

# The Mathematics Enthusiast

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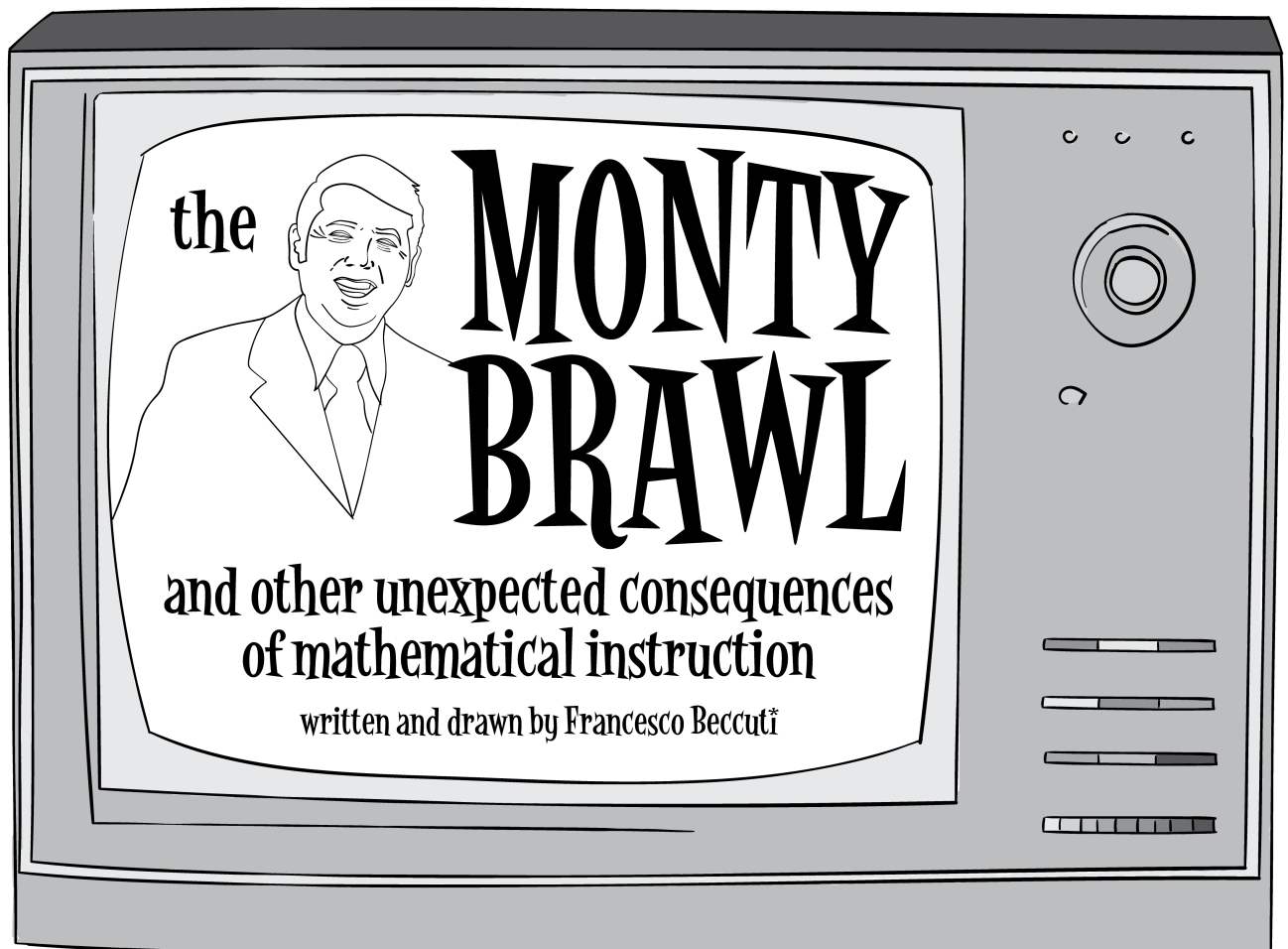
## The Monty Brawl and other unexpected consequences of mathematical instruction

Francesco Beccuti

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THE DEVELOPMENT AND DISSEMINATION OF MATHEMATICAL KNOWLEDGE AND MATHEMATICAL COMPETENCES ARE DEEMED TO BE VITAL FOR THE ADVANCEMENT OF A PROSPEROUS TECHNO-SCIENTIFIC SOCIETY



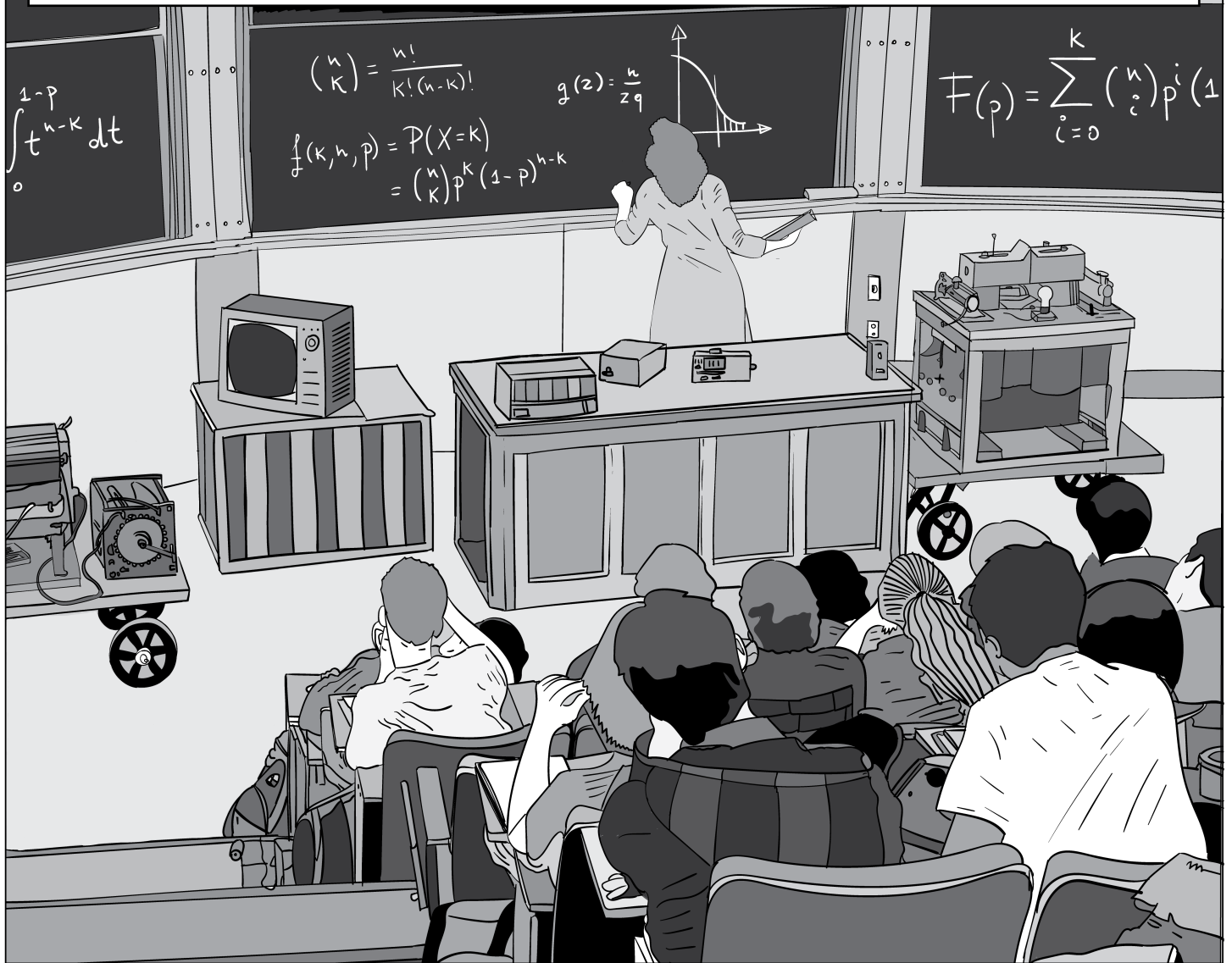
LEARNING MATHEMATICS IS CONSIDERED IMPORTANT FOR ACCESSING AND PERFORMING JOBS RELATED TO THE FUNCTIONING OF VARIOUS SECTORS OF SCIENCE AND INDUSTRY.



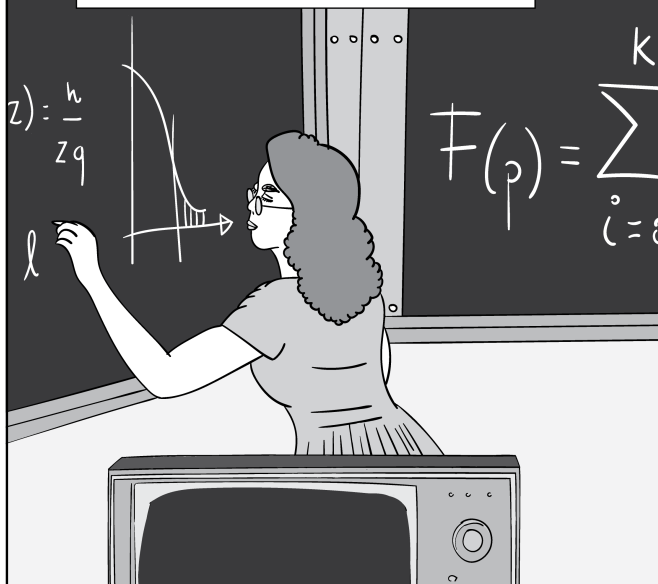
IN PARTICULAR, TEACHING TO HANDLE MATHEMATICALLY SITUATIONS INVOLVING UNCERTAINTY IS DEEMED FUNDAMENTAL FOR SUPPLYING THE WORKFORCE OF THE GROWING DATA-RELATED SECTORS AS WELL AS FOR FOSTERING PEOPLES' INFORMED PARTICIPATION TO SOCIETY.



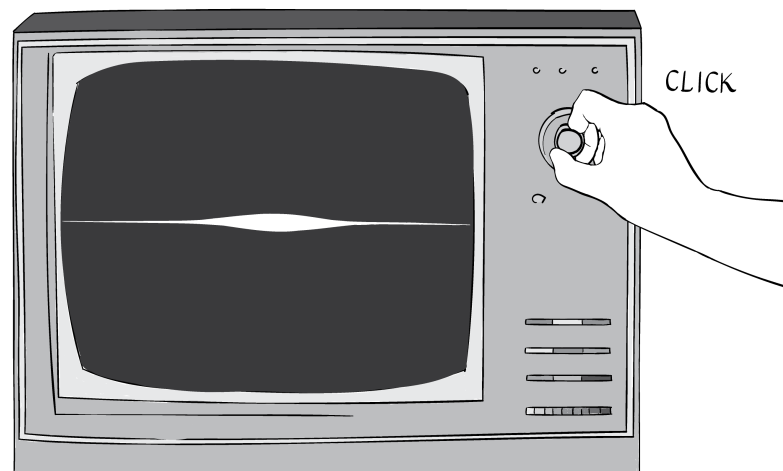
INDEED, IT IS OFTEN ASSUMED THAT THE LEARNING OF MATHEMATICAL PROBABILITY AND STATISTICS WITHIN EDUCATIONAL INSTITUTIONS WOULD GENERALLY PREPARE TO MANAGE UNCERTAIN SITUATIONS ARISING IN SCIENTIFIC, PROFESSIONAL AND CIVIC LIFE.



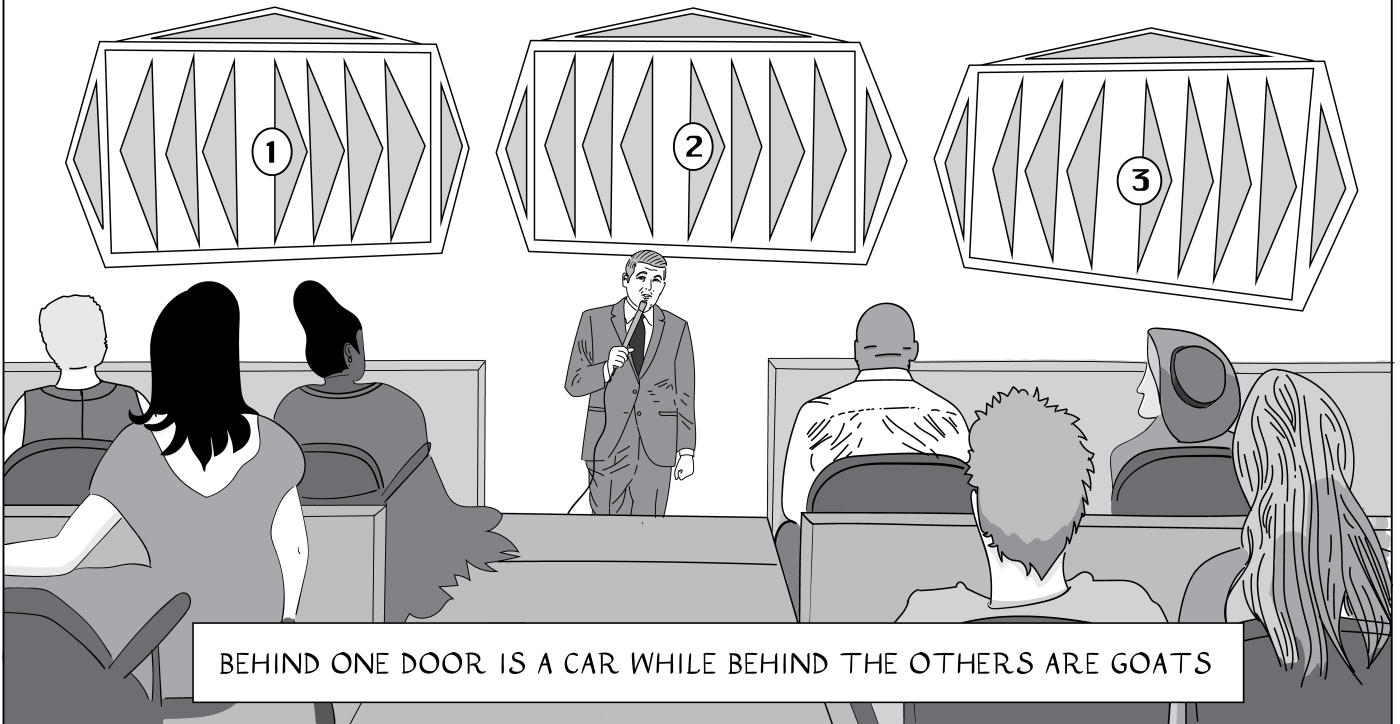
BUT IS THIS ALWAYS THE CASE?



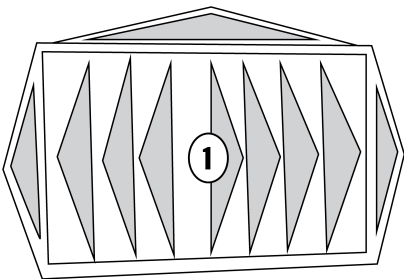
LET US CONSIDER THE STORY OF A NOTORIOUS BRAINTEASER: THE FAMOUS MONTY HALL PROBLEM.



SUPPOSE YOU'RE ON A GAME SHOW. THE HOST OF THE SHOW IS MONTY HALL AND HE GIVES YOU THE CHOICE OF THREE DOORS



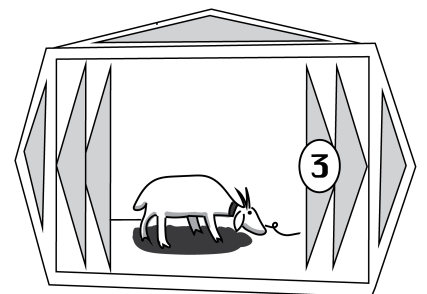
YOU CHOOSE A DOOR, SAY NO. 1



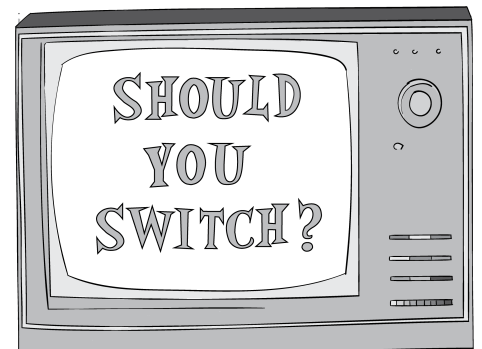
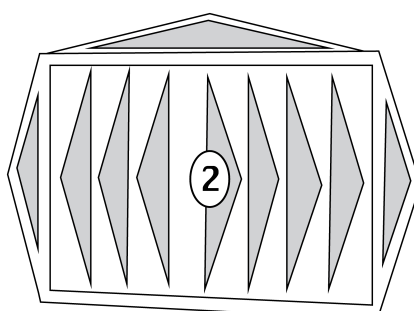
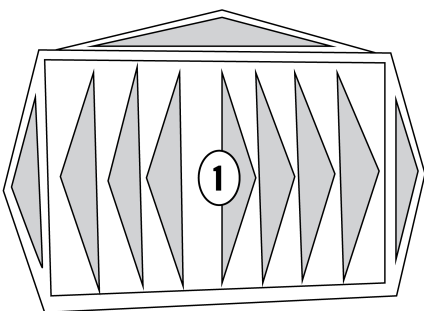
MONTY, WHO KNOWS WHAT'S BEHIND THE DOORS, OPENS ANOTHER ONE



SAY NO. 3, WHICH HAS A GOAT



HE THEN SAYS TO YOU "DO YOU WANT TO STICK WITH DOOR NO. 1 OR DO YOU WANT TO PICK DOOR NO. 2 INSTEAD?"



SO, SHOULD YOU SWITCH?



IT TURNS OUT THAT MOST PEOPLE TEND TO STICK WITH THEIR ORIGINAL CHOICE RATHER THAN PICKING THE REMAINING DOOR.

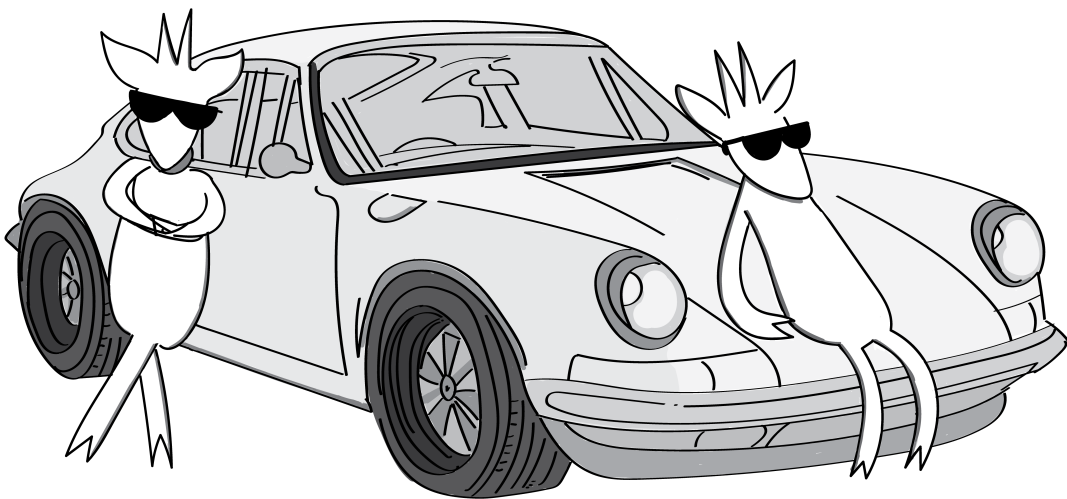


THIS IS THE CASE EVEN WHEN IT IS MADE EXPLICIT THAT THE HOST OF THE SHOW IS NOT TRYING TO TRICK THE PLAYER AND THAT HE WOULD OFFER A SWITCH BOTH IF THE PLAYER INITIALLY CHOSE A WINNING DOOR AND IF SHE CHOSE A LOSING DOOR.

INDEED, AT FIRST GLANCE IT SEEMS THAT, AFTER MONTY OPENS A DOOR, THERE IS NO ADVANTAGE IN CHANGING ONE'S ORIGINAL CHOICE.



HOWEVER, SWITCHING IS THE BEST OPTION IF ONE AIMS TO WIN THE CAR. THIS IS BECAUSE, IF YOU SWITCH, THEN YOU WIN THE CAR IF AND ONLY IF YOUR FIRST CHOICE WAS A DOOR HIDING A GOAT. CONVERSELY, IF YOU STICK WITH YOUR INITIAL CHOICE, YOU WIN IF AND ONLY IF THE LATTER WAS A DOOR HIDING THE CAR ALREADY. BUT IT IS MORE LIKELY THAT YOUR INITIAL CHOICE WAS FOR A DOOR HIDING A GOAT AND NOT THE CAR.

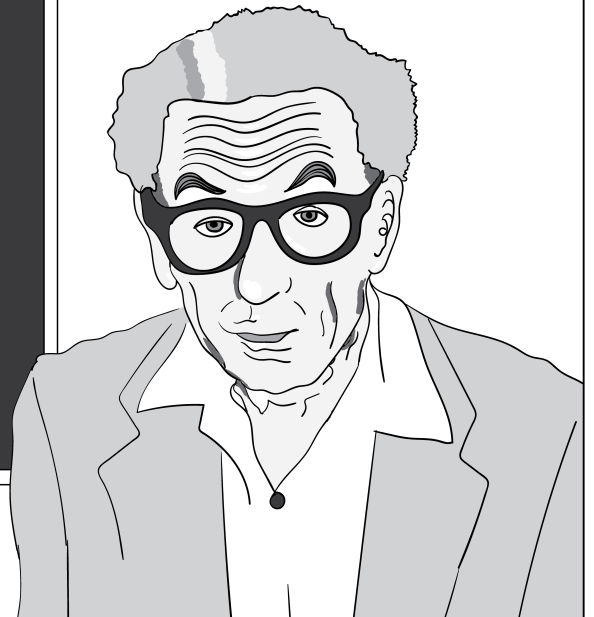


THUS, PROVIDED THAT MONTY OFFERS YOU THE OPTION TO SWITCH REGARDLESS OF YOUR INITIAL CHOICE, THEN SWITCHING ALLOWS YOU TO WIN THE CAR TWO TIMES OUT OF THREE. THIS ARGUMENT IS TYPICALLY FORMALIZED IN MATHEMATICAL TEXTBOOKS BY ARGUING FROM USUAL DEFINITIONS AND RULES OF PROBABILITY THEORY.

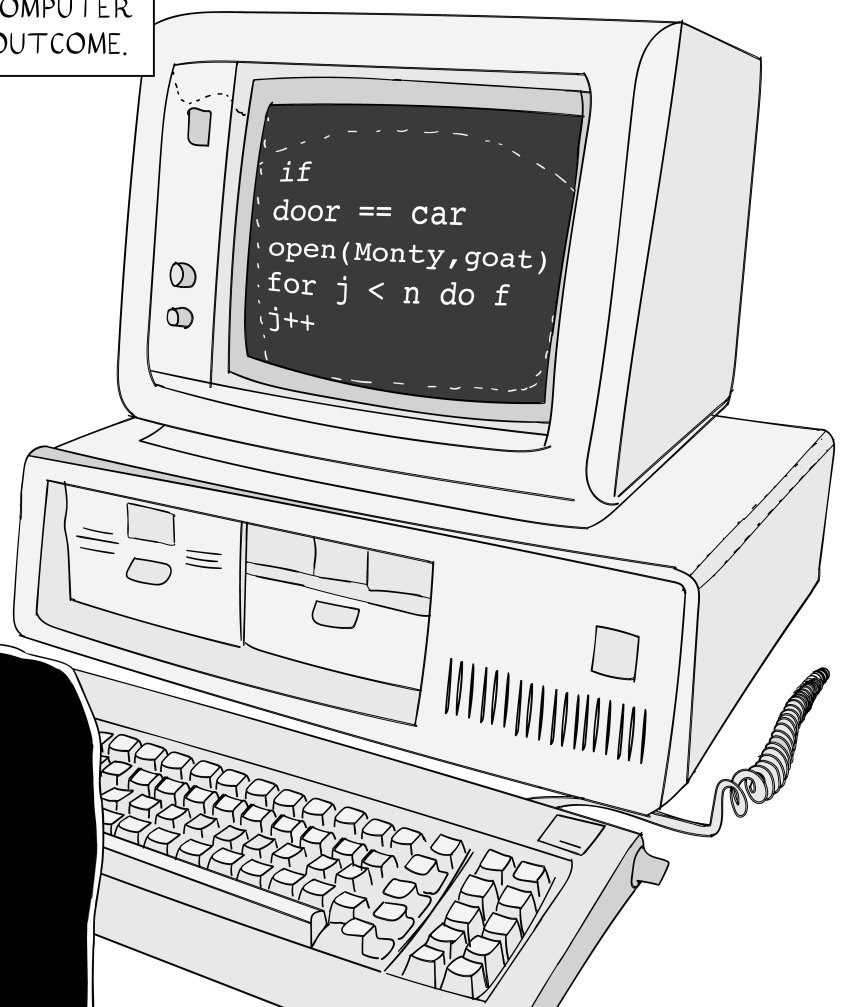
UNCONVINCED? WELL... YOU ARE NOT ALONE. EVEN CELEBRATED MATHEMATICIAN PAUL ERDŐS WAS FAMOUSLY PUZZLED FOR MONTHS BY THE MONTY HALL PROBLEM AND REMAINED SKEPTICAL OF THE FORMAL ARGUMENTS.

$$\begin{array}{l}
 P(M_3 | C_1, X_1) = 1/2 \\
 P(M_3 | C_2, X_1) = 1 \\
 P(M_3 | C_3, X_1) = 0 \\
 P(C_i) = 1/3
 \end{array}
 \quad
 \begin{array}{l}
 P(A, B, C) = P(A|B, C) \cdot P(B, C) \\
 = P(B|A, C) \cdot P(A, C) \\
 = P(C|A, B) \cdot P(A, B) \\
 = P(A, B|C) \cdot P(C) \\
 = P(A, C|B) \cdot P(B) \\
 = P(B, C|A) \cdot P(A)
 \end{array}$$

$$P(C_2 | M_3, X_1) = \frac{P(M_3, X_1 | C_2) \cdot P(C_2)}{P(M_3, X_1)} = 2/3$$



APPARENTLY, ERDŐS WAS PERSUADED ONLY AFTER HE WAS SHOWN REPEATED COMPUTER SIMULATIONS OF THE PROBLEM'S OUTCOME.

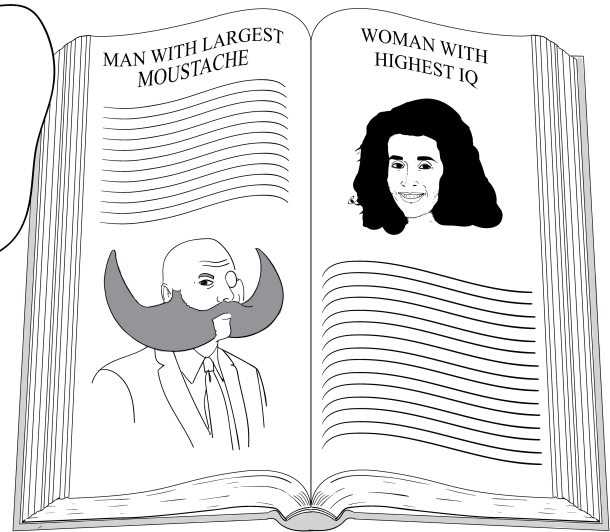




THE MONTY HALL PROBLEM WAS INITIALLY POPULARIZED BY COLUMNIST MARYLIN VOS SAVANT ON THE PARADE MAGAZINE IN 1990.

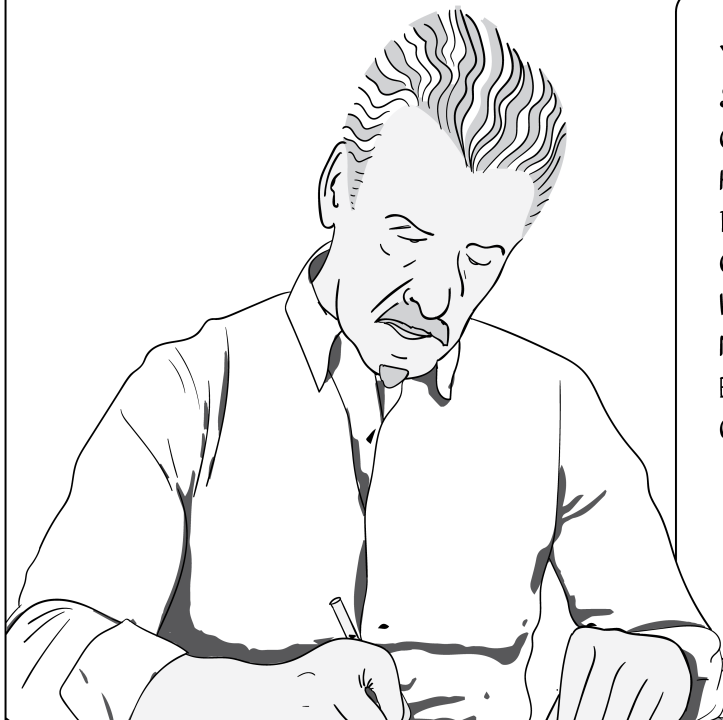
MARYLIN ADVERTIZED HERSELF AS THE PERSON WITH THE HIGHEST INTELLIGENCE QUOTIENT AND WAS LISTED AS SUCH IN THE FAMOUS GUINNESS BOOK OF WORLD RECORD.

YES, YOU SHOULD SWITCH. THE FIRST DOOR HAS A  $\frac{1}{3}$  CHANCE OF WINNING, BUT THE SECOND DOOR HAS A  $\frac{2}{3}$  CHANCE...



THUS, PERHAPS UNSURPRISINGLY, HER PROPOSED ANSWER TO THE MONTY HALL PROBLEM ATTRACTED MUCH CRITICISM.

ACCORDING TO MARILYN, NEARLY ONE THOUSAND PEOPLE HOLDING DOCTORATES WROTE TO HER, MOST CLAIMING THAT SHE WAS WRONG.



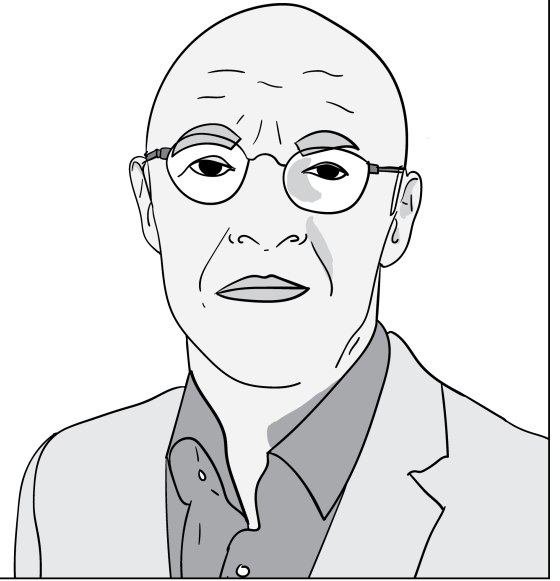
YOU BLEW IT, AND YOU BLEW IT BIG! SINCE YOU SEEM TO HAVE DIFFICULTY GRASPING THE BASIC PRINCIPLE AT WORK HERE, I'LL EXPLAIN. AFTER THE HOST REVEALS A GOAT, YOU NOW HAVE A ONE-IN-TWO CHANCE OF BEING CORRECT. WHETHER YOU CHANGE YOUR SELECTION OR NOT, THE ODDS ARE THE SAME. THERE IS ENOUGH MATHEMATICAL ILLITERACY IN THIS COUNTRY, AND WE DON'T NEED THE WORLD'S HIGHEST IQ PROPAGATING MORE. SHAME!

SCOTT SMITH, PH.D.  
UNIVERSITY OF FLORIDA

Note: The images accompanying the written words of mathematicians involved in the Monty Brawl are products of my imagination. They are intended for sole purposes of storytelling and are not meant to bear any resemblance to the physical appearance of the writers nor to depict them in a negative light.

LET ME EXPLAIN. IF ONE DOOR IS SHOWN TO BE A LOSER, THAT INFORMATION CHANGES THE PROBABILITY OF EITHER REMAINING CHOICE — NEITHER OF WHICH HAS ANY REASON TO BE MORE LIKELY — TO  $1/2$ . AS A PROFESSIONAL MATHEMATICIAN, I'M VERY CONCERNED WITH THE GENERAL PUBLIC'S LACK OF MATHEMATICAL SKILLS. PLEASE HELP BY CONFESSING YOUR ERROR AND IN THE FUTURE BEING MORE CAREFUL.

ROBERT SACHS, PH.D.  
GEORGE MASON UNIVERSITY



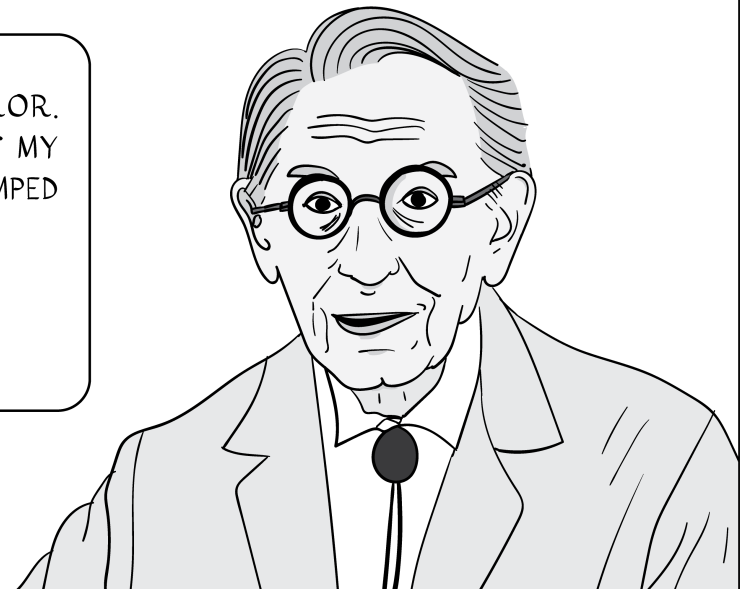
MAY I SUGGEST THAT YOU OBTAIN AND REFER TO A STANDARD TEXTBOOK ON PROBABILITY BEFORE YOU TRY TO ANSWER A QUESTION OF THIS TYPE AGAIN?

CHARLES REID, PH.D.  
UNIVERSITY OF FLORIDA



YOUR ANSWER TO THE QUESTION IS IN ERROR. BUT IF IT IS ANY CONSOLATION, MANY OF MY ACADEMIC COLLEAGUES HAVE ALSO BEEN STUMPED BY THIS PROBLEM.

BARRY PASTERNAK, PH.D.  
CALIFORNIA FACULTY ASSOCIATION





I AM IN SHOCK THAT AFTER BEING CORRECTED BY AT LEAST THREE MATHEMATICIANS, YOU STILL DO NOT SEE YOUR MISTAKE.

KENT FORD  
DICKINSON STATE UNIVERSITY

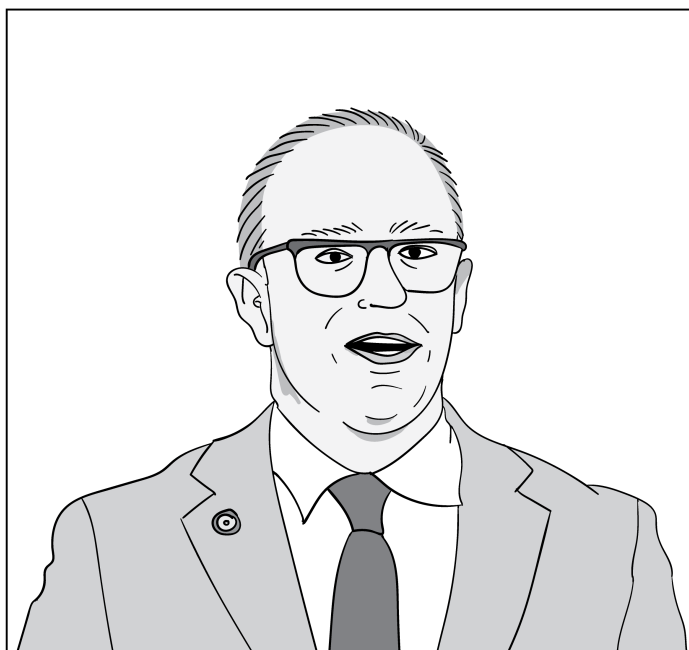
YOU ARE UTTERLY INCORRECT ABOUT THE GAME SHOW QUESTION, AND I HOPE THIS CONTROVERSY WILL CALL SOME PUBLIC ATTENTION TO THE SERIOUS NATIONAL CRISIS IN MATHEMATICAL EDUCATION. IF YOU CAN ADMIT YOUR ERROR, YOU WILL HAVE CONTRIBUTED CONSTRUCTIVELY TOWARDS THE SOLUTION OF A DEPLORABLE SITUATION. HOW MANY IRATE MATHEMATICIANS ARE NEEDED TO GET YOU TO CHANGE YOUR MIND?

E. RAY BOBO, PH.D.  
GEORGETOWN UNIVERSITY



YOU MADE A MISTAKE, BUT LOOK AT THE POSITIVE SIDE. IF ALL THOSE PH.D.'S WERE WRONG, THE COUNTRY WOULD BE IN SOME VERY SERIOUS TROUBLE.

EVERETT HARMAN, PH.D.  
U.S. ARMY RESEARCH INSTITUTE



OF COURSE NOT ALL MATHEMATICIANS WROTE THE SAME.



YOU ARE INDEED CORRECT. MY COLLEAGUES AT WORK HAD A BALL WITH THIS PROBLEM, AND I DARE SAY THAT MOST OF THEM, INCLUDING ME AT FIRST, THOUGHT YOU WERE WRONG!

SETH KALSON, PH.D.  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

I PUT MY SOLUTION OF THE PROBLEM ON THE BULLETIN BOARD IN THE PHYSICS DEPARTMENT OFFICE AT THE NAVAL ACADEMY, FOLLOWING IT WITH A DECLARATION THAT YOU WERE RIGHT. ALL MORNING I TOOK A LOT OF CRITICISM AND ABUSE FROM MY COLLEAGUES, BUT BY LATE IN THE AFTERNOON MOST OF THEM CAME AROUND. I EVEN WON A FREE DINNER FROM ONE OVERCONFIDENT PROFESSOR.

EUGENE MOSCA, PH.D.  
U.S. NAVAL ACADEMY, ANNAPOLIS, MARYLAND



AND THE MONTY BRAWL INITIATED BY MARILYN PROMPTED MANY SCHOOL TEACHERS TO TRY OUT THE EXPERIMENT BY ENGAGING THEIR STUDENTS WITH CUPS AND PENNIES.

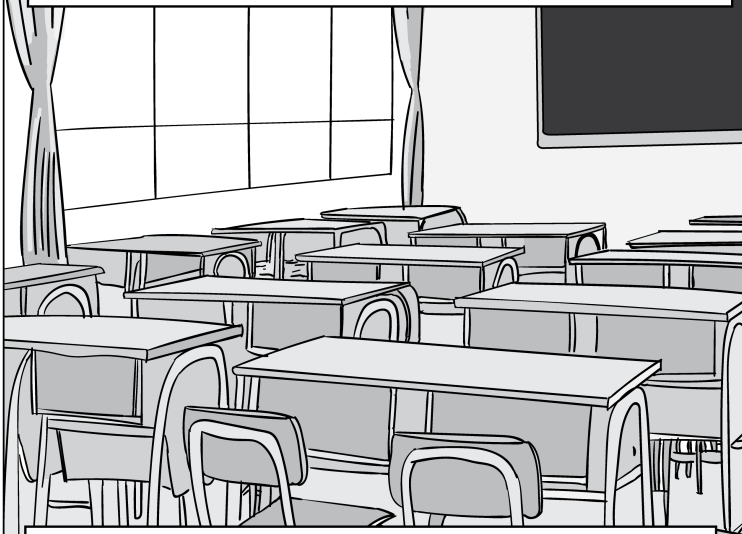


OUR CLASS, WITH UNBRIDLED ENTHUSIASM, IS PROUD TO ANNOUNCE THAT OUR DATA SUPPORT YOUR POSITION. THANK YOU SO MUCH FOR YOUR FAITH IN AMERICA'S EDUCATORS TO SOLVE THIS.

JACKIE CHARLES,  
HENRY GRADY ELEMENTARY, TAMPA, FLORIDA

AS ONE OF THE ANGRY MATHEMATICIANS SUGGESTED, IT SEEMS THAT THE MONTY BRAWL DOES TELL US SOMETHING ABOUT THE STATE OF THE SYSTEM OF INSTRUCTION.

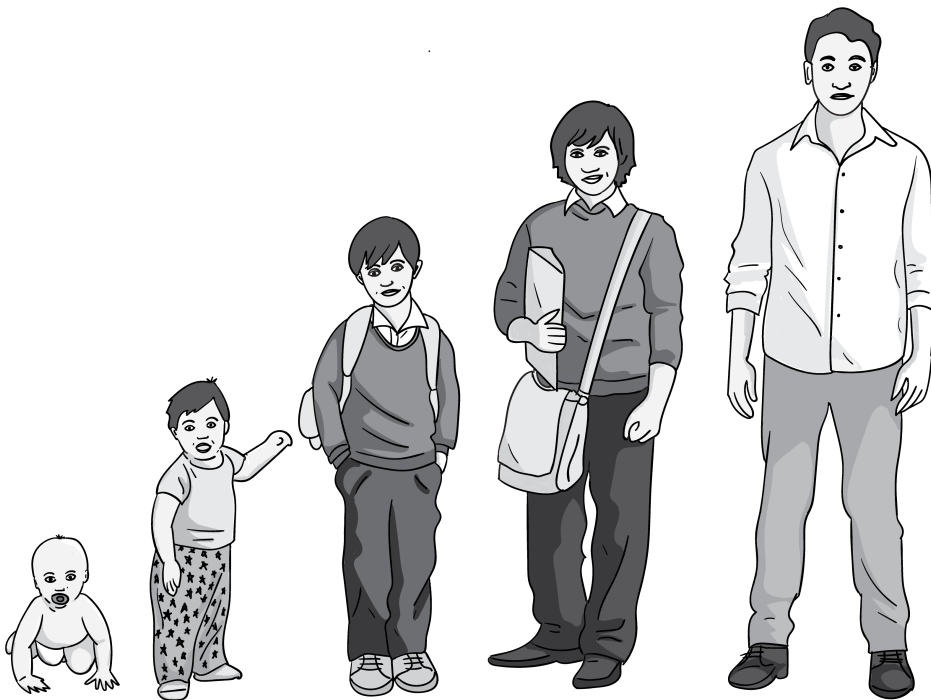
THIS IS THE TENDENCY TO CONSIDER A SET OF EVENTS AS EQUIPROBABLE, IN A SIMILAR WAY AS THE OUTCOMES OF AN IDEALIZED TOSS OF AN IDEALIZED COIN ARE CONSIDERED AS EQUALLY LIKELY IN SCHOOL MATHEMATICS WORD PROBLEMS.



DEEMING UNCONSEQUENTIAL TO SWITCH DOORS IN THE MONTY HALL PROBLEM SEEMS TO BE EVIDENCE OF A MORE GENERAL EQUIPROBABILITY EFFECT.

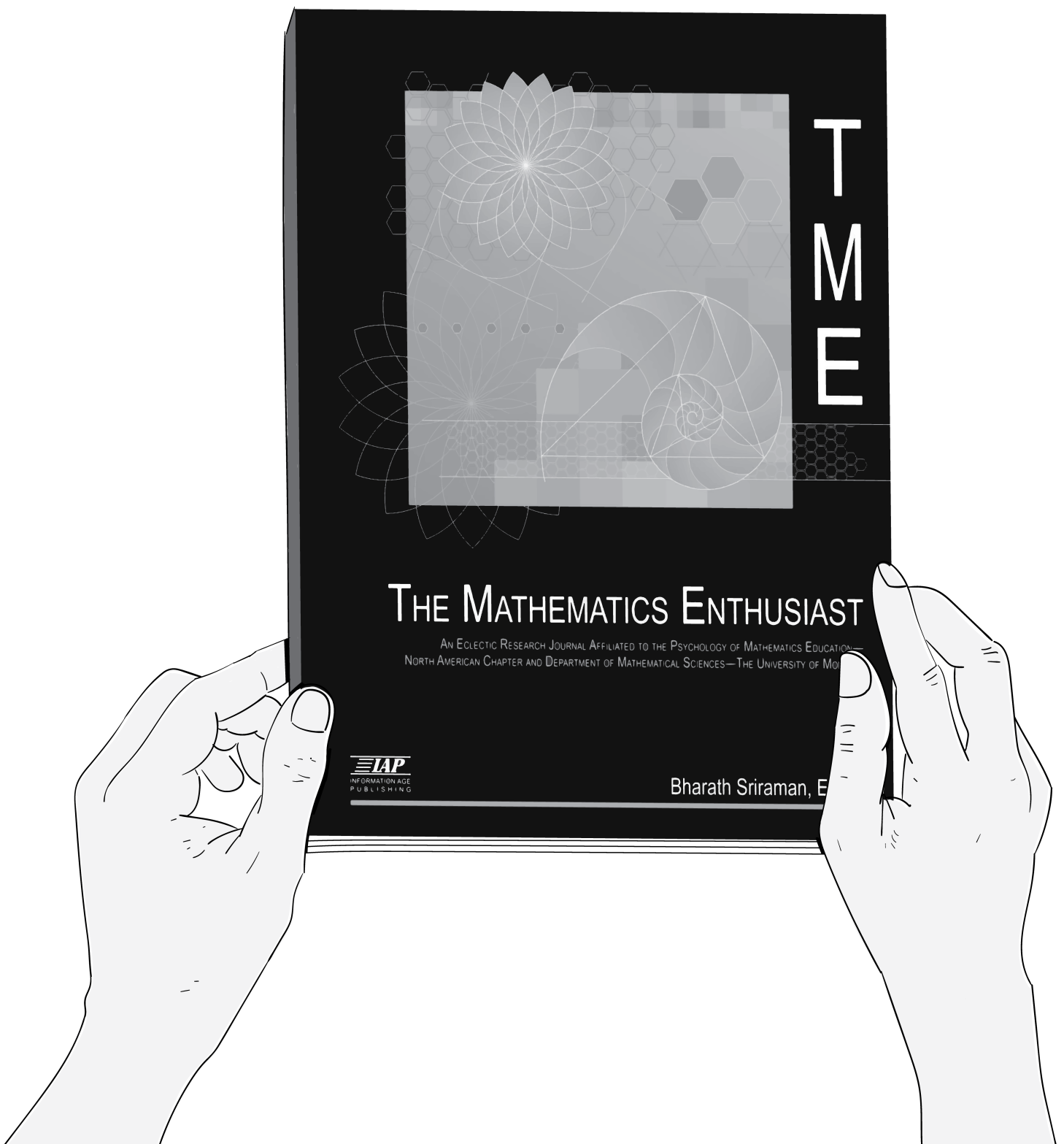


THIS EFFECT CORRELATES WITH AGE AND WITH PROGRESS IN FORMAL EDUCATION.



PEOPLE HAVING HAD A DEEP EDUCATION IN MATHEMATICS SEEM TO BE PARTICULARLY SUBJECT TO THE EQUIPROBABILITY EFFECT. INDEED MOST PROBLEMS ENCOUNTERED IN PROBABILITY CLASSROOM HAVE TO DO WITH IDEALIZED EVENTS WHOSE OUTCOME CAN BE USUALLY CONSIDERED EQUIPROBABLE.

IF THE MONTY HALL PROBLEM IS ONLY AN ABSTRACT PUZZLE, OTHER MORE CONCRETE CASES IN WHICH PEOPLE AND EXPERTS WERE LED ASTRAY AS A CONSEQUENCE OF THEIR EDUCATION IN PROBABILITY AND STATISTICS WERE DISCUSSED IN THIS JOURNAL.



## The case of Malcom and Janet Collins



IN 1964 IN CALIFORNIA, A PURSE WAS STOLEN FROM A LADY IN AN ALLEY. WITNESSES TO THE EVENT LATER TESTIFIED THAT THE BURGLARY WAS COMMITTED BY A YOUNG WHITE WOMAN WITH A DARK BLOND PONYTAIL WHO RAN AWAY IN A YELLOW CAR DRIVEN BY A BLACK MAN WEARING A BEARD AND MUSTACHE.

JANET AND MALCOM COLLINS WERE ARRESTED PRIMARILY BECAUSE THEY RESIDED IN THE NEARBIES AND SOMEWHAT RESEMBLED THE WITNESSES' DESCRIPTION.



AN UNKNOWN COLLEGE MATHEMATICAL INSTRUCTOR WAS CALLED TO TESTIFY AT THE TRIAL AND ASSERTED THAT THE PROBABILITY OF THE COLLINSSES BEING INNOCENT WAS JUST ONE OVER 12 MILLION. HE OBTAINED THIS FIGURE BY MULTIPLICATING THE FOLLOWING NUMBERS, WHICH HE TOOK TO BE THE ODDS OF THE SINGLE EVENTS INTO WHICH THE DESCRIPTION GIVEN BY THE WITNESSES MAY BE DECOMPOSED.



BLACK MAN WITH BEARD	$1/10$
MAN WITH MOUSTACHE	$1/4$
WOMAN WITH PONY TAIL	$1/10$
WHITE WOMAN WITH BLOND HAIR	$1/3$
YELLOW MOTOR CAR	$1/10$
INTERRACIAL COUPLE IN A CAR	$1/1000$

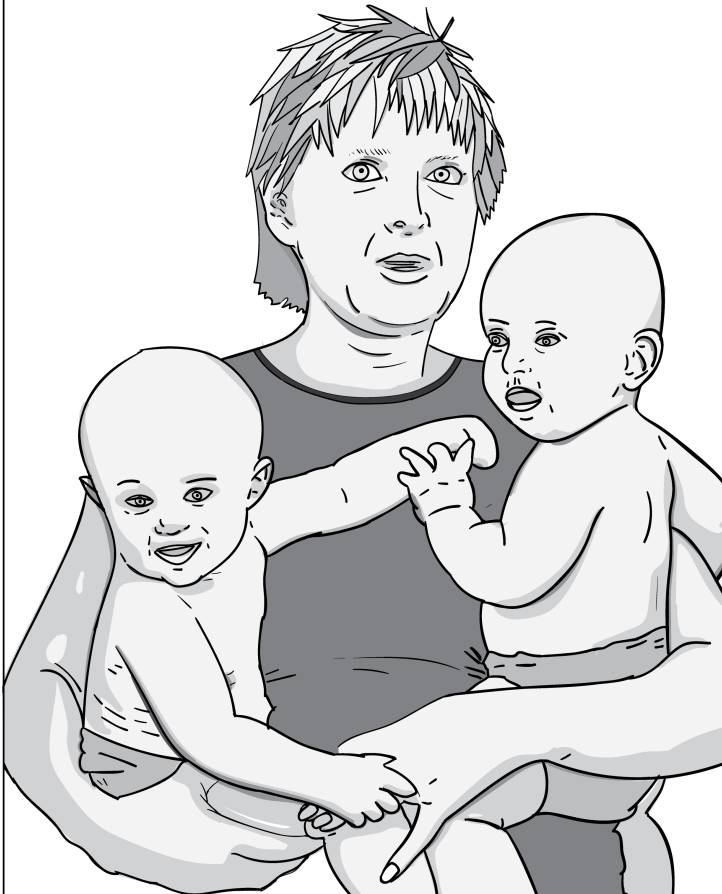
CLEARLY, THESE ODDS WERE JUST ASSUMED WITHOUT FURTHER EXPLANATION. MOREOVER, EVEN IF THESE NUMBERS WERE CORRECT, THE EVENTS TO WHICH THEY REFER ARE NOT INDEPENDENT AND HENCE THIS MULTIPLICATION IS NOT ALLOWED.

FURTHERMORE, DESPITE THE FACT THAT IT WAS IMPROBABLE THAT ONE SPECIFIC COUPLE FITTED ALL THESE FEATURES, THERE BEING MORE THAN ONE COUPLE FITTING SUCH FEATURES (IN THE NEIGHBORHOOD OR IN THE WHOLE OF CALIFORNIA PERHAPS) WAS ACTUALLY NOT IMPROBABLE. SIMILARLY, THE FACT THAT ONE PARTICULAR PERSON WINS THE LOTTERY IS VERY SMALL, BUT THE FACT THAT SOMEONE AT ALL WINS SHOULD ACTUALLY BE HIGH.



THE SUPREME COURT OF CALIFORNIA LATER STATED THAT "THE TESTIMONY AS TO MATHEMATICAL PROBABILITY INFECTED THE CASE WITH FATAL ERROR".

## The case of Sally Clark



SALLY CLARK WAS A BRITISH SOLICITOR. SHE HAD TWO CHILDREN WHO BOTH DIED IN INFANCY WITHOUT ANY APPARENT EXPLANATION. ONE IN 1996 AND ONE TWO YEARS LATER. THE FIRST DEATH WAS CONSIDERED A CASE OF SUDDENT INFANT DEATH SYNDROME (SIDS) WHICH IS DEEMED A RARE BUT NOT IMPLAUSIBLE OCCURRENCE. THE SECOND DEATH HOWEVER RAISED SUSPICIONS AND CAUSED SALLY TO UNDERGO TRIAL FOR DOUBLE MURDER.

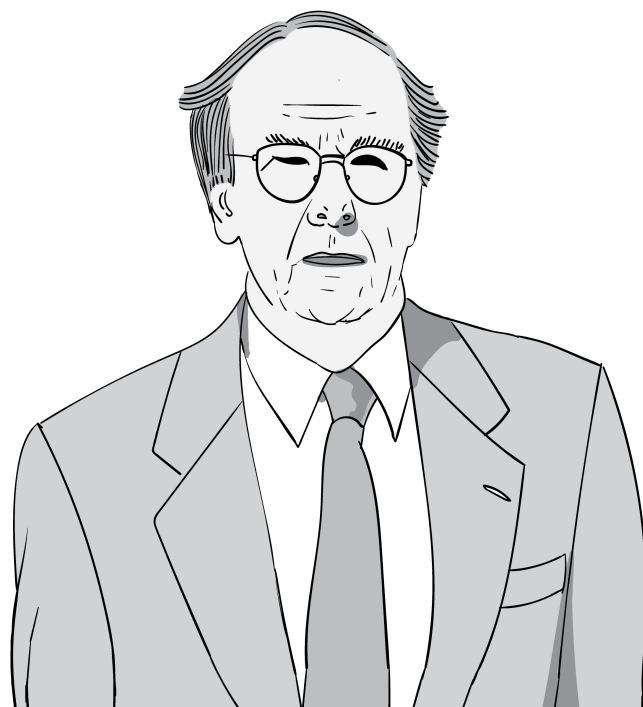


ACCORDING TO THE EXPERT SUMMONED BY THE COURT, SIR ROY MEADOW, THE ODDS AGAINST THE OCCURRENCE OF TWO SIDS IN THE SAME FAMILY WERE 73 MILLION TO ONE.

MEADOW FIRST ESTIMATED THE CHANCE OF HAVING A CHILD DIE OF SIDS AS ONE IN 8543. THEN, HE MULTIPLIED THIS NUMBER BY ITSELF AND OBTAINED THE FIGURE ABOVE.

AGAIN, EVEN IF THE FIRST ESTIMATE WERE CORRECT, THE TWO EVENTS ARE NOT INDEPENDENT (SINCE IT WAS KNOWN, SAY, THAT SIDS HAS A GENETIC COMPONENT) AND HENCE MULTIPLYING IS NOT CORRECT.

FURTHERMORE, WHILE THE PROBABILITY OF ONE PARTICULAR FAMILY INCURRING IN TWO SIDS IS VERY LOW, THE PROBABILITY OF THERE BEING ONE SUCH FAMILY SOMEWHERE IN BRITAIN (OR IN THE WORLD) IS HIGH.



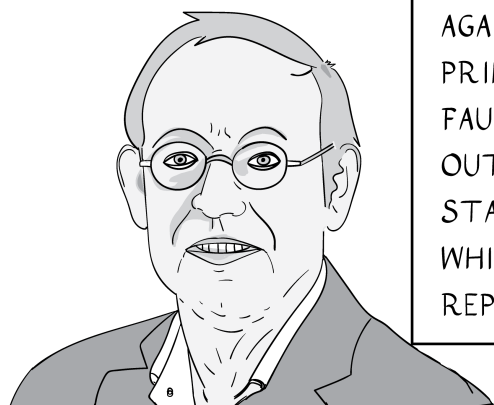
IN ANY CASE, SALLY CLARK WAS JUDGED GUILTY AND SHE SPENT THREE YEARS IN JAIL. SHE WAS LATER DISCHARGED ON APPEAL BUT NEVER EMOTIONALLY RECOVERED. SHE DIED FOUR YEARS LATER OF ALCOHOL POISONING.



THE ROYAL STATISTICAL SOCIETY OF THE UNITED KINGDOM DEEMED THIS A CASE OF AN "EXPERT WITNESS MAKING A SERIOUS STATISTICAL ERROR, ONE WHICH MAY HAVE HAD A PROFOUND EFFECT ON THE OUTCOME OF THE CASE".

# The case of Lucia de Berk

LUCIA DE BERK WAS A NURSE IN THE NETHERLANDS WHO IN 2001 WAS CHARGED WITH MULTIPLE MURDERS AFTER IT WAS DISCOVERED THAT MANY PATIENTS HAD DIED DURING HER HOSPITAL SHIFTS.



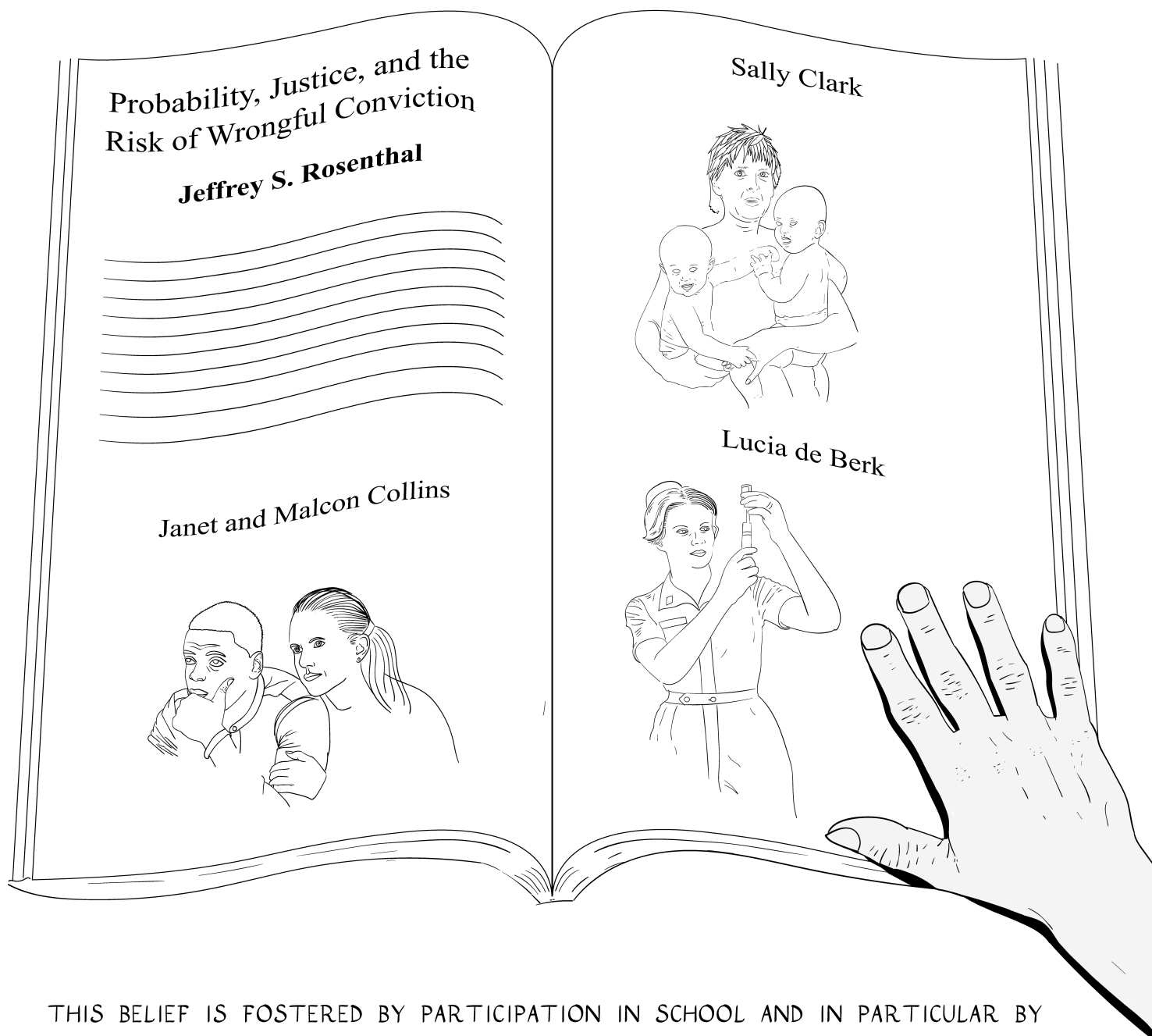
AGAIN, SHE WAS CONVICTED PRIMARILY ON THE BASIS OF A FAULTY CALCULATION CARRIED OUT BY PROSECUTION STATISTICIAN HANK ELFFERS WHICH WAS REPEATEDLY REPORTED BY THE PRESS.



LUCIA DE BERK WAS FINALLY DISCHARGED AFTER SEVEN YEARS. ACCORDING TO STATISTICIAN RICHARD GILL "THE MAGICAL POWER OF THE BIG NUMBER LED EVERYONE AT AN EARLY STAGE TO BE TOTALLY CONVINCED OF LUCIA'S GUILT".



OF COURSE, NOTHING PREVENTS THE FACT THAT LUCIA DE BERK, SALLY CLARK OR THE COLLINSSES WERE ACTUALLY GUILTY. NEVERTHELESS, IT SURELY WAS A MISTAKE TO CONDEMN THEM SOLELY OR PRIMARILY ON THE BASIS OF THE PROBABILISTIC ARGUMENTS PUT FORWARD BY THEIR PROSECUTORS. IN ALL THREE CASES WHAT HAPPENED CAN BE INTERPRETED AS A STEMMING FROM THE BELIEF THAT THESE CASES COULD BE RESOLVED BY MEANS OF NUMBERS AND OF MATHEMATICAL ARGUMENTS PROVIDED BY MATHEMATICAL EXPERTS.



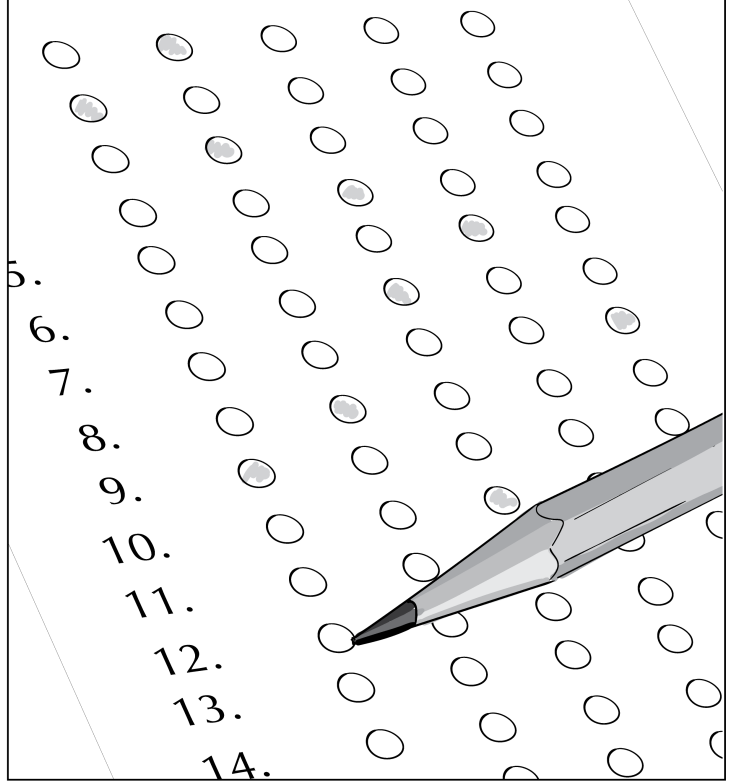
THIS BELIEF IS FOSTERED BY PARTICIPATION IN SCHOOL AND IN PARTICULAR BY ENGAGEMENT WITH REALISTIC WORD PROBLEMS HAVING TO DO WITH UNCERTAINTY. IN SUCH PROBLEMS EVERY EVENT CAN BE ASSIGNED A PROBABILITY, OFTEN EVENTS ARE INDEPENDENT TO ONE ANOTHER AND MATHEMATICAL CALCULATIONS ALWAYS LEAD TO THE ONLY CORRECT ANSWER.

BUT SO WHAT?



MATHEMATICAL INSTRUCTION DEALING WITH PROBLEMS OF UNCERTAINTY HAS SOMETIMES UNWANTED EFFECTS...

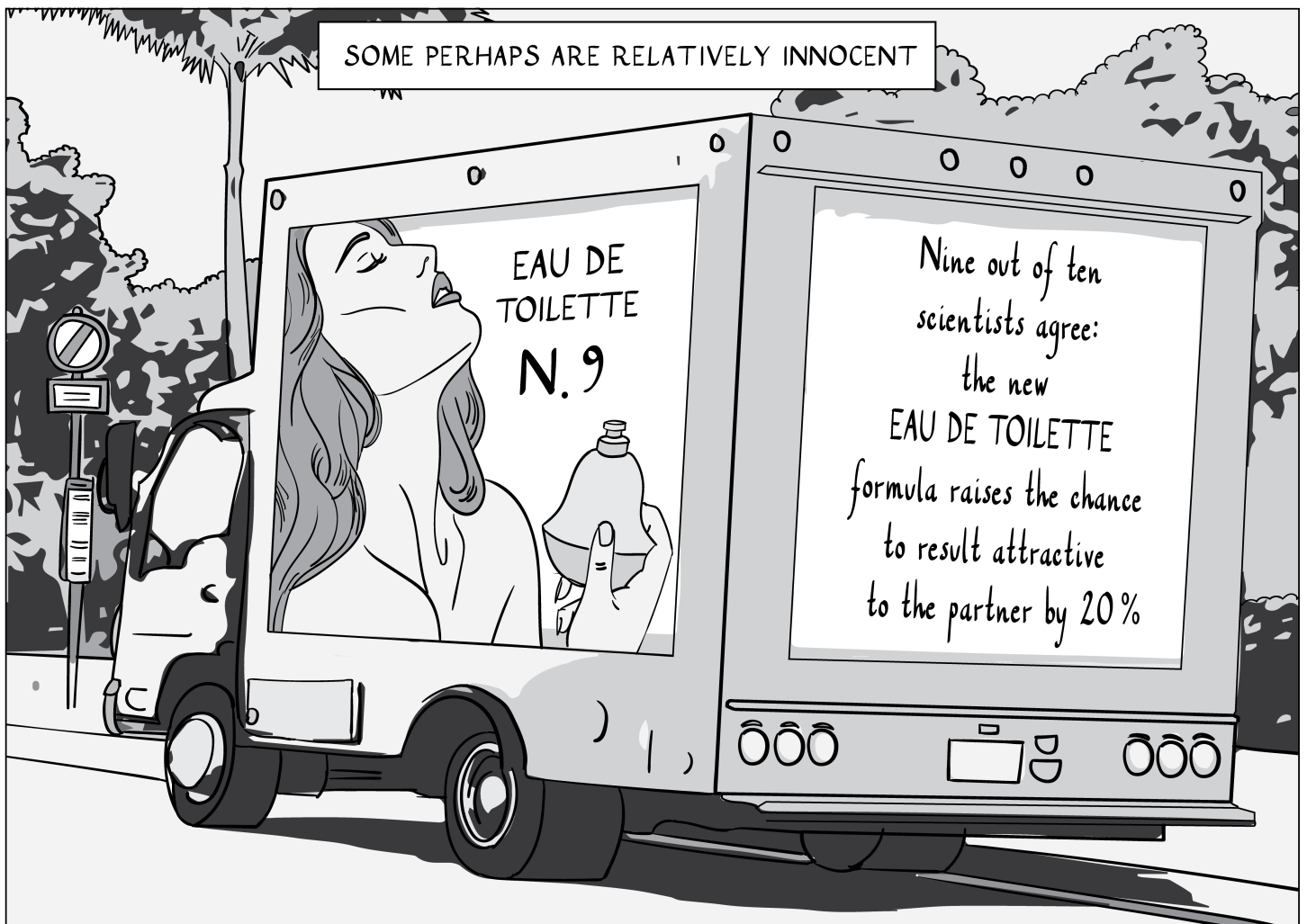
BOTH IN THE CONTEXT OF PUZZLES, TESTS OR PROBLEM SOLVING,



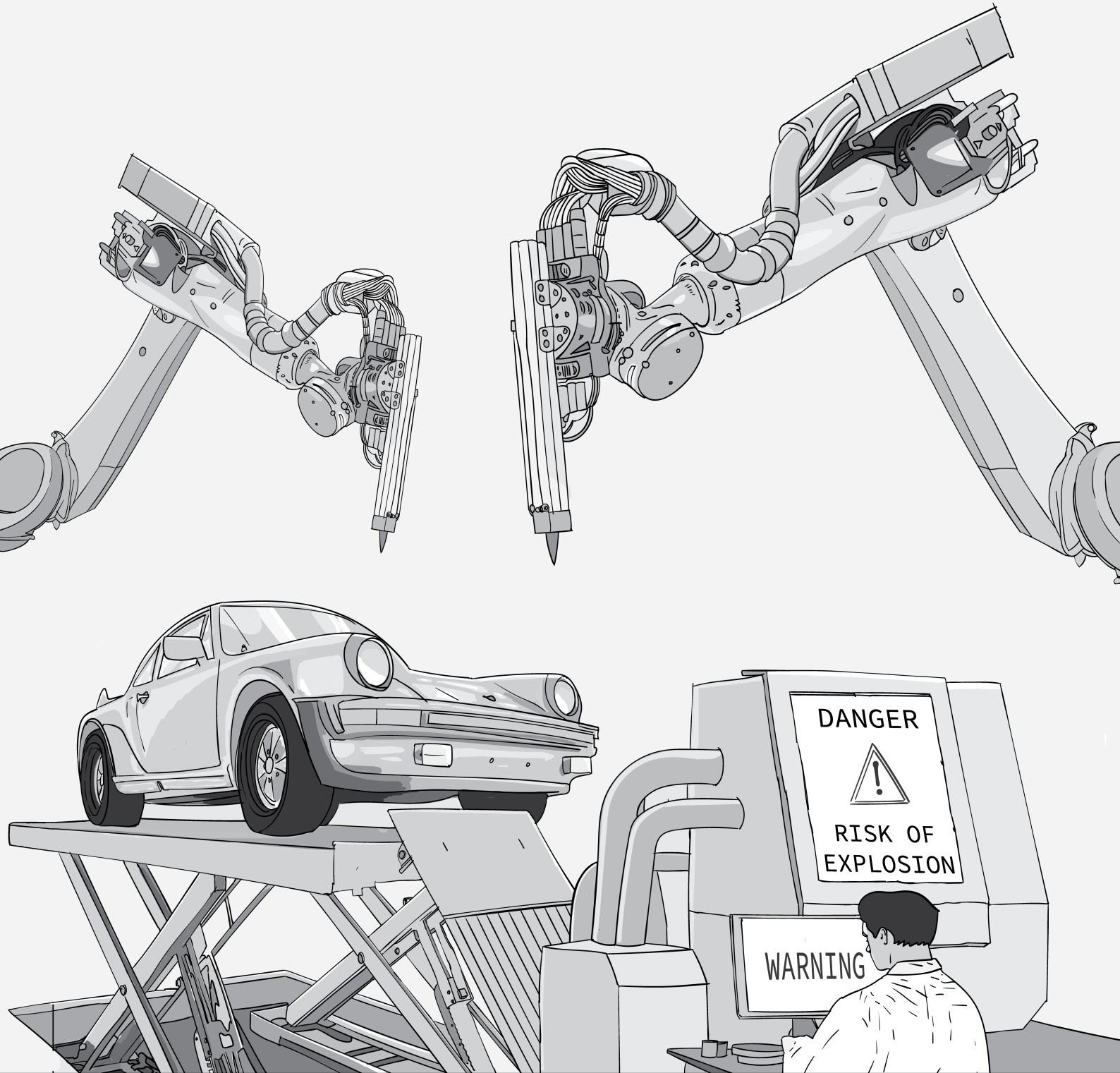
AS WELL AS IN MORE IMPACTFUL WORLDLY SCENARIOS



OTHER TYPICAL SITUATIONS COME TO MIND WHEN ONE REFLECTS ON THE MEDIATED RELATIONSHIP BETWEEN EXPERTS AND THE PUBLIC INVOLVING THE USAGE OF CONCEPTS AND ARGUMENTS FROM STATISTICS AND PROBABILITY ACQUIRED IN SCHOOL.

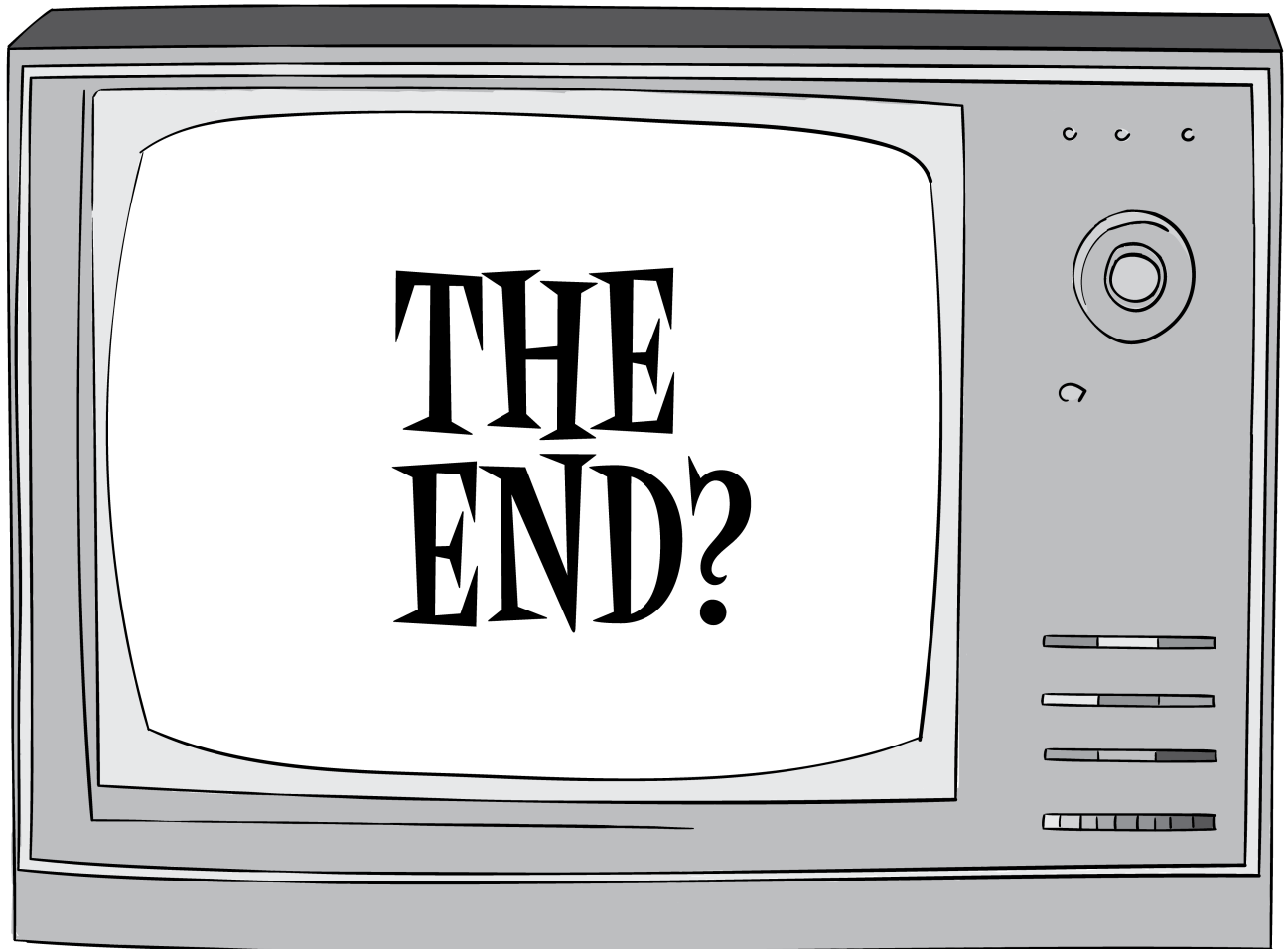


TEACHING TO TREAT SITUATIONS INVOLVING UNCERTAINTY WITH FORMAL MATHEMATICAL TOOLS IS IMPORTANT FOR THE DEVELOPMENT AND THE MAINTAINANCE OF A TECHNICAL, SCIENTIFIC AND INDUSTRIAL SOCIETY.



HOWEVER, IT SEEMS THAT FORMAL INSTRUCTION IN PROBABILITY AND STATISTICS CAN SOMETIMES LEAD TO MISTAKE, TO BIASED JUDGEMENT AS WELL AS, IN A FEW CASES, EVEN TO PERSONAL OR SOCIAL DISASTER.







## ACKNOWLEDGMENTS

I wish to thank Melissa Andrade-Molina for having been, with her own work, the main inspiration for commencing this project, as well as for the comments and suggestions she kindly provided at a later stage. Furthermore, I wish to thank Jordan Hill and Veronica Perez for having motivated me to complete this work as well as for their corrections on a previous version of this article. A final thank to Francesco M. Saettone and Alberto Botto Poala for reviewing a last version of this article.

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## APPENDIX

Following Gill (2011), a simple but rigorous solution to the Monty Hall problem relies on finding the unconditional probability of winning by switching. This requires the assumption that the probability of initially choosing a winning door is  $1/3$ , which is justified by the configuration of the problem itself. Indeed, the fact that there are two goats and one car and that each of these three is hidden behind one of three identical doors (and that no other information is given) justifies the assumption that the initial subjective probability of choosing a door hiding the car is  $1/3$ , while the initial subjective probability of choosing a door hiding a goat is  $2/3$ . From the statement of the problem, we know that Monty opens a door which is not the one hiding the car and which is not the player's initial choice. Thus, it follows trivially that the player wins by switching if and only if she initially chose a losing door. Therefore,  $2/3$  is the subjective probability of winning by switching. This also simply follows by enumeration of all the possible cases, as summarized in the following table.

player's initial choice	position of the car	door opened by Monty	player stays	player switches
1	1	2 or 3	WIN	LOSE
1	2	3	LOSE	WIN
1	3	2	LOSE	WIN
2	1	3	LOSE	WIN
2	2	1 or 3	WIN	LOSE
2	3	1	LOSE	WIN
3	1	2	LOSE	WIN
3	2	1	LOSE	WIN
3	3	1 or 2	WIN	LOSE

According to Gill, this simple argument succeeds in convincing everyone except writers of basic textbooks in probability and statistics. Indeed, in these, the solution of the Monty Hall problem is often given in terms of conditional probability, usually as an exercise on the application of Bayes' theorem and by employing further assumptions (i.e., that the car was hidden at random behind the doors and that, when Monty has a choice between opening two doors, he opens one at random). These assumptions are nonetheless justifiable by the undeterminateness of the formulation of the problem and arguably correspond better to a frequentist philosophy of probability. Thus, such assumptions were explicitly programmed in the code reproduced in the next page. This is a code written in the language Python which simulates one million rounds of the Monty Hall problem and counts how many times the player wins by switching at all rounds.

```

import random

NumRounds = 1000000
wins = 0
random.seed()
switch = True

for i in range(NumRounds) :

    doors = ["goat", "goat", "car"]
    random.shuffle(doors)
    player_choice = random.randrange(3)

    if random.choice([True, False]):
        for j, content in enumerate(doors):
            if j != player_choice and content == "goat":
                monty_opens = j
                break
    else:
        for j, content in reversed(list(enumerate(doors))):
            if j != player_choice and content == "goat":
                monty_opens = j
                break

    if switch:
        for j, content in enumerate(doors):
            if j != player_choice and j != monty_opens:
                switch_to = j
    else:
        switch_to = player_choice

    if doors[switch_to] == "car":
        wins += 1

print("\n\n We simulated ", NumRounds, "rounds of the game \
and the player won", wins, "times.")

```

The following is an example of the program's output, which can help us in corroborating our intuition that switching is the optimal strategy for the Monty Hall problem.

```
We simulated 1000000 rounds of the game and the player won 666784 times.
```

Notice that, by changing the value of `NumRounds` to any integer one can virtually simulate any number of rounds (provided one has enough computational power at disposal). Furthermore, changing the variable `switch` to `False` would simulate the situation in which the player always stays with her initial choice of door.

