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Math and Democracy

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Abstract

Math and Democracy is a math class containing topics such as voting theory, weighted voting, apportionment, and gerrymandering. It was first designed by Erika Ward for math master's students, mostly educators, but then adapted separately by both Erika Ward and Kim Roth for a general audience of undergraduates. The course contains materials that can be explored in mathematics classes from those for non-majors through graduate students. As such, it serves students from all majors and allows for discussion of fairness, racial justice, and politics while exploring mathematics that non-major students might not otherwise encounter. This article serves as a guide to resources and activities for teaching similar courses and also as a call to talk about issues of race and justice in the math classroom.

1. Introduction

In the past decade, we have seen the way that mathematical tools and techniques are being used in gerrymandering, discussions about how we conduct elections, and the ways we collect and use data to shape society coming into prominence in a number of ways. These can serve as an entry point for students who may not otherwise be interested in mathematics (but are interested in politics and societal issues), but also are a novel set of problems for those who are interested in mathematics to approach with their mathematical

tools. As such, it makes an appealing topic both for non-majors courses, and for providing a different sort of applied space to major or graduate students. But since it brings together ideas from a variety of mathematical disciplines, and requires some understanding of the political questions, it isn't obvious where one might begin. Here we describe the ways we came to teach these courses, providing resources, starting points, and reasons to do so to others who are interested in implementing their own classes, or or incorporating these topics into existing classes.

When I (Erika) first heard about the work that the Metric Geometry and Gerrymandering Group (MGGG) [7] was kicking off in 2017, I found it exciting and energizing. The state of the world, and in politics in particular, was making teaching abstract mathematics feel a bit pointless (while knowing and being able to explain that it's not), and the idea of bringing mathematical tools to the crisis in voting, gerrymandering, and fairness felt important and timely. I was able to attend two of their conferences, and was excited to bring what I learned back to students.

I incorporated gerrymandering into the non-majors class that I was already teaching (a non-algebra-based topics in math class that already included voting theory). Then, in the summer of 2019, I was given the opportunity to develop a new class for our master's program (a M.A. whose audience was primarily educators). I taught my first version of the course then — covering voting, gerrymandering, ecological inference, apportionment, other measures of fairness, and ideas around big data. In spring of 2022, I was able to teach a version aimed at undergraduates as part of Jacksonville University's core seminar series. Students took Mathematics and Democracy to fulfill the core requirement in either mathematics or social science, without significant background in either. This time around we covered apportionment, some data interpretation topics, voting methods, gerrymandering, and ecological inference. I'll be adapting this course to serve the Boston College core curriculum in the Fall of 2023.

I (Kim) heard about Erika's work with the MGGG and the subsequent class on a discussion group for academic knitters. I had even borrowed the Squaretopia activity (available at [8]) discussed further in Section 3 for use in my Quantitative Methods class. Juniata College had a curriculum overhaul in 2018 which resulted in a change in the quantitative requirement, so we needed some new math courses to fulfill the new formal reasoning requirement.

I had taught some voting theory in a Math in Society course at my previous job and some apportionment in our Quantitative Methods course at Juniata. I had enjoyed teaching the democracy related content in both courses, so I contacted Erika to look at her syllabus for adaptation. (Suggestions on materials for these topics and more can be found in the appendix in Section A.)

Mathematics and Democracy is a 100-level course fulfilling the formal reasoning requirement at Juniata. It contains in order of appearance reviews of percentages, voting theory, weighted voting, apportionment, gerrymandering, some appealing to voters, sampling/polling, and the census. It is not a prerequisite for any other math course. Instead of exams, it has structured projects at the end of each section, including a final reflection on fairness. Because one of the features of our new curriculum is that courses like formal reasoning should compare their epistemologies to another field, we talk about political science substantively including inviting in guest lecturers as discussed in Section 4.

Overall we encourage students to think about fairness in both general and specific. In particular many of the topics like voting with Arrow's theorem and apportionment with its various paradoxes include multiple options of measuring how good a system is and that there is no one best system. Kim often asks her class "Fairness how? Fairness for whom?". This leads to a discussion of social, political and racial issues that is often missing from mathematics classes. Such discussions both increase the relevance of the topic to students and their sense of understanding power and agency.

Finding that math is useful in many ways, that it lends insight to issues that seem opaque, or that it creates tools to help untangle difficult problems, gives students ways and reasons to care about math where that's sometimes lacking in more traditional math classes. Particularly when we're thinking about courses for non-majors, that difference can be transformational — the difference between shrugging and walking away from math, and beginning to see math as something that they can do, and perhaps even somewhere they belong.

Here, we describe a first day activity that helps to connect fundamental ways of thinking in mathematics to politics, then provide a discussion of some of the topics that can be included in a class like ours. We then discuss the ways that connections with other disciplines increase opportunities for bringing in

guest lectures. As we both relied heavily on projects, we discuss those and provide suggestions. Finally, we turn to the underlying value of incorporating racial justice and related issues into mathematics classes. This will, we hope, give other instructors places to start and reasons to do so.

2. First Day: The Constitution

On the first day of class, I (Erika) handed out copies of the Constitution to groups of students, and explained that since it was the foundational document for our government, we needed to figure out what was and wasn't in there. I put a skeleton of the structure up on the white board (a list of articles and their sections) and then asked students from different groups to put up a sentence or phrase or two for each of them. What topic does it cover? Is there an easy summary to give? I encouraged them to start in different places and aim for coverage - no one needed to read everything, but some group needed to read each part. This took about 15 minutes.

Then as a class we went through and found the parts we thought were important to our course goals — where elections were discussed, and the census. We noted that some parts didn't sound like what we do now, which was an excellent segue to amendments to the Constitution. We split these up among the groups more systematically, and again, they put a summary of each up on the white boards. We took a few minutes to note which parts of our understanding were brought into alignment by the amendments (who can vote, for example) and which parts still seemed to be missing (i.e. anything about how elections are actually run).

There's a tendency when talking about all of these topics to point at some ideas as being in the Constitution, and in a one hour talk, that's a totally understandable matter of efficiency (I do it, too!). But as mathematicians who care about definitions, seeing how things are actually set out, and modified, is an important exercise. (It's not often seen as the take home message for non-majors math classes, but I'd argue that "words matter" is a mathematical value as much as it belongs to other disciplines, too.) As citizens who are bombarded by messages with varying degrees of support, looking at the foundations is an important practice, too. With a whole semester set in front of us, I'm glad we took the time. It also got the students actively discussing and writing on the board, without being explicitly — or scarily — mathy, which was important to me for setting the classroom environment.

I took photos of the boards, made those available on our LMS, and made sure we referred back to what was and wasn't in the Constitution as we began different units of the class. In places I supplemented with excerpts from *OMG WTF Does the Constitution Actually Say?: A Non-Boring Guide to How Our Democracy is Supposed to Work* by Ben Sheehan [11], a modern plain-text rendering of the Constitution. There's also *What Does the Constitution Say?: A Kid's Guide to How Our Democracy Works* by the same author[12], with less swearing, as it's aimed at a younger audience.

3. Course Content

Voting theory, weighted voting, and apportionment have appeared in mathematics and society books for many years. I (Kim) have personally taught them out of three different books beginning in the early 2000s [2, 13, 4]; Erika had been teaching voting methods from *The Heart of Mathematics* [1]. Gerrymandering, however, is not in those books in the same depth. The references and the appendix in section A contain many resources for learning and teaching gerrymandering and the other content of math and democracy.

As I (Erika) began teaching gerrymandering, I relied heavily on the resources put out by the MGGG [8]. These include their "squaretopia" exercise, which asks students to district a "state" that is made up entirely of square districts. They're asked to district four different ways: to try to make nicely shaped districts, to be as fair as possible, and then to try to draw districts that give the most advantage each of the parties represented. The simplification allows students to delve in and get hands on-experience.

From there, I encourage students to sort the districts they drew along a spectrum from "good" to "bad" shapes, and come up with ways to assign scores to them that reflect that. They often develop analogues to the perimeter measure, and the Polsby-Popper score, and sometimes head off in interesting directions (I had a group working on developing a measure of "tentacleness"). The MGGG also has produced materials indicating the "square" version of a number of common district scores, and I've supplemented what my students come up with when they don't generate a lot of variety. I encourage a discussion of what the scores they have developed do and don't measure (what "bad" districts still score well?).

We then discuss the additional difficulties of working with actual maps. There are suddenly weird shapes, county and municipal boundaries, rivers, train

tracks, and varying population density! There are a lot of possible measures to discuss, and a class (or instructor) who is interested in getting deep into it has a lot of possible ground to cover. But you can keep it pretty simple, and think about a handful of simple and common measures [15]. For curious students, the Mandelbrot survey paper [6] that discusses the fractal nature of coastlines is surprisingly readable.

It's possible to go deeper, too, in considering real-world data. For students with a bit of experience / bravery around playing with more technical software, GIS software (Geographic Information Systems, the software that's used for most technical mapping endeavors) is freely available and students are able to load in real-world data (typically shapefiles of existing or proposed district maps) and experiment with various measures of compactness. QGIS (`https://qgis.org`) is the most widely used free GIS software. If your institution has a geography department, I'd suggest a conversation about what they're using (and possible collaboration!). When I had master's level students, one explored various proposed district maps using GIS software as the basis for their final project. Outside of class, I've had an undergraduate student do a research project comparing the Florida district maps before and after the state was forced to redistrict in 2016. These took some willingness to experiment with the interface and go digging for the data they wanted, but didn't require technical coding skills.

While the discussion of shape as a way to detect gerrymandering is compelling and relevant, we're also seeing advances in gerrymandering that result in subtler district shapes. There's more interest in ways to think about the fairness of district maps without appealing only to the shapes, and several of those are accessible—and worth discussing!

Efficiency gap (and the idea of "wasted votes") is a way to quantify the effect of districts that are drawn to overwhelmingly vote for one party or the other [14]. In first-year courses, I bring this up as a part of discussion of fairness. What role do we want districts that a party can consider "safe" to play in our democracy? That's also a good place to discuss the limits of mathematics, and the interface between mathematical tools and questions of democracy — mathematics can help us measure, illuminate, and maybe eliminate (or minimize) things we decide are unwanted, but society needs to make the value judgements (decide what kinds of fairness we want to promote). In the master-level version of the course, a student delved deeper, and analyzed the Jacksonville, Florida city council districts using the efficiency gap.

Another way to approach the question of fair districting is to ask, since we know the demographic breakdown of each district, and the results of a vote in each district, what can we tell about whether different demographic groups had distinct preferences? And if there was a preference, were districts drawn in a way to suppress that? This question, an example of ecological inference, was commonly asked in court cases trying to apply the Voting Rights Act, and led to the development of new statistical approaches to better answer them. The MGGG resources [8] have materials that explore these ideas both in the Bush v. Gore election (looking at absentee ballot data in Florida) and in a simplified, fictional setting (considering Sneeches). There are clear and immediate applications to racial gerrymandering, and current events. This material provides an opening to investigate the limits of what we can know from data more generally, including a discussion of Simpson's paradox and Robinson's paradox.

Sampling, polling, and margin of error are standard introductory statistics topics, something Kim teaches often, and fit well into the Mathematics and Democracy theme by discussing political polling in particular. The census, which relates to issues of sampling, is more of a throw-away line in teaching statistics - when you teach sampling you say when the sample equals the population you have a census. However, the United States Census has some rich math and statistics content. There is statistical content in the difficulties of a census and how differential undercount is estimated after the census is complete. Discussing how people are missed by the census and that other people choose to not complete the census are both things students have not often considered. Additionally there is mathematical content about data privacy and how the census can release block data and still protect privacy of the individual. Discussions of these complexities leads to discussions of both privacy and transparency and encourages the students to think about the trade-offs between them.

4. Guest Lecturers

Neither of us is a political scientist, so we have both utilized guest lecturers in our classes. We find that having guest lectures adds substantially to the course both in depth and breadth. It also helps to situate mathematics in conversation with other disciplines, helping to clarify the applications and implications of mathematical content.

In Kim's general education class, part of a general education course goals to compare epistemologies, so the guest lectures are a great chance for the students to learn from experts mainly from the Politics department, but also from the Psychology, Peace Studies, and History departments. Politics topics have included the politics of gerrymandering, proportional representation by party, legal rulings about voting and gerrymandering, European democracies and more history of the census and the three-fifths clause of the Constitution. Other topics have included measuring the rise in authoritarianism, election integrity, and dictators and the cold war.

The Politics department has an invited colloquium speaker each semester that I have also had speak to the class when available. The students and I really enjoy the guest lectures, they add a lot of depth to political side of the course. Because I have no budget to pay guest lecturers, I repay them with an offer to guest lecture in their classes and cookies. Interacting with the guest lecturers has had a few side effects for the course. The Politics department has decided that Math and Democracy should be cross-listed in politics. The course is also being included in a new minor in Race, Ethnicity, & Identity Studies.

Erika also had a guest lecture from the Political Science faculty, and it added substantially to the course. He focused on political polling, and how we interpret that data. Students responded very positively to seeing the material they were thinking about discussed from other perspectives. I've also seen this effect when I've shown students videos about voting methods or gerrymandering – it makes the material feel more real to have it discussed by people who didn't learn about it in the classroom with them.

There are other opportunities in the material to foster collaboration with other departments, too. The questions of what sort of data and support helps make gerrymandering court cases is a different perspective on the material that I'd love to bring into the classroom. I've also thought about inviting someone from Communications to talk about data and story-telling. There are topics where inviting someone in to discuss the historical (and/or social) perspective could lend depth to the material.

5. Projects

At Juniata, my (Kim's) class projects are on voting theory, apportionment, gerrymandering, and fairness in democracy and the census. All of the projects

involve some discussion of fairness and students are asked to justify why they think what they prefer is most fair politically and mathematically. Given that there is difficulty or even impossibility of picking the most fair method in most topics, as mentioned before we talk about fairness "how" and fairness "to whom" in class.

The voting theory project involves taking a recent election and comparing the current method of voting and its result to a simulated preference table of votes (as needed) and another method. The apportionment project involves taking state data from a census year and reapportioning house seats by a quota method and a divisor method of their choice, and talking about which states benefit. The gerrymandering project involves taking a state, checking what the rules for redistricting there are, and commenting on how the redistricting after the 2020 election went/is going, measuring approximate geometric measures of shape on two districts in the state, and talking about what seems more fair. The final reflection involves looking at one of the ways the census impacts a particular state, how the census went in that state in 2020, and what students have learned about mathematical and political fairness in the course overall.

I (Erika) used some similar projects (and students chose 3 or 4 from six to complete). There is an apportionment project and a gerrymandering project, both similar to Kim's, a chance to investigate a question of their choice with linear regression, an opportunity to read further about data science's social implications (added after students' very positive response to watching one of Cathy O'Neil's talks [10]), a request to make an argument for replacing plurality voting in US presidential elections (with mathematical support), and one working with some fictionalized data regarding ecological inference.

6. Racial Justice & Politics

Racial justice and equity issues arise naturally as we discuss voting and the census. From the first day, looking at the Constitution, the three-fifths clause jumps out as both a place where things have changed, and a place where troubling structures are written into our founding documents. These issues continue to be illuminated as we think about enumeration in the census, where systematic undercount is a problem, and in the timeline of enfranchisement for voting. Native Americans were among the last to achieve the right to vote, and the United States is still struggling with the question of how those convicted of crimes should engage in voting.

The Voting Rights Act fundamentally changed the way that districts are drawn, both requiring that districts have equal populations, and that minority populations be given the opportunity to elect candidates of their choosing. We have continued to see these issues play out since the 1960s and into current times - supreme court decisions have changed the scope of the VRA, and what sorts of gerrymandering are legal is still being debated today (along with other attempts to regulate who should be allowed to vote, and how difficult that should be).

My (Erika's) class also spent some time discussing linear regression (this is helpful as a prerequisite for some of the ecological inference discussion). While there are a tremendous number of data sets available, I chose a set of demographic data from the Bronx, divided by zip code, adapted from *Statistics, Part 1, Graphing and Averages*, by Jonathan Osler, [5], and shared via Radical Math. I also adapted one of Dave Kung's sample social justice activities [3], to talk about data and story-telling while practicing basic percentages. Kim did, too!

More broadly, we'd like to encourage the mathematical community to collectively be open to choosing data that engages with social, political, and racial issues, in classes like this and in others. One of the arguments we make for general-audience math classes is that math is useful. Opening doors that help students see how math can be used to address questions that they could care deeply about is motivating. While we can pick apart the material at the intersection of mathematics and democracy and present it in very sterile terms, it's also steeped in a history that is deeply discriminatory. Acknowledging that, and moreover digging into that, not only sparks students' interest, but gives them tools to start to understand things that may have seemed opaque or untouchable.

Mathematics is a tool of agency — it helps us to understand what is and is not significant, and to identify where the levers of power are. Exploring mathematics in context, and engaging with difficult or controversial issues (in a classroom that's operating with good norms around safety and respect [9]) helps students to see how to make the change they want to see in the world they inhabit.

7. Conclusion

This course manages to surprise students from two directions. Some are surprised that math has anything to do with all of this democracy stuff, and so they now have an entry into math through a topic they're interested in. Others are surprised about the same thing in the other direction, and so they let their interest in math take them into an understanding of our government and collective choice, and how that is (and isn't) working. Either way, that surprise engenders curiosity, and that's fundamental to setting out on any exploration.

The authentic way that discussions of race and power arise in these courses is an advantage. Students have not often seen these discussions in math classes. It increases the student perception of the utility of the math in this course and their interest.

The multiple ways to measure fairness, that sometimes there is not one fairest method, and that fairness can depend on from whose perspective it is fair adds another layer for students. Students often exit high school with an impression that math has one right answer. Here is an explicit example of multiple ways to measure and lack of a right answer in some cases. This realization is an important part of moving toward a better understanding of what math is and how it operates.

As mathematicians, we know that math shows up everywhere; really leaning into that for a course like this can be incredibly rewarding. It also means operating at the edges of our knowledge in ways that can be uncomfortable. That said, it's some of the most fulfilling teaching we've done, with students who are interested, indignant, and fired up. It's worth it to find the way that you, personally, balance being prepared enough against knowing that your students will ask questions you couldn't have predicted. We hope we've also indicated that there are enough topics under the "Math and Democracy" umbrella to start where you are most comfortable. You can begin your journey with the bibliography and Appendix A, where we've collected materials we've found useful. Leaning into that exploration together is pretty fundamental to both mathematics and to democracy as pursuits.

Engaging with politics and social issues in a math class can also have us operating on the edges of our comfort zones in different ways. As mathematicians we often don't have training in navigating the kind of discussions and issues these classes can raise, and it's smart to realize the limits of our

expertise. I (Erika) had some trepidation about appearing partisan in the classroom, especially as the boundaries of what is and is not political have shifted rapidly over the past several years. I've found that a couple of things have served me well, at least thus far: first, approaching the things I don't know with curiosity and questions, much as we would unknown mathematics content, and paying attention to the norms that I (and my students) collectively establish, both explicitly and implicitly as class progresses. (See [9] for more on setting classroom norms.)

Moreover, I haven't had the kinds of interactions with students that I feared. I have had students ask in various ways whether it's really wrong for people in power to use it to gerrymander, but that's easily redirected to the course content – what is fair? Fair for whom? What do we want our democracy to look like? What is, gerrymandering, really, and is that what we want to have happen? I've also had students thank me for the space for discussion and the atmosphere in the classroom. I think many of us can be better at this than we fear we might be.

Similarly I (Kim) was worried about discussing both politics and race in class. However I too have found that the interactions with students have been almost entirely positive. I think the historical frame starting with the Constitution and including part of the bill language for apportionment helps as well as the constant emphasis on fairness and how it is not a one answer question. Teaching this class has helped me get more comfortable at addressing social and political issues and influenced my teaching in my other math and statistics courses.

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[4] David Lippman, *Math In Society*, 2.6 edition, online textbook, July 2022. Available at http://www.opentextbookstore.com/mathinsociety/, last accessed on July 30, 2023.

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[15] Erika L. Ward, "Shapes Determine Elections", *Frontiers for Young Minds*, 9:663461 (October 14, 2021). doi:10.3389/frym.2021.663461

A. Other Resources

There's an ever-growing array of resources out there. This is a compendium of ones that either Kim or Erika has found useful in thinking about these courses.

A.1. Voting

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- CGP Grey, Politics in the Animal Kingdom videos, https://www.cgpgrey.com/politics-in-the-animal-kingdom,accessed on December 1, 2022. [These videos offer an overview of different voting systems with a focus on the limitations of "first past the post" voting.]
- Jonathan Hodge and Richard Klima, *The Mathematics of Voting and Elections: A Hands-on approach*, 2nd edition, American Mathematical Society, Providence, RI, 2018.

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A.4. The Census

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A.5. Fairness

- Mira Bernstein and Moon Duchin, "A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap", *Notices of the American Mathematical Society*, Volume **64** Number 9 (October 2017), pages 1020-1024.
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A.7. Other Resources

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- Alan D. Taylor and Allison M. Pacelli, *Mathematics and Politics: Strategy, Voting, Power, and Proof*, Springer, New York City, 2010.