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Abstract: The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition for students in the United States and Canada and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; AMS, 2020; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985). Students who earn the five highest scores on the examination are named Putnam Fellows. Since its inception in 1938, only 306 individuals have won the competition and a select few have won multiple times. Clearly, being named a Putnam Fellow is a remarkable achievement and therefore, understanding the factors and characteristics that contribute to their success is important for students interested in mathematics and STEM-related fields.

Twenty-five males who were named Putnam Fellows either four, three, or two times were recruited for the study. A 17-item questionnaire was created from various research sources (Campbell, 1996a, 1996b; Campbell & Wu, 1996; DeFranco, 1996), and used to collect information around four broad areas—personal experiences, formal educational experiences, the affective domain and the cognitive domain. Qualitative research techniques were used to analyze the data. The results indicated that four subcategories of personal experiences, five subcategories of formal educational experiences, seven subcategories involving the affective domain, and three subcategories of the cognitive domain all played an important role in the development of Putnam Fellows.

Future research recommendations should examine the factors and characteristics of female Putnam winners and ways to promote and support them as well as the role that Pólya-like heuristics play in the development of Putnam winners.

Keywords: Putnam Competition; characteristics of Putnam Fellows; mathematical problem solving; mathematics contests

1 Introduction

> The William Lowell Putnam Mathematical Competition, since its inception in 1938, has had a substantial impact on the field of mathematics in the United States and Canada. …While there have been many different reasons for the remarkable expansion of mathematics during the past forty years, we believe that the challenge provided by the Putnam Competition has led many gifted college students into serious involvement with mathematics, and our profession is the richer for it (as cited in Mathematical Association of America, 1980, p. vii).

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The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition for undergraduate college and university students in the United States and Canada and as noted above, has had a profound impact on the field of mathematics. Over the years, the Putnam Competition has grown to include, annually, more than 4,200 students representing 570 institutions (Krusemeyer & Ullman, 2020) and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; AMS, 2020; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985).

Participants in the Putnam Competition have been generally quite successful in a number of mathematics and STEM-related fields upon completion of their undergraduate and graduate studies and have made significant contributions to industry and-or academia as mathematicians, physicists, computer scientists, engineers, and chemists. In addition, Putnam participants have also received some of the most prestigious awards in their respective fields including: the Nobel Prize in Physics, the National Medal of Science, the Fields Medal, the Abel Prize, and the Albert Einstein Award in theoretical physics (Alexanderson, 2004; AMS, 2020; Gallian, 2004, 2017, 2018; Grossman, 2002; MAA, 2008). Further, some have served as presidents of the American Mathematical Society or the Mathematical Association of America (Gallian, 2004, 2017, 2018), as well as members of the National Academy of Sciences, the American Academy of Arts and Sciences, and the National Academy of Engineering (Alexanderson, 2004; Gallian, 2004, 2017, 2018).

Students who earn the five highest scores on the examination are named Putnam Fellows and since its inception in 1938, only 306 individuals have received such recognition. Since students are permitted to enter the competition up to four times during their undergraduate programs, a select few have been named Putnam Fellows multiple times[2]. Given the number of students who compete in the Putnam Examination each year, to be named a Putnam Fellow is a remarkable accomplishment while being named a Putnam Fellow multiple times is an extraordinary achievement. Clearly, such individuals would be categorized as problem-solving experts.

Over the past 50 years there has been extensive research in the general area of problem solving at the K-12 level as well as in the area of problem-solving expertise among Ph.D. mathematicians

[2] As of 2023, eight (8) students have been named a Putnam Fellow four times; 25 students have been named a Putnam Fellow three times; 50 students have been named a Putnam Fellow two times; and 223 individuals have earned the title of Putnam Fellow one time.

(DeFranco, 1987, 1996; Schoenfeld, 1985). Yet, since the inception of the Putnam Competition there is little research on the competition, its participants, and in particular, the select few recognized as Putnam Fellows.

Therefore, given their contributions to mathematics and other STEM-related fields, and their success on the Putnam Examination, it would be beneficial to examine the factors and characteristics that have contributed to the development and success of Putnam Fellows.

2 Background of the Study

In a 1921 edition of the *Harvard Graduates' Magazine*, William Lowell Putnam II communicated a deep conviction in the merits of academic intercollegiate competition. He likened academic competitions to athletic competitions, debates, and chess competitions, believing academic competitions have potential to stimulate students' interests in the academic disciplines. After Putnam's death in 1923, his wife, Elizabeth Lowell Putnam established the *William Lowell Putnam Intercollegiate Memorial Fund* to support intercollegiate competition (Arney & Rosenstein, 2001; Birkhoff, 1965; Bush, 1965; Gallian, 2004, 2017, 2018; MAA, 1980, 2008). This memorial fund provided support for a one-time Harvard vs. Yale academic competition in 1928 in the field of English and, five years later, a one-time Harvard vs. West Point mathematics competition (Arney, 1994; Arney & Rosenstein, 2001; Birkhoff, 1965; Gallian, 2004, 2017, 2018; MAA, 1980, 2008; Page & Robbins, 1984). Elizabeth Lowell Putnam's death spurred the Putnam children to consult with Harvard mathematician George Birkhoff in 1935 to fulfill their father's vision through the establishment of an annual undergraduate mathematics competition in the United States and Canada (Birkhoff, 1965; MAA, 1980, 2008).

In designing the Putnam Competition, Birkhoff identified four underlying principles: a) the mathematics competition would be open to both teams and individuals, b) the competition would be administered by the Mathematical Association of America (MAA), c) prizes would be awarded to teams and individuals, and d) one of the top five competitors would be awarded a graduate fellowship at Harvard University or Radcliffe College (Birkhoff, 1965; Bush 1965; MAA, 1938).

The University of Toronto won the first Putnam Competition, and it was the practice that the institution creating the questions for the Putnam Examination (which was Harvard University in the first year, and the winning institution in subsequent years) was disqualified from participation. This practice was discontinued after World War II (Birkhoff, 1965; Bush, 1965) at which point mathematicians Pólya, Radó, and Kaplansky, were asked to construct the questions on the Putnam Examination (Birkhoff, 1965). Questions on earlier Putnam Examinations were drawn from the areas of calculus, higher algebra, elementary differential equations, and analytic geometry (MAA, 1938); however, Pólya, Radó, and Kaplansky wanted to design mathematics problems that tested students' ingenuity in devising and using algorithms, as well as performing logical analysis (Birkhoff, 1965).

Until 1961, the number of questions on the Putnam Examination varied from 11 – 14 (Gallian, 2004, 2017, 2018; MAA, 1980). Currently, the Putnam Examination consists of 12 problems, each worth 10 points, for a maximum score of 120 points (Bush, 1965; Gallian, 2004, 2017, 2018; MAA 1985, 2008). To underscore the difficulty of the Putnam Examination, there have been only five perfect scores achieved between 1938 and 2021, which occurred once in 1987, twice in 1988, once in 2010, and once in 2019 (Gallian, 2017, 2018; Kedlaya, 2022). Further, the range of the median exam scores, each year from 1967-2018, was 0-19. During the same time period, the average of the exam median scores was 4.346 while the median of the median scores was 2. In addition, each year from 1996-2018, the average of the exam mean scores was 8.3 (https://www.d.umn.edu/~jgallian/Putnam18.pdf). (For a more detailed and nuanced understanding on the grading of the Putnam exam, see https://kskedlaya.org/putnam-archive/conversation.pdf.)

The Director of the William Lowell Putnam Competition, after consulting with several individuals, selects the members of the Problems Committee, which traditionally has consisted of three individuals who create the problems and who serve staggered three-year terms (Reznick, 1994). In 2020, the Problems Committee increased to four question composers (Krusemeyer, Ullman, & Zeitz, 2021) and in 2021, five problem writers contributed to the pool of problems for the Putnam Examination (MAA, 2022d). The examination includes two, three-hour test periods administered in the morning and afternoon (MAA, 1938). Members of the Putnam Grading Committee are recruited from the professors who proctor the Putnam Examination at the participating colleges and universities (April 20, 2022 e-mail correspondence). The Director of the William Lowell Putnam Competition manages the scoring process, which includes approximately 40 graders from participating institutions (MAA, 2016).

To enter a team in the Putnam Competition, any college or university within North America may submit the names of three students from the institution to the Secretary of the MAA (Cairns, 1938a; MAA, 2008) and those institutions with less than three or more than three participants compete as individuals (MAA, 1938). Over the years, the number of students and colleges and universities entering the competition has grown from 163 participants from 67 institutions in 1938 (Bush 1965; Cairns, 1938b) to 4,623 students from 568 colleges and universities in 2018 (MAA, 2019). Although there have been an increasing number of teams participating in the competition, the 390 top five winning teams have come from 43 institutions of higher education, while 184 of these winning teams have come from four of the 43 colleges and universities (see Appendix A) (Gallian, 2004, 2017, 2018; MAA, 2022d, 2022e, 2022f, 2023). In addition to publishing the names of the institutions that have winning teams, the MAA also publishes the names of the Putnam Fellows in alphabetical order (Cairns, 1938b; MAA, 1938). The contestants with the five highest scores, whether competing as part of a team of three people or as individuals, are designated as Putnam Fellows. The 306 Putnam Fellows, who have collectively won the Putnam Examination 430 times, represent 56 different colleges and universities (see Appendix B) (Gallian, 2004, 2017, 2018; MAA, 2022d, 2022e, 2022f, 2023).

During the second annual William Lowell Putnam Mathematical Competition in 1939, a team consisting of three women from Mississippi Woman's College won third place on the exam (Cairns, 1939). In recent years the number of women participating in the competition has increased and in 1992 the MAA awarded the Elizabeth Lowell Putnam Prize "to a woman whose performance on the Competition has been deemed particularly meritorious" (Klosinski, Alexanderson, & Larson, 1993, p. 757). During the last 31 years, 19 women have been awarded this honor and counting repeated winners, these individuals have received this award a total of 26 times. Among the 19 recipients, two students have earned this distinction three times and three individuals have won this award twice. Furthermore, the women who were awarded the Elizabeth Lowell Putnam Prize in 1996, 2002, 2003, and 2004 were also Putnam Fellows those same years (Gallian, 2004, 2017, 2018; MAA, 2022d, 2022e, 2022f, 2023).

Over the years there have been a number of mathematics competitions and contests that have served to discover mathematically talented students in our country and serve as valuable experiences for future Putnam winners. Mathematical competitions and contests have been sponsored mainly by the MAA at the elementary, middle and high school levels since 1950. In

1972, the MAA agreed to sponsor the first United States of America Mathematical Olympiad [USAMO] (Greitzer, 1973; Turner, 1978). The purpose of the USAMO was to identify mathematically talented and computationally fluent students who possessed mathematical creativity and inventiveness (Greitzer, 1973). In 1974, the Mathematics Olympiad Summer Program [MOSP] was created to help prepare students to become potential members of the U.S. team at the International Mathematics Olympiad [IMO] (MAA, 2022b, 2022c). Today, approximately 28 students who achieve the highest scores at the USAMO are invited to participate in the MOSP (MAA, 2022b, 2022c) and in the end, six high school students who earn the highest scores during the MOSP represent the U.S. at the IMO (MAA, 2022a).

The importance of mathematical competitions and in particular, the IMO to the development of Putnam Fellows cannot be overstated. According to Gallian, "Unlike the early years of the Putnam competition, in the past twenty-five years or so many of those who have done exceptionally well in the Putnam competition have participated as high school students in problem solving summer training camps in the United States and elsewhere in preparation for the annual International Mathematical Olympiad (IMO)…. Between 2010 and 2018 all but three Putnam Fellows were IMO Gold medalists (https://www.d.umn.edu/~jgallian/Putnam18.pdf)."

2.1 Framework for studying mathematically talented students

In order to examine the characteristics of mathematically talented students, Campbell and Wu (1996) conducted a number of research studies that examined the Mathematics Olympiad programs in the United States as well as in other countries. As part of their investigation of mathematics achievement in pre-collegiate students, Campbell and Wu (1996) adapted the Walberg Educational Productivity Model as the theoretical framework for their Mathematics Olympiad studies (Walberg, 1984a, 1984b, 1986; Walberg, as cited in Campbell & Wu, 1996). In doing so, Campbell and Wu (1996) subsumed five of the global factors of the Walberg nine-factor educational productivity model (Campbell & Wu, 1996) and expanded the number of variables within some of the factors in their model. Finally, as part of a research study of the American Mathematics Olympians, Campbell (1996a, 1996b) examined the factors that contribute to or impede the development of the Olympians' talent in mathematics and investigated the contributions Olympians made to the fields of mathematics and science. The study found that the four most important factors that contribute to the Olympians' mathematical talent include: the home, the school, the Olympiad Program, and mentoring (Campbell, 1996b).

Therefore, Campbell and Wu's (1996) adaptation of the Walberg Educational Productivity Model and Campbell's research (1996a, 1996b) on Mathematical Olympians was used as a basis to develop a questionnaire to examine the factors and characteristics that led to the development and success of multiple-winner Putnam Fellows.

3 Research Question

This study investigated the factors and characteristics that contributed to the development and success of the Putnam Fellows' exceptional abilities as problem solvers. In particular, this study proposed to answer the following question:

> What experiences do Putnam Fellows identify as contributing to their development as multiple-winner Putnam Fellows as well as to their success on the Putnam Examination?

4 Methodology

The study took place during the spring, summer, and fall months of 2014. A phenomenological study was conducted whereby in-depth, intensive, and iterative interviews were used to investigate the lived experiences (Rossman & Rallis, 2003) of the subjects in the study.

4.1 Subjects

A list of 74 individuals, who have been named a Putnam Fellow multiple times throughout the 74 Putnam Competitions held between 1938 and 2013 inclusive, was generated from the MAA's results published annually in *The American Mathematical Monthly*. The Putnam Fellows on the list were contacted beginning with the four-time competition winners, followed by the three-time and two-time winners, through e-mail, mail, and/or telephone invitations requesting their participation in this study. Of the 74 individuals, five people could not be located and 12 individuals are deceased. The remaining 57 Putnam Fellows, male and female, were contacted of which 25 individuals expressed a willingness to participate in the study; of the remaining Putnam Fellows, five people declined to participate; and 27 individuals did not respond. The 25 participants in the study were all males and attended nine colleges and universities in the United States and Canada at the time they were named Putnam Fellows. Further, of the 25 individuals who participated in the study, five have been named Putnam Fellows four times; seven have earned this distinction three times, and 13 have won this award two times.

4.2 Questionnaire

A 17-item questionnaire (Appendix C), which was designed using other instruments as sources (Campbell 1996a, 1996b; Campbell & Wu, 1996; DeFranco 1996), was created along four dimensions (i.e., personal experiences, formal educational experiences, affective domain (i.e., beliefs about mathematical problem-solving skills), and cognitive domain) and used to uncover information about the factors and characteristics that have contributed to the development of the subjects in the study.

4.3 Data Collection

Data collection began in May 2014 and ended in November 2014. During this period, each Putnam Fellow was asked to participate in an in-depth interview that took the form of a purposeful dialogue, as part of the phenomenological data-gathering process (Erlandson, Harris, Skipper, & Allen, 1993; Rossman & Rallis, 2003; Seidman, as cited in Rossman & Rallis, 2003). Prior to the interview, the 17-item questionnaire was sent to each participant either through e-mail or through postal mail, which allowed the participant the opportunity to review and reflect on the questions prior to the interview.

Eight subjects consented to participate in audio-recorded interviews, while the remaining 17 subjects elected to provide written responses to the questionnaire via e-mail. Each of the eight subjects who were audio-recorded participated in an in-depth, semi-structured interview that took the form of a dialogue or conversation (Erlandson, Harris, Skipper, & Allen, 1993). The eight participants who were interviewed over the telephone or on the computer through Skype, were audio-recorded using Peizo software and the interviews ranged in length from 45 to 90 minutes. The 17 participants, who elected to answer the interview questionnaire in writing, returned their written responses through e-mail. The written responses were formatted into Microsoft Word documents that ranged in length from two to four pages. To clarify or elaborate on participants' responses, two follow-up telephone calls and six follow-up e-mail communications were made to the participants. The two follow-up telephone calls were with a 4-time Putnam Fellow and a 3-time Putnam Fellow, while the six follow-up e-mail communications were with a 4-time Putnam Fellow, a 3-time Putnam Fellow, and four 2-time Putnam Fellows.

4.4 Data Analysis

The audio-recorded data from the eight subjects who participated in the telephone and Skype interviews were transcribed and this information along with the written responses from the 17 subjects who responded to the questionnaire in writing were used to answer the research question. The data was analyzed qualitatively due to the exploratory and descriptive nature of the study. The process of data analysis occurred concurrently with the process of data collection, which allowed the researcher to regulate data collection strategies and test out emerging ideas against the new data that was collected (Marshall & Rossman, 1995).

To answer the research question, responses from the interviews were transcribed and each statement was printed in a text matrix. The statements within the text matrices were color-coded according to the categories of personal experiences, formal educational experiences, the affective domain, and the cognitive domain. (See Appendix D, Figure 1 for an example of the line-by-line coding of the interview data). A within-case analysis was employed for each participant to identify themes and patterns with respect to the factors and characteristics that contribute to the success of the subjects. In order to accomplish this, the rows within the coded matrix for each individual participant were sorted by the categories and subcategories of personal experiences, formal educational experiences and the affective and cognitive domains. Building upon each within-case analysis and the data entered into the text matrices, a cross-case analysis was conducted to identify similarities and differences across participants with respect to the categories and subcategories. Finally, themes and patterns that emerged across cases were organized into summary tables, one for each subcategory of personal experiences, formal educational experiences, and the affective and cognitive domains.

A peer debriefer was employed to examine samples of the text to help build credibility for the study and to check the inter-rater reliability of the coding (Thomas, 2006). The peer debriefer was a professor of mathematics education with expertise in qualitative analysis. The researcher and the peer debriefer scheduled sessions to develop techniques for coding the data, examine the coded data, provide opportunities to ask probing questions about the data, and discuss different explanations or alternative coding of the data (Erlandson et al., 1993).

5 Results and Discussion

In order to uncover the characteristics and experiences that Putnam Fellows identify as contributing to their development as Putnam Fellows as well as contributing to their success on the Putnam Examination, 25 male Putnam Fellows who attended nine different colleges and universities in the United States and Canada at the time they were named multiple-winner Putnam Fellows were selected to participate in this study. Data was collected using a questionnaire over a period of seven months and included oral and written interviews and responses. The questionnaire solicited responses around four broad categories: a) personal experiences, b) formal educational experiences, c) affective variables, and d) cognitive variables. The data was analyzed qualitatively and the results of the data analysis are reported next with respect to each of the four broad categories outlined above.

5.1 Personal experiences

In this study, participants were asked to reflect on their personal experiences beginning with their earliest childhood memories, including stories retold by family members and family friends, and continuing up until the time they first participated in the Putnam Competition.

In addressing themes within this category, four subcategories of personal experiences emerged and were examined. These included: a) being raised in households that were conducive to learning, b) having influential family members and family friends, c) showing an interest in and a talent for mathematics at a young age, and d) having access to educational resources.

First, 22 subjects (n=22) indicated growing up in households that were conducive to learning and being raised by parents who valued academic achievement and who provided encouragement and support in learning mathematics as a key component to their success. Second, participants expressed having influential family members and family friends who helped them learn mathematics throughout their childhood (n=12).

Third, participants believed that having an interest in and talent for mathematics at a young age was influential in their success on the Putnam Examination (n=15). For example, Participant 3 noted,

> It was in third grade I first became aware of my interest in and talent for mathematics. The next year, while in fourth grade, I was part of a 'play' being put on by the school, involving classes at all levels. While waiting for rehearsal one day, a ninth-grade girl somehow became aware of my interests and taught me the simplest case of the binomial theorem – the formula for (x + y)^2.

For many of the Putnam Fellows their interest in mathematics and their ability to successfully do mathematics continued on throughout their academic careers.

Finally, Putnam Fellows reported having access to educational resources such as mathematics textbooks, puzzle books, and encyclopedias at home during their childhood (n=11). For example, Participant 5 said, "The main support was valuing academic achievement, and connecting me with a couple of math folks. She [mother] helped me get my own copy of Hardy and Wright's, *An Introduction to the Theory of Numbers,* when I was about 12." Many participants believed that access to mathematical resources provided a rich environment for learning mathematics.

The importance of the family and the home environment are factors that played an important role in the development of the Putnam Fellows' mathematical talent, which subsequently contributed to their success on the Putnam Competition. These findings are consistent with Campbell's (1996b) American Mathematical Olympiad research studies. According to Campbell (1996b), the participants in his study attributed their home environment as being critical to the development of their mathematical talent and found that most Olympians grew up in households with supportive and resourceful parents, where a stimulating learning environment existed, and learning was highly valued. Similarly, the participants in this study reported growing up in households surrounded by family members and family friends who valued academics and provided encouragement and support in helping them learn mathematics throughout their childhood.

5.2 Formal educational experiences

In this study, participants were asked to reflect on their formal educational experiences beginning with their earliest memories of schooling, including stories retold by family members and family friends, and continuing through their participation in the Putnam Competition.

In addressing themes within the formal educational experiences of the participants in this study, five subcategories of experiences emerged as themes and were examined. These included: a) having influential teachers and other individuals in academics and other formal educational settings, b) participating in mathematics contests and competitions prior to the Putnam Competition, c) having access to released mathematics contest and competition problems and solutions, d) participating in extracurricular mathematics training, and e) not receiving coaching or preparatory classes through their college or university.

First, subjects indicated having influential teachers, coaches, and mentors in academics during their formative K-12 years (n=19). For example, Participant 3 stated, "In grades eight and nine I had a fabulous math teacher, Miss XXXX, who was incredibly effective and inspiring for all ability levels. She gave me a wonderful foundation in algebra and geometry."

Next, participants expressed the importance of taking part in mathematical contests and competitions (e.g., the American Mathematics Competitions, the United States of America Mathematical Olympiad, and the International Mathematical Olympiad), throughout the childhood years leading up to their participation in the Putnam Competition (n=19). As noted by Participant 12,

> I was on math teams every year from seventh to 12th grade. I took the AHSME [American High School Mathematics Examination] those same years. I took the AIME [American Invitational Mathematics Examination] in 11th and 12th grades, and the USAMO [United States of America Mathematical Olympiad] in 10th, 11th, and 12th grades. I participated in the International Mathematical Olympiad after 11th and 12th grades.

Third, participants responded that solving Putnam problems from previous years' examinations was a valuable way to prepare for the Putnam Competition (n=13) while the subjects reported taking part in extracurricular mathematics training (e.g., after school, on weekends, and during the summer months) in addition to their normal high school program of studies (n=23). On this topic Participant 11 indicated,

> Well those Math Olympiad programs [Mathematical Olympiad Summer Program]. So the summer after my eighth grade year all the way through high school. And so of course, I had an enormous advantage over anybody who didn't have that background, when it came to the Putnam.

Finally, it should be noted that 40% (n=10) of the participants reported that their college or university did not offer any coaching or practice sessions as preparation for the Putnam Competition.

The importance of having strong K-12 mathematics teachers and university mathematics professors, participating in mathematical contests and competitions, having access to and practicing previous Putnam Examination problems, and receiving extracurricular mathematics training are all factors that played an essential role in the development of the Putnam Fellows' mathematical talent, which subsequently contributed to their success on the Putnam Competition. These findings are consistent with Campbell's (1996b) American Mathematical Olympiad research studies, which found that the Olympiad training program was an important stimulant to the development of the Olympians' success in mathematics and fostered the development of their mathematical talent. As noted above, this is very similar to the Putnam Fellows' beliefs in this study.

5.3 Affective domain

The affective domain has been described using a number of constructs, which include beliefs, attitudes, and emotions (Hart, 1989; McLeod, 1989, 1992; McLeod, as cited in Feldman, 2003). In addressing themes within the affective domain, seven subcategories of affect emerged as themes and were examined. These included: a) beliefs about confidence in solving Putnam problems, b) beliefs about natural ability, aptitude, or talent in mathematics, c) beliefs about having an interest in mathematics and liking mathematics, d) beliefs about the role that intuition plays in solving Putnam problems, e) beliefs about talent being innate or taught, f) feelings experienced when solving Putnam problems, and g) motivation for success on the Putnam Examination.

First, subjects indicated their belief in being confident in their ability to solve Putnam problems (n=16). For example, Participant 19 responded, "Yes, and this is important. Being confident helps you focus on approaches to the problems that are more likely to lead to solutions."

Second, participants expressed that possessing a natural ability, aptitude, or talent in mathematics as factors that contribute to winning the Putnam Competition (n=15). As noted by Participant 12, "There are three things: a natural problem-solving ability, adequate knowledge of math, and practice solving problems."

Third, subjects believed that having a strong interest in mathematics and enjoying doing mathematics are characteristics that contributed to their success (n=9). Fourth, Putnam Fellows believed that intuition plays an important role in solving Putnam problems (n=19). For example, Participant 25 stated,

> What is intuition? Insofar as it denotes the kind of non-rigorous 'hunches' used to supplement mathematical reasoning, it plays a role everywhere: in reading the

> problem, selecting an approach, finding ways to translate vague ideas into math, deciding which steps are worth writing and which are too obvious, and even scouring the final proof for the scent of logic gone awry.

Fifth, subjects thought that their extraordinary talent as a Putnam Fellow is partly innate and partly due to effective teaching (n=16). On this subject Participant 3 stated,

> It [talent] is certainly partly innate, but given this it can certainly be 'cultivated'. By the latter term I suppose I mean it can be taught, but really, I mean that it can be greatly improved with practice – and with learning and studying more mathematics. One often gets lots of extra 'practice' helping friends with their homework.

Sixth, participants indicated experiencing a certain aesthetic or positive feelings after solving a Putnam problem (n=18). As noted by Participant 22,

> I feel excitement and joy. And if the solution is nice, I feel a sense of aesthetic beauty. The smell of excitement, a sense of accomplishment you know when I solve a difficult problem. I think back when I was in high school, or in college doing the Putnam, you know solving these problems, doing well on these contests were definitely the high joys of my life at that point. It was a very exhilarating experience to do well on these competitions.

Finally, individuals in the study reported extrinsic motivation as an incentive for their success on the Putnam Examination (n=8).

Schoenfeld (1985) realized that mathematical behavior on a problem, which appears to be solely cognitive in nature, may in fact be influenced by affective components. As noted by DeFranco (1996), "beliefs regarding problem solving (e.g., perseverance, confidence, motivation, interest, etc.) contribute significantly to an individual's performance on a problem" (p. 205). Overall, the Putnam Fellows in this study clearly felt confident in their ability to solve Putnam problems and felt motivated to do well on the problems.

5.4 Cognitive domain

In addressing themes with respect to the cognitive domain and the Putnam Examination, participants indicated three subcategories that emerged as themes and were examined. These included: a) using alternative methods to rework previously solved mathematics problems, b) recognizing Putnam problems as being similar to previously solved mathematics problems, and

c) examining analogous cases as a strategy to get a feel for Putnam problems.

First, subjects indicated using alternative methods to rework previously solved mathematics problems in their everyday work, but not during the Putnam Competition because of the time constraint (n=18). As noted by Participant 15,

> I guess I do it right away. Sometimes it will happen right away depending on the time or often it will happen later when I'm not working directly, I'm just musing about it or I'm walking down the street and I'm thinking about it and I realize another idea might work or sometimes it will happen when I'm working on another problem and suddenly I realize this idea or I'm thinking about something else and I also may realize that something else is related and that's when I'll have the alternate solution, not quite intentionally going after alternative solutions all the time, but sometimes it just comes after the fact.

Second, participants expressed an ability to recognize similarities between Putnam problems and other types of mathematics problems they have previously solved (n=12).

Finally, individuals in the study reported examining analogous cases as a strategy to get a feel for a Putnam problem (n=15). For example, Participant 9 stated,

> Okay, the first thing to do is ask, does it look someway familiar, is it like something I've seen before? If so, then that is a strong hand as to what direction to go. And after that it's very important to know that I understand the question and in particular, probably write down some special cases and see whether they work, and how they work, and quite often once you go through two or three cases, you see a general pattern, which leads you to the proof of the full statement, so that's I guess what I would say. It's as close as I could look to a strategy. In general, anything more than that, it would depend on the particular type of problem.

As noted by Participant 11,

> Well, first of all I try to recognize that it is something that I've seen before, which as I've said I can very often do. Sometimes it takes a little bit of work before you see how it relates to something you've seen before, but usually, or maybe for half the Putnam problems, I look at it and I'm pretty sure that it's like a particular thing that I've seen before. After that, if you've read this book *How to Solve It* then you know what the basic things are. You try to specialize it; you try to generalize it; you try to think of an analogous problem; you can write hypotheses. You know the drill, so I do all those things.

The importance of using alternative methods to rework previously solved mathematics problems, recognizing similarities between Putnam problems and other mathematics problems, and examining individual cases of a problem or recalling analogous cases as a strategy to get a feel for a Putnam problem are factors that played an essential role in the development of the Putnam Fellows' ability to solve Putnam problems. These ideas are synonymous with the "heuristics" outlined by Pólya (1945) in his book, *How to Solve It*. In addition, these findings are consistent with the characteristics of "expert" problem solvers as well as the strategies used by expert problem solvers to solve problems (DeFranco, 1996; Schoenfeld, 1992).

6 **Conclusion**

The purpose of this study was to examine the factors and characteristics that contribute to an individual becoming a multiple winner Putnam Fellow, and in turn use the findings to help K-16 mathematics instructors foster a climate that promotes problem-solving success. A questionnaire solicited responses around four broad categories: a) personal experiences, b) formal educational experiences, c) affective variables, and d) cognitive variables. And while personal experiences provided a foundation for the development of Putnam Fellows, the results from the three remaining categories may provide mathematics teachers and instructors some insight in preparing students to become better problem solvers and one day be part of the Putnam experience.

First, schools and universities need to be able to identify mathematically talented students and take an active role in nurturing their talent. School districts, and in particular schools, need to support gifted and talented programs, after school math enrichment programs and math clubs that can help identify, encourage, and support mathematically talented students at all levels. In addition, mentors and coaches with the appropriate mathematical experience should be available to work with mathematically talented students at all levels. Further, educators also need to encourage students to pursue extracurricular opportunities in mathematics and recommend or provide them with additional sources of material for practice.

For example, Participant 10 stated,

> In my public high school Mr. XXXX regularly took students to compete in local math competitions, as well as encouraging them to participate in national math contests. A friend of mine recommended my name to him. When I won the first contest I went to, he did everything he could to encourage me to do more,

> including giving me many old exams from a variety of sources. I went through all of them and practiced lots of problems.

Next, the participants in this study believed that hard work and time spent solving problems played a critical role in their success in mathematics competitions, which subsequently led them to be named Putnam Fellows. Mathematics instructors at all levels should emphasize the importance of problem solving and encourage students to believe in their abilities, which in turn will help students gain more confidence and achieve greater success.

Finally, problem solving has been a core component of reform initiatives in the United States mathematics curriculum for many decades. At the center of such initiatives are the use of Pólya-type heuristic techniques, which are important to the development of students' problem-solving abilities. In addition, many participants expressed the idea that Pólya-like heuristics are part of the toolbox they use to solve difficult mathematics problems. As noted by Participant 11 above, "…if you've read this book *How to Solve It* then you know what the basic things are. You try to specialize it; you try to generalize it; you try to think of an analogous problem; you can write hypotheses. You know the drill, so I do all those things." Clearly, teachers can play an important role in developing a student's talent for solving problems as well as teach students the problem-solving skills necessary to successfully solve problems.

Future research should examine the factors and characteristics of female Putnam winners as well as ways to promote and support future female participants in the Putnam Competition. In addition, future research should examine the role that Pólya-like heuristics play in the development of Putnam winners. Such research may provide opportunities for students for future careers involving mathematics as well as the newly emerging STEM-related fields.

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Appendix A

Winning Teams by College or University

F = Fall; S = Spring; T = Tie Score

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
Brooklyn College (6)	1939 1941 1948	1952 1963	1946		
California Institute of Technology (33)	1950 1962 1964 1971 1972 1973 1975 1976 1983 2010	1959 1967 1979	1958 F 1961 1974 1977 1982 2009 2011	1957 1963 1998 2000 2003	1965 1970 1978 1988 1996 2004 2008 2013
Carnegie Institute of Technology Carnegie Mellon University (9)	2016	2011 2013 2015	1949 1987	1946	2012 2014
Case Institute of Technology Case Western Reserve University (4)	1978			1964 1976 T	1959
City College of New York (5)		1953 T		1942 1948 T 1949 1951	
Columbia University (6)		1956 1957	1938 1940 T 1947	2018	
Cooper Union Institute of Technology The Cooper Union (2)			1940 T 1951		

College or University (Total Number of Top Five Winning Teams)	**First Place**	**Second Place**	**Third Place**	**Fourth Place**	**Fifth Place**
Cornell University (9)	1951 1954	1953 T 1994 1995	1957	1958 F	1960 1992
Dartmouth College (2)		1962			1961
Duke University (12)	1993 1996 2000	1990 1997	1999 2001 2002 2003 2004 2005		2007
Harvard University (67)	1947 1949 1953 1955 1956 1957 1958 F 1965 1966 1982 1985 1986 1987 1988 1989 1990 1991 1992 1994 1995 1997 1998 2001 2002	1950 1951 1954 1958 S 1960 1980 1993 1999 2003 2006 2009 2014 2017 2019 2022	1948 1952 1962 1964 1967 1971 1972 1981 1984 1996 2000 2010 2016 2021	1959 1961 1969 1973 1978 2013	1975 2015

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
	2005 2007 2008 2011 2012 2018				
Harvey Mudd College (2)			1991		2003
Illinois Institute of Technology (1)				1970	
Kenyon College (1)				1955	
Massachusetts Institute of Technology (52)	1968 1969 1979 2003 2004 2009 2013 2014 2015 2017 2019 2021 2022	1939 1946 1961 1964 1965 1966 1970 1998 2000 2001 2010 2012 2018	1941 1942 1954 1960 1975 1994 1995 2006 2007 2008	1952 1956 1967 1974 1976 T 1993 1997 2005 2016	1963 1971 1972 1977 1986 1987 2011
McGill University (1)				1948 T	
Miami University (1)			1993		
Michigan State University (5)	1961 1963 1967			1960 1968	
Mississippi Women's College (1)			1939		

College or University **(Total Number of Top Five Winning Teams)**	**First Place**	**Second Place**	**Third Place**	**Fourth Place**	**Fifth Place**
New York University (1)			1950		
Oberlin College (1)		1972			
Polytechnic Institute of Brooklyn (3)	1958 S 1959				1958 F
Princeton University (32)	2006	1981 1985 1987 1988 1989 1996 2002 2004 2005 2007 2008 2016 2021	1976 1979 1997 1998 2015 2017	1965 1975 1977 1983 1984 1992 1994	1966 1973 1982 1995 2009
Queen's University (3)	1952		1956	1962	
Rensselaer Polytechnic Institute (1)			2014		
Rice University (4)		1969	1988	1985	1989
Stanford University (16)			2013 2019 2022	1979 1981 1991 2007 2008 2009 2011 2015 2021	2001 2002 2016 2018

College or University (Total Number of Top Five Winning Teams)	**First Place**	**Second Place**	**Third Place**	**Fourth Place**	**Fifth Place**
Stony Brook University (1)				2012	
Swarthmore College (1)				1972	
University of British Columbia (2)		1973			1974
University of California, Berkeley (11)	1960	1938	1985 1986	1953 1987 2001 2002 2010	1964 1980
University of California, Davis (3)	1984 T	1977		1971	
University of California, Los Angeles (7)			1968 2012 2018	2019	1962 2017 2021
University of Chicago (11)	1970	1971 1974 1975	1966 1969 1973	1980	1983 1999 2006
University of Kansas (1)					1968
University of Manitoba (1)				1958 S	
University of Maryland, College Park (3)			1980	2022	1981
University of Michigan (4)				1966 1999	1967 1993
University of Pennsylvania (3)		1941	1963	1947	

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
University of Toronto (19)	1938 1940 1942 1946	1948 1949 1955 1958 F 1992	1958 S 1959 1965 1970	1950 1954 1995 2006 2017	2000
University of Waterloo (20)	1974 1999	1968 1982 1991	1978 1983 1989 1990 1992	1988 2004 2014	1979 1985 1994 1998 2005 2010 2019
Washington University, St. Louis (11)	1977 1980 1981 1984 T	1976 1978 1983 1986		1996	1990 1997
Yale University (12)		1940 1942 1947	1955	1982 1986 1989 1990	1969 1984 1991 2022

43 Colleges and Universities Total Number of Top Five Wins	First Place	Second Place	Third Place	Fourth Place	Fifth Place
390	83	82	82	80	63

Appendix B

Number of Putnam Fellows by College or University

College or University	One Win	Two Wins	Three Wins	Four Wins	Total Wins
Armstrong State College	1				1
Brooklyn College	3	1			5

College or University	One Win	Two Wins	Three Wins	Four Wins	Total Wins
California Institute of Technology	10	6		1	26
Carnegie Institute of Technology Carnegie Mellon	3		1		6
Case Western Reserve University	2	1			4
City College of New York	4		2		10
College of Saint Thomas	1				1
Columbia University	1	2	1		8
Cooper Union	1				1
Cornell University	3	1			5
Dartmouth College	2				2
Duke University	4	1			6
Fort Hays Kansas State College	1				1
George Washington University	1				1
Harvard University	51	17	7	1	110
Kenyon College		1			2
Massachusetts Institute of Technology	43	9	6	3	91
McGill University	1				1
Michigan State University	2		1		5
New York University	3				3
Polytechnic Institute of Brooklyn	3				3
Princeton University	10	3	2		22
Purdue University		1			2
Queen’s University	1				1
Reed College	1				1
Rice University	3				3
Rose-Hulman Institute of Technology	1				1
San Diego State College	1				1

College or University	One Win	Two Wins	Three Wins	Four Wins	Total Wins
Simon Fraser University	1				1
Stanford University	1		1		4
Swarthmore College	1				1
Union College	1				1
University of Alberta		1			2
University of British Columbia	1				1
University of California, Berkeley	6	2	2		16
University of California, Davis	2				2
University of California, Los Angeles	2				2
University of California, Santa Barbara		1			2
University of Chicago	6	2			10
University of Detroit	1				1
University of Manitoba	1				1
University of Maryland, College Park	1				1
University of Minnesota, Minneapolis	3				3
University of Missouri, Rolla	1				1
University of North Carolina	1				1
University of Pennsylvania	3				3
University of Pittsburgh	1				1
University of Santa Clara	1				1
University of Toronto	14	1	1	1	23
University of Virginia	1				1
University of Washington, Seattle	1				1
University of Waterloo	6		1		9
Washington University, St. Louis	4	1			6
Wesleyan University	1				1
Williams College	1				1

College or University	One Win	Two Wins	Three Wins	Four Wins	Total Wins
Yale University	5	1	1		10

Total Number of Colleges and Universities					Total Wins
56	224	52	26	6	430

Appendix C

Interview Questionnaire

1. What role (e.g., financial, parental influence, psychological support, help with schoolwork, access to educational resources, progress monitoring and time management, conducive home atmosphere, etc.) did your parents play in your success as a Putnam Fellow?

2. Can you tell me a story about an event or an individual who influenced you to become a Putnam Fellow?

3. Can you describe a teacher or teachers who have influenced you to help become a Putnam Fellow?

4. Beyond traditional mathematics classes, did you participate in any enrichment classes or summer programs in mathematics?

5. Did you participate in mathematics competitions (e.g., MATHCOUNTS, the United States of America or Canadian Mathematical Olympiad, etc.) throughout your formal education?

6. How did you prepare to take the Putnam Examination? Did you participate in practice sessions or receive coaching as preparation for the Putnam Competition? Did your college or university offer preparatory classes for the Putnam Competition? Please explain.

7. Are you confident in your ability to solve Putnam problems? Please explain.

8. What do you feel (e.g., excitement, aesthetic joy, fear, etc.) when you solve a Putnam problem?

9. When you took the Putnam Examinations, what motivated you to be successful? Please explain.

10. Please describe the qualities, characteristics, or factors that you think contribute to an individual becoming a Putnam Fellow.

11. After solving a mathematics problem, when do you rework and use or not use alternative methods to solve the problem? Why?

12. When you first read a Putnam problem, what general strategies or techniques do you think you would use to help you toward the solution of the problem?

13. Typically when mathematicians begin to solve problems they examine analogous cases to get a feel for the problem. When you solve a Putnam problem, do you use a similar strategy or are there other methods you employ?

14. Do you still do Putnam problems? Why? Why not?

15. What role does intuition play in solving Putnam problems?

16. Do you believe your extraordinary talent as a Putnam Fellow is innate or can it be taught?

17. To be a multiple Putnam Competition winner is an extraordinary achievement. Is there anything you would like to share that I did not ask that might shed light on your success as a Putnam Fellow?

Appendix D

Figure 1

A Sample of the Line-by-Line Coding of the Interview Data

Participant	**Question 4: Can you describe a teacher or teachers who have influenced you to help become a Putnam Fellow?**	**Code**
17	I had many teachers and coaches who helped me over the years; none specifically helped with the Putnam exam. I should single out XXXX, the head coach of the IMO (International Mathematical Olympiad) team for the XXX years in which I participated. He had been working with me on writing up cleaner solutions; at the XXX IMO, he read over one solution of mine, sighed, and remarked that this was what people's papers looked like right before they went to start winning Putnams for XXX. His remark gave me a lot of confidence.	FE1 FE2 AD1

Coding Legend

Red: Personal Experiences

Blue: Formal Educational Experiences

Green: The Affective Domain

Purple: The Cognitive Domain

FE1: The participant had helpful teachers and coaches.

FE2: The subject participated in mathematical competitions.

AD1: The participant had a lot of confidence.