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A New Variant of Rama Distribution with Simulation Study and Application to Blood Cancer Data

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Abstract:

In this paper, we propose a new lifetime distribution with flexibility in modeling than its parent distribution. The new distribution is a variant of the Rama distribution having a positive shift parameter. We call the proposed distribution Shifted Rama (SR) distribution. Mathematical and statistical characteristics such as crude moments, central moment, coefficient of variation, index of dispersion, conditional moment, mean residual life function, mean deviation, Bonferroni and Lorenz curve, and the order statistics are derived. Furthermore, reliability measures like survival function, hazard function have been derived. Estimation techniques namely; the maximum likelihood, least squares, weighted least squares, maximum product spacing, Cramer-von-Mises, Anderson-Darling and the right-tailed Anderson-Darling estimations are used. To demonstrate the applicability of the distribution, a numerical example was the blood cancer data from Ministry Hospital in Saudi Arabia. Based on the results, the proposed distribution performed better than the competing distributions. Simulation of the Estimates of the parameters based on the classical methods considered are obtained, and result showed that the maximum likelihood estimator gave the best classical estimates of the parameters compared to other methods considered.

Keywords: *Rama Distribution, Shifted Rama Distribution, Shifted Exponential distribution, mean residual life function, conditional mean, order statistics, Bonferonni, Lorenz curve.*

Introduction

The inability of a particular probability distribution to fit all kinds of datasets has brought about the great need for the development or introduction of new distributions by researchers of different fields. Traditionally, a data could be transformed to follow a particular probability distribution of interest, but this act may destroy the originality of the data. Therefore, the need for a probability distribution that can best fit the data. This makes modelling of evolving life events easy for researchers.

Shanker (2017a) proposed a one parameter lifetime distribution named Rama distribution and its application. The distribution is based on a two-component mixture of an exponential distribution having scale parameter θ and a Gamma distribution having shape parameter 4 and scale parameter θ with their mixing proportion $\dfrac{\theta^3}{\theta^3+6}$. The Rama distribution was conducted with lifetime data from biomedical science and engineering. Other related distribution are as follows; Sujatha Distribution by Shanker, R. (2016a), Pranav distribution by

Shukla, K.K. (2018), Shukla distribution by Shukla and Shanker (2019), Ishita distribution by Shanker and Shukla (2017), Akash distribution by Shanker, R. (2015a), Rani Distribution by Shanker, R. (2017b), Chris-Jerry distribution by Onyekwere, C.K. and Obulezi, O.J. (2022), XGamma distribution by Sen, S., Maiti, S. and Chandra, N. (2016), Arhadana distribution by Shanker, R. (2016b), Shanker distribution by Shanker, R. (2015b). Each of the cited distributions above follows the pattern of Lindley distribution with each having a unique mixing proportion.

This literature will also consist of some extensions that has been done on the Rama distribution, the following are some authors and the distribution they proposed as an extension of the Rama distribution; Shanker, R., Abebe, B., Tesfay, M. & Eyob, T. (2019) proposed a two-parameter power Rama distribution with properties and Applications, Onyekwere, K.C., Osuji, G.A., & Enogwe, S.U. (2020) proposed an inverted power Rama Distribution capable of modelling real life data with upside down bathtub shape and heavy tails, Shanker, R. & Eyom, T. (2019) proposed a two-parameter weighted Rama Distribution with properties and application, Maryam, M. & Kannan, R. (2021) proposed a new generalization of Rama distribution with application to Machinery Data known as the Alpha Power Transformed Rama. Other related extension carried out on some of the stated related probability distribution are; the Inverted Power Ishita distribution by Frederick, A.O., Osuji, G.A. & Onyekwere, C.K. (2022), Marshal-Olkins Chris-Jerry distribution by Obulezi, O., Anabike, I., Oyo, O., Igbokwe, C. & Etaga, H. (2023), Inverse Hamza Distribution: properties and application to lifetime data by Frank, O.I., Obiora-Ilouno, H.O. & Frederick, O.A. (2023), Kumaraswamy Chris-Jerry by Obulezi, J.O., Anabike, I.C., Okoye, G.C., Igbokwe, C.P., Etaga, H.O. & Onyekwere, C.K. (2023) and so on.

Many probability distributions do not support values equal to or less than zero, mostly continuous probability distributions, unless one shifts the data by adding some constant, and then removing that shift from the fitted distribution. The application of the Rama distribution is basically on dataset with values greater than zero, and this has led to the development of its extension in this paper. Due to the above, we propose the Shifted Rama Distribution.

The rest of this paper is organized as follows; in section 2 & 3, we derive the Shifted Rama (SR) distribution and present the pdf, cdf, survival and hazard rate functions and their associated plots. In section 4, we derive some useful mathematical properties. In section 5, we obtained or estimated the parameters using some methods. In section 6, we apply the proposed distribution to real life data set on cancer and employ simulation study. In section 7, the article is concluded.

The Rama Distribution

Definition 1

Let $X \sim$ Rama (θ) due to (Shanker, 2017), hence the pdf and cdf of the Rama distribution is given below;

$$f(x) = \frac{\theta^4}{\theta^3+6}\left(1+x^3\right)e^{-\theta x}$$

x>0, θ>0 (1)

and

$$F(x) = 1-\left[1+\frac{\theta^3x^3+3\theta^2x^2+6\theta x}{\theta^3+6}\right]e^{-\theta x}$$

x>0, θ>0 (2)

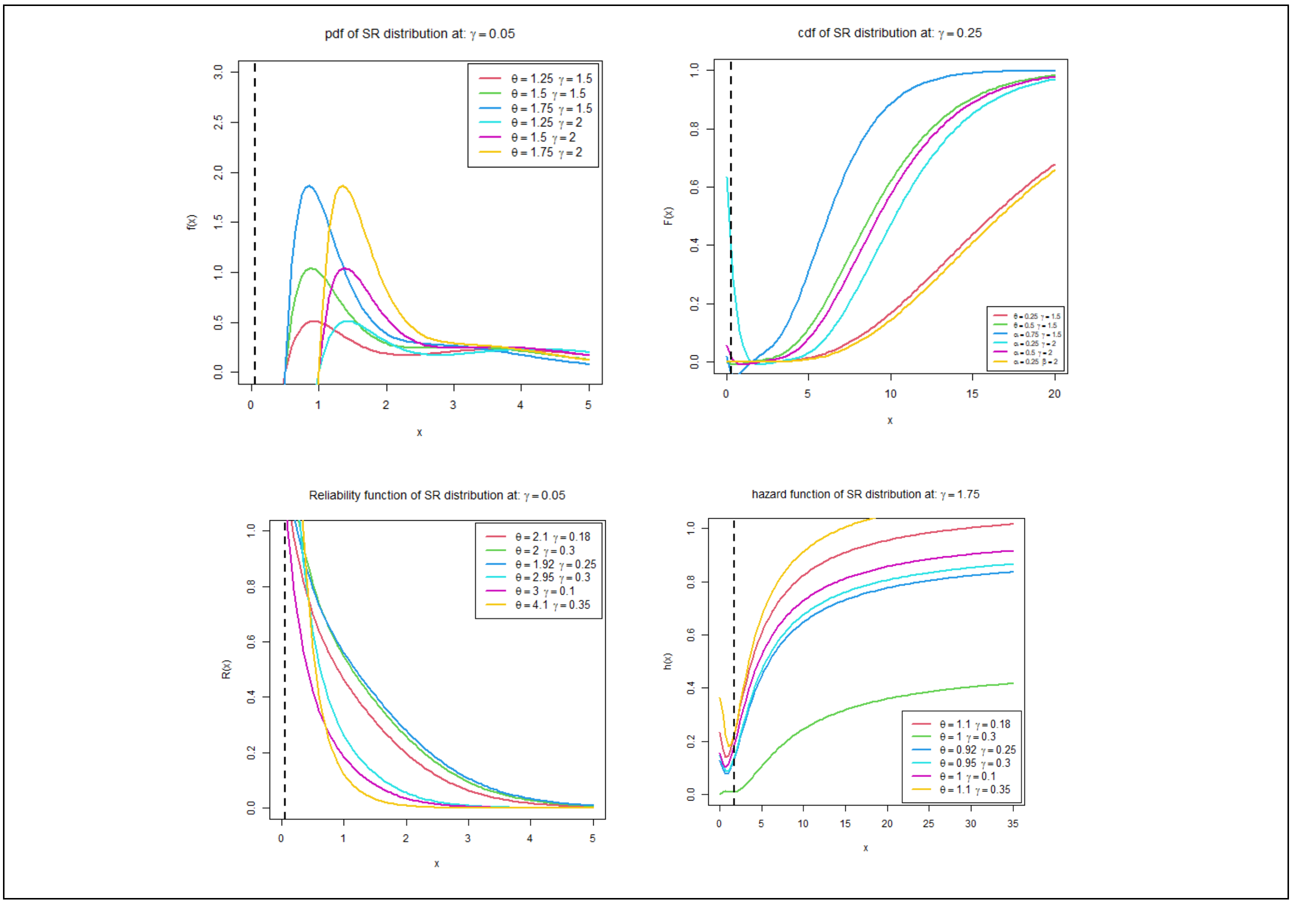

Figure 1. The Plots of the Probability Density Function, Cumulative Function, Reliability Function, and the Hazard Rate Function of the Shifted Rama Distribution

The Shifted Rama (SR) Distribution

Definition 2

Let $Y \sim$ shifted Rama (θ, γ), say X is a function of Y as stated in the mathematical expression below;

where $X = Y + \gamma$,

then $Y = X - \gamma$

Definition 3

The pdf and cdf of the Shifted Rama Distribution is given below as;

$$f_{SR}(x) = \frac{\theta^4}{\theta^3 + 6}\left(1 + (x-\gamma)^3\right)e^{-\theta(x-\gamma)}$$

$$x > \gamma,\ \theta > 0 \qquad (3)$$

$$F_{SR}(x) = 1 - \left[1 + \frac{\theta^3(x-\gamma)^3 + 3\theta^2(x-\gamma)^2 + 6\theta(x-\gamma)}{\theta^3 + 6}\right]e^{-\theta(x-\gamma)}$$

$$x > \gamma, \theta > 0 \qquad (4)$$

The Survival Rate and Hazard Rate Function, denoted by S(x) and h(x) respectively are given below;

$$S(x)=\left[1+\frac{\theta^3(x-\gamma)^3+3\theta^2(x-\gamma)^2+6\theta(x-\gamma)}{\theta^3+6}\right]e^{-\theta(x-\gamma)} \tag{5}$$

$$h(x)=\frac{\theta^4\left(1+(x-\gamma)^3\right)}{\theta^3+6+\theta^3(x-\gamma)^3+3\theta^2(x-\gamma)^2+6\theta(x-\gamma)} \tag{6}$$

Mathematical Properties of the Shifted Rama Distribution

In this section we derive some useful mathematical properties of the proposed Shifted Rama Distribution.

Definition 1

Let $X \sim SR(\theta,\gamma)$, the rth crude moment of a continuous probability distribution is obtained using the expression below;

$$E(X^r)=\int_0^\infty x^r f(x)dx \tag{7}$$

Inserting eqn. (3) into eqn. (7), we have

$$=\frac{\theta^4}{\theta^3+6}\int_\gamma^\infty x^r\left(1+(x-\gamma)^3\right)e^{-\theta(x-\gamma)}dx \tag{8}$$

Therefore, the rth crude moment of the Shifted Rama Distribution is given as;

$$E(X^r)=\sum_{k=0}^{r}\binom{r}{k}\gamma^{r-k}\theta^{-k}\frac{1}{(\theta^3+6)}\left(\theta^3\Gamma(k+1)+\Gamma(k+4)\right) \tag{9}$$

Definition 2

Let $X \sim SR(\theta,\gamma)$, the 1st, 2nd, 3rd, and 4th moment about the origin are given below,

$$\mu_1'=\frac{\gamma\theta(\theta^3+6)+(\theta^3+24)}{\theta(\theta^3+6)} \tag{10}$$

$$\mu_2'=\frac{\theta^2\gamma^2(\theta^3+6)+2\theta\gamma(\theta^3+24)+(2\theta^3+120)}{\theta^2(\theta^3+6)} \tag{11}$$

$$\mu_3'=\frac{\theta^3\gamma^3(\theta^3+6)+3\theta^2\gamma^2(\theta^3+24)+3\theta\gamma(2\theta^3+120)+(6\theta^3+720)}{\theta^3(\theta^3+6)} \tag{12}$$

$$\mu_4'=\frac{\theta^4\gamma^4(\theta^3+6)+4\theta^3\gamma^3(\theta^3+24)+6\theta^2\gamma^2(2\theta^3+120)+4\theta\gamma(6\theta^3+720)+(\theta^3+5040)}{\theta^4(\theta^3+6)} \tag{13}$$

Definition 3

Let $X \sim SR(\theta,\gamma)$, the 2nd central moment, known as the variance is obtained using the mathematical expression below,

$$E(X-E(X))^2 = E(X^2)-(E(X))^2 \tag{14}$$

Inserting eqn. (10) and eqn. (11) into eqn. (14) we have;

$$= \frac{\theta^2\gamma^2(\theta^3+6)+2\theta\gamma(\theta^3+24)+(2\theta^3+120)}{\theta^2(\theta^3+6)} - \left(\frac{\gamma\theta(\theta^3+6)+(\theta^3+24)}{\theta(\theta^3+6)}\right)^2 \tag{15}$$

Therefore, the variance of the Shifted Rama Distribution is given as;

$$\mu_2 = \frac{(\gamma^2-1)(\theta^8+12\theta^5+36\theta^2)+(\gamma-1)(2\theta^7+60\theta+288\theta)+144}{\theta^2(\theta^3+6)^2} \tag{16}$$

Definition 4

Let $X \sim SR(\theta,\gamma)$, the coefficient of variation is obtained using the mathematical expression below,

$$C.V = \frac{\sigma}{\mu_1^{'}} \tag{17}$$

Inserting eqn. (10) and the square root of eqn. (16) into eqn. (17) we have;

$$= \left(\frac{(\gamma^2-1)(\theta^8+12\theta^5+36\theta^2)+(\gamma-1)(2\theta^7+60\theta+288\theta)+144}{\theta^2(\theta^3+6)^2}\right)^{\frac{1}{2}} \div \frac{\gamma\theta(\theta^3+6)+(\theta^3+24)}{\theta(\theta^3+6)} \tag{18}$$

Therefore, the coefficient of variation of the Shifted Rama Distribution is given as;

$$C.V = \frac{((\gamma^2-1)(\theta^8+12\theta^5+36\theta^2)+(\gamma-1)(2\theta^7+60\theta+288\theta)+144)^{\frac{1}{2}}}{(\gamma\theta(\theta^3+6)+(\theta^3+24))} \tag{19}$$

Definition 5

Let $X \sim SR(\theta,\gamma)$, the index of dispersion is obtained using the mathematical expression below,

$$I.D = \frac{\sigma^2}{\mu_1^{'}} \tag{20}$$

Inserting eqn. (10) and eqn. (16) into eqn. (20) we have;

$$= \frac{(\gamma^2-1)(\theta^8+12\theta^5+36\theta^2)+(\gamma-1)(2\theta^7+60\theta+288\theta)+144}{\theta^2(\theta^3+6)^2} \div \frac{\gamma\theta(\theta^3+6)+(\theta^3+24)}{\theta(\theta^3+6)} \tag{21}$$

$$I.D = \frac{(\gamma^2-1)(\theta^8+12\theta^5+36\theta^2)+(\gamma-1)(2\theta^7+60\theta+288\theta)+144}{\theta(\theta^3+6)(\gamma\theta(\theta^3+6)+(\theta^3+24))} \tag{22}$$

Definition 6

Let $X \sim SR(\theta,\gamma)$, the conditional moment, which is useful in the derivation of the mean residual life function of a component as well as the mean deviation, is obtained using the mathematical expression below,

$$E(X^n/X>x) = \frac{1}{(1-F(x))} J_n(x) \tag{23}$$

where

$$J_n(x) = \int_x^\infty t^n f_{SR}(t) dt \quad (24)$$

Substituting the (3) into (24) above, we have

$$= \frac{\theta^4}{\theta^3 + 6} \int_{x+\gamma}^\infty t^n \left(1 + (t-\gamma)^3\right) e^{-\theta(t-\gamma)} dt \quad (25)$$

$$\text{Let } k = \theta(t-\gamma), t = \frac{k}{\theta} + \gamma, \frac{dt}{dk} = \frac{1}{\theta}$$

$$= \frac{\theta^4}{\theta^3 + 6} \int_{\theta x}^\infty \left(\frac{k}{\theta} + \gamma\right)^n \left(1 + \left(\frac{k}{\theta}\right)^3\right) e^{-k} \frac{dk}{\theta} \quad (26)$$

$$= \frac{1}{\theta^n(\theta^3 + 6)} \int_{\theta x}^\infty (k + \theta\gamma)^n (\theta^3 + k^3) e^{-k} dk \quad (27)$$

$$= \frac{1}{\theta^n(\theta^3 + 6)} \int_{\theta x}^\infty \sum_{j=0}^n \binom{n}{j} k^j \theta^{n-j} \gamma^{n-j} (\theta^3 + k^3) e^{-k} dk \quad (28)$$

$$= \sum_{j=0}^n \binom{n}{j} \gamma^{n-j} \frac{1}{\theta^j(\theta^3 + 6)} \left(\theta^3 \int_{\theta x}^\infty k^j e^{-k} dk + \int_{\theta x}^\infty k^{j+3} e^{-k} dx\right),$$

where

$$\int_z^\infty u^{a-1} e^{-u} du = \Gamma(a, z)$$

$$J_n(x) = \sum_{j=0}^n \binom{n}{j} \gamma^{n-j} \frac{1}{\theta^j(\theta^3 + 6)} \left(\theta^3 \Gamma(j+1, \theta x) + \Gamma(j+4, \theta x)\right) \quad (29)$$

and

$$1 - F(x) = \left(1 + \frac{\theta^3(x-\gamma)^3 + 3\theta^2(x-\gamma)^2 + 6\theta(x-\gamma)}{\theta^3 + 6}\right) e^{-\theta(x-\gamma)}, \quad (30)$$

therefore

$$E(X^n / X > x) = \frac{\sum_{j=0}^n \binom{n}{j} \frac{\gamma^{n-j}}{\theta^j} \left(\theta^3 \Gamma(j+1, \theta x) + \Gamma(j+4, \theta x)\right)}{\left((\theta^3 + 6) + \theta^3(x-\gamma)^3 + 3\theta^2(x-\gamma)^2 + 6\theta(x-\gamma)\right) e^{-\theta(x-\gamma)}} \quad (31)$$

Definition 7

The mean residual life function, which comes as a result of the numerous interest of experimenters to know the additional lifetime given that a component has survived until a certain amount of time. The mean residual life function refers to the expected remaining life, $X - x$, given that the item has survived up to time x, is required. Let $X \sim SR(\theta,\gamma)$, the mean residual life function is obtained using the mathematical expression below

$$M_X(x) = E(X - x/X > x) - x \qquad (32)$$

Putting $n = 1$ in (30) and substituting the result into (31), we obtain MRL function as

$$m_X(x) = \frac{\left[\theta\gamma\left(\theta^3\Gamma(1,\theta x) + \Gamma(4,\theta x)\right) + \left(\theta^3\Gamma(2,\theta x) + \Gamma(5,\theta x)\right)\right]}{\theta\left((\theta^3 + 6) + \theta^3(x-\gamma)^3 + 3\theta^2(x-\gamma)^2 + 6\theta(x-\gamma)\right)e^{-\theta(x-\gamma)}} \qquad (33)$$

Definition 8

Here, we will be obtaining the mean deviation about the mean, denoted by μ_d, and the mean deviation about the median, denoted by M_d, of the shifted Rama distribution. Both are derived using the formula below;

$$\mu_d = \int_0^\infty |x - \mu| f_{SR}(x)dx = 2\mu F(\mu) - 2\mu + 2J_1(\mu) \qquad (34)$$

and

$$M_d = \int_0^\infty |x - M| f_{SR}(x)dx = 2J_1(M) - 2\mu \qquad (35)$$

respectively. By replacing x with μ and M in (4) and (29), yields the following

$$\mu_d = 2\left[\frac{\theta^3\gamma(\Gamma(1,\theta\mu) + \Gamma(4,\theta\mu)) + \theta^2(\Gamma(2,\theta\mu) + \Gamma(5,\theta\mu)) - (\theta^3 + 6)\mu - \mu e^{-(\mu-\gamma)}\left(\theta^3(\mu-\gamma)^3 + 3\theta^2(\mu-\gamma)^2 + 6\theta(\mu-\gamma)\right)}{\theta^3 + 6}\right] \qquad (36)$$

$$M_d = \left[\frac{2\left[\theta^3\gamma(\Gamma(1,\theta M) + \Gamma(4,\theta M)) + \theta^2(\Gamma(2,\theta M) + \Gamma(5,\theta M))\right] - (\theta^3 + 6)\mu}{\theta^3 + 6}\right] \qquad (37)$$

Definition 9

Bonferroni curve proposed by Bonferroni, C.E. (1930), and Lorenz curve proposed by Lorenz, M.O. (1905), have applications in the fields of economics, reliability, demography, insurance, medicine among others. So if $X \sim SR(\theta,\gamma)$, the Bonferroni and Lorenz curves are respectively given by

$$B(p) = \frac{1}{p\mu}\int_0^q x f_{SR}(x)dx \qquad (38)$$

and

$$L(p) = \frac{1}{\mu}\int_0^q x f_{SR}(x)dx \qquad (39)$$

where $\mu = E(X)$ and $q = F^{-1}(p)$, $0 < p < 1$. Hence, for the shifted Rama pdf (3) one gets, substituting eqn. (3) into eqn. (38) and (39)

$$B(p)=\frac{\theta^3\left(1-qe^{-q}-e^{-q}\right)+\left(24-q^4e^{-q}-4q^3e^{-q}-12q^2e^{-q}-24qe^{-q}-24e^{-q}\right)+\gamma\theta^4\left(1-e^{-q}\right)+\gamma\theta\left(6-q^3e^{-q}-3q^2e^{-q}-6qe^{-q}-6e^{-q}\right)}{p\theta\mu\left(\theta^3+6\right)} \quad (40)$$

and

$$L(p)=\frac{\theta^3\left(1-qe^{-q}-e^{-q}\right)+\left(24-q^4e^{-q}-4q^3e^{-q}-12q^2e^{-q}-24qe^{-q}-24e^{-q}\right)+\gamma\theta^4\left(1-e^{-q}\right)+\gamma\theta\left(6-q^3e^{-q}-3q^2e^{-q}-6qe^{-q}-6e^{-q}\right)}{\theta\mu\left(\theta^3+6\right)} \quad (41)$$

Definition 10

Let $X \sim SR(\theta,\gamma)$, the Renyi Entropy due to Renyi, A. (1961) is obtained using the mathematical expression below,

$$E_\rho=\frac{1}{1-\rho}\log\left(\int_0^\infty f_{SR}^{\rho}(x)dx\right) \quad (42)$$

Hence we obtain the Renyi entropy of the shifted Rama distribution by substituting eqn. (3) into eqn. (42), giving the expression below

$$E_\rho=\frac{1}{1-\rho}\log\left(\int_\gamma^\infty \frac{\theta^{4\rho}}{\left(\theta^3+6\right)^\rho}\left(1+(x-\gamma)^3\right)^\rho e^{-\theta\rho(x-\gamma)}dx\right) \quad (43)$$

$$E_\rho=\frac{1}{1-\rho}\log\left(\int_\gamma^\infty \frac{\theta^{4\rho}}{\left(\theta^3+6\right)^\rho}\sum_{j=0}^{\rho}\binom{\rho}{j}(x-\gamma)^{3\rho-3j}e^{-\theta\rho(x-\gamma)}dx\right) \quad (44)$$

$$E_\rho=\frac{1}{1-\rho}\log\left(\int_\gamma^\infty \frac{\theta^{4\rho}}{\left(\theta^3+6\right)^\rho}\sum_{j=0}^{\rho}\sum_{k=0}^{3(\rho-j)}\binom{\rho}{j}\binom{3(\rho-j)}{k}\gamma^{3\rho-3j-k}(-1)^{k+1}e^{\theta\rho\gamma}x^k e^{-\theta\rho x}dx\right)$$

$$E_\rho=\frac{1}{1-\rho}\log\left(\sum_{j=0}^{\rho}\sum_{k=0}^{3(\rho-j)}\frac{\theta^{4\rho}}{\left(\theta^3+6\right)^\rho}\binom{\rho}{j}\binom{3(\rho-j)}{k}(-1)^{k+1}\gamma^{3\rho-3j-k}e^{\theta\rho\gamma}\int_\gamma^\infty x^k e^{-\theta\rho x}dx\right) \quad (45)$$

Using the upper incomplete gamma function, which is expressed as

$$\Gamma(s,x)=\int_x^\infty t^{s-1}e^{-t}dt \quad (46)$$

to solve for; $\frac{1}{(\theta\rho)^{k+1}}\int_{\theta\rho\gamma}^\infty w^{k+1-1}e^{-w}dw$, we have that;

$$\frac{1}{(\theta\rho)^{k+1}}\int_{\theta\rho\gamma}^\infty w^{k+1-1}e^{-w}dw=\frac{\Gamma(k+1,\theta\rho\gamma)}{(\theta\rho)^{k+1}}$$

$$E_\rho=\frac{1}{1-\rho}\log\left(\sum_{j=0}^{\rho}\sum_{k=0}^{3(\rho-j)}\frac{\theta^{4\rho}}{\left(\theta^3+6\right)^\rho}\binom{\rho}{j}\binom{3(\rho-j)}{k}(-1)^{k+1}\gamma^{3\rho-3j-k}e^{\theta\rho\gamma}\frac{\Gamma(k+1,\theta\rho\gamma)}{(\theta\rho)^{k+1}}\right) \quad (47)$$

Definition 11

Suppose $X_1, X_2, ..., X_n$ is a random sample of $X_{(r)}$; $(r = 1,2,...,n)$ are the rth order statistics obtained by arranging $X_{(r)}$ in ascending order of magnitude $\ni X_1 \leq X_2 \leq ... \leq X_r$ and $X_1 = \min(X_1, X_2, ..., X_r), X_r = \max(X_1, X_2, ..., X_r)$, then the probability density function of the rth order statistics is given by

$$f_{r:n}(x,\lambda,\theta) = \frac{n!}{(r-1)!(n-r)!} f_{SR}(x;\gamma,\theta)[F_{SR}(x;\gamma,\theta)]^{r-1}[1 - F_{SR}(x;\gamma,\theta)]^{n-r} \tag{48}$$

$$f_{r:n}(x,\lambda,\theta) = \frac{n!}{(r-1)!(n-r)!} \sum_{k=0}^{n-r} \binom{n-r}{k} (-1)^k f_{SR}(x;\gamma,\theta)[F_{SR}(x;\gamma,\theta)]^{r+k-1} \tag{49}$$

where $f_{SR}(x;\gamma,\theta)$ and $F_{SR}(x;\gamma,\theta)$ are the pdf and cdf of SR distribution respectively. Hence we have;

$$f_{r:n}(x,\lambda,\theta) = \begin{array}{c} \dfrac{n!}{(r-1)!(n-r)!} \displaystyle\sum_{k=0}^{n-r} \binom{n-r}{k} (-1)^k \left(\dfrac{\theta^4}{\theta^3+6} \left(1 + (x-\gamma)^3\right) e^{-\theta(x-\gamma)} \right) \\ \left(1 - \left[1 + \dfrac{\theta^3 (x-\gamma)^3 + 3\theta^2 (x-\gamma)^2 + 6\theta(x-\gamma)}{\theta^3+6} \right] e^{-\theta(x-\gamma)} \right)^{r+k-1} \end{array} \tag{50}$$

$$f_{r:n}(x,\lambda,\theta) = \frac{n!}{(r-1)!(n-r)!} \sum_{k=0}^{n-r} \binom{n-r}{k} (-1)^k \left[\begin{array}{c} \left(\theta^3 \left(1 + (x-\gamma)^3\right) e^{-\theta(x-\gamma)}\right) \\ \dfrac{\left(\left((\theta^3+6) - \left(\theta^3 + 6 + \theta^3 (x-\gamma)^3 + 3\theta^2 (x-\gamma)^2 + 6\theta(x-\gamma)\right) e^{-\theta(x-\gamma)}\right)\right)^{r+k-1}}{(\theta^3+6)^{r+k}} \end{array} \right] \tag{51}$$

The cumulative distribution of the rth order statistics is given by;

$$F_{r:n}(x,\lambda,\theta) = \sum_{j=k}^{n} \sum_{l=0}^{n-j} \binom{n}{j} \binom{n-j}{l} (-1)^l F_{SRD}^{j+l}(x;\gamma,\theta) \tag{52}$$

Hence we have;

$$F_{r:n}(x,\lambda,\theta) = \sum_{j=k}^{n} \sum_{l=0}^{n-j} \binom{n}{j} \binom{n-j}{l} (-1)^l \left[1 - \left[1 + \frac{\theta^3 (x-\gamma)^3 + 3\theta^2 (x-\gamma)^2 + 6\theta(x-\gamma)}{\theta^3+6} \right] e^{-\theta(x-\gamma)} \right]^{j+l} \tag{53}$$

Classical Methods of Estimation

In this section, the parameters are going to be estimated using; the maximum likelihood function, the Least squares estimation, the weighted least squares estimation, maximum product spacing estimation, cramer von mises estimation, Anderson Darling estimation, and right-tailed Anderson Darling estimation.

Definition 1

(Maximum Likelihood Estimation), Let be n random samples drawn from SR distribution, the likelihood function is given as;

$$\left.\begin{aligned}
&LLf(x;\gamma,\theta)=\prod_{i=1}^{n} f(x;\gamma,\theta)\\
&=\prod_{i=1}^{n}\frac{\theta^4}{\theta^3+6}\left(1+(x-\gamma)^3\right)e^{-\theta(x-\gamma)}\\
&=\frac{\theta^4}{\theta^3+6}\prod_{i=1}^{n}\left(1+(x-\gamma)^3\right)e^{-\theta\sum_{i=1}^{n}(x-\gamma)}\\
&\ln(\mathrm{LLf}(\mathrm{x};\gamma,\theta))=4n\ln\theta-n\ln\left(\theta^3+6\right)+\sum_{i=1}^{n}\ln\left(1+(x_i-\gamma)^3\right)-\theta\sum_{i=1}^{n}(x_i-\gamma)\\
&\frac{\partial\ln l}{\partial\theta}=\frac{4n}{\theta}-\frac{3n\theta^2}{\theta^3+6}-\sum_{i=1}^{n}(x_i-\gamma)=0\\
&\frac{4\theta^3-3\theta^2+24}{\theta\left(\theta^3+6\right)}=\frac{\sum_{i=1}^{n}(x_i-\gamma)}{n}\\
&\frac{\partial\ln l}{\partial\gamma}=-3\sum_{i=1}^{n}\frac{(x-\gamma)^2}{\left(1+(x-\gamma)^3\right)}+n\theta=0\\
&\sum_{i=1}^{n}\frac{(x-\gamma)^2}{\left(1+(x-\gamma)^3\right)}=\frac{n\theta}{3}
\end{aligned}\right\}\tag{54}$$

θ and γ has no closed-form solution, hence will be solve iteratively in R using Newton-Raphson's iterative algorithm.

we obtain approximate confidence intervals of the parameters based on the asymptotic distribution of the MLEs of the unknown parameters $\Phi=(\theta,\gamma)$. The asymptotic variances and covariances of the MLE for parameters θ and γ are given by elements of the inverse of the Fisher information matrix. It is not easy to obtain the exact mathematical expressions for the above-mentioned equations. Therefore, we give the approximate (observed) asymptotic variance-covariance matrix for the MLE, which is obtained by dropping the expectation operator E

$$\mathrm{I}_{ij}^{-1}(\theta,\gamma)=\begin{bmatrix}\frac{\partial^2\psi}{\partial\theta^2} & \frac{\partial^2\psi}{\partial\theta\partial\gamma}\\ \frac{\partial^2\psi}{\partial\gamma\partial\theta} & \frac{\partial^2\psi}{\partial\gamma^2}\end{bmatrix}^{-1}=\begin{bmatrix}\mathrm{var}\left(\hat{\theta}\right) & \mathrm{cov}\left(\hat{\theta},\hat{\gamma}\right)\\ \mathrm{cov}\left(\hat{\gamma},\hat{\theta}\right) & \mathrm{var}\left(\hat{\gamma}\right)\end{bmatrix}\tag{55}$$

where

$$\left.\begin{aligned}
&\frac{\partial^2\ln l}{\partial\theta^2}=\frac{-4n}{\theta^2}+\frac{3n\left(\theta^4+12\theta\right)}{\left(\theta^3+6\right)^2}\\
&\frac{\partial^2\ln l}{\partial\theta\partial\gamma}=n\\
&\frac{\partial^2\ln l}{\partial\gamma^2}=\sum_{i=1}^{n}\frac{6(x-\gamma)-3(x-\gamma)^4}{\left(1+(x-\gamma)^3\right)^2}\\
&\frac{\partial^2\ln l}{\partial\gamma\partial\theta}=n
\end{aligned}\right\}\tag{56}$$

Approximate confidence intervals for θ and γ can be obtained. Hence, a 100(1−τ)% confidence intervals for the parameters θ and γ are

$$\hat{\theta} \pm Z_{\frac{\tau}{2}}\sqrt{\operatorname{var}(\hat{\theta})}; \hat{\gamma} \pm Z_{\frac{\tau}{2}}\sqrt{\operatorname{var}(\hat{\gamma})} \tag{57}$$

where $Z_{\frac{\tau}{2}}$ is the percentile standard normal distribution with right-tailed probability.

Definition 2

(Least Squares Estimation (LSE)). The Least Squares Estimation due to Swain, J.J., Venkatraman, S. & Wilson, J.R. (1988) to estimate the parameters of Beta distribution. Using the deductions from the work of Swain, J.J., Venkatraman, S. & Wilson, J.R. (1988), we write

$$\begin{aligned} E\left[F(x_{i:n} \setminus \theta,\gamma)\right] &= \frac{i}{n+1} \\ V\left[F(x_{i:n} \setminus \theta,\gamma)\right] &= \frac{i(n-i+1)}{(n+1)^2(n+2)} \end{aligned} \tag{58}$$

The least squares estimates $\hat{\theta}_{LSE}$ and $\hat{\gamma}_{LSE}$ of the parameters θ and γ are obtained by minimizing the function L(θ,γ) with respect to θ and γ

$$L(\theta,\gamma) = \underset{(\theta)}{\arg\min} \sum_{i=1}^{n}\left[F(x_{i:n} \mid \theta,\gamma) - \frac{i}{n+1}\right]^2 \tag{59}$$

The estimates are obtained by solving the following non-linear equations

$$\left.\begin{aligned} \sum_{i=1}^{n}\left[F(x_{i:n} \mid \theta,\gamma) - \frac{i}{n+1}\right]^2 \Delta_1(x_{i:n} \mid \theta,\gamma) = 0 \\ \sum_{i=1}^{n}\left[F(x_{i:n} \mid \theta,\gamma) - \frac{i}{n+1}\right]^2 \Delta_2(x_{i:n} \mid \theta,\gamma) = 0 \end{aligned}\right\} \tag{60}$$

where

$$\left.\begin{aligned} \Delta_1(x_{i:n} \setminus \theta,\gamma) &= -(x-\gamma)e^{-\theta(x-\gamma)}\left[\begin{aligned} &1+\left(\frac{\theta^3(x-\gamma)^3+3\theta^2(x-\gamma)^2+6\theta(x-\gamma)}{\theta^3+6}\right)- \\ &\frac{18\theta^2(x-\gamma)^2+(36\theta-3\theta^4)(x-\gamma)+(36-12\theta^3)}{\theta^3+6} \end{aligned}\right] \\ \Delta_2(x_{i:n} \setminus \theta,\gamma) &= e^{-\theta(x-\gamma)}\left[\frac{\theta(\theta^3+6)+\theta^4(x-\gamma)^3-6\theta}{\theta^3+6}\right] \end{aligned}\right\} \tag{61}$$

Definition 3

(Weighted Least Squares Estimation (WLSE)). The weighted least squares estimates $\hat{\theta}_{WLSE}$ and $\hat{\gamma}_{WLSE}$ of SRD distribution parameters θ and γ are obtained by minimizing the function W(θ,γ) with respect to θ and γ.

$$W(\theta,\gamma)=\underset{(\theta,\gamma)}{\arg\min}\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(x_{i:n}\mid\theta)-\frac{i}{n+1}\right]^2 \quad (62)$$

Solving the following non-linear equation yields the estimate

$$\left.\begin{aligned}\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(x_{i:n}\mid\theta,\gamma)-\frac{i}{n+1}\right]^2\Delta_1(x_{i:n}\mid\theta)=0\\ \sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(x_{i:n}\mid\theta,\gamma)-\frac{i}{n+1}\right]^2\Delta_2(x_{i:n}\mid\theta)=0\end{aligned}\right\} \quad (63)$$

Where $\Delta_1(x_{i:n}\mid\theta)$ and $\Delta_2(x_{i:n}\mid\theta)$ are as defined in (61) respectively

Definition 4

(Maximum Product Spacing Estimation (MPSE)). A good substitute for the greatest likelihood approach is the maximum product spacing method, which approximates the Kullback-Leibler information measure. Let us now suppose that the data are ordered in an increasing manner. Then, the maximum product spacing for the SHW is given as follows

$$Gs(\theta,\gamma\mid data)=\left(\prod_{i=1}^{n+1}D_l(x_i;\theta,\gamma)\right)^{\frac{1}{n+1}} \quad (64)$$

where $D_l(x_{i;}\theta,\gamma)=F(x_i;\theta,\gamma)-F(x_{i-1};\theta,\gamma), i=1,2,3,\ldots,n$

$$H(\theta,\gamma)=\frac{1}{n+1}\sum_{i=1}^{n+1}\ln D_l(\theta,\gamma) \quad (65)$$

By taking the first derivative of the function H(ϑ) with respect to θ and γ, and solving the resulting nonlinear equations, at $\frac{\partial H(\phi)}{\partial\theta}=0, and\,\frac{\partial H(\phi)}{\partial\gamma}=0$, where $\phi=(\theta,\gamma)$, we obtain the value of the parameter estimates.

Definition 5

(Cramér-von-Mises Estimation (CVME)). The Cramér-von-Mises estimates $\hat{\theta}_{CVME}, and\hat{\gamma}_{CVME}$ of the SR distribution parameters θ and γ are obtained by minimizing the function C(θ,γ) with respect to θ and γ.

$$C(\theta,\gamma)=\underset{(\theta,\gamma)}{\arg\min}\left\{\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}\mid\theta,\gamma)-\frac{2i-1}{2n}\right]^2\right\} \quad (66)$$

The estimates are obtained by solving the following non-linear equations

$$\left.\begin{aligned}\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\theta,\gamma\right)-\frac{2i-1}{2n}\right]\Delta_1\left(x_{i:n}\mid\theta,\gamma\right)=0\\ \sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\theta,\gamma\right)-\frac{2i-1}{2n}\right]\Delta_2\left(x_{i:n}\mid\theta,\gamma\right)=0\end{aligned}\right\} \tag{67}$$

where $\Delta_1\left(x_{i:n}\mid\theta\right)$ and $\Delta_2\left(x_{i:n}\mid\theta\right)$ are as defined in (61) respectively

Definition 6

(Anderson-Darling Estimation (ADE)). The Anderson-Darling estimates $\hat{\theta}_{CVME}, and \hat{\gamma}_{CVME}$ of the SHR distribution parameters θ and γ are obtained by minimizing the function A(θ,γ) with respect to θ and γ.

$$A(\theta,\gamma)=\underset{(\theta,\gamma)}{\arg\min}\sum_{i=1}^{n}(2i-1)\left\{\ln F\left(x_{i:n}\mid\theta,\gamma\right)+\ln\left[1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)\right]\right\}. \tag{68}$$

The estimates are obtained by solving the following sets of non-linear equations

$$\left.\begin{aligned}\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_1\left(x_{i:n}\mid\theta,\gamma\right)}{F\left(x_{i:n}\mid\theta,\gamma\right)}-\frac{\Delta_1\left(x_{n+1-i:n}\mid\theta,\gamma\right)}{1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)}\right]=0\\ \sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_2\left(x_{i:n}\mid\theta,\gamma\right)}{F\left(x_{i:n}\mid\theta,\gamma\right)}-\frac{\Delta_2\left(x_{n+1-i:n}\mid\theta,\gamma\right)}{1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)}\right]=0\end{aligned}\right\} \tag{69}$$

where $\Delta_1\left(x_{i:n}\mid\theta\right)$ and $\Delta_2\left(x_{i:n}\mid\theta\right)$ are as defined in (61) respectively.

Definition 7

(Right-Tailed Anderson-Darling Estimation (RTADE)). The Right-Tailed Anderson-Darling estimates $\hat{\theta}_{RTADE} and \hat{\gamma}_{RTADE}$, of the SHR distribution parameters θ and γ are obtained by minimizing the function R(θ,γ) with respect to θ and γ

$$.R(\theta,\gamma)=\underset{(\theta,\gamma)}{\arg\min}\left\{\frac{n}{2}-2\sum_{i=1}^{n}F\left(x_{i:n}\mid\theta,\gamma\right)-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\ln\left[1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)\right]\right\}. \tag{70}$$

The estimates can be obtained by solving the following set of non-linear equations

$$\left.\begin{aligned}-2\sum_{i=1}^{n}\frac{\Delta_1\left(x_{i:n}\mid\theta,\gamma\right)}{F\left(x_{i:n}\mid\theta,\gamma\right)}+\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_1\left(x_{n+1-i:n}\mid\theta,\gamma\right)}{1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)}\right]=0\\ -2\sum_{i=1}^{n}\frac{\Delta_2\left(x_{i:n}\mid\theta,\gamma\right)}{F\left(x_{i:n}\mid\theta,\gamma\right)}+\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_2\left(x_{n+1-i:n}\mid\theta,\gamma\right)}{1-F\left(x_{n+1-i:n}\mid\theta,\gamma\right)}\right]=0\end{aligned}\right\} \tag{71}$$

Where $\Delta_1\left(x_{i:n}\mid\theta\right)$ and $\Delta_2\left(x_{i:n}\mid\theta\right)$ is as defined in (61) respectively. The estimates given in (54), (59), (62), (64), (65), (66), (69) and (70) are obtained using **optim()** function in R with the Newton-Raphson iterative algorithm.

Application

In this section, we apply the proposed distribution to one real life data sets and simulation study in order to determine its usefulness and fitness for use.

Data Set on Blood Cancer Leukemia

The following data represent 40 patients suffering from blood cancer (leukemia) from one of Ministry of Health Hospitals in Saudi Arabia (Abouammah, Ahmed & Khalique, 2000).

Table 1. Data Regarding Blood Cancer (Leukemia) Patients from Health Hospital in Saudi Arabia

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036
2.162	2.211	2.37	2.532	2.693	2.805	2.91	2.912	3.192
3.263	3.348	3.348	3.427	3.499	3.534	3.767	3.751	3.858
3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753
4.929	4.973	5.074	5.381					

We demonstrate that the proposed Shifted Rama (SR) distribution is superior by comparing its model performance and fitness with those of the Odd Exponential Pareto-Lomax distribution (OEPL), Shifted Exponential (SHE) distribution, Rama Distribution (RD), Odd Generalized Exponential Distribution (OGE), and Two-parameter Odoma Distribution (TPOD) using data on blood cancer (Leukemia) from one the health hospital in Saudi Arabia as shown in Table 2. From the analytical measures of fitness, the model with the smaller values of log-likelihood (LL), the Akaike information criterion (AIC), the Bayesian information criterion (BIC), Consistent Akaike Information Criterion, Hannan-Quinn Information Criterion, and Kolmogorov–Smirnov (K-S) statistics, is best among others. See Uzoma and Obulezi (Uzoma& Obulezi, 2016) for relevant modification on model performance criteria using Bayesian Information Criterion (BIC).

Table 2. The Analytical Measures of Model Performance and MLE Estimates for the Fitted Distributions Using Data on Blood Cancer (Leukemia) from One the Health Hospital in Saudi Arabia

Dist.	Para	Estimate	LL	AIC	CAIC	BIC	HQIC	K-S	P-value
SRD	θ	1.18502	-70.51	143.0288	143.1341	144.7177	143.6394	0.1323	0.4856
	γ	0.315							
RD	θ	1.10022	-73.3	148.6007	148.7059	150.2896	149.2113	0.13938	0.4188
TPOD	θ	1.45224	-71.05	146.1241	146.4484	149.5019	147.3454	0.21608	0.04773
	γ	0.30289							
SHE	θ	0.3539	-81.55	165.1012	165.2065	166.7901	165.7119	0.27874	0.003996
	γ	0.315							
OGE	α	0.59435	-68.38	154.8779	156.0208	161.6334	157.3205	0.2264	0.03312
	β	2.3527							
	θ	2.42968							
	λ	4.29412							
OEPL	α	0.72491	-79.31	75.8889	77.03167	82.6443	78.33139	0.14739	0.3499
	β	0.93247							
	θ	2.18895							
	λ	8.6296							

From table 2, the SR distribution has a better fit to the data on blood cancer (Leukemia) from one the health hospital in Saudi Arabia, since its probability value is the largest among other probabilities that are greater than 0.05. Again, SR distribution out performs other distributions in the comparison of best fit of the real life data.

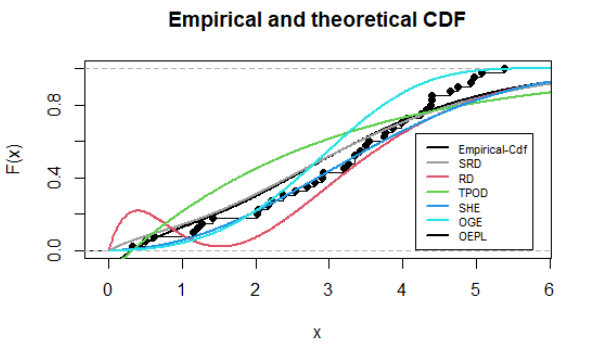

Figure 2. Empirical and Theoretical CDF

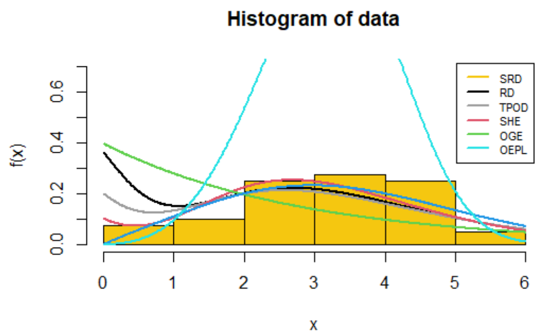

Figure 3. Histogram of Data

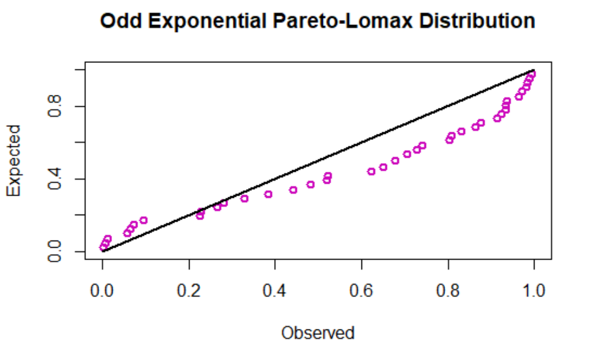

Figure 4. Odd Exponential Pareto-Lomax Distribution

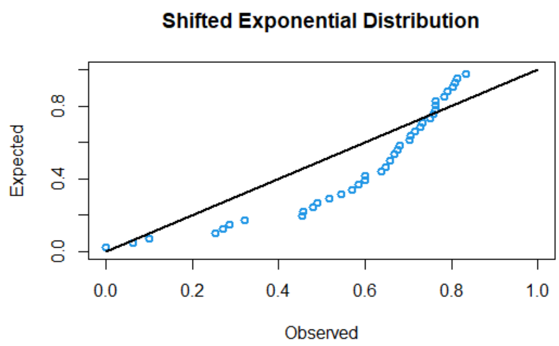

Figure 5. Shifted Exponential Distribution

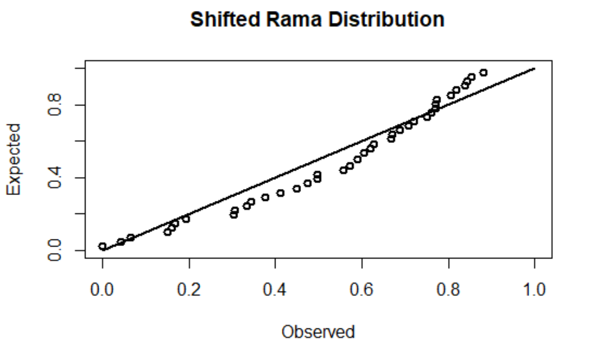

Figure 6. Shifted Rama Distribution

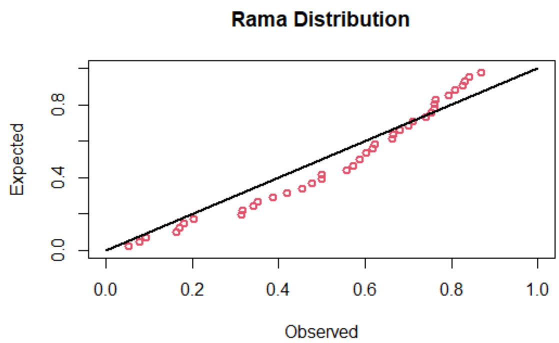

Figure 7. Rama Distribution

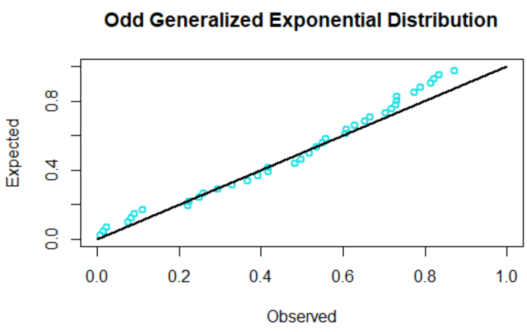

Figure 8. Odd Generalized Exponential Distribution

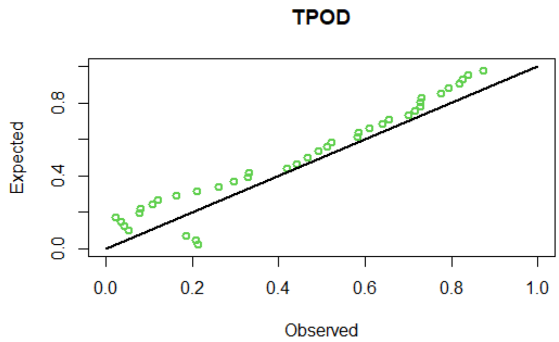

Figure 9. TPOD

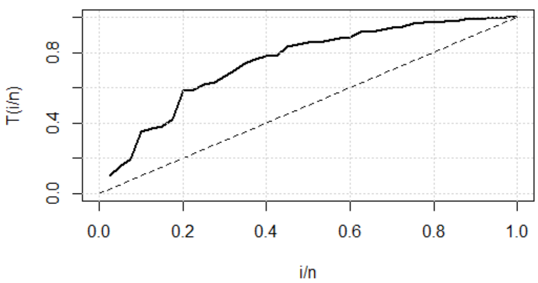

Figure 10.

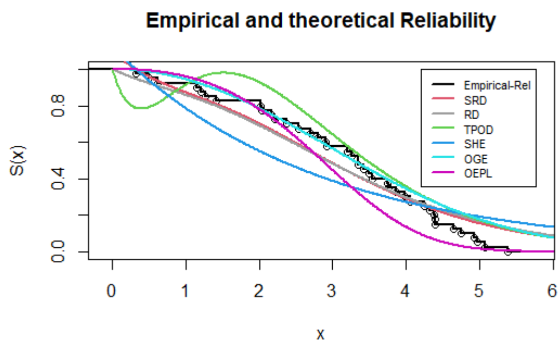

Figure 11. Empirical and Theoretical Reliability

Simulation Study and Comparisons

In this subsection, we simulate data for the Shifted Rama distribution to compare the performance of the Non-Bayesian estimation methods discussed in the previous section. we generate 1000 data from the Shifted Rama distribution by considering the initial parameter values as

- $\theta = 0.10, \gamma = 0.15$
- $\theta = 0.10, \gamma = 0.25$
- $\theta = 0.10, \gamma = 0.50$
- $\theta = 0.20, \gamma = 0.15$
- $\theta = 0.20, \gamma = 0.25$
- $\theta = 0.20, \gamma = 0.50$

and sample sizes n = 25,50,75,100. For each estimate $\hat{\phi} = \left(\hat{\theta}, \hat{\gamma}\right)$ we compute the Bias and Root Mean Squared Error(RMSE) respectively as

$$Bias\left(\hat{\phi}\right) = \frac{1}{B}\sum_{i=1}^{B}\left(\hat{\phi}_i - \phi\right), \qquad (72)$$

$$RMSE\left(\hat{\phi}\right) = \sqrt{\frac{1}{B}\sum_{i=1}^{B}\left(\hat{\phi}_i - \phi\right)} \qquad (73)$$

For the Non-Bayesian procedure, we employed the Newton-Raphson algorithm for finding the desired estimates.

Table 3: The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with Increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.10, \gamma = 0.15)$

θ=0.10; γ=0.15									
Method									
Method	parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.04293	0.00249	0.03159	0.00126	0.02643	0.00088	0.02284	0.00065
	γ	11.36051	146.0109	9.17012	93.73105	8.0208	71.77587	7.32957	59.72599
LSE	θ	0.00611	0.00026	0.00349	0.00012	0.00277	0.00008	0.00321	0.00006
	γ	2.69116	31.5659	1.45698	12.17969	1.1141	7.60362	1.12855	6.27329
WLSE	θ	0.00131	0.00037	0.00054	0.00019	0.00072	0.00013	0.00061	0.00009
	γ	1.44854	43.63456	0.3205	18.74138	0.08536	12.10619	0.35966	9.60267
MPSE	θ	0.00031	0.00032	0.00115	0.00015	0.00118	0.0001	0.00026	0.00007
	γ	0.89425	33.68069	0.00109	14.1077	0.16116	9.08558	0.17193	6.77104
CVME	θ	0.00462	0.00045	0.00341	0.00021	0.00257	0.00014	0.00075	0.00009
	γ	0.62809	39.55162	0.6666	18.498	0.5468	12.18467	0.12725	9.2331
ADE	θ	0.08862	0.00878	0.08488	0.00837	0.08565	0.00847	0.08279	0.00817
	γ	99550.39	1.29E+10	113937.7	1.75E+10	132481.4	2.32E+10	136251	2.51E+10
RTADE	θ	0.09723	0.00977	0.09976	0.01001	0.09958	0.00998	0.10008	0.01002
	γ	131113.2	2.14E+10	164266.6	3.17E+10	187269.5	4.15E+10	198365.2	4.61E+10

Table 4. The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.10, \gamma = 0.25)$

θ=0.10; γ=0.25									
Method									
Method	Parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.04269	0.00241	0.03125	0.00123	0.02713	0.00092	0.0231	0.00065
	γ	11.14475	139.7491	9.23113	94.59003	8.14628	73.73526	7.34162	59.80687
LSE	θ	0.0059	0.00023	0.00401	0.00012	0.00293	0.00008	0.00282	0.00005
	γ	2.74346	29.7681	1.54283	12.83937	1.23369	8.14707	1.01853	5.59765
WLSE	θ	0.00062	0.00038	0.00001	0.00018	0.00005	0.00012	0.00007	0.00008
	γ	1.38912	39.87338	0.4351	18.97709	0.40829	13.2744	0.20801	8.85719
MPSE	θ	0.00019	0.00033	0.00066	0.00015	0.00061	0.0001	0.00037	0.00007
	γ	0.91917	31.48257	0.08602	14.1482	0.12711	9.52892	0.00624	6.54185
CVME	θ	0.00556	0.00047	0.00296	0.0002	0.00201	0.00013	0.00137	0.00009
	γ	0.74221	36.39348	0.59687	18.53634	0.28228	12.81739	0.30267	8.70955
ADE	θ	0.09354	0.00938	0.09628	0.00963	0.09657	0.00964	0.09746	0.00974
	γ	108695.6	1.43E+10	136279.4	2.21E+10	154910.9	2.82E+10	168947.6	3.26E+10
RTADE	θ	0.08879	0.00891	0.08583	0.00859	0.08613	0.00861	0.08733	0.00877
	γ	116966.2	1.92E+10	138865.3	2.71E+10	156206.7	3.36E+10	164585.1	3.79E+10

Table 5. The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with Increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.10, \gamma = 0.50)$

θ=0.10; γ=0.50									
Method									
Method	Parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.0443	0.00261	0.03123	0.00125	0.02686	0.00088	0.02344	0.00068
	γ	11.41914	145.9721	9.06465	91.71651	8.17104	73.42365	7.31746	59.70148
LSE	θ	0.00536	0.00026	0.00394	0.00012	0.0031	0.00007	0.0029	0.00006
	γ	2.59485	30.84321	1.66409	13.29604	1.21139	7.29224	1.16068	6.39368
WLSE	θ	0.00017	0.00043	0.00003	0.00019	0	0.00011	0.00038	0.00008
	γ	1.41499	44.02354	0.62037	20.99603	0.36746	12.28194	0.43757	9.78322
MPSE	θ	0.00065	0.00037	0.00071	0.00016	0.00059	0.00009	0.00011	0.00007
	γ	0.87343	33.44018	0.22414	15.63581	0.07548	8.76625	0.19912	6.98913
CVME	θ	0.0059	0.00053	0.00295	0.00022	0.00199	0.00012	0.00107	0.00009
	γ	0.67651	39.31231	0.41897	20.06209	0.32742	12.04031	0.07379	9.47142
ADE	θ	0.09418	0.00933	0.09476	0.0094	0.0949	0.00942	0.09504	0.00943
	γ	103632.7	1.31E+10	126624.3	1.92E+10	146561.8	2.55E+10	159589.7	3E+10
RTADE	θ	0.09447	0.00951	0.09658	0.0097	0.09843	0.00984	0.09819	0.00983
	γ	136094.8	2.22E+10	165171.9	3.2E+10	194875.6	4.39E+10	211412	5.21E+10

Table 6. The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with Increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.20, \gamma = 0.15)$

θ=0.20; γ=0.15									
Method									
Method	Parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.08689	0.01045	0.06189	0.00492	0.05245	0.00341	0.04473	0.00247
	γ	5.64616	35.98199	4.51351	23.1082	3.98425	17.8534	3.53308	14.07442
LSE	θ	0.00948	0.00114	0.0082	0.00049	0.00552	0.00032	0.00555	0.00022
	γ	1.17222	7.61247	0.85954	3.60895	0.5843	2.10855	0.55075	1.5005
WLSE	θ	0.0011	0.00168	0.00173	0.00072	0.00057	0.00051	0.00039	0.00033
	γ	0.46518	10.19241	0.44108	5.19	0.25373	3.43621	0.15591	2.34954
MPSE	θ	0.00329	0.00153	0.00028	0.0006	0.00119	0.00041	0.00051	0.00026
	γ	0.18035	8.02906	0.19518	3.74478	0.05273	2.45323	0.05162	1.6632
CVME	θ	0.01341	0.0021	0.00425	0.0008	0.00347	0.00054	0.00263	0.00035
	γ	0.58211	9.59942	0.08829	4.858	0.10362	3.30185	0.11101	2.30572
ADE	θ	0.1965	0.03914	0.1993	0.03979	0.19944	0.03984	0.19975	0.03991
	γ	55526.86	3.54E+09	68886.55	5.31E+09	77620.81	6.75E+09	85282.55	8.17E+09
RTADE	θ	0.18419	0.03716	0.18388	0.03709	0.18551	0.03752	0.18489	0.03735
	γ	66436.82	5.51E+09	79604.47	7.87E+09	85747.14	9.25E+09	94416.85	1.11E+10

Table 7. The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with Increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.20, \gamma = 0.25)$

θ=0.20; γ=0.25									
Method									
Method	Parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.08659	0.01004	0.06052	0.00471	0.05179	0.00341	0.04616	0.00268
	γ	5.63957	36.09532	4.43481	22.18768	3.93381	17.55344	3.5929	14.70746
LSE	θ	0.00972	0.00101	0.00782	0.00049	0.00541	0.00033	0.005	0.00024
	γ	1.17633	7.68672	0.8238	3.17585	0.56825	2.16073	0.51767	1.55278
WLSE	θ	0.00231	0.00171	0.00027	0.00081	0.00047	0.00048	0.00016	0.00035
	γ	0.38247	10.83833	0.26109	4.80638	0.15276	3.21789	0.15448	2.36981
MPSE	θ	0.00395	0.00149	0.00175	0.00068	0.00184	0.0004	0.00111	0.00029
	γ	0.13818	8.67791	0.07418	3.54538	0.0117	2.38333	0.02008	1.73119
CVME	θ	0.01479	0.00216	0.00648	0.00093	0.00441	0.00053	0.00281	0.00037
	γ	0.66973	10.3438	0.27731	4.68177	0.19062	3.16893	0.10826	2.33911
ADE	θ	0.19111	0.03797	0.18948	0.03757	0.18773	0.03721	0.18945	0.0376
	γ	53396.51	3.45E+09	64594.03	5.02E+09	71657.49	6.13E+09	77401.57	7.14E+09
RTADE	θ	0.19279	0.03856	0.19759	0.03954	0.1983	0.03961	0.19873	0.0397
	γ	70054.3	5.76E+09	85959.81	8.58E+09	96085.58	1.07E+10	104707.7	1.27E+10

Table 8. The Analytical Observation of the Decreasing Trend in Average Bias and RMSE of the MLE and Other Bayesian Estimates for Shifted Rama Distribution with Increasing Sample Size and Assigning Values to the Two Parameter $(\theta = 0.20, \gamma = 0.50)$

θ=0.20; γ=0.50									
Method									
Method	Parameter	n=25		n=50		n=75		n=100	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
MLE	θ	0.08696	0.0101	0.06182	0.00488	0.05332	0.00355	0.04619	0.00264
	γ	5.64993	36.11846	4.43226	22.19087	3.99166	17.93474	3.63685	14.87217
LSE	θ	0.00978	0.00103	0.00745	0.00049	0.00494	0.00032	0.00493	0.00024
	γ	1.20086	7.43225	0.86308	3.43158	0.55717	1.92614	0.48555	1.54746
WLSE	θ	0.00109	0.00164	0.00029	0.0007	0.00083	0.00045	0.0002	0.00037
	γ	0.50277	10.07902	0.35687	5.04253	0.15387	2.87237	0.13591	2.4946
MPSE	θ	0.00281	0.00148	0.00162	0.00059	0.00215	0.00038	0.0008	0.0003
	γ	0.26507	7.93151	0.13548	3.72137	0.00355	2.10523	0.02007	1.79772
CVME	θ	0.01294	0.00197	0.00572	0.00079	0.00482	0.0005	0.00282	0.00039
	γ	0.52178	9.26527	0.16666	4.79895	0.19362	2.82777	0.13236	2.42997
ADE	θ	0.19961	0.0399	0.19903	0.03977	0.19927	0.0398	0.19971	0.03989
	γ	56108.23	3.61E+09	68063.76	5.39E+09	77359.49	6.8E+09	84748.4	8.04E+09
RTADE	θ	0.17866	0.03571	0.1729	0.0345	0.16841	0.03369	0.17137	0.03419
	γ	60420.87	4.85E+09	71808.9	6.89E+09	77019.39	8.22E+09	86123.8	9.79E+09

The following conclusions can be drawn from the simulation results that for a constant theta and an increase in the change of gamma values:

1. As sample size increases, the MLE estimator showed consistency in reduction in the values of the bias and RMSE, demonstrating improved accuracy in model parameter estimation.

2. As sample size increases, the LSE and CVME showed stability in the consistency of the reduction on the bias and RMSE values as the value of gamma increased.

3. For all sample sizes, the estimators' bias is positive

Conclusion

A new two-parameter distribution which is an extension of the Rama distribution, called the Shift Rama Distribution is proposed and studied. The mathematical properties of the new distribution including crude moments, central moment, coefficient of variation, index of dispersion, conditional moment, mean residual life function, mean deviation, Bonferroni and Lorenz curve, and the order statistics are derived. The cdf and hazard rate are increasing function and therefore can be deployed to study decreasing and increasing lifetime events. To estimate and study the parameters, seven methods are considered and comparisons are made. The techniques looked at include maximum likelihood estimation, maximum product spacing, least squares, weighted least squares, Cramer von mises, Anderson-Darling and the right-tailed Anderson-Darling, and a simulation of parameter based of the classical methods are done and studied. Application to real life data on blood cancer (Leukemia) from one of the health hospitals in Saudi Arabia was also illustrated. From the analytical measures of fitness and performance due to the p-value of the K-S statistics, the proposed Shifted Rama distribution is preferred to the other distributions compared.

Competing Interests

Authors have declared that no competing interests exist.

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