# Assessing the Impact of Parallel Burnout Fires on Flank Rate of Spread 

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## RATE OF SPREAD

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Assessing the Impact of Parallel Burnout Fires on Flank Rate of Spread
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The effects of flank-parallel suppression fires on the local rate of spread (ROS) of freely burning headfires through fully cured homogeneous grass fuels are assessed. Data sets include: one thermal image stack of a prescribed burn recorded by drone, and a suite of simulation experiments carried out in Wildland Urban Interface Fire Dynamics Simulator (WFDS). A new approach to computing ROS, curvature proxy driven normals to convex polylines, was developed to carry out this analysis. ROS time series depicting flank acceleration of the prescribed burn and simulation experiments, observable under coarse and fine directional classification schemes respectively, are the primary results. Pixelwise ROS magnitude and direction sensitivites to combined temperature threshold and curvature proxy localization parameter selection are also included.

## 1 Introduction

The costs of a wildland fire can be measured in terms of the burned acreage, destroyed infrastructure, quantity of released $\mathrm{CO}_{2}$, lives lost, and the monetary cost of controlling its spread [1]. In recent years the western United States has been experiencing longer fire seasons with both more and larger wildland fires [2, 3]. Given this, it is more important than ever that we understand wildland fire behavior, and thus can make accurate predictions about fire growth, intensity, and the effects of fire suppression tactics.

Since the 1990s there has been a proliferation of computational models of varying complexity. These have been categorized as physical (modeling fire directly through chemical and physical processes), empirical (modeling fire spread via statistical inference from lab and field based observations), and simulation (typically modeling fire spread through a GIS described landscape with spread driven by physical, empirical, or general mathematical model) [4, 5, 6]. While none of these models can perfectly capture fire behavior, an understanding of both the behaviors that they do capture - and those that are missed - is necessary for their refinement, and before they are broadly applied by fire managers [7].

A common fire suppression tactic employed by fire managers is that of backfiring or burnout firing - fighting fire with fire. Prior to the initiation of a suppression fire, control lines - boundaries set away from the wildfire where some combination of fuels removal and natural terrain barriers are believed to be sufficent for preventing its spread beyond them - are established. Subsequently, suppression fires are lit inside the control lines to consume fuels ahead of the wildfire, effectively improving the control line. In addition, suppression fires may dynamically interact with the wildfire influencing its behavior - changing its direction, rate of spread (ROS), or reducing its intensity [8].

The dynamic interaction of freely spreading wildfires and suppression fires is an active area of fire science research. Backing fires, suppression fires intended to spread towards the wind or down slope, have been studied extensively $[9,10$, 11, 12]. Firelines meeting at an oblique angle, sometimes called a "jump fire" or "junction fire," have also been studied both numerically and experimentally $[13,14,15,16]$. However, the dynamic interaction of parallel firelines spreading perpendicular to the wind have only recently been experimentally studied [17]. Furthermore, at the time that this work began, the interaction between the flank of a head fire and a parallel backing fire did not seem to have been studied.

A 2017 experimental grass fire conducted at the Sycan Marsh in Oregon demonstrated the dynamic interaction between the flanks of a wind driven headfire and suppression fires lit parallel to its flanks. This fire was one of the twelve prescribed burns recorded with a nadir facing infrared (IR) camera mounted to a drone by Moran et al, but not one of the four for which they reported results [18].

We hypothesize first, that the visually apparent flank accelerations observed by animating the Sycan prescribed burn image stack can be captured numerically (e.g. observe an increase in flank ROS through time), and second that
fully physics-based fire simulation will also show a measurable flank acceleration in the presence of a flank parallel suppression fire. To test these hypotheses, we developed a new method for obtaining ROS data from thermal imagery, and conducted an experimental suite of full physics simulations in Wildland Urban Interface Fire Dynamics Simulator (WFDS).

Thermal imagery is fast becoming an invaluable tool for the study of fire behavior driven by the increasing quality and affordability of IR cameras [19]. Once recorded, analyzing the thermal imagery is a two-step process, each of which must be carefully considered and conducted. Prior to extracting metrics of fire behavior, the raw imagery must be preprocessed to account for the camera's location, orientation, and movement. The second step is to employ suitable methods of extracting the relevant fire behavior metrics. In the present work, we are particularly concerned with obtaining a robust ROS dataset.

Caution in obtaining ROS data is warranted, as ROS computations are quite sensitive to algorithm choice, burning temperature thresholds, and the incorporation of direction of spread [20]. When evaluating previous work done to obtain ROS data from IR imagery, we assessed the scale at which the method was developed, (lab, medium, field, landscape), the fuels complex being consumed, the research goals directing its development, and the specifics of determining the direction of spread $[18,21,22,23,24,25,26]$. Given the data we were working with and our research goals, we developed a new approach to measuring ROS tailored to our needs.

In this work we treat ROS as a local (i.e. pointwise) vector quantity describing the evolution of a fire front (a planar curve) through time, and conceptually adopt the definition of Richards [27]: ". . . rate of spread at a point on a general two-dimensional fire perimeter is the rate of expansion in a direction normal to the perimeter instantaneously occurring at the point and time in question." However, given our thermal imagery's spatial ( $\approx 0.25 \mathrm{~m} / \mathrm{px}$ ) and temporal $(0.5-1 \mathrm{~Hz})$ resolutions, we felt a naive application of this conceptual model (identifying ROS vectors as pointwise normals to a given fire perimeter terminating on the successive perimeter) would mischaracterize the fire's behavior. The issue at hand is a mismatch between the spatial and temporal resolutions of our data: the spatial resolution is fine enough to capture some of the inherently noisy local structure of the fire perimeters, but the temporal resolution is too coarse for naively normal vectors to accurately represent the fire behavior.

Our solution to the resolution mismatch is to generate spread vectors which are locally normal to the fire perimeter's global shape regardless of any local noise. We conceptualize the global shape of fire perimeters from our data as smooth convex planar curves, and take each perimeter's convex hull boundary as an initial estimate of this conceptual ideal. These initial estimates are adjusted near the image frame's edge when several heuristic criteria indicate that they deviate from the fire's global shape as it extends beyond the image frame due to localized spread variability. Direction of spread is then computed to be normal to the adjusted convex hull boundary at its endpoints, and continuously varied from the start direction to the end direction according to a polyline curvature
proxy. We address parameter sensitivities, both burning temperature threshold and curvature driven normal localization, by specifying a reasonable range of values for each and producing spread vector collections for every combination and reporting outlier-resistant statistics for the aggregate collection.

Sections 2.1 and 2.2 provide detailed information on our data sources, the Sycan prescribed burn and suite of WFDS simulation experiments respectively. Section 2.4 addresses our new approach to computing ROS including its fundamental assumptions, the production of ROS vectors from curvature proxy driven normals and the scope of the our parameter sensitivity study. Our results and their discussion are presented in Section 3 followed by our conclusions, including a discussion of this study's limitations and directions for future work, in Section 4.

## 2 Methods

Our thermal imagery came from two sources - the Sycan prescribed burn (Section 2.1) and a suite of WFDS simulation experiments (Section 2.2). A sequence of pixelwise fire perimeters described by polylines was extracted from every thermal image stack (Section 2.3), and ROS data was generated from each sequence of perimeters using curvature proxy driven normals (Section 2.4) to determine direction of spread.

### 2.1 Sycan Prescribed Burn

The Sycan prescribed burn was performed on 10/18/2017 between roughly $23: 55$ and 23:59 GMT. The plot (Figure 2.1) was situated in a flat grass field bounded closely by an east-west roadway to the south and a northeast-southwest roadway to the west. Nearby weather stations and anemometers indicate the weather was a cloudy $65^{\circ} \mathrm{F}$ with winds out of the southwest and west southwest averaging between $4.5 \mathrm{~m} / \mathrm{s}$ and $6.7 \mathrm{~m} / \mathrm{s}$ and gusting up to $8.9 \mathrm{~m} / \mathrm{s}$. Direct fuel measurements were not taken, but the grasses were reported to be fully cured, in the highest $25 \%$ cover class, averaging 0.70 m and ranging to 1 m tall featuring vertically varied bulk density, while total bulk density and fuel moisture content were estimated to be $0.8 \mathrm{~kg} / \mathrm{m}^{2}$ and $6-8 \%$ respectively. Figure 2.2 shows grass similar to that present at the burn location. A hovering drone centered over the burn plot carrying a gimbaled FLIR XT thermal camera fitted with a custom neutral density filter recorded the burn from a height of 120 m with a nadir facing camera. The camera (with neutral density filter) was calibrated to measure temperatures from $100-1100^{\circ} \mathrm{C}$, and at a height of 120 m its $(640 \times 512)$ px images have a nominal spatial resolution of $0.227 \mathrm{~m} / \mathrm{px}$ [18]. The temporal resolution of the recording was variable, averaging 0.64 Hz . Metadata from the original images included precise time stamps, which were used to compute the time steps used in ROS computations. Figure 2.3 contains a selection of the stabilized images.


Figure 2.1: Precise location of the Sycan prescribed burn (Grass Plot E, Sycan Marsh, Oregon). A frame from the IR image stack overlays a digital terrain model of the surrounding marshland.


Figure 2.2: Similar Grass to that present in Plot E


Figure 2.3: A selection of frames from the Sycan IR image stack. Frames are labeled by elapsed time (s). The head fire crosses the image frame and suppression fires can be seen in the top-right and bottom-left corners.

### 2.2 Simulations

### 2.2.1 The WFDS Model

WFDS employs a Large Eddy Simulation (LES) Computational Fluid Dynamics model for fluid flow which accounts for sub-grid-scale (SGS) objects modeled by Lagrangian particles, the mixing and combustion of lumped chemical species, the effects of convective and radiant heat transfer, and the pyrolysis of solid and vegetative fuels [28, 29]. The model is based on a simplification of the NavierStokes equations for fluid flow developed by Rehm and Bahm [30], which is appropriate for low mach number flows. Grid cell (voxel) values in LES simula-
tions can be thought of as representing the mean value for each cell, and these are evolved according to the numerical solution (computed from a predictor corrector scheme) to the equations developed by Rehm and Bahm including the modeling of SGS turbulence by gradient diffusion. Mass and thermal diffusivity are modeled by constant Schmidt and Pradtl numbers, whose values were derived from smoke plume simulation experiments, while the Deardorff model is used for turbulent viscosity.

Combustion is modeled by tracking the advection, diffusion, and mixing of lumped quantities (air, fuel, products) of specific chemical species in fixed (mass fraction) proportions. Chemical reactions are assumed to occur infinitely fast by default, though finite rate chemistry governed by an Arrhenius equation may be specified. Every lumped quantity extant in a cell is assumed to exist unmixed with the others at the beginning of each timestep, and SGS mixing and combustion across the time step is modeled. The mixed quantity of the limiting reactant determines the total amount of combustion which occured - and the lumped species, heat released, and radiant energy are updated accordingly.

Vegetation is modeled by a single Lagrangian particle in each grid cell where it is specified to exist, as a thermally thin solid fuel element. The parametrization of this Lagrangian particle includes its physical characteristics as an obstruction to flow, and the necessary specifications for the the thermal degradation model. Thermal degradation is modeled in three Arrhenius rate controlled steps: endothermic drying, endothermic pyrolysis, and exothermic char oxidation.

### 2.2.2 WFDS Simulation Experiments

Simulation experiments were designed to test the effect of a suppression fire lit parallel to the flank of a wind driven grass fire. Control simulations consisted of a freely burning grass fire, and treatment simulations were identical to the control except for the inclusion of a suppression fire. Temperature values were extracted every second throughout the fuel bed at heights of $0,0.25,0.5$, and 1 m . The pixel values in our final image stack were assigned the maximum of these four values to best mimic an IR camera's view of the simulation because we expect the highest temperature would dominate given the $T^{4}$ term in the Stefan-Boltzman Law [31].

The total simulation domain was $(150 \mathrm{~m} \times 200 \mathrm{~m} \times 58 \mathrm{~m})$ described by $x \in$ $[-25,125] \mathrm{m}, y \in[-75,125] \mathrm{m}$, and $z \in[0,58] \mathrm{m}$. Situated inside the domain was a $100 \mathrm{~m} \times 100 \mathrm{~m}$ section of grass fuels, which was given a 25 m berth from the domain edge on three sides and a 75 m berth on the windward side. To allow for parallel computation, the domain was broken into 31 meshes each containing roughly the same number of voxels.

The simulation domain size and mesh resolutions were chosen to minimize the errors attributable to each given our available computational resources and time constraints. A sufficently large domain minimizes the effect of errors introduced to the wind flow through the non-physical boundaries, and furthermore provides space for the development of any fire-generated wind flows which could
influence the observed fire behavior. On the other hand, a fine grid size is necessary where combustion occurs to produce realistic fire behavior [29, 32]. However, a large domain with a uniformly fine grid is very computationally expensive. Thus, in order to produce optimal simulation results given our time and computational constraints, we found it necessary to vary the grid resolution according to the mesh location. Table 2.1 contains the meshing scheme details and Figures 2.4-2.5 provide a visual summary from a top-down ((x,y)-plane) and side-on ((y,z)-plane) perpsective.

| Fuel Area |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\#$ Meshes | $x$-domain | $y$-domain | $z$-domain | voxel size |  |  |  |
| 16 | $[0,100]$ | $[0,100]$ | $[0,4]$ | 0.25 m |  |  |  |
| 4 | $[-25,125]$ | $[-25,125]$ | $[4,8]$ | 0.5 m |  |  |  |
| 1 | $[-25,125]$ | $[-25,125]$ | $[8,18]$ | 1 m |  |  |  |
| 1 | $[-25,125]$ | $[-25,125]$ | $[18,58]$ | 2 m |  |  |  |
|  |  |  |  |  |  |  |  |
| \# Meshes | 25 m buffer |  |  |  |  | $z$-domain | voxel size |
| 5 | $([-25,125] \times[-25,125]) \backslash([0,100] \times[0,100])$ | $[0,4]$ | 0.5 m |  |  |  |  |
| Wind Development Area |  |  |  |  |  |  |  |
| $\#$ Meshes | $x$-domain | $y$-domain | $z$-domain | voxel size |  |  |  |
| 2 | $[-25,125]$ | $[-75,-25]$ | $[0,6]$ | 0.5 m |  |  |  |
| 1 | $[-25,125]$ | $[-75,-25]$ | $[6,18]$ | 1 m |  |  |  |
| 1 | $[-25,125]$ | $[-75,-25]$ | $[18,58]$ | 2 m |  |  |  |

Table 2.1: Simulation Domain Mesh Locations and Grid Resolutions


Figure 2.4: Top-down depiction of the mesh placements $(z \in[0,4])$, and the burner locations $(z=0)$


Figure 2.5: Side-on depiction of mesh grid resolutions $(x=50)$
The grass was parameterized to nearly match the 2007 Australian grass fire simulation work of Mell et al [32] — with adjustments to both more closely match the reported Sycan Marsh conditions, and to ensure flank fire existence and spread. Specifically, the grass was uniformly 1 m tall, with a vertically varied bulk density. The bottom 0.5 m of grass was assigned a bulk density of $0.9 \mathrm{~kg} / \mathrm{m}^{2}$, and the top 0.5 m was assigned a bulk density of $0.7 \mathrm{~kg} / \mathrm{m}^{2}$ resulting in a total bulk density of $0.8 \mathrm{~kg} / \mathrm{m}^{2}$. In addition, fuel moisture content was reduced to $5 \%$, and heat of combustion was increased to $17700 \mathrm{~kJ} / \mathrm{kg}[33,34]$.

The winds were imposed as a boundary condition from the YMIN boundary and held constant in each simulation at values of $4,6,8,10$, and $12 \mathrm{~m} / \mathrm{s}$. Additionaly, the winds were given an atmospheric profile varying the wind speed with height according to $u=u_{0}\left(z / z_{0}\right)^{0.143}$ where the reference wind height, $z_{0}$, was set to $2 \mathrm{~m}[29]$. All other non-ground boundaries were open, meaning ambient pressure and temperature conditions are assumed to exist beyond the boundary with flows through the boundary governed by the pressure gradient [28, 29].

In all simulations the grass was ignited by a burner encompassing a $6 \mathrm{~m} \times 6 \mathrm{~m}$ square block situated slightly off center from the windward edge of the grass field covering $[64,70] \times[0,6]$. This configuration was chosen because it quickly formed a clear wind driven head fire with well-defined flanks. The burner was set to ignite at time $T=1$, and to be fully turned off at $T=7$, with a 1 second ramp to and from full power in order to avoid numerical instabilities in the simulations. Treatment simulations included a second burner which initiated a suppression fire lit parallel to the flanks of the head fire. Each treatment burner ran at full power for 3 seconds with the same 1 second ramps, while their locations and ignition times, which varied with wind speed, are summarized in Table 2.2.

| Wind Speed | Location | Ignition Time |
| :---: | :---: | :---: |
| $4 \mathrm{~m} / \mathrm{s}$ | $[39,40] \times[5,40]$ | 45 |
| $6 \mathrm{~m} / \mathrm{s}$ | $[39,40] \times[5,40]$ | 35 |
| $8 \mathrm{~m} / \mathrm{s}$ | $[39,40] \times[5,50]$ | 30 |
| $10 \mathrm{~m} / \mathrm{s}$ | $[39,40] \times[5,50]$ | 30 |
| $12 \mathrm{~m} / \mathrm{s}$ | $[39,40] \times[5,100]$ | 25 |

Table 2.2: Suppression Burner Ignition Times and Locations

### 2.3 Fire Perimeter Identification



Figure 2.6: Sycan burn image and its associated perimeter
The original image stacks are lightly processed, illustrated in Figure 2.7, in order to identify a pixelwise fire perimeter according to a burning temperature threshold. First, max filtering is performed pixelwise through time. Letting $T(x, y, t)$ represent the temperature assigned to the pixel at location $(x, y)$ and time step $t$, we represent our max filter by

$$
T_{\max }\left(x, y, t^{*}\right)=\max _{t \leq t^{*}} T(x, y, t) .
$$

The max filter ensures that burnt pixels, which cool after the fire front passes over them, remain above the temperature threshold. Next the max filtered images are blurred with a Gaussian filter, $\sigma=3$, to ensure continuity of the fire perimeter [35]. The final image processing step is to assign pixels with a value greater than the burning temperature threshold a value of 1 and those below a value of 0 . The resulting binary images define the pixelwise fire perimeters, which we convert to a polyline representation (contour) for further analysis $[36,37]$. Contouring can produce a collection of polylines for a given image frame. Initial processing selects the longest perimeter from each collection by default, and further processing is performed by filtering the shorter polylines with a selection polygon covering the area of interest.


Blurred


Stacked


Binary Threshold


Figure 2.7: Perimeter identification image processing steps
As ROS algorithms are sensitive to threshold choice [20], we generated results across a visually identified range of thresholds $-T \in[300,500]$ and $T \in[200,300]$ every $20^{\circ} \mathrm{C}$ for the simulation and Sycan data respectively.

### 2.4 ROS Computations

Our ROS computations are designed to determine local, or point-wise, ROS such that the direction of spread is approximately normal to its global shape, conceptualized as a smooth convex planar curve, while preserving local spread variability. We begin this section by establishing a common understanding of the geometric space, objects, and concepts underpinning our methods of spread vector generation in Section 2.4.1; cover the approximation of the fire's global shape with a convex polyline in Section 2.4.3; and finally present the generation of spread vectors by curvature proxy driven normals in Section 2.4.4, which effectively smooths the polyline approximation.

### 2.4.1 Preliminary Geometric Notions

The geometric space we operate within is a consequence of the source of our fire perimeters - $\mathbb{R}^{2}$ equipped with the standard Euclidean metric, and scaled
such that coordinates and distances are measured in pixels. Points will be identified by lower case letters and specified in coordinates as $p=(x, y)$, vectors by bold lower case letters and specified in coordinates as $\mathbf{p}=\langle x, y\rangle$, lines by $\stackrel{\leftrightarrow}{6}$ where the object(s) $\bullet$ define a line, polylines and polygons by capital letters and specified by sequences of points. The convex hull of a collection of points is the smallest convex polygon containing the collection, and the interior angles of convex polygons all measure less than $180^{\circ}$ or $\pi$ radians. We note here that we assume all object and function definitions given below are well defined, given the scope of this work, and dispense with any consideration of edge cases or degenerate results.

Distances may be computed between points, denoted $d(p, q)$, between sets of points, defined by $d(X, Y)=\min _{x \in X, y \in Y} d(x, y)$, and along a polyline which we denote by $d_{P}(q)$, and $d_{P}(p, q)$.

Given a pair of points, $p, q \in \mathbb{R}^{2}$, we will write $\overline{p q}$ for the line segment between $p$ and $q, \overleftrightarrow{p q}$ for the line containing $p$ and $q$, and $\overrightarrow{p q}$ when specifying a vector by its initial point $p$ and terminal point $q$. We note here that while $\overrightarrow{p q} \in$ $\mathbb{R}_{p}^{2}$, the tangent space at $p$, we implicitly apply the vector space isomorphism $L_{p}: \mathbb{R}_{p}^{2} \rightarrow \mathbb{R}^{2}$ defined by $\overrightarrow{p q} \mapsto \mathbf{q}-\mathbf{p}$ as necessary, e.g. we may write $\overrightarrow{p q} \cdot \overrightarrow{r s}$, without explicitly applying $L_{p}$ and $L_{r}$ [38].

All angle measurements are derived from Equation 2.1, where u,v are vectors, and $\theta \in \mathbb{R} / 2 \pi \mathbb{Z}$ is the angle between them. We implicitly map every $\theta$ to its representative in the specified intervals.

$$
\begin{equation*}
\cos (\theta)=\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} \tag{2.1}
\end{equation*}
$$

For convenience we define several useful " $\theta$ " functions here. $\dot{\theta}: \mathbb{R}^{2} \times \mathbb{R}^{2} \rightarrow[0, \pi]$ computes the absolute value of $\theta$ for $\theta \in(-\pi, \pi]$.

$$
\begin{equation*}
\dot{\theta}(\mathbf{u}, \mathbf{v})=\arccos \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\right) \tag{2.2}
\end{equation*}
$$

$\hat{\theta}: \mathbb{R}^{2} \times \mathbb{R}^{2} \rightarrow[0, \pi]$, which we conceptually think of as measuring the sharpness of the angle between two vectors.

$$
\begin{equation*}
\hat{\theta}(\mathbf{u}, \mathbf{v})=\pi-\dot{\theta}(\mathbf{u}, \mathbf{v}) \tag{2.3}
\end{equation*}
$$

We also make use of signed angles with $\theta: \mathbb{R}^{2} \rightarrow(-\pi, \pi]$ defined by

$$
\begin{equation*}
\theta(\langle x, y\rangle)=\operatorname{sgn}(y) \dot{\theta}\left(\langle x, y\rangle, \mathbf{e}_{1}\right) \tag{2.4}
\end{equation*}
$$

and the signed angle relative to $\mathbf{v}, \theta_{\mathbf{v}}: \mathbb{R}^{2} \rightarrow(-\pi, \pi]$, is simply

$$
\begin{equation*}
\theta_{\mathbf{v}}(\mathbf{u})=\theta(\mathbf{u})-\theta(\mathbf{v}) \tag{2.5}
\end{equation*}
$$

A polyline, $P=P\left(v_{1}, \ldots, v_{n}\right)$, is specified by a sequence of vertices (points), includes all points of $\cup_{i=1}^{n-1} \overline{v_{i} v_{i+1}}$, and has a length, $\|P\|=\sum_{i=1}^{n-1} d\left(v_{i}, v_{i+1}\right)$. We call $v_{1}$ its start (vertex), and $v_{n}$ its end (vertex). When forming vectors from
the sequence of vertices $\left\{v_{i}\right\}$, we will write $\mathbf{v}_{i}^{+}=\overrightarrow{v_{i} v_{i+1}}$ and $\mathbf{v}_{i}^{-}=\overrightarrow{v_{i} v_{i-1}}$. Given a point $q \in \overline{v_{k} v_{k+1}}$, we write $d_{P}(q)$ for the distance along $P$ to $q$ and define it by $d_{P}(q)=\left\|P\left(v_{1}, \ldots, v_{k}, q\right)\right\|$. The distance between two points, $p, q \in P$, along $P$ is $d_{P}(p, q)=\left|d_{P}(p)-d_{P}(q)\right|$. We write $P\left(d_{P}(q)\right)=q$ for the inverse operation of obtaining a point at a given distance along $P$. Normalized versions are written with a tilde, for example $\widetilde{d_{P}}(q)=\frac{d_{P}(q)}{\|P\|}$ and $\dot{P}\left(\widetilde{d_{P}}(q)\right)=q$.

A polygon is also specified by a sequence of vertices $G=\left(v_{1}, \ldots, v_{n}\right)$, and includes all points of its interior and boundary, $\partial G$, which is the polyline $\partial G=$ $\left(v_{1}, \ldots, v_{n}, v_{1}\right)$. When $P=\left(v_{1}, \ldots, v_{n}\right)$ is a polyline, and $G=\left(v_{1}, \ldots, v_{n}\right)$ is its convex hull, we say that $P$ is convex.

### 2.4.2 Assumptions

Our approach to computing ROS was not developed for general use, but instead to best approximate direction of spread for the data sets it can be applied to. Thus, we present here a basic set of assumptions about the fire spread being studied, which we feel should be met prior to an application of these methods ${ }^{1}$.

## List 2.1: Basic Assumptions

1. The convex hull of the fire perimeter is a good approximation to its global shape.
2. The general trend of fire spread appears to be in a direction normal to the global fire shape.
3. The time step between successive perimeters is small enough to justify approximation of local fire spread paths by straight lines.
4. The fire perimeter polylines begin and end on the frame of the image.
5. The start and end points of the first perimeter of the entire sequence correspond to flank sections of the fire perimeter.

### 2.4.3 Global Shape Approximation

On its face, approximating a fire's global shape by the boundary of its convex hull should be simple, however the fire's interaction with the image frame complicates this process. The fire's interaction with the corners of the image frame need to be handled first, which will allow us to then extract an initial convex polyline estimate. In most cases, the initial estimate will then serve as our final polyline approximation to the fire's global shape, but each of these must be assessed for significant changes in direction near the image frame. When it appears that these changes reflect near frame spread variability, and the initial

[^0]estimate thus misrepresents the fire's global shape, slight adjustments are made to improve the initial convex polyline estimate.

As a fire expands, the start or end vertex of a fire perimeter may move from one frame edge to an adjacent one across a time step. We describe this interaction between the fire and the image frame as corner consumption. We make one assumption about how a fire will consume image frame corners that any single time step will consume at most one corner. Edge cases which violate this assumption must be known in advance, as they are handled not by internal logic, but with additional command line arguments.

A corner $c$ consumed at time step $t$ by $P_{t}$ will be appended to the vertex list of $P_{k}$ for $k \geq t$ prior to forming the convex hull $H_{k}$. The boundary of $H_{k}$ will then trace the image frame between the start and end of $P_{k}$ when the corners have been appended in the proper order. Consumed corners are tracked with global variables and kept in a sequence which ensures that $P\left(c_{1}, \ldots, c_{n}\right)$ traces along the image frame's edges. Thus, we need only check which image edge the end vertex lies on to decide between appending $c_{1}, \ldots, c_{n}$, or $c_{n}, \ldots, c_{1}$. The boundary of the resulting convex hull decomposes into two contiguous sections, one which traces the boundary of the image frame, and the other representing the fire boundary's global shape.

At this point our initial polyline approximation is

$$
\begin{equation*}
\partial H-\partial F, \tag{2.6}
\end{equation*}
$$

where $H$ is the adjusted convex polygon formed from the fire's perimeter accounting for corner consumption, and $F$ is a rectangular polygon representing the image frame. In practice making this computation directly is unreliable. It is, however, easily worked around. The work around consists of buffering $\partial F$ by 0.01 px to form a clipping polygon, clipping $\partial H$ (reliably removing the section which traces $\partial F$ ), and replacing the resulting start and end vertices with their original values.

Spread variability near the frame's edge may significantly alter direction of spread near the image frame's edge to reflect this variability, rather than the fire's true global shape as it extends beyond the image. The goal here is to recognize when this occurs and correct the start or end of the initial convex polyline to more accurately capture the fire's global shape. In practice, these adjustments are made infrequently because several (tuneable) heuristic criteria must be met to trigger their application. The four tuneable variables and their default values are: the frame offset percentage ( $\delta=0.05=5 \%$ ), the frame tolerance angle $\left(\theta_{F}=\frac{\pi}{36}=5^{\circ}\right)$, the flank tolerance angle $\left(\theta_{f}=\frac{\pi}{12}=15^{\circ}\right)$, and the strict flank tolerance angle $\left(\theta_{\boldsymbol{f}}=\frac{\pi}{36}=5^{\circ}\right)$.

The most basic criteria, that the convex polyline should not rapidly change direction near the frame edge, is assessed last. The adjustment addresses this directly, but only after verifying the critical implicit assumption that the section under consideration is part of the fire's flank. We verify this with the following edge labeling scheme, whose heuristic tests adjust according to a categorization of the fire perimeter's current frame interactions.

The core concept underpinning the edge labeling scheme is that for each time step every image frame edge exists in one of three (ordered) states: an unknown or vertex free state $(\mathcal{U})$, a head fire state $(\mathcal{H})$, when the headfire intersects with the edge, and a flank state $(\mathcal{F})$, when one or both flanks intersect with the edge ${ }^{2}$. These states arise naturally from Basic Assumptions 1, 2, 4, and 5. Assumptions 4 and 5 force the initial start and end vertices to lie on the fire's flanks. Accurate edge label initialization is therefore guaranteed, which the heuristic tests depend on. Assumptions 1 and 2 state that the fire's growth pattern is normal to its global convex shape, ensuring that the edge states respect the practical ${ }^{3}$ ordering $\mathcal{U} \prec \mathcal{H} \prec \mathcal{F}$. In other words, edges labeled $\mathcal{U}$ can transition to $\mathcal{H}$ or $\mathcal{F}$, edges labeled $\mathcal{H}$ can transition to $\mathcal{F}$, and edges labeled $\mathcal{F}$ are fixed.

The perimeter's frame interaction status is similarly categorized by three successive states (full $\prec$ mixed $\prec$ single flank), and we assume here that all three states exist in the data ${ }^{4}$. A full status indicates the presence of both flanks, and transitions to mixed when a corner is consumed and the new edge fails to earn an $\mathcal{F}$ label. We interpret this to mean that the head fire has passed beyond the image edge, and the current fire perimeter traces one flank and a portion of the head fire. Once the entirety of the head fire section passes beyond the frame, when an $\mathcal{H}$ - or $\mathcal{U}$-labeled edge earns (i.e. passes a $\theta_{\boldsymbol{f}}$ test) an $\mathcal{F}$ label, the perimeter status transitions to single flank. We note here that the heuristic tests require a consistent orientation of the fire perimeters, which is accomplished in pre-processing.

The edge labeling scheme is carried out by forming representative test vectors near the image frame's edges. Given a polyline $P=\left(v_{1}, \ldots, v_{n}\right)$ and frame offset percentage $\delta$, the start and end test vectors ( $\mathbf{v}_{\alpha}, \mathbf{v}_{\omega}$ respectively) are defined in Equation 2.7 where $\delta_{x}$ is defined as $\delta_{\alpha}=\delta$ or $\delta_{\omega}=1-\delta$

$$
\begin{equation*}
\mathbf{v}_{x}=\mathbf{v}_{k}^{+} \quad: \quad \widetilde{P}\left(\delta_{x}\right) \in \overline{v_{k} v_{k+1}} . \tag{2.7}
\end{equation*}
$$

The test vectors are offset from the frame by $\delta$, to minimize the likelihood that their direction is influenced by near frame spread variability, while still being near the edge to maximize the likelihood that they are representative of the appropriate edge label. Enforcing a consistent perimeter orientation and having $\mathbf{v}_{\alpha}$ point away from the start edge while $\mathbf{v}_{\omega}$ points toward the end edge ensures that the values $\dot{\theta}\left(\mathbf{v}_{x}, \mathbf{u}_{x}\right)$ are meaningful for each test called for by the current perimeter status and trigger. The edge labeling scheme is summarized in Table 2.3 for consecutive perimeters $P=\left(v_{1}, \ldots, v_{n}\right)$ and $Q=\left(u_{1}, \ldots, u_{n}\right)$ assuming end vertices lie on $\mathcal{H}$-labeled edges when the perimeter status is mixed.

[^1]| Perimeter Status | Trigger | Test | True | False |
| :---: | :---: | :---: | :---: | :---: |
| Full | $u_{1} \in E_{\mathcal{U}}$ | $\dot{\theta}\left(\mathbf{v}_{\alpha}, \mathbf{u}_{\alpha}\right)<\theta_{f}$ | $E_{\mathcal{F}}$ and Full | - |
| Full | $u_{m} \in E_{\mathcal{U}}$ | $\dot{\theta}\left(\mathbf{v}_{\omega}, \mathbf{u}_{\omega}\right)<\theta_{f}$ | $E_{\mathcal{F}}$ and Full | $E_{\mathcal{H}}$ and Mixed |
| Mixed | Mixed Status | $\dot{\theta}\left(\mathbf{v}_{\alpha}, \mathbf{u}_{\omega}\right)<\theta_{\boldsymbol{f}}$ | Total Flank | Mixed |

Table 2.3: Edge Labeling Scheme

Perimeter starts and ends identified as existing on a flank section are assessed for excessive near frame noise, and adjusted as necessary. This is accomplished by removing vertices from the relevant end until the near frame change in direction relative to that end's test vector is less than the frame tolerance angle $\left(\theta_{F}\right)$. The inital or final line segment is then extended back to the frame edge.

### 2.4.4 Curvature Proxy Driven Normals

Curvature proxy driven normals were developed to bridge the gap between the adjusted convex hull boundary obtained above, and the smooth convex planar curve we conceptualize as the fire's global shape. Intuitively curvature ( $\kappa$ ) measures the local rate of change in direction of normal vectors to a curve as it is traversed and is defined for plane curves in Equation 2.8 where $T$ is the unit normal tangent vector, and $d s$ is the arclength parametrization of the curve [39]

$$
\begin{equation*}
\kappa=\left\|\frac{d T}{d s}\right\| . \tag{2.8}
\end{equation*}
$$

Accordingly, the curvature at every vertex of a polyline is undefined and zero at all other points. This starkly contrasts the curvature of our conceptual ideal, which is everywhere defined and strictly positive ${ }^{56}$. In the remainder of this section we present the precise definitions of $\hat{\kappa}_{l}$ and $\theta\left(\mathbf{n}_{\hat{\kappa}}(p)\right)$, a family of strictly positive heuristic curvature proxies for convex polylines and the direction of the curvature proxy driven normal at $p$ respectively, and finish with the details of our parameter sensitivity study of curvature driven normals.

Given a convex polyline, $P=P\left(v_{1}, \ldots, v_{n}\right)$, and $p \in P$, we define $\hat{\kappa_{l}}(p)$ to be

$$
\begin{equation*}
\hat{\kappa}_{l}(p)=\sum_{i=2}^{n-1} \hat{\theta}\left(\mathbf{v}_{i}^{-}, \mathbf{v}_{i}^{+}\right)\left(1-\widetilde{d_{P}}\left(p, v_{i}\right)\right)^{l} . \tag{2.9}
\end{equation*}
$$

The construction of $\hat{\kappa}_{l}$ was guided by the intuitive meaning of curvature. It distributes the measurable changes in direction (the $\hat{\theta}$ values of non-boundary vertices) across the entirety of $P$, and the weight function $w_{l}(p, v)=\left(1-\widetilde{d_{P}}(p, v)\right)^{l}$ localizes this distribution. The parameter $l$ controls the degree of localization, and will be ommitted from our notation when an arbitrary fixed positive value may be assumed or its value should be clear from context.

[^2]Figure 2.8 illustrates the computation of $\hat{\kappa}(p)$ for a single point $p$ marked with an "x" on the convex polylines whose vertices are marked by blue circles. The size of each circle indicates its weight and its darkness indicates its $\hat{\theta}$ value in the computation of $\hat{\kappa}$. Figure 2.9 displays the same convex polylines, colored in their entirety according to $\hat{\kappa}$.


Figure 2.8: $\hat{\kappa}(p)$ for a selection of adusted convex Sycan perimeters


Figure 2.9: $\hat{\kappa}(P)$ for a selection of adjusted convex Sycan perimeters

Curvature driven normals are normal to $P$ at $v_{1}$ and $v_{n}$ while their directions are otherwise shifted from $\theta\left(\mathbf{n}_{\hat{\kappa}}\left(v_{1}\right)\right)$ to $\theta\left(\mathbf{n}_{\hat{\kappa}}\left(v_{n}\right)\right)$ according to the fraction of total curvature between each given point and $v_{1}$. More precisely, given a counterclockwise oriented convex polyline $P, p=\widetilde{P}(t)$, and $\theta_{s}$, (the direction of the outward facing normal to $\left.\overline{v_{1}, v_{2}}\right)$ the direction of a curvature proxy driven normal to $P$ at $p$ is:

$$
\begin{equation*}
\theta\left(\mathbf{n}_{\hat{\kappa}}(p)\right)=\theta_{s}+\left(\frac{\int_{0}^{t} \hat{\kappa}(\widetilde{P}(s)) \mathrm{d} s}{\int_{0}^{1} \hat{\kappa}(\widetilde{P}(s)) \mathrm{d} s}\right)\left(\sum_{i=2}^{n-1} \hat{\theta}\left(\mathbf{v}_{i}^{-}, \mathbf{v}_{i}^{+}\right)\right) . \tag{2.10}
\end{equation*}
$$

In practice the integrals in the scaling term are approximated by finite sums over a collection of evenly-spaced sample points from $P$.

Given a sequential pair of fire perimeters $(P, Q)$ and the inner perimeter's adjusted convex hull boundary $\left(H_{P}\right)$, we generate ROS vectors from $P$ to $Q$ according to the map given in Equation 2.11, which is defined in Equations 2.12-2.13 and elaborated on in List 2.2. In this work spread vectors are formed approximately every 0.5 px distance along $H_{P}$, and the integrals in Equation 2.10 are computed from curvature proxy values spaced approximately every 0.05 px along $H_{P}$.

$$
\begin{gather*}
\operatorname{ROS}: H_{P} \rightarrow T P . \\
h \mapsto \overrightarrow{p q} .  \tag{2.11}\\
\mathcal{P}=P \cap \overleftrightarrow{\mathbf{n}_{\hat{\kappa}}(h)}, \text { and } \mathcal{Q}=Q \cap \overleftrightarrow{\mathbf{n}_{\hat{\kappa}}(h)}  \tag{2.12}\\
p=\underset{x \in \mathcal{P}}{\operatorname{argmin}} d(x, h), \text { and } q=\underset{x \in \mathcal{Q}}{\operatorname{argmin}} d(x, h) . \tag{2.13}
\end{gather*}
$$

List 2.2: ROS Computational Steps

1. Compute $\mathbf{n}_{\hat{\kappa}}(h)$, the curvature driven normal at $h$.
2. Compute candidate sets of initial and terminal points $(\mathcal{P}, \mathcal{Q})$ by finding the intersection of the line containing $\mathbf{n}_{\hat{\kappa}}(h)$ with $P$ and $Q$ respectively (Equation 2.12).
3. Select the nearest point to $h$ from each candidate set to form the spread vector $\overrightarrow{p q}$ (Equation 2.13).

A parameter sensitivity study was conducted to aid in our selection of the localization parameter $l$ and to compare the accuracy of curvature driven normals with naive normals (encoded as $l=-1$ ). Tables 2.4 and 2.5 summarize the extent of the study.

| Parameter | Value Range | Value Sequence Relation | Units |
| :---: | :---: | :---: | :---: |
| Resolution | $25-400$ | $a_{n}=2 * a_{n-1}$ | $\mathrm{px} / \mathrm{U}$ |
| Simplification Tolerance | $0-2$ | $a_{n}=0.25 * 2^{n-1} n>0$ | px |
| Start Position | $0-0.5$ | $a_{n}=a_{n-1}+0.05$ | $\widetilde{d_{P}}\left(v_{1}, s\right)$ |
| End Position | $0-0.8$ | $a_{n}=a_{n-1}+0.05$ | $\widetilde{d_{P}}\left(v_{n}, e\right)$ |
| $l$ | $-1,25-500$ | $a_{n}=a_{n-1}+25$ | - |

Table 2.4: General Sensitivity Study Exploratory Parameters
${ }^{\mathrm{a}} T P=\coprod_{p \in P} \mathbb{R}_{p}^{2}$ i.e. vectors in $\mathbb{R}^{2}$ with initial points in $P$.

| Parent Function | $a$-Value Range | $\Delta a$ | Domain |
| :---: | :---: | :---: | :---: |
| $a \sin (x)$ | $0.5-5$ | 0.5 | $[0, \pi]$ |
| $e^{(a x)}$ | $0.1-1$ | 0.1 | $[-5,3.5]$ |
| $a x^{2}$ | $0.2-2$ | 0.2 | $[-5,5]$ |

Table 2.5: Sensitivity Study Functions
Every function in this study was approximated by the convex hull boundary of a binary pixelwise contour. The pixelwise representations of each function were generated at a range of resolutions (measured in pixels per unit), and their convex hull approximations were simplified across a range of tolerances (measured in pixels). The simplified convex hull approximations were sliced into fractional parts ranging from $20-100 \%$ of the simplified perimeter by combining the start and end positions found in Table 2.4. The start and end position combinations were chosen in an upper triangular fashion to eliminate redundancy due to the symmetry of $a \sin (x)$ and $a x^{2}$ around $x=\pi / 2$ and $x=0$. A collection of 2001 evenly spaced curvature driven normal directions were generated for every localization parameter in the study on each slice, and the root mean squared error (RMSE), mean absolute error, and maximum absolute error were recorded in addition to the parameters defining the slice and a selection of its easily computable characterstics (e.g. number of vertices, total length).

We assessd the sensitivity study results by first computing the ratios $R_{i}(l, t)$ defined by Equations 2.14 and 2.15 for each error measure $(E)$ where $l$ is the degree of localization and $t$ is the simplification tolerance, (all other exploratory parameters fixed i.e. resolution, function, start point, and end point), and observing their distributions across various subsets of the data set.

$$
\begin{gather*}
R_{i}(l, t)=\frac{E(l, t)}{D_{i}(l, t)}  \tag{2.14}\\
D_{1}(l, t)=\min _{l, t} E(l, t), D_{2}(l, t)=\min _{l} E(l, t) \\
D_{3}(l, t)=\min _{t} E(-1, t), D_{4}(l, t)=E(-1, t) \tag{2.15}
\end{gather*}
$$

Transforming each recorded error measure into the four ratios $R_{i}$ normalizes the data to measure relative performance, and we interpret the fixing of $t$ in $R_{2}$ and $R_{4}$ to represent a fixed "quality" of function approximation. $R_{1}$ and $R_{2}$ measure performance relative to optimal and have a lower boundary of 1.0. The optimal performance normalizer is selected from all $105 l, t$ combinations in the case of $R_{1}$, while $R_{2}$ 's optimal performance normalizer is selected from all $21 l$ values after fixing the simplification tolerance $t$ in addition to the other exploratory parameters. $R_{3}$ and $R_{4}$ on the other hand measure performance relative to naive normals $(l=-1)$ and have a lower boundary of 0.0 .
$R_{3}$ 's normalizer is the optimal naive normal performance across the five $t$ values, and $R_{4}$ normalizes by the naive normal performance when $t$ is fixed.

The $R_{i}$ distributions were evaluated for their performance within $\left(R_{2}, R_{4}\right)$ and across $\left(R_{1}, R_{3}\right)$ simplification thresholds. Well-performing distributions were characterized by peaks near the lower boundary and a more rapid convergence towards zero in comparison with their peers. Poor-performing distributions on the other hand were typically flatter in comparison, and if peaked would often rise from near zero at the boundary. When evaluating $R_{3}$ and $R_{4}$, we looked for the rapid convergence towards zero of a well-performing distribution to occur before 1 - indicating that those distributions were rarely outperformed by naive normals.

We assessed over 500 of these distribution collections and found that naive normals were rarely optimal. Unfortunately, there was no clear pattern for selecting an optimal $l, t$ combination. However, at low resolutions and high simplification tolerances, which we interpret as being most representative of our data, localization values of 25 and 50 did appear to stand out. We then repeated a smaller version of the study, varying $l$ values from $5-50$. Based on those results we produced spread vectors for every multiple of 5 in [25,50] for inclusion in our final results.

## 3 Results and Discussion

The first goal of this work is to numerically capture the visually apparent flankparallel supression fire effects observed during the Sycan prescribed burn. The second goal is to observe the effect of flank-parallel suppression fires in fully physics-based fire simulation. In each case, we seek to identify measurable flank fire acceleration as evidenced by an increase in ROS through time. To achieve this, we identified the need for a robust ROS data set to accurately characterize each fire's local behavior - especially its direction of spread. We developed curvature proxy driven normals to determine direction of spread according to the local properties of a pixelwise fire perimeter's idealized global shape irrespective of any pixel-level noise.

Our process for measuring ROS produces a collection of vectors in space, whose coordinates are measured in pixels. Vector magnitudes are scaled to represent ROS as measured in $\mathrm{m} / \mathrm{s}$ given the known spatial and temporal resolutions of each image stack. Vector direction is measured in radians by $\theta \in(-\pi, \pi)$, and computed relative to forward spread. Throughout this section we use both a fine and a coarse categorization of vectors according to their direction of spread, which are both summarized in Table 3.1, and use dashed and solid lines to distinguish between treatment and control simulations respectively when they appear together in the same figure.

| Coarse Categorization |  |  |
| :---: | :---: | :---: |
| Spread Direction | $\theta$-range | Color |
| Forward | $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ | Blue |
| Left Flank | $\left(\frac{\pi}{4}, \pi\right)$ | Red |
| Right Flank | $\left(\frac{-\pi}{4},-\pi\right)$ | Green |
| Fine Categorization |  |  |
| Categories | $\theta$-ranges $(+),(-)$ | Color |
| $\pm 1$ | $\left(\frac{\pi}{18}, \frac{3 \pi}{18}\right),\left(\frac{-3 \pi}{18}, \frac{-\pi}{18}\right)$ | Blue |
| $\pm 2$ | $\left(\frac{3 \pi}{18}, \frac{5 \pi}{18}\right),\left(\frac{-5 \pi}{18}, \frac{-3 \pi}{18}\right)$ | Green |
| $\pm 3$ | $\left(\frac{5 \pi}{18}, \frac{7 \pi}{18}\right),\left(\frac{-7 \pi}{18}, \frac{-5 \pi}{18}\right)$ | Red |
| $\pm 4$ | $\left(\frac{7 \pi}{18}, \frac{9 \pi}{18}\right),\left(\frac{-9 \pi}{18}, \frac{-7 \pi}{18}\right)$ | Purple |

Table 3.1: Vector Classification and Color Schemes
The remainder of the section has been organized into three subsections, which are sequenced to increase (both within and between each subsection) in complexity and the degree of data processing. In Section 3.1 we examine the results directly as vectors in space and the relationship between their direction and magnitude. From this relatively raw unprocessed perspective, we intend to provide assurance that spread vectors derived from curvature proxy driven normals accurately capture fire behavior. Section 3.2 contains our primary results: ROS time series. Here spread vectors are aggregated by direction and time
step across all localization and temperature threshold parameter combinations before reporting the quantile statistics of their magnitudes. We proceed from individual simulations with coarse directional classifications for which only the median and $95^{\text {th }}$ percentile are presented to the smoothed differences of treatment and control simulations across 20 evenly spaced percentiles from $50^{\text {th }}$ to $97.5^{\text {th }}$. We conclude by transitioning from temporal to spatial aggregation in Section 3.3, where we provide pseudocolored images of pixelwise medians and IQRs for both direction and magnitude and an examination of the distribution of pixelwise IQRs for every data set.

### 3.1 Vectors and Scatter Plots

We present the simplest case of manufactured elliptical spread first, as an initial sanity test and baseline of comparison, before proceeding to the more complicated simulation output and prescribed burn results. Gaussian curves (Equation 3.1) have been fit to several data sets to emphasize their overall arched shape. The tables containing curve fit parameters and their RMSEs serve to summarize the results for the simulations not depicted with figures, and as a point of comparison between the simulation and prescribed burn results. The curve fit parameters $a$ (the vertical scaling factor) and $b$ (the axis of symmetry) are the primary comparison points, while $c$ (the spread factor) is included for completeness. We note here that temperature threshold and localization parameters are fixed throughout this subsection at $l=25, T_{\text {sim }}=400^{\circ} \mathrm{C}$, and $T_{\text {Sycan }}=240^{\circ} \mathrm{C}$.

$$
\begin{equation*}
f(x)=a \exp \left(-\frac{(x-b)^{2}}{2 c^{2}}\right) \tag{3.1}
\end{equation*}
$$

Figure 3.1 displays the spread vectors generated by the method of curvature driven normals for a pair of manufactured elliptical spread polylines. The manufactured growth was by 1 unit on the semi-minor axis, and by 2 units on the semi-major axis. The forward vector magnitudes ranged from 1.44 units to 2.00 units with a mean of 1.75 units, while both sets of flanking vector magnitudes ranged from 0.99 units to 1.44 units with a mean of 1.13 units.


Figure 3.1: Manufactured Elliptical Growth: Vectors in Space (left), Direction vs Magnitude (right)

We remark that the total range of spread values is exactly in agreement with the amount of manufactured growth, and find the average magnitudes to be reasonable given the coarse classification bounds from Table 3.1. We found these results compelling enough to consider the sanity test passed, and proceed with assessing the more complicated spread patterns generated from our simulations and the Sycan prescribed burn.

Figure 3.2 contains results for the $13^{\text {th }}$ time step of $12 \mathrm{~m} / \mathrm{s}$ control simulation output. The overall shape of the data in the scatter plot is similar to that in Figure 3.1 as emphasized by the Gaussian curve.


Figure 3.2: Control Simulation: $12 \mathrm{~m} / \mathrm{s}$ winds, $t=13 \mathrm{~s}$
In Figure 3.3 spread vectors for the entire $10 \mathrm{~m} / \mathrm{s}$ control simulation are depicted on the left and their scatter plot, including a Gaussian curve fit, is in the center. On the right we have Gaussian curves for each simulation (blue) and Sycan (red). Tables 3.2 and 3.3 summarize the Gaussian curve fitting results for this section's fixed temperature threshold and localization parameters in terms of RMSE, and the curve fit parameters $a, b$, and $c$.


Figure 3.3: Control Simulation: $10 \mathrm{~m} / \mathrm{s}$ winds, all time steps and Gaussian curve fits for all simulations and Sycan at middle thresholds

We observe first that the simulation's global shape appears to be convex,
and that the division into forward and flank sections appears to be an adequate characterization across the $10 \mathrm{~m} / \mathrm{s}$ simulation's entirety. Turning our attention to the scatter plot, we see that it is generally arch shaped with a peak near 0 , and includes some large outliers in the forward section, corresponding to the visible "bursts" of spread visible in the spatial plot. Then, we can see that every Gaussian curve fit to a simulation is centered near 0 , that the treatment and control pairs are nearly identical for the slower wind speeds, and that the peak of the Sycan curve fit is shifted to the right.

|  | Treatment |  |  |  |  | $c$ | Control |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wind Speed | RMSE | $a$ | $b$ | $c$ | RMSE | $a$ | $b$ | $c$ |
| $4 \mathrm{~m} / \mathrm{s}$ | 0.14 | 0.63 | -0.06 | 1.31 | 0.13 | 0.61 | -0.03 | 1.31 |
| $6 \mathrm{~m} / \mathrm{s}$ | 0.20 | 1.08 | -0.04 | 1.00 | 0.17 | 1.02 | -0.01 | 1.00 |
| $8 \mathrm{~m} / \mathrm{s}$ | 0.34 | 1.65 | -0.04 | 0.90 | 0.28 | 1.61 | -0.01 | 0.88 |
| $10 \mathrm{~m} / \mathrm{s}$ | 0.31 | 2.38 | -0.04 | 0.80 | 0.33 | 2.19 | -0.01 | 0.82 |
| $12 \mathrm{~m} / \mathrm{s}$ | 0.33 | 2.51 | -0.03 | 0.83 | 0.42 | 2.69 | -0.03 | 0.81 |

Table 3.2: Gaussian Curve Fitting: Simulations
Figure 3.4 displays the Sycan spread vectors colored according to their coarse classification and orange flank-parallel suppression fire vectors (which are not included in any further analysis). Figure 3.5 displays scatter plots for the Sycan burn, which subdivide the data into the time frames $(0,40),(40,65)$, and $(65,146)$ according to a visual assessment of the time series presented in Figure 3.8.


Figure 3.4: Sycan Spread Vectors in Space
We note here that fitting smoothed parametric splines[35] to the adjusted convex hull perimeters was explored as an alternative to curvature proxy driven normals with the Sycan data set. The spread vectors created normal to the smoothed parametric splines differed significantly from curvature proxy driven normals across the head fire sections where they were visually less directionally coherent through time.


Figure 3.5: Sycan Scatter Plots $(\mathrm{L}: t \in[0,40) \mathrm{C}: t \in(40,65) \mathrm{R}: t \in(65,146))$

In comparison to the simulation scatter plots, the general arch shape is not evident, but we note the relative dearth of points representing forward ROS less than $0.25 \mathrm{~m} / \mathrm{s}$ during the first time frame, and less than $0.5 \mathrm{~m} / \mathrm{s}$, during the second, while the third time frame's scatter plot is clearly not arch shaped and is instead responsible for the shifted peak in Figure 3.3.

At first glance, it appears that the classification of Sycan spread vectors in Figure 3.4 is generally accurate, but ruined by edge effects as the fire burned past the image frame resulting in a "phantom trail" of blue vectors traveling up the image's left edge. However, cross-referencing Figure 3.4 against Figure 2.3 shows that the "phantom trail" appears to clearly reflect reality.

| Time Span | RMSE | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: | :---: |
| $(0,146)$ | 0.49 | 0.92 | -0.31 | 1.32 |
| $(0,40)$ | 0.29 | 0.90 | 0.00 | 0.86 |
| $(40,65)$ | 0.40 | 1.13 | 0.05 | 1.08 |
| $(65,146)$ | 0.50 | 1.24 | -0.41 | 1.08 |

Table 3.3: Gaussian Curve Fitting: Sycan

### 3.2 Time Series

Our primary results regarding the production of measurable flank fire accelerations in the presence of flank-parallel suppression firelines are contained here. Figures 3.8 and 3.9 capture the flank acceleration of the Sycan prescribed burn, then (aided by the fine categorization defined in Table 3.1) Figures 3.12, 3.13, and 3.14 capture accelerations towards the suppression fire in the simulation experiments, with one exception.

After aggregating spread vectors by direction at each timestep, we found that the ROS distributions were frequently non-Gaussian (e.g. multi-peaked and/or skewed). Thus, our time series are constructed from quantile statistics, specifically the median and $95^{\text {th }}$ percentile. Figures $3.6-3.9$ each contain two plots. Plots on the left further aggregate spread vectors across temperature thresholds and localization values then report the relevant quantile statistics with a shaded $95 \%$ confidence interval computed from 1000 bootstrap samples. While plots on the right display a distinct time series for each combination of temperature threshold and localization value. The $95 \%$ confidence intervals are frequently almost imperceptible. Thus, Figures 3.10 and 3.11 , which display the difference in ROS between the treatment and control simulations, were computed directly. These differenced time series feature a $\pm 1 \mathrm{px}= \pm 0.25 \mathrm{~m} / \mathrm{s}$ window shaded in gray and display time relative to suppression fire ignition.


Figure 3.6: $8 \mathrm{~m} / \mathrm{s}$ median time series


Figure $3.7: 8 \mathrm{~m} / \mathrm{s} 95$ th percentile time series

Features of note in Figure 3.6 include the oscillation of the right flank above all other flanks from 35 s on, and the upward trend in forward ROS over the first $\approx 40 \mathrm{~s}$, a feature shared by both the 4 and $6 \mathrm{~m} / \mathrm{s}$ simulations. The most striking feature is displayed in Figure 3.6, the simultaneous spike in both forward and right flank ROS around 40s.


Figure 3.8: Sycan median time series


Figure 3.9: Sycan 95th percentile time series

Turning our attention to the Sycan prescribed burn time series (Figures 3.8 and 3.9), we can see that they captured the right flank accelleration that we were expecting to find. The first feature of note though, is the burst of ROS in all directions which occurs from about the $40^{\text {th }}$ to $60^{\text {th }}$ second. After this, we see the left flank drop off to nearly zero spread, while the right flank, and forward spread burst forward together, after which the right flank maintains a relatively high ROS until it merges with the suppression fire, at which point, the left flank experiences a short burst in the opposite direction towards its suppression fire.


Figure 3.10: Treatment ROS - Control ROS, shaded section indicates +/1pixel/time step


Figure 3.11: Treatment ROS - Control ROS, shaded section indicates +/1pixel/time step

Figures 3.10 and 3.11 make it appear that across all wind speeds the addition of a suppression fire parallel to the flank had almost no impact on the flank rates of spread because most differences were contained inside of a $\pm 1 \mathrm{px}$ window. The few places where the flank spread rates differed by more than a pixel at any one time step were all at the $95^{\text {th }}$ percentile, which may be attributable to flank
spread vectors at the $95^{\text {th }}$ percentile being quite close to the forward threshold direction, as evidenced by the arch shaped scatter plots. In contrast, there were some signficant differences in forward spread rates, especially at the higher wind speeds, which displayed significant variation prior to ignition of the suppression fire. However, there is a pattern of increasing flank ROS in the treatment simulation after, ignition of the suppression fire, and at the $95^{\text {th }}$ percentile both of the 4 and $6 \mathrm{~m} / \mathrm{s}$ right flank differences exit the 1 px window.

At their core, figures $3.12,3.13$, and 3.14 are quantile difference time series. They were computed in the same fashion as Figures 3.10 and 3.11 prior to being smoothed. In addition, the data was subdivided according to the fine categorization of Table 3.1, and results are included for 20 evenly spaced quantile time series from the interval [0.5, 0.975] (distinguished by darkness) for each directional category. Smoothing was done with a one-dimensional Gaussian filter $(\sigma=3)$ to highlight the patterns that emerged from the finer categorization [35]. We note that while smoothing does highlight the overall pattern, it also reduces the absolute magnitudes. The patterns are clearest at wind speeds of 8 and $10 \mathrm{~m} / \mathrm{s}$, depicted in 3.12 and 3.14 respectively.


Figure 3.12: Smoothed treatment-control fine categorization $8 \mathrm{~m} / \mathrm{s}$
The negative $\theta$ categories are on the right, or near suppression fire side, and depicted with dotted curves. The influence of the suppression fire becomes clear when evaluating how the difference in spread between treatment and control simulations varies between "equally left and right" sub categories i.e. when comparing dotted lines to solid lines of the same color. The $8 \mathrm{~m} / \mathrm{s}$ simulation makes the pattern exceptionally easy to spot because it displays increased spread across its entire suppression fire side over about the final 5 seconds. We note that the strongest effect is seen in the near forward categories of $\pm 1,2$ and the flat character of the first 10 seconds. While there is some variation prior to
ignition of the suppression fire, the greatest effect comes around 10 seconds after ignition, and corresponds with the paired forward and flank spike in Figure 3.7.


Figure 3.13: Smoothed treatment-control fine categorization 4 and $6 \mathrm{~m} / \mathrm{s}$


Figure 3.14: Smoothed treatment-control fine categorization 10 and $12 \mathrm{~m} / \mathrm{s}$

The general pattern of fire side accelleration persists across all wind speeds except for the $12 \mathrm{~m} / \mathrm{s}$ simulation. In addition, the categories which display the clearest difference vary with wind speed such that slower wind speeds experience
a more pure flank, accelleration, while at higher speeds, categories $\pm 1,2$, display the clearest pattern.

### 3.3 Images

While the vector classification scheme of Table 3.1 was used extensively in sections 3.2 and 3.1, we abandon it here in favor of the more granular pixelwise aggregation of spread vectors. Figures 3.15 and 3.16 summarize each fire's progress through space in two images - one for ROS and another for direction. In addition to these summary images, we have also included and summarized the uncertainty introduced by aggregating across a range of temperature thresholds and localization values. Figure 3.17 displays all of the distribution estimates for both ROS and direction uncertainties. The tail space of some uncertainty distribution estimates was cut off in the figure to enhance its legibility - this information is summarized in Tables 3.4 and 3.5.


Figure 3.15: Pixelwise ROS and Direction: 10m/s Treatment


Figure 3.16: Pixelwise ROS and Direction: Sycan Prescribed Burn

In Figure 3.15 we can visually observe the magnetic pull of the suppression fire in a high wind speed situatuation. In this simulation, rather than driving a significant expansion perpendicular to forward spread, you can see the high rate of spread section curve to right. In addition we note the minimal amount of uncertainty introduced through the temperature threshold and localization parameters. Almost all of the the $\theta$ IQRs are less then 0.056 radians, which is approximately $3.2^{\circ}$. And while there are some high uncertatinty spots in terms of ROS, the vast majority are less than $0.5 \mathrm{~m} / \mathrm{s}$, which is a $\pm 1 \mathrm{px}$ window. The Sycan fire (Figure 3.16) displays even less uncertainty, and we note that directional uncertainty appears concentrated just to the left and right of forward spread in both images.


Figure 3.17: Kernel Density Estimates of pixelwise ROS and Direction IQRs

|  | Percentile |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Fire | 80 | 90 | 95 | 97.5 |
| Control $4 \mathrm{~m} / \mathrm{s}$ | 0.23 | 0.25 | 0.26 | 0.34 |
| Treatment $4 \mathrm{~m} / \mathrm{s}$ | 0.24 | 0.25 | 0.26 | 0.31 |
| Control $6 \mathrm{~m} / \mathrm{s}$ | 0.25 | 0.29 | 0.42 | 0.51 |
| Treatment $6 \mathrm{~m} / \mathrm{s}$ | 0.25 | 0.30 | 0.42 | 0.53 |
| Control $8 \mathrm{~m} / \mathrm{s}$ | 0.25 | 0.41 | 0.59 | 0.80 |
| Treatment $8 \mathrm{~m} / \mathrm{s}$ | 0.27 | 0.48 | 0.75 | 1.11 |
| Control $10 \mathrm{~m} / \mathrm{s}$ | 0.30 | 0.52 | 0.82 | 1.20 |
| Treatment $10 \mathrm{~m} / \mathrm{s}$ | 0.33 | 0.56 | 0.88 | 1.28 |
| Control $12 \mathrm{~m} / \mathrm{s}$ | 0.41 | 0.72 | 1.07 | 1.47 |
| Treatment $12 \mathrm{~m} / \mathrm{s}$ | 0.36 | 0.59 | 0.88 | 1.20 |
| Sycan | 0.44 | 0.70 | 0.95 | 1.19 |

Table 3.4: ROS IQR KDE Tail Space

|  | Percentile |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Fire | 80 | 90 | 95 | 97.5 |
| Control $4 \mathrm{~m} / \mathrm{s}$ | 0.04 | 0.05 | 0.06 | 0.06 |
| Treatment $4 \mathrm{~m} / \mathrm{s}$ | 0.04 | 0.05 | 0.06 | 0.07 |
| Control $6 \mathrm{~m} / \mathrm{s}$ | 0.04 | 0.05 | 0.06 | 0.07 |
| Treatment $6 \mathrm{~m} / \mathrm{s}$ | 0.04 | 0.05 | 0.06 | 0.07 |
| Control $8 \mathrm{~m} / \mathrm{s}$ | 0.06 | 0.08 | 0.10 | 0.12 |
| Treatment $8 \mathrm{~m} / \mathrm{s}$ | 0.07 | 0.09 | 0.12 | 0.15 |
| Control $10 \mathrm{~m} / \mathrm{s}$ | 0.07 | 0.09 | 0.12 | 0.17 |
| Treatment $10 \mathrm{~m} / \mathrm{s}$ | 0.07 | 0.09 | 0.12 | 0.15 |
| Control $12 \mathrm{~m} / \mathrm{s}$ | 0.08 | 0.11 | 0.14 | 0.18 |
| Treatment $12 \mathrm{~m} / \mathrm{s}$ | 0.07 | 0.10 | 0.13 | 0.16 |
| Sycan | 0.04 | 0.06 | 0.09 | 0.11 |

Table 3.5: Direction IQR KDE Tail Space

## 4 Conclusions

In this work we have developed a new approach to computing a fire's ROS (curvature proxy driven normals) which we stress tested against manufactured elliptical fire spread, the drone captured IR image stack from a prescribed grass fire, and a suite of simulation experiments completed in WFDS. Our results largely agreed with the theoretical values associated with the manufactured elliptical fire spread, captured the dynamic fireline interaction present in the prescribed burn image stack, and determined direction of spread with a level of consistency that allowed us to tease out the subtle influence of the suppression fire in our simulation experiments. While these initial results are promising, this project was limited in time and scope, and the method of curvature proxy driven normals would benefit from both expanded testing and further refinement.

Expanded testing should include more complicated spread patterns as the simulation and prescribed burn spread patterns in this study were relatively simple. We feel it is vitally important (as the scope of this work included only one live fire) that further testing include live-fire thermal imagery at a variety of spatial scales traveling through a variety of fuels complexes. More complicated spread patterns could be obtained via simulation as well. We would suggest exploring (individually or in combination) gusting or turning winds, non-homogenous fuel beds, and varied ignition patterns (including multiple suppression fires as was the case during the Sycan prescribed burn).

While we were able to detect a subtle influence of the suppression fire on the head fire's direction and ROS in our simulation experiments, we feel this paled in comparison to what we observed in the Sycan data set. We feel this indicates two needs for future work - experimental prescribed burns including flank parallel suppression fires to better characterize their expected influence on a freely burning headfire, and further simulation studies of flank parallel suppression fires to better understand the degree to which they capture this interaction.

We suggest two paths for refining the computation of ROS from curvature proxy driven normals: reducing the limitations imposed by its basic assumptions, and development of a method for selection of the localization parameter based upon easily computable convex polyline characteristics. While the first three basic assumptions are fundamental to the method (convex global shape, spread appears normal to the global shape, and sufficiently small time steps), the fourth and fifth assumptions (starts and ends correspond to flank sections initially and exist on the image frame) are (potentially) necessary only as the method is currently implemented. Their removal could reduce or eliminate manual preprocessing and extend the method's applicable scope. We suggest the development of an automated flank identifcation method as a starting point. Expanding the parameter sensitivity study conducted in this work to include a broader array of convex functions would be a natural starting point for developing a localization parameter selection algorithm. The expanded parameter study could then be used to build a naive Bayes classifier from kernel density estimates of the relative performance error ratios across a selection of features.

The naive Bayes classifier, might roughly look like the following:

$$
l=\underset{L}{\operatorname{argmax}} \prod_{F} P\left(R_{4}(l)<0.5 \mid f\right),
$$

where $L$ is a collection of localization values and $F$ is a collection of polyline features (resolution, simplification tolerance, and error measurement fixed). Polyline features which we posit may be useful to such a classifier include total change in direction and/or the mean and variance in 100 evenly-spaced $\hat{\kappa_{2}}$ values.

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[^0]:    ${ }^{1}$ After minimal preprocessing - our simulation perimiters were cropped by $\approx 15 \mathrm{px}$ on the windward edge to conform with all assumptions.

[^1]:    ${ }^{2}$ The component perimeters of a single fireline broken by extending beyond the image frame are processed independently.
    ${ }^{3}$ We say this is a practical ordering because $\mathcal{F}$-edges can technicially transition to a vertex free (irrelevant) state.
    ${ }^{4}$ Nearly guaranteed when no time step consumes more than one corner.

[^2]:    ${ }^{5}$ Strictly positive curvature is technically a stronger condition than smooth and convex.
    ${ }^{6}$ Signed curvature may be strictly negative, depending on parametrization.

