Adaptive Concurrent Non-Malleability with Bare Public-Keys

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Abstract

Coin-tossing (CT) is one of the earliest and most fundamental protocol problems in the literature [10, 2, 58, 12, 52]. In this work, we formalize and construct (constant-round) concurrent non-malleable coin-tossing (CNMCT) in the bare public-key (BPK) model. The CNMCT protocol can, in particular, be used to transform CNM zero-knowledge (CNMZK) in the common random string (CRS) model into the BPK model *with full adaptive input (statements and language) selection*. Here, full adaptive input selection in the public-key model means that the concurrent man-in-the-middle (CMIM) adversary can adaptively set statements to all sessions at any point of the concurrent execution evolution (not necessarily at the beginning of each session), and can set the underlying language based upon honest players' public-keys.

1 Introduction

Concurrent non-malleability is central to independence of protocol players' actions and to security against concurrent man-in-the-middle (CMIM) attacks [30]. In the CMIM setting, polynomially many concurrent executing instances (sessions) of a protocol $\langle L, R\rangle$ take place in an asynchronous setting (appropriate for environments such as over the Internet), and all the unauthenticated communication channels (among all the concurrent sessions) are controlled by a probabilistic polynomial-time (PPT) CMIM adversary $\mathcal{A}$. Specifically, the polynomially many executing instances of $\langle P, V\rangle$ can be viewed to be divided into two parts: the CMIM left part, in which the CMIM adversary $\mathcal{A}$ interacts with polynomially many instances of the prover P by playing the role of the verifier V, where the interactions with each instance of P is referred to as a left session; And the CMIM right part, in which the CMIM adversary $\mathcal{A}$ simultaneously interacts with polynomially many instances of the verifier V by playing the role of the prover, where the interactions with each instance of V is referred to as a right session. In this setting, honest players are assumed oblivious to each other's existence, nor do they generally know the topology of the network, and thus cannot coordinate their executions. The CMIM adversary $\mathcal{A}$ (controlling the communication channels) can do whatever it wishes. When we consider "CNM with adaptive input selection", we allow $\mathcal{A}$ to also adaptively set input to each (particularly left) session.

Unfortunately, in the stringent CMIM setting, large classes of cryptographic functionalities cannot be securely implemented round-efficiently [15, 62, 60]. In such cases, some setup assumptions are necessary. Establishing the general feasibility of round-efficient concurrent non-malleable cryptography with adaptive input selection, with as minimal as possible setups, has been a central problem which has attracted intensive research efforts. More discussions on related works are referred to in Appendix A.

Our Results: In this work, we investigate coin-tossing and zero-knowledge (with focus on coin-tossing), both of which are central and fundamental to modern cryptography, in the BPK model (a very weak form of PKI setting) introduced in [14]. Specifically, we define and construct (constant-round) CNMCT in the BPK model. The formulation and protocol construction of CNMCT are of independent interest, but we also show, specifically, how it can be used as a compiler that transforms CNMZK in the CRS model into the BPK model *with full adaptive input selection*.

2 Preliminaries

In this section, we briefly recall the building tools, the CMIM setting in the BPK model, and present the motivation for full adaptive input selection. More details are referred to Appendix B.

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2.1 Building Tools

Pseudorandom functions (PRF) can be constructed under any one-way function (OWF) [42, 40]. A OWF $f : \{0,1\}^* \to \{0,1\}^*$ is called *linear*, if for sufficiently large n and any $x \in \{0,1\}^n$ $|f(x)| = O(n)$, where $|f(x)|$ denotes the length of $f(x)$.

Non-interactive statistically-binding commitments can be based on any one-way permutation (OWP) [10, 44]. Practical perfectly-binding non-interactive (string) commitment scheme can be based on the decisional Diffie-Hellman (DDH) assumption. A commitment scheme C is called *linear*, if for sufficiently large n and any string $x \in \{0,1\}^n$ both $|C(x,s)|$ (i.e., the length of the commitment to x using random coins s) and $|s|$ (i.e., the length of s) are bounded by $O(n)$. In particular, the perfectly-binding commitment scheme over prime order groups based on [33] is *linear*.

Very roughly, adaptive tag-based one-left-many-right non-malleable statistical zero-knowledge argument of knowledge (NMSZKAOK) is non-malleable against any one-left-many-right PPT man-in-the-middle adversary $\mathcal{A}$ who involves one left session with the prover and many right sessions with verifiers; each session is indexed by a string (called a tag) and $\mathcal{A}$ is allowed to set the input and tag to the left session (besides those of right sessions). Then, the security says that for any PPT one-left-many-right MIM adversary $\mathcal{A}$, there exists an (expected) polynomial-time simulator S such that S outputs a simulated transcript that is *statistically* indistinguishable from the real view of $\mathcal{A}$; Moreover, for any successful right session on a input in the simulated transcript w.r.t. a tag different from that of the left session, a valid $\mathcal{NP}$-witness (to the statement set adaptively by $\mathcal{A}$ to this session) is also extracted.

The Pass-Rosen ZK (PRZK, in short) [72, 73], *with some specified length parameters* $l(n)$ where $l(\cdot)$ is a positive polynomial and n is the security parameter, is a *constant-round* adaptive tag-based one-left-many-right NMSZKAOK. Furthermore, PRZK is *public-coin* and can be *perfect* ZK. In [72, 73], the tag and input length is just specified to be the security parameter n (in this case, the length parameter is specified to be $l(n) \geq 2n^3 + n$), and the works [72, 73] did not explicitly consider adaptive input and tag selection for the one left-session. But a closer investigation shows that the PRZK can be extended to work for tags of length $O(n)$ (and inputs of length $poly(n)$) with length parameter $l(n) \geq O(n^3)$ (the actual length parameter $l(n)$ is specific to the tag length), and for the more general case of adaptive left-session tag and input selection.

2.2 The CMIM Setting in the BPK Model

Briefly speaking, a protocol in the BPK model simply assumes that all players have each deposited a public-key in a public file before any interaction takes place among the users. Note that, no assumption is made on whether the public-keys deposited are unique or valid (i.e., public keys can even be "non-sensical," where no corresponding secret-keys exist or are known) [14]. That is, no trusted third party is assumed, and the underlying communication network is assumed to be adversarially asynchronous. In many cryptographic settings, availability of a public key infrastructure (PKI) is assumed or required and in these settings the BPK model is, both, natural and attractive (note that the BPK model is, in fact, a weaker version of PKI where in the later added key certification is assumed). It was pointed out that BPK is, in fact, applicable to interactive systems in general [65].

We next briefly describe the CMIM setting in the BPK model for any two-party protocol $\langle L, R \rangle$. When it comes to zero-knowledge protocols, L stands for the prover P and R stands for the verifier V. Throughout this work, we abuse the notations L and R. Specifically, L stands for the left-player (e.g., a prover) and in some context we may explicitly indicates L to be a language, R stands for the right-player (e.g., a verifier) and in some context we may explicitly indicates R to be a relation.

We say a class of $\mathcal{NP}$-languages $\mathcal{L}$ is *admissible* to an interactive proof (IP) protocol $\langle L, R \rangle$ (e.g., a ZK protocol), if the protocol can work (or, be instantiated) for any language $L \in \mathcal{L}$. Typically, $\mathcal{L}$ can be the set of all $\mathcal{NP}$-languages or the set of any languages admitting Σ-protocols [20] (in the latter case $\langle L, R \rangle$ could be instantiated efficiently without going through general $\mathcal{NP}$-reductions).

Each player in the BPK model works in two stages: the *key-generation stage* in which it generates and registers a public-key in a public file F; and the *proof* stage between two players specified by a key pair (PK_L, PK_R) in F. The file F output at the end of the key-generation stage is denoted as $\{PK_I^{(1)}, PK_I^{(2)}, \cdots, PK_I^{(poly(n))}\}$ that is to be used and *remain intact* during the proof stage, where

$PK_I^{(j)}$ denotes a left-player key if $I = L$ or a right-player key if $I = R$. We also denote by $\mathcal{R}_{KEY}^I$ the $\mathcal{NP}$-relation validating the key pair (PK_I, SK_I), i.e., whether SK_I is a valid secret-key w.r.t. PK_I. For an IP protocol $\langle L, R\rangle$ *with adaptive language selection in the BPK model*, there exists also a PPT language-selecting machine, denoted $\mathcal{M}_{\mathcal{L}}$, that on inputs $(1^n, F)$ outputs the description of an $\mathcal{NP}$-relation $\mathcal{R}_L$ for a language $L \in \mathcal{L}$, on which the proof stage is to be conducted. We require that given the description of $\mathcal{R}_L$, the admissibility of L (i.e., the membership of $L \in \mathcal{L}$) can be efficiently decided.

The CMIM adversary $\mathcal{A}$. In the key-generation stage, on 1^n and some auxiliary input $z \in \{0,1\}^*$ and a pair of honestly generated public-keys (PK_L, PK_R), $\mathcal{A}$ outputs a set of public-keys F' (and an admissible language $L \in \mathcal{L}$, if $\langle L, R\rangle$ is an IP protocol). Then the public file F for the proof stage is set to be $F' \cup \{PK_L, PK_R\}$. Here, we remark that, in general, the input to $\mathcal{A}$ in order to generate F' could be a set of public-keys generated by many left and right players, rather than a single pair of honestly generated public-keys (PK_L, PK_R). Our CNM analysis works also for this general case, to be addressed later (at the end of Section 4). The formulation with a single pair of honestly generated public-keys is only for presentation simplicity.

In the proof stage, $\mathcal{A}$ can concurrently interact with any polynomial number of instances of the honest left-player of public-key PK_L in the *left CMIM interaction part*. The interactions with each instance of the honest left-player is called a *left session*, in which $\mathcal{A}$ plays the role of the right-player with a public-key $PK_R^{(j)} \in F$; *Simultaneously*, $\mathcal{A}$ interacts with any polynomial number of instances of the honest right-player of public-key PK_R in the *right CMIM interaction part*. The interactions with each instance of the honest right-player is called a *right session*, in which $\mathcal{A}$ plays the role of the left-player with a public-key $PK_L^{(j)} \in F$. For CMIM adversary with full adaptive input selection, $\mathcal{A}$ can further set statements to all sessions *on the fly at any point of the concurrent session run and adaptively base this action on its view*. A CMIM adversary is called s-CMIM adversary, for a positive polynomial $s(\cdot)$, if the adversary involves, on security parameter 1^n, at most $s(n)$ concurrent sessions in each CMIM interaction part and registers at most $s(n)$ public-keys in F'.

For any $(PK_L, SK_L) \in \mathcal{R}_{KEY}^L$ and $(PK_R, SK_R) \in \mathcal{R}_{KEY}^R$, we denote by $view_{\mathcal{A}}^{L(SK_L),R(SK_R)}(1^n, z, PK_L, PK_R)$ the random variable describing the view of $\mathcal{A}$ specific to (PK_L, PK_R), which includes its random tape, the auxiliary string $z \in \{0,1\}^*$, the (specific) (PK_L, PK_R), and all messages it receives from the instances of $L(1^n, SK_L)$ and $R(1^n, SK_R)$ in the proof stages.

We next describe a more detailed experiment, referred to as $\mathsf{Expt}_{CMIM}^{\mathcal{A}}(1^n, z)$, for the full adaptive input selection CMIM setting w.r.t. an IP protocol $\langle P, V\rangle$ and an $s(n)$-CMIM adversary $\mathcal{A}$.

Honest player key generation. $(PK_P, SK_P) \longleftarrow P_1(1^n)$, $(PK_V, SK_V) \longleftarrow V_1(1^n)$, where P_1 (resp., V_1) denotes the key-generation stage of P (resp., V).

Preprocessing stage of the CMIM. $\mathcal{A}(1^n, PK_P, PK_V, z)$ outputs $(F', \mathcal{R}_L, \tau)$, where F' consists of a list of, at most $s(n)$, public-keys, $\mathcal{R}_L$ specifies an admissible language $L \in \mathcal{L}$, and $\tau \in \{0,1\}^*$ is some auxiliary information to be transferred to the proof stage of $\mathcal{A}$. Then, the public file and language to be used in the proof stage are set to be : $F = F' \cup \{PK_P, PK_V\}$ and $L \in \mathcal{L}$.

Proof stage of the CMIM. At *any time* during this stage, $\mathcal{A}$ can do one of the following actions.

- Deliver to V a message for an already started right session, or deliver to P a message for an already started left session.
- Full adaptive statement selection: $\mathcal{A}$ sets a statement $\tilde{x}_i \in \{0,1\}^{poly(n)}$ for the i-th left or right session, $1 \leq i \leq s(n)$. We stress that if $\tilde{x}_i \in \{0,1\}^{poly(n)}$ is set to the i-th left session, it is required that $\tilde{x}_i \in L \cup \{0,1\}^{poly(n)}$, i.e., $\mathcal{A}$ is restricted to set only true statements for left sessions (otherwise, the experiment may render an $\mathcal{NP}$-membership oracle to $\mathcal{A}$), and then a witness $\tilde{w}_i$ such that $(\tilde{x}_i, \tilde{w}_i) \in \mathcal{R}_L$ is given (e.g., by an exponential-time machine [55]) to the prover instance of P. Though the adversary is allowed to set statement at any point of the concurrent execution evolution, whenever at some point the subsequent activities of an honest player in a session may utilize the statement of the session while the adversary did not provide it, the honest player just simply aborts the session.
- Session initiation: $\mathcal{A}$ starts a new i-th left (resp., right) session, $1 \leq i \leq s(n)$ by indicating a key $PK_V^{(j)} \in F$ (resp., $PK_P^{(j)} \in F$) to the honest prover P of public-key PK_P (resp., the honest verifier V of public-key PK_V). Then, P (resp., V) initiates a new session with the verifier of $PK_V^{(j)}$ (resp., the prover of $PK_P^{(j)}$) pretended by the CMIM $\mathcal{A}$. We remark that full

adaptive input selection and session initiation can be merged, if the statement to a session is mandated to be presented at the start of the session.

- Output a special "end attack" symbol within time polynomial in n.

We remark that the original specification of the BPK model does allow key registrations for both verifiers and provers. But, when dealing with some specific cryptographic problems, e.g., ZK and commitments, or adversaries without the capability of full adaptive input selection, all previous works in the BPK model only require verifiers to register public-keys. Note also that we strengthen the BPK model by allowing adaptive language selection.

2.3 Motivation for CNM with Full Adaptive Input Selection

In the traditional formulations of CNM with adaptive statement selection, the CMIM adversary $\mathcal{A}$ is required (limited) to set the statement to each (either left or right) session at the *beginning* of that session. Also, most previous works in the BPK model implicitly assume that the underlying language is fixed and cannot be adaptively set by the adversary. We note that such requirements, on input (i.e., statements and language) selection, could limit the applicability of CNMZK or the power of the CMIM adversary in certain natural settings. We give some concrete examples below. For presentation simplicity, throughout this work, we use "input" to refer to "both statements and language" in the public-key setting, and only to "statements" in other settings.

Though we can always mandate each honest prover (and also the CMIM adversary) to determine the statement to be proved at the beginning of each session, such a requirement limits the applicability of CNMZK in some natural settings. For example, when ZK is used for identification, the statement being proved is prover's identity information. However, in many scenarios (e.g., E-commerce over the Internet), for privacy preserving reasons an honest prover would like not to hastily reveal the statement to be proved (e.g., its digital identity like a card number) until the session run has successfully reached a certain point (even just the last round, so that the identity information is revealed only for successfully completed sessions). Note that for such privacy-preserving honest provers, the CMIM adversary can in particular set statements to right sessions not necessarily at the beginning of each right session. Though an honest prover can also send a commitment to its identity (i.e., the statement being proved) on the top of the session run to preserve its privacy, but such an approach incurs additional complexity to the system and is less common in practice. In such settings, it is thus more desirable to achieve CNMZK with full adaptive input selection. Note that most existing CNMZK protocols, in the plain model or in the BPK model, are composed of several sub-protocols, where the statement to be proved is only used in the last sub-protocol [7, 70, 56, 55].

Consider any protocol resulted from the composition of a coin-tossing protocol (referred to as the CT sub-protocol) and a protocol in the common random string (CRS) model (referred to as the CRS sub-protocol). In most prevalent cases, the input to the CRS sub-protocol is also the input to the entire composed protocol, and can be set after the CT sub-protocol is finished and even just at the last round of the composed protocol. In particular, ZK protocols resulted from such composition well fit the scenario of privacy preserving identifications as clarified above. We remark that an adaptive adversary in the CRS model is allowed to set statements and language (particularly if the protocol works for any language in $\mathcal{NP}$) based on the CRS. Though we can always mandate honest players to determine their inputs on the top of each session run of the composed protocol, from our view it is still desirable for the composed protocol to be applicable to some privacy-preserving scenarios (with statements to be revealed not necessarily on the top of each session run) and to remain CNM secure against the *more powerful* CMIM adversaries who can set statements and language being proved based upon the output of the CT-protocol. Also note that, if the above CRS sub-protocol is an IP protocol working for a set of admissible languages without going through $\mathcal{NP}$-reductions, the composed protocol is also without going through $\mathcal{NP}$-reductions.

For cryptographic protocols running concurrently in the public-key model, it is a far more realistic strategy for an adversary to mix the public-key structure as part of the underlying languages (on which the protocols are conducted). This issue of adaptive language selection may not be applicable to protocols in the BPK model that work only for a fixed pre-determined language (e.g., a generic $\mathcal{NP}$-complete language). But in real executions when the protocols in the BPK model can be instantiated

to work for a set of admissible languages (e.g., languages that admit Σ-protocols *particularly without going through general $\mathcal{NP}$-reductions*), as demonstrated in [75, 76, 78] and in this work, adaptive language selection based upon honest players' public-keys can render strictly stronger power to the CMIM adversary. In this work, as in [78], we strengthen the BPK model by allowing adaptive language selection based upon honest players' public-keys. To further justify adaptive language selection in the public-key setting, we demonstrate a concrete attack (presented in Appendix C) on the CNMZK protocol proposed in [24]. This attack allows a CMIM, *capable adaptive language selection based on honest players' public-keys*, to successfully convince the honest verifier of some $\mathcal{NP}$ statements but without knowing any witnesses to the statements being proved.

In contrast, by CMIM with *full* adaptive input selection (CNM-FINS in short), we mean that a CMIM adversary can set statements to both left sessions and right sessions, besides adaptive language selection in the public-key setting; furthermore, the adversary does not necessarily set the statement to each session at the beginning of the session; Rather, the statement may be set on the way of the session, and is based on the whole transcript evolution. By CMIM with predetermined left-session inputs but full adaptive input selection on the right, we refer to that the statements to left sessions are fixed and the CMIM adversary only sets statements to right sessions in the above fully adaptive manner (besides adaptive language selection in the public-key setting).

3 Formulations and Discussions of CNMZK and CNMCT in the Public-Key Model

On formulating CNMZK in the public-key model. Motivated by concrete attacks against existing protocols in the BPK model, we highlight a key difference between CNM in the standard model and CNM in the public-key model, which standard simulation/extraction formulation of CNM does not capture. For the CMIM setting in the standard model, honest verifiers are PPT algorithms. In this case, traditional CNM formulation only considers the extra advantages the CMIM can get from concurrent left sessions, as the actions of honest verifiers in right sessions can be efficiently and perfectly emulated. But, for the CMIM setting in the public-key model, honest verifiers possess secret values (i.e, secret-keys) that can *not* be efficiently computed out. That is, for protocols in the public-key model, the CMIM adversary can get extra advantages both from the left sessions and *from the right sessions*. To emulate the actions of honest verifiers in the public-key model, as clarified in [78], the simulator/extractor has to simulate the key-generation stages of honest verifiers and thus possesses the secret-keys of the simulated verifiers; otherwise, concurrent transcript simulation and knowledge extraction w.r.t. the real public-keys of honest verifiers in this setting amounts to constant-round CNMZK in the plain model (by viewing verifiers' public-keys as protocol inputs) [78]. However, for simulation/extraction w.r.t. simulated public-keys, traditional knowledge extraction [4, 9] does not guarantee that the CMIM adversary does indeed "know" the extracted witnesses to successful right sessions. Specifically, the knowledge extracted may be dependent on (even just equal to) the secret-keys possessed by the simulator/extractor itself in order to emulate honest verifiers.

With the above key difference in mind, we investigate reformulating CNMZK in the public-key model. Besides the ability of simulation/extraction, we require that for any CMIM adversary the witnesses extracted for (different) statements of successful right sessions are "*independent*" of the secret-keys used by the simulator/extractor S (who emulates honest verifiers in the simulation/extraction). Such property is named concurrent non-malleable knowledge-extraction independence (CNMKEI), which is an extension of the formulation of knowledge extraction independence (KEI) [78] into the more complicated CMIM setting. The formal definition of CNMZK in the BPK model, together with discussions and clarifications, is presented in Appendix C. Below, we mainly formulate CNMCT in the BPK model, as is the focus of this work. We note that the CNMZK [70] in the BPK model seems also to be CNMKEI secure, though the KEI issue was not explicitly formulated there.

On the subtleties of achieving CNM with full adaptive input selection. We briefly note that no previous ZK protocols in the BPK model or the plain model were proved to be CNM secure against even CMIM with predetermined left-session inputs but full adaptive input selection on the right, let alone to be CNM secure against CMIM with full adaptive input selection. Specifically, the standard

simulation-extraction paradigm (e.g., [58, 2, 72, 73, 7, 70, 71, 56, 55]) for showing CNM security fails, in general, when the CMIM is allowed the capability of full adaptive statement selection for right sessions.

In more detail, the standard simulation-extraction paradigm for establishing CNM security works as follows: the simulator first outputs an indistinguishable simulated transcript; and then extracts the witnesses to (different) inputs of successful right sessions appearing in the simulated transcript, *one by one sequentially*, by applying some assured underlying knowledge-extractor. This paradigm can work for CMIM adversary with the capability of traditional adaptive input selection, as the input to each right session is fixed at the beginning of the right session; Thus, applying knowledge-extractor on a right session does not change the statement of that session. But, for CMIM adversary of fully adaptive input selection, the standard simulation-extraction paradigm fails in general. The reason is, when we apply knowledge-extractor on a successful right session, the statement of this session may however also be changed, which means that the extractor may never extract the witness to the same statement appearing and being fixed in the simulated transcript.

3.1 Formulation and Discussion of CNMCT in the BPK Model

Legitimate CRS-simulating algorithm $\mathcal{M}_{CRS}$. Let $(r, \tau_r) \longleftarrow \mathcal{M}_{CRS}(1^n)$, where $\mathcal{M}_{CRS}$ is a PPT algorithm. The PPT algorithm $\mathcal{M}_{CRS}$ is called a legitimate CRS-simulating algorithm with respect to a polynomial-time computable CRS-trapdoorness validating relation $\mathcal{R}_{CRS}$, if the distribution of its first output, i.e., r, is computationally indistinguishable from U_n (the uniform distribution over strings of length n), and $\mathcal{R}_{CRS}(r, \tau_r) = 1$ for all outputs of $\mathcal{M}_{CRS}$ (typically, τ_r is some trapdoor information about r). For a positive polynomial $s(\cdot)$, we denote by $(\{r_1, r_2, \cdot, r_{s(n)}\}, \{\tau_{r_1}, \tau_{r_2}, \cdots, \tau_{r_{s(n)}}\}) \longleftarrow \mathcal{M}_{CRS}^{s(n)}(1^n)$ the output of the experiment of running $\mathcal{M}_{CRS}(1^n)$ *independently* $s(n)$ times, where for any i, $1 \leq i \leq s(n)$, (r_i, τ_{r_i}) denotes the output of the i-th independent execution of $\mathcal{M}_{CRS}$.

$\mathcal{M}_{CRS}$ trivially achievable distribution. Let G be a set of pairs of integers $\{(i_1, j_1), (i_2, j_2), \cdots, (i_t, j_t)\}$, where $1 \leq i_1 < i_2 < \cdots < i_t \leq s(n)$ and $1 \leq j_1, j_2, \cdots, j_t \leq s(n)$ are distinct integers, and $0 \leq t \leq s(n)$ such that G is defined to be the empty set when $t = 0$. Let $\mathcal{M}_{s,n,G}$ be the probability distribution over $(\{0,1\}^n)^{2s(n)}$, obtained by first generating $2s(n) - t$ n-bit strings $\{x_m, y_k | m \in \{1, 2, \cdots, s(n)\}, k \in \{1, 2, \cdots, s(n)\} - \{j_1, j_2, \cdots, j_t\}\}$, by running $\mathcal{M}(1^n)$ independently $2s(n) - t$ times, and then defining $y_{j_d} = x_{i_d}$ for $1 \leq d \leq t$ and taking $(x_1, x_2, \cdots, x_{s(n)}, y_1, y_2, \cdots, y_{s(n)})$ as the output. A probability distribution over $(\{0,1\}^n)^{2s(n)}$ is called *$\mathcal{M}$-trivially achievable*, if it is a convex combination of $U_{s,n,G}$ over all G's.

Definition 3.1 (concurrently non-malleable coin-tossing CNMCT) *Let $\Pi = \langle L, R \rangle$ be a two-party protocol in the BPK model, where $L = (L_{KEY}, L_{PROOF})$ and $R = (R_{KEY}, R_{PROOF})$. We say that Π is a concurrently non-malleable coin-tossing protocol in the BPK model w.r.t. some key-validating relations $\mathcal{R}_{KEY}^L$ and $\mathcal{R}_{KEY}^R$, if for any PPT $s(n)$-CMIM adversary $\mathcal{A}$ in the BPK model there exists a probabilistic (expected) polynomial-time algorithm $S = (S_{KEY}, S_{PROOF})$ such that, for any sufficiently large n, any auxiliary input $z \in \{0,1\}^*$, any PPT CRS-simulating algorithm $\mathcal{M}_{CRS}$ and any polynomial-time computable (CRS-trapdoor validating) relation $\mathcal{R}_{CRS}$, and any polynomial-time computable (SK-independence distinguishing) relation $\mathcal{R}$ (with components drawn from $\{0,1\}^* \cup \{\perp\}$), the following hold, in accordance with the experiment $\mathbf{Expt}_{\mathbf{CNMCT}}(1^n, z)$ described below (page 7):*

- **Simulatability.** *The following ensembles are computationally indistinguishable:* $\{S(1^n, z, PK_L, PK_R, SK_R)\}_{1^n, PK_L \in \mathcal{K}_L, (PK_R, SK_R) \in \mathcal{R}_{KEY}^R, z \in \{0,1\}^*}$ *and* $\{view_{\mathcal{A}}^{L(SK_L), R(SK_R)}(1^n, z, PK_L, PK_R)\}_{1^n, PK_L \in \mathcal{K}_L, (PK_R, SK_R) \in \mathcal{R}_{KEY}^R, z \in \{0,1\}^*}$.
- **Strategy-restricted and predefinable randomness.** *With overwhelming probability, both the distribution of (R_L, sta_L) and that of (R_R, sta_R) are identical to the distribution of $\mathcal{M}_{CRS}^{s(n)}(1^n)$; furthermore, the distribution of (R_L, R_R) is $\mathcal{M}$-trivially achievable.*
- **Secret-key independence.** $|\Pr[\mathcal{R}(SK_R, str, sta) = 1] - \Pr[\mathcal{R}(SK'_R, str, sta) = 1]|$ *is negligible.*

The probabilities are taken over the randomness of S in the key-generation stage (i.e., the randomness for generating (PK_R, SK_R, SK'_R)) and in all proof stages, the randomness of L_{KEY}, the randomness of $\mathcal{M}_{CRS}$, and the randomness of $\mathcal{A}$.

$\mathbf{Expt}_{\mathbf{CNMCT}}(1^n, z)$

Honest left-player key-generation:
$(PK_L, SK_L) \longleftarrow L_{KEY}(1^n)$. Denote by $\mathcal{K}_L$ the set of all legitimate public-keys generated by $L_{KEY}(1^n)$.

The simulator $S = (S_{KEY}, S_{PROOF})$:
$(PK_R, SK_R, SK'_R) \longleftarrow S_{KEY}(1^n)$, where the distribution of (PK_R, SK_R) is identical with that of the output of R_{KEY}, $\mathcal{R}^R_{KEY}(PK_R, SK_R) = \mathcal{R}^R_{KEY}(PK_R, SK'_R) = 1$ and the distributions of SK_R and SK'_R are identical and *independent*.

$(str, sta) \longleftarrow S^{\mathcal{A}(1^n,\ PK_L,\ PK_R,\ z)}_{PROOF}(1^n, z, PK_L, PK_R, SK_R)$. That is, on inputs $(1^n, z, PK_L, PK_R, SK_R)$ and with oracle access to $\mathcal{A}(1^n, PK_L, PK_R, z)$ (by providing random coins to $\mathcal{A}$ and running $\mathcal{A}$ as a subroutine), the simulator S outputs a simulated transcript *str* and some state information *sta*. Denote by $R_L = \{R_L^{(1)}, R_L^{(2)}, \cdots, R_L^{(s(n))}\}$ the set of outputs of the $s(n)$ left sessions in *str* and by $R_R = \{R_R^{(1)}, R_R^{(2)}, \cdots, R_R^{(s(n))}\}$ the set of outputs of the $s(n)$ right sessions in *str*. The state information *sta* consists, among others, of two sub-sets (of $s(n)$ components each): $sta_L = \{sta_L^{(1)}, sta_L^{(2)}, \cdots, sta_L^{(s(n))}\}$ and $sta_R = \{sta_R^{(1)}, sta_R^{(2)}, \cdots, sta_R^{(s(n))})\}$. *Note that S does not know secret-key SK_L of honest left player, that is, S can emulate the honest left-player only from its public-key PK_L.*

For any $z \in \{0,1\}^*$, any $PK_L \in \mathcal{K}_L$ and $(PK_R, SK_R) \in \mathcal{R}^R_{KEY}$, we denote by $S(1^n, z, PK_L, PK_R, SK_R)$ the random variable *str* specific to (z, PK_L, PK_R, SK_R).

Discussion on the CNMCT definition. The property of strategy-restricted and predefinable randomness is a formal definition of the following informal statements: *the coin-tossing output of each left (resp., right) session is either independent of the outputs of all other sessions OR copied from the output of one right (resp., left) session on the opposite CMIM part; furthermore, the output of each session in one CMIM part can be copied into the opposite CMIM part at most once. Moreover, the simulator S sets and controls, at the same time* in an online fashion*, the coin-tossing outputs of* all *left and right sessions in the simulated transcript (in the sense that S knows the corresponding trapdoor information of* all *the coin-tossing outputs appearing in the simulated transcript).* To justify this strategy-restricted randomness property, we consider some alternative formulations: One formulation is to require that all coin-tossing outputs are independent random strings. Such formulation rules out the natural copying strategy by definition, and thus is too strong to capture naturally secure protocols. On the other hand, in order to allow the copying strategy to the CMIM, an alternative relaxed formulation is to only require that the coin-tossing output of each *individual* session is random. But, this alternative formalization may be too weak to rule out naturally insecure protocols. For instance, consider that the CMIM manages to set the outputs of some sessions to be maliciously correlated (e.g., the XOR of two right-session outputs equals one left-session output, though each individual session output is random itself), or even just to be identical.

The ability of S in online setting (and learning the trapdoor information of) all coin-tossing outputs is critical to transforming CNM protocols *with straight-line simulation/extraction* in the CRS model into the BPK model *with full adaptive input selection.*

The secret-key independence property is necessary to guarantee that $\mathcal{A}$ knows what it claims to know in its CMIM attack in the BPK model, as (str, sta) implies *straight-line* knowledge-extraction.

In comparison, the Barak's approach [2] deals with *stand-alone* NMCT *in the plain model.* The NMCT formulation in [2] essentially says that the right-session coin-tossing output is either identical to or independent of the coin-tossing output of the left-session. But, the NMCT formulation [2] does not require online learning the trapdoor information of the coin-tossing outputs (in the simulated transcript that is indistinguishable from the real view of MIM). When composing an NMZK in the CRS model with the NMCT protocol of [2], the (stand-alone) non-malleability of the whole composed protocol is still proved via standard knowledge-extraction technique with rewindings, which particularly implies that the NMZK implied by [2] is not w.r.t. full adaptive input selection. This suggests that our approach for composing protocols in the CRS model with CNMCT may be considered more modular or powerful.

On achieving CNMZK with full adaptive input selection in the BPK model. When com-

posing CNMCT with the robust non-interactive zero-knowledge (NIZK) of [23], which is also universal composable ZK (UCZK) against static adversaries [16], or the UCZK protocol of [13, 16] in the CRS model, CNMCT implies CNMZK argument of knowledge with full adaptive input selection in the BPK model. Note that both of the ZK protocols in [23, 13, 16] enjoy straight-line simulation/extraction in the CRS model, where straight-line simulation/extraction is enabled by the trapdoor information (of the CRS) possessed by the simulator/extractor which simulates CRS in the security analysis.

Specifically, we can view the composed protocol as a special version of coin-tossing, and note that in this case (str, sta) implies knowledge-extraction. Then, the property of simulatability and the property of strategy-restricted and predetermined randomness of CNMCT imply simulation-extraction *against CMIM capable of full adaptive input selection*, by viewing $\mathcal{M}_{CRS}$ as the CRS simulator of the underlying ZK protocols in the CRS model. The CNMKEI property of the composed protocol is derived from the property of secret-key independence of CNMCT. In the rest of this paper, we focus on achieving constant-round CNMCT protocols in the BPK model.

4 Constant-Round CNM Coin-Tossing in the BPK Model

High-level overview of the CNMCT construction. We design a coin-tossing mechanism in the BPK model, which allows each player to set the coin-tossing output whenever it learns its peers's secret-key. The starting point is the basic Blum-Lindell coin-tossing [10, 58]: the left-player L commits a random string σ, using randomness s_σ, to $c = C(\sigma, s_\sigma)$ with a statistically-binding commitment scheme C; The right-player R responds with a random string r_r; L sends back $r = \sigma \oplus r_l$ and proves the knowledge of (σ, s_σ). To render the simulator the ability of online setting coin-tossing outputs against malicious right-players, R proves its knowledge of its secret-key SK_R (using the key-pair technique of [69]), and L accordingly proves the knowledge of either (σ, s_σ) or SK_R. To render the ability of online setting coin-tossing outputs against malicious left-players, L registers $c = C(\sigma, s_\sigma)$ as its public-key and treats σ as the seed of a pseudorandom function PRF; L firstly sends r'_l that commits to $r_l = PRF_\sigma(r'_l)$; after receiving r_r from R, it returns back $r = r_l \oplus r_r$ and proves the knowledge of either its secret-key $SK_L = (\sigma, s_\sigma)$ (such that $r = r_r \oplus PRF_\sigma(r'_l)$) or the right-player's secret-key SK_R. To ensure *correct* knowledge-extraction against CMIM in the BPK model, the underlying proof of knowledge is implemented with the composed protocol of statistically-binding commitment and PRZK, referred to as commit-then-PRZK [73, 70, 57]. The commit-then-PRZK is regular witness indistinguishable (actually, as shown by Proposition B.1 in Appendix B, page 22, the composition of statistically-binding commitment and any strong witness indistinguishable protocol [40, 41] is itself regular WI), and is also non-malleable witness indistinguishable argument-of-knowledge (NMWIAOK) [70].

The constant-round CNMCT protocol $\langle L, R\rangle$ in the BPK model, is depicted in Figure 1 (page 9). Here, for presentation simplicity, we often write L and R to denote the left and right players directly without explicitly indicating the key-generation algorithm and the proof algorithm (which are implicitly clear from the context). Note that in the CNMCT construction, the protocol of commit-then-PRZK is used as a building block tool and is composed with other sub-protocols, and that the left-tag of PRZK in Stage-5 is set interactively. The actual statements to be proved by commit-then-PRZK and PRZK (in Stage-1 and Stage-5) are achieved by applying $\mathcal{NP}$-reductions, while the tags remaining the same. Note that the tags of the underlying PRZK in Stage-1 and Stage-5 can be equal, both of which are $O(n)$ as both the OWF f and the statistically-binding commitment scheme C are required to be linear. For PRZK to work with the CNMCT construction, we should require the length parameter $l(n) \geq O(n^3)$ which are specific to the underlying tools f and C (in particular, $l(n) = n^4$ sufficies for this work).

We reminder some differences between the uses of commit-then-PRZK in [70] and in this work. Above all, commit-then-PRZK is used in [70] to achieve CNMZK in the BPK model, while commit-then-PRZK is used for achieving CNMCT in this work. Secondly, when achieving tag-based NMWIAOK with commit-then-PRZK in [70], commit-then-PRZK is also accompanied with a one-time strong signature, which is however waived for our purpose of achieving CNMCT in the BPK model (see the notes in page 32). Thirdly, the CNMZK in [70] involves *merely* the sequential composition of two NMWIAOK protocols, while in the CNMCT construction commit-then-PRZK is composed with other protocols (say, the interactions in Stages 2-4). Finally, the left-tag and statement of commit-then-PRZK in Stage-5 is set interactively with our CNMCT construction, while in the CNMZK of [70] the tag and the statement

Right-player key registration: Let $f : \{0,1\}^* \to \{0,1\}^*$ be a linear one-way function. On a security parameter n, the right-player R (actually R_{KEY}) randomly selects s_0, s_1 from $\{0,1\}^n$, computes $y_0 = f(s_0)$, $y_1 = f(s_1)$. R publishes $PK_R = (y_0, y_1)$ as its public-key, and keeps $SK_R = s_b$ as its secret-key for a random bit $b \in \{0,1\}$ while discarding $SK' = s_{1-b}$. Define $\mathcal{R}^R_{KEY} = \{((y_0, y_1), x) | y_0 = f(x) \vee y_1 = f(x)\}$, and $\mathcal{K}_R$ the corresponding $\mathcal{NP}$-language.

Left-player key registration: Let C be a non-interactive statistically-binding *linear* commitment scheme. Each left-player L (actually L_{KEY}) selects $\sigma \in \{0,1\}^n$ and $s_\sigma \in \{0,1\}^{poly(n)}$ uniformly at random, computes $c = C(\sigma, s_\sigma)$ (i.e., committing to σ using randomness s_σ). Set $PK_L = c$ and $SK_L = (\sigma, s_\sigma)$, where σ serves as the random seed of a pseudorandom function PRF. Define $\mathcal{K}_L = \{c | \exists (x, s) \ s.t. \ \ c = C(x, s)\}$.

Stage-1. The right-player R (actually R_{PROOF}) computes and sends $c_{sk} = C(SK_R, s_{sk})$, where C is the statistically-binding commitment scheme and s_{sk} is the randomness used for commitment; Define $\mathcal{L}_{SK} = \{((y_0, y_1), c_{sk}) | \exists (s_{sk}, SK) \ s.t. \ c_{sk} = C(SK, s_{sk}) \wedge (y_0 = f(SK) \vee y_1 = f(SK))\}$. Then, R proves to the left-player L the knowledge of (SK_R, s_{sk}) such that $((PK_R, c_{sk}), (SK_R, c_{sk})) \in \mathcal{R}_{\mathcal{L}_{SK}}$, by running the PRZK for $\mathcal{NP}$ with the tag set to be $(PK_L, PK_R = (y_0, y_1))$ that is referred to as the *right* tag. The composed protocol of statistically-binding commitment and PRZK is called *commit-then-PRZK*.

Stage-2. The left player L (actually L_{PROOF}) randomly selects $r'_l \leftarrow \{0,1\}^n$, and sends r'_l to R.

Stage-3. The right player R randomly selects $r_r \leftarrow \{0,1\}^n$ and sends r_r to the left player.

Stage-4. The left player computes $r_l = PRF_\sigma(r'_l)$ (where σ is the random seed of PRF committed in L's public-key PK_L), and sends $r = r_l \oplus r_r$ to the right player.

Stage-5. L computes and sends $c_{crs} = C(\sigma || s_\sigma, s_{crs})$, where "||" denotes the operation of string concatenation. Define $\mathcal{L}_{CRS} = \{(PK_L = C(\sigma, s_\sigma), PK_R = (y_0, y_1), r'_l, r_r, r, c_{crs}) | \exists (x, s, s_{crs}) \ s.t. \ c_{crs} = C(x || s, s_{crs}) \wedge [(PK_L = C(x, s) \wedge PRF_x(r'_l) = r \oplus r_r) \vee y_0 = f(x) \vee y_1 = f(x)]\}$. Then, L proves to R the knowledge $(\sigma, s_\sigma, s_{crs})$ such that $((PK_L, PK_R, r'_l, r_r, r, c_{crs}), (\sigma, s_\sigma, s_{crs})) \in \mathcal{R}_{\mathcal{L}_{CRS}}$, by running the PRZK for $\mathcal{NP}$ with the tag set to be (PK_L, r_r, r) that is referred to as the *left* tag. That is, L proves to R that either the value committed in c_{crs} is $SK_L = (\sigma, s_\sigma)$ such that $PRF_\sigma(r'_l) = r \oplus r_r$ OR the n-bit prefix of the committed value is the preimage of either y_0 or y_1. W.l.o.g., we can assume the left-tag (PK_L, r_r, r) and the right-tag (PK_L, y_0, y_1) are of the same length (e.g., f is simply a one-way permutation).

The result of the protocol is the string r. We will use the convention that if one of the parties aborts (or fails to provide a valid proof) then the other party determines the result of the protocol.

Figure 1: Constant-round CNMCT in the BPK model

to the second NMWIAOK are fixed. In view of these differences and the subtleties of CNMCT in the BPK model, we preferred to present the CNMCT construction and analysis from scratch.

Theorem 4.1 *Under linear OWF, (non-interactive) linear statistically-binding commitments, and PRZK, the protocol* $\Pi = \langle L, R \rangle$ *depicted in Figure 1 is a constant-round CNMCT protocol in the BPK model.*

The proof details of Theorem 4.1 are given in Appendix D. We present the analysis outline below.

The (high-level) description of the CNM simulation. For any s-CMIM adversary $\mathcal{A}$ in the BPK model, consider a mental simulator M who, on input $(1^n, z, PK_L, PK_R, SK_R, F')$, additionally knows secret-keys corresponding to all public-keys registered by $\mathcal{A}$ in F'. For any i, $1 \leq i \leq s(n)$, in the i-th left-session w.r.t. a (right-player) public-key $PK^{(j)}_R \in F = F' \cup \{PK_L, PK_R\}$, the Stage-4 message $r^{(i)}$ and the state-information are set to be $(S^{(i)}_L, \tau^{(i)}_L)$ by running the CRS-simulating algorithm $\mathcal{M}_{CRS}(1^n)$; then M commits the secret-key $SK^{(j)}_R$ (assumed known to it) to $c^{(i)}_{crs}$ and finishes the PRZK with $SK^{(j)}_R$ as the witness in Stage-5. For the i-th right-session w.r.t. $PK^{(j)}_L$ (with $SK^{(j)}_L = (\sigma^{(j)}, s^{(j)}_\sigma)$), after receiving Stage-2 message $\tilde{r}^{(i)\prime}_l$, M runs $\mathcal{M}_{CRS}(1^n)$ to get the output denoted $(S^{(i)}_R, \tau^{(i)}_R)$, sends $r^{(i)}_r = PRF_{\sigma^{(j)}}(\tilde{r}^{(i)\prime}_l) \oplus S^{(i)}_R$ at Stage-3. Here, the notation of m denotes a message sent by the simulator (emulating honest players), and $\tilde{m}$ denotes the arbitrary message sent by $\mathcal{A}$. To build up the simulator S from scratch, we resort to the key-coverage techniques of [14, 5, 43]. Specifically, $S(s_b)$ with simulated $SK_R = s_b$, works in at most $s(n) + 1$ repetitions. In each simulation repetition, it either successfully finishes the simulation or "covers" a new public-key. But, key-coverage in the complex CMIM setting with bare public-keys turns out to be much more complicated and subtler.

The CNM simulation is described in Figure 2 (page 11). Note that in Case-R2 of right-session simulation w.r.t. the uncovered left-player key $PK_L^{(j)} = PK_L$, S does not try to extract the secret-key of PK_L. In the following analysis, we show that in this case, with overwhelming probability, the tag of Stage-5 of this successful right session is identical to that of Stage-5 of a left-session. As the tag of Stage-5 of a session consists of the session output (i.e., the coin-tossing output), this implies that the session output of the right-session is identical to that of one of left-sessions. Moreover, we show that with overwhelming probability each left-session output can appear, as session output, in at most one successful right-session. In the unlikely event that $\mathcal{A}$ finishes a right session and the Stage-1 of a left-session simultaneously, both of which are w.r.t. uncovered public-keys, extracting SK_R in left simulation part takes priority.

Simulatability. Assuming truly random first output of $\mathcal{M}_{CRS}$ (the analysis to the pseudorandom case is direct), there are two differences between the output of the mental simulator M and the real view of $\mathcal{A}$: (1) Truly random (in simulation) vs. pseudorandom (in real execution) Stage-4 messages of left-sessions. The distinguishable gap caused by such difference can be ruled out, using hybrid arguments, by the pseudorandomness of PRF and the hiding property of PK_L that commits to the seed of PRF; (2) Witness difference in Stage-5 of left sessions: M always uses the (right-player) secret-key $SK_R^{(j)}$, while the honest left-player L always uses SK_L in real execution. The second difference can be ruled out, using hybrid arguments, by the regular WI property of commit-then-PRZK. For the simulator $S(s_b)$ from scratch with key-coverage, the subtle point here is: the value extracted from successful Stage-5 of a right session w.r.t. an uncovered $PK_L^{(j)}$, by the argument of knowledge (AOK) of PRZK, may not necessarily be $SK_L^{(j)}$, but may possibly be the preimage of $y_b = f(s_b)$ (due to the one-wayness of y_{1-b}, the value extracted cannot be the preimage of y_{1-b}). This is called *key-coverage failure* (i.e., the Case-R2 failure in Figure 2). All left (to establish the simulatability property) is to show that key-coverage failure occurs with negligible probability, which is also the core of the whole analysis and is heavily rely upon the one-left-many-right non-malleability of PRZK.

We first present some observations on commit-then-PRZK *with restricted input selection and indistinguishable auxiliary information*. Specifically, consider the following experiments: $\text{EXPT}(1^n, w^b, aux^b)$, where $w^b \in \{0,1\}^n$ for $b \in \{0,1\}$. In $\text{EXPT}(1^n, w^b, aux^b)$, the commit-then-PRZK for $\mathcal{NP}$ is run concurrently, and an m-CMIM adversary $\mathcal{A}$ for some polynomial $m(\cdot)$, possessing auxiliary information aux^b, can set the inputs and tags to prover instances in left sessions with the restriction: for any x_i, $1 \leq i \leq m(n)$, set by $\mathcal{A}$ for the i-th left session, the fixed value w^b is always a valid $\mathcal{NP}$-witness. Denote by $trans^b$ the transcript of the experiment $\text{EXPT}(1^n, w^b, aux^b)$, and by $\widehat{W}^b = \{\hat{w}_1^b, \cdots, \hat{w}_{s(n)}^b\}$ the witnesses encoded (determined) by the statistically-binding commitments (at the beginning) of *successful* right sessions (of the commit-then-PRZK) in $trans^b$ *with tags different from those of left-sessions*. By a series of hybrid arguments, we can get: if $\{aux^0\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ and $\{aux^1\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ are indistinguishable, the ensembles $\{(trans^0, \widehat{W}^0)\}$ and $\{(trans^1, \widehat{W}^1)\}$, indexed by $\{n \in N, w^0 \in \{0,1\}^n, w^1 \in \{0,1\}^n\}$, are also indistinguishable.

Denote by $\mathcal{C}_b^k$ the set of covered key-pairs, corresponding to public-keys in $F - \{PK_R\}$, which is used by $S(1^n, s_b)$ in its k-th simulation repetition. Note that $\mathcal{C}_b^k$ does not include the simulated (PK_R, SK_R) now. The key observation here is: by viewing the messages involving $SK_R = s_b$ from the simulator S (in Stage-1 of right sessions, or Stage-5 of left sessions *in case $\mathcal{A}$ impersonates the honest right-player of PK_R*) in the simulation as from the instances of the prover $P(1^n, s_b)$ of commit-then-PRZK, and viewing the interactions of Stage-1 of left-sessions and Stage-5 of right-sessions in the simulation (in which the underlying CMIM adversary $\mathcal{A}$ serves as the prover of commit-then-PRZK) are relayed by another PPT algorithm $\hat{S}$ between the underlying CMIM adversary $\mathcal{A}$ and external commit-then-PRZK verifiers (who actually just send random coins, as PRZK is *public-coin*), the k-th simulation repetition actually amounts to the experiment of $\text{EXPT}(1^n, w^b, aux^b)$, with w^b set to be s_b and aux^b set to be $\mathcal{C}_b^k$ and $\hat{S}$ playing the role of CMIM. Here, a point of worthy noting is: though commit-then-PRZK is composed with other interactions (say, the interactions at Stage-2, Stage-3 and Stage-4), all interactions other than the interactions with the prover $P(s_b)$ of commit-then-PRZK can be internally emulated by $\hat{S}$. By inductive steps, we can get $\{\mathcal{C}_0^k\}_{n,s_0,s_1}$ and $\{\mathcal{C}_1^k\}_{n,s_0,s_1}$ are indistinguishable, for any k, $1 \leq k \leq s(n)+1$. Suppose key-coverage failure occurs in the successful i-th right session w.r.t. an uncovered $PK_L^{(j)}$ during

External honest left-player key-generation: Let $(PK_L, SK_L) \longleftarrow L_{KEY}(1^n)$, where $PK_L = c$ and $SK_L = (\sigma, s_\sigma)$ such that $\sigma \in \{0,1\}^n$ and $s_\sigma \in \{0,1\}^{t(n)}$ and $c = C(\sigma, s_\sigma)$. This captures the fact that S does not know SK_L and can emulate the honest left-player with the same public-key PK_L.

Public-key file generation:
$S_{KEY}(1^n)$ perfectly emulates the key-generation stage of the honest right-player, getting $PK_R = (y_0 = f(s_0), y_1 = f(s_1))$ and $SK_R = s_b$ and $SK'_R = s_{1-b}$ for a random bit b. Then, S_{KEY} runs $\mathcal{A}(1^n, PK_L, PK_R, z)$ to get (F', τ), where F' is a set of at most $s(n)$ public-keys and τ is the state information to be used by the proof stage of $\mathcal{A}$. The public-key file to be used in the proof-stage is $F = F' \cup \{PK_L, PK_R\}$.

$\mathcal{S} \leftarrow \{(PK_R, SK_R)\}$ (i.e. initiate the set of *covered* keys $\mathcal{S}$ to be $\{(PK_R, SK_R)\}$).
On input $(1^n, F', PK_L, PK_R, SK_R, \tau)$ and running $\mathcal{A}(PK_L, PK_R, F', \tau)$ as a subroutine, the following process is run by S_{PROOF} repeatedly at most $s(n)+1$ times. In each simulation repetition, S uses fresh randomness and tries to either end with a successful simulation or cover a new public-key in $F - \mathcal{S}$.

Straight-line left simulation:

In the i-th left concurrent session (ordered by the time-step in which the first round of each left-session is played) between S and $\mathcal{A}$ in the left CMIM interaction part w.r.t. a public-key $PK_R^{(j)} = (y_0^{(j)}, y_1^{(j)}) \in \mathcal{K}_R$, $1 \le i, j \le s(n)$, S acts as follows:

In case $\mathcal{A}$ successfully finishes Stage-1 and $PK_R^{(j)} \in F' - \mathcal{S}$, the simulator ends the current repetition of simulation trial, and starts to extract a secret-key $SK_R^{(j)}$ such that $\mathcal{R}^R_{KEY}(PK_R^{(j)}, SK_R^{(j)}) = 1$, which is guaranteed by the AOK property of PRZK. Then, let $\mathcal{S} \leftarrow \mathcal{S} \cup \{(PK_R^{(j)}, SK_R^{(j)})\}$, and move to the next repetition (with the accumulated covered-key set $\mathcal{S}$).

In case $\mathcal{A}$ successfully finishes Stage-1 and $PK_R^{(j)} \in \mathcal{S}$ (i.e., S has already learnt the secret-key $SK_R^{(j)}$), S randomly selects $r_l^{(i)\prime} \leftarrow \{0,1\}^n$ and sends $r_l^{(i)\prime}$ to $\mathcal{A}$ at Stage-2. After receiving Stage-3 message, denoted $\tilde{r}_r^{(i)}$, from $\mathcal{A}$, S invokes $\mathcal{M}_{CRS}(1^n)$ and gets the output denoted $(S_L^{(i)}, \tau_L^{(i)})$. S then sends $r^{(i)} = S_L^{(i)}$ as the Stage-4 message (rather than sending back $r^{(i)} = PRF_\sigma(r_l^{(i)\prime}) \oplus \tilde{r}_r^{(i)}$ as the honest left-player does), and sets $sta_L^{(i)} = \tau_L^{(i)}$. In Stage-5, S computes and sends $c_{crs}^{(i)} = C(SK_R^{(j)} || 0^{t(n)}, s_{crs}^{(i)})$ to $\mathcal{A}$ (rather than sending back $c_{crs}^{(i)} = C(\sigma || s_\sigma)$ as the honest left-player does), where $t(n)$ is the length of s_σ in SK_L. Finally, S finishes the PRZK of Stage-5 with $(SK_R^{(j)}, s_{crs}^{(i)})$ as its witness and $(PK_L, \tilde{r}_r^{(i)}, S_L^{(i)})$ as the tag.

Straight-line right simulation:

In the i-th right concurrent session (ordered by the time-step in which the first round of each right-session is played) between S and $\mathcal{A}$ in the right CMIM interaction part with respect to a public-key $PK_L^{(j)} = c^{(j)} \in \mathcal{K}_L$, $1 \le i, j \le s(n)$, S acts as follows:

S perfectly emulates honest right-player in Stage-1 of any right session, with SK_R as the witness to commit-then-PRZK and $(PK_L^{(j)}, PK_R)$ as the tag.

Case-R1: If $PK_L^{(j)} \in \mathcal{S}$ (i.e., S has already learnt the secret-key $SK_L^{(j)} = (\sigma^{(j)}, s_\sigma^{(j)})$), after receiving $\tilde{r}_l^{(i)\prime}$ from $\mathcal{A}$ at Stage-2, S runs $\mathcal{M}_{CRS}(1^n)$ and gets the output denoted $(S_R^{(i)}, \tau_R^{(i)})$, and then computes and sends $PRF_{\sigma^{(j)}}(\tilde{r}_l^{(i)\prime}) \oplus S_R^{(i)}$ as Stage-3 message, and goes further as the honest right-player does.

Case-R2: If $PK_L^{(j)} \notin \mathcal{S} \cup \{PK_L\}$, and $\mathcal{A}$ successfully finishes the i-th right session, then S ends the current repetition of simulation trial, and starts to extract a secret-key $SK_L^{(j)}$ such that $\mathcal{R}^L_{KEY}(PK_L^{(j)}, SK_L^{(j)}) = 1$. *In case S fails to extract such $SK_L^{(j)}$, S stops the simulation, and outputs a special symbol $\perp$ indicating simulation failure. Such simulation failure is called Case-R2 failure.* In case S successfully extracts such $SK_L^{(j)}$, then let $\mathcal{S} \leftarrow \mathcal{S} \cup \{(PK_L^{(j)}, SK_L^{(j)})\}$, and move to the next repetition. If $PK_L^{(j)} = PK_L$, S just works as the honest right-player does.

Setting sta_R: For successful i-th right session, if the Stage-4 message $\tilde{r}^{(i)}$ is $S_R^{(i)}$ or $S_L^{(k)}$ for some k, $1 \le k \le s(n)$, then $sta_R^{(i)}$ is set accordingly to $\tau_R^{(i)}$ or $\tau_L^{(k)}$; otherwise, $sta_R^{(i)}$ is set to be $\perp$.

Figure 2: The CNM simulation

the k-th simulation repetition, by the tag-setting mechanism, the Stage-5 tag used by $\mathcal{A}$ in the i-th right session must be *distinct* (i.e., different from all tags used by the simulator for Stage-1 of right sessions and Stage-5 of left sessions). This means that the value committed to $\tilde{c}_{crs}^{(i)}$, and extracted efficiently, cannot be the preimage of y_b (as otherwise the indistinguishability between $\mathcal{C}_0^k$ and $\mathcal{C}_k^1$ is violated), from which key-coverage failure is ruled out.

Secret-key independence. For any pair (s_0, s_1) in the (simulated right-player) key-generation

stage, denote by (str^b, sta^b) the output of $S(1^n, s_b)$ with $SK_R = s_b$. Suppose the secret-key independence property does not hold, there must exist a bit $\alpha \in \{0,1\}$ such that the difference between $\Pr[\mathcal{R}(s_\alpha, str^0, sta^0) = 1 | S \text{ uses } s_0 \text{ in generating } (str^0, sta^0)]$ and $\Pr[\mathcal{R}(s_\alpha, str^1, sta^1) = 1 | S \text{ uses } s_1 \text{ in generating } (str^1, sta^1)]$ is non-negligible. This implies (s_α, str^0, sta^0) and (s_α, str^1, sta^1) are distinguishable. But, the preceding analysis has already established that the ensembles $\{(str^0, sta^0)\}$ and $\{(str^1, sta^1)\}$, indexed by $\{n \in N, s_0 \in \{0,1\}^n, s_1 \in \{0,1\}^n\}$, are indistinguishable.

Strategy-restricted and predefinable randomness. This is essentially to show, with overwhelming probability, for any i, the output of the successful i-th right session w.r.t. $PK_L^{(j)}$ is either $S_R^{(i)}$ or $S_L^{(k)}$ for some k, $1 \leq i, k \leq s(n)$; furthermore, any left-session output $S_L^{(k)}$ can be the output for at most *one* successful right session.

As key-coverage failure occurs with negligible probability, we get $PK_L^{(j)} \in \mathcal{C}_b \cup \{PK_R, PK_L\}$, where $\mathcal{C}_b$ denotes the set of extracted-keys (corresponding to public-keys in $F - \{PK_R\}$) used by $S(s_b)$ in its last simulation repetition. If $PK_L^{(j)} = PK_L$, we show that the Stage-5 tag of the successful i-th right session must be identical to that of Stage-5 of a left session, which means the coin-tossing output is identical to the output of the left-session (note that each Stage-5 tag contains coin-tossing output of the session). Otherwise (i.e., the Stage-5 tag of the i-th right session is different from Stage-5 tags of all left-sessions), the Stage-5 tag of the i-th right session is *distinct*, which violates one of the following (by considering the possibilities of the value committed to $\tilde{c}_{crs}^{(i)}$ that can be efficiently extracted): one-wayness of PK_L (note S never uses SK_L in simulation), one-wayness of y_{1-b}, the one-left-many-right non-malleability of PRZK (by the above analysis of key-coverage failure).

For the case of $PK_L^{(j)} \neq PK_L$, similar analysis shows that the coin-tossing output is either $S_R^{(i)}$ or $S_L^{(k)}$ for some k. Otherwise, by the tag setting mechanism, the Stage-5 tag used by $\mathcal{A}$ in the successful i-th right session must be *distinct* (recall Stage-5 tags of left sessions always include PK_L). Again, we consider the value committed to $\tilde{c}_{crs}^{(i)}$ that can be extracted by the AOK property of PRZK. As S always sets $r_r^{(i)} = PRF_{SK_L^{(j)}}(\tilde{r}_l^{(i)\prime}) \oplus S_R^{(i)}$, suppose the coin-tossing output is not $S_R^{(i)}$ then the value committed to $\tilde{c}_{crs}^{(i)}$ cannot be $SK_L^{(j)}$, as otherwise the $\mathcal{NP}$-statement to be proved by PRZK in Stage-5 of the i-th right session is false. But, the value committed to $\tilde{c}_{crs}^{(i)}$ also cannot be the preimage of either y_{1-b} (which violates the one-wayness of y_{1-b}) or y_b (which violates the one-left-many-right non-malleability of PRZK by the above analysis of key-coverage failure).

Finally, suppose there are two successful right sessions that are of the same left-session output $S_L^{(k)}$, one of the two sessions, referred to as the i_b-th right session, must be of *distinct* Stage-5 tag. This implies that the public-key $PK_L^{(j)}$ used by $\mathcal{A}$ in the i_b-th right session is covered and is *not* PK_L, as any right-session w.r.t. PK_L is of a tag identical to that of one left-session. For the value committed to $\tilde{c}_{crs}^{(i_b)}$ (at the beginning of Stage-5 of the i_b-th right session), we can show it is neither $SK_L^{(j)}$ (as, otherwise, the $\mathcal{NP}$-statement being proved by PRZK in the Stage-5 of the i_b-th right session is false) nor the preimage of y_{1-b} (due to the one-wayness of f); Also, the value committed to $\tilde{c}_{crs}^{(i_b)}$ cannot be the preimage of y_b in accordance with the analysis of key-coverage failure.

Remark. The above security analysis can be straightforwardly extended to the general case of multiple public-keys input to the key-generation stage of $\mathcal{A}$. The differences caused by "truly random vs. pseudorandom Stage-4 messages" and "witness difference of Stage-5 of left sessions" in the general case can be ruled out by simple hybrid arguments. That "key-coverage failure" occurs still with negligible probability in the general case is from the observation: the analysis w.r.t. $\text{EXPT}(1^n, w^b, aux^b)$, for commit-then-PRZK with restricted input selection and indistinguishable auxiliary inputs, holds also w.r.t. the extended experiment $\text{EXPT}(1^n, \overline{w}^b, aux^b)$, where $\overline{w}^b$ is a fixed vector, such that for any x_i set by $\mathcal{A}$ for the i-th left session of commit-then-PRZK, there always exists a component in $\overline{w}^b$ that is a valid $\mathcal{NP}$-witness for x_i. This establishes the simulatability property in the general case, from which other properties in the general case are also straightforwardly derived. The reader is referred to Appendix D for details. We prefer the simplified analysis w.r.t. a single public-key pair (PK_L, PK_R) as it allows us to focus upon the most essential parts of the analysis.

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A Related Works

The concept of non-malleability is introduced by Dolve, Dwork and Naor in the seminal work of [30]. The work of [30] also presents non-constant-round non-malleable commitment and zero-knowledge protocols. CNMZK with a poly-logarithmic round complexity is achieved in the plain model [7]. Constant-round non-malleable coin-tossing protocol in the plain model (and accordingly, constant-round non-malleable zero-knowledge arguments for $\mathcal{NP}$ and commitment schemes by combining the result of [23]) is achieved by Barak [2]. The non-malleable coin-tossing protocol of [2] employs non-black-box techniques (introduced in [1]) in a critical way. Parallel coin-tossing, which can be viewed as a restricted version of concurrent non-malleable coin-tossing, was studied in [58, 52] in the plain model. In particular, parallel non-malleable coin-tossing (PNMCT) was studied in [52]. We were ever informed of the work [52] only after our this work is finished. In backward comparison, the work [52] considers parallel non-malleable coin-tossing in the setting of secure multi-party computation, assuming *synchronous* network and *perfectly secure* communication channels. For our CNMCT formulation and the PNMCT formulation of [52], besides the secret-key independence property formulated for CNMCT in the public-key model, we also strengthen the "strategy-restricted randomness" property by requiring that each left-session output can appear as session output in *at most* one right-session. The work [52] only formalizes that the right-session output may be one left-session output, without forbidding multiple right-sessions have the same session output identical to one left-session output (see more detailed discussion in Section 3).

A large number of concurrent non-malleable (and the strongest, universal composable) cryptographic protocols are developed in the common reference/random string model, where a common reference/random string is selected trustily by a trusted third party and is known to all players (e.g., [27, 39, 28, 23, 16, 21], etc). In particular, concurrent non-malleability for any functionality can be implemented in the common random string (CRS) model [23, 16].

There are some works that deal with the specific CNMZK protocols in the plain model or in the BPK model [7, 70, 71, 56, 55]. In particular, (non-constant round) CNMZK with adaptive input selection is recently achieved in [55]. But, no previous protocols in the BPK model or the plain model are known to be CNM secure against even CMIM with predetermined left-session inputs but *full* adaptive input selection on the right (where the statements to left sessions are predetermined and the CMIM adversary can only set statements to right sessions in the manner of *full* adaptive input selection, particularly not necessarily at the beginning of each right session), needless to say to be CNM secure against CMIM with *full* adaptive input selection. Also, the more basic CNMCT protocols were not studied there.

B Basic Definitions and Tools

We use standard notations and conventions below for writing probabilistic algorithms, experiments and interactive protocols. If A is a probabilistic algorithm, then $A(x_1, x_2, \cdots ; r)$ is the result of running A on inputs $x_1, x_2, \cdots$ and coins r. We let $y \leftarrow A(x_1, x_2, \cdots)$ denote the experiment of picking r at random and letting y be $A(x_1, x_2, \cdots ; r)$. If S is a finite set then $x \leftarrow S$ is the operation of picking an element uniformly from S. If α is neither an algorithm nor a set then $x \leftarrow \alpha$ is a simple assignment statement. By $[R_1; \cdots ; R_n : v]$ we denote the set of values of v that a random variable can assume, due to the distribution determined by the sequence of random processes $R_1, R_2, \cdots, R_n$. By $\Pr[R_1; \cdots ; R_n : E]$ we denote the probability of event E, after the ordered execution of random processes $R_1, \cdots, R_n$.

Let $\langle P, V \rangle$ be a probabilistic interactive protocol, then the notation $(y_1, y_2) \leftarrow \langle P(x_1), V(x_2) \rangle (x)$ denotes the random process of running interactive protocol $\langle P, V \rangle$ on common input x, where P has private input x_1, V has private input x_2, y_1 is P's output and y_2 is V's output. We assume w.l.o.g. that the output of both parties P and V at the end of an execution of the protocol $\langle P, V \rangle$ contains a transcript of the communication exchanged between P and V during such execution.

The security of cryptographic primitives and tools, presented throughout this work, is defined with respect to uniform polynomial-time algorithms (equivalently, polynomial-size circuits). When it comes to non-uniform security, we refer to non-uniform polynomial-time algorithms (equivalently, families of polynomial-size circuits).

On a security parameter n (also written as 1^n), a function $\mu(\cdot)$ is negligible if for every polynomial $p(\cdot)$, there exists a value N such that for all $n > N$ it holds that $\mu(n) < 1/p(n)$. Let $X = \{X(n, z)\}_{n \in N, z \in \{0,1\}^*}$ and $Y = \{Y(n, z)\}_{n \in N, z \in \{0,1\}^*}$ be distribution ensembles. Then we say that X and Y are computationally (resp., statistically) indistinguishable, if for every probabilistic polynomial-time (resp., any, even power-unbounded) algorithm D, for all sufficiently large n's, and every $z \in \{0,1\}^*$, $|\Pr[D(n, z, X(n, z)) = 1] - \Pr[D(n, z, Y(n, z)) = 1]|$ is negligible in n.

Definition B.1 (one-way function) *A function $f : \{0,1\}^* \longrightarrow \{0,1\}^*$ is called a one-way function (OWF) if the following conditions hold:*

1. *Easy to compute: There exists a (deterministic) polynomial-time algorithm A such that on input x algorithm A outputs $f(x)$ (i.e., $A(x) = f(x)$).*
2. *Hard to invert: For every probabilistic polynomial-time PPT algorithm A', every positive polynomial $p(\cdot)$, and all sufficiently large n's, it holds $\Pr[A'(f(U_n), 1^n) \in f^{-1}(f(U_n))] < \frac{1}{p(n)}$, where U_n denotes a random variable uniformly distributed over $\{0,1\}^n$.*

Definition B.2 (interactive argument/proof system) *A pair of interactive machines, $\langle P, V \rangle$, is called an interactive argument system for a language $\mathcal{L}$ if both are probabilistic polynomial-time (PPT) machines and the following conditions hold:*

- *Completeness. For every $x \in \mathcal{L}$, there exists a string w such that for every string z, $\Pr[\langle P(w), V(z) \rangle (x) = 1] = 1$.*
- *Soundness. For every polynomial-time interactive machine P^*, and for all sufficiently large n's and every $x \notin \mathcal{L}$ of length n and every w and z, $\Pr[\langle P^*(w), V(z) \rangle (x) = 1]$ is negligible in n.*

An interactive protocol is called a proof *for $\mathcal{L}$, if the soundness condition holds against any (even power-unbounded) P^* (rather than only PPT P^*). An interactive system is called a public-coin system if at each round the prescribed verifier can only toss coins and send their outcome to the prover.*

Definition B.3 (witness indistinguishability WI [37]) *Let $\langle P, V \rangle$ be an interactive system for a language $\mathcal{L} \in \mathcal{NP}$, and let $\mathcal{R}_{\mathcal{L}}$ be the fixed $\mathcal{NP}$ witness relation for $\mathcal{L}$. That is, $x \in \mathcal{L}$ if there exists a w such that $(x, w) \in \mathcal{R}_{\mathcal{L}}$. We denote by $view^{P(w)}_{V^*(z)}(x)$ a random variable describing the transcript of all messages exchanged between a (possibly malicious) PPT verifier V^* and the honest prover P in an execution of the protocol on common input x, when P has auxiliary input w and V^* has auxiliary input z. We say that $\langle P, V \rangle$ is witness indistinguishable for $\mathcal{R}_{\mathcal{L}}$ if for every PPT interactive*

machine V^*, and every two sequences $W^1 = \{w_x^1\}_{x\in L}$ and $W^2 = \{w_x^2\}_{x\in L}$ for sufficiently long x, so that $(x, w_x^1) \in \mathcal{R}_\mathcal{L}$ and $(x, w_x^2) \in \mathcal{R}_\mathcal{L}$, the following two probability distributions are computationally indistinguishable by any non-uniform polynomial-time algorithm: $\{x, view_{V^*(z)}^{P(w_x^1)}(x)\}_{x\in\mathcal{L}, z\in\{0,1\}^*}$ and $\{x, view_{V^*(z)}^{P(w_x^2)}(x)\}_{x\in\mathcal{L}, z\in\{0,1\}^*}$. Namely, for every non-uniform polynomial-time distinguishing algorithm D, every polynomial $p(\cdot)$, all sufficiently long $x \in \mathcal{L}$, and all $z \in \{0,1\}^*$, it holds that

$$|\Pr[D(x, z, view_{V^*(z)}^{P(w_x^1)}(x) = 1] - \Pr[D(x, z, view_{V^*(z)}^{P(w_x^2)}(x) = 1]| < \frac{1}{p(|x|)}$$

It is interesting to note that the WI property preserves against adaptive concurrent composition [37, 36, 38, 23].

Definition B.4 (strong witness indistinguishability SWI [40]) *Let $\langle P, V\rangle$ and all other notations be as in Definition B.3. We say that $\langle P, V\rangle$ is* strongly witness-indistinguishable for $\mathcal{R}_\mathcal{L}$ *if for every PPT interactive machine V^* and for every two probability ensembles $\{X_n^1, Y_n^1, Z_n^1\}_{n\in N}$ and $\{X_n^2, Y_n^2, Z_n^2\}_{n\in N}$, such that each $\{X_n^i, Y_n^i, Z_n^i\}_{n\in N}$ ranges over $(\mathcal{R}_\mathcal{L} \times \{0,1\}^*) \cap (\{0,1\}^n \times \{0,1\}^* \times \{0,1\}^*)$, the following holds: If $\{X_n^1, Z_n^1\}_{n\in N}$ and $\{X_n^2, Z_n^2\}_{n\in N}$ are computationally indistinguishable, then so are $\{\langle P(Y_n^1), V^*(Z_n^1)\rangle(X_n^1)\}_{n\in N}$ and $\{\langle P(Y_n^2), V^*(Z_n^2)\rangle(X_n^2)\}_{n\in N}$.*

WI vs. SWI: It is clarified in [41] that the notion of SWI actually refers to issues that are fundamentally different from WI. Specifically, the issue is whether the interaction with the prover helps V^* to distinguish some auxiliary information (which is indistinguishable without such an interaction). Significantly different from WI, SWI does *not* preserve under concurrent composition. More details about SWI are referred to [41]. An interesting observation, as clarified later, is: the protocol composing commitments and SWI can be itself regular WI. Also note that any zero-knowledge protocol is itself SWI [41].

Definition B.5 (zero-knowledge ZK [46, 40]) *Let $\langle P, V\rangle$ be an interactive system for a language $\mathcal{L} \in \mathcal{NP}$, and let $\mathcal{R}_\mathcal{L}$ be the fixed $\mathcal{NP}$ witness relation for $\mathcal{L}$. That is, $x \in \mathcal{L}$ if there exists a w such that $(x, w) \in \mathcal{R}_\mathcal{L}$. We denote by $view_{V^*(z)}^{P(w)}(x)$ a random variable describing the contents of the random tape of V^* and the messages V^* receives from P during an execution of the protocol on common input x, when P has auxiliary input w and V^* has auxiliary input z. Then we say that $\langle P, V\rangle$ is zero-knowledge if for every probabilistic polynomial-time interactive machine V^* there exists a probabilistic (expected) polynomial-time oracle machine S, such that for all sufficiently long $x \in \mathcal{L}$ the ensembles $\{view_{V^*}^{P(w)}(x)\}_{x\in\mathcal{L}}$ and $\{S^{V^*}(x)\}_{x\in\mathcal{L}}$ are computationally indistinguishable. Machine S is called a ZK simulator for $\langle P, V\rangle$. The protocol is called statistical ZK if the above two ensembles are statistically close (i.e., the variation distance is eventually smaller than $\frac{1}{p(|x|)}$ for any positive polynomial p). The protocol is called perfect ZK if the above two ensembles are actually identical (i.e., except for negligible probabilities, the two ensembles are equal).*

Definition B.6 (system for argument/proof of knowledge [40, 9]) *Let $\mathcal{R}$ be a binary relation and $\kappa : N \to [0,1]$. We say that a probabilistic polynomial-time (PPT) interactive machine V is a knowledge verifier for the relation $\mathcal{R}$ with knowledge error κ if the following two conditions hold:*

- *Non-triviality: There exists an interactive machine P such that for every $(x, w) \in \mathcal{R}$ all possible interactions of V with P on common input x and auxiliary input w are accepting.*
- *Validity (with error κ): There exists a polynomial $q(\cdot)$ and a probabilistic oracle machine K such that for every interactive machine P^*, every $x \in \mathcal{L}_\mathcal{R}$, and every $w, r \in \{0,1\}^*$, machine K satisfies the following condition:*

  Denote by $p(x, w, r)$ the probability that the interactive machine V accepts, on input x, when interacting with the prover specified by $P_{x,w,r}^*$ (where $P_{x,w,r}^*$ denotes the strategy of P^* on common input x, auxiliary input w and random-tape r). If $p(x, w, r) > \kappa(|x|)$, then, on input x and with

oracle access to $P^*_{x,w,r}$, machine K outputs a solution $w' \in \mathcal{R}(x)$ within an expected number of steps bounded by

$$\frac{q(|x|)}{p(x,w,r) - \kappa(|x|)}$$

The oracle machine K is called a knowledge extractor.

An interactive argument/proof system $\langle P, V\rangle$ such that V is a knowledge verifier for a relation $\mathcal{R}$ and P is a machine satisfying the non-triviality condition (with respect to V and $\mathcal{R}$) is called a system for argument/proof of knowledge (AOK/POK) for the relation $\mathcal{R}$.

The above definition of POK is with respect to *deterministic* prover strategy. POK also can be defined with respect to *probabilistic* prover strategy. It is recently shown that the two definitions are equivalent for all natural cases (e.g., POK for $\mathcal{NP}$-relations) [9].

Σ-protocols are very useful cryptographic tools that are 3-round public-coin protocols satisfying a special honest-verifier zero-knowledge (SHVZK) property and a special soundness property in the sense of knowledge extraction.

Definition B.7 (Σ-protocol [20]) *A 3-round public-coin protocol $\langle P, V\rangle$ is said to be a Σ-protocol for an $\mathcal{NP}$-language with relation R_L if the following hold:*

- *Completeness. If P, V follow the protocol, the verifier always accepts.*
- *Special soundness. From any common input x of length $poly(n)$ and any pair of accepting conversations on input x, (a, e, z) and (a, e', z') where $e \neq e'$, one can efficiently compute w such that $(x, w) \in R_L$. Here a, e, z stand for the first, the second and the third message respectively and e is assumed to be a string of length k (such that 1^k is polynomially related to the security parameter 1^n) selected uniformly at random in $\{0,1\}^k$.*
- *Special honest verifier zero-knowledge (SHVZK). There exists a probabilistic polynomial-time (PPT) simulator S, which on input x (where there exists a w such that $(x, w) \in R_L$) and a random challenge string $\hat{e}$, outputs an accepting conversation of the form $(\hat{a}, \hat{e}, \hat{z})$, with the probability distribution that is indistinguishable from that of the real conversation (a, e, z) between the honest $P(w)$ and V on input x.*

A very large number of Σ-protocols have been developed in the literature. Most Σ-protocols for number-theoretical languages are practical and without going through general $\mathcal{NP}$-reductions. For a good survey of Σ-protocols and their applications, the reader is referred to [20].

Definition B.8 (pseudorandom functions PRF) *On a security parameter n, let $d(\cdot)$ and $r(\cdot)$ be two positive polynomials in n. We say that*

$$\{f_s : \{0,1\}^{d(n)} \longrightarrow \{0,1\}^{r(n)}\}_{s\in\{0,1\}^n}$$

is a pseudorandom function ensemble if the following two conditions hold:

1. *Efficient evaluation: There exists a polynomial-time algorithm that on input s and $x \in \{0,1\}^{d(|s|)}$ returns $f_s(x)$.*
2. *Pseudorandomness: For every probabilistic polynomial-time oracle machine A, every polynomial $p(\cdot)$, and all sufficiently large n's, it holds:*

$$|\Pr[A^{F_n}(1^n) = 1] - \Pr[A^{H_n}(1^n) = 1]| < \frac{1}{p(n)}$$

where F_n is a random variable uniformly distributed over the multi-set $\{f_s\}_{s\in\{0,1\}^n}$, and H_n is uniformly distributed among all functions mapping $d(n)$-bit-long strings to $r(n)$-bit-long strings.

PRFs can be constructed under any one-way function [42, 40]. The current most practical PRFs are the Naor-Reingold implementations under the factoring (Blum integers) or the decisional Diffie-Hellman hardness assumptions [68]. The computational complexity of computing the value of the Naor-Reingold functions at a given point is about two modular exponentiations and can be further reduced to only two multiple products modulo a prime (without any exponentiations!) with natural preprocessing, which is great for practices involving PRFs.

Definition B.9 (statistically/perfectly binding bit commitment scheme) *A pair of PPT interactive machines, $\langle P, V\rangle$, is called a perfectly binding bit commitment scheme, if it satisfies the following:*

Completeness. *For any security parameter n, and any bit $b \in \{0,1\}$, it holds that* $\Pr[(\alpha,\beta) \leftarrow \langle P(b), V\rangle(1^n); (t,(t,v)) \leftarrow \langle P(\alpha), V(\beta)\rangle(1^n) : v = b] = 1$.

Computationally hiding. *For all sufficiently large n's, any PPT adversary V^*, the following two probability distributions are computationally indistinguishable:* $[(\alpha,\beta) \leftarrow \langle P(0), V^*\rangle(1^n) : \beta]$ *and* $[(\alpha',\beta') \leftarrow \langle P(1), V^*\rangle(1^n) : \beta']$.

Perfectly Binding. *For all sufficiently large n's, and* any *adversary P^*, the following probability is negligible (or equals 0 for perfectly-binding commitments):* $\Pr[(\alpha,\beta) \leftarrow \langle P^*, V\rangle(1^n); (t,(t,v)) \leftarrow \langle P^*(\alpha), V(\beta)\rangle(1^n); (t',(t',v')) \leftarrow \langle P^*(\alpha), V(\beta)\rangle(1^n) : v, v' \in \{0,1\} \bigwedge v \neq v']$.

*That is, no (*even computational power unbounded*) adversary P^* can decommit the same transcript of the commitment stage both to 0 and 1.*

Below, we recall some classic perfectly-binding commitment schemes.

One-round perfectly-binding (computationally-hiding) commitments can be based on any one-way permutation OWP [10, 44]. Loosely speaking, given a OWP f with a hard-core predict b (cf. [40]), on a security parameter n one commits a bit σ by uniformly selecting $x \in \{0,1\}^n$ and sending $(f(x), b(x) \oplus \sigma)$ as a commitment, while keeping x as the decommitment information.

Statistically-binding commitments can be based on any one-way function (OWF) but run in two rounds [66, 50]. On a security parameter n, let $PRG : \{0,1\}^n \longrightarrow \{0,1\}^{3n}$ be a pseudorandom generator, the Naor's OWF-based two-round public-coin perfectly-binding commitment scheme works as follows: In the first round, the commitment receiver sends a random string $R \in \{0,1\}^{3n}$ to the committer. In the second round, the committer uniformly selects a string $s \in \{0,1\}^n$ at first; then to commit a bit 0 the committer sends $PRG(s)$ as the commitment; to commit a bit 1 the committer sends $PRG(s) \oplus R$ as the commitment.

One-round perfectly-binding (computationally-hiding) commitments can be based on any one-way permutation OWP. Loosely speaking, given a OWP f with a hard-core predict b, on a security parameter n one commits a bit σ by uniformly selecting $x \in \{0,1\}^n$ and sending $(f(x), b(x) \oplus \sigma)$ as a commitment, while keeping x as the decommitment information.

For practical perfectly-binding commitment scheme, in this work we use the DDH-based ElGamal non-interactive commitment scheme [33]. To commit to a value $v \in Z_q$, the committer randomly selects $u, r \in Z_q$, computes $h = g^u \mod p$ and sends $(h, \bar{g} = g^r, \bar{h} = g^v h^r)$ as the commitment. The decommitment information is (r, v). Upon receiving the commitment $(h, \bar{g}, \bar{h})$, the receiver checks that $h, \bar{g}, \bar{h}$ are elements of order q in Z_p^*. It is easy to see that the commitment scheme is of perfectly-binding. The computational hiding property is from the DDH assumption on the subgroup of order q of Z_p^*. We also note that Micciancio and Petrank presented another implementation of DDH-based perfectly-binding commitment scheme with advanced security properties [63].

A commitment scheme C is called *linear*, if for sufficiently large n and any string $x \in \{0,1\}^n$ both $|C(x,s)|$ (i.e., the length of the commitment to x using random coins s) and $|s|$ (i.e., the length of s) are bounded by $O(n)$.

Commit-then-SWI: Consider the following protocol composing a statistically-binding commitment and SWI:

Common input: $x \in \mathcal{L}$ for an $\mathcal{NP}$-language $\mathcal{L}$ with corresponding $\mathcal{NP}$-relation $\mathcal{R}_{\mathcal{L}}$.

Prover auxiliary input: w such that $(x, w) \in \mathcal{R}_{\mathcal{L}}$.

The protocol: consisting of two stages:

Stage-1: The prover P computes and sends $c_w = C(w, r_w)$, where C is a statistically-binding commitment and r_w is the randomness used for commitment.

Stage-2: Define a new language $\mathcal{L}' = \{(x, c_w) | \exists (w, r_w) \; s.t. \; c_w = C(w, r_w) \wedge \mathcal{R}_{\mathcal{L}}(x, w) = 1\}$. Then, P proves to V that it knows a witness to $(x, c_w) \in \mathcal{L}'$, by running a SWI protocol for $\mathcal{NP}$.

One interesting observation for the above commit-then-SWI protocol is that commit-then-SWI is itself a regular WI for $\mathcal{L}$.

Proposition B.1 *Commit-then-SWI is itself a regular WI for the language $\mathcal{L}$.*

Proof (of Proposition B.1). For any PPT malicious verifier V^*, possessing some auxiliary input $z \in \{0,1\}^*$, and for any $x \in \mathcal{L}$ and two (possibly different) witnesses (w_0, w_1) such that $(x, w_b) \in \mathcal{R}_{\mathcal{L}}$ for both $b \in \{0,1\}$, consider the executions of commit-then-SWI: $\langle P(w_0), V^*(z)\rangle(x)$ and $\langle P(w_1), V^*(z)\rangle(x)$.

Note that for $\langle P(w_b), V^*(z)\rangle(x)$, $b \in \{0,1\}$, the input to SWI of Stage-2 is $(x, c_{w_b} = C(w_b, r_{w_b}))$, and the auxiliary input to V^* at the beginning of Stage-2 is (x, c_{w_b}, z). Note that (x, c_{w_0}, z) is indistinguishable from (x, c_{w_1}, z). Then, the regular WI property of the whole composed protocol is followed from the SWI property of Stage-2. □

B.1 Adaptive tag-based one-left-many-right non-malleable statistical zero-knowledge argument of knowledge (SZKAOK)

Let $\{\langle P_{TAG}, V_{TAG}\rangle(1^n)\}_{n \in N, TAG \in \{0,1\}^{O(n)}}$ be a family of argument systems for an $\mathcal{NP}$-language $\mathcal{L}$ specified by $\mathcal{NP}$-relation $\mathcal{R}_{\mathcal{L}}$. For each security parameter n and $TAG \in \{0,1\}^{O(n)}$, $\langle P_{TAG}, V_{TAG}\rangle(1^n)$ is an instance of the protocol $\langle P, V\rangle$, which is indexed by TAG and works for inputs in $\mathcal{L} \cup \{0,1\}^{p(n)}$, where $p(\cdot)$ is some polynomial.

We consider an experiment $\text{EXPE}(1^n, x, TAG, z)$, where 1^n is the security parameter, $x \in \mathcal{L} \cup \{0,1\}^{p(n)}$, $TAG \in \{0,1\}^{O(n)}$ and $z \in \{0,1\}^*$. (The input (x, TAG) captures the predetermined input and tag of the prover instance in the following left MIM part, and the string $z \in \{0,1\}^*$ captures the auxiliary input to the following MIM adversary $\mathcal{A}$.) In the experiment $\text{EXPE}(1^n, x, TAG, z)$, on input $(1^n, x, TAG, z)$, an adaptive input-selecting one-left-many-right MIM adversary $\mathcal{A}$ is simultaneously participating in two interaction parts:

The left MIM part: in which $\mathcal{A}$ chooses $(\tilde{x}^l, \widetilde{TAG}^l)$ based on its view from both the left session and all right sessions, satisfying that: the membership of $\tilde{x}^l \in \mathcal{L} \cup \{0,1\}^{p(n)}$ can be efficiently checked (otherwise, the experiment may render an $\mathcal{NP}$-membership oracle to $\mathcal{A}$). and $\widetilde{TAG}^l \in \{0,1\}^{O(n)}$; In case $\tilde{x}^l \in \mathcal{L} \cup \{0,1\}^{p(n)}$ (that can be efficiently checked), then a witness $\tilde{w}^l$ such that $(\tilde{x}^l, \tilde{w}^l) \in \mathcal{R}_{\mathcal{L}}$ is given to the prover instance $P_{\widetilde{TAG}^l}$, and $\mathcal{A}$ interacts, playing the role of the verifier $V_{\widetilde{TAG}^l}$, with the prover instance $P_{\widetilde{TAG}^l}(\tilde{x}, \tilde{w})$ on common input $\tilde{x}^l$. The interactions with $P_{\widetilde{TAG}^l}(\tilde{x}^l, \tilde{w}^l)$ is called the left session. Note that, $\mathcal{A}$ can just set $(\tilde{x}^l, \widetilde{TAG}^l)$ to be (x, TAG), which captures the case of predetermined input and tag to left session.

The right CMIM part: in which $\mathcal{A}$ *concurrently* interacts with $s(n)$, for a polynomial $s(\cdot)$, verifier instances: $V_{\widetilde{TAG}_1^r}(\tilde{x}_1^r)$, $V_{\widetilde{TAG}_2^r}(\tilde{x}_2^r)$, $\cdots$, $V_{\widetilde{TAG}_{s(n)}^r}(\tilde{x}_{s(n)}^r)$, where $(\widetilde{TAG}_i^r, \tilde{x}_i^r)$, $1 \le i \le s(n)$, are set by $\mathcal{A}$ (at the beginning of each session) adaptively based on its view (in both the left session and all the right sessions) satisfying $\tilde{x}_i^r \in \{0,1\}^{p(n)}$ and $\widetilde{TAG}_i^r \in \{0,1\}^{O(n)}$. The interactions with the instance $V_{\widetilde{TAG}_i^r}(\tilde{x}_i^r)$ is called the i-th right session, in which $\mathcal{A}$ plays the role of $P_{\widetilde{TAG}_i^r}$.

Denote by $view_{\mathcal{A}}(1^n, x, TAG, z)$ the random variable describing the view of $\mathcal{A}$ in the above experiment EXPE$(1^n, x, TAG, z)$, which includes the input $(1^n, x, TAG, z)$, its random tape, and all messages received in the one left session and the $s(n)$ right sessions.

Then, we say that the family of argument systems $\{\langle P_{TAG}, V_{TAG}\rangle(1^n)\}_{n\in N, TAG\in\{0,1\}^{O(n)}}$ is adaptive tag-based one-left-many-right non-malleable SZKAOK with respect to tags of length $O(n)$, if for any PPT adaptive input-selecting one-left-many-right MIM adversary $\mathcal{A}$ defined above, there exists an expected polynomial-time algorithm S, such that for any sufficiently large n, any $x \in \mathcal{L} \cup \{0,1\}^{p(n)}$ and $TAG \in \{0,1\}^{O(n)}$, and any $z \in \{0,1\}^*$, the output of $S(1^n, x, TAG, z)$ consists of two parts (str, sta) such that the following hold, where we denote by $S_1(1^n, x, TAG, z)$ (the distribution of) its first output str.

- **Statistical simulatability.** The following ensembles are statistically indistinguishable: $\{view_{\mathcal{A}}(1^n, x, TAG, z)\}_{n\in N, x\in\mathcal{L}\cup\{0,1\}^{p(n)}, TAG\in\{0,1\}^{O(n)}, z\in\{0,1\}^*}$ and $\{S_1(1^n, x, TAG, z)\}_{n\in N, x\in\mathcal{L}\cup\{0,1\}^{p(n)}, TAG\in\{0,1\}^{O(n)}, z\in\{0,1\}^*}$
- **Knowledge extraction.** sta consists of a set of $s(n)$ strings, $\{w_1, w_2, \cdots, w_{s(n)}\}$, satisfying the following:
  - For any i, $1 \le i \le s(n)$, if the i-th right session in str is aborted or with a tag identical to that of the left session, then $w_i = \perp$;
  - Otherwise, i.e., the i-th right session in str is successful with $\widetilde{TAG}_i^r \neq \widetilde{TAG}^l$, then $(\tilde{x}_i^r, w_i) \in \mathcal{R}_{\mathcal{L}}$, where $\tilde{x}_i^r$ is the input to the i-th right session in str.

Pass-Rosen ZK (PRZK). The PRZK (with some specified length parameter $l(n) \ge O(n^3)$) developed in [72, 73] is the only known *constant-round* adaptive tag-based one-left-many-right non-malleable SZKAOK. Furthermore, PRZK is *public-coin* and can be of *perfect* ZK. We note that in [72, 73], the tag length is just specified to be the security parameter n (in this case, the length parameter is specified as $l(n) \ge 2n^3 + n$), but a closer investigation shows that the PRZK can be extended to work for tags of length $O(n)$ and inputs of length $poly(n)$. The works of [72, 73] do not explicitly consider adaptive input and tag selection for the one left-session, but a closer investigation shows that the security analysis presented in [72, 73] works also for this more general case.

C Formulation and Discussion of CNMZK in the Public-Key Model

Definition C.1 (CNMZK in the public-key model) *We say that a protocol $\langle P, V\rangle$, where $P = (P_{KEY}, P_{PROOF})$ and $V = (V_{KEY}, V_{PROOF})$, is* concurrently non-malleable zero-knowledge *(against CMIM capable of full adaptive input selection) in the BPK model w.r.t. a class of admissible languages $\mathcal{L}$ and some key-validating relations $\mathcal{R}_{KEY}^P$ and $\mathcal{R}_{KEY}^V$, if for any positive polynomial $s(\cdot)$, any s-CMIM adversary $\mathcal{A}$ (defined in accordance with the experiment $\mathsf{Expt}_{CMIM}^{\mathcal{A}}(1^n, z)$ in Section 2), there exist a pair of (expected) polynomial-time algorithms $S = (S_{KEY}, S_{PROOF})$ (the simulator) and E (the extractor) such that for any sufficiently large n, any auxiliary input $z \in \{0,1\}^*$, and any polynomial-time computable relation $\mathcal{R}$ (with components drawn from $\{0,1\}^* \cup \{\perp\}$), the following hold, in accordance with the experiment $\mathbf{Expt}_{\mathbf{CNMZK}}(1^n, z)$ described below (page 24):*

- **Simulatability.** *The following ensembles are indistinguishable: $\{S(1^n, PK_P, PK_V, SK_V, z)\}_{PK_P\in\mathcal{K}_P, (PK_V, SK_V)\in\mathcal{R}_{KEY}^V, z\in\{0,1\}^*}$ and $\{view_{\mathcal{A}}^{P(SK_P), V(SK_V)}(1^n, PK_P, PK_V, z)\}_{PK_P\in\mathcal{K}_P, (PK,SK)\in R_{KEY}, z\in\{0,1\}^*}$.*
- **Secret-key independent knowledge-extraction.** *Specifically, E outputs a list of strings $\widehat{W} = (\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_{s(n)})$, satisfying the following:*
  - *$\hat{w}_i$ is set to be $\perp$, if the i-th right session in str is not accepting (due to abortion or verifier verification failure) or the common input of the i-th right session is identical to that of one of left sessions, where $1 \le i \le s(n)$.*

$\mathbf{Expt}_{\mathbf{CNMZK}}(1^n, z)$

Honest prover key-generation:
$(PK_P, SK_P) \longleftarrow P_{KEY}(1^n)$. Denote by $\mathcal{K}_P$ the set of all legitimate public-keys generated by $P_{KEY}(1^n)$.

The simulator $S = (S_{KEY}, S_{PROOF})$:
$(PK_V, SK_V, SK'_V) \longleftarrow S_{KEY}(1^n)$, where the distribution of (PK_V, SK_V) is identical with that of the output of V_{KEY}, $\mathcal{R}^V_{KEY}(PK_V, SK_V) = \mathcal{R}^V_{KEY}(PK_V, SK'_V) = 1$ and the distributions of SK_V and SK'_V are identical and *independent*.

$(str, sta) \longleftarrow S^{\mathcal{A}(1^n, PK_P, PK_V, z)}_{PROOF}(1^n, PK_P, PK_V, SK_V, z)$. That is, on inputs $(1^n, PK_P, PK_V, SK_V, z)$ and with oracle access to $\mathcal{A}(1^n, PK_P, PK_V, z)$ (by providing random coins to $\mathcal{A}$ and running $\mathcal{A}$ as a subroutine) the simulator S outputs a simulated transcript str, and some state information sta to be transformed to the knowledge-extractor E. *Note that S does not know the secret-key SK_P of honest prover, that is, S can emulate the honest prover only from its public-key PK_P.*

For any $PK_P \in \mathcal{K}_P$ and $(PK_V, SK_V) \in \mathcal{R}^V_{KEY}$ and any $z \in \{0,1\}^*$, we denote by $S(1^n, PK_P, PK_V, SK_V, z)$ the random variable describing the first output (i.e., str) of $S^{\mathcal{A}(1^n, PK_P, PK_V, z)}_{PROOF}(1^n, PK_P, PK_V, SK_V, z)$.

The knowledge-extractor E:
$\widehat{W} \longleftarrow E(1^n, sta, str)$. On (sta, str), E outputs a list of witnesses to (different right) statements whose validations are successfully conveyed in right sessions in str, where each of these statements is different from the statements of left sessions.

- Correct knowledge-extraction for (individual) statements: *In all other cases, with overwhelming probability $(\hat{x}_i, \hat{w}_i) \in \mathcal{R}_L$, where $\hat{x}_i$ is the statement set by $\mathcal{A}$ for the i-th right session in str and $\mathcal{R}_L$ is the $\mathcal{NP}$-relation for the admissible language $L \in \mathcal{L}$ (set by P^* for proof stages) that may be dependent upon the public-keys of honest players.*
- concurrent non-malleable knowledge extraction independence (CNMKEI): $\Pr[\mathcal{R}(SK_V, \widehat{W}, str) = 1]$ *is negligibly close to* $\Pr[\mathcal{R}(SK'_V, \widehat{W}, str) = 1]$.

The probabilities are taken over the randomness of P_{KEY}, the randomness of S, the randomness of E, and the randomness of $\mathcal{A}$.

The above CNMZK formulation in the BPK model implies both concurrent ZK for concurrent prover security in the BPK model (as S emulates the honest prover without knowing its secret-key), and concurrent knowledge-extraction (CKE) for concurrent verifier security in the public-key model formulated in [78]. The above CNMZK formulation can also be trivially extended to a tag-based formalization version.

The CNMZK formulation follows the traditional simulation-extraction approach, and extends the formulation of knowledge-extraction independence (KEI) [78] into the more complex CMIM setting. The reader is referred to [78] for detailed clarifications and justification of the KEI formulation. We remark that, as clarified, mandating the CNMKEI property is crucial for correctly formulating CNM security in the public-key model. We also remind that, as clarified in [78] for the KEI formulation, the CNMKEI formulation implicitly assumes that verifier's public-key corresponds to multiple secret-keys (in the sense protocols with a single public-key for each verifier may trivially *not* satisfy the KEI property), which however can typically be achieved with the common key-pair trick [69]. In general, cryptography literature should welcome diversified approaches for modeling and achieving security goals of cryptographic systems, particularly witnessed by the evolution history of public-key encryption.

We briefly note that no previous protocols in the BPK model were proved to be CNM-secure against even CMIM with predetermined left-session inputs but full adaptive input selection on the right (i.e., the inputs to left sessions are predetermined and the CMIM adversary only sets inputs to right sessions in the *fully* adaptive way, particularly not necessarily at the beginning of each right session), needless to say to be CNM secure against CMIM with *full* adaptive input selection. Specifically, the standard simulation-extraction paradigm for showing CNM security fails, in general, when the CMIM adversary is allowed the capability of full adaptive input selection. In particular, we show a concrete attack on the protocol proposed in [24]. This attack allows an CMIM adversary, *capable adaptive language selection*

based upon the public-key of honest players, to successfully convince the honest verifier of some $\mathcal{NP}$ statements but without knowing any witness to the statement being proved.

C.1 CMIM attacks on the CNMZK proposed in [24]

Let us first recall the protocol structure of the protocol of [24].

Key-generation. Let (KG_0, Sig_0, Ver_0) and (KG_1, Sig_1, Ver_1) be two signature schemes that secure against adaptive chosen message attacks. On a security parameter 1^n, each verifier V randomly generates two pair $(verk_0, sigk_0)$ and $(verk_1, sigk_1)$ by running KG_0 and KG_1 respectively, where $verk$ is the signature verification key and $sigk$ is the signing key. V publishes $(verk_0, verk_1)$ as its public-key while keeping $sigk_b$ as its secret-key for a randomly chosen b from $\{0,1\}$ (V discards $sigk_{1-b}$). The prover does not possess public-key.

Common input. An element $x \in \mathcal{L}$ of length $poly(n)$, where $\mathcal{L}$ is an $\mathcal{NP}$-language that admits Σ-protocols.

The main-body of the protocol. The main-body of the protocol consists of the following three phases:

Phase-1. The verifier V proves to P that it knows either $sigk_0$ or $sigk_1$, by executing the (partial witness-independent) Σ_{OR}-protocol [17] on $(verk_0, verk_1)$ in which V plays the role of knowledge prover. Denote by a_V, e_V, z_V, the first-round, the second-round and the third-round message of the Σ_{OR}-protocol of this phase respectively. Here e_V is the random challenge sent by the prover to the verifier.

If V successfully finishes the Σ_{OR}-protocol of this phase and P accepts, then goto Phase-2. Otherwise, P aborts.

Phase-2. P generates a key pair (sk, vk) for a one-time strong signature scheme. Let COM be a commitment scheme. The prover randomly selects random strings $s, r \in \{0,1\}^{poly(n)}$, and computes $C = COM(s, r)$ (that is, P commits to s using randomness r). Finally, P sends (C, vk) to the verifier V.

Phase-3. By running a Σ_{OR}-protocol, P proves to V that it knows either a witness w for $x \in \mathcal{L}$ OR the value committed in C is a signature on the message of vk under either $verk_0$ or $verk_1$. Denote by a_P, e_P, z_P, the first-round, the second-round and the third-round message of the Σ_{OR} of Phase-3. Finally, P computes a one-time strong signature δ on the whole transcript with the signing key sk generated in Phase-2.

Verifier's decision. V accepts if and only if the Σ_{OR}-protocol of Phase-3 is accepting, and δ is a valid signature on the whole transcript under vk.

Note: The actual implementation of the DDL protocol combines rounds of the above protocol. But, it is easy to see that round-combination does not invalidate the following attacks.

C.1.1 The CMIM attack

We show a special CMIM attack in which the adversary $\mathcal{A}$ only participate the right concurrent interactions with honest verifiers (i.e., there are no concurrent left interactions in which $\mathcal{A}$ concurrently interacts with honest provers).

The following CMIM attack enables $\mathcal{A}$ to malleate the interactions of Phase-1 of one session into a successful conversation of another concurrent session for different (but verifier's public-key related) statements without knowing any corresponding $\mathcal{NP}$-witnesses.

Let $\hat{L}$ be any $\mathcal{NP}$-language admitting a Σ-protocol that is denoted by $\Sigma_{\hat{L}}$ (*in particular, $\hat{L}$ can be an empty set*). For an honest verifier V with its public-key $PK = (verk_0, verk_1)$, we define a new language $\mathcal{L} = \{(\hat{x}, verk_0, verk_1) | \exists w \ s.t. \ (\hat{x}, w) \in \mathcal{R}_{\hat{L}} \text{ OR } w = sigk_b \text{ for } b \in \{0,1\}\}$. Note that for any string $\hat{x}$ (whether $\hat{x} \in \hat{L}$ or not), the statement "$(\hat{x}, verk_0, verk_1) \in \mathcal{L}$" is always true as $PK = (verk_0, verk_1)$ is

honestly generated. Also note that $\mathcal{L}$ is a language that admits Σ-protocols (as Σ_{OR}-protocol is itself a Σ-protocol). Now, we describe the concurrent interleaving and malleating attack, in which $\mathcal{A}$ successfully convinces the honest verifier of the statement "$(\hat{x}, verk_0, verk_1) \in \mathcal{L}$" for *any arbitrary* $poly(n)$-bit string $\hat{x}$ (*even when* $\hat{x} \notin \hat{L}$) by concurrently interacting with V (with public-key $(verk_0, verk_1)$) in two sessions as follows.

1. $\mathcal{A}$ initiates the first session with V. After receiving the first-round message, denoted by a'_V, of the Σ_{OR}-protocol of Phase-1 of the first session on common input $(verk_0, verk_1)$ (i.e., V's public-key), $\mathcal{A}$ suspends the first session.

2. $\mathcal{A}$ initiates a second session with V, and works just as the honest prover does in Phase-1 and Phase-2 of the second session. We denote by C, vk the Phase-2 message of the second session, where C is the commitment to a random string and vk is the verification key of the one-time strong signature scheme generated by $\mathcal{A}$ (*note that* $\mathcal{A}$ *knows the corresponding signing key* sk *as* (vk, sk) *is generated by itself*). When $\mathcal{A}$ moves into Phase-3 of the second session and needs to send V the first-round message, denoted by a_P, of the Σ_{OR}-protocol of Phase-3 of the second session *on common input* $(\hat{x}, verk_0, verk_1)$, $\mathcal{A}$ does the following:

   - $\mathcal{A}$ first runs the SHVZK simulator of $\Sigma_{\hat{L}}$ (i.e., the Σ-protocol for $\hat{L}$) [20] on $\hat{x}$ to get a simulated conversation, denoted by $(a_{\hat{x}}, e_{\hat{x}}, z_{\hat{x}})$, for the (*possibly false*) statement "$\hat{x} \in \hat{L}$".
   - $\mathcal{A}$ runs the SHVZK simulator of the Σ-protocol for showing that the value committed in C is a signature on vk under one of $(verk_0, verk_1)$ to get a simulated conversation, denoted by (a_C, e_C, z_C).
   - $\mathcal{A}$ sets $a_P = (a_{\hat{x}}, a'_V, a_C)$ and sends a_P to V as the first-round message of the Σ_{OR}-protocol of Phase-3 of the second session, where a'_V is the one received by $\mathcal{A}$ in the first session.
   - After receiving the second-round message of Phase-3 of the second session, i.e., the random challenge e_P from V, $\mathcal{A}$ suspends the second session.

3. $\mathcal{A}$ continues the first session, and sends $e'_V = e_P \oplus e_{\hat{x}} \oplus e_C$ as the second-round message of the Σ_{OR}-protocol of Phase-1 of the first session.

4. After receiving the third-round message of the Σ_{OR}-protocol of Phase-1 of the first session, denoted by z'_V, $\mathcal{A}$ suspends the first session again.

5. $\mathcal{A}$ continues the execution of the second session again, sends to $z_P = ((e_{\hat{x}}, z_{\hat{x}}), (e'_V, z'_V), (e_C, z_C))$ to V as the third-round message of the Σ_{OR}-protocol of the second session.

6. Finally, $\mathcal{A}$ applies sk on the whole transcript *of the second session* to get a (one-time strong) signature δ, and sends δ to V

Note that $(a_{\hat{x}}, e_{\hat{x}}, z_{\hat{x}})$ is an accepting conversation for the (possibly false) statement "$\hat{x} \in \hat{L}$", (a'_V, e'_V, z'_V) is an accepting conversation for showing the knowledge of either $sigk_0$ or $sigk_1$, (a_C, e_C, z_C) is an accepting conversation for showing that the value committed in C is a signature on vk under one of $(verk_0, verk_1)$. Furthermore, $e_{\hat{x}} \oplus e'_V \oplus e_C = e_P$, and δ is a valid (one-time strong) signature on the transcript of the second session.This means that, from the viewpoint of V, $\mathcal{A}$ successfully convinced V of the statement "$(\hat{x}, verk_0, verk_1) \in \mathcal{L}$" in the second session *but without knowing any corresponding* $\mathcal{NP}$*-witness*!

D Proof Details of Theorem 4.1

The description of the simulator. On security parameter 1^n, for any positive polynomial $s(\cdot)$ and any PPT $s(n)$-CMIM adversary $\mathcal{A}$ in the BPK model with auxiliary information $z \in \{0,1\}^*$, the simulator $S = (S_{KEY}, S_{PROOF})$, with respect to the honest left-player key-registration algorithm L_{KEY} and a CRS simulating algorithm $\mathcal{M}_{CRS}$, is re-depicted in Figure 2 (page 11) in order to ease reference.

In the description, the notation of m denotes a message sent by the simulator (emulating honest players), and $\tilde{m}$ denotes the arbitrary message sent by the CMIM-adversary $\mathcal{A}$.

Notes on the CNM simulation: For any i, $1 \leq i \leq s(n)$, if in the i-th left (resp., right) session of the simulation $\mathcal{A}$ does not act accordingly or fails to provide a valid proof, then S aborts that session, and sets the output just to be $S_L^{(i)}$ (resp., $S_R^{(i)}$) and the state information to be $\tau_L^{(i)}$ (resp., $\tau_R^{(i)}$).

Note that in the right-session simulation, when a successful right-session is w.r.t. a left-player key $PK_L^{(j)} = PK_L$ the simulator does not try to extract the secret-key of PK_L. In the following analysis, we show that in this case, with overwhelming probability, the tag of Stage-5 of this successful right session is identical to that of Stage-5 of a left-session. As the tag of Stage-5 of a session consists of the session output (i.e., the coin-tossing output), this implies that the session output of this right-session is identical to that of one of left-sessions. Moreover, we show that with overwhelming probability each left-session output can appear, as session output, in at most one successful right-session.

In the unlikely event that $\mathcal{A}$ finishes a right session and the Stage-1 of a left-session simultaneously, both of which are w.r.t. uncovered public-keys, extracting SK_R in left simulation part takes priority (in this case, SK_L extraction in right simulation part is ignored in the current simulation repetition).

During any (of the at most $s(n)+1$) simulation repetition, if S does not encounter secret-key extraction and does not stop due to Case-R1 failure or Case-R2 failure, then S stops whenever $\mathcal{A}$ stops, and sets str to be F and the view of $\mathcal{A}$ in this simulation repetition and $sta = (sta_L, sta_R)$ to be the according state-information.

Analysis of the CNM simulation

In order to establish the CNM security of the coin-tossing protocol depicted in Figure 1, according to the CNMCT definition of Definition 3.1, we need to show the following properties of the CNM simulator S described in Figure 2:

- S works in expected polynomial-time.
- The simulatability property, i.e., the output of S is computationally indistinguishable from the view of $\mathcal{A}$ in real CMIM attack.
- The property of strategy-restricted and predefinable randomness.
- The secret-key independence property.

In the following, we analyze the above four properties of the CNM simulator S case by case.

- S works in expected polynomial-time

Note that S works for at most $s(n)+1$ repetitions. Then, pending on the ability of S to extract secret-key of uncovered public-keys in expected polynomial-time during each repetition (equivalently, within running-time inversely propositional to the probability of secret-key extraction event occurs), S will work in expected polynomial-time. The technique for covering public-keys follows that of [14, 5]. Below, we specify the secret-key extraction procedures in more details.

Right-player key coverage. Whenever S needs to extract the secret-key $SK_R^{(j)}$ corresponding to an uncovered public-key $PK_R^{(j)}$, due to successful Stage-1 of the i-th left session during the k-th simulation repetition w.r.t. covered key set $\mathcal{S}^{(k)}$, $1 \leq i, j \leq s(n)$ and $1 \leq k \leq s(n)+1$, we combine the CMIM adversary $\mathcal{A}$ and the simulation other than Stage-1 of the i-th left session (i.e., the public file F, the covered key set $\mathcal{S}^{(k)}$, the randomness $r_{\mathcal{A}}$ of $\mathcal{A}$, and the randomness $r_{\mathcal{S}}$ used by S except for that to be used in Stage-1 of the i-th left session) into an imaginary (deterministic) knowledge prover $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)}, r_{\mathcal{A}}, r_{\mathcal{S}})}$. Note that, by the description of the CNM simulation depicted in Figure 2, the Stage-1 of the i-th left session is the *first successful* Stage-1 of a left session finished by $\mathcal{A}$ (during the k-th simulation repetition) with respect to an uncovered public-key not in $\mathcal{S}^{(k)}$. The knowledge-prover $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)}, r_{\mathcal{A}}, r_{\mathcal{S}})}$ only interacts with a stand-alone knowledge-verifier of commit-then-PRZK, by running $\mathcal{A}$ internally and mimicking S with respect to $\mathcal{S}^{(k)}$ but with the following exceptions: (1) the messages belonging to the Stage-1 of the i-th left session are relayed between the internal $\mathcal{A}$ and the external

stand-alone knowledge-verifier of PRZK; (2) $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)},r_{\mathcal{A}},r_{\mathcal{S}})}$ ignores the events of secret-key extraction in right simulation part, i.e., successful right sessions with respect to uncovered (left-player) public-keys; (3) whenever $\mathcal{A}$ (run internally by $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)},r_{\mathcal{A}},r_{\mathcal{S}})}$) successfully finishes, *for the first time*, Stage-1 of a left session w.r.t. an uncovered (right-player) public-key not in $\mathcal{S}^{(k)}$, $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)},r_{\mathcal{A}},r_{\mathcal{S}})}$ just stops.

For any intermediate $\mathcal{S}^{(k)}$ used in the k-th simulation repetition, any $PK^{(j)}_R \notin \mathcal{S}^{(k)}$, any randomness $r_{\mathcal{A}}$ of $\mathcal{A}$ and any randomness $r_{\mathcal{S}}$ used by S except for that to be used in Stage-1 of the i-th left session, denote by p the probability (taken over the coins used by $\mathcal{S}$ for Stage-1 of the i-th left session) that the public-key used by $\mathcal{A}$ in Stage-1 of the i-th left session is $PK^{(j)}_R$, and furthermore, the Stage-1 of the i-th left session is the *first* successful execution of PRZK w.r.t. an uncovered public-key during the simulation of $\mathcal{S}$ w.r.t. covered-key set $\mathcal{S}^{(k)}$. That is, p is the probability, taken over the coins used by $\mathcal{S}$ for Stage-1 of the i-th left session (but for fixed other coins), of the event that $\mathcal{S}$ needs to cover $PK^{(j)}_R \notin \mathcal{S}^{(k)}$ in the i-th left session in its simulation w.r.t. $\mathcal{S}^{(k)}$. Clearly, with probability at least p, the knowledge prover $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)},r_{\mathcal{A}},r_{\mathcal{S}})}$ successfully convinces the stand-alone knowledge verifier of $PK^{(j)}_R$. By the AOK property of PRZK and applying the knowledge-extractor on $\hat{P}^{(i,j)}_{(\mathcal{S}^{(k)},r_{\mathcal{A}},r_{\mathcal{S}})}$, the secret-key $SK^{(j)}_R$ will be extracted within running-time inversely propositional to p. Here, when p is negligible, standard technique, originally proposed in [43] and then deliberated in [58], has to be applied here (to estimate the value of p) in order to make sure expected polynomial-time knowledge-extraction. In more detail, the running-time of the naive approach to directly applying knowledge-extractor whenever such events occur is bounded by $T(n) = p \cdot \frac{q(n)}{p-\kappa(n)}$, where $\kappa(n)$ is the knowledge-error and $q(\cdot)$ is the polynomial related to the running time of the knowledge-extractor that is $\frac{q(n)}{p-\kappa(n)}$. The subtle point is: when p is negligible, $T(n)$ is not necessarily to be polynomial in n. The reader is referred to [43, 58] for the technical details of dealing with this issue.

Left-player key coverage.

The coverage procedure for uncovered (left-player) public-keys used by $\mathcal{A}$ in successful Stage-5 of right sessions can be described accordingly, similar to above right-player key coverage. The key point to note here is: for a successful right session with respect to an uncovered (left-player) public-key $PK^{(j)}_L$, the value extracted in expected polynomial-time is not necessarily to be the secret-key $SK^{(j)}_L$, though the value extracted must be either $SK^{(j)}_L$ or SK_R (i.e., the preimage of either y_0 or y_1) , where $PK_R = (y_0, y_1)$ is the simulated (right-player) public-key. That is, S may abort due to Case-R2 failure (*though it works in expected polynomial-time*). We show, in the following analysis of the simulatability property, Case-R2 failure occurs with at most negligible probability.

- Simulatability

For presentation simplicity, in the following analysis of simulatability we assume the first output of $\mathcal{M}_{CRS}$ is truly random string of length n, i.e., all $S^{(i)}_L$'s and $S^{(i)}_R$'s are truly random strings. The extension of the simulatability analysis to the case of pseudorandom output of $\mathcal{M}_{CRS}$ is direct.

Assuming truly random output of $\mathcal{M}_{CRS}$, there are three differences between the simulated transcript output by S and the view of $\mathcal{A}$ in real CMIM attack against the honest left-player of PK_L and the honest right-player of PK_R:

Truly random vs. pseudorandom Stage-4 messages: In simulation, the simulator S sends truly random string $r^{(i)} = S^{(i)}_L$ at Stage-4 of the i-th left session, for any i, $1 \leq i \leq s(n)$. But, the honest left-player sends a pseudorandom Stage-4 message, i.e., $r^{(i)} = PRF_\sigma(r^{(i)\prime}_l) \oplus \tilde{r}^{(i)}_r$, where $r^{(i)\prime}_l$ and $\tilde{r}^{(i)}_r$ are the Stage-2 and Stage-3 messages of the i-th left session.

Witness difference of Stage-5 of left sessions: For any i-th left session w.r.t. a public-key $PK^{(j)}_R \in \mathcal{S}$, the witness used by S in the commit-then-PRZK of Stage-5 is always the extracted secret-key $SK^{(j)}_R$, while the witness used by the honest left-player is always its secret-key SK_L.

Case-R2 failure: S may stop with simulation failure, due to invalid secret-key extraction in Case-R2 in the right simulation part.

We first show that, conditioned on Case-R2 failure does not occur, the output of S is indistinguishable from the real view of $\mathcal{A}$. Specifically, we have the following lemma:

Lemma D.1 *Conditioned on Case-R2 failure does not occur, the following ensembles are indistinguishable:* $\{S(1^n, z, PK_L, PK_R, SK_R)\}_{1^n, PK_L \in \mathcal{K}_L, (PK_R, SK_R) \in \mathcal{R}^R_{KEY}, z \in \{0,1\}^*}$ *(defined in Definition 3.1) and* $\{view_{\mathcal{A}}^{L(SK_L), R(SK_R)}(1^n, z, PK_L, PK_R)\}_{1^n, PK_L \in \mathcal{K}_L, (PK_R, SK_R) \in \mathcal{R}^R_{KEY}, z \in \{0,1\}^*}$ *(defined in accordance with the experiment* $\mathsf{Expt}^{\mathcal{A}}_{CMIM}(1^n, z)$ *described in Section 2).*

Proof (of Lemma D.1). We first note that, conditioned on Case-R2 failure does not occur and assuming the truly random output of $\mathcal{M}_{CRS}$, S perfectly emulates the honest right-player of PK_R in right simulation part.

The left two differences all are w.r.t. left session simulation. Intuitively, in real interaction the seed σ of PRF is committed into left-player public-key PK_L and is re-committed and proved concurrently in Stage-5 of left sessions, the CMIM adversary may potentially gain some knowledge about the random seed σ by concurrent interaction, which enabling it to set its Stage-3 messages of left sessions maliciously depending on the output of PRF_σ. Note that in real interaction, the Stage-4 messages sent by honest left-player are determined by the PRF seed and the Stage-2 messages. Thus, the Stage-4 messages of left sessions in real interaction may be distinguishable from truly random strings as sent by the simulator S in simulation. The still indistinguishability between the simulated transcript and the real view of $\mathcal{A}$ is proved by hybrid arguments.

We consider a hybrid mental experiment $\mathcal{H}$. $\mathcal{H}$ mimics $S(1^n, z, PK_L, PK_R, SK_R)$, with additionally possessing $SK_L = (\sigma, s_\sigma)$ and with the following exception: At Stage-4 of any left session, $\mathcal{H}$ just emulates the honest left-player by setting the Stage-4 message $r^{(i)}$ to be $PRF_\sigma(r_l^{(i)\prime}) \oplus \tilde{r}_r^{(i)}$ (rather than sending $S_L^{(i)}$ as S does); In Stage-5 of any left session w.r.t. a covered key $PK_R^{(j)}$ (for which $\mathcal{H}$ has already learnt the corresponding secret-key $SK_L^{(j)}$), $\mathcal{H}$ still emulates S by using the extracted secret-key $SK_R^{(j)}$ as the witness (specifically, it commits to $SK_R^{(j)}||0^t$ and finishes PRZK accordingly as the simulator S does).

The difference between the view of $\mathcal{A}$ in $\mathcal{H}$ and the view of $\mathcal{A}$ in the simulation of S lies in the difference of Stage-4 messages of left sessions. Suppose that the view of $\mathcal{A}$ in $\mathcal{H}$ is distinguishable from the view of $\mathcal{A}$ in the simulation of S, then it implies that there exists a PPT algorithm D that, given the commitment of the PRF seed, i.e., $PK_L = C(\sigma, s_\sigma)$, can distinguish the output of PRF_σ from truly random strings. Specifically, on input PK_L, D emulates $\mathcal{H}$ or S by having oracle access to PRF_σ or a truly random function; Whenever it needs to send Stage-4 message in a left session, it just queries its oracle with the Stage-2 message. Clearly, if the oracle is PRF_σ, then D perfectly emulates $\mathcal{H}$, otherwise (i.e., the oracle is a truly random function), it perfectly emulates the simulation of S.

So, we conclude that if the view of $\mathcal{A}$ in $\mathcal{H}$ is distinguishable from the view of $\mathcal{A}$ in the simulation of S, then the PPT algorithm D that, given the commitment of the PRF seed σ, can distinguish the output of PRF_σ from that of truly random function. Consider the case that D, given the commitment $c = C(\sigma)$, has oracle access to an independent $PRF_{\sigma'}$ of an independent random seed σ' or a truly random function. Due to the pseudorandomness of PRF, the output of $D(c)$ with oracle access to $PRF_{\sigma'}$ is indistinguishable from the output of $D(c)$ with oracle access to a truly random function. It implies that D, given the commitment $c = C(\sigma)$, can distinguish the output of PRF_σ and the output of $PRF_{\sigma'}$, where σ and σ' are independent random seeds. But, this violates the computational hiding property of the commitment scheme C. Specifically, given two random strings of length n, (s_0, s_1), and a commitment $c_b = C(s_b)$ for a random bit b, the algorithm D can be used to distinguish the value committed in c_b, which violates the computational hiding property of C.

Now, we consider the difference between the output of $\mathcal{H}$ and the view of $\mathcal{A}$ in real execution. Recall that, as we have shown the view of $\mathcal{A}$ in $\mathcal{H}$ is indistinguishable from that in the simulation and we have assumed Case-R2 failure does not occur in the simulation of S, Case-R2 failure can occur in $\mathcal{H}$ with

at most negligible probability. Then, the difference between the output of $\mathcal{H}$ and the view of $\mathcal{A}$ in real execution lies in the witnesses used in Stage-5 of left sessions. Specifically, $\mathcal{H}$ still uses the extracted right-player secret-keys in Stage-5 of left sessions, while the honest left-player always uses its secret-key SK_L in Stage-5 of left sessions in real execution. By hybrid arguments, the difference can be reduced to violate the regular WI property of commit-then-PRZK. Note that commit-then-PRZK is itself regular WI for $\mathcal{NP}$ (actually, any commit-then-SWI is itself regular WI).

In more detail, we consider the mental experiment $M_b, b \in \{0,1\}$. On input $\{(PK_L, SK_L), (PK_R, SK_R)\}$ and public file F, and auxiliary information z to the CMIM adversary $\mathcal{A}$ [1], the mental M_b also takes as input all secret-keys corresponding to right-player public-keys in the public file F (in case the corresponding secret-keys exist). M_b runs the CMIM adversary $\mathcal{A}$ as follows:

1. M_b emulates the honest right-player of PK_R (with SK_R as the witness) in right sessions. In particular, M just sends truly random Stage-3 messages in all right sessions, and ignores knowledge-extraction of left-player secret-keys in right sessions (i.e., in case $\mathcal{A}$ successfully finishes a right session w.r.t an uncovered public-key $PK_L^{(j)}$, M_b ignores the need of secret-key extraction and just moves on);

2. For any i, j, $1 \leq i \leq s(n)$ and $1 \leq j \leq s(n)+1$, in the i-th left session w.r.t. right-player public-key $PK_R^{(j)}$, M_b emulates the honest left-player of PK_L until Stage-4 (in particular, it sets the Stage-4 message $r^{(i)}$ to be $PRF_\sigma(r_l^{(i)\prime}) \oplus \tilde{r}_r^{(i)}$), but with the following exception in Stage-5:

   - If $b = 0$, then M_b just emulates the honest left-player in Stage-5 of the left session, with SK_L as its witness.
   - If $b = 1$, M_b still emulates the simulator by using the secret-key $SK_R^{(j)}$, for which we assume it exists and M knows, as the witness in Stage-5. Specifically, it commits to $SK_R^{(j)}||0^t$ and finishes PRZK accordingly as the simulator S does.

It's easy to see that the output of M_0 is identical to the real view of $\mathcal{A}$ in real execution, and the output of M_1 is indistinguishable from the output of $\mathcal{H}$. Then, suppose the real view of $\mathcal{A}$ in real execution is distinguishable from the output of $\mathcal{H}$, by hybrid arguments we can break the regular WI of commit-then-PRZK. □

Now, we show that Case-R2 failure indeed occurs with negligible probability, from which the simulatability of the CNM simulation is established.

Lemma D.2 *Case-R2 failure occurs with negligible probability.*

Proof (of Lemma D.2). Suppose Case-R2 failure occurs with non-negligible probability. That is, for some polynomial $p(n)$ and infinitely many n's, with probability of $\frac{1}{p(n)}$ there exist k, i, j, $1 \leq k \leq s(n)+1$ and $1 \leq i, j \leq s(n)$, such that in the k-th simulation repetition $\mathcal{A}$ successfully finishes the i-th right session with respect to an uncovered public-key $PK_L^{(j)} \notin \mathcal{S} \cup \{PK_L\}$, furthermore, the k-th simulation repetition is the *first* one encountering Case-R2 failure and the i-th right session is the *first* successful session w.r.t. an uncovered public-key not in $\mathcal{S} \cup \{PK_L\}$ during the k-th simulation repetition, but the simulator fails in extracting the corresponding secret-key $SK_L^{(j)}$. Recall that S makes at most $s(n)+1$ simulation trials (repetitions) and each simulation trial uses fresh randomness in the proof stages; S starts knowledge-extraction whenever it encounters a successful session w.r.t. an uncovered public-key different from PK_L; Whenever Case-R2 failure occurs S aborts the whole simulation, which implies that the k-th simulation repetition is also the *last* simulation trial.

Note that, *by the AOK property of PRZK* (we can combine the k-th simulation repetition except for the Stage-5 of the i-th right session into a stand-alone knowledge prover of the PRZK), in this case the simulator still extracts some value that is determined by the statistically-binding commitment $\tilde{c}_{crs}^{(i)}$ at the start of Stage-5 of the i-th right session. According to the AOK property of PRZK, there are two possibilities for the value committed to $\tilde{c}_{crs}^{(i)}$ and extracted by S assuming Case-R2 failure.

[1] Recall that, in accordance with the definition of CNMCT, z is a priori information of $\mathcal{A}$ that is independent from the public file F (in particular, PK_L and PK_R).

Case-1. The value committed is the preimage of y_{1-b}. Recall that $PK_R = (y_0, y_1)$ is the simulated public-key of honest right player, with $SK_R = s_b$ for a random bit b such that $y_b = f(s_b)$.

Case-2. The value committed is the preimage of y_b.

Due to the one-wayness of the OWF f, it is easy to see that Case-1 can occur only with negligible probability. Specifically, consider the case that y_{1-b} is given to the simulator, rather than generated by the simulator itself.

Below, we show that Case-2 occurs also with negligible probability, from which Lemma D.2 is then established.

We consider the following two experiments: $E(1^n, z, PK_L, PK_R, s_b)$, where $s_b = SK_R$ and $b \in \{0,1\}$. The experiment $E(1^n, z, PK_L, PK_R, s_b)$ consists of two phases (or algorithms), denoted by E_1 and E_2: In the first phase, E_1 just runs $S(1^n, z, PK_L, PK_R, s_b)$ until S stops. Denote by $\mathcal{C}_b$ the set of extracted-keys, corresponding to public-keys in $F - \{PK_R\}$, which are extracted and used by S in its *last* simulation trial (recall that the *first* simulation repetition encountering Case-R2 failure is also the last simulation repetition). Specifically, suppose S uses $SK_R = s_b$ in the simulation and stops in the k-th simulation repetition with respect to covered-key set, denoted $\mathcal{S}_b^{(k)}$, then $\mathcal{C}_b = \mathcal{S}_b^{(k)} - \{(PK_R, SK_R)\}$. Note that $\mathcal{C}_b$ does not include (PK_R, SK_R) now. The set $\mathcal{C}_b$, the public-key file $F = F' \cup \{PK_L, PK_R\}$ and the state information τ are passed on to E_2, where (F', τ) is the output of (the key-generation stage of) the underlying CMIM adversary $\mathcal{A}(1^n, z, PK_L, PK_R)$ (run by S).

Then, in the second phase of the experiment E, a PPT algorithm $E_2(1^n, \mathcal{C}_b, F, \tau)$ runs (the proof-stage of) the CMIM adversary $\mathcal{A}(1^n, F, \tau)$ and (re)mimics the simulation of S (to be precise, S_{PROOF}) at its last simulation trial w.r.t. the set of covered-keys $\mathcal{C}_b$, but with the following exceptions (note that E_2 does not take $s_b = SK_R$ as input): (1) E_2 externally interacts with the prover of commit-then-PRZK $P(1^n, s_b)$: Whenever S needs to give a Stage-1 proof of a right session on $PK_R = (y_0, y_1)$, or needs to give a Stage-5 proof of a left session with respect to PK_R [2] on input $(PK_L, PK_R, (r_l^{(i)\prime}, \tilde{r}_r^{(i)}, r^{(i)}))$, E_2 just sets the corresponding input, i.e., PK_R or $(PK_L, PK_R, (r_l^{(i)\prime}, \tilde{r}_r^{(i)}, r_l^{(i)}))$,[3] as well as the according left or right tag, to $P(1^n, s_b)$, and then relays messages between $P(1^n, S_b)$ and the CMIM adversary $\mathcal{A}$; (2) In case $\mathcal{A}$ successfully finishes Stage-1 of a left session with respect to an uncovered public-key not in $\mathcal{C}_b \cup \{PK_R\}$ in the run of $E_2(1^n, \mathcal{C}_b, F, \tau)$, E_2 just stops.

Now, suppose Case-2 of Case-R2 failure occurs with non-negligible probability. That is, with non-negligible probability, the simulator S aborts due to Case-R2 failure in its last simulation trial with respect to the covered public-key set $\mathcal{C}_b$, and the value committed in $\tilde{c}_{crs}^{(i)}$ (in the successful i-th right session, for some i, $1 \leq i \leq s(n)$, w.r.t. an uncovered public-key $PK_L^{(j)} \notin \mathcal{C}_b \cup \{PK_L, PK_R\}$ during the simulation trial w.r.t. $\mathcal{C}_b$) is the preimage of y_b. Recall that, the successful i-th right session is also the first successful session w.r.t. an uncovered public-key different from PK_L during the simulation trial w.r.t. $\mathcal{C}_b$. It is easy to see that, with also non-negligible probability, the value committed in $\tilde{c}_{crs}^{(i)}$ in the i-th right successful session (which is also the first successful session w.r.t. an uncovered public-key not in $\mathcal{C}_b \cup \{PK_L, PK_R\}$) under the run of $E_2(1^n, \mathcal{C}_b, F, \tau)$ is the preimage of y_b. We will use this fact to violate the one-left-many-right simulation/extraction of commit-then-PRZK with adaptively setting input and tag for the one left-session, where the simulator/extractor of commit-then-PRZK first commits to 0 and then runs the one-left-many-right simulator/extractor of PRZK.

Before proceeding the analysis, we first present some observations on commit-then-PRZK with restricted input selection and indistinguishable auxiliary information. Consider the following experiments: $\text{EXPT}(1^n, w^b, aux^b)$, where $w^b \in \{0,1\}^n$ for $b \in \{0,1\}$. In $\text{EXPT}(1^n, w^b, aux^b)$, the commit-then-PRZK for $\mathcal{NP}$ is run concurrently, and a many-left-many-right CMIM adversary $\mathcal{A}$, possessing auxiliary information aux^b and involving at most $m(n)$ left-sessions and at most $m(n)$ right-sessions simultaneously where $m(\cdot)$ is a positive polynomial, can set the inputs and tags to prover instances of left sessions

[2] Note that left sessions may be with respect to the simulated public-key PK_R, i.e., the CMIM adversary may impersonate the honest right-player of PK_R in left sessions.

[3] Actually, the $\mathcal{NP}$-statements reduced from them for the $\mathcal{NP}$-Complete language for which commit-then-PRZK actually works.

with the following restriction: for any x_i, $1 \leq i \leq m(n)$, set by $\mathcal{A}$ for the i-th left session of commit-then-PRZK, the fixed value w^b is always a valid $\mathcal{NP}$-witness. In other words, although $\mathcal{A}$ has the power of adaptive input selection for provers, but there exists fixed witness-pair (w^0, w^1) for all inputs selected by $\mathcal{A}$. Such adversary is called *restricted* input-selecting CMIM-adversary. Denote by $trans^b$ the transcript of the experiment $\text{EXPT}(1^n, w^b, aux^b)$ (i.e., the view of $\mathcal{A}$ in $\text{EXPT}(1^n, w^b, aux^b)$), and by $\widehat{W}^b = \{\hat{w}_1^b, \cdots, \hat{w}_{m(n)}^b\}$ the witnesses encoded (determined) by the statistically-binding commitments (at the beginning) of successful right sessions in $trans^b$ (as in [73], in the unlikely event that a statistically-binding commitment does not uniquely determine a witness, the corresponding witness is set to be "$\perp$"); For a right session that aborts or the tag of the underlying PRZK is identical to that in one of left sessions, $\hat{w}_i^b$ is set to be a special symbol $\perp$. We want to show the following proposition:

Proposition D.1 *If the ensembles* $\{aux^0\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ *and* $\{aux^1\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ *are indistinguishable, then the ensembles* $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ *in accordance with* $\text{EXPT}(1^n, w^0, aux^0)$ *and* $\{(trans^1, \widehat{W}^1)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ *in accordance with* $\text{EXPT}(1^n, w^1, aux^1)$ *are also indistinguishable.*

Proof (of Proposition D.1): This is established by investigating a series of experiments.

First consider two experiments $\text{EXPT}_1^n(1^n, w^b, aux^b)$, where $b \in \{0, 1\}$. In $\text{EXPT}_1^n(1^n, w^b, aux^b)$, a *one-left-many-right* restricted input-selecting MIM adversary $\mathcal{A}$, possessing auxiliary information aux^b, interacts with the prover instance of commit-then-PRZK in one left session and sets the input x of the left session such that $(x, w^b) \in \mathcal{R}_{\mathcal{L}}$, and concurrently interacts with many honest verifier instances on the right. From the one-many simulation/extraction SZKAOK property of PRZK (with adaptively setting input and tag for the one left session) and computational-hiding property of the underlying statistically-binding commitments, by hybrid arguments, we can conclude that if aux^0 is indistinguishable from aux^1, then $\mathcal{A}$'s views and the witnesses encoded (actually *extracted*) in the two experiments, i.e., $(trans^0, \widehat{W}^0)$ and $(trans^1, \widehat{W}^1)$), are indistinguishable.

In more details (as shown in [73]), due to the one-many simulation/extraction perfect ZKAOK property of PRZK, for any bit $b \in \{0, 1\}$ $(trans^b, \widehat{W}^b)$ in $\text{EXPT}_1^n(1^n, w^b, aux^b)$ is identical to $(trans^b, \widehat{W}^b)$ in a modified version of $\text{EXPT}_1^n(1^n, w^b, aux^b)$, called commit($w^b$)-then-simulatedPRZK, in which the one left-session and many right-sessions are emulated by a PPT algorithm (i.e., the one-left-many-right simulator/extractor guaranteed by PRZK) with witness extraction for successful right-sessions of different tags (but the witness w^b is still committed to the statistically-binding commitment of the left session). Then, for this experiment, due to the computational hiding property of the statistically-binding commitment scheme used in commit-then-PRZK, $(trans^b, \widehat{W}^b)$ of the commit(w^b)-then-simulatedPRZK experiment is computationally indistinguishable from that of the commit(0)-then-simulatedPRZK experiment in which "0" (rather than w^b) is committed to the statistically-binding commitment of the one left session. Note that the commit(0)-then-simulatedPRZK experiment can be performed by a merely PPT algorithm. The reader is referred to [73] for more details of the hybrid arguments of this step. Here, we point out that the hybrid arguments of [73] is actually w.r.t. a *strengthened* version of commit-then-PRZK, which is referred as *signed* commit-then-PRZK here. Roughly speaking, the tag of the underlying PRZK is set to be the public-key of a signature scheme, and the protocol transcript of commit-then-PRZK is in turn signed by the prover. Some advantages of signed commit-then-PRZK are: it can work for tags of length $poly(n)$ (rather than $O(n)$ as required by the underlying PRZK), and it can satisfy some stronger non-malleability requirements w.r.t. session transcripts (rather than only session tags or inputs). We note this signature-based trick is unnecessary for our purpose in this work. In particular, the tags of the underlying PRZK in our CNMCT constructions are indeed of length $O(n)$, and our CNM definitions (for ZK and CT) are based on the normal formulation approach that is not w.r.t. session transcripts.[4]

Now we consider the following two experiments: $\text{EXPT}(1^n, w, aux^b)$, where $b \in \{0, 1\}$ and $w \in \{w^0, w^1\}$. In $\text{EXPT}(1^n, w, aux^b)$, a many-left-many-right restricted input-selecting MIM adversary $\mathcal{A}$,

[4]The extension to session-transcript based formulations and protocol implementations of CNMCT and CNMZK are left for future explorations.

possessing auxiliary information aux^b, interacts concurrently with many prover instances on the left (such that w is always a witness for inputs selected adaptively by $\mathcal{A}$ for left sessions), and interacts with many honest verifier instances on the right. Then, the indistinguishability between the ensembles $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ and $\{(trans^1, \widehat{W}^1)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ is direct from the indistinguishability between $\{aux^0\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ and $\{aux^1\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ and the adaptive one-left-many-right simulation-extractability of PRZK. Specifically, this is derived by a simple reduction to the above one-left-many-right case. Note that according to the definition of indistinguishability between *ensembles*, (w, aux^0) and (w, aux^1) are indistinguishable. *Actually,* (w^0, w^1, aux^0) *and* (w^0, w^1, aux^1) *are indistinguishable.* Also, note that all sessions in $\text{EXPT}(1^n, w, aux^b)$ can be emulated internally by a PPT algorithm given (w, aux^b).

In more details, we consider the experiments $\text{EXPT}_1^n(1^n, w, aux^b)$, where $b \in \{0,1\}$ and $w \in \{w^0, w^1\}$. In the experiment $\text{EXPT}_1^n(1^n, w, aux^b)$, a one-left-many-right MIM adversary $\mathcal{A}'$ that on auxiliary input (w, aux^b) mimics the CMIM adversary $\mathcal{A}$ (of the auxiliary input aux^b) in the above experiment $\text{EXPT}(1^n, w, aux^b)$, with the following modifications: all left-sessions except for the first left-session are perfectly emulated by $\mathcal{A}'$ by using w as the witness, and $\mathcal{A}'$ externally interacts with the commit-then-PRZK prover in the first left-session; $\mathcal{A}'$ outputs a simulated view of $\mathcal{A}$ that is identical to the view of $\mathcal{A}$ in the experiment $\text{EXPT}(1^n, w, aux^b)$. By the adaptive one-left-many-right simulation-extractability of PRZK, the view of $\mathcal{A}$ and the corresponding witnesses encoded by the statistically-binding commitments under the run of $\mathcal{A}'(w, aux^b)$ are indistinguishable from the outputs of the PPT simulator/extractor guaranteed by PRZK, with auxiliary input (w, aux^b), in the commit(0)-then-simulatedPRZK experiment. As (w, aux^0) and (w, aux^1) are indistinguishable, we conclude that $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ in accordance with the experiment $\text{EXPT}(1^n, w, aux^0)$ and $\{(trans^1, \widehat{W}^1)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ in accordance with $\text{EXPT}(1^n, w, aux^1)$ are indistinguishable.

We return back to investigate the experiments: $\text{EXPT}(1^n, w^b, aux^b)$ with respect to many-left-many-right restricted input-selecting MIM adversary $\mathcal{A}$. Firstly, the distribution ensemble of $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ in accordance with $\text{EXPT}(1^n, w^0, aux^0)$ and the distribution ensemble of $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ *in accordance with* $\text{EXPT}(1^n, w^0, aux^1)$ are indistinguishable, if $\{aux_0\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ and $\{aux_1\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ are indistinguishable, where $\text{EXPT}(1^n, w^0, aux^1)$ denotes a hybrid experiment in which the CMIM adversary possesses auxiliary information aux^1 while concurrently interacting on the left with many prover instances of the fixed witness w^0. Then, by a simple hybrid argument to the one-left-many-right case, we get that the distribution ensemble $\{(trans^0, \widehat{W}^0)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ in accordance with $\text{EXPT}(1^n, w^0, aux^1)$ is indistinguishable from the distribution ensemble of $\{(trans^1, \widehat{W}^1)\}_{n\in N, w^0\in\{0,1\}^n, w^1\in\{0,1\}^n}$ in accordance with $\text{EXPT}(1^n, w^1, aux^1)$. In more detail, if the above ensembles are distinguishable, then the difference can be reduced, by hybrid arguments, to the difference of witnesses used in only one left session. Note that, all sessions other than the one left session can be emulated internally by a PPT algorithm given (w^0, w^1, aux^1). We remark that, here, posing the *restricted* input selection requirement in $\text{EXPT}(1^n, w^b, aux^b)$, i.e., the fixed w_0 and w_1 are always the valid witnesses to all statements set by the CMIM adversary for left-sessions, is critical for the above hybrid arguments to go through.

Proposition D.1 follows. □

Now, we return back to the experiment $E(1^n, z, PK_L, PK_R, s_b)$ for finishing the proof of Lemma D.2. We first prove that $\{\mathcal{C}_0\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ is indistinguishable from $\{\mathcal{C}_1\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ according to the analysis of Proposition D.1, where $\mathcal{C}_b$, $b \in \{0,1\}$, is the set of extracted-keys (corresponding to public-keys in $F - \{PK_R\}$) that is used by the simulator S in its last simulation repetition. Equivalently, $\mathcal{C}_b$ is generated by E_1 and is passed on to E_2. Note that $s_b = SK_R$ is the simulated secret-key used by S (equivalently, E_1). For presentation simplicity, in the following description we simply refer to S, E_1 and E_2 as $S(1^n, s_b)$, $E_1(1^n, s_b)$ and $E_2(1^n, \mathcal{C}_b)$. Actually, we can show that for any k, $1 \le k \le s(n)+1$, if the distribution ensemble of the set of extracted-keys used in the $(k-1)$-th simulation repetition of $S(1^n, s_0)$ using $SK_R = s_0$, denoted $\{\mathcal{C}_0^{k-1}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$, is indistinguishable from that of $\{\mathcal{C}_1^{k-1}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ (the set of extracted-keys used in the $(k-1)$-th simulation repetition of $S(1^n, s_1)$), then the distribution ensembles of $\{\mathcal{C}_0^k\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{\mathcal{C}_1^k\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are also indistinguishable.

We consider another PPT algorithm $\hat{S}(1^n, \mathcal{C}_b^{k-1})$ that mimics $E_2(1^n, \mathcal{C}_b^{k-1})$ (with externally interacting with the commit-then-PRZK prover $P(1^n, s_b)$) but with the following modifications: the interactions of Stage-1 of left-sessions and Stage-5 of right-sessions, in which the underlying CMIM adversary $\mathcal{A}$ serves as the prover of commit-then-PRZK, are relayed by $\hat{S}$ between the underlying CMIM adversary $\mathcal{A}$ and external commit-then-PRZK verifiers (who actually just send random coins, as PRZK is actually *public-coin*). We remark that the run of $\hat{S}(1^n, \mathcal{C}_b^{k-1})$ actually amounts to the experiment $\text{EXPT}(1^n, w^b, aux^b)$ defined in Proposition D.1, where $2s(n)$ amounts to $m(n)$ as $\hat{S}$ can involve at most $2s(n)$ sessions in each (left or right) CMIM interaction part, s_b amounts to w^b, $\mathcal{C}_b^{k-1}$ amounts to aux^b and the interactions with the at most $2s(n)$ instances of the commit-then-PRZK prover $P(1^n, s_b)$ amount to the left-sessions and the interactions between $\hat{S}$ (actually $\mathcal{A}$) and the at most $2s(n)$ instances of the commit-then-PRZK verifier amount to right-sessions. Here, a point of worthy noting is: though commit-then-PRZK is composed with other interactions (say, the interactions at Stage-2, Stage-3 and Stage-4), all interactions other than the interactions with the prover $P(s_b)$ of commit-then-PRZK (i.e., the left-sessions of $\hat{S}$) can be internally emulated by $\hat{S}$, though Stage-1 interactions of left-sessions and Stage-5 interactions of right-sessions (which correspond to the right-sessions of $\hat{S}$ and are just public coins) are not internally emulated by $\hat{S}$. By applying Proposition D.1, we get that if the ensembles $\{\mathcal{C}_0^{k-1}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{\mathcal{C}_1^{k-1}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are distinguishable, $\{\mathcal{C}_0^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{\mathcal{C}_1^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are also distinguishable. Finally, note that $\mathcal{C}_0^0$ and $\mathcal{C}_1^0$ (the set of extracted-keys corresponding to $F - \{PK_R\}$ at the beginning of the simulation) are identical, i.e., both of them are the empty set. By inductive steps, we get that the distribution ensembles of $\{\mathcal{C}_0^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{\mathcal{C}_1^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are indistinguishable for any k, $1 \leq k \leq s(n)+1$. *Here, we note that the above analysis (for showing the indistinguishability between* $\{\mathcal{C}_0^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ *and* $\{\mathcal{C}_1^{k}\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$*, for any* k*,* $1 \leq k \leq s(n)+1$*) works also for the case that* $\mathcal{C}_0^0 = \mathcal{C}_1^0 = \{s_\alpha\}$*, where* $\alpha \in \{0,1\}$ *and* $s_\alpha \in \{s_0, s_1\}$ *is a fixed value and is independent of* s_b*. This property will be used in the subsequent analysis of secret-key independence.*

But, suppose Case-2 of Case-R2 failure occurs with non-negligible probability. Then, with also non-negligible probability, the value committed to the statistically-binding commitment (at the beginning) of a (actually the first) successful right-session of commit-then-PRZK w.r.t. an uncovered public-key not in $\mathcal{C}_b \cup \{PK_L, PK_R\}$ under the run of the CMIM algorithm $\hat{S}$ (equivalently, $E_2(1^n, \mathcal{C}_b)$), with auxiliary input $\mathcal{C}_b$ where $b \in \{0,1\}$ and $\mathcal{C}_0$ and $\mathcal{C}_1$ are indistinguishable, is the preimage of y_b. Suppose this right-session is the i-th right-session run by $\hat{S}(1^n, s_b)$, $1 \leq i \leq 2s(n)$, it can be directly checked that, with overwhelming probability, the tag used by Stage-5 of this i-th right session, denoted $(PK_L^{(j)}, r_r^{(i)}, \tilde{r}^{(i)})$ where $PK_L^{(j)} \notin \mathcal{C}_b \cup \{PK_L, PK_R\}$ and $r_r^{(i)}$ is a random n-bit string, must be different from the tags used by the prover $P(1^n, s_b)$ of commit-then-PRZK. Recall that the tags of Stage-1 of right sessions (*run by* $P(s_b)$) is of the form $(\cdot, y_0, y_1)$ and the tags of Stage-5 of left sessions (*run by* $P(s_b)$) is of the form $(PK_L, \cdot, \cdot)$. This means that, by concurrently interacting with the prover $P(s_b)$ of commit-then-PRZK in left-sessions and with the commit-then-PRZK verifier instances in the right-sessions, $\hat{S}(1^n, \mathcal{C}_b)$ can successfully commit the preimage of y_b in a successful right session that is of a tag different from all the tags of the left-session interactions with $P(1^n, s_b)$ and is actually the first right-session w.r.t an uncovered public-key not in $\mathcal{C}_b \cup \{PK_R, PK_L\}$, which violates Proposition D.1. This shows that Case-2 of Case-R2 failure can occur also with negligible probability. Thus, Case-R2 failure can occur with at most negligible probability. This finishes the proof of Lemma D.2, from which the simulatability of the CNM simulation depicted in Figure 2 is then established. □

Next, before proceeding the analysis of the property of strategy-restricted and pre-definable randomness, we first investigate the property of secret-key independence which is essentially implied by the above analysis of Lemma D.2 and Proposition D.1.

- Secret-key independence

Specifically, we need to show that $\Pr[\mathcal{R}(SK_R, str, sta) = 1]$ is negligibly close to $\Pr[\mathcal{R}(SK'_R, str, sta) = 1]$ for any polynomial-time computable relation $\mathcal{R}$. In more details, for any pair (s_0, s_1) in the (simulated right-player) key-generation stage, denote by (str^b, sta^b) the output of $S(1^n, s_b)$ when it is using $SK_R = s_b$. Then, $\Pr[\mathcal{R}(SK, str, sta) = 1] = \frac{1}{2}\Pr[\mathcal{R}(s_0, str^0, sta^0) = 1 | S \text{ uses } SK_R =$

s_0 in generating $(str^0, sta^0)] + \frac{1}{2}\Pr[\mathcal{R}(s_1, str^1, sta^1) = 1|S$ uses $SK_R = s_1$ in generating $(str^1, sta^1)]$, and $\Pr[\mathcal{R}(SK'_R, str, sta) = 1] = \frac{1}{2}\Pr[\mathcal{R}(s_0, str^1, sta^1) = 1|S$ uses $SK_R = s_1$in generating $(str^1, sta^1)] + \frac{1}{2}\Pr[\mathcal{R}(s_1, str^0, sta^0) = 1|S$ uses $SK_R = s_0$ in generating $(str^0, sta^0)]$. Suppose the secret-key independence property does not hold, it implies that there exists a bit $\alpha \in \{0,1\}$ such that the difference between $\Pr[\mathcal{R}(s_\alpha, str^0, sta^0) = 1|S$ uses s_0 in generating $(str^0, sta^0)]$ and $\Pr[\mathcal{R}(s_\alpha, str^1, sta^1) = 1|S$ uses s_1 in generating $(str^1, sta^1)]$ is non-negligible. It implies that (s_α, str^0, sta^0) and (s_α, str^1, sta^1) are distinguishable. But, note that the analysis of Lemma D.2 and Proposition D.1 has already established that the distribution ensembles of $\{S(1^n, s_0) = (str^0, sta^0)\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{S(1^n, s_1) = (str^1, sta^1)\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are indistinguishable. Specifically, the distribution ensembles of the sets of extracted-keys corresponding to the public-keys in $F - \{PK_R\}$, $\{\mathcal{C}_0\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{\mathcal{C}_1\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ used by $S(1^n, s_b)$ for $b \in \{0,1\}$ in the last simulation repetition, are indistinguishable, and then the indistinguishability between the ensembles $\{(str^0, sta^0)\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ and $\{(str^1, sta^1)\}_{n\in N, s_0\in\{0,1\}^n, s_1\in\{0,1\}^n}$ are from Proposition D.1.

- Strategy-restricted and predefinable randomness

Now, we proceed to show the strategy-restricted and predefinable randomness property of the CNM simulator S depicted in Figure 2. Denote by $R_L = \{R_L^{(1)}, R_L^{(2)}, \cdots R_L^{(s(n))}\}$ the coin-tossing outputs of the $s(n)$ left sessions in str (i.e., the first output of S), and by $sta_L = \{sta_L^{(1)}, sta_L^{(2)}, \cdots, sta_L^{(s(n))}\}$ the state information corresponding to R_L included in sta (i.e., the second output of S). Similarly, denote by $R_R = \{R_R^{(1)}, R_R^{(2)}, \cdots R_R^{(s(n))}\}$ the coin-tossing outputs of the $s(n)$ right sessions in str, and by $sta_L = \{sta_R^{(1)}, sta_R^{(2)}, \cdots, sta_R^{(s(n))}\}$ the state information for R_R. We want to show that, with overwhelming probability, both the distribution of (R_L, sta_L) and that of (R_R, sta_R) are identical to that of $\mathcal{M}_{CRS}^{s(n)}(1^n)$. Recall that, $(\{r_1, r_2, \cdot, r_{s(n)}\}, \{\tau_{r_1}, \tau_{r_2}, \cdots, \tau_{r_{s(n)}}\}) \longleftarrow \mathcal{M}_{CRS}^{s(n)}(1^n)$ denotes the output of the experiment of running $\mathcal{M}_{CRS}(1^n)$ *independently* $s(n)$ times.

Note that, according to the CNM simulation described in Figure 2, for any i, $1 \le i \le s(n)$, the output of the i-th left session, i.e., $R_L^{(i)}$, in the simulation is always $S_L^{(i)}$ and $sta_L^{(i)}$ is always $\tau_L^{(i)}$, where $(S_L^{(i)}, \tau_L^{(i)})$ is the output of an independent run of $\mathcal{M}_{CRS}(1^n)$. It is directly followed that the distribution of (R_L, sta_L) is identical to that of $\mathcal{M}_{CRS}^{s(n)}(1^n)$.

The complicated point here is to show that, with overwhelming probability, the distribution of (R_R, sta_R) is also identical to that of $\mathcal{M}_{CRS}^{s(n)}(1^n)$. According to the CNM simulation depicted in Figure 2, if we can prove that, with overwhelming probability, for any i, $1 \le i \le s(n)$, the coin-tossing output of the successful i-th right session $R_R^{(i)}$ is either $S_R^{(i)}$ or $R_L^{(k)} = S_L^{(k)}$ for some k, $1 \le k \le s(n)$; furthermore, any left-session output $S_L^{(k)}$ can be the coin-tossing output for at most *one* successful right session (which implies the coin-tossing outputs of successful right sessions are independent), then the distribution of (R_R, sta_R) is also identical to that of $\mathcal{M}_{CRS}^{s(n)}(1^n)$. In the following description, for presentation simplicity, we sometimes omit some unlikely events occurring with negligible probability.

For any i, $1 \le i \le s(n)$, we consider the successful i-th right session with respect to a public-key $PK_L^{(j)}$. As we have shown that Case-R2 failure occurs with negligible probability, we get $PK_L^{(j)} \in \mathcal{C}_b \cup \{PK_R, PK_L\}$, where $\mathcal{C}_b$ is the set of extracted-keys (corresponding to public-keys in $F - \{PK_R\}$) used by $S(s_b)$ in its last simulation repetition.

We first observe that, if $PK_L^{(j)} = PK_L$ then with overwhelming probability the tag of Stage-5 of the successful i-th right session must be identical to that of Stage-5 of a left session simulated by the simulator S. Recall that all the Stage-5 tags of right sessions are different strings, as they contain random Stage-3 strings sent by the simulator. This means that Stage-5 tags of right sessions are also different from Stage-1 tags of right sessions simulated by S (note that all Stage-1 tags of right sessions consist of the fixed PR_R). Now, suppose the Stage-5 tag of the successful i-th right session is also different from the Stage-5 tags of all left sessions simulated by S, then it implies that the tag used by the CMIM adversary for Stage-5 of the i-th right session is different from all tags used by the simulator (particularly, the prover $P(s_b)$ of commit-then-PRZK run by $E_2(1^n, \mathcal{C}_b)$ or $\hat{S}(1^n, \mathcal{C}_b)$ in the analysis of Lemma D.2).

By the AOK property of PRZK, it implies that the value committed to $\tilde{c}_{crs}^{(i)}$ (sent by $\mathcal{A}$ in Stage-5 of the i-th right session) can be extracted. We consider the possibilities of the value committed to $\tilde{c}_{crs}^{(i)}$:

- By the one-wayness of y_{1-b} the value committed cannot be the preimage of y_{1-b};
- According to the analysis of Lemma D.2, the value also cannot be the preimage of y_b.

Thus, the value committed (that can be extracted) will be the secret-key of PK_L, which however violates the one-wayness of PK_L as the simulator never knows and uses the secret-key of PK_L in its simulation. Thus, we conclude that, if a successful right session is w.r.t. PK_L, the tag used by $\mathcal{A}$ for commit-then-PRZK of Stage-5 must be identical to that of one left-session simulated by S. As the Stage-5 tag consists of the coin-tossing output, i.e., the Stage-4 message, this means that the coin-tossing output of the i-th right session must be $R_L^{(k)} = S_L^{(k)}$ for some k, $1 \leq k \leq s(n)$.

Now, we consider the case $PK_L^{(j)} \neq PK_L$ but $PK_L^{(j)} \in \mathcal{C}_b \cup \{PK_R\}$. In this case, S has already learnt the corresponding secret-key $SK_L^{(j)}$. Now, suppose the coin-tossing output of the successful i-th right session is neither $S_R^{(i)}$ nor $R_L^{(k)} = S_L^{(k)}$ for all k, $1 \leq k \leq s(n)$. This implies that the Stage-5 tag used by $\mathcal{A}$ in the successful i-th right session is different from Stage-5 tags of all left sessions [5] as well as the Stage-1 tags of all right sessions simulated by S. Again, by the AOK property of PRZK, we consider the value committed to $\tilde{c}_{crs}^{(i)}$: According to the simulation of S, it always sets Stage-3 message $r_r^{(i)}$ of right session to be $PRF_{SK_L^{(j)}}(\tilde{r}_l^{(i)\prime}) \oplus S_R^{(i)}$, where $\tilde{r}_l^{(i)\prime}$ is the Stage-2 message of the i-th right session sent by the CMIM adversary $\mathcal{A}$. Suppose the coin-tossing output of the successful i-th right session is not $S_R^{(i)}$, then (by the AOK of PRZK) the value committed to $\tilde{c}_{crs}^{(i)}$ cannot be $SK_L^{(j)}$, as otherwise (with overwhelming probability) the $\mathcal{NP}$-statement to be proved by PRZK in Stage-5 of the i-th right session is false. This means that the value committed to $\tilde{c}_{crs}^{(i)}$ will be the preimage of either y_{1-b} or y_b. But, each case reaches the contradiction: committing to the preimage of y_{1-b} is impossible due to the one-wayness of y_{1-b}; committing to the preimage of y_b violates the one-left-many-right non-malleability of PRZK as demonstrated in the analysis of Lemma D.2. So, we conclude that, with overwhelming probability, for any successful right session the coin-tossing output is either the independent value $S_R^{(i)}$ or $S_L^{(k)}$ for some k, $1 \leq k \leq s(n)$ (i.e., the coin-tossing output of one left session).

To finally establish the property of strategy-restricted and predefinable randomness, we need to further show, for any $S_L^{(k)}$ it can occur as Stage-4 message (i.e., the coin-tossing output) for *at most one successful right session*. Suppose there are i_0, i_1, $1 \leq i_0 \neq i_1 \leq s(n)$, such that both of the i_0-th right session and the i_1-th right session are successful with the same Stage-4 message $S_L^{(k)}$. Recall that the Stage-5 tag of each of the two right sessions includes the same $S_L^{(k)}$ as well as a random Stage-3 message sent by the simulator; Also note that the $S_L^{(k)}$ can appear as a part of Stage-5 tag, as well as coin-tossing output, for at most one left session, as all coin-tossing outputs (i.e., Stage-4 messages) of left sessions are independent random strings output by $\mathcal{M}_{CRS}$. This implies that, with overwhelming probability, there must exist a bit b such that the Stage-5 tag of the i_b-th right session is different from all Stage-5 tags of left sessions (run by the simulator) and Stage-1 tags of right sessions (run by the simulator). According to above clarifications and analysis, with overwhelming probability, the (left-player) public-key $PK_L^{(j)}$ used by $\mathcal{A}$ in the i_b-th successful right session is *covered* and is *not* PK_L (as any right-session w.r.t. PK_L is of tag identical to that of one left-session), and the value committed in $\tilde{c}_{crs}^{(i_b)}$ is neither the secret-key of the *covered* public-key $PK_L^{(j)}$ (as, otherwise, the $\mathcal{NP}$-statement successfully proved by PRZK in the Stage-5 of the i_b-th right-session is actually false) nor the preimage of y_{1-b} (due to the one-wayness of f); Also, the value committed cannot be the preimage of y_b in accordance with the analysis of Lemma D.2. Contradiction is reached in either case.

The proof of Theorem 4.1 is finished. □

On extension to multiple public-keys input to the key-generation stage of $\mathcal{A}$. For the general case of multiple public-keys input to the key generation stage of $\mathcal{A}$, in the definition

[5] Note that all Stage-5 tags of left sessions are of the form $(PK_L, \cdot, \cdot)$, and the Stage-5 tag of the successful i-th right session is of the form $(PK_L^{(j)}, \cdot, \cdot)$ for $PK_L^{(j)} \neq PK_L$.

of CNMCT each of the values (PK_L, SK_L) and (PK_R, SK_R, SK'_R) will be changed to be a vector $(\overline{PK}_L = \{PK_L^{(1)}, \cdots, PK_L^{(k)}\}, \overline{SK}_L = \{SK_L^{(1)}, \cdots, SK_L^{(k)}\}$ and $((\overline{PK}_R = \{PK_R^{(1)}, \cdots, PK_R^{(k)}\}, \overline{SK}_R = \{SK_R^{(1)}, \cdots, SK_R^{(k)}\}, \overline{SK}'_R = \{SK_R^{(1)\prime}, \cdots, SK_R^{(k)\prime}\})$, where k is polynomial in n. Here, for presentation simplicity, we have assumed there are equal number of honest left and right players. We remark that, as the simulator does not know the secret-keys of honest left-players and only simulates key-generations of honest right-players, we do not need to take secret-keys of honest left-players into account in the formulation of secret-key independence. We note that the proof of Theorem 4.1 can be easily extended to this general case.

The proof of Lemma D.1 can be extended to this general case by simple hybrid arguments. Specifically, we consider the two differences dealt with by the proof Lemma D.1 in this general case:

- Truly random vs. pseudorandom Stage-4 messages: For the general case, we need to show, with respect to the hybrid experiment $\mathcal{H}$ in the proof of Lemma D.1, that given $\overline{PK_L} = \{PK_L^{(1)} = C(\sigma_1, s_{\sigma_1}), \cdots, PK_L^{(k)} = C(\sigma_k, s_{\sigma_k})\}$, no efficient algorithm A can distinguish the interactions with a vector of PRFs $(PRF_{\sigma_1}, \cdots, PRF_{\sigma_k})$ or a vector of truly random functions $(H_1, \cdots, H_k)$. Suppose it is not the case, we consider a series of hybrid experiment $\mathcal{H}_1, \cdots, \mathcal{H}_k$, where in $\mathcal{H}_i$, $1 \leq i \leq k$, an efficient algorithm A_i interacts with a vector $(PRF_{\sigma_1}, \cdots, PRF_{\sigma_i}, H_{i+1}, \cdots, H_k)$. By hybrid argument, there must exist an i, such that the output of A_i and A_{i+1} is distinguishable. This shows that, given $\overline{PK_L} = \{PK_L^{(1)}, \cdots, PK_L^{(k)})\}$, there exists an efficient algorithm A', which encodes $(\sigma_1, \cdots, \sigma_i)$ and runs A_i as a subroutine, can distinguish the interactions with $PRF_{\sigma_{i+1}}$ or a truly random function, which however in turn violates the computational hiding property of C as shown in the proof of Lemma D.1.

- Witness difference of Stage-5 of left sessions: Specifically, for the general case, for any i-th left session specified by an *honest* left-player public-key $PK_L^{(i)}$ and a covered right-player public-key $PK_R^{(j)} \in \mathcal{S}$, the witness used by S in the commit-then-PRZK of Stage-5 is always the extracted secret-key $SK_R^{(j)}$, while the witness used by the honest left-player is always its secret-key SK_L^i. The analysis of Lemma D.1 can be straightforwardly extended to this general case, by the regular WI property of commit-then-PRZK which holds under concurrent self composition.

Now, we consider the proof of Lemma D.2 in the general case. In the general case, the simulator S simulates a vector of right-player public-key and secret-key pairs $\{(SK_R^{(1)}, SK_R^{(1)\prime}), \cdots, (SK_R^{(k)}, SK_R^{(k\prime)})\}$, and only uses the vector $\overline{SK}_R = \{SK_R^{(1)}, \cdots, SK_R^{(k)}\}$ in its simulation. The Case-R2 failure in this general case says that the witness extracted for a successful right session, specified by an uncovered public-key $PK_L^{(j)}$ and honest right-player public-key $PK_R^{(i)}$, is not $SK_L^{(j)}$ but $SK_R^{(i)}$ (note that according to the one-wayness of f, the extracted witness cannot be $SK_R^{(i)\prime}$). That Case-R2 failure occurs with negligible probability in the general case is from the following observation on an extended experiment of commit-then-PRZK with restricted input selection and indistinguishable auxiliary inputs.

Consider the following experiments: $\text{EXPT}(1^n, \overline{w}^b, aux^b)$, where $\overline{w}^b = (w_1^b, \cdots, w_k^b) \in (\{0,1\}^n)^k$ for $b \in \{0,1\}$. Here, one of $(\overline{w}^0, \overline{w}^1)$ corresponds to $\overline{SK}_R$, and one corresponds to $\overline{SK}'_R$ in the simulation of S. In $\text{EXPT}(1^n, \overline{w}^b, aux^b)$, the commit-then-PRZK for $\mathcal{NP}$ is run concurrently, and a many-left-many-right CMIM adversary $\mathcal{A}$, possessing auxiliary information aux^b and involving at most $m(n)$ left-sessions and at most $m(n)$ right-sessions simultaneously, can set the inputs and tags to prover instances of left sessions with the following restriction: for any x_i, $1 \leq i \leq m(n)$, set by $\mathcal{A}$ for the i-th left session of commit-then-PRZK, there always exists a component $w_j^b \in \overline{w}^b$ for some j, $1 \leq j \leq k$ (different x_i may correspond to different w_j^b), such that w_j^b is a valid $\mathcal{NP}$-witness for x_i. In the simulation of S, for any statement involving the simulated public-key $PK_R^{(i)}$, the secret-key $SK^{(i)} \in \overline{SK}_R$ used by S is always a valid $\mathcal{NP}$-witness. Straightforward investigation shows that Proposition D.1 still holds with respect to the (extended) experiment $\text{EXPT}(1^n, \overline{w}^b, aux^b)$. This establishes the simulatability property in the general case, from which the properties of secret-key independence, and strategy-restricted and predefinable randomness, in the general case are also straightforwardly derived.