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Analytical Study of Two Serial Channels with Priority and Reneging

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Abstract— Most of the studies of queuing theory, which are useful in our daily life has been investigated by many researchers. The present research is the study of pre-emptive priority queuing system consisting two serial channels in stochastic environment. The impatient behavior of customer's will be discussed with exponential service distribution and Poisson arrivals. Higher priority customers have pre-emptive priority over the low priority customers. The G.F. technique is used to derive the performance measures of high & low priority queues and assuming FCFS discipline in busy schedule of higher priority class. Also evaluate queue behavior graphically and discussed a special case at the end which shows utilization of channels.
Keywords- priority, reneging, serial channels, impatient behavior

I. INTRODUCTION

Several real-world systems can be designed, capacity planned, behavior assessed, and modified with the use of queueing theory. In the most basic configurations, it is believed that all queueing system users are homogeneous and have similar needs and credentials to services. This is not applicable to plenty of real-world systems. Due to various factors, some customers or classes of customers are given a higher priority when it comes to reaching the servers than other groups. These explanations may vary. A smart choice of priorities can greatly enhance the financial gain obtained from running a corresponding system and income-generating enterprises. As a result, priority queueing models have received a huge interest from scholars. Problems related to queues in series was studied by O'Brien [1], Jackson [2] with the assumption no reneging at any server without completing the service in the system. It was Barrer [3] who used the concept of Markovian reneging in their research. Later on, Singh M. [4] studied serial queue network model with impatient customers. Further T. P. Singh & Arti [5,6] extended the work by discussed the effect of reneging on various performance measures of queues in stochastic and fuzzy environment. In [7] Satyabir Singh et al introduced a connection between serial and non- serial service channels with balking and reneging. Meenu Gupta [8] et al analyzed impatient behavior of customer in stochastic situation with the assumption of finite waiting space multichannel queuing system and derived time-independent solution. Recently, Meenu Gupta [9] et al analyzed a queuing model with balking, reneging and queue discipline for service is pre-emptive priority. Saini V [10] studied a feedback queue model consisting two serial servers with impatient customers.

In this paper we extend the work done by T.P. Singh [5,6] et al by analyzing a queuing model comprised of two servers in series. This type of situation commonly occurs in our day to life and most of the time we notice priority is given to someone by ignoring others. Graphical representation of the study is given for understanding the role of reneging in the system.

II. PRACTICAL ENHANCEMENT

The practical utility of the purposed model can be seen in banking sector, amusement park, theatres, educational institutions, saloons, offices etc. if we take an example of amusement park in which at entry there is a ticket counter where there are two types of customers, one is high priority (kids, senior citizens, VIP's,) and second is low priority (adults). If place is overcrowded then few of the people leave without taking the service due to shortage of time & some urgent works.

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Those who succeed in collecting tickets for entry go inside park and enjoy the service accordingly they need.

III. ASSUMPTIONS

- A customer who wants service join the system at C_1 after that go to next server for next phase
- Priority is allowed only at C_1
- Calling population is infinite
- If customer gets impatience due to lack of time and services then he may leave the system
- Arrival and service pattern follow Poisson distribution.

IV. MODEL DESCRIPTION

This model consists two service channels C_1 & C_2 which are in series. Both type of customers low and high priority arrives in the system for getting service with arrival rate λ_{1L} & λ_{1H} and service rate at C_1 is μ_{1L} & μ_{1H} respectively. If high and low priority type customer gets impatient due to long queue length or slow service rates or shortage of time or pre-emptive priority is given to high over the low then he will leave the system with reneging rate r_{1H} and r_{1L} respectively. After getting service at C_1 customer visit to next server C_2 where service rate is μ_2 and finally exit the system.

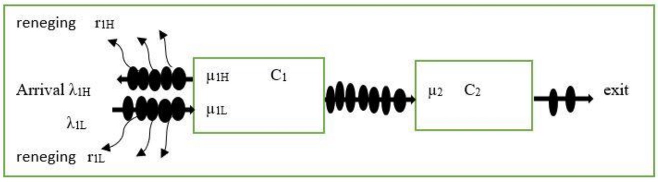

Figure1. Purposed Priority Queue Model

V. STEADY - STATE ANALYSIS

Define Probability function $P\eta_{1L}, \eta_{1H}, \eta_2$ (t) and $\eta_{1L}, \eta_{1H}, \eta_2$ number of customers in queues Q_{1H}, Q_{1L}, Q_2 in front of servers C_1, C_2 respectively, where $\eta_{1L}, \eta_{1H}, \eta_2 \geq 0$

In Steady-State, Differential Difference equation is defined as

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+r_{1H}+\mu_2) P_{\eta_{1L},\eta_{1H},\eta_2} = \lambda_{1L} P_{\eta_{1L}-1,\eta_{1H},\eta_2} + \lambda_{1H} P_{\eta_{1L},\eta_{1H}-1,\eta_2} + \mu_{1H} P_{\eta_{1L},\eta_{1H}+1,\eta_2-1} + r_{1H} P_{\eta_{1L},\eta_{1H}+1,\eta_2} + \mu_2 P_{\eta_{1L},\eta_{1H},\eta_2+1}$$

$$\eta_{1L}, \eta_{1H}, \eta_2 > 0 \quad (1)$$

$\eta_{1L} = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+r_{1H}+\mu_2) P_{0,\eta_{1H},\eta_2} = \lambda_{1H} P_{0,\eta_{1H}-1,\eta_2} + \mu_{1H} P_{0,\eta_{1H}+1,\eta_2-1} + r_{1H} P_{0,\eta_{1H}+1,\eta_2} + \mu_2 P_{0,\eta_{1H},\eta_2+1} \quad (2)$$

$\eta_{1H} = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1L}+r_{1L}+\mu_2) P_{\eta_{1L},0,\eta_2} = \lambda_{1L} P_{\eta_{1L}-1,0,\eta_2} + \mu_{1H} P_{\eta_{1L},1,\eta_2-1} + r_{1H} P_{\eta_{1L},1,\eta_2} + \mu_{1L} P_{\eta_{1L}+1,0,\eta_2-1} + r_{1L} P_{\eta_{1L}+1,0,\eta_2} + \mu_2 P_{\eta_{1L},0,\eta_2+1} \quad (3)$$

$\eta_2 = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+r_{1H}) P_{\eta_{1L},\eta_{1H},0} = \lambda_{1L} P_{\eta_{1L}-1,\eta_{1H},0} + \lambda_{1H} P_{\eta_{1L},\eta_{1H}-1,0} + r_{1H} P_{\eta_{1L},\eta_{1H}+1,0} + \mu_2 P_{\eta_{1L},\eta_{1H},1} \quad (4)$$

$\eta_{1L} = \eta_{1H} = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_2) P_{0,0,\eta_2} = \mu_{1H} P_{0,1,\eta_2-1} + r_{1H} P_{0,1,\eta_2} + \mu_{1L} P_{1,0,\eta_2-1} + r_{1L} P_{1,0,\eta_2} + \mu_2 P_{0,0,\eta_2+1} \quad (5)$$

$\eta_{1L} = \eta_2 = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+r_{1H}) P_{0,\eta_{1H},0} = \lambda_{1H} P_{0,\eta_{1H}-1,0} + r_{1H} P_{0,\eta_{1H}+1,0} + \mu_2 P_{0,\eta_{1H},1} \quad (6)$$

$\eta_{1H} = \eta_2 = 0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1L}+r_{1L}) P_{\eta_{1L},0,0} = \lambda_{1L} P_{\eta_{1L}-1,0,0} + r_{1H} P_{\eta_{1L},1,0} + r_{1L} P_{\eta_{1L}+1,0,0} + \mu_2 P_{\eta_{1L},0,1} \quad (7)$$

$\eta_{1L} = \eta_{1H} = \eta_2 = 0$

$$(\lambda_{1L}+\lambda_{1H}) P_{0,0,0} = r_{1H} P_{0,1,0} + r_{1L} P_{1,0,0} + \mu_2 P_{0,0,1} \quad (8)$$

VI. SOLUTION METHODOLOGY

To solve the above differential equations, we use the g.f and p.g.f as:

$$G(X,Y,Z) = \sum_{\eta_{1L}=0}^{\infty}\sum_{\eta_{1H}=0}^{\infty}\sum_{\eta_2=0}^{\infty} P_{\eta_{1L},\eta_{1H},\eta_2} X^{\eta_{1L}} Y^{\eta_{1H}} Z^{\eta_2},$$

$|X|=1, |Y|=1, |Z|=1 \quad (9)$

Also define the partial generating functions as:

$$G_{\eta_{1H},\eta_2}(X) = \sum_{\eta_{1L}=0}^{\infty} P_{\eta_{1L},\eta_{1H},\eta_2} X^{\eta_{1L}} \quad (10)$$

$$G_{\eta_2}(X,Y) = \sum_{\eta_{1H}=0}^{\infty} G_{\eta_{1H},\eta_2}(X)\, Y^{\eta_{1H}} \quad (11)$$

$$G(X,Y,Z) = \sum_{\eta_2=0}^{\infty} G_{\eta_2}(X,Y)\, Z^{\eta_2} \quad (12)$$

And adopt solution methodology given by T.P. Singh et al, we get

$$G(X,Y,Z) = \frac{G_1\left[\mu_{1H}\left(1-\frac{Z}{Y}\right)-\mu_{1L}\left(1-\frac{Z}{X}\right)+r_{1H}\left(1-\frac{1}{Y}\right)-r_{1L}\left(1-\frac{1}{X}\right)\right]+\mu_2\left(1-\frac{1}{Z}\right)G_3+\left[\mu_{1L}\left(1-\frac{Z}{X}\right)+r_{1L}\left(1-\frac{1}{X}\right)\right]G_2}{\lambda_{1L}(1-X)+\lambda_{1H}(1-Y)+\mu_{1H}\left(1-\frac{Z}{Y}\right)+r_{1H}\left(1-\frac{1}{Y}\right)+\mu_2\left(1-\frac{1}{Z}\right)} \quad (13)$$

Where $G_3 = G_0(X,Y), G_1 = G_0(X,Z), G_2 = G_{0,0}(Z)$

Since, equation (13) is in indeterminate form when $G(X,Y,Z) = 1$ *and* $|X| = 1, |Y| = 1, |Z| = 1$. Now apply L' Hospital Rule to remove indeterminate form

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Differentiating w. r. t. X and taking Y=1, Z=1

$$-\mu_{1L}G_1 - G_1 r_{1L} + \mu_{1L}G_2 + G_2 r_{1L} = -\lambda_{1L} \quad (14)$$

Differentiating w. r. t. Y and taking X=1, Z=1

$$\mu_{1H}G_1 + G_1 r_{1H} = -\lambda_{1H} + \mu_{1H} + r_{1H} \quad (15)$$

Differentiating w. r. t. Z and taking X=1, Y=1

$$\mu_2 G_3 - \mu_{1H}G_1 + \mu_{1L}G_1 - \mu_{1L}G_2 = -\mu_{1H} + \mu_2 \quad (16)$$

After solving equations (14) – (16) we get

$$G_3 = 1 - \frac{\lambda_{1L}\mu_{1L}}{\mu_2(r_{1L}+\mu_{1L})} - \frac{\lambda_{1H}\mu_{1H}}{\mu_2(r_{1H}+\mu_{1H})} \quad (17)$$

$$G_1 = 1 - \frac{\lambda_{1H}}{(r_{1H}+\mu_{1H})} \quad (18)$$

$$G_2 = 1 - \frac{\lambda_{1L}}{(r_{1L}+\mu_{1L})} - \frac{\lambda_{1H}}{(r_{1H}+\mu_{1H})} \quad (19)$$

$$\gamma_1 = \frac{\lambda_{1H}}{(r_{1H}+\mu_{1H})} \quad (20)$$

$$\gamma_2 = \frac{\lambda_{1L}}{(r_{1L}+\mu_{1L})} + \frac{\lambda_{1H}}{(r_{1H}+\mu_{1H})} \quad (21)$$

$$\gamma_3 = \frac{\lambda_{1L}\mu_{1L}}{\mu_2(r_{1L}+\mu_{1L})} + \frac{\lambda_{1H}\mu_{1H}}{\mu_2(r_{1H}+\mu_{1H})} \quad (22)$$

The steady-state solution of the model is

$$P_{\eta 1L,\eta 1H,\eta 2} = \gamma_1^{\eta 1H}\gamma_2^{\eta 1L}\gamma_3^{\eta 2}(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)$$

The solution of the model exist if $\gamma_1, \gamma_2, \gamma_3 < 1$

Partial queue lengths

$$L_1 = \frac{\gamma_1}{(1-\gamma_1)}$$

$$L_2 = \frac{\gamma_2}{(1-\gamma_2)}$$

$$L_3 = \frac{\gamma_3}{(1-\gamma_3)}$$

Mean Queue Length (L) = $\frac{\gamma_1}{(1-\gamma_1)} + \frac{\gamma_2}{(1-\gamma_2)} + \frac{\gamma_3}{(1-\gamma_3)}$

VII. NUMERIC BEHAVIOUR ANALYSIS

Considering numerical values as:

$$\lambda_{1H} = 5, \lambda_{1L} = 3, \mu_{1L} = 10, \mu_{1H} = 14, \mu_2 = 18, r_{1H} = 3, r_{1L} = 2$$

We find the values of utilization factor and queue length at both the server as follows:

TABLE 1. TRAFFIC INTENSITY, PARTIAL AND MEAN QUEUE LENGTH AT BOTH SERVER WITH VARIANCE IN RENEGING RATE OF HIGH PRIORITY CUSTOMERS

r_{1H}	γ_1	γ_2	γ_3	L_1	L_2	L_3	L
1	.3333	.5833	.3980	.4999	1.3998	.6611	2.5608
1.5	.3226	.5726	.3897	.4762	1.3397	.6385	2.4544
2	.3125	.5625	.3819	.4545	1.2857	.6179	2.3581
2.5	.3030	.5530	.3745	.4347	1.2371	.5987	2.2705
3	.2941	.5441	.3676	.4166	1.1935	.5813	2.1914
3.5	.2857	.5357	.3610	.3999	1.1538	.5649	2.1186
4	.2778	.5278	.3548	.3846	1.1177	.5499	2.0522
4.5	.2702	.5203	.3490	.3702	1.0846	.5361	1.9909
5	.2632	.5132	.3435	.3572	1.0542	.5232	1.9346

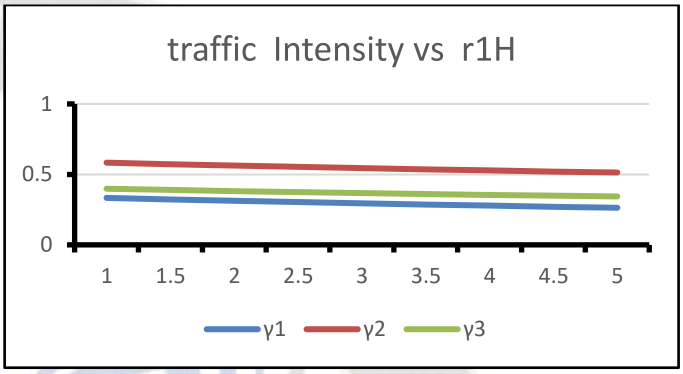

Figure2. $\gamma_1, \gamma_2, \gamma_3$ vs r_{1H}

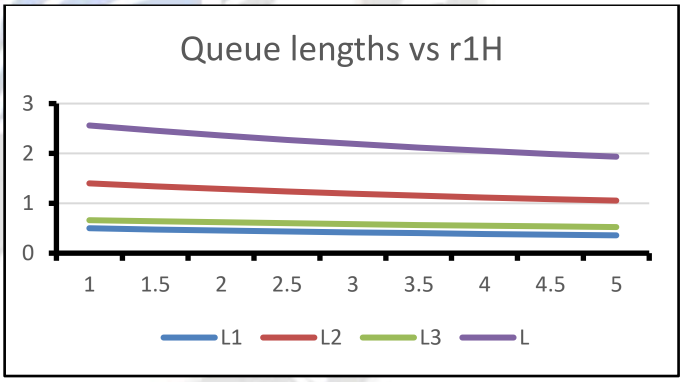

Figure 3: L_1, L_2, L_3, L vs r_{1H}

TABLE 2: TRAFFIC INTENSITY, PARTIAL AND MEAN QUEUE LENGTH AT BOTH SERVER WITH VARIANCE IN RENEGING RATE OF LOW PRIORITY CUSTOMERS

r_{1L}	γ_1	γ_2	γ_3	L_1	L_2	L_3	L
1	.2941	.5668	.3803	.4166	1.3084	.6137	2.3387
1.5	.2941	.5549	.3737	.4166	1.2467	.5967	2.2599
2	.2941	.5441	.3677	.4166	1.1935	.5815	2.1916
2.5	.2941	.5341	.3621	.4166	1.1464	.5676	2.1306
3	.2941	.5249	.3570	.4166	1.1048	.5552	2.0766
3.5	.2941	.5163	.3522	.4166	1.0674	.5437	2.0277
4	.2941	.5084	.3478	.4166	1.0342	.5333	1.9841
4.5	.2941	.5009	.3437	.4166	1.0036	.5237	1.9439
5	.2941	.4941	.3399	.4166	.9767	.5149	1.9082

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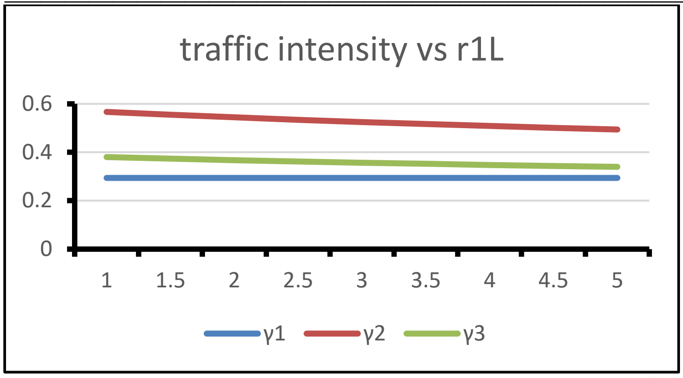

Figure4: $\gamma_1, \gamma_2, \gamma_3$ vs r_{1L}

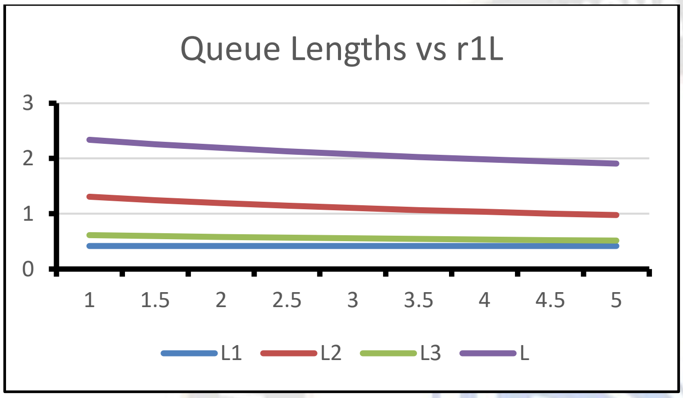

Figure 5: L_1, L_2, L_3, L vs r_{1L}

TABLE 3: TRAFFIC INTENSITY, PARTIAL AND MEAN QUEUE LENGTH AT BOTH SERVER WITH VARIANCE IN SERVICE RATE OF HIGH PRIORITY CUSTOMERS

μ_{1H}	γ_1	γ_2	γ_3	L_1	L_2	L_3	L
10	.3846	.6346	.3525	.6249	1.7367	.5444	2.9060
11	.3571	.6071	.3570	.5555	1.5452	.5552	2.6559
12	.3333	.5833	.3610	.4999	1.3998	.5649	2.4646
13	.3125	.5625	.3645	.4545	1.2857	.5736	2.3138
14	.2941	.5441	.3676	.4166	1.1935	.5813	2.1914
15	.2777	.5277	.3702	.3845	1.1173	.5878	2.0896
16	.2632	.5132	.3727	.3572	1.0542	.5941	2.0055
17	.25	.5	.3749	.3333	1	.5997	1.9330
18	.2381	.4881	.3769	.3125	.9535	.6049	1.8708

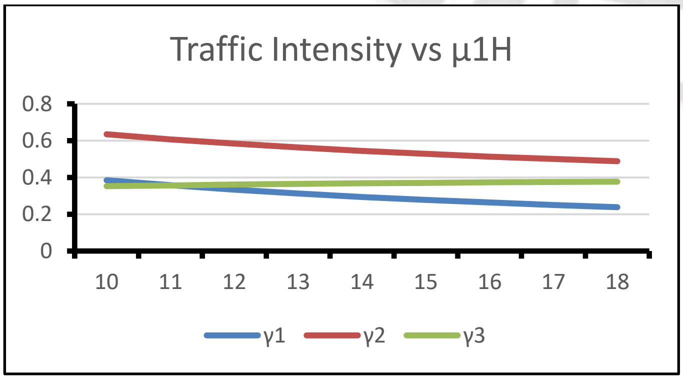

Figure 6: $\gamma_1, \gamma_2, \gamma_3$ vs μ_{1H}

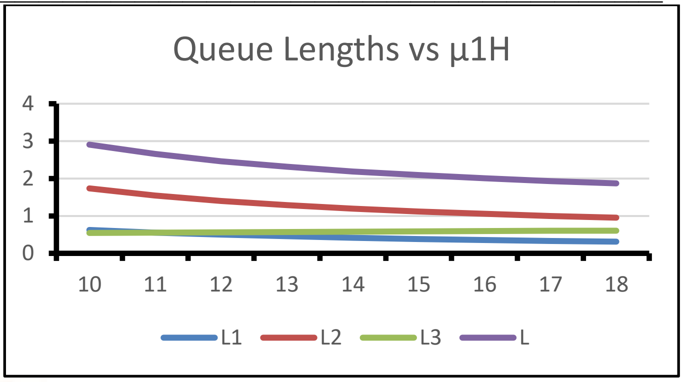

Figure 7: L_1, L_2, L_3, L vs μ_{1H}

TABLE 4: TRAFFIC INTENSITY, PARTIAL AND MEAN QUEUE LENGTH AT BOTH SERVER WITH VARIANCE IN SERVICE RATE OF LOW PRIORITY CUSTOMERS

μ_{1L}	γ_1	γ_2	γ_3	L_1	L_2	L_3	L
7	.2941	.6274	.3584	.4166	1.6838	.5586	2.659
8	.2941	.5941	.3621	.4166	1.4636	.5676	2.4478
9	.2941	.5668	.3652	.4166	1.3084	.5753	2.3003
10	.2941	.5441	.3677	.4166	1.1935	.5815	2.1916
11	.2941	.5249	.3698	.4166	1.1048	.5868	2.1082
12	.2941	.5084	.3717	.4166	1.0342	.5916	2.0424
13	.2941	.4941	.3732	.4166	.9767	.5954	1.9887
14	.2941	.4816	.3746	.4166	.9290	.5990	1.9446
15	.2941	.4706	.3759	.4166	.8889	.6023	1.9078

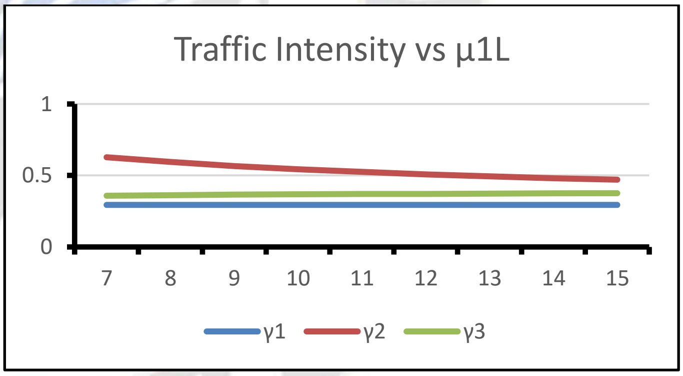

Figure 8: $\gamma_1, \gamma_2, \gamma_3$ vs μ_{1L}

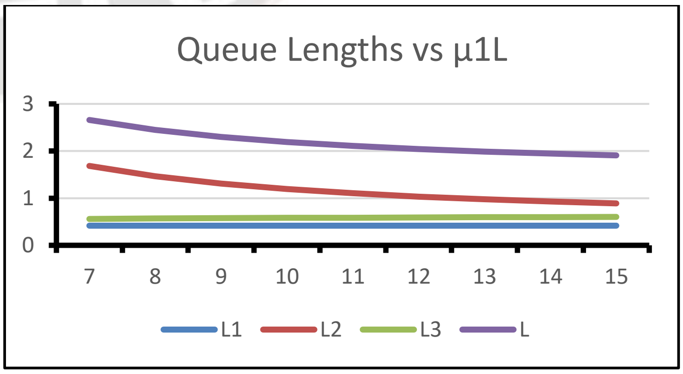

Figure 9: L_1, L_2, L_3, L vs μ_{1L}

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VIII. RESULTS

- From above numerical calculation, it is clear that change in reneging and service rate effect the queue lengths and utilization factor in the system. Table 1 and Figure 2, 3 indicates that increase in reneging rate of high priority customers results constant decrease in traffic intensity and queue lengths at each server. But increase in reneging rate of low priority customers have no effect on queue length and utilization of server by high priority customers at C_1. It can be seen graphically in figure 4,5.
- If we increase service rate for higher priority class at first server shows reduction in congestion and marginal queue lengths at C_1. Traffic intensity & queue length at second server increases slowly. figure 6,7 shows it. Table 4, figure 8,9 results increase in service rate of low priority class have no change in queue length of higher class. But length of queue increases at second server.

A. *Special case*

If priority is not given to the customers, then result resembles with results given by T. P. Singh et al [2014] and we get

$$\lambda_{1L} = 0, \lambda_{1H} = \lambda, r_{1L} = r_{1H} = r_1, \mu_{1L} = \mu_{1H} = \mu_1$$

$$L_1 = L_2 = \frac{\lambda}{(r_1 + \mu_1 - \lambda)}$$

$$L_3 = \frac{\lambda \mu_1}{\mu_2 (r_1 + \mu_1) - \lambda \mu_1}$$

$$L = \frac{\lambda}{(r_1 + \mu_1 - \lambda)} + \frac{\lambda \mu_1}{\mu_2 (r_1 + \mu_1) - \lambda \mu_1}$$

If we take, $\lambda = 5, r_1 = 3, \mu_1 = 14, \mu_2 = 18$

Then we get values as,

$$\gamma_1 = \gamma_2 = .2941, \gamma_3 = .2287,$$
$$L_1 = L_2 = .4166, L_3 = .2966, L = .7132$$

IX. CONCLUSION

In this paper, we studied a queue network model containing two servers in series. Impatient behavior of customer with reneging rate of both type of Priority customer at entry server in the system were discussed. We find increase in reneging rate of low and high priority customer queues due to some urgent calls or slow service rate then the queue lengths at both the server increases with slow speed. From comparison of numerical calculation and special case, it is clear that if priority discipline is not considered in the system, then there is only 29% & 22% use of first and second server and mean length of queue decreases. That requires the change in planning and designing of the corresponding system for maximum utilization of the service facilities. The results of the research further carried in more complex real-life situations with multiple servers to demonstrate the effect of priority in queueing network system.

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