# A novel k-out-of-n Oblivious Transfer Protocols Based on Bilinear <br> <br> Pairings <br> <br> Pairings <br> Yalin Chen ${ }^{1}$, ${ }^{*}$ Jue-Sam Chou ${ }^{2}$, Xian-Wu Hou ${ }^{3}$ <br> ${ }^{1}$ Institute of Information Systems and Applications, National Tsing Hua University, Taiwan, R.O.C d949702@oz.nthu.edu.tw Tel: 886+3+5738997, Fax: 886+3+5723694 <br> ${ }^{2}$ Department of Information Management, Nanhua University, Taiwan, R.O.C jschou@mail.nhu.edu.tw <br> ${ }^{3}$ Department of Information Management, Nanhua University, Taiwan, R.O.C g4161509@mail2.nhu.edu.tw 

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#### Abstract

Low bandwidth consumption is an important issue in a busy commercial network whereas time may not be so crucial, for example, the end-of-day financial settlement for commercial transactions in a day. In this paper, we construct a secure and low bandwidth-consumption k-out-of-n oblivious transfer scheme based on bilinear pairings. We analyze the security and efficiency of our scheme and conclude that our scheme is more secure and efficient in communication bandwidth consumption than most of the other existing oblivious transfer schemes that we know.


## 1. Introduction

Since Oblivious transfer (OT) has an important feature that the sender cannot know which part of the transmitted messages the receiver will obtain and the receiver cannot learn extra messages other than the ones he chosen in advance, it has become an important primitive for designing secure protocol to provide privacy protection. The original OT was proposed by Rabin [20] in 1981. In the scheme, Alice sends a bit to Bob and Bob only has $1 / 2$ probability to obtain the bit. Subsequently, many flavors of OT schemes were proposed such as, 1-out-of -2 OT $\left(\mathrm{OT}_{2}^{1}\right)$ [1,2,17,21,22], 1-out-of-n $\mathrm{OT}\left(\mathrm{OT}_{\mathrm{n}}^{1}\right)[13,24,25]$, k-out-of-n OT ( $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$ ) [5,8,9,11,12,18], adaptive
$\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}[10,15]$, and non-interactive OT [23,26,27]. In 1985, Even et al.[22] first proposed a general 1-out-of-2 OT $\left(\mathrm{OT}_{2}^{1}\right)$ scheme, in which the sender sends two messages to the receiver, and the receiver can receive only one of them. In 1987, Crepeau [3] proved that Rabin's OT [20] and Even et al.'s $\mathrm{OT}_{2}^{1}$ [22] are computationally equivalent. The extension of $\mathrm{OT}_{2}^{1}$ is 1-out-of-n $\mathrm{OT}\left(\mathrm{OT}_{\mathrm{n}}^{1}\right)$ in which the sender sends n messages to the receiver, and the receiver can learn only one of them. The more general form is k-out-of-n OT $\left(\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}\right)$, in which the sender sends n messages to the receiver, and the receiver can obtain $k$ of them. Most previous $\mathrm{OT}_{\mathrm{n}}^{1}$ schemes cannot be used to construct an $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$ scheme easily. Hence, for constructing an $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$ scheme, an $\mathrm{OT}_{\mathrm{n}}^{1}$ scheme must be run k times. In an adaptive $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$, the sender sends n messages to the receiver, and the receiver can learn k of them in an adaptive manner. Another form of OT is non-interactive OT. It is a variation of interactive OT scheme. In it, the receiver doesn't need to communicate with the sender since he had chosen the message wished in advance in the setup phase.

During 1999 to 2001, Naor et al. proposed some related OT works such as, adaptive $\mathrm{OT}_{k}^{n}$ [18], proxy $\mathrm{OT}_{1}^{2}$ [31], distributed $\mathrm{OT}_{k}^{n}$ [14], efficient $\mathrm{OT}_{1}^{n}$ [32], and efficient $\mathrm{OT}_{k}^{n}$ [339]. In 2000, Naor et al. [14] proposed two efficient distributed $\mathrm{OT}_{1}^{2}$ schemes in which the sender Alice's task is to distribute his messages among a set of servers and the chooser's task is to make contact with $k(k<n)$ servers to get one of these messages. They claimed that their scheme can protect both parties in an information theoretic sense. However in 2007, Ghodosi [8] showed that their schemes fails since they don't protect the chooser/sender in the information theoretic sense. In 2002, Mu et al. [26] proposed three m-out-of n OT schemes. Two of them are interactive and the other can be used to construt either interactive or non-interactive. They claimed that their schemes are complete, robust, and flexible. However in 2006, Ghodosi et al. [29] showed that their schemes fail to satisfy the requirement of the oblivious transfer. In 2004, Wang et al.[5], presented an efficient $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$ scheme which can conceal all sender's secrets and greatly reduce the sender's communication cost.

In 2005, Huang et al.[9], proposed an efficient t -out-of-n OT. They claimed that their scheme is efficient than all existing OT schemes. However, their scheme has three rounds. This means their scheme is less efficient in the number of passes. In the same year, Zhang et al.s’[12], proposed two efficient t-out-of-n OT schemes. Both are based on DDH assumption. They claimed that both of their schemes are provably secure under the Decisional Diffie - Hellman (DDH) assumption. However, we found that $\mathrm{C}_{\mathrm{k}-1} \neq\left(-\delta_{1}-\delta_{2}, \ldots,-\delta_{k}\right)$ in their first scheme. Also, in 2005, Chu, et al [4] proposed two efficient k-out-of-n oblivious transfer schemes with adaptive and non-adaptive, respectively. Their schemes are mainly based on the discrete logarithm problem. They claimed that their schemes are more efficient than all the previous proposed. In 2006, Parakh [1] proposed an $\mathrm{OT}_{2}^{1}$ scheme based on Elliptic Curve Cryptography (ECC). But we found that A can decide whether $B$ can obtain his secret $n_{A}$ by first assuming that $\mathrm{P}_{\mathrm{A}}=\mathrm{P}_{\mathrm{B}}$. Under this assumption, he can obtain R in step 2 and compute $\mathrm{n}_{\mathrm{A}} \mathrm{R}$. Then, he can compute $\mathrm{K}=\mathrm{Q}$ since $\mathrm{n}_{\mathrm{B}}\left(\mathrm{n}_{\mathrm{A}} \mathrm{P}_{\mathrm{A}}\right)=\mathrm{n}_{\mathrm{A}}\left(\mathrm{n}_{\mathrm{B}} \mathrm{P}_{\mathrm{B}}\right)$ in step 5 a . That is, although A doesn't know $n_{B}$, he can compute $K$, and $n_{B} K$ (in step $5 b$ ) $=n_{B} n_{A} R=n_{A}\left(n_{B} R\right)$, as $B$ does, where $\left(n_{B} R\right)$ is trasferred in step2 by $B$. Therefore, if $A$ can obtain $Z_{B}=\operatorname{Pn}_{A}$ after the result of step 5 b, A knows that $P_{A}=P_{B}$ and $n_{A}$ can be obtained by $B$. This violates B's secrecy. In the same year, Kim, et al.[23] proposed a new secure verifiable non-interactive oblivious transfer protocol using RSA. They claimed their scheme has the function of authenticating the sender and anyone can't deny the message he sent. But we found their protocol is vulnerable to the impersonation attack. Since if an adversary E intercepts the messages sent from Alice, modifies $\mathrm{X}_{\mathrm{A}}$ to $\mathrm{X}^{\prime}{ }_{\mathrm{A}}$ $\left(\equiv\left(\mathrm{X}^{\prime}{ }_{0}, \mathrm{X}^{\prime}{ }_{1}\right)\right.$ ), and then sends $\left(\mathrm{X}^{\prime}{ }_{A}, \mathrm{M}_{\mathrm{A}}, \mathrm{C}_{\mathrm{A}}\right)$ to Bob. Bob will verify him as authentic by using his private key $d_{B}$ and the sender's public key $e_{A}$ to decrypt $C_{A}$. Since $C_{A}$ is the signature of $\mathrm{M}_{\mathrm{A}}$ encrypted by Bob's public key, it has no relationship with $\mathrm{X}_{\mathrm{A}}$ '. Hence, E can therefore successfully impersonate Alice. Moreover, in their scheme, there are two modulus, $\mathrm{n}_{\mathrm{A}}$ and $\mathrm{n}_{\mathrm{B}}$. If they are not properly used, for example, if $\mathrm{n}_{\mathrm{A}}>\mathrm{n}_{\mathrm{B}}$, it will incur the re-blocking problem [30]. Also, in 2006, Zhang et al.[11] proposed two efficient t-out-of-n oblivious transfer schemes. They claimed that both of their schemes are efficient. However, it needs three rounds in the first scheme. In 2009, Jing et al. proposed two non-interactive $\mathrm{OT}_{1}^{n}$ schemes [34]. However in their protocols, when R wants to select one of the n messages sent from S every time, he has to interact with the third party T to obtain the choice-related secret key $\mathrm{x}_{\mathrm{i}}$. This makes their scheme somewhat inconvenient and inconsistent with the meaning of non-interactive protocols as indicated in the title. (This phenomenon can be found in other proposed non-interactive OT schemes as well.) Also in 2009, Chang [28]
presented a robust $\mathrm{OT}_{k}^{n}$ scheme using both the RSA blind signature and Chinese

Remainder Theorem. However, we found that their scheme fails since the sender Alice is able to decide which part of the sent messages were chosen by the chooser Bob. We will describe this in Section 3.2.

Although, there are so many OT schemes proposed. However, due to lack of considering possible attacks that maybe encounter in an open network, all of them need run under a secure channel. This incurs extra communication overhead. Hence, for efficiency consideration in communicational bandwidth consumption, in this paper we propose a novel bilinear pairing based $\mathrm{OT}_{\mathrm{n}}^{\mathrm{k}}$ scheme which not only is secure but also possesses the property of low bandwidth consumption.

The rest of this paper is organized as follows. In Section 2, we show the relational work. In Section 3, we review Chang et al.'s protocol [28] then show their weakness. In Section 4, we present our protocol. And show the security analysis and the performance comparisons with other proposed works in Section 5. Finally, a conclusion is given in Section 6.

## 2 Preliminary

In 2001, bilinear pairings, namely the Weil pairing and the Tate pairing, defined on elliptic curves were proved and applied to cryptography by Boneh and Franklin in 2001 [7]. Since then, many protocols have been designed based on the Weil pairing [6,7,16] for easier key distribution consideration. In this section, we will briefly describe the basic definitions and properties of bilinear pairings.

### 2.1 Bilinear Pairings

Let P be a generator of $\mathrm{G}_{1}$ that is a cyclic group whose order is a prime q , and $\mathrm{G}_{2}$ be a cyclic multiplicative group of the same order q . It is assumed that the discrete logarithm problem (DLP) in both $\mathrm{G}_{1}$ and $\mathrm{G}_{2}$ is difficult. The security level of bilinear pairing is equal to the discrete logarithm problem [7] and a bilinear pairings is a map e:
$G_{1 \times} \times G_{1} \rightarrow G_{2}$ with the following conditions.
(1) Bilinear: $e(a P, b Q)=e(P, Q)^{a b}$, for any $a, b \in Z_{q}{ }^{*}$ and $P, Q \in G_{1}$.
(2) Computable: There is an efficient algorithm to compute ( $P, Q$ ) for all, $P, Q \in G_{1}$.
(3) Non - degenerate: there exists $P \in G_{1}$ and $Q \in G_{1}$ such that $e(P, Q) \neq 1$ in $\mathrm{G}_{2}$.

After showing what is a bilinear map, we first introduce the following problems in $\mathrm{G}_{1}$ :

- Discrete Logarithm Problem (DLP): Given two group elements $P$ and $Q$, find an integer $n$, such that $Q=n P$ whenever such an integer exists.
- Decision Diffie-Hellman Problem (DDHP): For $a, b, c \in Z_{q}{ }^{*}$, given $P, a P$, $b P, c P$, decide whether $c \equiv a b \bmod q$.
- Computational Diffie-Hellman Problem (CDHP): For $a, b \in Z_{q}{ }^{*}$, given $P$, $a P, b P$, compute $a b P$.
- Decisional Bilinear Diffie-Hellman Problem (DBDHP): Given $P, a P, b P, c P$ for $a, b, c \in Z_{q}^{*}$ and $z \in \mathrm{G} 2$, decide whether $z=e(P, P)^{\mathrm{abc}}$.


## 3. Review of Chang et al.'s protocol

In 2009, Chang et al. proposed a robust $\mathrm{OT}_{k}^{n}$ scheme based on CRT, hoping that their scheme can achieve the requirements of general $\mathrm{OT}_{k}^{n}$ schemes. However, we found that their scheme can not satisfy the chooser's privacy. In the following, we first review the scheme in Section 3.1 then show the weakness in Section 3.2.

### 3.1 Review

We roughly list the steps of the protocol in following (see [28] for more details).

Step 1: After receiving the request sent by Bob for all messages $a_{1}, a_{2}, \ldots, a_{n}$, Alice selects $n$ relatively prime integers, $d_{1}, d_{2}, \ldots, d_{n}$, and computes $D=$ $\mathrm{d}_{1} * \mathrm{~d}_{2} * \ldots * \mathrm{~d}_{\mathrm{n}}$. He then constructs the congruence system

$$
\mathrm{C} \equiv \mathrm{a}_{1} \bmod \mathrm{~d}_{1}, \mathrm{C} \equiv \mathrm{a}_{2} \bmod \mathrm{~d}_{2}, \ldots, \mathrm{C} \equiv \mathrm{a}_{\mathrm{n}} \bmod \mathrm{~d}_{\mathrm{n}}
$$

Furthermore, Alice computes the following values:

$$
\begin{aligned}
& \mathrm{T}_{1}=\mathrm{d}_{1}^{\mathrm{e}} \bmod \mathrm{~N}, \mathrm{~T}_{2}=\mathrm{d}_{2}^{\mathrm{e}} \bmod \mathrm{~N}, \ldots, \mathrm{~T}_{\mathrm{n}}=\mathrm{d}_{\mathrm{n}}^{\mathrm{e}} \bmod \mathrm{~N}, \\
& \text { using the public key } \mathrm{e} .
\end{aligned}
$$

Finally, Alice publishes $C$ and the pairs of $\left(\mathrm{ID}_{\mathrm{i}}, \mathrm{T}_{\mathrm{i}}\right)$, for $\mathrm{i}=1$ to n , in the public board.

Step 2: If Bob wants to learn the information possessed by Alice, then Bob must select $t$ pairs of $\left(\mathrm{ID}_{j}^{\prime}, \mathrm{T}_{\mathrm{j}}^{\prime}\right)$, for $\mathrm{j}=1$ to t , from the public board and generate t corresponding random numbers $r_{1}, r_{2}, \ldots, r_{t}$, for each pair of ( ID $_{j}^{\prime}, T_{j}^{\prime}$ ) first. Bob subsequently computes the following:

$$
\alpha_{1}=\mathrm{r}_{1}^{\mathrm{e}} * \mathrm{~T}_{1}^{\prime} \bmod \mathrm{N}, \alpha_{2}=\mathrm{r}_{2}^{\mathrm{e}} * \mathrm{~T}_{2}^{\prime} \bmod \mathrm{N}, \ldots, \alpha_{\mathrm{t}}=\mathrm{r}_{\mathrm{t}}^{\mathrm{e}} * \mathrm{~T}_{\mathrm{t}}^{\prime} \bmod \mathrm{N},
$$ using Alice's public key e and then sends $\left\{\alpha_{1}, \alpha_{2}, \ldots, \alpha_{t}\right\}$ back to Alice.

Step 3: Upon receiving the messages sent by Bob, Alice employs the private key d to compute $\beta_{1}=\alpha_{1}^{\mathrm{d}} \bmod \mathrm{N}, \beta_{2}=\alpha_{2}^{\mathrm{d}} \bmod \mathrm{N}, \ldots, \beta_{\mathrm{t}}=\alpha_{\mathrm{t}}^{\mathrm{d}} \bmod \mathrm{N}$, and then sends the results $\left\{\beta_{1}, \beta_{2}, \ldots, \beta_{\mathrm{t}}\right\}$ to Bob.

Step 4: After receiving the messages sent by Alice, Bob computes the following values:
$\mathrm{d}_{1}{ }^{\prime}=\mathrm{r}_{1}^{-1} * \beta_{1} \bmod \mathrm{~N}, \mathrm{~d}_{2}{ }^{\prime}=\mathrm{r}_{2}^{-1} * \beta_{2} \bmod \mathrm{~N}, \ldots, \mathrm{~d}_{\mathrm{t}}{ }^{\prime}=\mathrm{r}_{\mathrm{t}}^{-1} * \beta_{\mathrm{t}} \bmod \mathrm{N}$

Consequently, Bob learns the demanded messages successfully by computing $\mathrm{b}_{1}=\mathrm{C} \bmod \mathrm{d}_{1}{ }^{\prime}, \mathrm{b}_{2}=\mathrm{C} \bmod \mathrm{d}_{2}^{\prime}, \ldots, \mathrm{b}_{\mathrm{t}}=\mathrm{C} \bmod \mathrm{d}_{\mathrm{t}}^{\prime}$.

### 3.2 Weaknesses

Although, Chang et al. claimed that their scheme can achieve the requirements of general $\mathrm{OT}_{k}^{n}$ schemes. However, we found that the privacy of Bob has been violated. Since Bob has committed his $t$ choices to the $t$ values of $\alpha$ and the $n d_{i}$ values are selected by Alice. Hence, after computing the $t$ values of $\beta_{j} s$, Alice can use each of the $\mathrm{nd} \mathrm{d}_{\mathrm{i}}^{-1}$ s to compute $\mathrm{r}_{\mathrm{k}}=\beta_{\mathrm{j}} * \mathrm{~d}_{\mathrm{i}}$, for $\mathrm{k}=1$ to $\mathrm{n} * \mathrm{t}$. And for each $\mathrm{r}_{\mathrm{k}}$, Alice computes the n
$\alpha_{i}=\left(\mathrm{r}_{\mathrm{j}} * \mathrm{~d}_{\mathrm{i}}\right)^{\mathrm{e}}$ values to compare with the t committed values, where $1 \leqq \mathrm{i} \leqq \mathrm{n}$ and $1 \leqq \mathrm{j}$ $\leqq t$. Hence, we can easily see that it needs $n * t$ multiplications to obtain $r_{k}, n^{2} * t$ multiplications to obtain $\left(r_{j} * d_{t}\right)$, and $n^{2} * t$ exponentiation operations to determine $\alpha_{j}$ values, e.g., $\alpha_{j}=\left(r_{k} * d_{t}\right)^{e}$. Therefore, totally with at most $n^{2} * t+n * t$ multiplications and $\mathrm{n}^{2} * \mathrm{t}$ exponentiations, Alice can decide which t values Bob selected. This violates Bob's privacy.

## 4. The proposed k-out-of-n OT scheme

In this session, we present a k-out-of n OT scheme based on bilinear pairing. Our scheme consists of two phases: (1) setup phase, (2) data transfer phase. The details of our protocol are executed as follows and also illustrated in Figure 1.
(1) Setup phase

Initially, there is a public system parameter set, $\left\{\mathrm{G}_{1}, \mathrm{G}_{2}, \mathrm{q}, \mathrm{P}, \mathrm{e}, \mathrm{H}\right\}$, where $\mathrm{G}_{1}$ be a cyclic additive group generated by $P$ whose order is a prime $\mathrm{q}, \mathrm{G}_{2}$ be a cyclic multiplicative group of the same order q , e is a bilinear pairing mapping e: $G_{1 \times} G_{1} \rightarrow$ $G_{2}$, and H be a one-way hash function $\mathrm{H}:\{0,1\}^{*} \rightarrow \mathrm{G}_{1} . \mathrm{R}$ is the receiver and S is the sender.
(2) Data transfer phase.
(a). R randomly chooses integers $\mathrm{t}, \mathrm{w}$, and $\mathrm{n} \mathrm{s}_{\mathrm{i}} s$ (for $\mathrm{i}=1,2, \ldots, \mathrm{n}$ ). He then computes $A_{i}=w s_{i} P, B_{j}=t t_{\sigma_{j}} P($ for $j=1,2, \ldots, k)$ and $V=t P$, where $\sigma_{\mathrm{j}}$ is the k indices to represent $k$ of the $n$ random integers $S_{i}, i=1$ to $n$, chosen by $R$. Then R sends $\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}}, \mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{k}}$ and V to S .
(b). After receiving the above sent message form $\mathrm{R}, \mathrm{S}$ randomly chooses an integer r , computes $\mathrm{C}_{\mathrm{i}}=\mathrm{m}_{\mathrm{i}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{A}_{\mathrm{i}}, \mathrm{V}\right)^{\mathrm{r}}\right)$ and $\mathrm{D}_{\mathrm{j}}=\mathrm{r} \mathrm{B}_{\mathrm{j}}$. Then he sends $\mathrm{C}_{\mathrm{i}}$, $\mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{n}}, \mathrm{D}_{1}, \mathrm{D}_{2}, \ldots, \mathrm{D}_{\mathrm{k}}$ to R .
(c). After receiving $\mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{n}}, \mathrm{D}_{1}, \mathrm{D}_{2}, \ldots, \mathrm{D}_{\mathrm{k}}, \mathrm{R}$ computes $\mathrm{c}_{\sigma_{\mathrm{j}}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{D}_{\mathrm{j}}, \mathrm{P}\right)^{\mathrm{w}}\right)$ to obtain $\mathrm{m}_{\sigma \mathrm{j}}$.


Figure 1 Our k-out-of-n OT scheme

## 5. Security and Performance analysis

In this session, we analyze the security of our scheme in Section 5.1 and also compare its communicational cost with other related work in Section 5.2.

### 5.1 Security Analysis

In this section, we examine the security of our scheme by considering the following properties.
(1). Correctness: After receiving messages, $\mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{n}}, \mathrm{D}_{1}, \mathrm{D}_{2}, \ldots, \mathrm{D}_{\mathrm{k}}$, by using these received $k D_{j} s$ to decrypt the $n C_{i} s, R$ can correctly obtain the $k$ messages $m_{\sigma}$, $\mathrm{m}_{\sigma 2}, \ldots, \mathrm{~m}_{\sigma \mathrm{k}}$, which he had chosen by computing
$\mathrm{m}_{\mathrm{\sigma j}}=\mathrm{C}_{\mathrm{\sigma j}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{D}_{\mathrm{j}}, \mathrm{P}\right)^{\mathrm{w}}\right)=\mathrm{C}_{\mathrm{\sigma j}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{rB}_{\mathrm{j}}, \mathrm{P}\right)^{\mathrm{w}}\right)=\mathrm{C}_{\mathrm{\sigma j}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{rts}_{\mathrm{\sigma j}} \mathrm{P}, \mathrm{P}\right)^{\mathrm{w}}\right)=$ $\mathrm{C}_{\mathrm{\sigma j}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{ws}_{\sigma \mathrm{j}} \mathrm{P}, \mathrm{tP}\right)^{\mathrm{r}}\right)=\mathrm{C}_{\text {бj }} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{A}_{\sigma \mathrm{j}}, \mathrm{V}\right)^{\mathrm{r}}\right)$.
(2). Assurance of the sender's privacy: The sender sends message $\mathrm{m}_{\mathrm{i}}$ which is
protected by XORing $\mathrm{H}\left(\mathrm{e}\left(\mathrm{A}_{\mathrm{i}}, \mathrm{tP}\right)^{\mathrm{r}}\right)$. If R wants to obtain extra messages which he didn't choose, he need to know the number r. However, solving the random number $r$ from $D_{j}, j=1 . . \mathrm{k}$ is computationally infeasible due to the ECDLP assumption.
(3). Assurance of the receiver privacy: The receiver's $k$ choices in the $n S_{i}$ random numbers, $\mathrm{s}_{\mathrm{i}},(\mathrm{i}=1$ to n$), \mathrm{s}_{\sigma 1}, \mathrm{~s}_{\sigma 2}, \ldots, \mathrm{~s}_{\sigma \mathrm{k}}$, are enciphered in $\mathrm{B}_{\mathrm{j}}\left(=\mathrm{ts}_{\mathrm{\sigma j}} \mathrm{P}\right)$. After receiving $A_{1}, A_{2}, \ldots, A_{n}, B_{1}, B_{2}, \ldots, B_{k}$ and $V(=t P)$ from $R, S$ can not compute $\mathrm{s}_{\sigma 1}$, $\mathrm{s}_{\sigma 2}, \ldots, \mathrm{~s}_{\mathrm{\sigma j}}$ from $\mathrm{B}_{\mathrm{j}}$ in polynomial time due to the ECDLP assumption. Therefore, the sender can not know which k messages, $\mathrm{m}_{\sigma \mathrm{j}}$, for $\mathrm{j}=1$ to k , the receiver chose.
(4). Against dishonest receiver: In our k-out-of-n OT scheme, if R is dishonest in computing $\mathrm{B}_{\mathrm{j}}\left(=\mathrm{ts}_{\mathrm{\sigma j}} \mathrm{P}\right.$, for $\left.\mathrm{i}=1, \ldots, \mathrm{k}\right)$ or $\mathrm{V}=(\mathrm{tP}), \mathrm{R}$ will not be able to obtain the correct $\mathrm{m}_{\text {бj }}$ by computing $\mathrm{m}_{\sigma \mathrm{j}}=\mathrm{C}_{\text {бj }} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{D}_{\mathrm{j}}, \mathrm{P}\right)^{\mathrm{w}}\right)=\mathrm{C}_{\text {бj}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{rB}_{\mathrm{j}}, \mathrm{P}\right)^{\mathrm{w}}\right) \neq$ $\mathrm{C}_{\mathrm{\sigma j}} \oplus H\left(\mathrm{e}\left(\mathrm{A}_{\mathrm{j}}, \mathrm{V}\right)^{\mathrm{r}}\right)=\mathrm{C}_{\mathrm{\sigma j}} \oplus H\left(\mathrm{e}\left(\mathrm{ws}_{\sigma \mathrm{j}} \mathrm{P}, \mathrm{tP}\right)^{\mathrm{r}}\right)=\mathrm{C}_{\mathrm{\sigma j}} \oplus \mathrm{H}\left(\mathrm{e}\left(\mathrm{rts}_{\mathrm{\sigma j}} \mathrm{P}, \mathrm{P}\right)^{\mathrm{w}}\right)$, where t and $\mathrm{s}_{\mathrm{\sigma j}}$ are committed in $V$ and $A_{i}$ by $R$, respectively.

### 5.2 Performance Analysis

In this section, we first compare the efficiency in bandwidth consumption and then the computation cost of our scheme with Chu et al.s' [4], Zhang et al.s'[12], and Mu et al.s' [26] in Table 1 and Table 2, respectively.

If the computation in discrete $\log$ problem needs 1024 bits, the bilinear pairings only needs 160 bits to achieve the same security level. Based on this fact, we compare the communicational cost of our scheme with the others by considering three factors, (1) the number of needed rounds between $S$ and $R$, (2) the number of bits transfered from R to S , and (3) the number of bits from S to R . We show the result in Table 1.

Table 1: comparisons of needed rounds and transferred bits among proposed $\mathrm{OT}_{k}^{n}$ protocols

|  | Our scheme | Chu et al.s'[4] | Zhang et al.s'[12] | Mu et al.s'[26] |
| :--- | :--- | :--- | :--- | :---: |
| Needed <br> Rounds | 2 | 2 | 3 | 2 |
| Bits needed <br> from R to S | $(\mathrm{n}+\mathrm{k}+1)^{*} 160$ <br> bits | $\mathrm{k} * 1024$ bits | $(\mathrm{K}+3)^{*} 1024$ bits | $2 \mathrm{n} * 1024$ bits |
| Bits needed <br> from S to R | $(\mathrm{n}+\mathrm{k})^{*} 160$ <br> bits | (n+k+1)*1024 <br> bits | $2 \mathrm{n} * 1024$ bits | $\mathrm{n} * 1024$ bits |

From table 1, we can see that if we wish our scheme to be more efficient than the others such as [4], $(\mathrm{n}+\mathrm{k}+1)^{*} 160$ must be less than $\mathrm{k} * 1024$. That is $(\mathrm{n}+\mathrm{k}+1)^{*} 160 \leqq$ 1024 k . In other words, our scheme has the best performance in bandwidth consumption when $\mathrm{n} \leqq 5.4 \mathrm{k}-1$. Now, we compare the computational cost with the other three by using two factors: (1) the number of operations $S$ performs, and (2) the number of operations $R$ performs. We first show the definition of used notations in the following then show the result in Table 2. We first list the definitions of used notation.
$\mathrm{T}_{\text {Exp }}$ : the time needed by a modular exponentiation, $1\left(\mathrm{~T}_{\text {Exp }} \cong 240 \mathrm{~T}_{\text {Mul }}\right)$ [19]
$\mathrm{T}_{\mathrm{Mul}}$ : the time needed by a modular multiplication
$\mathrm{T}_{\mathrm{XOR}}$ : the time needed by a modular bit-XOR
$\mathrm{T}_{\mathrm{EC} \text { _Mul }}$ : the time needed by a scalar multipling a point on an elliptic curve(1

$$
\left.\mathrm{T}_{\mathrm{EC} \_\mathrm{Mul}} \cong 29 \mathrm{~T}_{\mathrm{Mul}}\right)[19]
$$

$\mathrm{T}_{\mathrm{bp}}$ : the computation time of a bilinear pairing
$\mathrm{T}_{\text {hash }}$ : the computation time of a hash function
$\mathrm{T}_{\text {enc }}$ : the computation time of an encryption under DLP assumption

## $\mathrm{T}_{\text {dec }}$ : the computation time of a decryption under DLP assumption

Table 2: comparisons of computational cost

|  | Our scheme | Chu et al.s'[4] | Zhang et al.s'[12] | Mu et al.s'[26] |
| :---: | :---: | :---: | :---: | :---: |
| Sender | $\begin{aligned} & \mathrm{n}\left(\mathrm{~T}_{\mathrm{bp}}+\mathrm{T}_{\mathrm{XOR}}\right)+\mathrm{k} \\ & \mathrm{~T}_{\mathrm{EC} \text { _Mul }}+\mathrm{n}_{\text {hash }} \end{aligned}$ | $\begin{aligned} & (\mathrm{n}+\mathrm{k}+1) \mathrm{T}_{\mathrm{Exp}} \\ & +\mathrm{nT}_{\mathrm{XOR}}+(\mathrm{n}+\mathrm{k}) \\ & \mathrm{T}_{\text {hash }} \end{aligned}$ | $3 \mathrm{n} \mathrm{T}_{\text {Exp }}+3 n \mathrm{~T}_{\text {Mul }}$ | $\begin{aligned} & 2 \mathrm{nT}_{\mathrm{Exp}}+\mathrm{nT}_{\mathrm{Mul}}+ \\ & \mathrm{nT}_{\mathrm{enc}} \\ & \text { or } \\ & 3 \mathrm{nT}_{\mathrm{Exp}+\mathrm{nT}_{\mathrm{Mul}}} \end{aligned}$ |
| Receiver | $\begin{aligned} & (2 \mathrm{n}+2 \mathrm{k}+1) \\ & \mathrm{T}_{\mathrm{EC} \_\mathrm{Mul}}+\mathrm{k}\left(\mathrm{~T}_{\mathrm{bp}}+\right. \\ & \left.\mathrm{T}_{\mathrm{XOR}}\right)+\mathrm{kT}_{\text {hash }} \end{aligned}$ | $\begin{aligned} & 2 \mathrm{kT}_{\mathrm{Mul}}+2 \mathrm{kT}_{\mathrm{Exp}} \\ & +\mathrm{kT}_{\mathrm{XOR}}+2 \mathrm{kT}_{\mathrm{hash}} \end{aligned}$ | $2 \mathrm{k}+3 \mathrm{~T}_{\text {Exp }}+\mathrm{kT} \mathrm{Mul}^{\text {Mul }}$ | $\begin{aligned} & \mathrm{kT}_{\mathrm{Exp}}+2 \mathrm{kT}_{\mathrm{Mul}}+ \\ & \mathrm{kT}_{\mathrm{dec}} \text { or } \\ & 2 \mathrm{kT}_{\mathrm{Exp}}+2 \mathrm{kT}_{\mathrm{Mul}} \end{aligned}$ |

From Table 1 and Table 2, we can see that our scheme maybe less efficient in computation time. However, it is more efficient in bandwidth consumption than the other proposed schemes.

## 6. Conclusion

In this paper, we propose a secure efficient k-out-of-n oblivious transfer scheme based on pairings to reduce the bandwidth consumption for both of the sender and the receiver. After our analysis, we conclude that our scheme is not only secure but also more efficient than all other existing $\mathrm{OT}_{k}^{n}$ schemes in bandwidth consumption which plays an important role for end-of-day settlement in a busy financial network.

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