A new approach for FCSRs

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Abstract. The Feedback with Carry Shift Registers (FCSRs) have been proposed as an alternative to Linear Feedback Shift Registers (LFSRs) for the design of stream ciphers. FCSRs have good statistical properties and they provide a built-in non-linearity. However, two attacks have shown that the current representations of FCSRs can introduce weaknesses in the cipher. We propose a new "ring" representation of FCSRs based upon matrix definition which generalizes the Galois and Fibonacci representations. Our approach preserves the statistical properties and circumvents the weaknesses of the Fibonacci and Galois representations. Moreover, the ring representation leads to automata with a quicker diffusion characteristic and better implementation results. As an application, we describe a new version of F-FCSR stream ciphers.

Keywords: Stream cipher, FCSRs, ℓ-sequence, ring FCSRs.

1 Introduction

The FCSRs have been proposed by Klapper and Goresky [14, 15, 13] as an alternative to LFSRs for the design of stream ciphers. FCSRs share many of the good properties of LFSRs: sequences with known period and good statistical properties. But unlike LFSRs, they provide an intrinsic resistance to algebraic and correlation attacks because of their quadratic feedback function. However, two recent results [5, 8] have shown weaknesses in stream ciphers using either the Fibonacci or Galois FCSR. Hell and Johansson [8] have exploited the bias in the carries behaviour of a Galois FCSR to mount a very powerful attack against the F-FCSR stream cipher [2, 3]. Fisher *et al.* [5] have considered an equivalent of the F-FCSR stream cipher based upon a Fibonacci FCSR to study the linear behavior of the induced system.

We present a new approach for FCSRs which we call the ring representation or ring FCSR. This representation is based on the adjacency matrix of the automaton graph. A ring FCSR can be viewed as a generalization of the Fibonacci and Galois representations. This structure has been widely studied for the LFSR case as shown in [20, 10, 18] and is a building block of the stream cipher Pomaranch when LFSRs are used [11]. However, this paper presents for the first time this structure in the FCSR case.

In a Fibonacci FCSR, we have a single feedback function which depends on multiple inputs. In a Galois FCSR, we have multiple feedback functions with one common input. A ring FCSR can be viewed as a tradeoff between the two previous representations. It has multiple feedback functions with different inputs. An example of a ring FCSR is shown in Fig. 1.

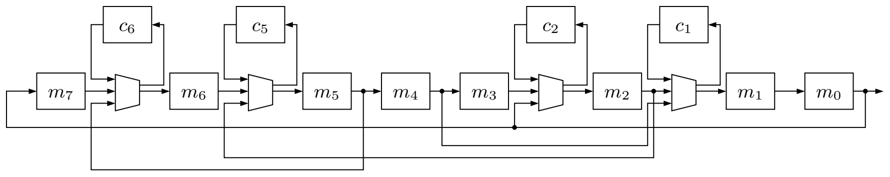


Fig. 1. An example of a ring FCSR ($q = -347$).

Ring FCSRs have many advantages over the previous representations. First, they keeps all the good and traditional properties of the FCSRs (known period, large entropy,...). Second, they can also be used to prevent the attack of Hell and Johansson [8]. Third, they have a better diffusion than the Galois or Fibonacci FCSR. Moreover, the ring representation allows the designer to tune the implementation of FCSRs.

Section 2 gives an overview on FCSRs theory and classical representations. The ring FCSR is presented in Section 3. We discuss the implementation properties in Section 4 and a new version of F-FCSR is proposed in Section 5.

2 Theoretical Background

First, we will recall some basic properties of 2-adic integers. For a more theoretical approach the reader can refer to [14, 16, 15, 7, 1].

2.1 2-adic numbers and period

A 2-adic integer is formally a power series $s = \sum_{i=0}^{\infty} s_i 2^i$, $s_i \in \{0,1\}$. This series always converges if we consider the 2-adic topology. The set of 2-adic integers is denoted by $\mathbb{Z}_2$. Addition and multiplication in $\mathbb{Z}_2$ can be performed by reporting the carries to the higher order terms, i.e. $2^n + 2^n = 2^{n+1}$ for all $n \in \mathbb{N}$. If there exists an integer N such that $s_n = 0$ for all $n \geq N$, then s is a positive integer. Every odd integer q has an inverse in $\mathbb{Z}_2$.

The following property gives a complete characterization of eventually periodic binary sequences in terms of 2-adic integers (see [7] for the proof).

Property 1 *Let $S = (s_n)_{n\in\mathbb{N}}$ be a binary sequence and let $s = \sum_{i=0}^{\infty} s_i 2^i$ be the corresponding 2-adic integer. The sequence S is eventually periodic if and only if there exist two numbers p and q in $\mathbb{Z}$, q odd, such that $s = p/q$.*
Moreover, S is strictly periodic if and only if $pq \leq 0$ and $|p| \leq |q|$. In this case, we have the relation $s_n = (p \cdot 2^{-n} \mod q) \mod 2$.

Then, the period of S is the order of 2 modulo q, i.e., the smallest integer T such that $2^T \equiv 1 \pmod{q}$. The period T is always less or equal to $|q| - 1$. If q is prime, then T divides $|q| - 1$. If $T = |q| - 1$, the corresponding sequence S is called an ℓ-sequence. As detailed in [14, 15, 7, 17], ℓ-sequences have many proved properties that could be compared to the ones of m-sequences: known period, good statistical properties, fast generation of sequences, etc. In summary, FCSRs have almost the same properties as LFSRs but they provide non-linear relations between inputs and outputs.

2.2 Galois FCSRs

A Galois FCSR (as shown in Fig. 2) consists of an n-bit main register $M = (m_0, \ldots, m_{n-1})$ with some fixed feedback positions $d_0, \ldots, d_{n-1}$. All the feedbacks are controlled by the feedback cell m_0, and $n-1$ binary carry cells $C = (c_0, \ldots, c_{n-2})$. At time t, an automaton in state (M, C) is updated in the following way:

1. Compute the sums $x_i = m_{i+1} + c_i d_i + m_0 d_i$ for all i such that $0 \leq i < n$ with $m_n = 0$ and $c_{n-1} = 0$ and where m_0 represents the feedback bit;
2. update the state as follows: $m_i \leftarrow x_i \mod 2$ for all $i \in [0..n-1]$ and $c_i \leftarrow x_i \operatorname{div} 2$ for $0 \leq i < n$ for all $i \in [0..n-2]$.

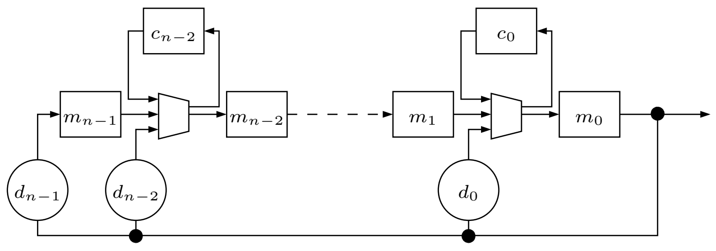


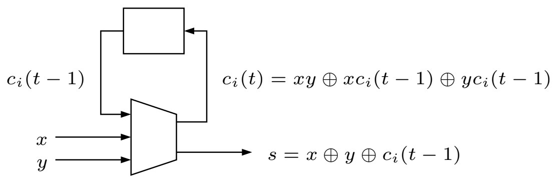
Fig. 2. A Galois FCSR.

Each cell is updated using an adder with carry as described in Fig. 3.

The reader can refer to [7] for a complete description of Galois FCSRs properties. We recall the main property presented in [4] where the content of each cell of M could be seen as a 2-adic integer.

Property 2 *Let* $q = 1 - 2\sum_{i=0}^{n-1} d_i 2^i$*, and* $r_i = \sum_{t=0}^{\infty} m_i(t)2^t$ *for* $0 \leq i < n$*;* r_i *is the 2-adic integer corresponding to the sequence observed in the i-th cell of the main register* M*. Then, for all* $0 \leq i < n$*, there exists* $p_i \in \mathbb{Z}$ *such that* $r_i = p_i/q$*.*

As shown on Fig. 2, there exist some correlations between the carries values and the feedback value. More precisely a single cell controls all the feedbacks. This property is the basis of the attack presented in [8].



$c_i(t-1)$
$c_i(t) = xy \oplus xc_i(t-1) \oplus yc_i(t-1)$
x
y
$s = x \oplus y \oplus c_i(t-1)$

Fig. 3. 2-bit adder with carry.

2.3 Fibonacci FCSRs

A Fibonacci FCSR (represented in Fig. 4) is composed of a main register $M = (m_0, \ldots, m_{n-1})$ with n binary cells. The binary feedback taps $(d_0, \ldots, d_{n-1})$ are associated to an additional carry register c of $w_H(d)$ binary cells, where $w_H(d)$ is the Hamming weight of $d = (1 + |q|)/2$.

An automaton in state (M, c) is updated in this way:

1. compute the sum $x = c + \sum_{i=0}^{n-1} m_i d_{n-1-i}$;
2. then, update the state: $M \leftarrow (m_1, \ldots, m_{n-1}, x \mod 2)$, $c \leftarrow x \text{ div } 2$.

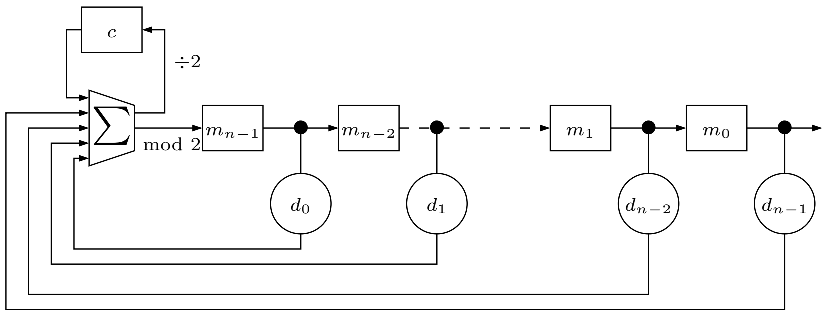


Fig. 4. A Fibonacci FCSR.

As for a Galois FCSR, the content of a particular cell m_i of a Fibonacci FCSR is a 2-adic integer: Property 2 holds also for Fibonacci FCSRs as shown in [7].

The cell m_{n-1} is the only element with a non-linear behaviour in a Fibonacci FCSR. As shown in [5], an attack can be carried out if a linear filter is used with a Fibonacci FCSR.

3 A new approach for FCSRs

Galois and Fibonacci FCSRs are two different automata with similar properties as seen in the previous section. In a Galois FCSR, the first cell m_0 modifies $w_H(d)$ cells of the main register. In a Fibonacci FCSR, the cell m_{n-1} is modified by $w_H(d)$ cells of the main register. The ring representation of FCSRs is a trade-off between these two setups. It is in fact a generalization of Galois and Fibonacci FCSRs.

Definition 1 *A ring FCSR is an automaton composed of a main shift register of n binary cells $m = (m_0, \dots, m_{n-1})$, and a carry register of n integer cells $c = (c_0, \dots, c_{n-1})$. It is updated using the following relations:*

$$\begin{cases} m(t+1) = Tm(t) + c(t) \mod 2 \\ c(t+1) = Tm(t) + c(t) \div 2 \end{cases} \tag{1}$$

where T is a $n \times n$ matrix with coefficients 0 or 1 in $\mathbb{Z}$, called transition matrix, of this form:

$$\begin{pmatrix} * & 1 & & & & \\ & * & 1 & & (*) & \\ & & * & 1 & & \\ & & & \ddots & \ddots & \\ & & (*) & & * & 1 \\ 1 & & & & & * \end{pmatrix}$$

Note that $\div 2$ *is the traditional expression:* $X \div 2 = \frac{X-(X \mod 2)}{2}$.

The main difference with Fibonacci and Galois FCSRs is that any cell can be used as a feedback for any other cell. This remark leads to a new way to draw FCSRs. For instance, the ring FCSR of Fig. 1 can be viewed as in Fig. 5.

3.1 Remarks on the transition matrix

According to Definition 1, we have the following property considering that $t_{i,j}$ is the element at the i-th row and j-th column:

$$T = (t_{i,j})_{0 \le i,j < n} \text{ with } t_{i,j} = \begin{cases} 1 & \text{if cell } m_j \text{ is used to update cell } m_i, \\ 0 & \text{otherwise.} \end{cases}$$

As the main register of a ring FCSR is by definition a shift register, the over-diagonal of the associated transition matrix is by definition full of ones, i.e. for all $0 \le i < n$ we have $t_{i,i+1 \mod n} = 1$. For example, the FCSR presented in Fig.1 can be described using the following transition matrix T_R (with $q = -347$):

$$T_R = \begin{pmatrix} 0&1&0&0&0&0&0&0 \\ 0&0&1&0&1&0&0&0 \\ 1&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&1&0&1 \\ 1&0&0&0&0&0&0&0 \end{pmatrix}$$

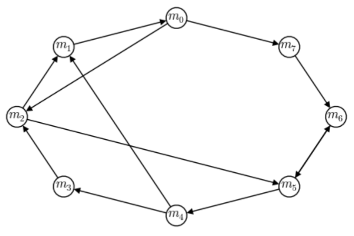


Fig. 5. A ring FCSR.

This notation agrees with the ones proposed in [7]. In particular, Galois FCSRs and Fibonacci FCSRs have respectively the following transition matrices T_G and T_F with the following form:

$$T_G = \begin{pmatrix} d_0 & 1 & & & & \\ d_1 & 0 & 1 & & (0) & \\ d_2 & & 0 & 1 & & \\ \vdots & & & \ddots & \ddots & \\ d_{n-2} & & (0) & & 0 & 1 \\ 1 & & & & & 0 \end{pmatrix} \qquad T_F = \begin{pmatrix} 0 & 1 & & & & \\ & 0 & 1 & & (0) & \\ & & 0 & 1 & & \\ & (0) & & \ddots & \ddots & \\ & & & & 0 & 1 \\ 1 & d_{n-2} & \dots & d_2 & d_1 & d_0 \end{pmatrix}$$

3.2 Characterizing the cells content

Definition 1 introduces the transition matrix of a ring FCSR. We explain now how the value q could be directly computed from a given matrix T.

Let $m_i(t)$ denote the content of each cell of the main register at time t and $M_i(t)$ the series observed in the i-th cell:

$$M_i(t) = \sum_{k \in \mathbb{N}} m_i(t+k)2^k.$$

Then, the ring FCSR is updated at each clock and the series observed in each cell verify the following relation according to Equation 1:

$$M(t+1) = TM(t) + c(t) \tag{2}$$

with $M(t) = (M_0(t), \cdots, M_{n-1}(t))$ and where $c(t) = (c_0(t), \cdots, c_{n-1}(t))$ is the carry register at clock t.

This new representation of the main register contents is fundamental in our approach. We ignore the general content of the main register and focus on the series produced by each cell of the main register.

Theorem 1 *The series $M_i(t)$ observed in the cells of the main register are 2-adic expansion of p_i/q with $p_i \in \mathbb{Z}$ and with $q = \det(I - 2T)$.*

Proof. According to the definition of $M_i(t)$ and to Definition 1, we can write the following relation: $M(t) = 2M(t+1) + m(t)$ where $m(t)$ is a binary vector of size n. Using Equation 2, we have:

$$(I - 2T) \cdot M(t) - 2 \cdot c(t) - m(t) = 0.$$

Considering the adjugate of $I - 2T$, we obtain:

$$\det(I - 2T) \cdot M(t) = Adj(I - 2T)(m(t) + 2 \cdot c(t)).$$

In this relation, the right member is a vector of integers $(p_0(t), \dots, p_{n-1}(t))$. And so we have $M_i(t) = p_i(t)/\det(I - 2T)$ by identifying the terms using Property 2.

Lemma 1 *With the notation of Theorem 1, if $q = \det(I - 2T)$ is prime, and if the order of 2 in $\mathbb{Z}/q\mathbb{Z}$ is maximal, then each M_i is an ℓ-sequence.*

A ring FSCR has all the properties induced by the ℓ-sequences if its transition matrix T is such that $q = \det(I - 2T)$ is prime and the order of 2 in $\mathbb{Z}/q\mathbb{Z}$ is maximal.

4 Implementation properties

We detail in this section the new implementation characteristics of the ring FCSRs. All the properties given in this section can be directly applied to LFSRs by replacing addition with carry by addition modulo 2.

Path/fan-out – The Galois FCSR is considered in many works [6, 7, 12] as the best representation for the hardware implementation of an FCSR. It has a better critical path, i.e, a shorter longuest path, than a Fibonacci FCSR. A drawback of the Galois representation is that the fan-out of the feedback cell m_0 is $w_H(d)$ with $d = (1 + |q|)/2$. At the opposite, the Fibonacci representation has a fan-out of 2. A ring FCSR allows the designer to tune both the critical path and the fan-out throught the choice of the transition matrix:

– the critical path is given by the row a_i with the largest number of 1s;
– the fan-out is given by the column b_i with the largest number of 1s.

We compare in Table 1 the critical path, the fan-out and the cost of the different representations of an FCSR. We have expressed the critical path as the number of adders crossed. The choice of the adder has also an impact on the path of a ring FCSR. A naive adder (Fig. 6) composed of a serialisation of generic adder leads to a path of $max(w_H(a_i)) - 1$ adders. However, it is possible to exploit the commutativity to perform additions in parallel. This reduces the critical path to $max(\lceil \log_2(w_H(a_i)) \rceil)$ adders.

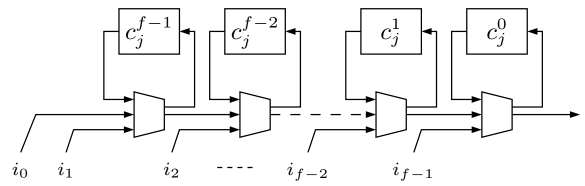


Fig. 6. A naive adder

For each given q, it should be possible to find a transition matrix corresponding to a critical path with only one adder and a fan-out equal to 2. This is the case of the ring FCSR given in Fig. 1.

	Fibonacci	Galois	Ring
Path	$\lceil \log_2(w_H(d)) \rceil$	1	$\max(\lceil \log_2(w_H(a_i)) \rceil)$
Fan-out	2	$w_H(d)$	$\max(w_H(b_i))$
Cost ($\#_{adders}$)	$w_H(d) - 1$	$w_H(d) - 1$	$w_H(T) - n$

Table 1. Comparison of the different representations.

Cost – An interesting perspective of the ring FCSR is that we can find implementation with fewer gates than in a Fibonacci/Galois representation. This possibility was first observed in [18]. However, the solution proposed in [18] is specific to LFSRs and it cannot be applied systematically to FCSRs as shown in appendix C. The number $\#_{adders}$ of 2-bit adders required in the different representations of an n-bit FCSR is compared in Table 1. Amongst the three representations, ring FCSR is the only one that allows to find an implementation with less than $(w_H(d)-1)$ 2-bit adders. For $q = -347$, a Galois or Fibonacci representation leads to $\#_{adders} = 4$. A ring representation with the matrix T_R defined by:

$$T_R = \begin{pmatrix} 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&1 \\ 0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&1&0&0&0&0&0 \end{pmatrix}$$

leads to an implementation with $\#_{adders} = 3$, a fan-out of 2 and a critical path of 1 adder.

Side-channel attacks – It seems possible to work out an equivalent of the side-channel attack of Joux and Delaunay [12] on Galois FCSR using the results of Hell and Johansson [8]. Such an attack would exploit the power consumption to recover the feedback m_0 (because of the excessive fan-out of the feedback cell) and therefore how the carry cells are modified. As the ring FCSR has a reduced fan-out and uncorrelated carries, it is a better alternative to prevent side-channel attacks.

5 F-FCSR based on ring representation

In this section, we propose a generic algorithm to construct F-FCSR stream ciphers based upon a ring FCSR with a linear filter. We give two particular examples which are F-FCSR-H v3 and F-FCSR-16 v3. Any designer using the proposed algorithm could generate its own stream cipher according to the following parameters:

- key length k and IV length v that will provide the corresponding size $n := k + v$ of the T matrix (usually $k = v$);
- the number u of bits output at each clock taken between 1 and $n/16$ to ensure a hard inversibility of the filter. Moreover for later design we require u to be a divisor of n;
- the number of willing feedbacks ℓ usually taken between $n/2 - 5$ and $n/2 + 5$ to ensure a sufficient non linear structure and a sufficiently weighted filter.

The algorithm is composed of 3 particular steps: the choice of the matrix T, the choice of the linear filter and the key/IV setup.

5.1 The choice of the matrix T

Using the requirements of Section 3, we pick a $n \times n$ random matrix T with the following requirements:

- the matrix must be composed of 0 and 1 and with a general weight equal to $n + \ell$;
- the over-diagonal must be full of 1 and $t_{n-1,0} = 1$ (to ensure a classical ring structure of the automaton);
- the number of ones for a given row or a given column must be at most two. This last condition allows a better diffusion induced by maximizing the number of cells reached by the feedbacks. It also provides uncorrelated carries and a fan-out bounded by 2.

For each picked matrix with the previous requirements, test if:

1. $\log_2(q) \geq n$; $\det(T) \neq 0$;
2. $q = \det(I - 2T)$ is prime; the order of 2 modulo q is $|q| - 1$.

The first condition ensures a non-degenerated matrix. The second ensures good statistical properties and a long period.

This matrix completely defines the ring FCSR. In this case, the diffusion (that will be faster than in Galois or Fibonacci FCSRs) corresponds to the diameter d of the graph associated to the ring FCSR. This value is the maximal distance between two cells of the main register. In other words, d is the distance after which all the cells of the main register have been influenced by at least one other cell through the feedbacks. It corresponds to the minimal number of clocks required to have all the cells of the main register influenced by at least one other cell. d should be sufficiently small for diffusion purpose.

5.2 The filter

As done in the previous versions of F-FCSR [3], the filter extracting the keystream is linear in order to break the 2-adic structure of the automaton through $\oplus$ operations. It also prevents linearization attacks over the set of 2-adic numbers. As previously done, the filter uses the cells of the main register which receive a feedback to prevent correlation attacks. The periodic structure of the filter in the previous F-FCSR has been exploited in [8] to generate particular subsystems from the main one. We break this structure in the following way:

– let $F = \{m_{f_0}, \cdots, m_{f_{\ell-1}}\}$ be the set of all the cells m_i that receive a feedback and indexed in this way: the row f_i of the matrix T has more than one 1 for $0 \leq i < \ell$, and $f_i < f_{i+1}$.
– The u bits of output are: $\forall\, 0 \leq i < u,\ z_i = \bigoplus_{j \equiv i \mod u} m_{f_j}$.

This new structure leads to a more "random" structure in the filtered cells choice avoiding extraction of simple subsystems at small time intervals.

5.3 Key and IV setup

As shown in [8], if at a given time, the FCSR is in a synchronized state (i.e. a state from which after a finite number of steps the automaton will return, i.e. a state belonging to the main cycle), next and previous states of the main cycle could be directly deduced using only multiplications over $\mathbb{Z}/q\mathbb{Z}$. Moreover, as shown in [4], a Galois FCSR is synchronized in at most $n + 4$ clocks, but in reality, few clocks are sufficient. So, to completely discard the weakness of the key and IV setup used in [8], we need to create a non-synchronized structure for the key and IV setup to prevent a direct key recovery attack. Thus, the new key and IV setup aims at keeping its states non-synchronized in order to create a transformation that is really hard to inverse.

However, using a ring FCSR leads to a new problem: we can not ensure the entropy of the automaton. In the case of F-FCSR using a Galois or a Fibonacci FCSR, taking the content of the carry register equal to zero prevents collisions (i.e. one point of the states graph with two preimages) and allows to maintain a constant entropy. This particular property comes from the particular structure of the adjoint matrix $(I - 2T)^*$ that has its first row composed of the successive powers of two in case of a Galois FSCR (in a Fibonacci FCSR, a similar property exists on the last row). In the ring case, no particular structure exists in $(I - 2T)^*$. Note that in this case the collisions search becomes an instance of the subset sum problem with a complexity equals to $2^{n/2}$ (forcing the carry to be null) or $2^{3n/2}$ (in the general case).

Thus, the new key and IV setup is based upon those two facts: to stay on non-synchronized states as long as possible and to limit the entropy loss. The key and IV setup consists first in connecting at u different places shift registers of length $r = n/u$ (this corresponds to add n binary cells $a_0, \ldots, a_{n-1}$ at different places as shown in Fig. 7)

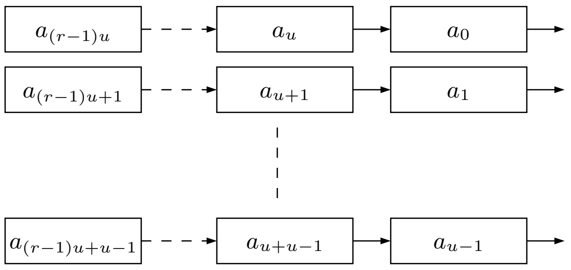


Fig. 7. Disposition of the cells $a_0, \ldots, a_{n-1}$ in u shift registers.

The u positions denoted by $J := \{j_0 < \cdots < j_{u-1}\}$ allowing to connect the u shift registers have been chosen such that for all $0 \leq i < u$, no adder exists between cells m_{j_i+1} and m_{j_i} (i.e. $w_H(R_{j_i}) = 1$ where R_{j_i} is the j_i^{th} row of the matrix T). Then, the shift register is connected between m_{j_i+1} and m_{j_i} using a 2-bit adder with carry (as shown in Fig. 8) modifying the content of m_{j_i} according the values of m_{j_i+1}, a_i and of the carry cell c_{j_i}.

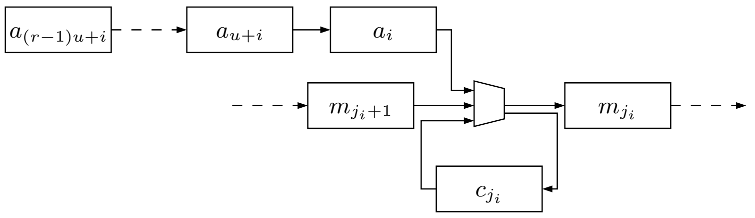


Fig. 8. Connection of a shift register in position j_i.

Using those new u simple shift registers inserted in the ring FCSR using a 2-bit adder, the key and IV setup works as follows:

– Initialize the cells $(a_0, \ldots, a_{n-1})$ with $(K \| 0^{n-k-v} \| IV)$, $M \leftarrow 0$, $C \leftarrow 0$.

– The FCSR is clocked r times. At each clock, the FCSR is filtered using F to produce a u bits vector $z_0, \ldots, z_{u-1}$ used to fill back $a_{(r-1)u}, \ldots, a_{(r-1)u+u-1}$: $a_{(r-1)u+i} \leftarrow z_i$ for $0 \leq i < u$.
– The FCSR is clocked $\max(r, d+4)$ times discarding the output.

The first step of the key and IV setup allows an initial diffusion of the key through the simple shift registers. The next r clocks induces a complete diffusion of the IV and of the key in the FCSR. The diffusion is complete at the end of the key and IV setup. Moreover, if an attacker is able to recover the state just at the end of the key and IV setup, he won't be able to use this information to recover the key because the new design is composed of non-synchronized states that are hard to inverse and hard to compute: for a given m_{k+1} bit value of the main register, the values c_k and m_k producing m_{k+1} are not unique leading to a combinatorial explosion when an attacker wants to recover a previous state.

As previously mentioned, this construction is not a bijection and behaves more like a random function. From this point, two attacks are essentially possible: direct collisions search and time memory data trade-off attack for collisions search built upon entropy loss. As mentioned before, direct collisions search has a cost of $2^{(n/2)}$ if the attacker is able to put at zero the carry bits. Due to the use of the ring FCSR that does not allow a direct control of the carry bits through the feedback bit, the probability to force to 0 the carry bits is about $2^{-\ell}$. Thus such an attack is more expensive than a key exhaustive search. In the other cases, the corresponding complexity is $2^{(3n/2)}$ preventing collisions search.

TMDTO attacks are possible if a sufficient quantity of entropy is lost. As studied in [19], considering that the key and IV setup are random function, the induced entropy loss is about 1 bit, so considering an initial entropy equal to n bits, the entropy after the key and IV setup is equal to $n-1$ bits. The question is now how to exploit this entropy loss for a collisions search in a TMDTO attack? A well-known study case is the attack proposed in [9] by J. Hong and W.H. Kim against the stream cipher MICKEY. Even if this attack seems to work, A. Rock has shown in [19] that the query complexity in the initial states space could not be significantly reduced and that the attacks based on the problem of entropy loss are less efficient than expected especially regarding the query complexity. So, we conjecture that considering that our key and IV setup behaves as a random function, the induced entropy loss is not sufficient to mount a complete TMDTO attack for collisions search taking into account the query complexity.

5.4 F-FCSR-H v3 and F-FCSR-16 v3

The details of the two constructions, especially the corresponding T matrices, are given respectively in Appendix A and B. The respective parameters are the following ones:

- For F-FCSR-H v3: $k = 80$, $v = 80$, $\ell = 82$, $n = 160$, $u = 8$, $d = 24$;
- For F-FCSR-16 v3: $k = 128$, $v = 128$, $\ell = 130$, $n = 256$, $u = 16$, $d = 28$.

These two automata have been chosen with an additional property: $(|q|-1)/2$ prime. This condition ensures maximal period for the outputted stream. However this is a hard condition to fill, so we don't put this in the classical requirements.

5.5 Resistance against known attacks

We do not discuss here the resistance against traditional attacks such as correlation and fast correlation attacks, guess and determine attacks, algebraic attacks, etc. The reader can refer to [3] for some details about resistance to these classical attacks. For a complete analysis against TMDTO attacks, the reader can refer to the Section 5.3. Now, we discuss the two main recent results [8] and [5] concerning attacks against FCSR and F-FCSR.

The attack presented in [8] against F-FCSR, which is based on a Galois FCSR, relies on the existence of correlations between the carries values and the feedback values. More precisely, the control of the m_0 bit leads to the control of the feedback values. If the feedback can be forced to 0 during t consecutive clocks, the behavior of the stream cipher becomes linear, and its synthesis becomes possible by solving of a really simple system. Due to ℓ-sequence properties, this behavior happens with a probability about 2^{-t} for a Galois FCSR. If instead a ring FCSR is used, this probability decreases to $2^{-t \cdot k}$ where k is the number of cells of the main register controlling a feedback. Thus for k values corresponding to most ring FCSR, the linear behavior probability becomes so small that the cost of the corresponding attack becomes higher than an exhaustive search. Analyzing further the attack in [8], we see that it is also based upon the fact that during t consecutive clocks the carries remains constant. We made an experiment with F-FCSR-H v3 to search for states for which carries does not change during transition to the next state. Looking over 2^{38} states, we found only 41 different states for which carries remains constant after one transition. We found none for which carries remains constant after two transitions.

In [5], the authors propose a linearization attack against a linearly filtered Fibonacci FCSR which does not work against any version of F-FCSR. This attack uses the linearity of the filter to build linear equations. In a Fibonacci FCSR, the carries values only influence one bit of the main register at each clock. Thus, if one could imagine to build a F-FCSR using a Fibonacci FCSR, such a generator would be subject to an attack where the control of the carries leads to the control of a part of the main register. Thus, we recommend to NOT use a Fibonacci FCSR in a linearly filtered stream cipher.

6 Conclusion and future work

In this paper, we present a new approach for FCSRs that unifies the previous representations. We can obtain with the ring representation a better diffusion characteristic and a better implementation for FCSRs. It can be used to prevent recent attacks against F-FCSR as shown in Section 5.

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A Description of the transition matrix for F-FCSR-H v3

Input parameters: $k = 80$ (key length), $v = 80$ (IV length), $\ell = 82$ (number of feedbacks), $n = 160$ (size of T), $u = 8$ (number of output bits), $d = 24$ (diameter of the graph).

We give here the description of the transition matrix $T = (t_{i,j})_{0 \leq i,j < 160}$ (see Fig. 9 for graphic representations):

– For all $0 \leq i < 160$, $t_{i,i+1 \mod 160} = 1$;

- For all $(i, j) \in S$, $t_{i,j} = 1$, where $S = \{$ (1, 121); (2, 133); (4, 44); (5, 82); (9, 38); (11, 40); (12, 54); (14, 105); (15, 42); (16, 63); (18, 80); (19, 136); (20, 2); (21, 35); (23, 28); (25, 137); (28, 131); (31, 102); (36, 41); (39, 138); (40, 31); (42, 126); (44, 127); (45, 77); (46, 110); (47, 86); (48, 93); (49, 45); (51, 17); (54, 8); (56, 7); (57, 150); (59, 25); (62, 51); (63, 129); (65, 130); (67, 122); (73, 148); (75, 18); (77, 46); (79, 26); (80, 117); (81, 1); (84, 72); (86, 60); (89, 15); (90, 89); (91, 73); (93, 12); (94, 84); (102, 141); (104, 142); (107, 71); (108, 152); (112, 92); (113, 83); (115, 23); (116, 32); (118, 50); (119, 43); (121, 34); (124, 13); (125, 74); (127, 149); (128, 90); (129, 57); (130, 103); (131, 134); (132, 155); (134, 98); (139, 24); (140, 61); (141, 104); (144, 48); (145, 14); (148, 112); (150, 59); (153, 39); (156, 22); (157, 107); (158, 30); (159, 78) $\}$;
- Otherwise, $t_{i,j} = 0$.

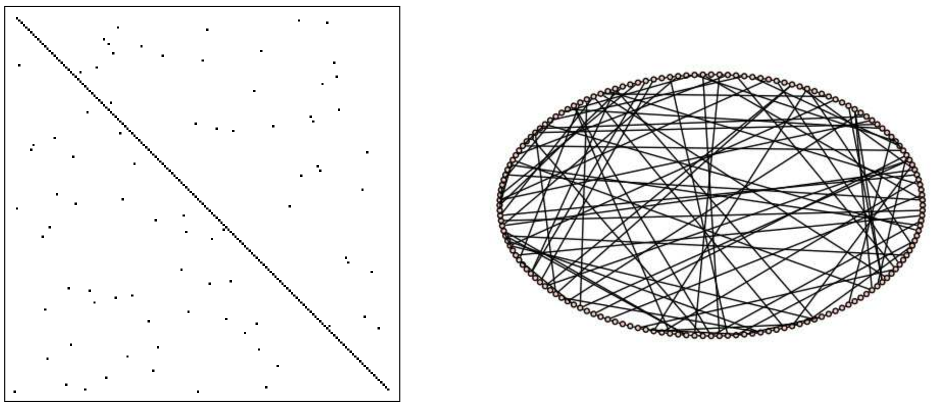

Fig. 9. Matrix representation and graph representation of the matrix T chosen for F-FCSR-H v3

- The corresponding q value is (in decimal notation):

$$q = 1741618736723237862812353996255699689552526450883$$

- The set J (for the first part of the Key/IV setup) is:

$$J = \{3, 22, 43, 64, 83, 103, 123, 143\}$$

- the 8 subfilters $F_0, \cdots, F_7$ are given by:

$$
\begin{aligned}
F_0&=\{1, 15, 28, 46, 59, 79, 93, 115, 128, 141, 158\}\\
F_1&=\{2, 16, 31, 47, 62, 80, 94, 116, 129, 144, 159\}\\
F_2&=\{4, 18, 36, 48, 63, 81, 102, 118, 130, 145\}\\
F_3&=\{5, 19, 39, 49, 65, 84, 104, 119, 131, 148\}\\
F_4&=\{9, 20, 40, 51, 67, 86, 107, 121, 132, 150\}\\
F_5&=\{11, 21, 42, 54, 73, 89, 108, 124, 134, 153\}\\
F_6&=\{12, 23, 44, 56, 75, 90, 112, 125, 139, 156\}\\
F_7&=\{14, 25, 45, 57, 77, 91, 113, 127, 140, 157\}
\end{aligned}
$$

B Description of the transition matrix for F-FCSR-16 v3

Input parameters: $k = 128$, $v = 128$, $\ell = 130$, $n = 256$, $u = 16$, $d = 28$.

We give here a description of the transition matrix $T = (t_{i,j})_{0 \le i,j < 256}$ (see Fig. 10 for graphic representations):

- For all $0 \le i < 256$, $t_{i,i+1 \mod 256} = 1$;
- For all $(i, j) \in S$, $t_{i,j} = 1$, where $S = \{$ (0, 52); (2, 150); (3, 2); (5, 169); (6, 89); (8, 100); (9, 1); (11, 156); (12, 9); (13, 46); (19, 146); (20, 206); (26, 204); (31, 254); (32, 151); (38, 144); (40, 108); (46, 167); (47, 198); (48, 70); (49, 98); (50, 213); (53, 214); (56, 87); (57, 55); (58, 162); (62, 160); (63, 13); (64, 192); (65, 59); (66, 12); (67, 207); (68, 209); (71, 229); (73, 84); (74, 199); (77, 168); (78, 122); (79, 35); (80, 154); (82, 153); (85, 188); (87, 51); (89, 4); (90, 49); (93, 231); (95, 224); (97, 249); (101, 208); (102, 120); (104, 218); (105, 8); (108, 77); (109, 68); (110, 250); (113, 237); (115, 252); (116, 17); (118, 73); (119, 182); (123, 29); (124, 234); (127, 138); (132, 190); (134, 244); (136, 219); (141, 228); (142, 205); (143, 58); (144, 230); (145, 210); (146, 44); (147, 137); (148, 130); (150, 79); (152, 111); (153, 172); (154, 141); (156, 78); (157, 131); (158, 110); (159, 127); (170, 189); (171, 112); (174, 217); (175, 7); (176, 187); (177, 40); (179, 118); (181, 195); (184, 48); (186, 64); (189, 246); (190, 47); (191, 37); (192, 211); (193, 85); (194, 181); (195, 61); (196, 54); (198, 222); (199, 83); (203, 105); (204, 201); (205, 43); (206, 139); (208, 20); (210, 242); (211, 124); (213, 253); (215, 243); (216, 69); (218, 176); (220, 30); (222, 19); (223, 232); (224, 239); (225, 220); (227, 102); (231, 185); (232, 15); (234, 152); (236, 62); (238, 245); (242, 197); (245, 235); (246, 171); (247, 67); (253, 26); (254, 202) $\}$;
- Otherwise, $t_{i,j} = 0$.

- The corresponding q value is (in hexadecimal notation):

$$
\begin{aligned}
q = (&B085834B6BFAE1541C54F7D84F42084C\\
&B0568496DDD0FEA5E99AA79C022023241)
\end{aligned}
$$

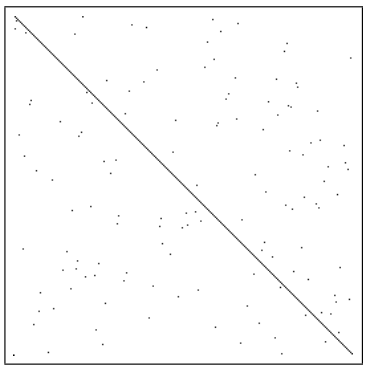
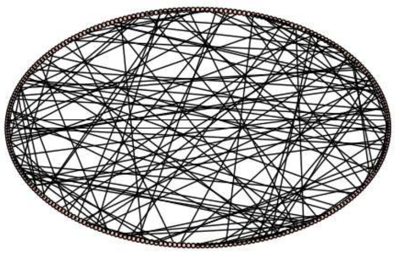

Fig. 10. Matrix representation and graph representation of the matrix T chosen for F-FCSR-16 v3

- The set J (for the first part of the Key/IV setup) is:

  $J = \{10, 27, 43, 59, 75, 91, 107, 122, 139, 155, 172, 187, 202, 219, 235, 251\}$

- the 16 subfilters $F_0, \cdots, F_{15}$ are given by:

$$\begin{aligned}
F_0 &= \{0, 40, 68, 101, 134, 158, 193, 218, 253\}\\
F_1 &= \{2, 46, 71, 102, 136, 159, 194, 220, 254\}\\
F_2 &= \{3, 47, 73, 104, 141, 170, 195, 222\}\\
F_3 &= \{5, 48, 74, 105, 142, 171, 196, 223\}\\
F_4 &= \{6, 49, 77, 108, 143, 174, 198, 224\}\\
F_5 &= \{8, 50, 78, 109, 144, 175, 199, 225\}\\
F_6 &= \{9, 53, 79, 110, 145, 176, 203, 227\}\\
F_7 &= \{11, 56, 80, 113, 146, 177, 204, 231\}\\
F_8 &= \{12, 57, 82, 115, 147, 179, 205, 232\}\\
F_9 &= \{13, 58, 85, 116, 148, 181, 206, 234\}\\
F_{10} &= \{19, 62, 87, 118, 150, 184, 208, 236\}\\
F_{11} &= \{20, 63, 89, 119, 152, 186, 210, 238\}\\
F_{12} &= \{26, 64, 90, 123, 153, 189, 211, 242\}\\
F_{13} &= \{31, 65, 93, 124, 154, 190, 213, 245\}\\
F_{14} &= \{32, 66, 95, 127, 156, 191, 215, 246\}\\
F_{15} &= \{38, 67, 97, 132, 157, 192, 216, 247\}
\end{aligned}$$

C Transvections

Mrugalski *and al.* have proposed in [18] a set of of transformations for LFSRs. The authors alter the structure of the conventional LFSRs but preserve the characteristic polynomial. They use several elementary mappings called EL corresponding to elementary transvections: if A is the original matrix, the transformation after one EL step is $A' = V_{(i,j)}TV_{(i,j)}^{-1}$ where $V_{(i,j)}$ represents the elementary transvection such that $V_{(i,j)} = (v_{k,l})$ with $v_{k,k} = 1$ for all possible k and $v_{i,j} = 1$. If we directly try to apply this technique to the case of FCSRs, we observe that the coefficients that appear in the transition matrix are no more only 0 and 1. For example, let consider $q = -53$, we have:

$$T = \begin{pmatrix} 1&1&0&0&0 \\ 1&0&1&0&0 \\ 0&0&0&1&0 \\ 1&0&0&0&1 \\ 1&0&0&0&0 \end{pmatrix}.$$

We could apply the elementary tranvection $V_{2,1}$ and we obtain:

$$V_{(2,1)}TV_{(2,1)}^{-1} = \begin{pmatrix} 1&0&0&0&0 \\ 1&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{pmatrix} \cdot \begin{pmatrix} 1&1&0&0&0 \\ 1&0&1&0&0 \\ 0&0&0&1&0 \\ 1&0&0&0&1 \\ 1&0&0&0&0 \end{pmatrix} \cdot \begin{pmatrix} 1&0&0&0&0 \\ -1&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{pmatrix} = \begin{pmatrix} 0&1&0&0&0 \\ 1&1&1&0&0 \\ 0&0&0&1&0 \\ 1&0&0&0&1 \\ 1&0&0&0&0 \end{pmatrix}$$

whereas, if we apply the elementary tranvection $V_{4,1}$, we obtain

$$V_{(4,1)}TV_{(4,1)}^{-1} = \begin{pmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 1&0&0&1&0 \\ 0&0&0&0&1 \end{pmatrix} \cdot \begin{pmatrix} 1&1&0&0&0 \\ 1&0&1&0&0 \\ 0&0&0&1&0 \\ 1&0&0&0&1 \\ 1&0&0&0&0 \end{pmatrix} \cdot \begin{pmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ -1&0&0&1&0 \\ 0&0&0&0&1 \end{pmatrix} = \begin{pmatrix} 1&1&0&0&0 \\ 1&1&1&0&0 \\ -1&0&0&1&0 \\ 2&0&0&0&1 \\ 1&0&0&0&0 \end{pmatrix}.$$