# Biclique Attack of the Full ARIA-256 

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#### Abstract

In this paper, combining the biclique cryptanalysis with the MITM attack, we present the first key recovery method for the full ARIA256 faster than brute-force. The attack requires $2^{80}$ chosen plaintexts, and the time complexity is about $2^{255.2}$ full-round ARIA encryptions in the processing phase.


keywords: Block Cipher; ARIA-256; Biclique structure

## 1 Introduction

The ARIA block cipher [1]was designed by a group of Korean experts in 2003, and was standardized as the Korean Standard in 2004. ARIA supports key length of 128,192 and 256 bits, these versions of ARIA are denoted as ARIA-128, ARIA-192 and ARIA-256. The number of rounds for these three versions are 12, 14 and 16 , respectively. In ref. [1], the designers of ARIA presented some cryptanalysis including both differential cryptanalysis, linear cryptanalysis, and some other known attacks . Later Biryukov et al. performed an evaluation of ARIA [2], however, they especially focused on truncated differential cryptanalysis and dedicated linear cryptanalysis. In ref. [3], Wu et al. firstly found some non-trivial 4-round impossible differentials which led to a 6 -round attack on ARIA. The security of ARIA against boomerang attack was presented by Fleischmann et al. in [4]. Li et al. firstly
found some 3 -round integral distinguishers by counting methods, which also led up to a 6 -round integral attack on ARIA-192 [5], recently, Li et al. [6] presented the meet-in-the-middle(MITM) attacks on the round-reduced ARIA, which led to a 8 -round attack on ARIA-256. For ARIA-256, the number of cryptanalyzed round did not increase since then.

ARIA with its wide-trail strategy was designed to withstand differential and linear cryptanalysis. ARIA is an involution SPN cipher and has involution substitution and diffusion layers by using operations such as multiplications in a finite field. In the diffusion layer of ARIA, a $16 \times 16$ binary matrix of the maximum branch number 8 is used to avoid some attacks well applied to the reduced round of Rijndael.

The basic concept of the MITM attack was proposed by Diffie and Hellman [7]. So far, this attack has been applied to several block ciphers. Furthermore, over the past few years, this attack has been improved in a line of preimage attacks on hash functions and several novel techniques are introduced.

Biclique cryptanalysis was first introduced for hash cryptanalysis [8]. Andrey Bogdanov* et al. carried over the concept of the independent bicilique to AES cryptanalysis [9], and constructed a 3 round biclique. Combining the biclique with the MITM attack, they presented the first key recovery method for the full AES-128 faster than brute-force. To construct the independent biclique, we need two independent related-key differentials. When it comes to the ciphers whose diffusion operation is fast, it is difficult to construct the biclique.

## Our Contribution:

In this paper, we combine the Biclique cryptanalysis with the MITM attack, and present the first key recovery method for the full ARIA-256 faster than brute-force.

1. Four 128-bit values $W_{0}, W_{1}, W_{2}, W_{3}$ are generated from the master key $M K$ by using a 3 -round 256 -bit Feistel cipher. We select those types of differentials of $M K$ such that the difference of $W_{1}$ is 0 . Then we can construct a 2-round biclique for ARIA-256. However, those types of differentials of $M K$ in other versions of ARIA are not exist.
2. Because the position of active-byte of the master key will affect the number of active-byte of the round key, we found a type of keys such that the round key have the least number of active-bytes in round 3 . Hence, the computation complexity can be decreased as well as possible.
3. The attack on ARIA-256 consists of two parts. Firstly, we construct a 2 -round biclique of dimension 8 for ARIA-256. Then, we attack the remaining 14 rounds by using MITM method. By matching only on a single byte, the computation complexity can be reduced. Table 1 gives the comparison between our result and earlier best result.

Table 1 Summary of Attacks on ARIA-256

| Rounds | Data | Complexity | Type of attack | Paper |
| :---: | :---: | :---: | :---: | :---: |
| 8 | $2^{56}$ | $2^{251.6}$ | MITM Attack | $[6]$ |
| 16 | $2^{80}$ | $2^{255.2}$ | Biclique Key Recovery Attack | This paper |

This paper is organized as follows. We introduce the biclique key recovery attack in Section 2 and give a brief description of ARIA in Section 3. In Section 4, we present a biclique key recovery attack for the full round ARIA256. Finally, section 5 summarizes this paper.

## 2 Biclique Key Recovery Attack

The main idea behind the biclique key recovery attack is based on the biclique structure which can be used to decrease the complexity of the MITM attack. It is apparent to see that the construction of biclique is an important step. The following sections will describe the steps of the biclique key recovery attack.

### 2.1 Biclique Structure

Now we introduce the notation of a biclique. Let $f$ be a subcipher that maps a plaintext $P$ to the internal state $S: f_{K}(P)=S$.

The 3-tuple $\left[\left\{P_{i}\right\},\left\{S_{j}\right\},\{K[i, j]\}\right]$ is called a $d$-dimensional Biclique, if

$$
S_{j}=f_{K[i, j]}\left(P_{i}\right)
$$

### 2.2 Biclique from Independent Related-Key Differentials

Definition 1.A byte is called an active-byte, if the input differential of it is non-zero. Otherwise, it is called a non-active-byte.

Definition 2.If the trail of a differential do not share active nonlinear components (such as active S-boxes in ARIA) with the trail of another differential, we call them independent related-key differentials.

Let the key $K[0,0]$ map plaintext $P_{0}$ to the internal state $S_{0}$, and consider two sets of $2^{d}$ related-key differentials each over $f$ with respect to the base computation $P_{0} \xrightarrow[f]{K[0,0]} S_{0}$.

- $\nabla_{i}$-differentials. A differential in the first set maps input difference $\nabla_{i}$ to an output difference 0 under key difference $\nabla_{i}^{K}$ :

$$
\nabla_{i} \xrightarrow[f]{\nabla_{i}^{K}} 0, \text { with } \nabla_{0}^{K}=0, \nabla_{0}=0
$$

- $\triangle_{j}$-differentials. A differential in the second set maps input difference 0 to an output difference $\triangle_{j}$ under key difference $\triangle_{j}^{K}$ :

$$
0 \xrightarrow[f]{\triangle_{j}^{K}} \triangle_{j}, \text { with } \triangle_{0}^{K}=0, \triangle_{0}=0
$$

If $\nabla_{i}$-differentials and $\triangle_{j}$-differentials are independent, then we can construct $2^{2 d}$ combined differentials:

$$
\nabla_{i} \xrightarrow[f]{\nabla_{i}^{K} \oplus \Delta_{j}^{K}} \triangle_{j}, \text { for all } i, j \in\left\{0, \ldots, 2^{d}-1\right\}
$$

Substituting $P_{0}, S_{0}$, and $K[0,0]$ to the combined differentials, one obtains:

$$
P_{0} \oplus \nabla_{i} \xrightarrow[f]{K[0,0] \oplus \nabla_{i}^{K} \oplus \triangle_{j}^{K}} S_{0} \oplus \triangle_{j}
$$

Finally, we put:

$$
\begin{gathered}
P_{i}=P_{0} \oplus \nabla_{i}, \\
S_{j}=S_{0} \oplus \triangle_{j}, \\
K[i, j]=K[0,0] \oplus \nabla_{i}^{K} \oplus \triangle_{j}^{K}
\end{gathered}
$$

and get exactly the definition of a $d$-dimensional biclique.

### 2.3 Biclique Key Recovery Attack

Based on the biclique from independent related-key differentials, we describe the biclique key recovery attack.

For each set of keys the adversary builds a structure of $2^{d}$ plaintexts $P_{i}$ and $2^{d}$ intermediate states $S_{j}$ with respect to the set of keys $K[i, j]$ which satisfies the following condition:

$$
S_{j}=f_{K[i, j]}\left(P_{i}\right)
$$

The adversary asks the oracle to encrypt plaintexts $P_{i}$ under the secret key $K_{\text {secret }}$, and obtains the $2^{d}$ ciphertexts $C_{i}$ :

$$
P_{i} \xrightarrow[e]{\text { oracle }} C_{i}
$$

If the texted key $K[i, j]$ satisfies the following condition:

$$
\begin{equation*}
S_{j} \xrightarrow[e_{1}]{K[i, j]} \vec{v}=\overleftarrow{v} \underset{e_{2}}{\stackrel{K[i, j]}{\longleftarrow}} C_{i} \tag{1}
\end{equation*}
$$

We consider it is a key candidate.

### 2.4 Matching with Precomputations

Here we describe the idea of matching with precomputations. This can be seen as an efficient way of checking equation (1).

Firstly, the adversary computes and stores in memory $2^{d+1}$ full computations

$$
\text { for all } j S_{j} \xrightarrow{K[0, j]} \vec{v} \text { and for all } i \overleftarrow{v} \stackrel{K[i, 0]}{\longleftarrow} C_{i}
$$

Then for particular $i, j$ he recomputes only those parts of state that differ from the stored ones. Hence, he can decrease the computational complexity.

## 3 Description of ARIA-256

The ARIA-256 block cipher is an involution SPN cipher and have involution substitution and diffusion layers. The diffusion layer uses a $16 \times 16$ binary matrix in $G F\left(2^{8}\right)$. The substitution layer consists of sixteen $8 \times 8$ S-boxes. The 128 bits state can be denoted as $4 \times 4$ matrix in $G F\left(2^{8}\right)$ as shown in Fig. 1.

Each round of the cipher consists of three parts, Round key addition $(A K)$, Substitution layer $(S L)$, Diffusion layer $(D L)$. Note that the diffusion layer of the last round is replaced by a round key addition.

| $x_{0}$ | $x_{4}$ | $x_{8}$ | $x_{12}$ |
| :--- | :--- | :--- | :--- |
| $x_{1}$ | $x_{5}$ | $x_{9}$ | $x_{13}$ |
| $x_{2}$ | $x_{6}$ | $x_{10}$ | $x_{14}$ |
| $x_{3}$ | $x_{7}$ | $x_{11}$ | $x_{15}$ |

Figure 1: The state of ARIA

$$
\begin{aligned}
& \begin{array}{l|l|l|l|l|l|l|l|l|l|l|l|l|l|l|l|}
\hline S_{1} & S_{2} & S_{1}^{-1} & S_{2}^{-1} & S_{1} & S_{2} & S_{1}^{-1} & S_{2}^{-1} & S_{1} & S_{2} & S_{1}^{-1} & S_{2}^{-1} & S_{1} & S_{2} & S_{1}^{-1} & S_{2}^{-1} \\
\hline
\end{array} \\
& \text { S-box layer type } 1
\end{aligned}
$$

S-box layer type 2

Figure 2: Two types of S-box layer

Substitution layer: nolinear substitution, the Substitution layer consists of two types of S-box layers. Type 1 is used in the odd rounds and type 2 is used in the even rounds as shown in Fig. 2.

Diffusion layer: $P: G F\left(2^{8}\right)^{16} \rightarrow G F\left(2^{8}\right)^{16}$ is given by

$$
\left(x_{0}, x_{1}, \ldots, x_{15}\right) \rightarrow\left(y_{0}, y_{1}, \ldots, y_{15}\right)
$$

where
$y_{0}=x_{3} \oplus x_{4} \oplus x_{6} \oplus x_{8} \oplus x_{9} \oplus x_{13} \oplus x_{14}, y_{8}=x_{0} \oplus x_{1} \oplus x_{4} \oplus x_{7} \oplus x_{10} \oplus x_{13} \oplus x_{15}$,
$y_{1}=x_{2} \oplus x_{5} \oplus x_{7} \oplus x_{8} \oplus x_{9} \oplus x_{12} \oplus x_{15}, y_{9}=x_{0} \oplus x_{1} \oplus x_{5} \oplus x_{6} \oplus x_{11} \oplus x_{12} \oplus x_{14}$,
$y_{2}=x_{1} \oplus x_{4} \oplus x_{6} \oplus x_{10} \oplus x_{11} \oplus x_{12} \oplus x_{15}, y_{10}=x_{2} \oplus x_{3} \oplus x_{5} \oplus x_{6} \oplus x_{8} \oplus x_{13} \oplus x_{15}$,
$y_{3}=x_{0} \oplus x_{5} \oplus x_{7} \oplus x_{10} \oplus x_{11} \oplus x_{13} \oplus x_{14}, y_{11}=x_{2} \oplus x_{3} \oplus x_{4} \oplus x_{7} \oplus x_{9} \oplus x_{12} \oplus x_{14}$,
$y_{4}=x_{0} \oplus x_{2} \oplus x_{5} \oplus x_{8} \oplus x_{11} \oplus x_{14} \oplus x_{15}, y_{12}=x_{1} \oplus x_{2} \oplus x_{6} \oplus x_{7} \oplus x_{9} \oplus x_{11} \oplus x_{12}$,
$y_{5}=x_{1} \oplus x_{3} \oplus x_{4} \oplus x_{9} \oplus x_{10} \oplus x_{14} \oplus x_{15}, y_{13}=x_{0} \oplus x_{3} \oplus x_{6} \oplus x_{7} \oplus x_{8} \oplus x_{10} \oplus x_{13}$,
$y_{6}=x_{0} \oplus x_{2} \oplus x_{7} \oplus x_{9} \oplus x_{10} \oplus x_{12} \oplus x_{13}, y_{14}=x_{0} \oplus x_{3} \oplus x_{4} \oplus x_{5} \oplus x_{9} \oplus x_{11} \oplus x_{14}$,
$y_{7}=x_{1} \oplus x_{3} \oplus x_{6} \oplus x_{8} \oplus x_{11} \oplus x_{12} \oplus x_{13}, y_{15}=x_{1} \oplus x_{2} \oplus x_{4} \oplus x_{5} \oplus x_{8} \oplus x_{10} \oplus x_{15}$,
We address two internal states in each round as follows in ARIA-256: \#1 is the state after $A K$ in round $1, \# 2$ is the state after $D L$ in round $1, \# 3$

| $\rightarrow K L$ |  |  |  | $\rightarrow K R \leftarrow$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $K L_{0}$ | $K L_{4}$ | KL | $K L_{12}$ | KR | KR | KR | ${ }^{R_{1}}$ |
| кц | KL | KL. | $K L_{13}$ | KR | KR | KR | KR |
| $K L_{2}$ | KL | $K L_{0}$ | $K L_{4}$ | KR | KR | $K R_{10}$ | KR |
| $K L_{s}$ | KL | KL | $K L_{15}$ | $K_{3}$ | $K R_{7}$ | $K R_{11}$ | $K R_{5}$ |

Figure 3: $M K$ of ARIA-256


Figure 4: Initialization part of key schedule
is the state after $A K$ in round $2, \ldots, \# 31$ is the state after the first $A K$ in round $16, \# 32$ is the state after the second $A K$ in round 16.

The 256 bites initial key $M K$ of ARIA- 256 can be rearranged as a $4 \times 8$ matrix as shown in Fig .3.

The key schedule of ARIA-256 consists of two parts, which are initialization and round key generation as follows:
(1) Initialization

In the initialization part, the master key $M K$ denote as follow: $M K=$ $K L \| K R$. Four 128-bit values $W_{0}, W_{1}, W_{2}, W_{3}$ are generated from the master key $M K$, by using a 3 -round 256 -bit Feistel cipher.

$$
\begin{array}{lrl}
W_{0} & =K L, & W_{2}
\end{array}=F_{e}\left(W_{1}, C K_{2}\right) \oplus W_{0} 0 .
$$

The initialization process is given in Fig. 4.
Here, $F_{o}$ and $F_{e}$ are even and odd round functions, respectively, The 128-bit keys $C K_{i}$ of the round functions are constants.
(2) Round key generation


Figure 5: Initialization key $M K[0,0]$

In the generation part, combining the four 18 -bits values $W_{0}, W_{1}, W_{2}$, $W_{3}$ to obtain encryption round keys $e k_{i}$ as follows:

$$
\begin{array}{ll}
e k_{1}=\left(W_{0}\right) \oplus\left(W_{1} \ggg 19\right), & e k_{2}=\left(W_{1}\right) \oplus\left(W_{2} \ggg 19\right), \\
e k_{3}=\left(W_{2}\right) \oplus\left(W_{3} \ggg 19\right), & e k_{4}=\left(W_{0} \ggg 19\right) \oplus\left(W_{3}\right), \\
e k_{5}=\left(W_{0}\right) \oplus\left(W_{1} \ggg 31\right), & e k_{6}=\left(W_{1}\right) \oplus\left(W_{2} \ggg 31\right), \\
e k_{7}=\left(W_{2}\right) \oplus\left(W_{3} \ggg 31\right), & e k_{8}=\left(W_{0} \ggg 31\right) \oplus\left(W_{3}\right), \\
e k_{9}=\left(W_{0}\right) \oplus\left(W_{1} \ggg 61\right), & e k_{10}=\left(W_{1}\right) \oplus\left(W_{2} \ggg 61\right), \\
e k_{11}=\left(W_{2}\right) \oplus\left(W_{3} \ggg 61\right), e k_{12}=\left(W_{0} \ggg 61\right) \oplus\left(W_{3}\right), \\
e k_{13}=\left(W_{0}\right) \oplus\left(W_{1} \lll 31\right), e k_{14}=\left(W_{1}\right) \oplus\left(W_{2} \lll 31\right), \\
e k_{15}=\left(W_{2}\right) \oplus\left(W_{3} \lll 31\right), e k_{16}=\left(W_{0} \lll 31\right) \oplus\left(W_{3}\right), \\
e k_{17}=\left(W_{0}\right) \oplus\left(W_{1} \lll 19\right) .
\end{array}
$$

## 4 Biclique Key Recovery Attack for the Full ARIA-256

In this section, combining the Biclique cryptanalysis with the MITM attack, we present the first key recovery method for the full ARIA- 256 faster than brute-force.

### 4.1 Key Partitioning

We define the key sets respect to the master key $M K$. The base keys $M K[0,0]=K L \| K R$ are all possible $2^{240} 32$-byte values with $K L_{0}=K L_{9}=$ 0 whereas the remaining 30 bytes run through all values as shown in Fig. 5.

The keys $\{M K[i, j]\}$ in a set are enumerated by all possible byte differences $i$ and $j$ with respect to the $K L$ value and $F_{o}\left(K L, C K_{1}\right) \oplus F_{o}(K L \oplus$


Figure 6: Key set $\{M K[i, j]\}$
$\left.T^{i, j}, C K_{1}\right)\left(T^{i, j}\right.$ denote the input differences of $\left.K L\right)$ value respect to the $K R$ value of the base key $M K[0,0]$ as shown in Fig. 6.

Hence, the ARIA-256 key space is partitioned into the $2^{240}$ sets of $2^{16}$ keys each.

### 4.2 2-Round Biclique of Dimension 8 for ARIA-256

In this section, we aim to construct a biclique such that the number of rounds contained is as long as possible. Because the diffusion layer of ARIA-256 has the maximum branch number 8,1 active-byte will transform to 16 activebytes after 2 round. We only can construct a 2 -round biclique from combined related-key differentials as described in section 2.2.

1. The adversary fixes $P_{0}=0$ and derives $S_{0}=f_{M K[0,0]}\left(P_{0}\right)$

Note that the diffusion layer of the last round is replaced by a round key addition, it is difficult to construct a long biclique on the bottom of ARIA256. Hence, we choose the biclique on the top of ARIA-256 which differs from [9].
2. Constructing key differences $\nabla_{i}^{K}$ and $\triangle_{j}^{K}$

The $\nabla_{i}$-differentials are based on the key difference $\nabla_{i}^{K}$, and $\triangle_{j}$-differentials are based on the key difference $\triangle_{j}^{K}$. The choice of $\nabla_{i}^{K}$ and $\triangle_{j}^{K}$ is dominated by two aspects. On one hand, in order to construct two independent relatedkey differentials, the number of active-bytes of the first 2 round key should be as few as possible. Note that the round keys $e k_{i}$ are generated by $W_{0}, W_{1}$, $W_{2}, W_{3}$. we choose those kind of differences which can make the differences of these four 128 -bits values as simple as possible. On the other hand, note that the position of active-byte of the master key will affect the number of active-bytes of the round key, in order to decrease the number of $S L$ and $D L$ bytes which need to be recomputed, we need a type of key that can makes


Figure 7: Initialization Key differentials $\nabla_{i}^{K}$ and $\triangle_{j}^{K}$
the round key have the least number of active-bytes in round 3 after round key generation.

According to the two aspects above, we choose the key as shown in Fig. 7. Key differences $\nabla_{i}^{K}$ are enumerated by all possible byte differences $i$ with respect to the $K L$ value and the $F_{o}\left(K L, C K_{1}\right) \oplus F_{o}\left(K L \oplus T^{i}, C K_{1}\right)\left(T^{i}\right.$ denote the input differences of $K L$ ) value respect to the $K R$ value of the base key $M K[0,0]$. Hence, the difference of $W_{1}$ is 0 and the difference of $W_{2}$ is the same as the difference of $W_{0}$. A similar property hold for $\triangle_{j}^{K}$. Furthermore, the number of active-bytes of $e k_{3}$ in $\nabla_{i}$-differentials are determined by the position of the 7 active-bytes of $W_{3}$, and the position of differences $i$ affect the position of the 7 active-bytes of $W_{3}$. We found $K L_{9}$ is the best position of active-byte that can make $e k_{3}$ have the least number of active-bytes.
3. Constructing $\nabla_{i}$-differentials and $\triangle_{j}$-differentials

We can obtain two independent related-key differentials $\nabla_{i}$-differentials and $\triangle_{j}$-differentials from key differentials $\nabla_{i}^{K}$ and $\triangle_{j}^{K}$ respectively. Both sets of differentials are depicted in Fig .8.

We put:

$$
\begin{gathered}
P_{i}=P_{0} \oplus \nabla_{i}, \\
S_{j}=S_{0} \oplus \triangle_{j} \\
M K[i, j]=M K[0,0] \oplus \nabla_{i}^{K} \oplus \triangle_{j}^{K}
\end{gathered}
$$

and get a 2-round biclique of dimension $8\left[\left\{P_{i}\right\},\left\{S_{j}\right\},\{M K[i, j]\}\right]$ for ARIA256.

Since the $\nabla_{i}$-differentials affects only 10 bytes of the plaintext, all the plaintexts share the same values in $P_{0,5,8,10,13,15}$. As a result, the data complexity does not exceed $2^{80}$.


Figure 8: $\nabla_{i}$-differentials and $\triangle_{j}$-differentials

### 4.3 Matching over 14 Rounds

Firstly, as shown in Fig. 11, we expand $\nabla_{i}$-differentials and $\triangle_{j}$-differentials to full-round differentials, and derive the overlapping byte from them. The tested key $M K[i, j]$ is a key candidate of the secret key $K_{\text {secret }}$ if

$$
\begin{equation*}
S_{j} \xrightarrow[e_{1}]{M K[i, j]} \vec{v}\left(\# 25_{0}\right)=\overleftarrow{v}\left(\# 25_{0}\right) \underset{e_{2}}{\stackrel{M K[i, j]}{\longleftarrow}} C_{i} \tag{2}
\end{equation*}
$$

(\#250 denote the first byte of \#25). Here, only matching a single byte, the computation complexity can be decreased.

We first precompute $S_{j} \xrightarrow{K[0, j]} \vec{v}$ and $\overleftarrow{v} \stackrel{K[i, 0]}{\rightleftarrows} C_{i}$ for $0 \leq i, j \leq 2^{8}-1$, and store the $2^{9}$ values as well as the intermediate states and subkeys in memory. Then let $i$ and $j$ run through $\left\{0,1, \ldots, 2^{8}-1\right\}$ respectively and check whether the condition (2) holds. In the matching process, we need only recompute those variables that differ from the ones stored in memory.

Next we evaluate the amount of recomputation in both directions. Because the round from 3 to 11 does not decrease the computation complexity, we omit these rounds as shown in Fig. 9 and Fig. 11.

## Recomputation cryptanalysis of forward direction:

As shown in Fig. 9, compute $S_{j} \xrightarrow{M K[i, j]} \vec{v}$ differs from the stored bytes of $S_{j} \xrightarrow{M K[0, j]}$ for all $i, j$. It is determined by the influence of the difference


Figure 9: Forward direction


Figure 10: Backward direction
between keys $M K[i, j]$ and $M K[0, j]$. The number byte of internal state which need not to be recomputed is $5+9=14$.

## Recomputation cryptanalysis of backward direction:

As shown in Fig. 10, compute $\overleftarrow{v} \stackrel{M K[i, j]}{\longleftarrow} C_{i}$ differs from the stored bytes of $\overleftarrow{v} \stackrel{M K[i, 0]}{\longleftarrow} C_{i}$ for all $i, j$. It is determined by the influence of the difference between keys $M K[i, j]$ and $M K[i, 0]$. The number byte of internal state which need not to be recomputed is $15+9+15=39$.

### 4.4 Complexities

Let us evaluate the full complexity of this approach. Since the full key space partition into the $2^{240}$ sets, we get the following equation:

$$
C_{\text {full }}=2^{n-2 d}\left(C_{\text {Biclique }}+C_{\text {precomp }}+C_{\text {recomp }}+C_{\text {falsepas }}\right)
$$

where:

- $C_{\text {Biclique }}$ is the complexity of constructing a single biclique.
- $C_{\text {precomp }}$ is the complexity of the precomputation.
- $C_{\text {recomp }}$ is the complexity of the internal variable $v 2^{16}$ times. It strongly depends on the diffusion properties.


Figure 11: Biclique differentipls and matching in ARIA-256

- $C_{\text {falsepas }}$ is the complexity generated by false positives, which have to be matched on other variables. If we match on a single byte, the number of false positives is about $2^{8}$. Each requires only a few operations to re-check.

The values $C_{\text {Biclique }}$ and $C_{\text {precomp }}$ together do not exceed $2^{8}$ calls of the full AIRA-256. The full key recovery complexity is dominated by $2^{n-2 d} \cdot C_{\text {recomp }}$. $S L$ and $D L$ are the major part of the round transformation and the key schedule. The number of bytes which need to be recomputed is $256-32-$ $14-39=171$ in the round transformation and 0 in the key schedule. And the total number of bytes is $16 * 16+3 * 16=304$. As a result, $C_{\text {recomp }}$ is equivalent to $2^{16} * 171 / 304=2^{15.17}$ runs of the full AIRA-256.

Hence, the full computational complexity amounts to about

$$
C_{\text {full }}=2^{240}\left(2^{8}+2^{15.17}+2^{8}\right)=2^{255.2} .
$$

## 5 Conclusion

In this paper, we construct a 2 -round biclique of dimension 8 for ARIA-256 by using a type of special master keys and tools like independent relatedkey differentials et al. We put forward the first key recovery attack on the full ARIA-256, and it has an advantage of $2^{0.8}$ over brute-force. The data complexity does not exceed $2^{80}$. Alternatively, this paper may inspire work on other block ciphers if the key schedule of them diffuse slowly.

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