kida donnant $\alpha = 1{,}65$ est parfaitement compatible avec les incertitudes expérimentales relevées par Belin et al. Le paramètre d'échelle σ est quant-à lui mesurable à partir des corrélations (2), (3) et (4). Chaque corrélation conduisant à une relation du type (20) compatible avec (25).

$$\sigma^\alpha = C \ln(Re_\lambda) + \ln(A) \tag{20}$$

$$\sigma^\alpha \sim \left[(1{,}06 \pm 0{,}08) \ln\left(\sqrt{Re_\lambda}\right)\right] + 0{,}00 \pm 0{,}20 \tag{25}$$

Cette dernière relation peut s'interpréter plus classiquement en introduisant la micro-échelle de Taylor λ et l'échelle de Kolmogorov η. On obtient alors à partir de (26) la relation particulièrement simple (27) :

$$\ln\left(\frac{\lambda}{\eta}\right) \cong \sigma^\alpha \tag{27}$$

Enfin, en ré-introduisant l'échelle intégrale de la turbulence L et l'échelle de mesure r, on déduit de (25) la corrélation équivalente (29) après avoir utilisé (28) :

$$\frac{L}{r} = \frac{L}{\lambda}\frac{\lambda}{\eta}\frac{\eta}{r} \sim \frac{L}{\lambda}\frac{\lambda}{\eta} \sim Re_\lambda^{3/2} \tag{28}$$

$$\sigma^\alpha = \frac{C}{3} \ln\left(\frac{L}{r}\right) + \ln(A), \quad \eta \lesssim r \lesssim 10\eta \tag{29}$$

Il apparaît alors la relation suivante entre la constante C et le paramètre d'intermittence « traditionnel »: $\mu = C/(3\cos(\pi\alpha/2)) = 0{,}21 \pm 0{,}01$ ce qui est compatible avec le résultat original de Kida $\mu \cong 0{,}2$.

1. Introduction

In this paper, we will show how recent numerical and experimental results in turbulence can be used to extend the validity range of Kida's Log-stable law with a stability index $\alpha = 1.65$ and an intermittency parameter $\mu = 0.21$ [1]. Log-stable laws are a generalisation of Oboukhov–Kolmogorov's lognormal law for the distribution of the dissipation of turbulence [2,3]. This law, which dates back to 1961 and was then used in the 'refined similarity hypothesis', was previously obtained to describe the distribution of the size of particles under pulverization [4]. It is a phenomenological modeling of Richardson's cascade of energy from the large scales of turbulence to the small scales. It has already been theoretically generalized to log-infinitely divisible law by Novikov [5] and others, see Castaing et al. [6] for instance for a unifying point of view and Schertzer et al. [7], for a comparison between log-Poisson and log-stable statistics and a discussion about the problem of the divergence of statistical moments. Let us pinpoint the fact that stable distributions, though less general than infinitely divisible distributions, are of much simpler use. As a matter of fact, they are defined by a Lévy measure following a simple power-law, i.e., by a simple exponent: their stability index. As for infinitely divisible distributions their spectral measures belong to a much wider functional space, making them both more difficult to calculate and to fit. Moreover, there exists a generalised central limit theorem, which can be invoked to justify the appearance of stable laws making them quite generic tools for modelling. Kida showed that a log-stable law of stability index $\alpha = 1.65$ and intermittency parameter $\mu = 0.2$ could fit the evolution of the moments of the velocity increment in the experiments of Anselmet et al. [1,8] up to order 18. Actually, validity of statistical models of intermittencies are to be taken with caution and different models are difficult to distinguish using the statistical moments of order 10 or inferior. Without resorting to statistical moments, Kida [9] showed that this law could describe the whole p.d.f. of the dissipation in the atmospheric boundary layer as measured by Stewart et al. [10]. In the first case, the Taylor-scale Reynolds number Re_λ was 852 and in the second case, global Reynolds number Re was varying from 6 to 16×10^6. Since the macroscopic length scale, i.e., the height of the probe above the ocean, was not very well defined, it is difficult to calculate the corresponding Taylor-scale Reynolds number precisely, but it can be estimated, using the relation $Re_\lambda \approx Re^{1/2}$, to be around 10^3.

In the present work, we will show how Kida's log-stable law can be used to fit results obtained from recent numerical simulations and experiments. Direct numerical simulations of homogeneous isotropic turbulence were obtained by Kerr for a Taylor-scale Reynolds number Re_λ ranging from 35 to 200 [11]. Experimental results were obtained by Belin et al. in an helium flow confined in a cylinder which is limited axially by two counter-rotating disks for Re_λ ranging from 200 to 2000 [12]. The latter value of the Taylor-scale Reynolds number has been limited to 750 in the present work because a transition analogous to a second order phase transition was detected by Tabeling and Willaime, in the same experiment, above $Re_\lambda = 700$ [13]. The study of this transition is not the topic of this note. We focus on the evolution for $p = 4, 5$ and 6 of the hyper-flatness factor H_p with the Taylor scale Reynolds number:

$$H_p(r) = \frac{G_p(r)}{(G_2(r))^{p/2}} = \frac{\langle |u(x+r) - u(x)|^p \rangle}{\langle |u(x+r) - u(x)|^2 \rangle^{p/2}} \tag{1}$$

where $G_p(r) = \langle |u(x+r) - u(x)|^p \rangle$.

Fig. 1 shows these evolutions. As previously explained, for $35 < Re_\lambda < 200$, experimental points are obtained by the numerical simulations of Kerr and for $200 < Re_\lambda < 750$, they come from the experimental work of Belin et al. Lines are the results of the correlations (2), (3) and (4) which are of the form $H_i = k_i Re_\lambda^{a_i}$. They were obtained for Re_λ ranging from 35 to 750. Uncertainties of these correlations are reported in Table 1.

$$H_4 = 0.95 Re_\lambda^{0.38} \tag{2}$$

$$H_5 = 1.07 Re_\lambda^{0.64} \tag{3}$$

$$H_6 = 0.99 Re_\lambda^{0.99} \tag{4}$$

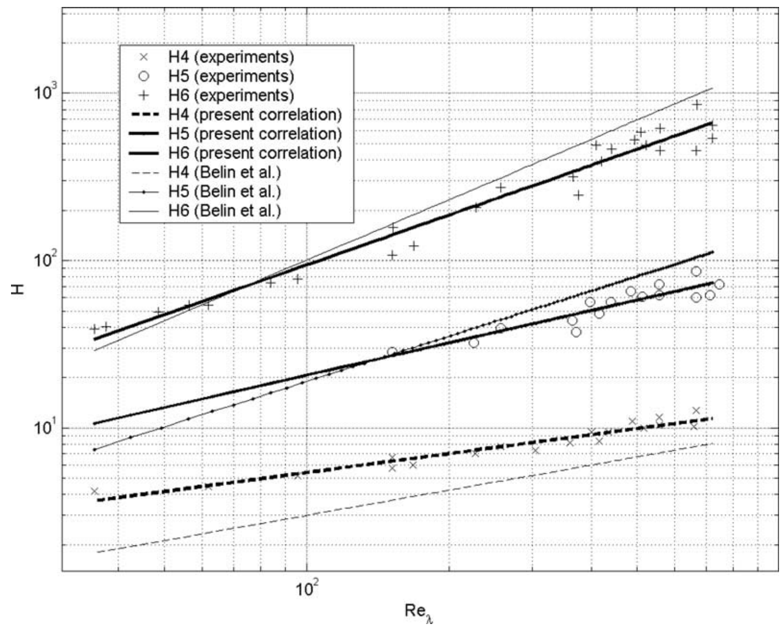

Fig. 1. Variations of the hyper-flatness factor H_4 (+: experiments, solid line: fitting of a power-law), H_5 (o: experiments, solid line: fitting of a power-law) and H_6 (x: experiments, dashed line: fitting of a power-law) with Taylor scale Reynolds number Re_λ.

Fig. 1. Variations des coefficients d'hyper-aplatissement en fonction du nombre de Reynolds.

These correlations are different from those of Belin et al. [12] which were obtained for Re_λ ranging from 200 to 700. Indeed, they found the following correlations:

$$H_4 = 0.3 Re_\lambda^{0.5\pm 0.1} \tag{5}$$

$$H_5 = 0.3 Re_\lambda^{0.9\pm 0.2} \tag{6}$$

$$H_6 = 0.4 Re_\lambda^{1.2\pm 0.2} \tag{7}$$

In relations (5), (6) and (7), the uncertainties on the value of the different exponents were not calculated but were estimated to be around 20%.

It was also noticed in [12] the following link between the Hyper-flatness factors:

$$H_4 = (0.99 \pm 0.05) H_6^{0.376\pm 0.015} \tag{8}$$

$$H_5 = (0.95 \pm 0.05) H_6^{0.67\pm 0.022} \tag{9}$$

The results oh this Note give the very close results:

$$H_4 = 0.96 H_6^{0.38} \tag{10}$$

$$H_5 = 1.08 H_6^{0.65} \tag{11}$$

We will show in Section 2 that Kida's log-stable law of index parameter $\alpha = 1.65$ leads to power-law (8) and (9) up to the measured uncertainty. In Section 3, we will make the hypothesis that the value of this index is independent of the value of the Reynolds number. We will then show that scaling laws (2), (3) and (4) or (5), (6) and (7) can be used to evaluate the scale parameter σ of the stable law. Correlations (2), (3) and (4), *a priori* lead to three different laws for this dependency. In order to get a sound model, these three laws should agree and merge in a common law. We will use the new correlations presented in this note to obtain a sound model. In Section 4, to conclude, we will show that the dependency of the scale parameter with the Taylor-scale Reynolds number can be interpreted in term of an intermittency parameter $\mu = 0.21$, close to the value of 0.2 found by Kida.

2. Stability index of Kida's log-stable law

The bridge between intermittency model based on velocity increment and intermittency model based on turbulence dissipation is the following hypothesis, due to Kolmogorov [3]. It is assumed that at high Reynolds number, the following relation holds:

$$\left|u(x+r) - u(x)\right| \sim \varepsilon_r^{1/3} r^{1/3} \tag{12}$$

where ε_r is the turbulence dissipation averaged over a ball of radius r. If its law is a log-stable law with skewness parameter $\beta = -1$ then the following moments can be written [14]:

$$\left\langle \varepsilon_r^q \right\rangle = \exp\left[q m_{\ln \varepsilon_r} - \frac{\sigma^\alpha}{\cos(\pi\alpha/2)} q^\alpha\right], \quad q > 0$$

where α is the stability index governing the decrease of the tail of the probability, σ is the scale parameter, analogous to the standard deviation of the normal law, and $m_{\ln \varepsilon_r}$ is the shift parameter governing, but not to be confounded with, the mean of the distribution. For $\alpha = 2$, the lognormal model is recovered and the scaling can be classically written as [2,3]:

$$\left\langle \varepsilon_r^q \right\rangle = \exp\left[q m_{\varepsilon_r} + \sigma^2 q^2\right]$$

We can then use Kolmogorov hypothesis (12) to write coefficients H_p as:

$$H_p = \frac{\langle \varepsilon_r^{p/3} \rangle}{\langle \varepsilon_r^{2/3} \rangle^{p/2}} = \exp(\xi_p) \tag{13}$$

where:

$$\xi_p = -\frac{\sigma^\alpha}{\cos(\pi\alpha/2)}\left\{\left(\frac{p}{3}\right)^\alpha - \frac{p}{2}\left(\frac{2}{3}\right)^\alpha\right\} \tag{14}$$

A priori H_p is r dependent but Belin et al. noticed a convergence of H_p in the near-dissipation range $\eta \lesssim r \lesssim 10\eta$ and these limit values are the values we have used.

It can be noticed that the following scaling relations hold:

$$H_4 \sim H_6^{\xi_4/\xi_6}, \qquad H_5 \sim H_6^{\xi_5/\xi_6}$$

Using data from Belin et al. [12], one obtains:

$$\frac{\xi_4}{\xi_6} = \frac{4^\alpha - 2\cdot 2^\alpha}{6^\alpha - 3\cdot 2^\alpha} = 0.376 \pm 0.015 \tag{15}$$

$$\frac{\xi_5}{\xi_6} = \frac{5^\alpha - \frac{5}{2}\cdot 2^\alpha}{6^\alpha - 3\cdot 2^\alpha} = 0.67 \pm 0.022 \tag{16}$$

In (15) and (16) the ratios ξ_4/ξ_6 and ξ_5/ξ_6 have been used to get rid of the scale parameter σ. Using as Kida, the value $\alpha = 1.65$ gives for the ratio ξ_4/ξ_6 the value 0.364 which is acceptable in term of reported uncertainty. This is not the case for the different models tested by Belin et al. (β-model, lognormal model, etc.). One gets for ξ_5/ξ_6 a value of 0.65 which is again acceptable in term of reported uncertainty.

Since these correlations were obtained over a wide range of Reynolds number and taking into account Kida's former results, we can make the hypothesis that $\alpha = 1.65$ is independent of the Reynolds number.

3. Scale parameter of Kida's log-stable law

In order to get the dependency of the scale parameter σ with Re_λ, we no longer consider the ratio of ξ_p. Relations (5), (6) and (7) of Belin et al. [12] yield:

$$\xi_4 = 0.5\ln(Re_\lambda) - 1.20 \tag{17}$$

$$\xi_5 = 0.9\ln(Re_\lambda) - 1.20 \tag{18}$$

$$\xi_6 = 1.2\ln(Re_\lambda) - 0.92 \tag{19}$$

By making use of (14) and by dividing the three relations (17), (18) and (19) by:

$$\frac{1}{\cos(\pi\alpha/2)}\left[\left(\frac{p}{3}\right)^\alpha - \frac{p}{2}\left(\frac{2}{3}\right)^\alpha\right], \quad p = 4, 5, 6$$

a relation of the following form is expected:

$$\sigma^\alpha = C\ln(Re_\lambda) + \ln(A) \tag{20}$$

where C and A are constants (in the log-normal model, C is classically supposed to be universal and there are assumptions that A could take into account deviations from universality [15]).

Actually, this leads to a common value for C equal to 0.73 but to three different values of $\ln(A)$: 0.82, 1.46 and 1.73, calculated respectively from ξ_4, ξ_5, and ξ_6. We therefore conclude that correlation (5), (6) and (7) from Belin et al. are not consistent with (20). In contrast the same process has been applied to the correlations (2), (3) and (4) which leads to better results. A detailed attention was paid to the measurement of the 95% confidence interval of the parameters of this linear regression using Student's law and results are summarized in Table 1. The results can be summarized as:

Table 1
Variations of the different parameters of the power-law fitting shown Fig. 1 with the order p of the hyper-flatness factor H_p. The corresponding values for the coefficients of the law (9) are given

Tableau 1
Variations des paramètres utilisés dans les corrélations de la Fig. 1

$H_i = k_i Re_\lambda^{a_i}$	H_4	H_5	H_6
Constant k	0.96	1.07	0.99
95% confidence interval	[0.72, 1.26]	[0.42, 1.75]	[0.65, 1.49]
Exponent a	0.38	0.64	0.99
95% confidence interval	[0.33, 0.43]	[0.49, 0.80]	[0.91, 1.06]
C	0.55	0.53	0.53
95% confidence interval	[0.48, 0.62]	[0.40, 0.65]	[0.49, 0.57]
$\ln(A)$	0.066	−0.056	0.0067
95% confidence interval	[−0.34, 0.48]	[−0.83, 0.72]	[−0.21,0.23]

$$\xi_4 = 0.38 \ln(Re_\lambda) - 0.045 \tag{21}$$

$$\xi_5 = 0.64 \ln(Re_\lambda) + 0.069 \tag{22}$$

$$\xi_6 = 0.99 \ln(Re_\lambda) - 0.013 \tag{23}$$

It can be noticed that $a_4 = 0.38$ and that this governing exponent of H_4 is very close to the value 0.37 ± 0.03 given by Kerr [11] over the range of Reynolds number $35 < Re_\lambda < 200$.

The three different power-law fittings are now collapsing to the same law (20) with a constant C which can be estimated to be around 0.53 and a parameter $\ln(A)$ which is close to zero. Uncertainty on the value of these parameters has been calculated and by identifying it with the 95% confidence interval size, the relative error can be said to be less than 8% for the constant C. However, for $\ln(A)$, since it seems to be zero, it is difficult to give a value of the relative error. Nevertheless the width of the 95% confidence interval for $\ln(A)$ is quite wide. The best performance is obtained with H_6 since the available data were the most numerous. Hopefully, this is compensated by the good collapse of the three fittings on the same law.

Lastly, it can be noticed that the hypothesis of the independence of the stability index α with the Reynolds number seems to be verified. Indeed, if α had varied with Re_λ, another form for relation (20) should have been obtained.

4. Conclusion

To conclude, the following law can be proposed to describe the evolution of the parameters of the log–stable law over the range of Reynolds number: $35 < Re_\lambda < 750$

$$\alpha = 1.65 \tag{24}$$

$$\sigma^\alpha = \left[(1.06 \pm 0.08) \ln\left(\sqrt{Re_\lambda}\right)\right] + 0.00 \pm 0.20 \tag{25}$$

This is in agreement with the data of Fig. 1.

Moreover, this can be related to Kida's measurement of the intermittency parameter. Firstly, let us notice that the ratio between Taylor scale λ and Kolmogorov scale η is equal to [16]:

$$\frac{\lambda}{\eta} = 15^{1/4} Re_\lambda^{1/2} \tag{26}$$

So, the scaling law (25) can be written as:

$$\ln\left(\frac{\lambda}{\eta}\right) \approx \sigma^\alpha \tag{27}$$

As the measurement scale r and the integral scale L have disappeared in (27) let us reintroduce them in order to get the same kind of scaling as proposed in [15] for the lognormal law or in [1] for the log-stable law:

$$\frac{L}{r} = \frac{L}{\lambda}\frac{\lambda}{\eta}\frac{\eta}{r} \sim \frac{L}{\lambda}\frac{\lambda}{\eta} \sim Re_\lambda^{3/2} \tag{28}$$

where the following facts have been used:

$$\frac{L}{\lambda} \sim Re_\lambda \quad \text{and} \quad 1 < \frac{r}{\eta} < 10 \quad \text{or} \quad \frac{r}{\eta} = \mathrm{O}(1)$$

This is valid in the near-dissipation range where measurements were reported but where scaling laws are to be taken with caution. (20) then reduces to:

$$\sigma^\alpha = \frac{C}{3}\ln\left(\frac{L}{r}\right) + \ln(A) \tag{29}$$

and $\mu = C/(3\cos(\pi\alpha/2))$ can be identified with the traditional intermittency parameter. We then get:

$$\mu = 0.21 \pm 0.01$$

This result is close to Kida's results $\mu \approx 0.2$. To compare with Belin et al. results (5), (6) and (7) lead to a value $\mu \approx 0.28$.

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