Practical Frameworks For h-Out-Of-n Oblivious Transfer With Security Against Covert and Malicious Adversaries

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Abstract—We present two practical frameworks for h-out-of-n oblivious transfer (OT_h^n). The first one is secure against covert adversaries who are not always willing to cheat at any price. The security is proven under the ideal/real simulation paradigm (we call such security fully-simulatable security against covert adversaries). The second one is secure against malicious adversaries who are always willing to cheat. It provides privacy and fully-simulatable security for the receiver and the sender respectively (we call such security half-simulatable security). The two frameworks can be implemented from the decisional Diffie-Hellman (DDH) assumption, the decisional N-th residuosity assumption, the decisional quadratic residuosity assumption and so on.

The DDH-based instantiation of our first framework, compared with existing practical protocols for oblivious transfer with fully-simulatable security against covert adversaries or malicious adversaries, costs the minimum communication rounds and the minimum computational overhead.

Though our second framework, compared with existing practical protocols with half-simulatable security against malicious adversaries, is not efficient. However, it first provides a way to deal with general OT_h^n on this security level. What is more, its DDH-based instantiation is more efficient than the existing practical protocols for oblivious transfer with standard security against malicious adversaries.

Index Terms—oblivious transfer (OT) protocols

1 Introduction

1.1 Oblivious transfer

Oblivious transfer (OT), first introduced by [40] and later defined in another way with equivalent effect [15] by [16], is a fundamental primitive in cryptography and a concrete problem in the filed of secure multi-party computation. Considerable cryptographic protocols can be built from it. Most remarkable, [24], [28], [30], [42] proves that any secure multi-party computation can be based on a secure oblivious transfer protocol. In this paper, we concern a variant of OT, i.e., h-out-of-n oblivious transfer (OT_h^n). OT_h^n deals with the following scenario. A sender holds n private messages, $m_1, m_2, \ldots, m_n$. A receiver holds h private positive integers $i_1, i_2, \ldots, i_h$, where $1 \leq i_1 < i_2 < \ldots < i_h \leqslant n$. The receiver expects to get the messages $m_{i_1}, m_{i_2}, \ldots, m_{i_h}$ without leaking any information about his private input, i.e., the h positive integers he holds. The sender expects all new knowledge learned by the receiver from their interaction is at most h messages. Obviously, OT that most literature refer to is OT_1^2 and can be viewed as a special case of OT_h^n.

It is known that, by using Goldreich's compiler [22], [24], we can gain a protocol for OT_h^n with security against malicious adversaries (a malicious adversary act in any arbitrary malicious way to learn as much extra information as possible) from a protocol for OT_h^n with security against semi-honest adversaries (a semi-honest adversary, on one side, honestly does everything told by a prescribed protocol; on the other side, records the messages he sees to deduce extra information which is not supposed to be known to him), which can be built from any collection of enhanced trapdoor permutations [22]. The security of the resulting protocol can be proven under the real/ideal simulation paradigm and so we say this security is fully-simulatable. The paradigm requires that for any adversary in the real world, there exists a corresponding adversary in the ideal world who can simulate him, where the ideal world holds a desirable security level. That is, for any malicious adversary, he can not do more harm in the real world than in the ideal world, which implies that the proven protocol is secure. However, the resulting protocol for OT_h^n mentioned above is prohibitively expensive for practical use, because it is embedded with so many invocations of zero-knowledge proof for NP and invocations of commitment scheme. Thus, directly constructing the protocol based on specific intractable assumptions seems more feasible.

1.2 Our Contribution

In this paper, we aim to construct practical and efficient frameworks for OT_h^n which can be instantiated from the decisional Diffie-Hellman (DDH) assumption, the decisional N-th residuosity (DNR) assumption, the decisional quadratic residuosity (DQR) assumption and so on. To this end, compared with the fully-simulatable security against malicious adversaries mentioned above

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— called standard security from this time on for simplicity, the security we guarantee is relaxed to some extent. Specifically, we construct two frameworks with the following two security level respectively.

- Fully-simulatable security against covert adversaries. This security, presented by [3], guarantees that if an adversary commits an effective cheating, the adversary can be caught with probability at least ϵ, where ϵ is called the deterrence factor. By effective cheating, we refer to a cheating in which it is possible that an adversary learns extra new knowledge. The security definition follows the real/ideal simulation paradigm and so is said to be fully-simulatable too. Therefore, compared with the standard security, the only relaxed point is that it does not require to rule out any possibility that the adversaries learns any extra knowledge in an execution of a protocol. In settings that the malicious adversaries are not always willing to cheat at any price, this security makes sense. For simplicity, we refer to this security as security against covert adversaries.

  In our first framework, if an adversary is caught in committing an effective cheating, he will learn no extra knowledge. What is more, the deterrence factor can be very close to 1, e.g., $\epsilon = 1 - 6.45 \times 10^{-9}$.
- Half-simulatable security against malicious adversaries. This security, also called security based on one-sided simulation by [26] and called security based on half-simulation by [7], guarantees that, in the case that one party is corrupted, it provides privacy to the honest party; in the case that another party is corrupted, it provides standard security to the honest party. Specifically, in the case that one party is corrupted, the security only requires to prove that the adversary controlling the corrupted party can't distinguish his views obtained in the distinct interactions with the honest party who uses distinct private inputs; in the case that another party is corrupted, the security proof requires the same as the standard security does. Therefore, compared with the standard security, the properties correctness and the independence of private inputs are not guaranteed any more in this security. However, this security indeed is useful in the computation in which only one party receives an output. For example, [17], [26], [36] take this security definition to design efficient protocols. In these works, the party receiving an output is provided with the privacy, and the other party is provided with the fully-simulatable security. In our second framework, this is also the case.

To our best knowledge, our first framework is the first known framework for OT_h^n with security against covert adversaries. Though a framework for OT_1^2 with half-simulatable security against malicious adversaries is given by [29], to our best knowledge, our second framework is the first known framework for general OT_h^n with this security level.

Our two frameworks have the following features.

First, practical. On one side, the securities of our frameworks are obtained without resorting to a random oracle. Though the schemes presented by [27], [38] are efficient, their security is proven under the Random Oracle Model. However, the Random Oracle Model seems risky. [10] shows that a scheme which is secure in the Random Oracle Model does not necessarily imply that a particular implementation of it (in the real world) is secure, or even that this scheme does not have any "structural flaws". [10] also shows an efficient implementation of the random oracle is impossible. Later, [31] finds that the random oracle instantiations proposed by Bellare and Rogaway from 1993 and 1996, and the ones implicit in IEEE P1363 and PKCS standards are weaker than a random oracle. What is worse, [31] shows that how the defects of the random oracle instantiations deadly damage the securities of the cryptographic schemes designed by [5], [6]. Therefore, considering practical use, our frameworks are better.

On the other side, the securities of our frameworks are obtained without resorting to a trusted common reference string (CRS). Though the framework for OT_1^2 presented by [39] and its adaptive instantiations presented by [19] hold higher security level that is secure against malicious adversaries even under universal composition, they don't work without a trusted CRS. How to provide a trusted CRS before the protocol running still is a unsolved problem. The known possible solutions, such as natural process suggested by [39], are only conjectures without formal proofs. What is worse, [9], [11] show that even given a authenticated communication channel, it is impossible to implement a universally composable protocol providing useful trusted CRS in the presence of malicious adversaries. Therefore, considering practical use, our frameworks are better.

Second, efficient. Our first framework costs four communication rounds. Fixing the deterrence factor to be $\frac{1}{2}$, this framework costs n public key encryption operations, h public key decryption operations. Compared with the existing practical protocols for OT with standard security or with security against covert adversaries, i.e., the protocols presented by [3], [7], [25], [32], [43], the DDH-based instantiation of our first framework is the most efficient one in the sense that it costs the minimum number of communication rounds and costs the minimum computational overhead. Please see Section 3.4 and Section 3.5 for the detailed comparisons.

Our second framework costs four communication rounds too. To be securely used in practice, our framework, in the worst case, costs the sender $40n$ public key encryption operations and costs the receiver $40h$ public key decryption operations. Compared with the existing practical protocols for OT with half-simulatable security against malicious adversaries, i.e., the protocols presented by [2], [29], [34]–[37], our second framework is

not efficient. However, it still makes sense since it does not require k is far less than n and is the first framework/protocol dealing with general OT_k^n on this security level. Compared with the existing practical protocols for OT with standard security, i.e., the protocols presented by [7], [25], [32], [43], the DDH-based instantiation of our second framework is the most efficient one in the sense mentioned above. Please see Section 4.3 and Section 4.4 for the details.

Third, abstract and modular. The framework is described using only one high-level cryptographic tool, i.e., a variant of smooth projective hash provided by [43] (for simplicity, denoted it by $SPHDHC_{t,h}$). This allows a simple and intuitive understanding of its security.

Fourth, generally realizable. [43] shows that $SPHDHC_{t,h}$ are realizable from a variety of specific assumptions, e.g., DDH, DNR, DQR. We remark that $SPHDHC_{t,h}$ also is possible to be realized from future assumptions. This makes our frameworks generally realizable. Considering the future progress in breaking a specific intractable problem, general realizability is vital to make frameworks live long. Because it suffices that replacing the instantiation based on the broken problem with that based on a unbroken problem.

1.3 Construction Overview

Our first framework is described with high-level as follows.

1) Let K, g be two predetermined positive integers. The receiver generates hash parameters and K instance vectors, then sends them to the sender after disordering each vector.
2) The receiver chooses g instance vectors at random to open.
3) The receiver opens the chosen instances and encodes his private input by reordering each unchosen vector.
4) The sender checks that the chosen vectors are generated in the legal way which guarantees that the receiver learns at most h message. If the check pass, the sender encrypts his private input (i.e., the n messages he holds) using the hash values of the instances of the unchosen vectors in the way indicated by the code of receiver's private input, and sends the ciphertexts together with some auxiliary information (i.e., the projective hash keys) to the receiver.
5) The receiver decrypts the ciphertexts with the help of the auxiliary information and gains the messages he expects.

Intuitively speaking, the receiver's security is implied by the property hard subset membership of $SPHDHC_{t,h}$. This property guarantees that the receiver can securely encode his private input by reordering each unchosen instance vector. The sender's security is implied by the cut-and-choose technique, which guarantees that the probability that the adversary controlling a corrupted receiver learns extra new knowledge is small enough.

Our second framework is obtained by modifying the first framework by letting K be a polynomial in security parameter and granting the sender to uniformly choose any instance vectors to open.

From some point of view, our frameworks and the framework given in [29] look alike, because all these frameworks use the tool smooth projective hash. However, in fact, our frameworks are different from that of [29] very much. First, the variants of smooth projective hash used by we and [29] are distinct. Second, we use cut-and-choose to guarantee the sender's security rather than turn to a property of a hash tool of some kind. Third, both our frameworks deal with general OT_h^n, and in particular, the security of our first framework is fully-simulatable, in contrast to the framework given in [29] only deals with OT_1^2 and its security is half-simulatable.

1.4 Related Work

The first step to construct practical and efficient protocol for OT is independently made by [36] and [2]. The protocols are for OT_1^2 and based on the DDH assumption. Later, using the tool smooth projective hash, [29] generalizes the ideas of the two previous works and presents a framework for OT_1^2. Besides the DDH assumption, [29] shows the framework can be instantiated from the DNR assumption or the DQR assumption.

There are some work aiming to build protocols for OT_h^n from the protocols for OT_1^2. [34] shows how to implement OT_h^n using $\log n$ invocations of OT_1^2. A similar implementation for adaptive OT_h^n can be seen in [35]. These works mentioned are later described in [37]. With the help of a random oracle, [27] shows how to extend k oblivious transfers (for some security parameter k) into many more, without much additional effort. [38] proposes an efficient adaptive h-out-n oblivious transfer (denoted by $OT_{h\times 1}^n$ in related literature) schemes under Random Oracle Model.

All works previously referred to are with half-simulatable security against malicious adversaries. The first protocols that is with standard security is presented by [7]. This protocol is for $OT_{k\times 1}^n$ and based on non-standard assumptions, i.e., the q-Power Decisional Diffie-Hellman assumption and the q-Strong Diffie-Hellman assumption. Using a blind identity-based encryption, [25] presents the first protocol that is based on a standard assumption and has standard security. In a little more detail, this protocol is for OT_h^n. Later, [32] presents a DDH-based protocol for OT_1^2 with standard security whose efficiency is over that of the two mentioned works. Recently, [43] presents a framework for OT_h^n with standard security, which also can be instantiated from all the assumptions used by [29], [32]. The DDH-based instantiation of this framework is the most efficient one of the protocols that work without appealing to for a random oracle or a set-up assumption, e.g., a trusted CRS.

Under CRS model, [39] presents a framework for OT_1^2 with higher security level that security is preserved even other arbitrary malicious protocols concurrently run with it. [39] shows that the framework can be instantiated from the DDH, DQR and worst-case lattice assumption. We remark that if a trusted CRS is available, the DDH-based instantiation of the framework is the most efficient protocol for OT_1^2. Later [19] strengthen the instantiation based on DDH and that based on DQR to be secure against adaptive malicious adversaries.

Using homomorphic encryption, [3] first presents a protocol for OT_1^2 with security against covert adversaries.

1.5 Organization

In Section 2, we describe the notations used in this paper, the security definitions of OT_h^n and a variant of smooth projective hash which is the basic tool we use. In Section 3, we construct the first framework for OT_h^n, prove its security and analyze its performance. In Section 4, we construct the second framework for OT_h^n, prove its security and analyze its performance. In Section **??**, we discuss the application of technique of cut-and-choose in these two frameworks.

2 PRELIMINARIES

2.1 Basic Notations

We denote an unspecified positive polynomial by $poly(.)$. We denote the set consists of all natural numbers by $\mathbb{N}$. For any $i \in \mathbb{N}$, $[i] \stackrel{def}{=} \{1, 2, \ldots, i\}$.

We denote security parameter used to measure security and complexity by k. A function $\mu(.)$ is negligible in k, if there exists a positive constant integer n_0, for any $poly(.)$ and any k which is greater than n_0 (for simplicity, we later call such k sufficiently large k), it holds that $\mu(k) < 1/poly(k)$. A probability ensemble $X \stackrel{def}{=} \{X(1^k, a)\}_{k\in\mathbb{N}, a\in\{0,1\}^*}$ is an infinite sequence of random variables indexed by (k, a), where a represents various types of inputs used to sample the instances according to the distribution of the random variable $X(1^k, a)$. Probability ensemble X is polynomial-time constructible, if there exists a probabilistic polynomial-time (PPT) sample algorithm $S_X(.)$ such that for any a, any k, the random variables $S_X(1^k, a)$ and $X(1^k, a)$ are identically distributed. We denote sampling an instance according to $X(1^k, a)$ by $\alpha \leftarrow S_X(1^k, a)$.

Let $X \stackrel{def}{=} \{X(1^k, a)\}_{k\in\mathbb{N}, a\in\{0,1\}^*}$ and $Y \stackrel{def}{=} \{Y(1^k, a)\}_{k\in\mathbb{N}, a\in\{0,1\}^*}$ be two probability ensembles. They are computationally indistinguishable, denoted by $X \stackrel{c}{=} Y$, if for any non-uniform PPT algorithm D with an infinite auxiliary information sequence $z = (z_k)_{k\in\mathbb{N}}$ (where each $z_k \in \{0,1\}^*$), there exists a negligible function $\mu(.)$ such that for any sufficiently large k, any a, it holds that

$$|Pr(D(1^k, X(1^k, a), a, z_k) = 1) - Pr(D(1^k, Y(1^k, a), a, z_k) = 1)| \leqslant \mu(k)$$

They are the same, denoted by $X = Y$, if for any sufficiently large k, any a, $X(1^k, a)$ and $Y(1^k, a)$ are defined in the same way. They are equal, denoted by $X \equiv Y$, if for any sufficiently large k, any a, the distributions of $X(1^k, a)$ and $Y(1^k, a)$ are identical. Obviously, $X = Y$ implies $X \equiv Y$ and $X \equiv Y$ implies $X \stackrel{c}{=} Y$.

Let $\vec{x}$ be a vector (note that arbitrary binary string can be viewed as a vector). We denote its i-th element by $\vec{x}\langle i\rangle$, denote its dimensionality by $\#\vec{x}$, denote its length in bits by $|\vec{x}|$. For any positive integers set I, any vector $\vec{x}$, $\vec{x}\langle I\rangle \stackrel{def}{=} (\vec{x}\langle i\rangle)_{i\in I, i\leq\#\vec{x}}$.

Let M be a probabilistic (interactive) Turing machine. By $M_r(.)$ we denote the output of M in an execution using randomness r.

Let $f : D \to R$. Let $D' \subseteq \{0,1\}^*$. Then $f(D') \stackrel{def}{=} \{f(x)|x \in D' \cap D\}$, $Range(f) \stackrel{def}{=} f(D)$.

Let $x \in_\chi Y$ denote sampling an instance x from domain Y according to a distribution law or a probability density function χ. Specifically, let $x \in_U Y$ denotes uniformly sampling an instance x from domain Y.

2.2 Security Against Covert Adversaries

Security against covert adversaries is presented by [3]. The key observation of this security is that in many real-world setting, such as business, political settings and playing remote games, it is overly pessimistic and unnecessary to assume that the adversaries are always ready to cheat at any price, which is assumed in traditional model of secure multiparty computation, e.g., [8], [22]. Therefore, it seems unnecessary that using a great amount of resource to preclude such covert adversaries from committing cheating. Instead, this security aims at catching the cheating of the covert adversaries with some probability, called deterrence factor and denoted by ε, rather than aims at eliminating any probability that the covert adversaries succeed in effective cheating, i.e., learn more new knowledge than that they deserve. [3] presents three versions of security against covert adversaries. In this paper, we take the strongest one, which implies the other two versions. For clarity and simplicity, we tailor it to the need of dealing with OT_h^n.

[3] shows that if $1 - \varepsilon$ is negligible, this security turns to be the standard security, i.e., fully-simulatable security against malicious adversaries; if $\varepsilon \geq 1/poly(k)$, this security implies fully-simulatable security against semi-honest adversaries. It is easy to deduce that, if $\varepsilon \in (0, 1)$ is a constant value — our first framework is this case, this security also implies fully-simulatable security against semi-honest adversaries.

2.2.1 Functionality Of OT_h^n

OT_h^n involves two parties, party P_1 (i.e., the sender) and party P_2 (i.e., the receiver). OT_h^n's functionality is formally defined as follows

$$\begin{aligned} f : \mathbb{N} \times \{0,1\}^* \times \{0,1\}^* &\rightarrow \{0,1\}^* \times \{0,1\}^* \\ f(1^k, \vec{m}, H) &= (\lambda, \vec{m}\langle H\rangle) \end{aligned}$$

and also be denoted by

$$(\vec{m}, H) \mapsto (\lambda, \vec{m}\langle H\rangle)$$

for simplicity, where

- k is the public security parameter.
- $\vec{m} \in (\{0,1\}^*)^n$ is P_1's private input, and each $|\vec{m}\langle i\rangle|$ is the same.
- $H \in \Psi \stackrel{def}{=} \{B | B \subseteq [n], \#B = h\}$ is P_2's private input.
- λ denotes an empty string and is supposed to be got by P_1. That is, P_1 is supposed to get nothing.
- $\vec{m}\langle H\rangle$ is supposed to be got by P_2.

Note that, the length of all parties' private input has to be identical in SMPC (please see [22] for the reason and related discussion). This means that $|\vec{m}| = |H|$ is required. Without loss of generality, in this paper, we assume $|\vec{m}| = |H|$ always holds, because padding can be easily used to meet this requirement.

Intuitively speaking, the security of OT_h^n requires that P_1 can't learn any new knowledge — typically, P_2's private input — from the interaction at all, and P_2 can't learn more than h messages held by P_1. To capture the security in a formal way, the concepts such as adversary, trusted third party, the ideal world and the real world were introduced. Since the security level here is to be secure against non-adaptive covert adversaries, we only refer to concepts related to this case.

2.2.2 Non-Adaptive Covert Adversary

Before engaging a protocol, a non-adaptive covert adversary $\mathcal{A}$ has to corrupt all parties listed in $I \subseteq [2]$. If $P_i \in \{P_1, P_2\}$ is not corrupted, P_i will strictly follow the prescribed protocol as an honest party. If party P_i is corrupted, P_i will be fully controlled by $\mathcal{A}$ as a corrupted party. In this case, P_i will have to pass all his knowledge to $\mathcal{A}$ before the execution of the protocol and follows $\mathcal{A}$'s instructions from then on — so there is a probability that P_i arbitrarily deviates from the prescribed protocol. In fact, after $\mathcal{A}$ finishing corruption, $\mathcal{A}$ and all corrupted parties have formed a coalition led by $\mathcal{A}$ to learn as much extra new knowledge, e.g. the honest parties' private inputs, as possible at acceptable price. Hereafter, they share knowledge with each other and coordinate their behavior. Without loss of generality, we can view this coalition as follows. All corrupted parties are dummy; $\mathcal{A}$ receives messages addressed to the members of the coalition and sends messages on behalf of the members.

2.2.3 OT_h^n In the ideal world

In the ideal world, there is an incorruptible trusted third party (TTP). All parties hand their private inputs to TTP. TTP computes f and sends back $f(.)\langle i\rangle$ to P_i. An execution of OT_h^n with deterrence factor $\epsilon \in (0,1]$ proceeds as follows.

1) Initial Inputs. All entities know the public security parameter k. Party P_1 holds a private input $\vec{m} \in (\{0,1\}^*)^n$. Party P_2 holds a private input $H \in \Psi$. Adversary $\mathcal{A}$ holds a name list $I \subseteq [2]$, a randomness $r_{\mathcal{A}} \in \{0,1\}^*$ and an infinite auxiliary input sequence $z = (z_k)_{k\in\mathbb{N}}$, where $z_k \in \{0,1\}^*$. Before proceeding to the next stage, $\mathcal{A}$ corrupts parties listed in I and learns $\vec{x}\langle I\rangle$, where $\vec{x} \stackrel{def}{=} (\vec{m}, H)$.
2) The parties submitting inputs to TTP. Each honest party P_i submits $\vec{x}\langle i\rangle$ to TTP. $\mathcal{A}$ submits an arbitrary string based on his knowledge to TTP for corrupted parties. The inputs TTP receives, denoted by $\vec{y}$, can be described as follows.

$$\vec{y}\langle i\rangle = \begin{cases} \vec{x}\langle i\rangle & \text{if } i \notin I \ , \\ \alpha_i & \text{if } i \in I. \end{cases}$$

where

$$\alpha_i \leftarrow \mathcal{A}(1^k, I, r_{\mathcal{A}}, z_k, \vec{x}\langle I\rangle, i)$$

$$\alpha_i \in \{\vec{x}\langle i\rangle\} \cup \{0,1\}^{|\vec{x}\langle i\rangle|} \cup \{abort_i\} \cup \{corrupted_i\} \cup \{cheat_i\}$$

Obviously, there is a probability that $\vec{x} \neq \vec{y}$.
3) TTP computing f. TTP checks $\vec{y}$ and takes the following actions in order.
   a) In case $\exists i(\vec{y}\langle i\rangle = abort_i)$. For $i \in I$, $\vec{w}\langle i\rangle \leftarrow \lambda$; for $i \in [n] - I$, $\vec{w}\langle i\rangle \leftarrow abort_i$. If there are multiple $abort_i$s, TTP chooses the one whose i is the smallest. TTP deals with multiple $corrupted_i$s and $cheat_i$s, which will meet later, in a similar way.
   b) In case $\exists i(\vec{y}\langle i\rangle = corrupted_i)$. For $i \in I$, $\vec{w}\langle i\rangle \leftarrow \lambda$; for $i \in [n] - I$, $\vec{w}\langle i\rangle \leftarrow corrupted_i$.
   c) In case $\exists i(\vec{y}\langle i\rangle = cheat_i)$. TTP does nothing.
   d) In other cases, $\vec{w} \leftarrow f(1^k, \vec{y})$.
4) TTP delivering results to the parties.
   a) In case $\exists i(\vec{y}\langle i\rangle = abort_i)$ or in case $\exists i(\vec{y}\langle i\rangle = corrupted_i)$. TTP sends each $\vec{w}\langle i\rangle$ to each honest party $P_i(i \in [n] - I)$. Finally, TTP halts.
   b) In case $\exists i(\vec{y}\langle i\rangle = cheat_i)$.
      - With probability ϵ, TTP sends $corrupted_i$ to $\mathcal{A}$ and honest parties. That is, TTP sets $\vec{w} \leftarrow (corrupted_i, corrupted_i)$. TTP sends each $\vec{w}\langle i\rangle$ to each honest party $P_i(i \in [n]-I)$, sends $\vec{w}\langle I\rangle$ to $\mathcal{A}$. Finally, TTP halts.
      - With probability $1 - \epsilon$, TTP sets $\vec{w}\langle I\rangle \leftarrow (\vec{x}\langle [n]-I\rangle, undetected)$ and sends $\vec{w}\langle I\rangle$ to $\mathcal{A}$. On receiving $\vec{w}\langle I\rangle$, $\mathcal{A}$ determines the results the honest parties will receive as follows.

$$c_j \leftarrow A(1^k, I, r_{\mathcal{A}}, z_k, \vec{x}\langle I\rangle, \vec{w}\langle I\rangle, j)$$

$\mathcal{A}$ sends them to TTP.
For each $j \in [n]-I$, TTP sets $\vec{w}\langle j\rangle \leftarrow c_j$ and sends $\vec{w}\langle j\rangle$ to P_j. Finally, TTP halts.

We remark that the inputs of honest parties are leaked to $\mathcal{A}$ only if the cheating committed by $\mathcal{A}$ is not detected.

c) In other cases, TTP first sends back the corrupted parties' results, i.e., sends $\vec{w}\langle I\rangle$ to $\mathcal{A}$. $\mathcal{A}$ computes

$$\beta \leftarrow A(1^k, I, r_{\mathcal{A}}, z_k, \vec{x}\langle I\rangle, \vec{w}\langle I\rangle)$$
$$\beta \in \{abor_i | i \in I\} \cup \{continue\}$$

and sends β to TTP.
TTP checks β and if $\beta \in \{abor_i | i \in I\}$, for each $i \in [n]-I$, TTP sets $\vec{w}\langle i\rangle \leftarrow \beta$. TTP sends each $\vec{w}\langle i\rangle (i \in [n]-I)$ to each honest party P_i. Finally, TTP halts.

5) Outputs. Each honest party P_i outputs $\vec{w}\langle i\rangle$. Each corrupted party outputs nothing (i.e., λ). The adversary outputs something generated by executing an arbitrary function of the information he gathers in the above stages. Without loss of generality, this can be assumed to be $(1^k, I, r_{\mathcal{A}}, z_k, \vec{x}\langle I\rangle, \vec{w}\langle I\rangle)$.

The whole ideal execution is denoted by $Ideal^{\epsilon}_{f,\mathcal{A}(z),I}(1^k, \vec{x}, r_{\mathcal{A}})$. The output of the execution is defined by the outputs of all parties and the adversary as follows.

$$Ideal^{\epsilon}_{f,\mathcal{A}(z),I}(1^k, \vec{x}, r_{\mathcal{A}})\langle i\rangle \stackrel{def}{=} \begin{cases} \mathcal{A}\text{'s output, i.e., } (1^k, I, r_{\mathcal{A}}, z_k, \vec{x}\langle I\rangle, \vec{w}\langle I\rangle), & i = 0; \\ P_i\text{'s output, i.e., } \lambda, & i \in I; \\ P_i\text{'s output, i.e., } \vec{w}\langle i\rangle, & i \in [n]-I. \end{cases}$$

Obviously, $Ideal^{\epsilon}_{f,\mathcal{A}(z),I}(1^k, \vec{x})$ is a random variable whose randomness is $r_{\mathcal{A}}$.

2.2.4 OT^n_h In The Real World

In the real world, there is not a TTP. Let π be a protocol for OT^n_h. A execution of π proceeds as follows.

1) Initial Inputs. Initial input held by each entity in real world is the same as that in the ideal world with the following exceptions. A randomness r_i is held by each party P_i. After finishing the corruption, in addition to knowledge $\mathcal{A}$ learns in the ideal world, the corrupted parties' randomness $\vec{r}\langle I\rangle$ is also learn by $\mathcal{A}$, where $\vec{r} \stackrel{def}{=} (r_1, r_2)$.
2) Computing f. In the real world, computing f is finished by all entities' interaction. Each honest party strictly follows the prescribed protocol π. The corrupted parties have to follow $\mathcal{A}$'s instructions and may arbitrarily deviate from π.
3) Outputs. Each honest party P_i always outputs what π instructs. Each corrupted party P_i outputs nothing. The adversary outputs something generated by executing an arbitrary function of the information he gathers during the execution of π. Without loss of generality, this can be assumed to be $(1^k, I, r_{\mathcal{A}}, \vec{r}\langle I\rangle, z_k, \vec{x}\langle I\rangle)$ and the messages addressed to the corrupted parties (We denote these messages by msg_I).

The whole real execution is denoted by $Real_{\pi,I,\mathcal{A}(z_k)}(1^k, \vec{x}, r_{\mathcal{A}}, \vec{r})$. The output of the execution is defined by the outputs of all parties and the adversary as follows.

$$Real_{\pi,I,\mathcal{A}(z_k)}(1^k, \vec{x}, r_{\mathcal{A}}, \vec{r})\langle i\rangle \stackrel{def}{=} \begin{cases} \mathcal{A}'s \text{ output, i.e., } (1^k, I, r_{\mathcal{A}}, \vec{r}\langle I\rangle, z_k, \vec{x}\langle I\rangle, msg_I), & i = 0; \\ P_i's \text{ output, i.e., } \lambda, & i \in I; \\ P_i's \text{ output, i.e., what instructed by } \pi, & i \in [n]-I. \end{cases}$$

Obviously, $Real_{\pi,I,\mathcal{A}(z_k)}(1^k, \vec{x})$ is a random variable whose randomnesses are $r_{\mathcal{A}}$ and $\vec{r}$.

2.2.5 Security definition

Loosely speaking, we say that protocol π securely computes OT^n_h in the presence of covert adversaries, if and only if, for any covert adversary $\mathcal{A}$, the knowledge $\mathcal{A}$ learns in the real world is not more than that he learns in the ideal world. In other words, if and only if, for any covert adversary $\mathcal{A}$, what harm $\mathcal{A}$ can do in the real world is not more than what harm he can do in the ideal world.

Definition 1 (security against covert adversaries)**.** *Let f denote the functionality of OT^n_h. Let π be a concrete protocol for OT^n_h. Let $\epsilon \in (0, 1]$. We say π securely computes f in the presence of covert adversaries with deterrence factor ϵ, if and only if for any non-uniform probabilistic polynomial-time adversary $\mathcal{A}$ with an infinite sequence $z = (z_k)_{k\in\mathbb{N}}$ in the real world, there exists a non-uniform probabilistic polynomial-time adversary $\mathcal{S}$ with the same sequence in the ideal world such that, for any $I \subseteq [2]$, the following equation holds.*

$$\{Real_{\pi,I,\mathcal{A}(z_k)}(1^k, \vec{m}, H)\}_{\substack{k\in\mathbb{N}, \vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}} \stackrel{c}{=} \{Ideal^{\epsilon}_{f,I,\mathcal{S}(z_k)}(1^k, \vec{m}, H)\}_{\substack{k\in\mathbb{N}, \vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}} \quad (1)$$

where the parameters input to the two probability ensembles are the same, $|\vec{m}| = |H|$ and each $\vec{m}\langle i\rangle$ is of the same length. The adversary $\mathcal{S}$ in the ideal world is called a simulator of the adversary $\mathcal{A}$ in the real world.

2.3 Half-Simulatable Security Against Malicious Adversaries

The standard security, i.e., fully-simulatable security against malicious adversaries, in the field of multi-party computation is first presented by [8] and also can be seen in [22]. Compared with the security against covert adversaries, the essential difference is that this security precludes any adversary from effective cheating. In other worlds, there are no choices of $cheat_i$ and $corrupted_i$

available for the adversaries in the ideal world. Hence, removing such choice and related processes in the ideal world, we gain the ideal execution of OT_h^n with standard security. Appropriately modifying Definition 1, we gain the definition of standard security.

Half-simulatable security against malicious adversaries, also called security based on one-sided simulation by [26] and called security based on half-simulation by [7], is a slightly weaker security against malicious adversaries than standard security. Specifically, if P_2 is corrupted, the security is proven in the same way as that under standard security; if P_1 is corrupted, the security is proven in a way that only guarantees that the adversaries know nothing about P_2's input. With respect to OT_h^n, the definition is specified as follows.

Definition 2 (half-simulatable security against malicious adversaries)**.** *Let f denote the functionality of OT_h^n. Let π be a concrete protocol for OT_h^n. We say π securely computes f in the presence of malicious adversaries under half simulation, if and only if the following holds.*

1) *In the case that P_2 (i.e., the receiver) is corrupted, for any non-uniform probabilistic polynomial-time adversary $\mathcal{A}$ with an infinite sequence $z = (z_k)_{k\in\mathbb{N}}$ in the real world, there exists an expected non-uniform probabilistic polynomial-time adversary $\mathcal{S}$ with the same sequence in the ideal world such that, the following equation holds.*

$$\{Real_{\pi,\{2\},\mathcal{A}(z_k)}(1^k,\vec{m},H)\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n\\ H\in\Psi,z_k\in\{0,1\}^*}} \stackrel{c}{=} \{Ideal_{f,\{2\},\mathcal{S}(z_k)}(1^k,\vec{m},H)\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n\\ H\in\Psi,z_k\in\{0,1\}^*}} \quad (2)$$

   where the parameters input to the two probability ensembles are the same, $|\vec{m}| = |H|$, and each $\vec{m}\langle i\rangle$ is of the same length.
2) *In the case that P_1 (i.e., the sender) is corrupted, for any non-uniform probabilistic polynomial-time adversary $\mathcal{A}$ with an infinite sequence $z = (z_k)_{k\in\mathbb{N}}$ in the real world, the following equation holds.*

$$\{View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k,\vec{m},H)\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n\\ H\in\Psi,z_k\in\{0,1\}^*}} \stackrel{c}{=} \{View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k,\vec{m},\widetilde{H})\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n\\ \widetilde{H}\in\Psi,z_k\in\{0,1\}^*}} \quad (3)$$

   Here, we use $View^{\mathcal{A}}_{\pi,I,\mathcal{A}(z_k)}(1^k,\vec{m},H)$ to denote the adversary's view in $Real_{\pi,I,\mathcal{A}(z_k)}(1^k,\vec{m},H)$. Note that the above equation doesn't require H to equal $\widetilde{H}$.

Half-simulatable security against malicious adversaries is widely considered in designing cryptographic protocols. For example, this security is previously considered by [2], [29], [36], [37] for OT and by [26] for the problem of set intersection. Compared with standard security, some security properties, such as independence of inputs and correctness, are not guaranteed. However, fully-simulatable security against semi-honest adversaries is indeed guaranteed. Most of all, half-simulatable security enables us to construct efficient protocols against malicious adversaries.

We point out that here we allow the simulator to run in expected polynomial-time, rather than require the simulator to run in strictly polynomial-time as half-simulatable security considered previously. We argue that this is justified, first, [4] shows that constant-round proof systems for sets outside $\mathcal{BPP}$ do not have strictly polynomial-time black-box simulators; second, in many cases (also when strictly polynomial-time simulators exist), the expected running time of the simulator provides a better bound than the worst-case running time of the simulator [23].

2.4 A Variant Of Smooth Projective Hash

Basic smooth projective hash is first presented by [14]. Though it is originally used to design chosen-ciphertext secure encryption schemes, now it and its variants are used as tools to solve other problems, for instance, password-based authenticated key exchange [20], oblivious transfer [29], [43] and extractable commitment [1]. In this paper, we use the variant of smooth projective hash presented by [43] to construct frameworks for OT_h^n. In this section, we introduce this tool.

For clarity in presentation, we assume $n = h+t$ always holds and introduce additional notations as follows. Let $R = \{(x,w)|x,w \in \{0,1\}^*\}$ be a relation, then $L_R \stackrel{def}{=} \{x|x \in \{0,1\}^*, \exists w((x,w)\in R)\}$, $R(x) \stackrel{def}{=} \{w|(x,w)\in R\}$. $\Pi \stackrel{def}{=} \{\pi|\pi : [n] \to [n], \pi \text{ is a permutation}\}$. Let $\pi \in \Pi$ (to comply with other literature, we also use π somewhere to denote a protocol without bringing any confusion). Let $\vec{x}$ be an arbitrary vector. By $\pi(\vec{x})$, we denote a vector resulted from applying π to $\vec{x}$. That is, $\vec{y} = \pi(\vec{x})$, if and only if $\forall i(i \in [d] \to \vec{x}\langle i\rangle = \vec{y}\langle\pi(i)\rangle) \wedge \forall i(i \notin [d] \to \vec{x}\langle i\rangle = \vec{y}\langle i\rangle)$ holds, where $d \stackrel{def}{=} \min(\#\vec{x}, n)$.

Definition 3 (t-smooth h-projective hash family that holds properties distinguishability, hard subset membership, feasible cheating)**.** *$\mathcal{H} = (PG, IS, DI, KG, Hash, pHash, Cheat)$ is an t-smooth h-projective hash family with witnesses and hard subset membership ($SPHDHC_{t,h}$), if and only if $\mathcal{H}$ is specified as follows*

- *The parameter-generator PG is a PPT algorithm that takes a security parameter k as input and outputs a family parameter Λ, i.e., $\Lambda \leftarrow PG(1^k)$. Λ will be used as a parameter to define three relations $R_\Lambda, \dot{R}_\Lambda$ and $\ddot{R}_\Lambda$, where $R_\Lambda = \dot{R}_\Lambda \cup \ddot{R}_\Lambda$. Moreover, $\dot{R}_\Lambda \cap \ddot{R}_\Lambda = \emptyset$ are supposed to hold.*
- *The instance-sampler IS is a PPT algorithm that takes a security parameter k, a family parameter Λ as input and outputs a vector $\vec{a}$, i.e., $\vec{a} \leftarrow IS(1^k, \Lambda)$.*
  Let $\vec{a} = ((\dot{x}_1,\dot{w}_1),\ldots,(\dot{x}_h,\dot{w}_h),(\ddot{x}_{h+1},\ddot{w}_{h+1}),\ldots,(\ddot{x}_n,\ddot{w}_n))^T$ be a vector generated by IS. We call each $\dot{x}_i$ or $\ddot{x}_i$ an instance of L_{R_Λ}. For each pair $(\dot{x}_i,\dot{w}_i)$ (resp., $(\ddot{x}_i,\ddot{w}_i)$), $\dot{w}_i$ (resp., $\ddot{w}_i$) is called a witness of $\dot{x}_i \in L_{\dot{R}_\Lambda}$ (resp., $\ddot{x}_i \in L_{\ddot{R}_\Lambda}$). Note that, by this way we indeed

have defined the relationship $R_\Lambda, \dot{R}_\Lambda$ and $\ddot{R}_\Lambda$ here. The properties smoothness and projection we will mention later make sure $\dot{R}_\Lambda \cap \ddot{R}_\Lambda = \emptyset$ holds.
For simplicity in formulation later, we introduce some additional notations here. For $\vec{a}$ mentioned above, $\vec{x}^{\vec{a}} \stackrel{def}{=} (\dot{x}_1, \ldots, \dot{x}_h, \ddot{x}_{h+1}, \ldots, \ddot{x}_n)^T$, $\vec{w}^{\vec{a}} \stackrel{def}{=} (\dot{w}_1, \ldots, \dot{w}_h, \ddot{w}_{h+1}, \ldots, \ddot{w}_n)^T$. What is more, we abuse notation $\in$ to some extent. We write $\vec{x} \in Range(IS(1^k, \Lambda))$ if and only if there exists a vector $\vec{x}^{\vec{a}}$ such that $\vec{x}^{\vec{a}} = \vec{x}$ and $\vec{a} \in Range(IS(1^k, \Lambda))$. We write $x \in Range(IS(1^k, \Lambda))$ if and only if there exists a vector $\vec{x}$ such that $\vec{x} \in Range(IS(1^k, \Lambda))$ and x is an entry of $\vec{x}$.

- *The distinguisher DI is a PPT algorithm that takes a security parameter k, a family parameter Λ and a pair strings (x, w) as input and outputs an indicator bit b, i.e., $b \leftarrow DI(1^k, \Lambda, x, w)$.*
- *The key generator KG is a PPT algorithm that takes a security parameter k, a family parameter Λ and an instance x as input and outputs a hash key and a projection key, i.e., $(hk, pk) \leftarrow KG(1^k, \Lambda, x)$.*
- *The hash $Hash$ is a PPT algorithm that takes a security parameter k, a family parameter Λ, an instance x and a hash key hk as input and outputs a value y, i.e., $y \leftarrow Hash(1^k, \Lambda, x, hk)$.*
- *The projection $pHash$ is a PPT algorithm that takes a security parameter k, a family parameter Λ, an instance x, a witness w of x and a projection key pk as input and outputs a value y, i.e., $y \leftarrow pHash(1^k, \Lambda, x, w, pk)$.*
- *The cheat $Cheat$ is a PPT algorithm that takes a security parameter k, a family parameter Λ as input and outputs n elements of $\dot{R}_\Lambda$, i.e., $((\dot{x}_1, \dot{w}_1), \ldots (\dot{x}_n, \dot{w}_n)) \leftarrow Cheat(1^k, \Lambda)$.*

and $\mathcal{H}$ has the following properties

1) *Projection. Intuitively speaking, it requires that for any instance $\dot{x} \in L_{\dot{R}_\Lambda}$, the hash value of $\dot{x}$ is obtainable with the help of its witness $\dot{w}$. That is, for any sufficiently large k, any $\Lambda \in Range(PG(1^k))$, any $(\dot{x}, \dot{w})$ generated by $IS(1^k, \Lambda)$, any $(hk, pk) \in Range(KG(1^k, \Lambda, \dot{x}))$, it holds that*

$$Hash(1^k, \Lambda, \dot{x}, hk) = pHash(1^k, \Lambda, \dot{x}, \dot{w}, pk)$$

2) *Smoothness. Intuitively speaking, it requires that for any instance vector $\vec{\dot{x}} \in L^t_{\dot{R}_\Lambda}$, the hash values of $\vec{\dot{x}}$ are random and unobtainable unless their hash keys are known. That is, for any $\pi \in \Pi$, the two probability ensembles $Sm_1 \stackrel{def}{=} \{Sm_1(1^k)\}_{k\in\mathbb{N}}$ and $Sm_2 \stackrel{def}{=} \{Sm_2(1^k)\}_{k\in\mathbb{N}}$, defined as follows, are computationally indistinguishable, i.e., $Sm_1 \stackrel{c}{=} Sm_2$.*
$SmGen_1(1^k)$: $\Lambda \leftarrow PG(1^k)$, $\vec{a} \leftarrow IS(1^k, \Lambda)$, $\vec{x} \leftarrow \vec{x}^{\vec{a}}$, for each $j \in [n]$ operates as follows: $(hk_j, pk_j) \leftarrow KG(1^k, \Lambda, \vec{x}\langle j\rangle)$, $y_j \leftarrow Hash(1^k, \Lambda, \vec{x}\langle j\rangle, hk_j)$, $\overrightarrow{xpky}\langle j\rangle \leftarrow (\vec{x}\langle j\rangle, pk_j, y_j)$. Finally outputs $(\Lambda, \overrightarrow{xpky})$.
$SmGen_2(1^k)$: compared with $SmGen_1(1^k)$, the only difference is that for each $j \in [n] - [h]$, $y_j \in_U Range(Hash(1^k, \Lambda, \vec{x}\langle j\rangle, .))$.
$Sm_i(1^k)$: $(\Lambda, \overrightarrow{xpky}) \leftarrow SmGen_i(1^k)$, $\widetilde{\overrightarrow{xpky}} \leftarrow \pi(\overrightarrow{xpky})$, finally outputs $(\Lambda, \widetilde{\overrightarrow{xpky}})$.

3) *Distinguishability. Intuitively speaking, it requires that the DI can distinguish the projective instances and smooth instances with the help of their witnesses. That is, it requires that the DI correctly computes the following function.*

$$\zeta : \mathbb{N} \times (\{0,1\}^*)^3 \to \{0,1\}$$

$$\zeta(1^k, \Lambda, x, w) = \begin{cases} 0 & \text{if } (x,w) \in \dot{R}_\Lambda, \\ 1 & \text{if } (x,w) \in \ddot{R}_\Lambda, \\ \text{undefined} & \text{otherwise .} \end{cases}$$

4) *Hard Subset Membership. Intuitively speaking, it requires that for any $\vec{x} \in Range(IS(1^k, \Lambda))$, $\vec{x}$ can be disordered without being detected. That is, for any $\pi \in \Pi$, the two probability ensembles $HSM_1 \stackrel{def}{=} \{HSM_1(1^k)\}_{k\in\mathbb{N}}$ and $HSM_2 \stackrel{def}{=} \{HSM_2(1^k)\}_{k\in\mathbb{N}}$, specified as follows, are computationally indistinguishable, i.e., $HSM_1 \stackrel{c}{=} HSM_2$.*
$HSM_1(1^k)$: $\Lambda \leftarrow PG(1^k)$, $\vec{a} \leftarrow IS(1^k, \Lambda)$, finally outputs $(\Lambda, \vec{x}^{\vec{a}})$.
$HSM_2(1^k)$: Operates as same as $HSM_1(1^k)$ with an exception that finally outputs $(\Lambda, \pi(\vec{x}^{\vec{a}}))$.

5) *Feasible Cheating. Intuitively speaking, it requires that there is a way to cheat to generate a $\vec{x}$ which is supposed to fall into $L^h_{\dot{R}_\Lambda} \times L^t_{\ddot{R}_\Lambda}$ but actually falls into $L^n_{\dot{R}_\Lambda}$ without being caught. That is, for any $\pi \in \Pi$, for any $\pi' \in \Pi$, the two probability ensembles HSM_2 and $HSM_3 \stackrel{def}{=} \{HSM_3(1^k)\}_{k\in\mathbb{N}}$ are computationally indistinguishable, i.e., $HSM_2 \stackrel{c}{=} HSM_3$, where HSM_2 is defined above and HSM_3 is defined as follows.*
$HSM_3(1^k)$:$\Lambda \leftarrow PG(1^k)$, $\vec{a} \leftarrow Cheat(1^k)$, finally outputs $(\Lambda, \pi'(\vec{x}^{\vec{a}}))$.

[43] shows that $SPHDHC_{t,h}$ can be instantiated under various hardness assumptions, e.g., the decisional Diffie-Hellman assumption, the decisional N-th residuosity assumption, the decisional quadratic residuosity assumption. Please see [43] for these instantiations.

3 Constructing A Framework For OT^n_h Against Covert Adversaries

In this section, we construct a framework for OT^n_h with security against covert adversaries. In the construction, we need a PPT algorithm, denoted by Γ, that takes two sets $B_1, B_2 \in \Psi$ as input and outputs a uniformly chosen permutation π such that $\pi(B_1) = B_2$, i.e., $\pi \leftarrow \Gamma(B_1, B_2)$. We give an example implementation of Γ as follows.

$\Gamma(B_1, B_2)$: First, $E \leftarrow \emptyset$, $C \leftarrow [n] - B_1$. Second, for each $j \in B_2$, operates as follows: $i \in_U B_1$, $B_1 \leftarrow B_1 - \{i\}$, $E \leftarrow E \cup \{j \rightleftharpoons i\}$. Third, $D \leftarrow [n] - B_2$, for each $j \in D$, operates as follows: $i \in_U C$, $C \leftarrow C - \{i\}$, $E \leftarrow E \cup \{j \rightleftharpoons i\}$. Fourth, define π in the way that $\pi(i) = j$ if and only if $j \rightleftharpoons i \in E$. Finally, outputs π.

3.1 The Detailed Framework For OT_h^n Against Covert Adversaries

- Common inputs: the public security parameter k, a $SPHDHC_{t,h}$ (where $n = h + t$) hash system $\mathcal{H}$, the number of instance vectors P_2 (i.e., the receiver) should generate K, the number of instance vectors P_2 should open g, where K, g are positive integer, $g < K$, and $K \leq poly(k)$.
- Private Inputs: Party P_1 (i.e., the sender) holds a private input $\vec{m} \in (\{0,1\}^*)^n$ and a randomness $r_1 \in \{0,1\}^*$. Party P_2 holds a private input $H \in \Psi$ and a randomness $r_2 \in \{0,1\}^*$. The adversary $\mathcal{A}$ holds a name list $I \subseteq [2]$ and a randomness $r_{\mathcal{A}} \in \{0,1\}^*$.
- Auxiliary Inputs: The adversary $\mathcal{A}$ holds an infinite auxiliary input sequence $z = (z_k)_{k\in\mathbb{N}}, z_k \in \{0,1\}^*$.

The protocol are described as follows.

- Convention: For clarity in description, we make conventions here about some trivial error-handlings, e.g., P_1 refusing to send P_2 some message which is supposed to be sent and P_1 sending an invalid message that P_2 can not process. Handling these errors is easy. It suffices that P_2 halting and outputting $abort_1$. P_1 can handle such errors in a similar way. In the following description, we will not explicitly iterate these details any more.
- Receiver's step (R1): P_2 generates hash parameters and samples instances.
  1) P_2 samples K instance vectors.
     P_2 does: $\Lambda \leftarrow PG(1^k)$; for each $i \in [K]$, $\vec{a}_i \leftarrow IS(1^k, \Lambda)$. For simply, let $\vec{a}_i \stackrel{def}{=} ((\dot{x}_1, \dot{w}_1), \ldots, (\dot{x}_h, \dot{w}_h), (\ddot{x}_{h+1}, \ddot{w}_{h+1}), \ldots, (\ddot{x}_n, \ddot{w}_n))^T$.
  2) P_2 disorders each instance vector.
     For each $i \in [K]$, P_2 uniformly chooses a permutation $\pi_i^1 \in_U \Pi$, then $\tilde{\vec{a}}_i \leftarrow \pi_i^1(\vec{a}_i)$.
  3) P_2 sends the instance vectors and the corresponding hash parameters to P_1.
     P_2 sends $(\Lambda, \tilde{\vec{x}}_1, \tilde{\vec{x}}_2, \ldots, \tilde{\vec{x}}_K)$ to P_1, where $\tilde{\vec{x}}_i \stackrel{def}{=} \vec{x}^{\tilde{a}_i}$ (correspondingly, $\tilde{\vec{w}}_i \stackrel{def}{=} \vec{w}^{\tilde{a}_i}$).
- Sender's step (S1): P_1 choose g instance vectors at random to open.
  P_1 uniformly choose a randomness $r \in_U \{0,1\}^K$ such that $g = \#\{i | r\langle i\rangle = 1, i \in [K]\}$, and sends r to P_2.
  r indicates that the instance vectors whose indexes fall into $CS \stackrel{def}{=} \{i | r\langle i\rangle = 1, i \in [K]\}$ (correspondingly, $\overline{CS} \stackrel{def}{=} [K] - CS$) are chosen by P_1 to be open. We name this r choice indicator from now on.
- Receiver's step (R2): P_2 opens the chosen instances to P_1, encodes and sends his private input to P_1.
  1) P_2 verifies that the choice indicator is legal, i.e., the number of 1-bits contained in r just is g. If r is not legal, P_2 halts and outputs $abort_1$; otherwise, P_2 proceeds to the next step.
  2) P_2 sends the witnesses of the chosen instance vectors to prove that the generated instance vectors are legal.
     P_2 sends $((i, j, \tilde{\vec{w}}_i\langle j\rangle))_{i\in CS, j\in J_i}$ to P_1, where $J_i \stackrel{def}{=} \{j | \tilde{\vec{x}}_i\langle j\rangle \in L_{\dot{R}_\Lambda}, j \in [n]\}$.
  3) P_2 encodes his private input and sends the resulting code to P_1.
     Let $G_i \stackrel{def}{=} \{j | \tilde{\vec{x}}_i\langle j\rangle \in L_{\dot{R}_\Lambda}, i \in \overline{CS}\}$. For each $i \in \overline{CS}$, P_2 does $\pi_i^2 \leftarrow \Gamma(G_i, H)$, sends $(\pi_i^2)_{i\in\overline{CS}}$ to P_1. That is, P_2 encodes his private input into sequences such as $\pi_i^2(\tilde{\vec{x}}_i)$ where $i \in \overline{CS}$.

  We comment that, to prove instance vectors' legality, there is no need for P_2 to send the witnesses of the projective instances of the chosen instance vectors to P_1, since P_1 only care whether each chosen instance vector contains enough smooth instances, which we will see later. We also comment that P_2 can send $((i, j, \tilde{\vec{w}}_i\langle j\rangle))_{i\in CS, j\in J_i}$ and $(\pi_i^2)_{i\in\overline{CS}}$ in one communication round.
- Sender's step (S2): P_1 checks the chosen instances, sends his private input to P_2 after encrypting them.
  1) P_1 verifies that each chosen instance vector is legal, i.e., for each chosen instance vector, the number of the entries belonging to $L_{\ddot{R}_\Lambda}$ is at least $n - h$.
     P_1 checks that, for each $i \in CS$, $\#J_i \geq n - h$, and for each $j \in J_i$, $DI(1^k, \Lambda, \tilde{\vec{x}}_i\langle j\rangle, \tilde{\vec{w}}_i\langle j\rangle)$ is 1. If P_2 does not send the opening or the check fails, P_1 halts and outputs $corrupted_2$, otherwise P_1 proceeds to the next step.
  2) P_1 reorders the entries of each unchosen instance vector in the way indicated by P_2.
     For each $i \in \overline{CS}$, P_1 does $\tilde{\tilde{\vec{x}}}_i \leftarrow \pi_i^2(\tilde{\vec{x}}_i)$.
  3) P_1 encrypts and sends his private input to P_2 together with some auxiliary messages.
     For each $i \in \overline{CS}$, $j \in [n]$, P_1 does: $(hk_{ij}, pk_{ij}) \leftarrow KG(1^k, \Lambda, \tilde{\tilde{\vec{x}}}_i\langle j\rangle)$, $\beta_{ij} \leftarrow Hash(1^k, \Lambda, \tilde{\tilde{\vec{x}}}_i\langle j\rangle, hk_{ij})$, $\vec{\beta}_i \stackrel{def}{=} (\beta_{i1}, \beta_{i2}, \ldots, \beta_{in})^T$, $\vec{c} \leftarrow \vec{m} \oplus (\oplus_{i\in\overline{CS}}\vec{\beta}_i)$, $\overrightarrow{pk}_i \stackrel{def}{=} (pk_{i1}, pk_{i2}, \ldots, pk_{in})^T$, sends $\vec{c}$ and $(\overrightarrow{pk}_i)_{i\in\overline{CS}}$ to P_2.
- Receiver's step (R3): P_2 decrypts the ciphertext $\vec{c}$ and gains the message he want.
  For each $i \in \overline{CS}$, $j \in H$, P_2 operates: $\beta'_{ij} \leftarrow pHash(1^k, \Lambda, \tilde{\tilde{\vec{x}}}_i\langle j\rangle, \tilde{\vec{w}}_i\langle j\rangle, \overrightarrow{pk}_i\langle j\rangle)$, $m'_j \leftarrow \vec{c}\langle j\rangle \oplus (\oplus_{i\in\overline{CS}}\beta'_{ij})$. Finally, P_2 gains the messages $(m'_j)_{j\in H}$.

3.2 The Correctness Of The Framework

In our framework, the main use of the witnesses of an instance $\dot{x} \in L_{\dot{R}_\Lambda}$ is to project and gain the hash value of $\dot{x}$. By this way, P_2 learns messages held by P_1. In contrast, with respect to an instance $\ddot{x} \in L_{\ddot{R}_\Lambda}$, the witness serves as a proof of $\ddot{x} \in L_{\ddot{R}_\Lambda}$. P_2 use the witnesses of $\ddot{x}$ to persuade P_1 to believe that P_2 is unable to gain the hash value of $\ddot{x}$. By this way, P_1 make sure that P_2 learns not more messages than that P_2 deserves.

Now let us check the correctness of the framework, i.e., the framework works in the case that P_1 and P_2 are

honest. For each $i \in \overline{CS}$, $j \in H$, we know

$$\vec{c}\langle j\rangle = \vec{m}\langle j\rangle \oplus (\oplus_{i\in\overline{CS}}\vec{\beta}_i\langle j\rangle)$$
$$m'_j = \vec{c}\langle j\rangle \oplus (\oplus_{i\in\overline{CS}}\beta'_{ij})$$

Because of the projection of $\mathcal{H}$, we know

$$\vec{\beta}_i\langle j\rangle = \beta'_{ij}$$

So we have

$$\vec{m}\langle j\rangle = m'_j$$

This means what P_2 gets is $\vec{m}\langle H\rangle$ that indeed is P_2 wants.

3.3 The Security Of The Framework

Regarding the security of the framework, we have the following theorem.

Theorem 4 (the framework is secure against covert adversaries)**.** *Assume that $\mathcal{H}$ is an t-smooth h-projective hash family that holds properties distinguishability, hard subset membership and feasible cheating. Then, the framework securely computes h-out-of-n oblivious transfer functionality in the presence of non-adaptive covert adversaries with deterrence factor $1 - 1/C_K^{K-g}$.*

Before proving Theorem 4, we first give an intuitive analysis as a warm-up. For the security of P_1, the framework should prevent P_2 from gaining more than h messages. Using cut and choose technique, P_1 can detect with probability $1-\frac{g}{K}$ that P_2 generates a instance vector containing more than h projective instances. Thus, the probability that P_2 cheats to obtain more than h messages seems to be $\frac{g}{K}$. Unexpectedly, the following theorem implies that this probability may be far less than $\frac{g}{K}$.

Theorem 5. *In the case that P_1 is honest and P_2 is corrupted, the probability that P_2 cheats to obtain more than h messages is at most $1/C_K^{K-g}$.*

Proof: According to the framework, there are the following necessary conditions for P_2 to learn extra messages.

1) P_2 has to generate at least one illegal $\vec{x}_i$ which contains more than h projective instances. If not, P_2 cann't correctly decrypt more than h entries of $\vec{c}$, because of the smoothness of $\mathcal{H}$. Without loss of generality, we assume that the illegal instance vectors are $\vec{x}_{l_1}, \vec{x}_{l_2}, \ldots, \vec{x}_{l_d}$.
2) All illegal instance vectors are lucky not to be chosen and all the unchosen instance vectors just are the illegal instance vectors, i.e., $\overline{CS} = \{l_1, l_2, \ldots, l_d\}$. We prove this claim in the following two cases.
   a) In the case that $\overline{CS} \neq \{l_1, l_2, \ldots, l_d\}$ and $\overline{CS} - \{l_1, l_2, \ldots, l_d\} = \emptyset$, there exists $j(j \in [d] \wedge l_j \in CS)$. So P_1 can detect P_2's cheating and so P_2 will gain nothing.
   b) In the case that $\overline{CS} \neq \{l_1, l_2, \ldots, l_d\}$ and $\overline{CS} - \{l_1, l_2, \ldots, l_d\} \neq \emptyset$, there exists $j(j \in \overline{CS} \wedge \vec{x}_j$ is legal). Because of the smoothness of $\mathcal{H}$, P_2 can't correctly decrypt more than h entries of $\vec{c}$.
3) The number of illegal instance vectors are just $K - g$, i.e., $d = K - g$. On one side, in the framework, P_1 choose g instance vectors to open, therefore the number of illegal instance vectors has to be no more than $K - g$ to avoid being caught. On the other side, following the analysis of the secondary condition, to learn extra messages for P_2, the number of illegal instance vectors has to be no less than $K - g$ to make that all d unchosen instance vectors are illegal possible.

Let us estimate the probability that both the second and the third necessary conditions are met. We have

$$\begin{aligned} Pr(\overline{CS} = \{l_1, l_2, \ldots, l_d\} \wedge d = K - g) &= 1/C_K^d \\ &= 1/C_K^{K-g} \end{aligned}$$

This means that the probability that P_2 cheats to obtain more than h messages is at most $1/C_K^{K-g}$. □

From the proof of Theorem 5, we know there is a possibility that P_2 commits a ineffective cheating in the sense that even if such cheating is not detected by P_1, P_2 still learns no extra new knowledge. For instance, P_2 commits a cheating that only meets the second necessary condition. What is more, such ineffective cheating even diminishes the amount of knowledge P_2 deserves. Therefore, the deterrence factor to P_2 is at least $1 - 1/C_K^{K-g}$. Following the properties of binomial factor, the maximum lower-bound of the deterrence factor can be achieved by setting g to be $K - \llcorner \frac{K+1}{2} \lrcorner$. This is in contrast to our intuition that the more instance vectors chosen to be open, the higher the deterrence factor is. The essential reason is that our intuition is apt to neglect the confusion induced by $SPHDHC_{t,h}$'s property smoothness. Since K, g are predetermined constant value, the distance between the deterrence factor and 1 can not be negligibly small. However, a very small distance is achievable. For example, setting K, g to be $30, 15$ respectively, we gain a deterrence factor $\epsilon = 1 - 6.45 \times 10^{-9}$.

For the security of P_2, the framework first should prevent P_1 from learning P_2's private input. There is a possible leakage of P_2's private input in Step R2 where P_2 encodes his private input. However, $SPHDHC_{t,h}$'s property hard subset membership guarantees that for any $\vec{x} \in Range(IS(1^k, \Lambda))$, any $\pi \in \Pi$, any PPT adversary $\mathcal{A}$, without being given π, it is negligible that the advantage of $\mathcal{A}$ identifying an entry of $\pi(\vec{x})$ falling into $L_{\dot{R}_\Lambda}$ (resp., $L_{\ddot{R}_\Lambda}$) with probability over prior knowledge h/n (resp., t/n). That is, seen from $\mathcal{A}$, every entry of $\pi(\vec{x})$ seems the same. This implies that P_2 encodes his private inputs without leaking any private information.

Besides cheating P_2 of private input, it seems there is another obvious attack that malicious P_1 sends invalid messages to P_2, e.g. pk_{ij} that $(hk_{ij}, pk_{ij}) \notin$

$Range(KG(1^k, \Lambda, x_{ij}))$. This attack in fact doesn't matter. Its effect is equal to the behaviors that are allowed in the ideal world too, e.g., P_1 altering his real input and P_1 refusing to participate in the computation.

Seen from intuition, it seems that there is no way for P_1 to cheat out the inputs of P_2. Indeed, our security proof later shows this intuition is correct. Therefore, deterrence factor to P_1 is 1. Moreover, recalling deterrence factor to P_1, we know that the deterrence factor of this framework is $1 - 1/C_K^{K-g}$.

We now proceed to prove Theorem 4. For notational clarity, we denote the parties and the adversary in the real world by P_1, P_2, $\mathcal{A}$, and denote the corresponding entities in the ideal world by P_1', P_2', $\mathcal{S}$. In the light of the parties corrupted by adversaries, there are four cases to be considered and we separately prove Theorem 4 in each case.

3.3.1 In The Case That Only P_1 Is Corrupted

In the case that only P_1 is corrupted, $\mathcal{A}$ takes the full control of P_1 in the real world. Correspondingly, $\mathcal{A}$'s simulator $\mathcal{S}$, takes the full control of P_1' in the ideal world, where $\mathcal{S}$ is constructed as follows.

- Initial input: $\mathcal{S}$ holds the same k, $I \stackrel{def}{=} \{1\}$, $z = (z_k)_{k\in\mathbb{N}}$, as $\mathcal{A}$. What is more, $\mathcal{S}$ holds a uniform distributed randomness $r_{\mathcal{S}} \in \{0,1\}^*$. The parties P_1' and P_1, whom $\mathcal{S}$ and $\mathcal{A}$ respectively are to corrupt, hold the same $\vec{m}$.
- $\mathcal{S}$ works as follows.
  - Step Sim1: $\mathcal{S}$ performs initialization in this step. Specifically, first, $\mathcal{S}$ corrupts P_1' and learns P_1''s private input $\vec{m}$. Second, let $\bar{\mathcal{A}}$ be a copy of $\mathcal{A}$, i.e., $\bar{\mathcal{A}} = \mathcal{A}$. $\mathcal{S}$ uses $\bar{\mathcal{A}}$ as a subroutine. $\mathcal{S}$ fixes the initial inputs of $\bar{\mathcal{A}}$ to be $\mathcal{S}$'s initial input with an exception that fixes the randomness of $\bar{\mathcal{A}}$ to be a uniformly distributed string. In other words, the initial inputs of $\bar{\mathcal{A}}$ is fixed to be k, I, z and $r_{\bar{\mathcal{A}}}$, where $r_{\bar{\mathcal{A}}} \in_U \{0,1\}^*$. Third, $\mathcal{S}$ activates $\bar{\mathcal{A}}$.
    We comment that, during what follows until $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ builds an environment for $\bar{\mathcal{A}}$ which simulates the real world. That is, $\mathcal{S}$ simultaneously disguises himself as P_1 and P_2 to interact with $\bar{\mathcal{A}}$.
    Fourth, before engaging in the framework for OT_h^n, $\bar{\mathcal{A}}$ sends a message indicating to corrupt P_1 as in the real world. $\mathcal{S}$ plays the role of P_1 to supplies $\bar{\mathcal{A}}$ with $\vec{m}$. Now, $\bar{\mathcal{A}}$ has engaged in the framework.
  - Convention: In the following interactions between $\bar{\mathcal{A}}$ and $\mathcal{S}$, if $\bar{\mathcal{A}}$ refuses to send some message which is supposed to be sent or $\bar{\mathcal{A}}$ sends an invalid message that $\mathcal{S}$ can not process, $\mathcal{S}$ sends $abort_1$ to the TTP, and halts with outputting whatever $\bar{\mathcal{A}}$ outputs. We make this convention here, and will not explicitly iterate these details any more.
  - Step Sim2: $\mathcal{S}$ repeats the following procedure, denoted by Υ, until the output of Υ is λ. In each repetition, all randomness used by $\mathcal{S}$ is fresh. On finishing the repetition, $\mathcal{S}$ proceeds to the next step.
    - Procedure Υ:
      1) $\mathcal{S}$ rewinds $\bar{\mathcal{A}}$ to the beginning of Step S1 of the framework.
      2) $\mathcal{S}$ uniformly chooses a choice indicator as an honest party P_1 does in Step S1 of the framework. $\mathcal{S}$ executes receiver's Step R1 of the framework with the following exception, for each $\bar{r}\langle i\rangle = 1$, $\mathcal{S}$ honestly generates instance vector $\vec{x}_i$; for each $\bar{r}\langle i\rangle = 0$, $\mathcal{S}$ call $Cheat(1^k, \Lambda)$ to generate instance vector $\vec{x}_i$.
      3) $\mathcal{S}$ receives a choice indicator r from $\bar{\mathcal{A}}$.
      4) If $r = \bar{r}$, $\alpha \leftarrow \lambda$; otherwise, $\alpha \leftarrow \perp$. Finally outputs α.

    Remark 6. *Step Sim2 is a key step to extract $\bar{\mathcal{A}}$'s real input. Note that, if event $r = \bar{r}$ happens, each entry of each unchosen instance vector is projective. $SPHDHC_{t,h}$'s property feasible cheating guarantees that $\bar{\mathcal{A}}$ can't detect this cheating. Recalling the proof of Theorem 5, we know $\mathcal{S}$ can succeed in extracting $\bar{\mathcal{A}}$'s real input.*
  - Step Sim3: Playing the role of P_2 who takes an arbitrary element of Ψ as private input, $\mathcal{S}$ honestly execute receiver's Step R2 to interact with $\bar{\mathcal{A}}$. On receiving $\vec{c}$, $(\overrightarrow{pk}_i)_{i\in\overline{CS}}$ sent by $\bar{\mathcal{A}}$, $\mathcal{S}$ decrypts every entry of $\vec{c}$, and gains $\bar{\mathcal{A}}$'s real input $\vec{m}'$.
  - Step Sim4: $\mathcal{S}$ sends $\vec{m}'$ to TTP and receives back a message λ. Note that the cases that $\bar{\mathcal{A}}$ induces honest receiver interacting with him not to obtain a result is considered in the convention, here the honest receiver can receive a result. Thus, $\mathcal{S}$ sends $continue$ to TTP.
  - Step Sim5: When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.

Lemma 7. *The simulator $\mathcal{S}$ is expected polynomial-time.*

Proof: First, let us focus on Step Sim2. Note that the number of legal choice indicators is C_K^g. So we have $Pr(r = \bar{r}) = 1/C_K^g$. Therefore, the expected number of repeating Υ is C_K^g which is a predetermined constant value. Then, Step Sim2 runs in expected polynomial-time.

Second, let us focus on other steps of $\mathcal{S}$. Obviously, these steps run in strict polynomial-time. Getting together the above analysis, $\mathcal{S}$ is expected polynomial-time. □

However, the security definition, i.e., Definition 1, requires a strictly polynomial-time simulator. Observers that, the reason why the above $\mathcal{S}$ doesn't run strictly polynomial-time is that the probability of Υ outputting $\perp$ is relatively too high. The following lemma guarantees that we can get a new version of Υ that runs in strictly polynomial-time and outputs $\perp$ with negligible probability.

Lemma 8. *Given a PPT machine M that outputs $\perp$ with probability p, where $p \in (0,1)$ is a constant value, we can have a PPT machine $\widetilde{M}$ such that*

$$\forall x \rightarrow Pr(\widetilde{M}(x) = \perp) = \mu(k)$$
$$\forall x \forall \alpha \rightarrow Pr(\widetilde{M}(x) = \alpha | \widetilde{M}(x) \neq \perp) = Pr(M(x) = \alpha | M(x) \neq \perp)$$

holds.

Proof: Let v be some polynomial that $v(k) > 1/p$. We construct $\widetilde{M}$ as follows.

$\widetilde{M}$: on receiving input x, repeats $M(x)$ at most v times. If $M(x)$ outputs something different from $\perp$ in a repetition, sets α to be this output, and stops repeating; otherwise, set α to be $\perp$, and continues repeating. Finally outputs α.

Let $M_i(x)$ be the output of $M(x)$ in i-th repetition. Let X_i be $0/1$ random variable defined as follows.

$$X_i \stackrel{def}{=} \begin{cases} 1, & M_i(x) = \perp; \\ 0, & M_i(x) \neq \perp. \end{cases}$$

Then, we know $E(X_i) = p$, $D(X_i) = p(1-p) = \sigma^2$.

$$\begin{aligned} Pr(\widetilde{M}(x) = \perp) &= Pr(\sum_i^v X_i < 1) \\ &= Pr(p - \frac{\sum_i^v X_i}{v} > p - \frac{1}{v}) \\ &\leq Pr(|p - \frac{\sum_i^v X_i}{v}| \geq p - \frac{1}{v}) \end{aligned}$$

Following Chernoff Bound (can be seen in [21]), we have

$$Pr(\widetilde{M}(x) = \perp) \leq \frac{2}{e^{\frac{v}{2\sigma^2}(p - \frac{1}{v})^2}}$$

Therefore, $Pr(\widetilde{M}(x) = \perp)$ is negligible. □

Modifying Step Sim2 as follows, we gain a new version of simulator.

- Step Sim2: $\mathcal{S}$ executes the new version of Υ guaranteed by Lemma 8. If Υ outputs $\perp$, $\mathcal{S}$ outputs $\perp$ and halts; otherwise, $\mathcal{S}$ proceeds to the next step.

Combining the proof of Lemma 7, we get the following proposition.

Proposition 9. *The new simulator $\mathcal{S}$ is polynomial-time.*

From now on, the simulator we refer to is the new version simulator. Note that $\mathcal{S}$ never sends $cheat_1$ to TTP. Thus, we actually construct a standard simulator for $\mathcal{A}$, and so provide a standard security to P_2. This means, first, as mentioned previously, the deterrence factor to $\mathcal{A}$ is 1 in this case that P_1 is corrupted. Second, we need to prove for any non-uniform probabilistic polynomial-time adversary $\mathcal{A}$ with an infinite sequence $z = (z_k)_{k \in \mathbb{N}}$ in the real world, the following equation holds.

$$\{Ideal_{f,\{1\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\}_{\substack{k \in \mathbb{N}, \vec{m} \in (\{0,1\}^*)^n \\ H \in \Psi, z_k \in \{0,1\}^*}} \stackrel{c}{=} \{Real_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\}_{\substack{k \in \mathbb{N}, \vec{m} \in (\{0,1\}^*)^n \\ H \in \Psi, z_k \in \{0,1\}^*}} \quad (4)$$

Proposition 10. *In the case that only P_1 is corrupted, the equation* (4) *holds.*

Before proving Proposition 10, we first prove some lemmas.

Lemma 11. *The output of $\mathcal{S}$ in the ideal world and the output of $\mathcal{A}$ in the real world are computationally indistinguishable, i.e., the following equation holds.*

$$\{Ideal_{f,\{1\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\langle 0 \rangle\}_{\substack{k \in \mathbb{N}, \vec{m} \in (\{0,1\}^*)^n \\ H \in \Psi, z_k \in \{0,1\}^*}} \stackrel{c}{=} \{Real_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\langle 0 \rangle\}_{\substack{k \in \mathbb{N}, \vec{m} \in (\{0,1\}^*)^n \\ H \in \Psi, z_k \in \{0,1\}^*}}$$

Proof: First, we claim that the outputs of $\mathcal{S}$ and $\bar{\mathcal{A}}$ are computationally indistinguishable. From the construction of $\mathcal{S}$, we know that the event that $\mathcal{S}$ doesn't take $\bar{\mathcal{A}}$'s output as its output arises only if the event of Υ outputting $\perp$ arises. Since the latter arises with negligible probability, this claim holds.

Second, we claim that the views of $\bar{\mathcal{A}}$ and $\mathcal{A}$ are computationally indistinguishable. The only difference between $\bar{\mathcal{A}}$'s view and $\mathcal{A}$'s is that the unchosen instance vectors $\bar{\mathcal{A}}$ sees are generated by calling $Cheat(1^k, \Lambda)$. $SPHDHC_{t,h}$'s property feasible cheating guarantees such instance vectors are indistinguishable from that generated honestly. Therefore, this claim holds.

Combining the above two claims, this lemma holds. □

Lemma 12. *Let $X \stackrel{def}{=} \{X(1^k, a)\}_{k \in \mathbb{N}, a \in \{0,1\}^*}$ and $Y \stackrel{def}{=} \{Y(1^k, a)\}_{k \in \mathbb{N}, a \in \{0,1\}^*}$ be two polynomial-time constructible probability ensembles, $X \stackrel{c}{=} Y$, $F \stackrel{def}{=} (f_k)_{k \in \mathbb{N}}$, $f_k : \{0,1\}^* \rightarrow \{0,1\}^*$ is polynomial-time computable, then*

$$F(X) \stackrel{c}{=} F(Y)$$

where $F(X) \stackrel{def}{=} \{f_k(X(1^k, a))\}_{k \in \mathbb{N}, a \in \{0,1\}^*}$, $F(Y) \stackrel{def}{=} \{f_k(Y(1^k, a))\}_{k \in \mathbb{N}, a \in \{0,1\}^*}$.

Proof: Assume that the proposition is false, then there exists a non-uniform PPT distinguisher D with an infinite sequence $z = (z_k)_{k \in \mathbb{N}}$, a polynomial $poly(.)$, an infinite positive integer set $G \subseteq \mathbb{N}$ such that, for each $k \in G$, it holds that

$$|Pr(D(1^k, z_k, a, f_k(X(1^k, a))) = 1) - Pr(D(1^k, z_k, a, f_k(Y(1^k, a))) = 1)| \geq 1/poly(k)$$

We can construct a distinguisher D' with the same infinite sequence $z = (z_k)_{k \in \mathbb{N}}$ for the ensembles X and Y as follows.

$D'(1^k, z_k, a, \gamma)$: $\delta \leftarrow f_k(\gamma)$, finally outputs $D(1^k, z_k, a, \delta)$.

Obviously, $D'(1^k, z_k, a, X(1^k, a)) = D(1^k, z_k, a, f_k(X(1^k, a)))$, $D'(1^k, z_k, a, Y(1^k, a)) = D(1^k, z_k, a, f_k(Y(1^k, a)))$. So we have

$$|Pr(D'(1^k, z_k, a, X(1^k, a)) = 1) - Pr(D'(1^k, z_k, a, Y(1^k, a)) = 1)| \geq 1/poly(k)$$

This contradicts the fact $X \stackrel{c}{=} Y$. □

Now we proceed to prove Proposition 10.

Proof: First let us focus on the real world. $\mathcal{A}$'s real input can be formulated as $\gamma \leftarrow A(1^k, \vec{m}, z_k, r_{\mathcal{A}}, r_1)$. Note that in this case, P_2's output is a determinate function of $\mathcal{A}$'s real input. Since $\mathcal{A}$'s real input is in its view, without loss of generality, we assume $\mathcal{A}$'s output, denoted by α, contains its real input. Therefore, P_2's output is a determinate function of $\mathcal{A}$'s output, where the function is given as follows.

$$g(\alpha) = \begin{cases} abort_1 & \text{if } \gamma \text{ cause } P_2 \text{ to output } abort_1, \\ corrupted_1 & \text{if } \gamma \text{ cause } P_2 \text{ to output } corrupted_1, \\ \gamma\langle H\rangle & otherwise. \end{cases}$$

Let $h(\alpha) \stackrel{def}{=} (\alpha, \lambda, g(\alpha))$. Then we have

$$Real_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H) \equiv h(Real_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle)$$

Similarly, in the ideal world, we have

$$Ideal_{f,\{1\},\mathcal{S}(z_k)}(1^k, \vec{m}, H) \stackrel{c}{=} h(Ideal_{f,\{1\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle)$$

We use $\stackrel{c}{=}$ not $\equiv$ here because there is a negligible probability that $\mathcal{S}$ outputs $\perp$, which makes $h(.)$ undefined.

Let $X(1^k, \vec{m}, H, z_k, \{1\}) \stackrel{def}{=} Real_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle$, $Y(1^k, \vec{m}, H, z_k, \{1\}) \stackrel{def}{=} Ideal_{f,\{1\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle$. Following Lemma 11, we get $X \stackrel{c}{=} Y$. Let $F \stackrel{def}{=} (h)_{k\in\mathbb{N}}$. According to Lemma 12, the proposition holds. □

Getting together Proposition 9 and Proposition 10, we know Theorem 4 is true in the case that P_1 is corrupted.

3.3.2 In The Case That Only P_2 Is Corrupted

In the case that only P_2 is corrupted, $\mathcal{A}$ takes the full control of P_2 in the real world. Correspondingly, $\mathcal{S}$ takes the full control of P_2' in the ideal world. We construct $\mathcal{S}$ as follows.

- Initial input: $\mathcal{S}$ holds the same k, $I \stackrel{def}{=} \{2\}$, $z = (z_k)_{k\in\mathbb{N}}$ as $\mathcal{A}$, and holds a uniformly distributed randomness $r_{\mathcal{S}} \in_U \{0,1\}^*$. The parties P_2' and P_2 hold the same private input H.
- $\mathcal{S}$ works as follows.
  - Step Sim1: In this step, $\mathcal{S}$ performs initialization in a similar way that the simulator does in the case that P_1 is corrupted. That is, $\mathcal{S}$ corrupts P_2' and learns P_2''s private input H. Then, $\mathcal{S}$ takes $\mathcal{A}$'s copy $\bar{\mathcal{A}}$ as a subroutine, fixes $\bar{\mathcal{A}}$'s initial input to be $k, I, z, r_{\bar{\mathcal{A}}}$ ($r_{\bar{\mathcal{A}}} \in_U \{0,1\}^*$), activates $\bar{\mathcal{A}}$ and supplies $\bar{\mathcal{A}}$ with H. Now, $\bar{\mathcal{A}}$ has engaged in the framework.
  - Convention: We make a convention here that is similar to that made in the previous case.
  - Step Sim2: On receiving $(\Lambda, \tilde{\vec{x}}_1, \tilde{\vec{x}}_2, \ldots, \tilde{\vec{x}}_K)$ sent from $\bar{\mathcal{A}}$, $\mathcal{S}$ rewinds $\bar{\mathcal{A}}$ C_K^g times to knows $\bar{\mathcal{A}}$'s distinct responses to distinct choice indicators. Specifically, at the i-th time, $\mathcal{S}$ works as follows.
    1) $\mathcal{S}$ chooses a choice indicator r never used. Note that there are just C_K^g distinct choice indicators in total.
    2) Playing the role of P_1, $\mathcal{S}$ sends r to $\bar{\mathcal{A}}$, records $\bar{\mathcal{A}}$'s response, i.e., a rejection, or a corresponding opening $((i, j, \tilde{\vec{w}}_i\langle j\rangle))_{i\in CS, j\in J_i}$.
    3) $\mathcal{S}$ rewinds $\bar{\mathcal{A}}$ to the beginning of Step R2 of the framework.
  - Step Sim3: Now $\mathcal{S}$ knows $\bar{\mathcal{A}}$'s all possible responses. $\mathcal{S}$ proceeds as follows based on these knowledge.
    - Case 1, no $\bar{\mathcal{A}}$'s response will cause honest party P_1 to output $corrupted_2$. $\mathcal{S}$ proceeds to Step Sim4.
    - Case 2, every $\bar{\mathcal{A}}$'s response will cause honest party P_1 to output $corrupted_2$. $\mathcal{S}$ sends $corrupted_2$ to TTP and receives back $corrupted_2$ from TTP. Then, playing the role of P_1, $\mathcal{S}$ honestly follows the sender's steps of the framework which begins from Step S1 to interact with $\bar{\mathcal{A}}$. When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.
    - Case 3, the rest case, i.e., the number of $\bar{\mathcal{A}}$'s responses that will cause honest party P_1 to output $corrupted_2$ is in the domain $[1, C_K^g - 1]$. $\mathcal{S}$ send $cheat_2$ to TTP. Since $\mathcal{S}$ knows $\bar{\mathcal{A}}$'s all responses, $\mathcal{S}$ knows the the probability that $\bar{\mathcal{A}}$ is being caught in cheating by honest party P_1 in the real world. We denote this probability by ϵ. It is easy to see that $0 < \epsilon < 1$. We remark that ϵ corresponds with the deterrence factor in the real world.
      - Case 3.1, with probability ϵ, TTP replies $\mathcal{S}$ with $corrupted_2$. $\mathcal{S}$ uniformly chooses one of the choice indicators that will cause $\bar{\mathcal{A}}$'s cheating to be caught and sends it to $\bar{\mathcal{A}}$. Then, playing the role of P_1, $\mathcal{S}$ honestly follows the sender's steps which begins from Step S2 to interact with $\bar{\mathcal{A}}$.
      - Case 3.2, with probability $1-\epsilon$, TTP replies $\mathcal{S}$ with $undetected$ and honest party P_1's private input $\vec{m}$. $\mathcal{S}$ determines the result that P_1' will receive to be λ, and sends λ to TTP. Playing the role of P_1 with private input $\vec{m}$, $\mathcal{S}$ honestly follows the sender's steps which begins from Step S1 to interact with $\bar{\mathcal{A}}$.

    When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.
  - Step Sim4: Playing the role of P_1, $\mathcal{S}$ honestly follows the sender's steps including Step S1, Step S2.1 and Step S2.2 to interact with $\bar{\mathcal{A}}$. During the interaction, $\mathcal{S}$ extracts $\bar{\mathcal{A}}$'s real input H' based on the instances' witnesses recorded in Step Sim2 and the codes $(\pi_i^2)_{i\in\overline{CS}}$ sent by

$\bar{\mathcal{A}}$. Specifically, $\mathcal{S}$ operates as follows: for each $i \in \overline{CS}$, $H'_i \leftarrow \emptyset$; for each $i \in \overline{CS}$ and $j \in [n]$, if $\tilde{\vec{x}}_i\langle j\rangle$ is not smooth, $H'_i \leftarrow H'_i \cup \{j\}$; finally, $H' \leftarrow \cap_{i\in\overline{CS}} H'_i$.

- Step Sim5: $\mathcal{S}$ sends H' to TTP and receives back $\vec{m}\langle H'\rangle$. $\mathcal{S}$ then sends *continue* to TTP. To proceed to interact with $\bar{\mathcal{A}}$, $\mathcal{S}$ fabricates $\vec{m}'$ as follows. For each $i \in H'$, $\vec{m}'\langle i\rangle \leftarrow \vec{m}\langle i\rangle$. For each $i \notin H'$, set $\vec{m}'\langle i\rangle$ to be an arbitrary message of appropriate length.
- Step Sim6: Playing the role of P_1 with private input $\vec{m}'$, $\mathcal{S}$ follows Step S2.3 to complete the interaction with $\bar{\mathcal{A}}$. When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.

Proposition 13. *The simulator $\mathcal{S}$ is polynomial-time.*

Looking at the construction of $\mathcal{S}$, each step is polynomial-time. Obviously, Proposition 13 is true.

Lemma 14. *If Case 1 of Step Sim3 happens, the output of $\mathcal{A}$ in the real world and that of $\bar{\mathcal{A}}$ in the ideal world are indistinguishable.*

Proof: Note that the only point makes $\bar{\mathcal{A}}$'s view different from $\mathcal{A}$'s view is that the ciphertext $\vec{c}'$ $\bar{\mathcal{A}}$ receives is generated by encrypting fabricated $\vec{m}'$ rather than party P_1's private input $\vec{m}$.

In case $\#H' = h$, $SPHDHC_{t,h}$'s property smoothness directly guarantees that $\bar{\mathcal{A}}$'s view and $\mathcal{A}$'s view are indistinguishable.

In case $\#H' < h$, $\bar{\mathcal{A}}$'s view and $\mathcal{A}$'s are indistinguishable too. This is because $SPHDHC_{t,h}$'s property smoothness guarantees that for each $j \in [n] - [H']$, what $\bar{\mathcal{A}}$, $\mathcal{A}$ respectively can get from $\vec{c}'\langle j\rangle$, $\vec{c}\langle j\rangle$ are values uniformly distributed over the same domain.

There is no case $\#H' > h$, since all instance vectors are legal.

Combining above analysis, we know that $\mathcal{A}$'s output and $\bar{\mathcal{A}}$'s output are indistinguishable. □

We remark that the proof of Lemma 14 also shows that both $\mathcal{A}$'s effective private input and $\bar{\mathcal{A}}$'s are H'.

Lemma 15. *If Case 2 or Case 3 of Step Sim3 happens, the outputs of $\mathcal{A}$ in the real world and $\bar{\mathcal{A}}$ in the ideal world are identical.*

Proof: Looking at the construction, we know that, in the case that $\mathcal{S}$ receives back $corruptted_2$ from TTP, $\mathcal{S}$ plays the role of honest party P_1 during the subsequent interaction with $\bar{\mathcal{A}}$ without needing to know $\vec{m}$. In the case that $\mathcal{S}$ receives back $undetected$ from TTP, $\mathcal{S}$ knows $\vec{m}$ and perfectly plays the role of honest party P_1. Therefore, $\mathcal{A}$'s view and $\bar{\mathcal{A}}$'s view are identical. This implies their outputs are identical. □

Lemma 16. *The output of the adversary $\mathcal{A}$ in the real world and that of the simulator $\mathcal{S}$ in the ideal world are computationally indistinguishable, i.e.,*

$$\{Real_{\pi,\{2\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}} \stackrel{c}{=}$$

$$\{Ideal^{\epsilon}_{f,\{2\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}}$$

Proof: First, we claim that the outputs of $\mathcal{S}$ and $\bar{\mathcal{A}}$ are identical. This follows from the fact that $\mathcal{S}$ always takes $\bar{\mathcal{A}}$'s output as his own output.

Second, we claim that the outputs of $\mathcal{A}$ and $\bar{\mathcal{A}}$ are computationally indistinguishable. Looking at Step Sim3 and Step Sim4, the probability that each case happens is identical to that in the real world. According to Lemma 14 and Lemma 15, this claim holds.

Combining the two claims, the lemma is true. □

Proposition 17. *In the case that only P_2 was corrupted, i.e., $I = \{2\}$, the equation* (1) *required by Definition 1 holds.*

Proof: Note that the honest parties P_1 and P'_1 end up with outputting nothing. Thus, the fact that the outputs of $\mathcal{S}$ and $\mathcal{A}$ are computationally indistinguishable, which is supported by Proposition 24, directly prove this proposition. □

Getting together Proposition 13 and Proposition 17, we know Theorem 4 is true in the case that P_2 is corrupted.

3.3.3 In Other Cases

In the case that both P_1 and P_2 are corrupted, $\mathcal{A}$ takes the full control of the two corrupted parties. In the ideal world, a similar situation also holds with respect to $\mathcal{S}$, P'_1 and P'_2. Like in previous cases, $\mathcal{S}$ uses $\mathcal{A}$'s copy, $\bar{\mathcal{A}}$, as a subroutine and builds a simulated environment for $\bar{\mathcal{A}}$. $\mathcal{S}$ provides $\bar{\mathcal{A}}$ with the initial inputs of P'_1 and P'_2 before $\bar{\mathcal{A}}$ engaging in the framework. When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs. Obviously, $\mathcal{S}$ runs in strictly polynomial-time and the equation (1) required by Definition 1 holds in this case.

In the case that none of P_1 and P_2 is corrupted. The simulator $\mathcal{S}$ is constructed as follows. $\mathcal{S}$ uses $\bar{\mathcal{A}}$, $\bar{P}_1$, $\bar{P}_2$ as subroutines, where $\bar{\mathcal{A}}$, $\bar{P}_1$, $\bar{P}_2$, respectively, are the copy of $\mathcal{A}$, P_1 and P_2. $\mathcal{S}$ fixes $\bar{\mathcal{A}}$'s initial inputs in the same way as in previous cases. $\mathcal{S}$ chooses an arbitrary $\bar{\vec{m}} \in (\{0,1\}^*)^n$ and a uniformly distributed randomness $\bar{r}_1$ as $\bar{P}_1$'s initial inputs. $\mathcal{S}$ chooses an arbitrary $\bar{H} \in \Psi$ and a uniformly distributed randomness $\bar{r}_2$ as $\bar{P}_2$'s initial inputs. $\mathcal{S}$ activates these subroutines and makes the communication between $\bar{P}_1$ and $\bar{P}_2$ available to $\bar{\mathcal{A}}$. Note that, in the case that none of P_1 and P_2 is corrupted, what adversaries can see in the real world is only the communication between honest parties. When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs. Obviously, $\mathcal{S}$ runs in strictly polynomial-time and the equation (1) required by Definition 1 holds in this case.

Getting together above analysis, we know Theorem 4 is true in the cases considered in this section.

3.4 The Communication Rounds

Step R3 is performed without communication. Each of the rest steps is performed in one round. Thus, the number of the total communication rounds is four.

Seen from practical use, there is no sense to compare our work to the works whose security are not guaranteed in the implementation or the works that can not be implemented at present. Therefore, we do not consider any protocols presented by [19], [27], [38], [39] in the rest of this paper.

Compared with existing protocols for OT_h^n with standard security or with security against covert adversaries, our framework is the most efficient one in communications rounds. When count the total communication rounds costed by a protocol, we count that of the modified version. In the modified version, the consecutive communications of the same direction are combined into one round. The protocol for $OT_{h\times 1}^n$ of [7] respectively costs one, two zero-knowledge proof of knowledge in initialization and in transfering a message, where each zero-knowledge proofs of knowledge is performed in four rounds. The whole protocol costs at least ten rounds. The protocol for OT_h^n of [25] costs one zero-knowledge proof of knowledge in initialization which is performed in three rounds at least, one protocol to extract a secret key corresponding to the identity of a message which is performed in four rounds, one zero-knowledge proof of knowledge in transfering a message which is performed in three rounds at least. We point out that the interactive proof of knowledge of a discrete logarithm modulo a prime, presented by [41] and taken as a zero-knowledge proof of knowledge protocol in [25], to our best knowledge, is not known to be zero-knowledge. However, turning to the techniques of Σ-protocol, [13] makes it zero-knowledge at cost of increment of three rounds in communication, which in turn induces the increment in communication rounds of the protocol of [25]. Taking all into consideration, this protocol costs at least ten rounds. The framework for OT_h^n of [43] costs six rounds.

Compared with existing protocols for OT_1^2 with standard security or with security against covert adversaries, our framework is also the most efficient one in communications rounds. The protocol presented by [32] can be viewed as a special DDH-based instantiation of the framework in [43], though modifying the protocol to some extent is needed. Therefore, there is no need to consider this protocol independently. In the rest of this paper, we will not repeat this point again. The protocol presented by [3] is secure against covert adversaries with deterrence factor $\frac{1}{2}$ and costs four rounds.

3.5 The Computational Overhead

We measure the computational overhead of a protocol mainly firstly in terms of the number of public key operations (i.e., operations based on trapdoor functions, or similar operations), because the overhead of public key operations, which depends on the length of their inputs, is greater than that of symmetric key operations (i.e. operations based on one-way functions) by orders of magnitude. However, in comparison of two protocols, if their overhead of public key operations are the same, the number of private key operations has to be taken into consideration. Please see [33] to know which cryptographic operation is public or private key operation.

As to our framework, the public key operations are $Hash(.)$ and $pHash(.)$. In Step S2, P_1 takes $n(K-g)$ invocations of $Hash(.)$ to encrypt his private input. In Step R3, P_2 takes $h(K-g)$ invocations of $pHash(.)$ to decrypt the messages he want. To compare with existing protocols in computational overhead, we instantiate our framework with the DDH-based $SPHDHC_{t,h}$ presented by [43] and set K, g to be $2, 1$ respectively. In the resulting DDH-based instantiation of our framework, the deterrence factor is $\frac{1}{2}$; P_1 costs n public key encryption operations, P_2 costs h public key decryption operations.

Compared with existing protocols for OT_h^n with standard security or with security against covert adversaries, our DDH-based instantiation is the most efficient one in computational overhead. The operations of the protocol in [7] are based on the non-standard assumptions, i.e., q-Power Decisional Diffie-Hellman and q-Strong Diffie-Hellman (q-SDH) assumptions, which both are associated with bilinear groups. [12] indicates that q-SDH-based operations are more expensive that standard-assumption-based operations. The operations of the protocol in [25] are based on Decisional Bilinear Diffie-Hellman (DBDH) assumption. Because bilinear curves are considerably more expensive than regular Elliptic curves [18] and DDH is obtainable from Elliptic curves, the operations in [7], [25] are considerably more expensive than DDH-based operations. The most efficient instantiation of the framework presented by [43] are DDH-based too. However, used in practice, the instantiation costs P_1 $40n$ public key encryption operations and costs P_2 $40h$ public key decryption operations in the worst case. The computational overhead in the average case is half of that in the worst case. Therefore, our DDH-based instantiation are the most efficient protocol for OT_h^n.

Compared with existing protocols for OT_1^2 with standard security or with security against covert adversaries, our DDH-based instantiation for OT_1^2 is also the most efficient one in computational overhead. The operations of the protocol for OT_1^2 in [3] are based on homomorphic encryption. Implementing the same functionality OT_1^2, the number of public operations of the protocol and that of our instantiation are equivalent. However, to our best knowledge, there is no homomorphic encryption scheme whose operations are more efficient than or are as efficient as that based on DDH.

We point out that the protocols presented in [7], [25], [32], [43] are secure against malicious adversaries, which leads to the security level of the protocols are higher than that of ours. However, as pointed out by [3], the adversaries are not always malicious and ready to cheat at any price in many real-world setting. Therefore, in such setting, the protocols given by [7], [25], [32], [43] seem too pessimistic and our framework makes better sense seen from efficiency.

3.6 Extensions

In our original framework, we require K, g to be predetermined constant values. However, K, g can be relaxed to be any functions in k with a limitation that C_K^g is polynomial in k. The reason why we place such limitation on K, g is to make sure that the simulator runs in strictly polynomial time.

4 Constructing A Framework For OT_h^n Against Malicious Adversaries

4.1 The Detailed Framework For OT_h^n Against Malicious Adversaries

In this section, we present a framework for OT_h^n against malicious adversaries. This framework is obtained by modifying the framework presented in Section 3.1 as follows.

- Common inputs: Remove the limitation that K is a predetermined constant. Instead, K is relaxed to be a function $K = poly(k)$. Remove the parameter g from the common inputs.
- Sender's step (S1): Remove the limitation placed on the way of P_1 choosing the choice indicator. Instead, P_1 is granted to uniformly choose choice indicator over $\{0,1\}^K$.
- Receiver's step (R2): Remove the step that P_2 checks the legality of the choice indicator.

It is easy to verify the correctness of the framework. So, we omit the details.

4.2 The Security Of The Framework

Theorem 18 (The framework is secure against malicious adversaries)**.** *Assume that $\mathcal{H}$ is a t-smooth h-projective hash family that holds properties distinguishability, hard subset membership and feasible cheating ($SPHDHC_{t,h}$). Then, the protocol securely computes h-out-of-n oblivious transfer functionality in the presence of malicious adversaries under half simulation.*

As previous, before proving Theorem 18, we first give an intuitive analysis as a warm-up. With respect to the security of P_1, we have Theorem 19.

Theorem 19. *In the case that P_1 is honest and P_2 is corrupted, the probability that P_2 cheats to obtain more than h messages is at most $1/2^K$.*

Proof: Compared with previous framework, the necessary conditions for P_2 to learn extra messages are the same ones mentioned in the proof of Theorem 5 except the third necessary condition. Let us estimate the probability that the second necessary condition is met. We have

$$\begin{aligned} Pr(\overline{CS} = \{l_1, l_2, \ldots, l_d\}) &= \frac{1}{2^d} \cdot \frac{1}{2^{K-d}} \\ &= 1/2^K \end{aligned}$$

This means that the probability that P_2 cheats to obtain more than h messages is at most $1/2^K$. □

With respect to the security of P_2, the intuitive analysis given to the previous framework also holds in this case. Therefore, seen from intuition, P_2's security is guaranteed by this new framework. Now, we proceed to prove Theorem 18.

4.2.1 *In The Case That Only P_1 Is Corrupted*

In the case that P_1 is corrupted, according to Definition 2, we don't need to provide a simulator $\mathcal{S}$ for the real adversary $\mathcal{A}$ as required by the ideal/real simulation paradigm. Instead, we only prove that the framework provides privacy to honest party P_2.

Proposition 20. *In the case that only P_1 was corrupted, i.e., $I = \{1\}$, the equation* (3) *required by Definition 2 holds.*

Proof: Looking at the framework, we know the only difference between $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)$ and $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, \widetilde{H})$ are the codes of honest party P_2's private input, i.e., π_i^2s and $\pi_i^2(\tilde{\vec{x}}_i)$s, where $i \in \overline{CS}$.

First, we claim that each π_i^2 in $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)$ or $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, \widetilde{H})$ is uniformly distributed. Observe that π_i^1 is uniformly distributed. This leads to that G_i is uniformly distributed. Therefore, for any image $j \in [n]$, its pre-image under π_i^2 is uniformly distributed. That is, π_i^2 is uniformly distributed.

Second, we claim that $\vec{x}_i \stackrel{c}{=} \pi_i^2(\tilde{\vec{x}}_i)$ holds in $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)$ or $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, \widetilde{H})$. Following $SPHDHC_{t,h}$'s property hard membership, we have $\vec{x}_i \stackrel{c}{=} \pi_i^1(\vec{x}_i) = \tilde{\vec{x}}_i$, and $\tilde{\vec{x}}_i \stackrel{c}{=} \pi_i^2(\tilde{\vec{x}}_i)$. Therefore, $\vec{x}_i \stackrel{c}{=} \pi_i^2(\tilde{\vec{x}}_i)$.

Combining the above two claims, we know the codes in $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)$ and $View^{\mathcal{A}}_{\pi,\{1\},\mathcal{A}(z_k)}(1^k, \vec{m}, \tilde{H})$ are indistinguishable. Thus the proposition holds. □

4.2.2 *In The Case That Only P_2 Is Corrupted*

In the case that only P_2 is corrupted, $\mathcal{A}$ takes the full control of P_2 in the real world. Correspondingly, $\mathcal{S}$ takes the full control of P_2' in the ideal world. We construct $\mathcal{S}$ as follows.

- Initial input: $\mathcal{S}$ holds the same k, $I \stackrel{def}{=} \{2\}$, $z = (z_k)_{k\in\mathbb{N}}$ as $\mathcal{A}$, and holds a uniformly distributed randomness $r_{\mathcal{S}} \in \{0,1\}^*$. The parties P_2' and P_2 hold the same private input H.
- $\mathcal{S}$ works as follows.
  - Step Sim1: In this step, $\mathcal{S}$ performs initialization in a similar way that the simulator for the first framework does in the case that P_1 is corrupted. That is, $\mathcal{S}$ corrupts P_2' and learns P_2''s private input H. Then, $\mathcal{S}$ takes $\mathcal{A}$'s copy $\bar{\mathcal{A}}$ as a subroutine, fixes $\bar{\mathcal{A}}$'s initial input to be $k, I, z, r_{\bar{\mathcal{A}}}$ ($r_{\bar{\mathcal{A}}} \in_U \{0,1\}^*$), activates $\bar{\mathcal{A}}$ and supplies $\bar{\mathcal{A}}$ with H. Now, $\bar{\mathcal{A}}$ has engaged in the framework.
  - Convention: In the following interactions between $\bar{\mathcal{A}}$ and $\mathcal{S}$, in case $\bar{\mathcal{A}}$ refuses to send some

message which is supposed to be sent or $\bar{\mathcal{A}}$ sends an invalid message that $\mathcal{S}$ can not process, $\mathcal{S}$ sends $abort_2$ to the TTP, and halts with outputting whatever $\bar{\mathcal{A}}$ outputs. We make this convention here, and will not explicitly iterate these details any more.

– Step Sim2: Playing the role of P_1, $\mathcal{S}$ honestly executes the sender's steps until reaching the beginning of Step S2.3. If Step S2.3 is reached, $\mathcal{S}$ records the choice indicator r and the messages, denoted by msg, which he sends to $\bar{\mathcal{A}}$. Then $\mathcal{S}$ proceeds to the next step. Otherwise, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.
– Step Sim3: $\mathcal{S}$ repeats the following procedure, denoted by Ξ, until the hash parameters and the instance vectors that $\bar{A}$ sends in Step R1 of the framework pass the check.
  Ξ: $\mathcal{S}$ rewinds $\bar{\mathcal{A}}$ to the beginning of Step R2, and honestly follows the sender's steps to interact with $\bar{\mathcal{A}}$ from Step S1 to the beginning of Step S2.3.
  In each repetition, the randomness $\mathcal{S}$ uses is fresh. Finishing the repetition, $\mathcal{S}$ records the choice indicator $\tilde{r}$ and the messages $\bar{\mathcal{A}}$ sent to open the chosen instance vectors in the last repetition.
– Step Sim4:
  * Case 1, $r = \tilde{r}$. $\mathcal{S}$ outputs $failure$ and halts.
  * Case 2, $r \neq \tilde{r}$ and $\forall i(r\langle i\rangle \neq \tilde{r}\langle i\rangle \rightarrow r\langle i\rangle = 1 \wedge \tilde{r}\langle i\rangle = 0)$. $\mathcal{S}$ runs from scratch.
  * Case 3, $r \neq \tilde{r}$ and $\exists i(r\langle i\rangle = 0 \wedge \tilde{r}\langle i\rangle = 1)$. $\mathcal{S}$ records arbitrary one of these is, denotes it by e, and proceeds to the next step.

  Remark 21. *The aim of Step Sim3 and Sim4 is to extract one of $\bar{\mathcal{A}}$'s possible private inputs. If Case 3 of Step Sim4 happens, $\mathcal{S}$ knows which instances in $\vec{\tilde{x}}_e$ are smooth. What is more, $\vec{\tilde{x}}_e$ is indeed a legal instance vector. This is because $\vec{\tilde{x}}_e$ passes the check executed by $\mathcal{S}$ in Step Sim3. Combing π_e^2 received in Step Sim2, $\mathcal{S}$ knows the private input corresponding to $\pi_e^2(\vec{\tilde{x}}_e)$ of $\bar{\mathcal{A}}$.*
  We stress the fact that this private input is one of $\bar{\mathcal{A}}$'s possible private inputs. Because the elements in $\{\pi_i^2(\vec{\tilde{x}}_i)|i \in [K], r\langle i\rangle = 0\}$ may correspond to distinct private inputs. However, as we will see in the proof of Proportion 24, this inconsistence does not bring any trouble to the correctness of our simulation because of $SPHDHC_{t,h}$'s property smoothness.
  Note that, $\bar{\mathcal{A}}$'s initial input is fixed by $\mathcal{S}$ in Step Sim1. So receiving the same messages, $\bar{\mathcal{A}}$ responds in the same way. Therefore, $\mathcal{S}$ can reproduce the same scenario as he meets in Step Sim2 by rewinding $\bar{\mathcal{A}}$ to the beginning of Step R2 and sending the message sent in Step Sim2.
– Step Sim5: $\mathcal{S}$ rewinds $\bar{\mathcal{A}}$ to the beginning of Step R2 of the framework, and sends msg previously recorded to $\bar{\mathcal{A}}$ in order. According to the analysis of Remark 21, $\mathcal{S}$ can extract one of $\bar{\mathcal{A}}$'s possible private inputs. We denote this one by H'_e. $\mathcal{S}$ does so and gets H'_e.
– Step Sim6: $\mathcal{S}$ sends H'_e to TTP and receives back $\vec{m}\langle H_e\rangle$. $\mathcal{S}$ then sends $continue$ to TTP. To proceed to interact with $\bar{\mathcal{A}}$, $\mathcal{S}$ fabricates $\vec{m}'$ as follows. For each $i \in H'_e$, $\vec{m}'\langle i\rangle \leftarrow \vec{m}\langle i\rangle$. For each $i \notin H'_e$, set $\vec{m}'\langle i\rangle$ to be an arbitrary message of appropriate length.
– Step Sim7: Playing the role of P_1 with private input $\vec{m}'$, $\mathcal{S}$ follows Step S2.3 to complete the interaction with $\bar{\mathcal{A}}$. When $\bar{\mathcal{A}}$ halts, $\mathcal{S}$ halts with outputting what $\bar{\mathcal{A}}$ outputs.

Lemma 22. *The simulator $\mathcal{S}$ is expected polynomial-time.*

Proof: First, let us focus on Step Sim3. In each repetition of Ξ, the chosen instance vectors are uniformly distributed. This leads to the probability that $\bar{\mathcal{A}}$ passes the check in each repetition is the same. Denote this probability by p. The expected time of Step Sim3 is

$$ExpTime_{Sim3} = (1/p) \cdot Time_{\Xi}$$

Under the same analysis, the probability that $\bar{\mathcal{A}}$ passes the check in Step Sim2 is p too. Then, the expected time that $\mathcal{S}$ runs once from Step Sim1 to the beginning of Step Sim4 is

$$\begin{aligned} OncExpTime_{Sim1\rightarrow Sim4} &\leq Time_{Sim1} + Time_{Sim2} \\ &\quad + p \cdot ExpTime_{Sim3} \\ &= Time_{Sim1} + Time_{Sim2} \\ &\quad + Time_{\Xi} \end{aligned}$$

Second, let us focus on Step Sim4, especially Case 2. Note that the initial input $\mathcal{S}$ holds is the same in each trial. Thus the probability that $\mathcal{S}$ runs from scratch in each trial is the same. We denote this probability by $1-q$. Then the expected time that $\mathcal{S}$ runs from Step Sim1 to the beginning of Step Sim5 is

$$\begin{aligned} ExpTime_{Sim1\rightarrow Sim5} &\leq (1 + 1/q) \\ &\quad \cdot (OncExpTime_{Sim1\rightarrow Sim4} \\ &\quad + Time_{Sim4}) \\ &= (1 + 1/q) \cdot (Time_{Sim1} + \\ &\quad Time_{Sim2} + Time_{\Xi} + Time_{Sim4}) \end{aligned}$$

The reason why there is 1 in $(1 + 1/q)$ is that $\mathcal{S}$ has to run from scratch at least one time in any case.

The expected running time of $\mathcal{S}$ in a whole execution is

$$\begin{aligned} ExpTime_{\mathcal{S}} &\leq ExpTime_{Sim1\rightarrow Sim5} + Time_{Sim5} \\ &\quad + Time_{Sim6} + Time_{Sim7} \\ &= (1 + 1/q) \cdot (Time_{Sim1} + Time_{Sim2} \\ &\quad + Time_{\Xi} + Time_{Sim4}) \\ &\quad + Time_{Sim5} + Time_{Sim6} + Time_{Sim7} \end{aligned} \quad (5)$$

Third, let us estimate the value of q, which is the probability that $\mathcal{S}$ does not run from scratch in a trial.

We denote this event by C. It's easy to see that event C happens, if and only if one of the following mutually disjoint events happens.

1) Event B happens, where B denotes the event that $\mathcal{S}$ halts before reaching Step Sim3.
2) Event $\bar{B}$ happens and $R = \tilde{R}$, where R and $\tilde{R}$ respectively denote the random variables which are defined as the choice indicators $\mathcal{S}$ records in Step Sim2 and Step Sim3.
3) Event $\bar{B}$ happens and there exists i such that $R\langle i\rangle = 0 \wedge \tilde{R}\langle i\rangle = 1$.

So

$$\begin{aligned} q =& Pr(C) \\ =& Pr(B) + Pr(\bar{B} \cap R = \tilde{R}) \\ & + Pr(\bar{B} \cap \exists i(R\langle i\rangle = 0 \wedge \tilde{R}\langle i\rangle = 1)) \\ =& Pr(B) + Pr(\bar{B}) \cdot (Pr(R = \tilde{R}|\bar{B}) \\ & + Pr(\exists i(R\langle i\rangle = 0 \wedge \tilde{R}\langle i\rangle = 1)|\bar{B})) \end{aligned} \tag{6}$$

Let $S_1 \stackrel{def}{=} \{(r,\tilde{r})|(r,\tilde{r}) \in (\{0,1\}^K)^2, r = \tilde{r}\}$, $S_2 \stackrel{def}{=} \{(r,\tilde{r})|(r,\tilde{r}) \in (\{0,1\}^K)^2, r \neq \tilde{r}, \forall i(r\langle i\rangle \neq \tilde{r}\langle i\rangle \rightarrow r\langle i\rangle = 1 \wedge \tilde{r}\langle i\rangle = 0)\}$, $S_3 \stackrel{def}{=} \{(r,\tilde{r})|(r,\tilde{r}) \in (\{0,1\}^K)^2, r \neq \tilde{r}, \exists i(i \in [K] \wedge r\langle i\rangle = 0 \wedge \tilde{r}\langle i\rangle = 1)\}$. It is easy to see that S_1, S_2, S_3 constitute a complete partition of $(\{0,1\}^K)^2$ and $\#S_1 = 2^K$, $\#S_2 = \#S_3 = (2^K \cdot 2^K - 2^K)/2$.

Since both R and $\tilde{R}$ are uniformly distributed, we have

$$Pr(R = \tilde{R}|\bar{B}) = \#S_1/\#(\{0,1\}^K)^2 = 1/2^K \tag{7}$$

and

$$\begin{aligned} Pr(\exists i(R\langle i\rangle = 0 \wedge \tilde{R}\langle i\rangle = 1)|\bar{B}) &= \#S_3/\#(\{0,1\}^K)^2 \\ &= 1/2 - 1/2^{K+1} \end{aligned} \tag{8}$$

Combining equation (6), (7) and (8), we have

$$\begin{aligned} q &= Pr(B) + Pr(\bar{B})(1/2 + 1/2^{K+1}) \\ &= 1/2 + 1/2^{K+1} + (1/2 - 1/2^{K+1})Pr(B) \\ &> 1/2 \end{aligned} \tag{9}$$

Combining equation (5) and (9), we have

$$\begin{aligned} ExpTime_{A'} < 3(&Time_{Sim1} + Time_{Sim2} \\ &+ Time_{\Xi} + Time_{Sim4}) \\ &+ Time_{Sim5} + Time_{Sim6} + Time_{Sim7} \end{aligned}$$

which means the expected running time of $\mathcal{S}$ is bounded by a polynomial. □

Lemma 23. *The probability that $\mathcal{S}$ outputs $failure$ is less than $1/2^{K-1}$.*

Proof: Let X be a random variable defined as the number of the trials in a whole execution. From the proof of Proposition 22, we know two facts. First, $Pr(X = i) = (1-q)^{i-1}q < 1/2^{i-1}$. Second, in each trial the event that $\mathcal{S}$ outputs $failure$ is the combined event of $\bar{B}$ and $R = \tilde{R}$, where the combined event happens with the following probability.

$$Pr(\bar{B} \cap R = \tilde{R}) = Pr(\bar{B})Pr(R = \tilde{R}|\bar{B}) \leq Pr(R = \tilde{R}|\bar{B})$$

Combining equation (7), this probability is not more than $1/2^K$. Therefore, the probability that $\mathcal{S}$ outputs $failure$ in a whole execution is

$$\begin{aligned} \sum_{i=1}^{\infty} Pr(X = i)Pr(\bar{B} \cap R = \tilde{R}) &< (1/2^K) \cdot \sum_{i=1}^{\infty} 1/2^{i-1} \\ &= 1/2^{K-1} \end{aligned}$$

□

Lemma 24. *The output of the adversary $\mathcal{A}$ in the real world and that of the simulator $\mathcal{S}$ in the ideal world are computationally indistinguishable, i.e.,*

$$\begin{aligned} &\{Real_{\pi,\{2\},\mathcal{A}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}} \stackrel{c}{=} \\ &\{Ideal_{f,\{2\},\mathcal{S}(z_k)}(1^k, \vec{m}, H)\langle 0\rangle\}_{\substack{k\in\mathbb{N},\vec{m}\in(\{0,1\}^*)^n \\ H\in\Psi, z_k\in\{0,1\}^*}} \end{aligned}$$

Proof: First, we claim that the outputs of $\mathcal{S}$ and $\bar{\mathcal{A}}$ are computationally indistinguishable. The only difference between the output of $\mathcal{S}$ and that of $\bar{\mathcal{A}}$ is that $\mathcal{S}$ may outputs $failure$. Since Lemma 23 shows that this difference arises with negligible probability, our claim holds.

Second, we claim that the outputs of $\mathcal{A}$ and $\bar{\mathcal{A}}$ are computationally indistinguishable. The only point makes $\bar{\mathcal{A}}$'s view different from $\mathcal{A}$'s view is that the ciphertext $\vec{c}'$ $\bar{\mathcal{A}}$ receives is generated by encrypting fabricated $\vec{m}'$ rather than party P_1's private input $\vec{m}$. This is the situation we meet in the proof of Lemma 14. Therefore, this proof can be done in a similar way with little modification.

Let $H' \stackrel{def}{=} \cap_{i\in\overline{CS}} H'_i$. The proof of the Lemma 14 shows H' is the effective private input of $\bar{\mathcal{A}}$ and $\mathcal{A}$. If $H'_e = H'$, the proof of the second claim can be done in the same way as the proof of the Lemma 14 is done. If $H'_e \supset H'$, for each $j \in H'_e - H'$, there exists $\pi_i^2(\tilde{\vec{x}}_i)$ whose j-th entry is a smooth instance, which leads to both $\vec{c}\langle j\rangle$ seen by $\mathcal{A}$ and $\vec{c}'\langle j\rangle$ seen by $\bar{\mathcal{A}}$ are strings uniformly distributed over the same domain. Therefore, $\mathcal{A}$'s view and $\bar{\mathcal{A}}$'s also are indistinguishable in this case.

Combining the above two claims, this lemma holds. □

Proposition 25. *In the case that P_2 was corrupted, i.e., $I = \{2\}$, the equation* (2) *required by Definition 2 holds.*

Proof: Note that the honest parties P_1 and P'_1 end up with outputting nothing. Thus, the fact that the outputs of $\mathcal{S}$ and $\mathcal{A}$ are computationally indistinguishable, which is supported by Lemma 24, directly proves this proposition. □

4.3 The Communication Rounds

It is easy to see that this framework costs four communication rounds. Since this framework is secure against malicious adversaries under half simulation, we only compare it with protocols that are secure on this level, and protocols that are secure on standard level.

Compared with the protocols for OT_1^2 with half-simulatable security, i.e., the protocols presented by [2],

[29], [36], our framework is not efficient in communications, since these protocols only cost two rounds. However, we argue that our framework still makes sense. Since the known protocols for OT_h^n on this security level, i.e., the protocols presented by [34], [35], [37], are not constant-round and only work when h is far less than n, e.g., $n = 10^6$, $h \leq 6$. Moreover, theses protocols seems too complicated to be understood well. To our best knowledge, our framework is the first protocol for general OT_h^n on this security level.

Compared with the protocols with standard security, the conclusions for the framework constructed in Section 3.1 still holds for this framework. Because the two framework cost the same number of communication rounds. Thus, this framework is more efficient than the protocols with standard security in communication rounds.

4.4 The Computational Overhead

In Step S2, P_1 takes $n \cdot \#\overline{CS}$ invocations of $Hash(.)$ to encrypt his private input. In Step R3, P_2 takes $h \cdot \#\overline{CS}$ invocations of $pHash(.)$ to decrypt the messages he deserves. The value of $\#\overline{CS}$ is K, $K/2$, respectively, in the worst case and in the average case. Thus, given the same problem (i.e., fixing the values of n and h), the efficiency only depends on the value of K. Lemma 23 shows that the simulator may fails with probability at most $1/2^{K-1}$ in the case that P_2 is corrupted. Thus, conditioning on the cryptographic primitives without being broken, the real world and the ideal world can be distinguished at most $1/2^{K-2}$. Setting K to be 40, we obtain such a probability 3.6×10^{-12}, which is secure enough to be used in practice. As a result, in the worst case, the computational overhead mainly consists of $40n$ invocations of $Hash()$ taken by P_1 and $40h$ invocations of $pHash()$ taken by P_2; in the worst case, the computational overhead mainly consists of $20n$ invocations of $Hash()$ taken by P_1 and $20h$ invocations of $pHash()$ taken by P_2.

Compared with the protocols with the same security level, the computational overhead of DDH-based instantiation of this framework is about 40 times as much as that of [2], [29], [36], and is about $\frac{40n}{Wh \log n}$ times as much as that of [34], [35], [37]. Therefore, our framework is not efficient. However, as pointed in Section 4.3, our framework still makes sense.

Compared with the protocols for OT_h^n with standard security, our DDH-based instantiation is the most efficient one. Following the analysis in Section 3.5, the operations of the protocol in [7], [25] are far more expensive than DDH-based operations. Though the DDH-based instantiation in [43] costs the same amount of pubic-key operations as ours, it costs two additional private-key operations, i.e., two commitment operations. Getting together these facts, the claim is true.

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