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Childhood health and educational investment under risk

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Abstract

A huge literature shows that childhood health and educational attainment are highly correlated. However, estimates for the effect of childhood health on educational attainment under risk generally confound the effect of liquidity constraints and of lack of insurance against risk. It is unclear whether the correlation between health and education under uninsured risk would remain if the capital markets were perfect and household faced no liquidity constraints. This paper fills in this lacuna in the literature. We develop a two period model of investment in education when future labor earnings are stochastically dependent on current investments in schooling and health. It is found that when there is uninsured risk, then parental investment in a child's education will be inefficient even in the presence of perfect capital markets. Under certain assumptions, there will be a positive correlation between childhood health and educational investment. Health inequalities will translate into educational inequalities in an environment of uninsured risk. We are able to show that when perfect insurance markets are present, investments in child health and schooling will be optimal. From the policy perspective, this argues for the development of insurance markets. The results also suggest that policy interventions that target higher levels of educational investment among the population need to account for the effect of childhood differences in health.

JEL Classification: O12, O16, O20

1 Introduction

Grantham-McGregor et al. (2007) estimate that more than 200 million children under 5 years of age fail to reach their potential in cognitive development because of poverty, poor health and nutrition, and deficient care. The positive correlation between health and education has also been highlighted by a number of empirical economic studies (see Grossman and Kaestner (1997) for a review). This correlation is robust even after controlling for different measures of socio-economic status, such as income and race, and regardless of whether health levels are measured by mortality rates, self-reported health status, or physiological indicators of health. More recently, Mayer-Foulkes (2003) provides evidence from Mexico that childhood nutrition and health as well as parental education have substantial and possibly increasing returns in the acquisition of education as measured by school permanence. The poor are less able to invest in human capital, and constituent elements for a low human capital trap or for a prolonged transition in intergenerational human capital accumulation are present in Mexico.

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The correlation between health and educational attainment has also been tackled theoretically. For instance, Edwards and Grossman (1977) develop a model of childhood health and cognitive development to study the effect of parents' income and other variables on a child's subsequent health and cognitive development. Their model is developed on the lines of analyses of inter-generational transfers (e.g., Becker (1967, 1974)) and allows them to analyze whether poor health in childhood adversely affects cognitive development of the child. Their key insight is that to understand the behavior of parents with respect to the child's health and development, it is important to distinguish low-income families from high-income families. In a recent paper, Galor and Mayer-Foulkes (2002) develop an overlapping generation model in which they focus on the effects of minimal health requirements for acquiring an education. They show that when families do not have enough resources to invest in the satisfaction of basic needs and health care, and finance is not available for this purpose, a poverty trap may exist with low health, education, and income. This approach adds to the literature that explains the persistence of poverty through the presence of credit constraints. However, both Edwards and Grossman (1977) and Galor and Mayer-Foulkes (2002) make no assumption that the returns to investment in child health and cognitive development are stochastic. Consequently, their approach is inappropriate in an environment of insurance market failures.

An entire branch of development economics deals with the effect of widespread risk and market failures on investment decisions by poor households in developing countries. It is argued that uninsured risk is a cause rather than just being an aspect of poverty. A number of studies have found that in the presence of uninsured risk, investment decisions by households will be inefficient. For instance, Jacoby and Skoufias (1997) estimate the effect of financial market imperfections on child school attendance using panel data from India. They find that seasonal fluctuations in school attendance are a form of self-insurance, i.e., households withdraw children from school when they experience a negative income shock. Moreover, a few empirical studies analyze the effect of childhood health on educational attainment within a stochastic setting (e.g., Alderman et al. (2001); Gan and Gong (2007)).

However, mathematical models of investment in human capital under risk typically focus on a household's decision to invest in education or health rather than situate the effect of risk on the interlinkages from education to health. Moreover, often, the effect of credit market failures on the health-education correlation cannot be distinguished from the effect of insurance market failures. This is the lacuna which this paper aims to fill up.

The model in this paper is developed by linking and extending existing models of investment in health and education. For instance, consider the seminal paper by Levhari and Weiss (1974) on the effect of risk on a household's decision to invest in education. Levhari and Weiss show that under the assumption of decreasing absolute risk averse preferences, investment in education will be positively correlated with a household's initial income level. In this paper, we follow Levhari and Weiss closely by adopting their framework of a two-period model where in the first period, a household invests in a child's education and, in the second period, it receives stochastically determined returns on that investment. However, we also extend the model developed by Levhari and Weiss by adding another dimension to human capital investment, that is, health.

Investment in health has been modelled in interesting ways in the literature. In Grossman's (1972) model, individuals demand health for two reasons. As a consumption commodity, health enters directly into their utility functions, and as an investment commodity, health determines the total amount of time available for work in the market sector of the economy, where consumers produce money earnings, and for work in the non-market or household sector, where they produce commodities that enter their utility function. Typically, risk is introduced into the consumption or investment versions of Grossman's (1972) model of demand for health (see Grossman (1999) for a review). For instance, Dardanoni and Wagstaff (1990) introduce uncertainty into the pure consumption version of Grossman's model, and Cropper (1977), Muurinen (1982), Dardanoni and Wagstaff (1987), Selden (1993), and Chang (1996) introduce uncertainty into the pure investment version of Grossman's model. In line with the latter group of studies, we ignore the consumption motive for spending on health and assume that individuals demand health purely as an investment commodity. This helps to focus our analysis.

As we will see below, our approach of analyzing the effect of health on education in a stochastic framework offers a new interpretation for the empirically observed positive correlation between health and education. Typically, the observed positive correlation between health and schooling is explained in one of the following three ways. First, it is argued that there is causal relationship that runs from increases in schooling to increases in health. Education improves health by raising economic conditions in per capita income terms so that a higher expenditure in health is possible and/or by increasing knowledge of health issues (Grossman 1975; Kenkel 1991; Rosenzweig and Schultz 1991). Second, it is argued that the causal relationship runs from increases in health to increases in schooling rather than the other way around. Healthier students are more efficient at studying (Perri 1984; Currie and Hyson 1999). Better health may also increase the demand for education because of longer life expectancy (Gan and Gong 2004). Third, it is argued that no causal relationship is implied by the correlation between health and education. Instead, differences in one or more third variables, such as physical and mental ability, parental characteristics, and time discount rates (Fuchs, 1982), affect both health and schooling in the same direction.

In this paper, we construct a sufficiently general framework to capture the interplay between health and cognitive development under risk. This approach allows us to infer that in the presence of uninsured risk, part of the correlation between health and education will reflect the effect of risk on household investment decisions. The remainder of the paper proceeds as follows. Section 2 presents and discusses the model. Section 3 analyzes the optimal level of health and education investments under perfect insurance markets. Section 4 concludes.

2 Conceptual framework

2.1 The model

Consider a unitary household model with an adult and a child. There are two time periods, the first (present) and the second (future). In the first period, adult income (Y) and child wages (W) are the two sources of household income. Also, in the first period, the adult makes the decision to invest in the child's education and health. In the second period, income accrues from the interest earned on household savings in the first period and as a function of the child's human capital (education and health) accumulated in the first period. The two period utility function is represented by $U(C_1, C_2)$, where C_1 denotes consumption in the first period and C_2 denotes consumption in the second period. We assume that the inter-temporal utility function $U(C_1, C_2)$ is additively separable over time and states of the world with discount rate $\beta(< 1)$ and is subject to an inter-temporal budget constraint relating assets in the current period to assets in the previous period, current expenditure and current income. Also, we assume that $U(C_1, C_2)$ is at least three times continuously differentiable and possesses everywhere positive and diminishing marginal utilities ($U'(C) > 0$; $U''(C) < 0$).

Investments in health and education are time-consuming. A is the proportion of the first period time devoted to schooling, and M is the proportion of the first period time devoted to health. We assume that the opportunity cost of child's time is the only cost associated with investments in health and schooling. There are no direct costs of schooling and health. Investment in health and education in the second period is identically zero. We ignore leisure—both of the child and of the adult. Future earnings $\pi(.)$ depend on current investments in health and education and on the future (unknown) state of the world θ where θ is a random variable with a known distribution. We assume that the income-generating function $\pi(.)$ is concave in its arguments.

This is illustrated as follows:

$$U(C_1, C_2) = U(C_1) + \beta EV(C_2)$$

where

$$C_2 = (Y + W \cdot (1 - A - M) - C_1) \cdot (1 + r) + \pi(D, H, \theta)$$

$$H = H_0 + M$$

$$D = D_0 + A$$

H_0 is the initial or childhood health endowment, and H is the stock of health in the second period. D_0 is the initial or childhood endowment of cognitive ability, and D is the stock of cognitive development in the second period. This is a simple representation of the maximization problem, as we do not (yet) allow for interplay between health and cognitive development. We will see later that many interesting results are obtained by allowing for alternative technologies for the second period income-generating function. For instance, we can either assume that health and education are jointly determined or assume that health and education are substitutes in the second period income-generating function.

Another crucial assumption that we make at this point is that the capital markets are perfect and the household faces no liquidity constraints. In essence, we are interested in capturing the effect of insurance market failures on household decision to invest in a risky asset. It is well established that when financial markets are complete, investment decisions are solely determined by rates of return. However, when markets are incomplete or imperfect, separation of consumption and human capital investment decisions no longer holds and investment decisions will be influenced by family resources.

In the context of our model, it is assumed that the child is born with an endowment of health and cognitive ability and these endowments as well as first period investments in the child's health and education together enter as arguments into the second period income-generating function. In what follows, we will simplify our model to focus on a household's decision to invest in the child's education and within the context of that maximization problem try to understand the impact of the child's health endowment on investment in education. We will initially assume that investment in education is risky and the risk is increasing in the level of investment in education. We will show that there will be a positive correlation between childhood health and household investment in the child's education.

Notice that this result is dependent on the assumption of increasing risk which is satisfied if the risk parameter in the income-generating function is multiplicative. If, however, the risk parameter in the income-generating function is additive, then that would be akin to the assumption of decreasing risk and the results would be different. We will discuss more on this later in the paper. For the moment, we assume that risk in the second period income-generating function is multiplicative, and thus, the assumption of increasing risk with an increase in household investment in education is satisfied.

Let us consider the second period income-generating function in some detail. The arguments of the second period income-generating function are the second period stocks of health and education. We have already noted the important assumption of multiplicative risk for our analysis. Now, we will also state some assumptions for the effects of changes in health and education stocks on the second period income. We assume that the marginal productivity of education and health is positive in income. Thus, we have that $\pi_D > 0$ and $\pi_H > 0$. We assume that $\pi_{HD\theta} = 0$. In other words, the change in marginal productivity of education with respect to health does not vary with the parameter of risk.

To sum up, a household is a portfolio investor that invests in financial assets (which have a safe and constant rate of return), education, and health. The household's maximization problem is represented as follows:

$$\text{Max}_{C_1 A, M}\ U(C_1) + \beta E V(C_2)$$

where

$$C_2 = (Y + W \cdot (1 - A - M) - C_1) \cdot (1 + r) + \pi(D, H, \theta)$$

This is a very general framework. We consider specific cases for analytical convenience. First, we are interested in capturing the effect of childhood endowment of health H_0 on the investment in education A by a household.

3 Optimal level of investment

In this section, we will present a definition of optimal level of investment in human capital and also analytically prove that in the presence of perfect insurance markets, household investment will be optimal. Optimality under perfect insurance and credit markets can be interpreted in terms of the first welfare theorem. The first welfare theorem suggests that competitive markets result in equilibrium in a Pareto efficient allocation so that no one can be made better off without at least one person being made worse off. When there is a credit or insurance market failure, the first welfare theorem fails to hold and household investment in risky assets will be at sub-optimal levels or inefficient.

Let us start with the definition of certainty. We follow Levhari and Weiss (1974) and interpret certainty in two alternative ways. First, we interpret it to mean that the future income of the individual is equal to its expected value. Alternatively, we interpret certainty to mean that the state of the world is known.

Mathematically, we represent the two definitions of certainty as follows:

a. $y_1 = E(y_1)$ or $\pi(A, H_0) = E(\pi(A, H_0, \theta))$

where $\pi(A, H_0)$ is the future earnings function of the individual.

And

b. State of the world is known, $\theta = \theta_0$

where θ_0 is chosen so as to equate the marginal productivity of investment in human capital under certainty with the expected marginal productivity under uncertainty.

$$E(\pi(A, H_0, \theta)) = \pi(A, H_0, \theta_0)$$

Next, we consider the effect of the presence of an insurance market. This is discussed as follows.

Proposition 1 *If a household has access to competitive insurance markets where they can contract at rate $\Delta = \theta - 1$ per unit of human capital, returns to human capital are continuous random variables, savings are interior, and human capital investment will be optimal.*

Proof. First, we show that expected profit of insurance company is zero. And second, we derive the condition that given the availability of an insurance contract, a household will maximize subject to the same constraints as it would were there no risk. The profit Π of the insurance company is defined as:

$$\prod = \Delta\pi(A, H_0)$$

Taking expectations,

$$E(\Pi) = E(\Delta)\pi(A, H_0) = 0$$

as $E(\Delta) = E(\theta) - 1 = 1 - 1 = 0$ and $\pi(A, H_0)$ is predetermined.

Next, we show that with the provision of insurance, the constraint in the household optimization problem is reduced to the one without risk.

Without insurance, the second period consumption is

$$C_2 = \pi(A, H_0)\theta + (Y + W(1 - A) - C_1) \cdot (1 + r) - \Delta\pi(A, H_0)$$

With insurance, the second period consumption is

$$\begin{aligned} C_2 &= \pi(A, H_0)\theta + (Y + W(1 - A) - C_1) \cdot (1 + r) - \Delta\pi(A, H_0) \\ &= \pi(A, H_0) + (Y + W(1 - A) - C_1) \cdot (1 + r) \end{aligned}$$

which is the consumption without risk. Thus, the human capital investment decisions will be optimal when insurance against risk is available.

We will now see that in the presence of uninsured risk, a household's decision to invest in a risky asset will be inefficient. The household investment will be at sub-optimal levels.

We assume for simplicity that initial childhood cognitive ability is zero and that household investment in child health is zero. Therefore, the household's decision problem is reduced to investment in the financial assets (safe) and investment in the child's education (risky). Under these assumptions, the arguments of our second period income-generating function are H_0 and A. The maximization problem is as follows:

$$\mathrm{Max}_{C_1,A}\ U(C_1) + \beta EV\ \left((Y + W(1-A) - C_1)\cdot(1+r) + \pi(A, H_0)\tilde{\theta}\right)$$

The first-order conditions are

$$U'(C_1) - \beta EV'(C_2)\cdot(1+r) = 0 \tag{1}$$

$$-W\beta(1+r)EV'(C_2) + \beta\pi_A EV'\left(C_2\tilde{\theta}\right) = 0 \tag{2}$$

From Eq. (1), we can infer that no gain can be achieved by transferring consumption from period to period. Also, from Eq. (2), we can infer that at the optimum, the expected marginal (in utility terms) rates of return from physical capital and education must be equal. We assume that an interior solution such that $0 < A < 1$ is attained.

Next, let us rewrite Eq. (2). This is as follows:

$$E(V'(\pi_A\theta - W(1+r)) = 0$$

This plus the definition of covariance yields

$$\mathrm{Cov}(\nu', \pi_A\theta - W(1+r)) + E(\nu')\cdot E(\pi_A\theta - W(1+r)) = 0$$

We can rearrange terms in the above equation to give us the following expression:

$$E(\pi_A\theta) = W(1+r) - \frac{\mathrm{Cov}\left(\nu', \pi_A\theta\right)}{EV'}$$

where

$$\mathrm{Cov}(\nu'; \pi_A\theta) < 0$$

This implies that the household invests in the child's education such that the expected return on that education is pushed above the return on physical capital. In other words, when the expected marginal rate of return from education exceeds that of physical capital, it is indicated that the individual views human capital as more risky on the margin. That is, a marginal increase in the level of investment in human capital increases the variance of future earnings and consumption.

Let us compare the above first-order conditions with those that would be obtained if the consumer faced no uncertainty with regard to an investment in education. When θ is non-stochastic or there is no risk, the appropriate first-order conditions are the following:

$$U'(C_1) - \beta(1+r)EV'(C_2) = 0$$

$$W(1+r) = \pi_A$$

Comparison of the above first-order condition with that under the uncertainty version of the model reveals that the latter differs from the former only through the presence of the covariance term $\mathrm{Cov}(\nu'; \pi_A\theta)$. It is interesting to note that the presence of this term in the first-order condition means that the educational investment in period 1 will depend inter alia on the initial health endowment, H_0. This is because $V'(.)$ is a function of H_0, and therefore, the covariance between V' and θ will also be a function of H_0.

Furthermore, by our assumptions that the market rate of interest is constant and investment in the financial market is risk free, we can imagine the household as an investor who makes a portfolio choice between investment in risk-free savings and risky education. This model is a variant of Arrow's (1965) model of portfolio selection.

To see this, rewrite the first-order Eq. (2) as the following:

$$E(V'(C_2)\cdot(\pi_A\theta - W(1+r))) = 0 \tag{3}$$

Let

$$\pi_A\theta - W(1+r) = G(\theta) \tag{4}$$

Combining (3) and (4), we get

$$E\,(V'(C_2)\cdot G(\theta)) = 0 \tag{5}$$

Looking at Eq. (5) above, resemblance to Arrow's first-order condition is apparent.

Arrow's model can be summarized as follows. An investor faces a choice between a risky asset and a risk-free asset. The return to the risk-free asset is zero. The return to the risky asset is a random variable X, which assumes both positive and negative values. Let A and a be, respectively, the initial wealth and the amount of wealth invested in the risky asset. The investor maximizes the expected utility $E(U(Y))$ of stochastic income $Y = A + aX$ by choosing an optimal portfolio. The investment in the risky asset is positive if and only if $E(X) > 0$. The first-order condition for an interior solution is $E(U'(Y)X) = 0$, and the income elasticity of the risky asset has the same sign as $E(U''(Y)X)$. The celebrated result is that the sign of $E(U''(Y)X)$ is positive and, therefore, the demand for the risky asset is a normal good, if the utility function $U(Y)$ exhibits decreasing absolute risk aversion. In proposition 1 below, we will assume decreasing absolute risk averse preferences and prove that an increase in childhood health increases the household investment in child's education. This result is an income effect coming from the fact that an increase in childhood health endowment implies some increase in expected future income. There is, however, a fundamental difference between this model and that of Arrow's. In Arrow's model, the distribution of the rate of return is independent of the amount invested. He called such an assumption stochastic constant returns to scale. In this model, there are two random variables: θ, the source of uncertainty, and $G(\theta)$, the rate of return to the risky asset. The assumption of stochastic constant returns to scale does not apply to the rate of return $G(\theta)$.

Proposition 2 *Under the assumption of decreasing absolute risk averse preferences, an increase in childhood health endowment will encourage investment in education when there is risk associated with the second period income-generating function.*

Proof. In the Appendix.

Next, we are interested in capturing the effect of childhood endowment of cognitive ability on a household's decision to invest in the child's health. For simplicity, we assume that $H_0 = 0$; $A = 0$ instead of assuming $D_0 = 0$; $M = 0$ (as we did for deriving the proposition 1). This implies that the arguments of our period 2 income-generating function are D_0 and M. The maximization problem is as follows:

$$\text{Max}_{C_1,M}\ U(C_1) + \beta EV\left((Y + W(1-M) - C_1)\cdot(1+r) + \pi(M, D_0)\tilde{\theta}\right)$$

We can prove the following proposition.

Proposition 3 *Under the assumption of decreasing absolute risk averse preferences, an increase in cognitive ability will encourage investment in health when there is risk associated with the second period income-generating function.*

Proof. In the Appendix.

In presenting our model, we have focused on the important cross-effects, namely, the effect of childhood health endowment on investment in education (proposition 2) and the effect of childhood cognitive ability on investment in health (proposition 3).

A crucial assumption that we make in proving the propositions 1 and 2 is that the rate of return on financial assets is constant. When the rate of return is variable, then an increase in rate of return could invert sign of the relation between investment in education and the initial health endowment.

4 Alternative technological assumptions

In this section, we will first follow Edwards and Grossman (1977) and develop a specification to capture the interdependence between health and cognitive development. Let us assume that the inputs into production of second period stock of education are the investment in education in the first period and the initial or childhood health endowment. Also, assume that the inputs into production of second period stock of health are the investment in health in the first period and the initial or childhood cognitive ability endowment. Clearly, this is a more general form of the production functions for health and education and allows for the interplay between health and education. It also allows for the possibility that a low childhood health endowment will adversely affect the cognitive development of the child.

Our context is as follows. The second period stock of health H is given by the initial childhood health endowment plus a production function for health that transforms the inputs into the child's health into the second period stock of health. The health production function is $I(M, H_0, D_0)$. Similarly, the second period stock of cognitive development D is given by the initial childhood cognitive ability plus a production function for education that transforms the inputs into the child's education into the second period stock of education. The education production function is $Q(A, H_0, D_0)$. We assume that $Q_{H_0} > 0$ and $I_{D_0} > 0$. These assumptions are akin to Cunha et al.'s (2005) assumptions of universal self-productivity or recursive productivity. The assumption $Q_{H_0} > 0$ implies that the level of initial childhood health has a positive effect on the production of the second period stock of cognitive development. We can think of health as enabling the formation of child quality in the early years and throughout youth, bringing the efficiency of education to a viable level, by raising skilled and unskilled labor efficiency and through longevity, itself influenced by early health, by lengthening the time during which education will yield a return. The assumption $I_{D_0} > 0$ implies that the level of initial cognitive ability has a positive effect on the production of the second period stock of health. We can think of this in terms of the efficiency effect discussed by Grossman (1999). The efficiency effect can take two forms: productive efficiency and allocative efficiency. Productive efficiency pertains to a situation in which the more educated obtain a larger health output from given amounts of endogenous (choice) inputs. Allocative efficiency pertains to a situation in which schooling increases information about the true effects of the input on health (Kenkel 2000). Allocative efficiency will improve health to the extent that it leads to the selection of a better input mix. Furthermore, our formulation is sufficiently general to allow cross-effects of initial endowments of health or schooling ability on investments into schooling and health, respectively. This is akin to the concept of universal direct complementarity of investments (Cunha et al. 2005) and implies that $\frac{\partial^2 Q}{\partial A \partial H_0} > 0$. This means that higher levels of childhood health increase the productivity of investment in education. The model now becomes

$$U(C_1, C_2) = U(C_1) + \beta EV(C_2)$$

where

$$C_2 = (Y + W \cdot (1\text{–}A\text{–}M)C_1) \cdot (1 + r) + \pi(D, H, \theta)$$

$$H = H_0 + I(M, H_0, D_0)$$

$$D = D_0 + Q(A, H_0, D_0)$$

where H denotes the stock of health in the second period. H_0 is the endowment of health that the child is born with in period 1. I is the technology that converts the investment in child health in the first period into net additions to health stock over the two period time framework. Similarly, D_0 is the endowment of cognitive ability that the child is born with in period 1. Q is the technology that converts the investment in child schooling in the first period into net additions to education stock over the two time periods.

This specification allows for interplay between health and development, and in particular, it allows for the possibility that low initial health levels will affect the realized level of cognitive development. We will analyze the effect of initial health endowment on subsequent cognitive development. Note that the initial stocks of health and development (H_0, D_0) are included in the production functions of both I and G. While no assumption is made at this time with respect to the directional effects of these initial stocks on I and G, this flexible specification allows for a number of possibilities. For example, the effect of medical care inputs on changes in health may be greater when an individual's stock of health is at a lower level (i.e., $\partial I/\partial H_0 < 0$). Or, children with greater inherited intellectual ability may augment that ability more easily (i.e., $\partial Q/\partial D_0 < 0$). Finally, we reiterate our assumption that risk in the second period income-generating function, $\pi(A, H, \theta)$, is multiplicative in nature. The assumption of multiplicative risk can be interpreted as capturing the effect of risk on the returns to (rather than stocks of) health and educational investments incorporated in the income-generating function.

In this context, under our assumption of perfect capital markets, we can imagine the household as an investor allocating his portfolio between health, education, and savings. The household maximization problem in this case becomes

$$\text{Max}_{C_1, A, M}\, U(C_1) + \beta EV(C_2)$$

where

$$C_2 = (Y + W \cdot (1 - A - M) - C_1) \cdot (1 + r) + \pi(D, H, \theta)$$

$$H = H_0 + I(M, H_0, D_0)$$

$$D = D_0 + Q(A, H_0, D_0)$$

The corresponding first-order conditions (FOC) are as follows:

$$U'(C_1) - \beta EV'(.) \cdot (1 + r) = 0$$

$$-\beta \cdot (1 + r) \cdot EV'(.) + \beta E(V'\theta) \cdot \pi_D Q_A = 0$$

$$-\beta \cdot (1 + r) \cdot EV'(.) + \beta E(V'\theta) \cdot \pi_H I_M = 0$$

Looking at the first FOC, we can infer that no gain can be achieved by transferring consumption from period to period. The second FOC means that at the optimum, the expected marginal (in utility terms) rates of return from physical capital and education must be equal. The third FOC means that at the optimum, the expected marginal (in utility terms) rates of return from physical capital and health must be equal. We assume that interior solutions such that $0 < A < 1$ and $0 < M < 1$ are attained. We can also show that the following proposition will hold for the above context.

Proposition 4 *If education and health are jointly determined inputs into the income-generating function in the second period, then there will be a positive correlation between initial health endowment and educational investment even in the absence of risk. Assuming decreasing absolute risk averse preferences, this positive correlation will be exacerbated in the presence of risk.*

Proof. Similar to earlier proofs for propositions 2 and 3.

In the above specification, we allowed for interdependence between health and education investments. Alternatively, let us assume that health and education investments are substitutes in the income-generating function. Such a specification, in general terms, would imply a functional form such as the following:

$$\pi(A + \gamma M)$$

In the above specification, health and education are assumed to be substitutes in the income-generating function subject to a scaling factor given by γ where $0 < \gamma < 1$.

Now, consider the maximization problem with health and education as substitutes in the income-generating function. Let us assume the maximization problem is

$$\text{Max}_{C_1, A}\, U(C_1) + \beta EV\left((Y + W \cdot (1 - A) - C_1) \cdot (1 + r) + \pi(f(A) + H_0\theta)\right)$$

The first-order conditions are as follows:

$$U'(C_1) - \beta EV'(C_2) \cdot (1 + r) = 0$$

$$-W\beta \cdot (1 + r) \cdot EV'(C_2) + \beta \pi_f f_A EV'(C_2) = 0$$

Rewriting the second of the two first-order conditions above, we get

$$E(\pi_A f_A) = W(1 + r) - \frac{Cov(v', \pi_f f_A)}{EV'}$$

where

$$\text{Cov}(v', \pi_f f_A) > 0$$

The covariance term in the above equation is positive, so that risk averse individuals push the expected rate of return below the safe rate, increasing their investment in education. This positive relation makes educational investment relatively more valuable than saving, which has a safe but uncorrelated return. The results above plus an assumption of declining absolute risk aversion lead us to the result that $\frac{dA}{dH_0} < 0$. This is proven by means of the proposition below.

Proposition 5 *If education and health enter as additive components of the income-generating function in the second period, then risk associated with the exogenous health endowment will cause the level of investment in education to decline with an increase in health endowment.*

Proof. In the Appendix. Similar to earlier proofs for propositions 2 and 3.

Given this formulation of uncertainty, those born with a low level of childhood health endowment will invest more in their education than those born with a high level of childhood health endowment (assuming similar initial levels of schooling ability). In other words, it is risky for those poor in health in this framework not to invest on education.

Let us look at an alternative specification for the second period income-generating function. The maximization problem is denoted as follows:

$$\text{Max}_{C_1,\, A}\ U(C_1) + \beta EV((Y + W \cdot (1 - A) - C_1) \cdot (1 + r) + \pi(f(A) + \pi(H_0)\theta))$$

The first-order conditions are as follows:

$$U'(C_1) - \beta EV'(C_2) \cdot (1 + r) = 0$$

$$-W\beta \cdot (1 + r) \cdot EV'(C_2) + \beta f_A EV'(C_2) = 0$$

Rewriting the second of the two first-order conditions above, we get

$$E(f_A) = W\,(1 + r) - \frac{\text{Cov}(v', f_A)}{EV'}$$

where

$$\text{Cov}(v', f') = 0$$

In this framework, the level of risk affects the household consumption decisions but household production decisions are left unaffected. The optimal level of educational investment will be given by equating the returns to education to the alternative cost of educational investment. The optimal level of education is not a function of the level of initial health endowment. This is the special case when $f_A\theta = 0$ uniformly or equivalently $f(A, \theta) = f(A) + \theta$

It is immediately seen from the first-order condition that

$\frac{f'}{W} = 1 + r$, for all states of the world.

For this specification, we can prove the following proposition.

Proposition 6 *If the returns to education are certain and separable from the returns to health, then the optimal level of investment in education will not vary with the level of exogenous health endowment, when there is risk associated with the stock of health.*

Proof. On similar lines as the earlier proofs for propositions 2 and 3.

5 Imperfect capital markets

Suppose that the capital market is imperfect. The household faces liquidity constraints and can invest only in health and in education. The household decision problem is reduced to that of investing in health or education rather than the earlier specification of investment in three assets, namely, savings, health, and education. The following proposition can be proven.

Proposition 7 *If the capital market is imperfect so that a household either invests in schooling or health but does not invest in financial assets, then a risk associated with the second period income-generating function will cause the separability assumption to fail and children born with a high health endowment will receive more investment in their health as well as education.*

Proof. In the Appendix.

6 Conclusions

We have seen above that uninsured risk will lead to inefficient household decisions. Specifically, there will be a positive correlation between childhood health endowment and household investment in a child's education in the presence of uninsured risk. In so far as this contributes to lower educational attainment and thus to lower future productivity, this self insurance strategy may perpetuate poverty and underdevelopment. The policy implications of the above analysis therefore point to the important role for development of insurance markets in developing countries. Risk reduction measures such as provision of health services, micro-insurance, and microfinance should be considered to reduce household vulnerability and help them cope better with the effects of shocks.

Appendix

6.1 Proof of proposition 2

$$\mathrm{Max}_{C_1,\,A}\ U(C_1)\ +\ \beta EV\ ((Y\ +W(1–A)–C_1)\cdot(1\ +\ r)\ +\ \pi(A,H_0)\theta)$$

The first-order conditions (FOCs) are as follows:

$$U'\ (C_1)–\beta(1\ +\ r)EV'(.)\ =\ 0$$

$$\beta EV'(.)\cdot(-W\ (1\ +\ r)+\ \pi_A\theta)\ =\ 0$$

Totally differentiating the above FOCs with respect to C_1, A, and H_0, we get

$$\begin{aligned}&\left(U''(C_1)+\beta(1+r)^2EV''(.)\right)\cdot dC_1\\&\quad+\left(\beta(1+r)^2W\cdot EV''(.)–\beta(1+r)\pi_AEV''(\theta)\right)\cdot dA+(-\beta(1\ +\ r)\pi_{H_0}EV''(\theta))\\&\quad\cdot dH_0\\&\quad=\ 0\end{aligned}$$

$$\begin{aligned}&(-\beta EV''(.)\cdot(1\ +\ r)\cdot(-W(1\ +\ r)+\ \pi_A\theta))\cdot dC_1\\&\quad+\ \left(\beta EV''(.)\cdot(-W(1\ +\ r)+\pi_A\theta)^2+\beta E(V'\theta)\pi_{AA}\right)\cdot dA\\&\quad+(\beta EV''(.)\cdot(-W(1+r)+\pi_A\theta)\cdot\pi_{H_0}\theta+\beta EV'(\theta)\pi_{AH_0})\cdot dH_0\\&\quad=\ 0\end{aligned}$$

We now make two alternative assumptions for the sign of π_{AH_0}. First (case A), we assume that $\pi_{AH_0}=0$, and second (case B), we assume that $\pi_{AH_0}>0$.

Case A. $\pi_{AH_0}=0$

The total differential equations from above are

$$\left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \cdot dC_1 + \left(\beta(1+r)^2 W \cdot EV''(.) - \beta(1+r)\pi_A EV''(\theta)\right) \cdot dA = (\beta(1+r)\pi_{H_0} EV''(\theta)) \cdot dH_0$$

$$(-\beta EV''(.) \cdot (1+r) \cdot (-W(1+r) + \pi_A \theta)) \cdot dC_1 + \left(\beta EV''(.) \cdot (-W(1+r) + \pi_A \theta)^2 + \beta E(V'\theta)\pi_{AA}\right) \cdot dA = (\beta E(V''\theta) \cdot \pi_{H_0} \cdot (-W(1+r) + \pi_A \theta)) \cdot dH_0$$

These can be expressed in matrix notation as follows:

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \cdot \begin{bmatrix} dC_1 \\ dA \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix} \cdot dH_0$$

where

a is $U''(C_1) + \beta(1+r)^2 EV''(.)$
b is $\beta(1+r)^2 W \cdot EV''(.) - \beta(1+r)\pi_A EV''(\theta)$
c is $\beta(1+r)\pi_{H_0} EV''(\theta)$
d is $-\beta EV''(.) \cdot (1+r) \cdot (-W(1+r) + \pi_A \theta)$
e is $\beta EV''(.) \cdot (-W(1+r) + \pi_A \theta)^2 + \beta E(V'\theta)\pi_{AA}$
f is $\beta E(V''\theta) \cdot \pi_{H_0} \cdot (-W(1+r) + \pi_A \theta)$
In other words,

$$\begin{bmatrix} dC_1 \\ dA \end{bmatrix} = \frac{1}{|H|} \cdot \begin{bmatrix} e & -b \\ -d & a \end{bmatrix} \cdot \begin{bmatrix} c \\ f \end{bmatrix} \cdot dH_0$$

where $|H| = ae - bd > 0$. This follows from the second-order conditions.
From the above, we have that

$$\text{sign}\ \frac{dA}{dH_0} = \text{sign}\ -dc + af$$

$$= \beta EV''(.) \cdot (1+r) \cdot (-W(1+r) + \pi_A \theta) \cdot \beta(1+r)\pi_{H_0} E(V''\theta) + \left(U''(C_1) + \beta(1+r)^2 \cdot EV''(.) \cdot \beta EV''(\theta) \cdot \pi_{H_0} \cdot (-\pi_A \theta + W(1+r)\right)$$

$$= \beta^2 EV''(.) \cdot (1+r)^2 \cdot E(V''\theta) \cdot \pi_{H_0} \cdot (-W(1+r) + \pi_A \theta) + U''(C_1) \cdot \beta E(V''(\theta)) \cdot \pi_{H_0} \cdot \left(-\pi_A \theta + W(1+r) + \beta(1+r)^2 \cdot EV''(.) \cdot (\beta E(V''\theta) \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A \theta)\right)$$

$$= U''(C_1) \cdot \beta E(V''(\theta)) \cdot \pi_{H_0} \cdot (-\pi_A \theta + W(1+r))$$

The sign of the term $E(V''\theta) \cdot (W(1+r) - \pi_A \theta)$ is derived as follows.
Using Arrow's definition of absolute risk aversion (R):

$$R = \frac{-V''}{V'}$$

with $R' < 0$ for all $C_2 \geq 0$ (via the assumption of declining absolute risk aversion).
Define $\tilde{C}_2$ to be the level of second period consumption that occurs if $\tilde{\theta}$ solves $\pi_A EV'\left(C_2\, \tilde{\theta}\right) = W(1+r)EV'(C_2)$, and define $\tilde{R}$ to be the value of R at $\tilde{C}_2$.
Given these definitions and the monotonicity of R and the concavity of $\pi(.)$ implies

$$R > \left(< \tilde{R}\right) \Leftrightarrow (W(1+r) - \pi_A \theta) > (< 0)$$

This implies the following:

$$\left(\tilde{R} - R\right) \cdot (W(1 + r) - \pi_A\theta) < 0$$

$$V''(W(1 + r) - \pi_A\theta) < -V' \tilde{R} (W (1 + r) - \pi_A\theta)$$

Taking expectations,

$$EV''(W(1 + r) - \pi_A\theta) < -\tilde{R}EV'(W (1 + r) - \pi_A\theta)$$

Right hand side of the above inequality is zero from the first-order conditions. Thus,

$$EV''(W(1 + r) - \pi_A\theta) < 0$$

This together with our equation derived before for

$$\text{sign}\frac{(dA)}{(dH_0) = \text{sign}-dc + af = \text{sign}\beta U\}(C_1) \cdot E(V\}\theta) \cdot \pi_{(H_0)}} \cdot (-\pi_A\theta + W(1+r))$$

gives us that

$$\frac{dA}{dH_0} > 0$$

Case B. If $\pi_{AH_0} > 0$

Again, rewriting the total differential equations derived previously, we have that

$$\begin{aligned}&\left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \cdot dC_1 + \left(\beta(1+r)^2 WEV''(.) - \beta(1+r)\pi_A EV''(\theta)\right)\\&\quad \cdot dA + (-\beta(1 + r)\pi_{H_0} EV''(\theta)) \cdot dH_0\\&\quad = 0\end{aligned}$$

$$\begin{aligned}&(-\beta EV''(.) \cdot (1 + r) \cdot (-W(1 + r) + \pi_A\theta)) \cdot dC_1\\&\quad + \left(\beta EV''(.) \cdot (-W(1 + r) + \pi_A\theta)^2 + \beta E(V'\theta)\pi_{AA}\right) \cdot dA\\&\quad + (\beta EV''(.) \cdot \pi_{H_0} \cdot (-W(1+r) + \pi_A\theta) + \beta E(V'\theta)\pi_{AH_0}) \cdot dH_0\\&\quad = 0\end{aligned}$$

Rearranging terms in the above equations, we get that

$$\begin{aligned}&\left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \cdot dC_1 + \left(\beta(1+r)^2 WEV''(.) - \beta(1+r)\pi_A EV''(\theta)\right)\\&\quad \cdot dA\\&\quad = \beta(1 + r)\pi_{H_0} EV''(\theta) \cdot dH_0\end{aligned}$$

$$\begin{aligned}&(-\beta EV''(.).(1 + r).(-W(1 + r) + \pi_A\theta)) \cdot dC_1 + \beta EV''(.)\\&\quad \cdot (-W(1 + r) + \pi_A\theta)^2 + \beta E(V'\theta)\pi_{AA}) \cdot dA\\&\quad = (\beta EV''(.) \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A\theta) - \beta E(V'\theta)\pi_{AH_0}) \cdot dH_0\end{aligned}$$

Expressing the above equations in matrix notation, we get that

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \cdot \begin{bmatrix} dC_1 \\ dA \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix} \cdot dH_0$$

where

a is $U''(C_1) + \beta(1+r)^2 EV''(.)$

b is $\beta(1+r)^2 WEV''(.) - \beta(1+r)\pi_A EV''(\theta)$

c is $\beta(1 + r)\pi_{H_0} EV''(\theta)$

d is $-\beta EV''(.) \cdot (1 + r) \cdot (-W(1 + r) + \pi_A\theta)$

e is $\beta EV''(.) \cdot (-W(1 + r) + \pi_A\theta)^2 + \beta E(V'\theta)\pi_{AA}$

f is $\beta EV''(.) \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A\theta) - \beta E(V'\theta)\pi_{AH_0}$

This can be rewritten as follows:

$$\begin{bmatrix} dC_1 \\ dA \end{bmatrix} = \frac{1}{|H|} \cdot \begin{bmatrix} e & -b \\ -d & a \end{bmatrix} \cdot \begin{bmatrix} c \\ f \end{bmatrix} \cdot dH_0$$

where $|H| = ae - bd > 0$ from the second-order conditions.

Solving the above for the sign of $\frac{dA}{dH_0}$, we get that

$$\begin{aligned} \text{sign } \frac{dA}{dH_0} &= \text{ sign} - dc + af \\ &= (\beta EV''(.) \cdot (1+r) \cdot (-W(1+r) + \pi_A\theta)) \\ &\quad \cdot (\beta E(V''\theta) \cdot \pi_{H_0} \cdot (1+r)) + \left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \\ &\quad \cdot (\beta E(V''\theta) \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A\theta) - \beta E(V'\theta)\pi_{AH_0}) \end{aligned}$$

$$= U''(C_1) \cdot \beta \cdot E(V''\theta) \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A\theta) - \left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \cdot \beta E(V'\theta) \cdot \pi_{AH_0}$$

The sign of the second term on the RHS of the equation above is negative from our assumption of $\pi_{AH_0} > 0$. Thus, even in the absence of risk, $\frac{dA}{dH_0} > 0$. In other words, in the absence of risk, the first term on the RHS will be zero but the second term on RHS will be positive. Thus, there will be a positive correlation between health and educational investment. When there is risk, the first term on the RHS will be positive and the correlation between education and health endowment will be exacerbated. This is proven below.

From Arrow's definition of absolute risk aversion,

$$R = \frac{-V'}{V'}$$

with $R' < 0$ for all $C_2 \geq 0$ (via the assumption of declining absolute risk aversion).

Also, define $\tilde{C}_2$ to be the level of second period consumption that occurs if $\tilde{\theta}$ solves $\pi_A EV'\left(C_2\tilde{\theta}\right) = \mathrm{W}(1+\mathrm{r})EV'(C_2)$, and define $\tilde{R}$ to be the value of R at $\tilde{C}_2$. Given these definitions and the monotonicity of R and the concavity of $\pi(.)$ implies

$$R > \left(< \tilde{R}\right) \Leftrightarrow (W(1+r) - \pi_A\theta) > (< 0)$$

This implies the following:

$$\left(\tilde{R} - R\right) \cdot (W(1+r) - \pi_A\theta) < 0$$

$$V\}(W(1+r) - \pi_A\theta) < -V'\tilde{R}(W(1+r) - \pi_A\theta)$$

Taking expectations,

$$EV\}(W(1+r) - \pi_A\theta) < -\tilde{R}EV'(W(1+r) - \pi_A\theta)$$

Right hand side of the above inequality is zero from the first-order conditions. Thus,

$$EV\}(W(1+r) - \pi_A\theta) < 0$$

This together with our equation derived before gives us that

$$\text{sign } \beta U''(C_1) \cdot \beta \cdot EV'' \cdot \pi_{H_0} \cdot (W(1+r) - \pi_A\theta) > 0$$

which implies that

$$\frac{dA}{dH_0} > 0$$

6.2 Proof of proposition 3

$$\text{Max}_{C_1, M}\; U\;(C_1) + \;\beta EV\;((Y\; + W(1\text{–}A)\text{–}C_1)\cdot(1\; + \;r)\; + \;\pi(M, D_0)\theta)$$

The first-order conditions (FOCs) are as follows:

$$U'(C_1)\;\text{–}\beta(1\; + \;r)EV'(C_2)\; = \;0$$

$$\beta EV'(C_2)\cdot(-W(1\; + \;r)\; + \;\pi_M\theta)\; = \;0$$

Totally differentiating the above FOCs with respect to C_1, M, and D_0, we get

$$\begin{aligned}&\left(U''(C_1) + \beta(1+r)^2EV''(C_2)\right)\cdot dC_1\\ &\quad + (-\beta(1+r)\pi_A EV''(C_2)\cdot(-W(1+r)+\pi_M\theta)\cdot dM\\ &\quad + (-\beta(1\; + \;r)EV''(C_2).\pi_{D_0}\theta)\cdot dD_0\\ &\quad = \;0\end{aligned}$$

$$\begin{aligned}&(-\beta EV''(C_2)\cdot(1\; + \;r)\cdot(-W(1\; + \;r) + \;\pi_M\theta))\cdot dC_1\\ &\quad + \;\left(\beta EV''(C_2)\cdot(-W(1\; + \;r)+\pi_M\theta)^2 + \beta E(V'C_2)\pi_{MM}\,\theta\right)\cdot dM\\ &\quad + \;(\beta EV''(C_2)\cdot(-W(1+r)+\pi_M\theta)\cdot\pi_{D_0}\theta + \beta EV'(C_2)\pi_{MD_0}\theta)\cdot dD_0\\ &\quad = 0\end{aligned}$$

For simplicity of analysis, assume that $\pi_{MD_0} = 0$. The total differential equations can thus be expressed as follows:

$$\begin{aligned}&\left(U''(C_1) + \beta(1+r)^2EV''(C_2)\right)\cdot dC_1\\ &\quad + (-\beta(1+r)\pi_A EV''(C_2)\cdot(-W(1+r)+\pi_M\theta)\cdot dM\\ &\quad = \;(\beta(1\; + \;r)EV''(C_2)\cdot\pi_{D_0}\theta)\cdot dD_0\end{aligned}$$

$$\begin{aligned}&(\beta EV''(C_2)\cdot(1\; + \;r)\cdot(-W(1\; + \;r) + \;\pi_M\theta))\cdot dC_1\\ &\quad + \;\left(\beta EV''(C_2)\cdot(-W(1\; + \;r)+\pi_M\theta)^2 + \beta E(V'\theta)\pi_{MM}\,\theta\right)\cdot dM\\ &\quad = \;(\beta EV''(C_2)\cdot(W(1+r)\text{–}\pi_M\theta)\cdot\pi_{D_0}\theta)\cdot dD_0\end{aligned}$$

Rewriting these equations in matrix form, we get the following:

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\cdot\begin{bmatrix} dC_1 \\ dM \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}\cdot dD_0$$

$$\begin{bmatrix} dC_1 \\ dM \end{bmatrix} = \frac{1}{|H|}\;\cdot\;\begin{bmatrix} e & -b \\ -d & a \end{bmatrix}\cdot\;\begin{bmatrix} c \\ f \end{bmatrix}\cdot dD_0$$

This implies that

$$\frac{dM}{dD_0} = \;\frac{1}{|H|}\;\cdot(-dc + af)$$

where $|H| = ae - bd > 0$ from the second-order conditions.

From the above analysis, we get that

$$\text{sign}\;\frac{dM}{dD_0} = \;\text{sign}\;(-dc + af)$$

where

$$-dc + af = (\beta EV''(C_2) \cdot (1 + r) \cdot (-W(1 + r) + \pi_M\theta)) \cdot (\beta(1+r)EV''(C_2)\pi_{D_0}\theta) +$$

$$\left(U''(C_1) + \beta(1+r)^2 EV''(C_2)\right) \cdot (\beta EV''(C_2)(-\pi_M\theta + W(1 + r))\pi_{D_0}\theta)$$

$$= U''(C_1)\beta EV''(C_2)(-\pi_M\theta + W(1 + r)\pi_{D_0}\theta$$

Sign of this term can be determined as follows. We use Arrow's definition of absolute risk aversion as noted in earlier proofs. Let the value of R which clears the first-order conditions under risk be denoted $\tilde{R}$. R is monotonically decreasing in consumption and the risk parameter θ. This together with the first-order conditions gives us

$$R > \left(< \tilde{R}\right) \Leftrightarrow (W(1 + r) - \pi_M\theta) > (< 0)$$

This implies the following:

$$\left(\tilde{R} - R\right) \cdot (W(1 + r) - \pi_M\theta) < 0$$

$$V\}(W(1 + r) - \pi_M\theta) < -V'\tilde{R}(W(1 + r) - \pi_M\theta)$$

Taking expectations,

$$EV\}(W(1 + r) - \pi_M\theta) < -\tilde{R}EV'(W(1 + r) - \pi_M\theta)$$

Right hand side of the above inequality is zero from the first-order conditions. Thus,

$$EV\}(W(1 + r) - \pi_M\theta) < 0$$

This together with our equation derived before for

$$\text{sign}\,\frac{dM}{dD_0} = \text{sign}\,(d - e) = \text{sign}\,(U''(C_1)\beta EV''\pi_{D_0}(-\pi_M\theta + W(1 + r))$$

which gives us the following:

$$\frac{dM}{dD_0} > 0$$

This result shows that those with higher cognitive ability will receive more investment in their health.

6.3 Proof of proposition 5

$$\text{Max}_{C_1, A}\, U(C_1) + \beta EV\,((Y + W(1-A) - C_1) \cdot (1 + r) + \pi(f(A) + H\theta))$$

The first-order conditions are the following:

$$U'(C_1) - \beta(1 + r)EV'(C_2) = 0$$

$$\beta EV'(.) \cdot (-W(1 + r) + \pi' f') = 0$$

Totally differentiating the above first-order conditions, we get the following:

$$\left(U''(C_1) + \beta(1+r)^2 EV''(.)\right) \cdot dC_1 + \left(-\beta(1+r)EV''(C_2) \cdot \left(-W(1+r) + \pi_f f_A\right)\right) \cdot dA + (-\beta(1 + r)EV''\theta.\pi_{H_0}) \cdot dH_0 = 0$$

$$\left(-\beta EV''(.)\cdot(1+r)\cdot\left(-W(1+r)+\pi_f f_A\right)\right)\cdot dC_1 + \left(\beta EV''(.)\cdot\left(-W(1+r)+\pi_f f_A\right)^2+\beta EV'(.)\pi_{ff}{f_A}^2\right) \cdot dA+\left(\beta EV''(.)\cdot\left(-W(1+r)+\pi_f f_A\right)\cdot\pi_{H_0}\theta\right)\cdot dH_0=0$$

Re-arranging in matrix form, we get the following:

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\cdot\begin{bmatrix} dC_1 \\ dA \end{bmatrix}=\begin{bmatrix} c \\ f \end{bmatrix}\cdot dH_0$$

where

a is $U''(C_1)+\beta(1+r)^2EV''(.)$
b is $-\beta(1+r)EV''(C_2)\cdot(-W(1+r)+\pi_{ff}A)$
c is $\beta(1+r)EV''\theta\cdot\pi_{H_0}$
d is $-\beta EV''(.)\cdot(1+r)\cdot(-W(1+r)+\pi_{ff}A)$
e is $(\beta EV''(.)\cdot(-W(1+r)+\pi_{ff}A)^2+\beta EV'(.)\pi_{ff}f_A^2)$
f is $\beta EV''(.)\cdot\left(-W(1+r)+\pi_f f_A\right)\cdot\pi_{H_0}\theta$

From the above, we get that

$$\frac{dA}{dH_0}=\frac{1}{|H|}\cdot(-dc+af)$$

where $|H|$ = ae − bd > 0 from the second-order conditions. Thus,

$$\text{sign}\,\frac{dA}{dH_0}=\text{sign}\,(-dc+af)$$

where

$$-dc+af=\left(\beta EV''(.)\cdot(1+r)\cdot\left(-W(1+r)+\pi_f f_A\right)\right)\cdot(\beta((1+r))EV''(\theta)\cdot\pi_{H_0}+\left(U''(C_1)+\beta(1+r)^2EV''(.)\right)\cdot\left(\beta EV''(.)\cdot\left(W(1+r)-\pi_f f_A\right)\cdot\pi_{H_0}\theta\right)$$

$$=U\}(C_1)\cdot\ \beta EV\}(.)\cdot\left(W(1+r)-\pi_f f_A\right)\cdot\pi_{H_0}\theta$$

Let the value of R which clears the first-order conditions under risk be denoted $\tilde{R}$. R is monotonically decreasing in consumption and the risk parameter θ. This together with the first-order conditions gives us

$$R>(<\tilde{R})\Leftrightarrow\left(W(1+r)-\pi_f f_A\right)<(>0)$$

This implies the following:

$$\left(\tilde{R}-R\right)\cdot\left(W(1+r)-\pi_f f_A\right)>0$$

$$V\}\left(W(1+r)-\pi_f f_A\right)-\pi_f f_A)>-\mathrm{R}V'\left(W(1+r)-\pi_f f_A\right)$$

Taking expectations;

$$EV\}\left(W(1+r)-\pi_f f_A\right)>-\tilde{R}EV'\left(W(1+r)-\pi_f f_A\right)$$

Right hand side of the above inequality is zero from the first-order conditions. Thus,

$$EV\}\left(W(1+r)-\pi_f f_A\right)>0$$

From the previous derivation, we know that

$$\text{sign}\ \frac{dA}{dH_0} = \text{sign}\ (-dc + af) = \text{sign}\ U''(C_1)\cdot\ \beta EV''(.)\cdot\left(W(1+r)-\pi_f f_A\right)\cdot\pi_{H_0}\theta$$

which implies that

$$\frac{dA}{dH_0} < 0$$

as from our assumption of diminishing marginal utility, $U''(C_1) < 0$

6.4 Proof of proposition 7

$$\text{Max}_{A,\ M}\ U(Y + W(1-A-M)) + \beta EV\ (\pi(A,H)\theta)$$

$$H\ =\ H_0\ +\ I(M,H_0)$$

The first-order conditions are given as follows:

$$-W\cdot U'(C_1)\ +\ \beta EV'(C_2\theta)\pi_A\ =\ 0$$

$$-W\cdot U'(C_1)\ +\ \beta EV'(C_2\theta)\pi_H\, I_M\ =\ 0$$

Totally differentiating the FOCs wrt A, M, and H_0, we get

$$\left[W^2\cdot U''(C_1)\ +\ \beta EV''(C_2\theta)\pi_A{}^2\right]\cdot dA + \left[W^2\cdot U''(C_1)\ +\ \beta EV''(C_2\theta)\pi_A\pi_H I_M\right]\cdot dM + \left[\beta EV''(C_2\theta)\pi_A\pi_H(1+I_{H_0})\right]\cdot dH_0 =\ 0$$

$$\left[W^2\cdot U''(C_1)\ +\ \beta EV''(C_2\theta)\pi_A\pi_H I_M\right]\cdot dA + \left[W^2\cdot U''(C_1)\ +\ \beta EV''(C_2\theta)\pi_H{}^2 I_M{}^2\right]\cdot dM + \left[\beta EV''(C_2\theta)\pi_H{}^2 I_M(1+I_{H_0})\right]\cdot dH_0 =\ 0$$

Rewriting the above equations in matrix notation, we get

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\cdot\begin{bmatrix} dA \\ dM \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}\cdot dH_0$$

Equating the first and second first-order conditions, we get the optimality condition that $\pi_A = \pi_H I_M$. Looking at the c and f terms above, we see that c is equal to f. Also,

$$\begin{bmatrix} dA \\ dM \end{bmatrix} = \frac{1}{H}\cdot\ \begin{bmatrix} e & -b \\ -d & a \end{bmatrix}\cdot\begin{bmatrix} c \\ f \end{bmatrix}\cdot dH_0$$

By the second-order conditions for a local optima, we get that $H > 0$

From Arrow's definition of absolute risk aversion,

$$R = \frac{-V\}}{V'}$$

with $R' < 0$ for all $C_2 \geq 0$ (via the assumption of declining absolute risk aversion).

Let the value of R which clears the first-order conditions under risk be denoted $\tilde{R}$. R is monotonically decreasing in consumption and the risk parameter θ. This together with the first-order conditions gives us

$$R\ >\ \left(<\ \tilde{R}\right)\Leftrightarrow(\pi_H I_M-\ \pi_A)\theta > (<\ 0)$$

implies

$$\left(\tilde{R}-R\right)\cdot(\pi_H I_M-\pi_A)\theta<0$$

$$V''(\pi_H I_M-\pi_A)\theta<-\tilde{R}V'(\pi_H I_M-\pi_A)\theta$$

Taking expectations,

$$EV\}((\pi_H I_M-\pi_A)\theta)>-\tilde{R}EV'((\pi_H I_M-\pi_A)\theta)$$

Right hand side of the above inequality is zero from the first-order conditions. Thus,

$$EV\}((\pi_H I_M-\pi_A)\theta)>0$$

From the previous derivation, we know that

$$\text{sign}\ \frac{dA}{dH_0}=\text{sign}\ c(e\text{–}b)$$

which implies that $\frac{dA}{dH_0}>0$. Similarly, $\frac{dM}{dH_0}>0$

hence proved.

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References

Alderman H, Behrman J, Lavy V, Menon R (2001) Child health and school enrollment: a longitudinal analysis. J Hum Resour 36:185–205

Arrow K (1965) The theory of risk aversion. In: Collected papers of Kenneth J (ed) Individual choice under certainty and uncertainty, vol. 3. The Belknap Press of Harvard University Press, Cambridge, 1984

Becker GS (1967). Human Capital and the Personal Distribution of Income: An Analytical Approach (University of Michigan, Ann Arbor, MI). Also available in: G.S. Becker (1993), Human Capital, 3rd edition (University of Chicago Press) 102–158

Becker GS (1974). A Theory of Social Interactions. Journal of Political Economy. Volume 82, No. 6, 1063–93

Chang F (1996) Uncertainty and investment in health. J Health Econ 15:369–376

Cropper ML (1977) Health, investment in health, and occupational choice. J Polit Econ 85(6):1273–1294

Cunha F, Heckman J, Lochner L, Masterov D (2005) Interpreting the evidence on life cycle skill formation., NBER Working Paper 11331

Currie J, Hyson R (1999) Is the impact of health shocks cushioned by socioeconomic status? The case of low birth weight. Am Econ Rev 89:245.50

Dardanoni V, Wagstaff A (1987) Uncertainty, inequalities in health and the demand for health. J Health Econ 6(1987): 283–290

Dardanoni V, Wagstaff A (1990) Uncertainty and the demand for medical care. Journal of Health Economics. Volume 9 (1), Pages 23–38, June

Edwards L, Grossman M (1977) An economic analysis of children's health and intellectual development., NBER Working Paper Series. Working Paper No. 180

Fuchs VR (1982) Time preference and health: an exploratory study. In: Fuchs VR (ed) Economic aspects of health. University of Chicago Press, Chicago, pp 93–120

Galor O, Mayer D (2002) Food for thought: basic needs and persistent educational inequality. PAHO, Washington DC, Paper prepared for the Pan American Health Organization, under Contract No. CS ASC-02/00010-0, 2002. Program on Public Policy and Health, Health and Human Development Division

Gan L, Gong G (2004) Mortality Risk and Educational Attainment of Black and White Men. National Bureau of Economic Research Working Paper No. 10381

Gan L, Gong G (2007) Estimating interdependence between health and education in a dynamic model., NBER Working Paper Series. Working Paper 12830

Grantham-McGregor S, Cheung VB, Cueto S, Glewwe P, Richter L, B. Strupp and the International Child Development Steering Group (2007) Child development in developing countries. Development potential in the first 5 years for children in developing countries. Lancet 369:60–70

Grossman M (1972a) The demand for health: a theoretical and empirical estimation. Columbia University Press for the National Bureau of Economic Research, New York

Grossman M (1972b) On the concept of health capital and the demand for health. J Polit Econ 80(2):223–255

Grossman M (1975) The correlation between health and schooling. In Terleckyj, Nestor E. (ed.), Household Production and Consumption. New York: Columbia University Press for the National Bureau of Economic Research
Grossman M, Kaestner R (1997) Effects of Education on Health. The Sociological Benefits of Education, ed. by J.R. Behrman and N. Stacey. University of Michigan Press, Ann Arbor, MI, pp. 69–124
Grossman M (1999) The Human Capital Model of the Demand for Health. National Bureau of Economic Research (NBER) Working Paper No. 7078. April
Jacoby H, Skoufias E (1997) Risk, financial markets, and human capital in a developing country. Rev Econ Stud 64(3): 311–335
Kenkel D (1991) Health behavior, health knowledge and schooling. J Polit Econ 99(2):287–305
Kenkel D (2000) Prevention. Handbook of Health Economics. Volume 1, Part B, Pages 1675–1720
Levhari D, Weiss Y (1974) The effect of risk on the investment in human capital. Am Econ Rev 64(6):950–963
Mayer-Foulkes D (2003) Market failures in health and education: investment for the young, Mexico
Muurinen JM (1982) Demand for health: a generalized Grossman model. J Health Econ 1(1):5–28
Perri T (1984) Health status and schooling decisions of young men. Econ Educ Rev 3(3):207–213
Rosenzweig MR, Schultz TP (1991) Who receives medical care? Income, implicit prices, and the distribution of medical services among pregnant women in the United States. J Hum Resour 26(3):473–508
Selden TM (1993) Uncertainty and health care spending by the poor: the health capital model revisited. J Health Econ 12:109–115

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