Fault Analysis of Grain Family of Stream Ciphers

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Abstract—In this paper, we present fault attack on Grain family of stream ciphers, an eStream finalist. The earlier fault attacks on Grain work on LFSR whereas our target for fault induction is the NFSR. Our attack requires a small number of faults to be injected; 150 only for Grain v1 and only 312 and 384 for Grain-128 and Grain-128a, respectively. The number of faults are much lesser than the earlier reported fault attacks; 1587 for Grain-128 and 1831 for Grain-128a.

Keywords: Stream Cipher, Grain-128, Grain v1, Grain 128a, Side Channel Attack, Fault Attack, NFSR Fault Attack

I. INTRODUCTION

Fault attack is a practical form of side channel attack. Here, the adversary induces faults in states or operations of the cipher and analyze fault-free and faulty ciphertexts/keystreams to break the system. Both block ([10], [11]) and stream ([16], [17], [18]) ciphers have been shown *very* weak under fault analysis. However, practically inducing faults according to certain models are still challenging and widely researched area, with few successes ([19], [20]).

The final portfolio of eStream [1] includes three hardware-efficient and four software-efficient stream ciphers [3]. Grain [13] proposed by Martin Hell et al. is one of the three hardware based ciphers enlisted in the portfolio. A mathematical attack based on dynamic cube attack [12] and few fault attacks ([4], [7], [8], [9]) are only known weaknesses of Grain. These attacks employ a reasonable fault model, induces faults cipher state and analyses fault-free and faulty keystreams to deduce secret *key* of the cipher using small number of faults and few minutes of computation in practical scenario.

The reported fault analysis ([4], [7], [8], [9]) target LFSR of Grain. The first fault attack on Grain family targeting LFSR was [4], which targets Grain-128. While [7], [8], [9] extends this work to other ciphers of the Grain family, Grain v1 and Grain128a targeting LFSR. In [9] the authors have discussed a few modifications so as to adapt these attacks under less restricted assumptions, such as incorporating multiple bit faults etc.

In this paper, faults are assumed to be induced at the NFSR of Grain. As already mentioned most of the research on fault analysis of Grain family of stream ciphers targets LFSR. This opens the question if only protecting LFSR will secure it from fault attacks. We show that the NFSR needs also be protected. Since, this means that both LFSR and NFSR can be targeted, the adversary essentially works on a less restricted fault model. LFSR and NFSR targeted attacks need to be modified to make it work on the entire state. We follow certain algorithms to deduce secret key of the cipher following fault induction respecting our assumed model. The attack is applied

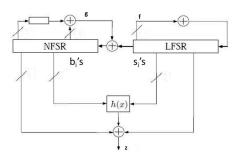


Fig. 1. Design of Grain family of ciphers

and reported here against Grain v1, Grain-128 and Grain-128a. We require reasonable number of faults and solution of at most 2^{nd} degree equations (mostly linear) to break the systems. A preliminary version of this work is published in [6] which analyzes Grain-128 only, the complete fault analysis of all the members of Grain family, Grain v1, Grain-128 and Grain-128a is presented in this paper.

This paper is organized as follows. Following this introduction, section II briefly discusses the specification of Grain family. We present our fault model in section III. The proposed attack against Grain family is presented in section IV. Section V summarizes performance analysis. Finally, section VI concludes the paper.

II. BACKGROUND

In this section, we briefly discuss the specification of the Grain family of stream ciphers.

Grain family of stream ciphers were introduced with Grain v1 ([14]). Grain-128 ([13]) is the latest member of the family.

The structure of Grain family of stream ciphers is shown in figure 1. Two left shift registers one linear, LFSR and one nonlinear, NFSR of equal length, n store internal state of the cipher. The LFSR and NFSR of Grain v1 is of 80 bits, while both Grain-128 and Grain-128a are of 128 bits. The LFSR and NFSR are updated by linear function f and nonlinear function f and nonlinear function f and nonlinear function f and LFSR produces the output keystream f at f cycle. Throughout this paper f refers to f modulo 2.

For Grain v1 these functions are defined as.

$$f = s_{i+62} + s_{i+51} + s_{i+38} + s_{i+23} + s_{i+13} + s_{i}$$

$$g = s_{i} + b_{i+62} + b_{i+60} + b_{i+52} + b_{i+45} + b_{i+37} + b_{i+33} + b_{i+28} + b_{i+21} + b_{i+14}$$

$$(1)$$

$$+b_{i+9} + b_{i} + b_{i+63}b_{i+60} + b_{i+37}b_{i+33} + b_{i+15}b_{i+9} + b_{i+60}b_{i+52}b_{i+45} + b_{i+33}b_{i+28}b_{i+21} + b_{i+63}b_{i+45}b_{i+28}b_{i+9} + b_{i+60}b_{i+52}b_{i+37}b_{i+33} + b_{i+63}b_{i+60}b_{i+21}b_{i+15} + b_{i+63}b_{i+60}b_{i+52}b_{i+45}b_{i+37} + b_{i+33}b_{i+28}b_{i+21}b_{i+15}b_{i+9} + b_{i+52}b_{i+45}b_{i+37}b_{i+33}b_{i+28}b_{i+21}$$

$$(2)$$

$$h = s_{i+25} + b_{i+63} + s_{i+3}s_{i+64} + s_{i+46}s_{i+64} + s_{i+64}b_{i+63}$$

$$h = s_{i+25} + b_{i+63} + s_{i+3}s_{i+64} + s_{i+46}s_{i+64} + s_{i+64}b_{i+63} + s_{i+3}s_{i+25}s_{i+46} + s_{i+3}s_{i+46}s_{i+64} + s_{i+3}s_{i+46}b_{i+63} + s_{i+25}s_{i+46}b_{i+63} + s_{i+46}s_{i+64}b_{i+63}$$

$$(3)$$

$$z_{i} = b_{i+1} + b_{i+2} + b_{i+4} + b_{i+10} + b_{31} + b_{43} + b_{56} + h$$

$$(4)$$

For Grain-128 these functions are defined as,

$$f = s_{i} + s_{i+7} + s_{i+38} + s_{i+70}$$

$$+ s_{i+81} + s_{i+96}$$

$$g = s_{i} + b_{i} + b_{i+26} + b_{i+56} + b_{i+91} + b_{i+96}$$

$$+ b_{i+3}b_{i+67} + b_{i+11}b_{i+13}$$

$$+ b_{i+17}b_{i+18} + b_{i+27}b_{i+59} + b_{i+40}b_{i+48}$$

$$+ b_{i+61}b_{i+65} + b_{i+68}b_{i+84}$$

$$(6)$$

$$h = b_{i+12}s_{i+8} + s_{i+13}s_{i+20} + b_{i+95}s_{i+42}$$

$$+ s_{i+60}s_{i+79} + b_{i+12}b_{i+95}s_{i+95}$$

$$z_{i} = b_{i+2} + b_{i+15} + b_{i+36} + b_{i+45} + b_{i+64}$$

$$(5)$$

Grain-128a was proposed to adapt Grain-128 in authentication [15]. For Grain-128a these functions are defined as,

 $+b_{i+73} + b_{i+89} + h + s_{i+93}$

$$f = s_{i} + s_{i+7} + s_{i+38} + s_{i+70}$$

$$+ s_{i+81} + s_{i+96}$$

$$g = s_{i} + b_{i} + b_{i+26} + b_{i+56} + b_{i+91} + b_{i+96}$$

$$+ b_{i+3}b_{i+67} + b_{i+11}b_{i+13} + b_{i+17}b_{i+18}$$

$$+ b_{i+27}b_{i+59} + b_{i+40}b_{i+48} + b_{i+61}b_{i+65}$$

$$+ b_{i+68}b_{i+84} + b_{i+88}b_{i+92}b_{i+93}b_{i+95}$$

$$+ b_{i+22}b_{i+24}b_{i+25}$$

$$+ b_{i+70}b_{i+78}b_{i+82}$$

$$(10)$$

$$h = b_{i+12}s_{i+8} + s_{i+13}s_{i+20} + b_{i+95}s_{i+42} + s_{i+60}s_{i+79} + b_{i+12}b_{i+95}s_{i+94}$$
(11)

$$y_{i} = s_{i+93} + b_{i+2} + b_{i+15} + b_{i+36} + b_{i+45} + b_{i+64} + b_{i+73} + b_{i+79} + h$$
(12)

$$z_i = y_{64+2i} (13)$$

It can be seen that in Grain-128a all output bits are not available for inspection, every second bit following 64^{th} output bit (y_i) are actually visible to the user. This certainly complicates the attack.

An *initialization* phase is carried out before generation of keystream bits. The n bit key, $k=(k_0,k_2,\ldots,k_n)$ and the m bit initialization vector $IV=(IV_0,IV_2,\ldots,IV_{m-1})$ are loaded in the NFSR and the LFSR respectively as, $b_i=k_i, 0\leq i\leq n$ and $s_i=IV_i, 0\leq i\leq m$, rest of the LFSR bits, $(s_m,s_{m+1},\ldots,s_{n-1})$ are loaded with 1. m has value

64 for Grain v1 and 96 for Grain-128 and Grain-128a. During initialization, the cipher is run for 2n rounds without producing any keystream. The output bit, z_i is XOR-ed with feedback bit of both the LFSR and the NFSR.

III. FAULT ANALYSIS MODEL

Our fault model creates faults in the NFSR. The following features are required for the attack.

- The adversary is able to induce faults at random positions of the NFSR of the Grain implementation (hardware or software). Hence, exact fault position is not known beforehand.
- 2) The fault affects *exactly one* bit of the NFSR at any cycle of operation. So, the fault amounts to flipping *exactly one bit* of the NFSR of the implementation.
- 3) A fault to an NFSR bit can be *reproduced* at any cycle of operation, once, it is created.
- 4) The attacker is able to determine and control the cycles of operation of the implementation, i.e., the *timing of the implementation is under control* of the attacker.

Flipping exactly one bit of the NFSR may seem to be a strong assumption, but can be achieved by triggering laser shots through the I/O signal for hardware implementations ([19], [20]). Also reproducing a fault at a particular location may seem difficult, but keeping parameters intact this can *possibly* be achieved.

IV. FAULT ANALYSIS OF GRAIN FAMILY OF STREAM CIPHERS

We inject faults at the NFSR of Grain ciphers and use differences in normal (z_{normal}) and faulty (z_{faulty}) keystreams to determine state of the cipher (i.e. $b_0, b_1, \ldots, b_n, s_0, s_1, \ldots, s_n$) at a target cycle T (referred to as *base point*) of operation following initialization. Output difference of faulty and fault-free keystream at iteration t after t is denoted by t, i.e., t i.e., t i.e., t i.e., t i.e., t is denoted by t i.e., t i.

We will follow the following five steps:

- 1) Determine fault position in the NFSR.
- 2) Pre-compute fault traces.

(8)

- 3) Determine NFSR bits, b_0, b_1, \ldots, b_n .
- 4) Determine LFSR bits, s_0, s_1, \ldots, s_n .
- 5) Invert states from cycle T to 0 to obtain the key.

The details of each step is described next.

A. Determine Fault Location in the NFSR

Basic Idea: The output z_i are given by equations 4, 8 and 13. A fault induced in the NFSR will according to equation of z_i produce different δ^t depending on the fault location at different t. For example, for Grain-128, faulting b_{89} will produce $\delta^t = 1$ for t = 0, 16, 25 etc. The observation is that a linear term of z_i will always produce $\delta^t = \delta(z_i) = 1$, while nonlinear terms lead to $\delta^t = \delta(z_i) = non-constant \neq 1$ for some input values following standard finite difference rules, when fault is moved to corresponding bits in the term, without affecting other bits in z_i in the process. It turns out that this constant δ^t pattern is unique

Algorithm 1 FormDeltaPattern(f)

```
stateSize \leftarrow n;
Grain v1
outputPos[] \leftarrow \{1, 2, 4, 10, 31, 43, 56, 63\};
Grain-128 and Grain-128a
outputPos[] \leftarrow \{2, 15, 36, 45, 64, 73, 79\};
for f=0 to stateSize do
  \sigma_f = \Phi
  for linearTerm \in outputPos do
     if f > linearTerm then
        mov \leftarrow (f - linearTerm);
       if onlyContains(outputPos, mov, linearTerm, f)
          \sigma_f \leftarrow \sigma_f \cup \{mov\};
        end if
     end if
  end for
end for
```

for faults in NFSR bits. This gives us algorithm 1 to find this unique pattern versus fault locations. During simulation, we check the distance(mov) of linear terms of z_i from induced fault locations and check that no other z_i input is corrupt (onlyContains), thus determining the pattern of δ^t . Algorithm onlyContains(array, movement, location, f) (algorithm 7) returns true if only location is corrupt after movement cycles of operation, among the elements of array for fault at f at base point. The $\delta^t = 1$, $(t = 0, 1, \ldots, n)$ pattern for faults at different NFSR positions is shown in tables I, III and II.

Once σ_i 's are known, during online attack phase, using algorithm 2 we determine the fault location by varying IV only randomly and reproducing the fault. For Grain-128a, the corresponding visible cycles of constant output are also given alongside $\delta(y_i)$. Note that no fault location can be obtained up to location 67.

B. Pre-compute Fault Traces

The purpose of this phase is to store all *possible* indices of corrupted b-bits at cycle t following fault injection at location f.

Basic Idea: Algorithm 3 is used to store fault traces at t^{th} cycle from fault induction at location f (FaultTraces(f)[t]). The idea here is that the a corruption in b bit positions of the feedback (equation of b_{i+n-1}) (FeedbackPositions) may corrupt b_{i+n-1} next cycle. Also, a corruption at b_i shifts left following cycle. Other locations remain fault-free. Note that s bits remain uncorrupted throughout by design.

C. Determine NFSR Bits

We now exploit the difference of fault-free and faulty output bits to determine values of NFSR bits. Both feedback and output equations are used in this phase. It is observed that output bit equation, z (equations 4, 8, 15) has nonconstant monomials with b-bits that always contain s-bits. Hence, linear

Algorithm 2 DetermineFaultlocation()

```
FaultLocation \leftarrow \{\};
NumIVs \leftarrow 0;
inc \leftarrow 100;
while size(FaultLocation) \neq 1 do
   NumIVs \leftarrow NumIVs + inc;
  for i = 0 to NumIVs do
     initialize Grain with random IV, with that fixed key;
     fault at the same location;
     form \delta^t, t = 0, 1, 2, \dots, n;
  end for
  for i = 0 to n do
     FaultLocation \leftarrow \{\}
     if all positions of \delta^t = 1 \ \forall \ j = 0, 1, 2, \dots, NumIVs
     is in \sigma_i then
        FaultLocation \leftarrow FaultLocation \cup \{i\}
     end if
  end for
end while
return FaultLocation
```

Table I Fault location vs. $\delta^t=1$ (σ), Fault Locations 0 to 79 of Grain v1

Fault	t =	Fault	t =
Loc.	ι –	Loc.	ι —
0		40	39, 38, 36, 30, 9
1	0	41	40, 39, 37, 31, 10
2	1, 0	42	41, 40, 38, 32, 11
3	2,1	43	42, 41, 39, 33, 12, 0
4	3, 2, 0	44	43, 42, 40, 34, 13, 1
5	4, 3, 1	45	44, 43, 41, 35, 14, 2
6	5, 4, 2	46	45, 44, 42, 36, 15, 3
7	6, 5, 3	47	46, 45, 43, 37, 16, 4
8	7, 6, 4	48	47, 46, 44, 38, 17, 5
9	8, 7, 5	49	48, 47, 45, 39, 18, 6
10	9, 8, 6, 0	50	49, 48, 46, 40, 19, 7
11	10, 9, 7, 1	51	50, 49, 47, 41, 20, 8
12	11, 10, 8, 2	52	51, 50, 48, 42, 21, 9
13	12, 11, 9, 3	53	52, 51, 49, 43, 22, 10
14	13, 12, 10, 4	54	53, 52, 50, 44, 23, 11
15	14, 13, 11, 5	55	54, 53, 51, 45, 24, 12
16	15, 14, 12, 6	56	55, 54, 52, 46, 25, 13, 0
17	16, 15, 13, 7	57	56, 55, 53, 47, 26, 14, 1
18	17, 16, 14, 8	58	57, 56, 54, 48, 27, 15, 2
19	18, 17, 15, 9	59	58, 57, 55, 49, 28, 16, 3
20	19, 18, 16, 10	60	59, 58, 56, 50, 29, 17, 4
21	20, 19, 17, 11	61	60, 59, 57, 51, 30, 18, 5
22	21, 20, 18, 12	62	61, 60, 58, 52, 31, 19, 6
23	22, 21, 19, 13	63	62, 61, 59, 53, 32, 20, 7, 0
24	23, 22, 20, 14	64	63, 62, 60, 54, 33, 21, 8, 1
25	24, 23, 21, 15	65	64, 63, 61, 55, 34, 22, 9, 2
26	25, 24, 22, 16	66	65, 64, 62, 56, 35, 23, 10, 3
27	26, 25, 23, 17	67	66, 65, 63, 57, 36, 24, 11, 4
28	27, 26, 24, 18	68	67, 66, 64, 58, 37, 25, 12, 5
29	28, 27, 25, 19	69	68, 67, 65, 59, 38, 26, 13, 6
30	29, 28, 26, 20	70	69, 68, 66, 60, 39, 27, 14, 7
31	30, 29, 27, 21, 0	71	70, 69, 67, 61, 40, 28, 15, 8
32	31, 30, 28, 22, 1	72	71, 70, 68, 62, 41, 29, 16, 9
33	32, 31, 29, 23, 2	73	72, 71, 69, 63, 42, 30, 17, 10
34	33, 32, 30, 24, 3	74	73, 72, 70, 64, 43, 31, 18, 11
35	34, 33, 31, 25, 4	75	74, 73, 71, 65, 44, 32, 19, 12
36	35, 34, 32, 26, 5	76	75, 74, 72, 66, 45, 33, 20, 13
37	36, 35, 33, 27, 6	77	76, 75, 73, 67, 46, 34, 21, 14
38	37, 36, 34, 28, 7	78	77, 76, 74, 68, 47, 35, 22, 15
39	38, 37, 35, 29, 8	79	78, 77, 75, 69, 48, 36, 23, 16

TABLE III FAULT LOCATION VS. $\delta^t = 1~(\sigma)$ OF GRAIN-128

Fault	t =	Fault	t =
Location		Location	
0		1	39,55,70,75,86,91,106,110,111,122,127
2	0,40,56,71,76,87,92,107,111,112,123,128	3	1,41,57,72,77,88,93,108,112,113,124
4	2,42,58,73,78,89,94,109	5	3,43,59,74,79,90,95,110
6	4,44,60,75,80,91,96,111	7	5,45,61,76,81,92,97,112
8	6,46,62,77,82,93,98,113	9	7,47,63,78,83,94,99,114
10	8,48,64,79,84,95,100,115	11	9,49,65,80,85,96,101,116
12	10,50,66,81,97,102	13	11,51,67,82,98,103
14	12,52,68,83	15	
		_	0,13,53,69,84
16	1,14,54,70,85	17	2,15,55,71,86
18	3,16,56,72,87	19	4,17,57,73,88
20	5,18,58,74,89	21	6,19,59,75,90
22	7,20,60,76,91	23	8,21,61,77,92
24	9,22,62,78,93	25	10,23,63,79,94
26	11,24,64,80,95	27	12,25,39,55,65,75,81,86,96
28	13,26,40,56,66,87,97	29	14,27,41,57,67,88,98
30	15,28,42,58,68,89,99	31	16,29,43,59,69,90,100
32	17,30,44,60,70,91,101	33	18,31,45,61,71,92,102
34	19,32,46,62,72,93,103	35	20,33,47,63,73,94,104
36	0,21,34,48,64,74,95,105	37	1,22,35,49,65,75,96,106
38	2,23,36,50,66,76,97,107	39	3,24,37,51,67,77,98,108
40	4,25,38,52,68,78,99,109	41	5,26,53,79
42	6,27,54,80	43	7,28,55,81
44	8,29,56,82	45	9,30,57,83
46	10,31,58,84	47	11,32,59,85
48	12,33,60,86	49	13,34,61,87
50	14,35,62,88	51	15,36,63,89
	1 1 1		1 1 1
52	16,37,64,90	53	17,38,65,91
54	18,39,66,92	55	19,40,67,93
56	20,41,68,94	57	21,39,42,70,75,110
58	22,40,43,71,76,111	59	23,41,44,72,77,112
60	24,42,45,73,78	61	25,43,46,74,79
62	26,44,47,80	63	27,45,48,81
64	0,28,46,49,82	65	1,29,47,50,83
66	2,30,48,51,84	67	3,31,49,52,85
68	4,32,50,53	69	5,51,49,52,65
70	1 1 1	71	
	6,52,55		7,53,56
72	8,54,57	73	0,9,55,58
74	1,10,56,59	75	2,11,57,60
76	3,12,58,61	77	4,13,59,62
78	5,14,60,63	79	6,15,61,64
80	7,16,62,65	81	8,17,63,66
82	9,18,64,67	83	10,19,65,68
84	11.20.66.69	85	12,21,67
86	13.22.68	87	14,23,69
88	15,24,70	89	0,16,25,71
90	1,17,26,72	91	
90	3,19,28,39,55,70,74,75,86,91	91	2,18,27,73 4,20,29,40,56,71,75,76,87,92
94	5,21,30,41,57,72,76,77,88,93	95	6,22,31,42,58,73,77,78,89,94
96	7,23,32,43,59,74,78,79,90,95	97	8,24,39,44,55,60,75,79,80,91,96
98	9,25,40,45,56,61,76,80,81,92,97	99	10,26,41,46,57,62,77,81,82,93,98
100	11,27,42,47,58,63,78,82,83,94,99	101	12,28,43,48,59,64,79,83,84,95,100
102	13,29,44,49,60,65,80,84,85,96,101	103	14,30,45,50,61,66,81,85,86,97,102
104	15,31,46,51,62,67,82,86,87,98,103	105	16,32,47,52,63,68,83,87,88,99,104
106	17,33,48,53,64,69,84,88,89,100,105	107	18,34,49,54,65,70,85,89,90,101,106
108	19,35,50,55,66,71,86,90,91,102,107	109	20,36,51,56,67,72,87,91,92,103,108
110	21,37,52,57,68,73,88,92,93,104,109	111	22,38,53,58,69,74,89,93,94,105,110
112	23,39,54,59,70,75,90,94,95,106,111	113	24,40,55,60,71,76,91,95,96,107,112
114	25,41,56,61,72,77,92,96,97,108,113	115	26,42,57,62,73,78,93,97,98,109,114
116	27,43,58,63,74,79,94,98,99,110,115	117	28,44,59,64,75,80,95,99,100,111,116
118	29,45,60,65,76,81,96,100,101,112,117	119	30,46,61,66,77,82,97,101,102,113,118
120	31,47,62,67,78,83,98,102,103,114,119	121	32,48,63,68,79,84,99,103,104,115,120
122	33,49,64,69,80,85,100,104,105,116,121	123	34,50,65,70,81,86,101,105,106,117,122
	35.51.66.71.82.87.102.106.107.118.123	125	36.52.67.72.83.88.103.107.108.119.124
124 126	35,51,66,71,82,87,102,106,107,118,123 37,53,68,73,84,89,104,108,109,120,125	125 127	36,52,67,72,83,88,103,107,108,119,124 38,54,69,74,85,90,105,109,110,121,126

TABLE II FAULT LOCATION VS. $\delta^{y_t}=1$ (σ) , Fault Locations 68 to 127 of Grain-128a

Fault	t: Visible	Fault	t : Visible
Loc.	Cycle	Loc.	Cycle
68	66:1	69	
70	68:2	71	
72	70:3	73	
74	72:4	75	
76	74:5	77	
78	76:6, 66:1	79	
80	78:7, 68:2	81	66:1
82	80:8, 70:3	83	68:2
84	82:9, 72:4	85	70:3
86	84:10, 74:5	87	72:4
88	86:11, 76:6	89	74:5
90	88:12, 78:7	91	76:6
92	90:13, 80:8	93	78:7
94	92:14, 82:9	95	80:8
96	94:15, 84:10	97	82:9
98	96:16, 86:11	99	84:10
100	98:17, 88:12	101	86:11
102	100:18, 90:13, 66:1	103	88:12
104	102:19, 92:14, 68:2	105	90:13
106	104:20, 94:15, 70:3	107	92:14
108	106:21, 96:16, 72:4	109	94:15
110	108:22, 98:17, 74:5	111	96:16, 66:1
112	110:23, 100:18, 76:6	113	98:17, 68:2
114	112:24, 102:19, 78:7	115	100:18, 70:3
116	114:25, 104:20, 80:8	117	102:19, 72:4
118	116:26, 106:21, 82:9	119	104:20, 74:5
120	118:27, 108:22, 84:10	121	106:21, 76:6
122	120:28, 110:23, 86:11	123	108:22, 78:7
124	122:29, 112:24, 88:12	125	110:23, 80:8
126	124:30, 114:25, 90:13	127	112:24, 82:9

Algorithm 3 FaultTrace(f)

```
Grain-128
48, 56, 59, 61, 65, 67, 68, 84, 91, 96};
Grain-128a
Feedback Positions
0,26,56,91,96,3,67,11,13,17,18,27,59,40,48,61,
65,68,84,88,92,93,95,22,24,25,70,78,82 };
Grain v1
feedBackPositions
\{62, 60, 52, 45, 37, 33, 28, 21, 14, 9, 0, 63, 15\};
FaultTrace[0] \leftarrow {f};
for i = 0 to 127 do
  FaultTrace[i] \leftarrow \{\};
  for element \in FaultTrace[i-1] do
    FaultTrace[i] \leftarrow FaultTrace[i] \cup \{element - 1\}
    if element \in FeedbackPositions then
       FaultTrace[i] \leftarrow FaultTrace[i] \cup \{n-1\}
    end if
  end for
end for
```

equations in b-bits are not obtainable. So we utilize linear b terms of z and feedback equation b_n (equations 2, 6 and 10) to determine values of b-bits.

Basic Idea: The idea is to move the induced fault to b_p or b_q if b_pb_q is a term in b_{i+n} by movement iterations. The observation is that if b_n is not faulted through any other feedback tap (seen by consulting FaultTraces table) at T + movement cycle, $\delta^{movement} = b_q$ or b_p , respectively. However, if fault into b_n is due to multiple linear or nonlinear taps of g, we get, $\delta^{movement} =$ a polynomial in g-bits = g. This polynomial will be affine if at most degree 2 monomials were corrupt. Thus, essentially we can have an affine difference in feedback. The target is now to move this difference uncorrupted to g.

We utilize linear b terms of z for this purpose. That is the feedback difference is moved to a linear b term of z, which is different for different versions of Grain. Note that here also we need to be sure that corruption of z comes through this term only (using onlyContains).

So, we use FaultTraces table first time during feedback through degree-2 monomial and the second time for output through linear b terms. We can determine P (Left Hand Side) during simulation, the actual output difference δ^t is obtained during online phase. P will now equal the difference of output bits (δ^t) after the movement to corresponding linear b term. These b values have actually moved from another locations. So, if we need to move the fault c cycles from the original location to effect b_p of b_n , we obtained b_{n+c} at base point. It can be executed for each degree-2 monomial in g and each linear term in g.

This procedure is summarized and parameterized according to equations in algorithm 4. Algorithm 5 constructs P throughout the process. Note that fault need to be moved from f, after movement shifts, to $feedbacklocation_b$. It output a single b position or a linear equation in b according as single or multiple taps corrupt b_n .

For Grain-128, the *base point* fault locations and number of bits obtained from that location in single b-bit is tabulated in table IV. The NFSR bits obtained for Grain-128a are tabulated in table V. The equations obtained for Grain v1 are tabulated in table VI.

Grain-128:The number of faults which give linear equations is, 125. Single bit P's obtainable are $b_3, b_4, \ldots, b_{127}$. Other 3 bits, b_0, b_1, b_2 need to be brute-forced. On an average of $\frac{0 \times 3 + 1 \times 8 + 2 \times 23 + 3 \times 31 + 4 \times 26 + 5 \times 37}{128} = 3.40$ b-bits can be obtained from a single fault at the NFSR and 56 faults are required to determine state bits, $b_0, b_1, \ldots, b_{127}$ of the NFSR. Number of induced faults can be reduced by injecting them at consecutive cycles. For example, from the fault at b_{67} at base point, we can obtain value of bit b_3 . A fault at the previous cycle at b_{67} will give value of b_3 at that cycle, which is the value of b_2 at $base\ point$.

Grain-128a: In algorithm 4 note the check (movement - 64)%2 == 0 for Grain-128a. This guarantees that corresponding output bit is *visible*. However, this means we need the value of (movement - 64)/2, which gives the cycle of the corresponding output bit. In this case it turns out that all the output bits are at cycle 0 for the equations obtained in table V. So, for faults at 7 locations of NFSR we obtain 7 b bits

Algorithm 4 Determine NFSRBits(f)

```
Grain-128
48, 59, 61, 65, 67, 68, 84}
Double feedBack b[]
\{3, 11, 13, 17, 18, 27, 40, 48, 59, 61, 65, 67, 68, 84\}
Double\_feedBack\_b\_corr[]
\{67, 13, 11, 18, 17, 59, 48, 40, 27, 65, 61, 3, 84, 68\}
Single\_output\_b[] \leftarrow \{2, 15, 36, 45, 64, 73, 89\}
Grain-128a
feedBack\_b[] \leftarrow
68, 84, 88, 92, 93, 95, 22, 24, 25, 70, 78, 82}
Double\_feedBack\_b[] \leftarrow \{3, 11, 17, 27, 40, 61, 68\}
Double\_feedBack\_b\_corr[] \leftarrow \{67, 13, 18, 59, 48, 65, 84\}
Single\_output\_b[] \leftarrow \{2, 15, 36, 45, 64, 73, 89\}
Grain v1
feedBack \ b[] \leftarrow
\{62, 60, 52, 45, 37, 33, 28, 21, 14, 9, 0, 63, 15\}
Double feedBack b[] \leftarrow \{63, 37, 15\}
Double\_feedBack\_b\_corr[] \leftarrow \{60, 33, 9\}
Single\_output\_b[] \leftarrow \{1, 2, 4, 10, 31, 43, 56\}
movement \leftarrow 0
for i = 0 to length(Double\_feedBack\_b) do
  if f \geq Double\_feedBack\_b[i] then
    movement \leftarrow (f - Double\_feedBack\_b[i])
                       ConstructFeedbackDiffEqn(f,
    egnLHS
                 \leftarrow
    Double\_feedBack\_b[i] - f,
    Double\_feedBack\_b[i])
    for j = 0 to length(Single\_output\_b) do
       movement
                               movement
       Single\_output\_b[j];
       movement \leftarrow disp + 79 - Single\_output\_b[j];
       //Grain v1
       if condition then
         Grain-128
                            onlyContains(feedBack b,
         condition
         movement, Single\_output\_b[j], f)
                    (Double\_feedBack\_b\_corr[i]
         movement \le 127)
         Grain-128a
         condition = (movement - 64)\%2
                 OnlyCorrupt(feedBack\_b, movement,
         Single\_output\_b[j], f)
         &&
                    (Double\_feedBack\_b\_corr[i]
         movement \leq 127)
         Grain v1
         condition
                             OnlyCorrupt(feedBack\_b,
         movement, Single\_output\_b[j], f)
                    (Double\_feedBack\_b\_corr[i]
         movement \leq 79)
         obtained eqnLHS = \delta^{movement}
         Grain-128a
         Cycle = (movement - 64)/2;
       end if
    end for
  end if
end for
```

Algorithm 5 ConstructFeedbackDiffEqn(f, movement, feedbacklocation)

```
linearTerms = \{\};
for element \in FaultTrace(f)[movement] do
  for i = 0 to length(feedBack\_b) do
    if element == feedBack\_b[i] then
      if element == Double feedBack b[k] for some
      k then
        linearTerms
                                    linearTerms
                                                    U
         {Double\_feedBack\_b\_corr[k]};
      else
        linearTerms = linearTerms \cup \{1\};
      end if
    end if
  end for
end for
return XOR of linearTerms;
```

at base point, say, T. Among the fault location 68 is only identifiable, so, 1 fault at b_{68} gives value of 1 b bit at cycle=T, corresponding to faults at T-disp cycle. Therefore, a fault at 68 in cycle T-disp-1, gives value of b_{84} at cycle T-1 which equals b_{83} at cycle T. This explains column 2 of table V with indices between 0 and 127. Note that i of table VI may be both positive and negative. It is seen experimentally that b_i for $0 \le i \le 84$ can be obtained in this way for Grain-128a. Rest 43 bits require a positive shift T-disp+i, the argument being similar. b bits are obtained from difference of this visible output in fault-free and faulty forms. Therefore, after this step we are able to find values of all b bits.

Grain v1: It is verified experimentally that there are fault-positions which could give linear equations involving b bits only (table VI). Therefore, a fault at b_{15} in cycle T-disp-1, gives value of b_9 at cycle T-1 which equals b_8 at cycle T. This explains column 2 of table VI with indices between 0 and 79 similar to Grain-128a and Grain-128. Note that i of table VI may be both positive and negative like Grain-128a. It is seen experimentally that with positive i, b_0 to b_{33} may be determined, while with negative i, up to $b_{33+36}=b_{69}$ can be obtained. Rest 10 b bits are to be determined brute-force, with complexity $O(2^{10})$.

D. Determine LFSR bits

Basic Idea: Here we need to obtain equations involving s-bits alone or s and b bits. The idea is to target terms of z containing both s and b indices. Now move fault to the one of the b locations in the term and find output difference as an equation involving s and b bits, provided other terms are not corrupted.

We explain it using Grain-128. The induced fault is propagated to any of the locations b_{12} or b_{95} without corrupting other b-bits of z_i (equation 8). It is an equation of the form,

TABLE IV
FAULT LOCATION VS. NFSR BITS OBTAINED FOR GRAIN-128

Fault Location	NFSR Bits Obtained	Fault Location	NFSR Bits Obtained
b_0		b_{43}	$b_{41}, b_{44}, b_{45}, b_{51}, b_{75}$
b_1		b_{44}	$b_{42}, b_{45}, b_{46}, b_{52}, b_{76}$
b_2		b_{45}	$b_{43}, b_{46}, b_{47}, b_{53}, b_{77}$
b_3	b ₆₇	b_{46}	$b_{44}, b_{47}, b_{48}, b_{54}, b_{78}$
b_4	b ₆₈	b_{47}	$b_{45}, b_{48}, b_{49}, b_{55}, b_{79}$
b_5	b ₆₉	b_{48}	$b_{40}, b_{46}, b_{49}, b_{56}, b_{80}$
b_6	b ₇₀	b_{49}	$b_{41}, b_{47}, b_{50}, b_{57}, b_{81}$
b_7	b_{71}	b_{50}	$b_{42}, b_{48}, b_{51}, b_{58}, b_{82}$
b_8	b_{72}	b_{51}	$b_{43}, b_{49}, b_{52}, b_{59}, b_{83}$
b ₉	b ₇₃	b_{52}	$b_{44}, b_{50}, b_{53}, b_{60}, b_{84}$
b_{10}	b ₇₄	b ₅₃	$b_{45}, b_{51}, b_{54}, b_{61}, b_{85}$
b_{11}	b_{13}, b_{75}	b_{54}	$b_{46}, b_{52}, b_{55}, b_{62}, b_{86}$
b_{12}	b_{14}, b_{76}	b_{55}	$b_{47}, b_{53}, b_{56}, b_{63}, b_{87}$
b_{13}	b_{11}, b_{15}, b_{77}	b_{56}	b_{48}, b_{88}
b_{14}	b_{12}, b_{16}, b_{78}	b_{57}	b_{49}, b_{89}
b_{15}	b_{13}, b_{17}, b_{79}	b ₅₈	b_{50}, b_{90}
b_{16}	b_{14}, b_{18}, b_{80}	b_{59}	b_{27}, b_{51}
b_{17}	$b_{15}, b_{18}, b_{19}, b_{81}$	b_{60}	b_{28}, b_{52}
b_{18}	$b_{16}, b_{17}, b_{19}, b_{20}, b_{82}$	b ₆₁	b_{29}, b_{53}, b_{65}
b_{19}	$b_{17}, b_{18}, b_{20}, b_{21}, b_{83}$	b_{62}	b_{30}, b_{54}, b_{66}
b_{20}	$b_{18}, b_{19}, b_{21}, b_{22}, b_{84}$	b_{63}	b_{31}, b_{55}, b_{67}
$\frac{b_{20}}{b_{21}}$	$b_{19}, b_{20}, b_{22}, b_{23}, b_{85}$	b_{64}	b_{32}, b_{56}, b_{68}
b_{22}	$b_{19}, b_{20}, b_{22}, b_{23}, b_{86}$ $b_{20}, b_{21}, b_{23}, b_{24}, b_{86}$	b_{65}	b_{33}, b_{61}, b_{69}
b_{23}	$b_{21}, b_{22}, b_{24}, b_{25}, b_{87}$	b_{66}	b_{34}, b_{62}, b_{70}
b_{24}	$b_{22}, b_{23}, b_{25}, b_{26}, b_{88}$	b_{67}	$b_3, b_{35}, b_{63}, b_{71}$
b_{25}	$b_{23}, b_{24}, b_{26}, b_{27}, b_{89}$	b ₆₈	$b_4, b_{36}, b_{64}, b_{84}$
b_{26}	$b_{24}, b_{27}, b_{28}, b_{90}$	b_{69}	$b_5, b_{37}, b_{65}, b_{85}$
b ₂₆			$b_6, b_{38}, b_{66}, b_{86}$
b ₂₇	$\begin{array}{c c} b_{25}, b_{28}, b_{29}, b_{59}, b_{91} \\ b_{26}, b_{29}, b_{30}, b_{60}, b_{92} \end{array}$	b ₇₀	b- b- b- b b
b ₂₈	1 L L L L L	b ₇₁	$b_7, b_{39}, b_{67}, b_{87}$
b ₂₉	$b_{27}, b_{30}, b_{31}, b_{61}, b_{93}$	b ₇₂	$b_8, b_{40}, b_{68}, b_{88}$
b ₃₀	$b_{28}, b_{31}, b_{32}, b_{62}, b_{94}$	b ₇₃	$b_9, b_{41}, b_{69}, b_{89}$
b ₃₁	$b_{29}, b_{32}, b_{33}, b_{63}, b_{95}$	b ₇₄	$b_{10}, b_{42}, b_{70}, b_{90}$
b ₃₂	$b_{30}, b_{33}, b_{34}, b_{64}, b_{96}$	b ₇₅	$b_{11}, b_{43}, b_{71}, b_{91}$
b ₃₃	$b_{31}, b_{34}, b_{35}, b_{65}, b_{97}$	b ₇₆	$b_{12}, b_{44}, b_{72}, b_{92}$
b ₃₄	$b_{32}, b_{35}, b_{36}, b_{66}, b_{98}$	b ₇₇	$b_{13}, b_{45}, b_{73}, b_{93}$
b ₃₅	$b_{33}, b_{36}, b_{37}, b_{67}, b_{99}$	b ₇₈	$b_{14}, b_{46}, b_{74}, b_{94}$
b ₃₆	$b_{34}, b_{37}, b_{38}, b_{68}, b_{100}$	b ₇₉	$b_{15}, b_{47}, b_{75}, b_{95}$
b ₃₇	$b_{35}, b_{38}, b_{39}, b_{69}, b_{101}$	b ₈₀	$b_{16}, b_{48}, b_{76}, b_{96}$
b ₃₈	$b_{36}, b_{39}, b_{40}, b_{70}, b_{102}$	b ₈₁	$b_{17}, b_{49}, b_{77}, b_{97}$
b_{39}	$b_{37}, b_{40}, b_{41}, b_{71}, b_{103}$	b_{82}	$b_{18}, b_{50}, b_{78}, b_{98}$
b_{40}	$b_{38}, b_{41}, b_{42}, b_{48}, b_{72}$	b ₈₃	$b_{19}, b_{51}, b_{79}, b_{99}$
b_{41}	$b_{39}, b_{42}, b_{43}, b_{49}, b_{73}$	b ₈₄	$b_{52}, b_{68}, b_{80}, b_{100}$
b_{42}	$b_{40}, b_{43}, b_{44}, b_{50}, b_{74}$	b ₈₅	$b_{53}, b_{69}, b_{81}, b_{101}$
b ₈₆	$b_{54}, b_{70}, b_{82}, b_{102}$	b ₈₇	$b_{55}, b_{71}, b_{83}, b_{103}$
b ₈₈	$b_{56}, b_{72}, b_{84}, b_{104}$	b ₈₉	$b_{57}, b_{73}, b_{85}, b_{105}$
b ₉₀	$b_{58}, b_{74}, b_{86}, b_{106}$	b ₉₁	b_{75}, b_{87}, b_{107}
b ₉₂	b_{76}, b_{88}, b_{108}	b ₉₃	b_{77}, b_{89}, b_{109}
b ₉₄	b_{78}, b_{90}, b_{110}	b ₉₅	b_{79}, b_{91}, b_{111}
b ₉₆	b_{80}, b_{92}, b_{112}	b ₉₇	b_{81}, b_{93}, b_{113}
b ₉₈	b_{82}, b_{94}, b_{114}	<i>b</i> ₉₉	b_{83}, b_{95}, b_{115}
b_{100}	b_{84}, b_{96}, b_{116}	b_{101}	b_{85}, b_{97}, b_{117}
b_{102}	b_{86}, b_{98}, b_{118}	b_{103}	b_{87}, b_{99}, b_{119}
b_{104}	b_{88}, b_{100}, b_{120}	b_{105}	b_{89}, b_{101}, b_{121}
b_{106}	b_{90}, b_{102}, b_{122}	b_{107}	b_{91}, b_{103}, b_{123}
b_{108}	b_{92}, b_{104}, b_{124}	b_{109}	b_{93}, b_{105}, b_{125}
b_{110}	b_{94}, b_{106}, b_{126}	b_{111}	b_{95}, b_{107}, b_{127}
b_{112}	b_{96}, b_{108}	b_{113}	b_{97}, b_{109}
b_{114}	b_{98}, b_{110}	b_{115}	b_{99}, b_{111}
b_{116}	b_{100}, b_{112}	$ b_{117}$	b_{101}, b_{113}
b_{118}	b_{102}, b_{114}	b_{119}	b_{103}, b_{115}
b_{120}	b_{104}, b_{116}	b_{121}	b_{105}, b_{117}
b_{122}	b_{106}, b_{118}	b_{123}	b_{107}, b_{119}
b_{124}	b_{108}, b_{120}	b_{125}	b_{109}, b_{121}
b_{126}	b_{110}, b_{122}	b_{127}	b_{111}, b_{123}

TABLE V
FAULT LOCATION VS. NFSR BITS OBTAINED FOR GRAIN-128A

Fault Location	NFSR Bit Obtained i^{th} Shift from T
3	b_{67}
11	b_{13}
17	b_{18-i}
27	b_{59-i}
40	b_{48-i}
61	b_{65-i}
68	b_{84-i}

TABLE VI FAULT LOCATION VS. NFSR BITS OBTAINED FOR GRAIN V1

Fault Location	NFSR Bit Obtained i^{th} Shift from T
15	b_{9-i}
37	b_{33-i}

 $s_p + s_q b_r = \delta^t$. So, if this b_r bit is 0, we have an equation in s_p , otherwise, we have a linear equation in s-bits.

Algorithm 6 describes and conditionally parameterizes the process.

Grain-128:Only 66 faults (at locations 12, 13, ..., 44, $95, 96, \ldots, 127$) in the NFSR gives equations. 33 equations on an average will involve a single s-bit. Other equations are stored. The equations obtained from faults in the NFSR are tabulated in table VII. LFSR bits are updated according to a linear feedback relation (equation 1). So, all s-bits at any cycle t after/before the base point can be written as linear combinations of the base point LFSR bits $s_0, s_1, \ldots, s_{127}$. So, essentially we need 128 linearly independent equations from faults at different cycles after/before the base point. We obtain equations involving LFSR bits following algorithm 6 after inducing faults at later/earlier cycles of operation. This process is continued till we obtain 128 linearly independent equations involving 128 s-bits at the base point. On an average, (128/33) * 66 = 256 faults need to be injected to obtain all LFSR bits. 128 linearly independent equations can be solved through Gaussian elimination in time $128^3 = O(2^{21})$.

Grain-128a: Here, the terms involving b as well as s bits in z_i are exactly same as Grain-128. The equations in LFSR bits obtained from faults in NFSR bit will be same as that tabulated in table VIII following algorithm 6. Again, the corresponding visible output bit is tabulated alongside.

Since, all b bits are known these equations are essentially linear equations at base point say, T. So, a fault at T-movement-1 cycle at location 102 gives the value of $s_8+b_{95}s_{94}$ at cycle T-1, which is equal to $s_{8-1}+b_{95-1}s_{94-1}$ at cycle T. This is tabulated alongside in table VIII 2^{nd} column with indices between 0 and 127. It is seen experimentally that this yields 128 linear equations in s bits when both s and s shifts from s are allowed with 256 faults in NFSR following same argument as Grain-128. It can be solved with about s time complexity using Guassian elimination.

Grain v1:The terms involving b as well as s bits are, $s_{i+64}b_{i+63}$, $s_{i+3}s_{i+46}b_{i+63}$, $s_{i+25}s_{i+46}b_{i+63}$ and $s_{i+46}s_{i+64}b_{i+63}$. Hence, $\delta^t = z_t + z_t^f$ with a bit-flip at b_{i+63} after *movement* gives the nonlinear equation for some

Algorithm 6 Determine LFSR Bits(f)

```
Grain-128, Grain-128a
Output b double s[] = \{12, 95\};
Output\_s\_double\_b[] = \{8, 42\};
outputPos[] = \{12, 95, 2, 15, 36, 45, 64, 73, 89\};
Grain v1
Output\_b\_double\_s[] = \{63\};
Output\_s\_double\_b[] = \{64\};
outputPos[] = \{1, 2, 4, 10, 31, 43, 56, 63\};
movement \leftarrow 0;
for i = 0 to 1 do
  movement \leftarrow f - Output\_b\_double\_s[i];
  if condition then
     Grain-128
    condition = OnlyContains(outputPos, movement,
     Output b double s[i], f) &&
     (Output\_s\_double\_b[i] + movement \le 127) \&\&
     (Output\_b\_double\_s[1-i] + movement) \le 127 \&\&
     (95 + movement) \le 127
     Grain-128a
    condition = OnlyContains(outputPos, movement,
    Output b double s[i], f) &&
     (Output\_s\_double\_b[i] + movement \le 127) \&\&
     (Output\_b\_double\_s[1-i] + movement) \le 127 \&\&
     (95 + movement) \le 127 \&\& (movement-64)\%2
    && (movement-64)/2 i, 0
     Grain v1
    condition = OnlyContains(outputPos, movement,
    Output\_b\_double\_s[i], f) \&\&
     (Output\_s\_double\_b[i] + movement \le 79) \&\&
     (Output\_b\_double\_s[1-i] + movement) \le 79
     Grain v1
    obtained equation,
    1 + s_{64} + s_{46}(s_{25} + s_{64})
    =\delta^{movement}
     Grain-128, Grain-128a
    obtained equation,
     SOutput\_s\_double\_b[i] + movement
    b_{Output\_b\_double\_s[1-i]+movement}.s_{95+movement}
    =\delta^{mov\overline{e}m\overline{e}nt}
  end if
```

Algorithm 7 onlyContains(array, movement, location, f)

end for

```
 \begin{aligned} &\textbf{for } element \in FaultTrace(f)[movement] \ \textbf{do} \\ &\textbf{for } i=0 \ \textbf{to } length(array) \ \textbf{do} \\ &\textbf{if } element \ != location \ \&\& \ element \ == \ array[i] \\ &\textbf{then} \\ &\textbf{return } \ false; \\ &\textbf{end if} \\ &\textbf{end for} \\ &\textbf{end for} \\ &\textbf{return } \ true; \end{aligned}
```

TABLE VII
FAULT LOCATION VS. LFSR, NFSR EQUATIONS OBTAINED FOR
GRAIN-128

Fault	Equations	Fault	Equations
Location		Location	
b_{12}	$s_8 + b_{95}.s_{95}$	b_{95}	$s_{42} + b_{12}.s_{95}$
b_{13}	$s_9 + b_{96}.s_{96}$	b_{96}	$s_{43} + b_{13}.s_{96}$
b_{14}	$s_{10} + b_{97}.s_{97}$	b_{97}	$s_{44} + b_{14}.s_{97}$
b_{15}	$s_{11} + b_{98}.s_{98}$	b_{98}	$s_{45} + b_{15}.s_{98}$
b_{16}	$s_{12} + b_{99}.s_{99}$	b_{99}	$s_{46} + b_{16}.s_{99}$
b_{17}	$s_{13} + b_{100}.s_{100}$	b_{100}	$s_{47} + b_{17}.s_{100}$
b_{18}	$s_{14} + b_{101}.s_{101}$	b_{101}	$s_{48} + b_{18}.s_{101}$
b_{19}	$s_{15} + b_{102}.s_{102}$	b_{102}	$s_{49} + b_{19}.s_{102}$
b_{20}	$s_{16} + b_{103}.s_{103}$	b_{103}	$s_{50} + b_{20}.s_{103}$
b_{21}	$s_{17} + b_{104}.s_{104}$	b_{104}	$s_{51} + b_{21}.s_{104}$
b_{22}	$s_{18} + b_{105}.s_{105}$	b_{105}	$s_{52} + b_{22}.s_{105}$
b_{23}	$s_{19} + b_{106}.s_{106}$	b_{106}	$s_{53} + b_{23}.s_{106}$
b_{24}	$s_{20} + b_{107}.s_{107}$	b_{107}	$s_{54} + b_{24}.s_{107}$
b_{25}	$s_{21} + b_{108}.s_{108}$	b_{108}	$s_{55} + b_{25}.s_{108}$
b_{26}	$s_{22} + b_{109}.s_{109}$	b_{109}	$s_{56} + b_{26}.s_{109}$
b_{27}	$s_{23} + b_{110}.s_{110}$	b_{110}	$s_{57} + b_{27}.s_{110}$
b_{28}	$s_{24} + b_{111}.s_{111}$	b_{111}	$s_{58} + b_{28}.s_{111}$
b_{29}	$s_{25} + b_{112}.s_{112}$	b_{112}	$s_{59} + b_{29}.s_{112}$
b_{30}	$s_{26} + b_{113}.s_{113}$	b_{113}	$s_{60} + b_{30}.s_{113}$
b_{31}	$s_{27} + b_{114}.s_{114}$	b_{114}	$s_{61} + b_{31}.s_{114}$
b_{32}	$s_{28} + b_{115}.s_{115}$	b_{115}	$s_{62} + b_{32}.s_{115}$
b_{33}	$s_{29} + b_{116}.s_{116}$	b_{116}	$s_{63} + b_{33}.s_{116}$
b_{34}	$s_{30} + b_{117}.s_{117}$	b_{117}	$s_{64} + b_{34}.s_{117}$
b_{35}	$s_{31} + b_{118}.s_{118}$	b_{118}	$s_{65} + b_{35}.s_{118}$
b_{36}	$s_{32} + b_{119}.s_{119}$	b_{119}	$s_{66} + b_{36}.s_{119}$
b_{37}	$s_{33} + b_{120}.s_{120}$	b_{120}	$s_{67} + b_{37}.s_{120}$
b_{38}	$s_{34} + b_{121}.s_{121}$	b_{121}	$s_{68} + b_{38}.s_{121}$
b_{39}	$s_{35} + b_{122}.s_{122}$	b_{122}	$s_{69} + b_{39}.s_{122}$
b_{40}	$s_{36} + b_{123}.s_{123}$	b_{123}	$s_{70} + b_{40}.s_{123}$
b_{41}	$s_{37} + b_{124}.s_{124}$	b_{124}	$s_{71} + b_{41}.s_{124}$
b_{42}	$s_{38} + b_{125}.s_{125}$	b_{125}	$s_{72} + b_{42}.s_{125}$
b_{43}	$s_{39} + b_{126}.s_{126}$	b_{126}	$s_{73} + b_{43}.s_{126}$
b_{44}	$s_{40} + b_{127}.s_{127}$	b_{127}	$s_{74} + b_{44}.s_{127}$

determinable t,

$$s_{i+64} + s_{i+3}s_{i+46} + s_{i+25}s_{i+46} + s_{i+46}s_{i+64} = \delta^t$$

$$s_{i+64} + s_{i+46}(s_{i+3} + s_{i+25} + s_{i+64}) = \delta^t$$

This equation becomes linear if s_{i+46} is known, otherwise, we have few small nonlinear equations in s variables. The equations in LFSR bits obtained from faults in NFSR bit is tabulated in table IX. As in the case of NFSR changing cycles of fault injection by i, gives a displacement of indices of bit-positions of equations by i, with indices between 0 and 79, this explains column 2 of table IX. Total number of faults required on average is, 80/5*5=80, as 80 equations are needed to solve 80 variables. The obtained small nonlinear equations can be solved using Grobner basis mathod, XL etc. with complexity at most $O(80^3) < O(2^{21})$, with about 160 faults required. Thus the complexity of the attack remains same as Grain-128 and Grain-128a.

E. Inverting Internal States

Till this phase we have successfully obtained full internal state of Grain ciphers at base pointi.e.,

TABLE VIII FAULT LOCATION VS. LFSR,NFSR BITS OBTAINED FOR GRAIN-128A

Fault Location	NFSR Bit Obtained	Output Cycle
	$/i^{th}$ Shift from T	
78	$s_{8-i} + b_{95-i}s_{94-i}$	1
80	$s_{8-i} + b_{95-i}s_{94-i}$	2
82	$s_{8-i} + b_{95-i}s_{94-i}$	3
84	$s_{8-i} + b_{95-i}s_{94-i}$	4
86	$s_{8-i} + b_{95-i}s_{94-i}$	5
88	$s_{8-i} + b_{95-i}s_{94-i}$	6
90	$s_{8-i} + b_{95-i}s_{94-i}$	7
92	$s_{8-i} + b_{95-i}s_{94-i}$	8
94	$s_{8-i} + b_{95-i}s_{94-i}$	9
96	$s_{8-i} + b_{95-i}s_{94-i}$	10
98	$s_{8-i} + b_{95-i}s_{94-i}$	11
100	$s_{8-i} + b_{95-i}s_{94-i}$	12
102	$s_{8-i} + b_{95-i}s_{94-i}$	13
104	$s_{8-i} + b_{95-i}s_{94-i}$	14
106	$s_{8-i} + b_{95-i}s_{94-i}$	15
108	$s_{8-i} + b_{95-i}s_{94-i}$	16
110	$s_{8-i} + b_{95-i}s_{94-i}$	17
112	$s_{8-i} + b_{95-i} s_{94-i}$	18
114	$s_{8-i} + b_{95-i}s_{94-i}$	19
116	$s_{8-i} + b_{95-i}s_{94-i}$	20
118	$s_{8-i} + b_{95-i}s_{94-i}$	21
120	$s_{8-i} + b_{95-i}s_{94-i}$	22
122	$s_{8-i} + b_{95-i}s_{94-i}$	23
124	$s_{8-i} + b_{95-i}s_{94-i}$	24
126	$s_{8-i} + b_{95-i}s_{94-i}$	25

TABLE IX FAULT LOCATION VS. LFSR,NFSR BITS OBTAINED FOR GRAIN V1

Fault I and a	MECD Dis Observed
Fault Location	NFSR Bit Obtained
	$/i^{th}$ Shift from T
63	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
64	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
65	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
66	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
67	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
68	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
69	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
70	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
71	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
72	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
73	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
74	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
75	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
76	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
77	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$
78	$1 + s_{64-i} + s_{46-i}(s_{25-i} + s_{64-i})$

 $(s_0^T, s_1^T, \dots, s_n^T, b_0^T, b_1^T, \dots, b_n^T)$ are known. Now, describe the procedure of obtaining key by inverting this state. This method is similar to that of [4]. We denote by f^t and g^t the feedback values at t^{th} cycle.

let. internal state of Grain after (t+1) cycles of operation. Then, according to Grain specification (refer to section 2),

$$s_i^{t+1} = s_{i+1}^t; i = 0, 1, \dots, n-2;
 s_{n-1}^{t+1} = f^t;
 b_i^{t+1} = b_{i+1}^t; i = 0, 1, \dots, n-2;
 b_{n-1}^{t+1} = g^t;$$
(14)
(15)

$$s_{n-1}^{t+1} = f^t; (15)$$

$$b_i^{t+1} = b_{i+1}^t; i = 0, 1, \dots, n-2;$$
 (16)

$$b_{n-1}^{t+1} = g^t; (17)$$

TABLE X FAULTS REQUIRED AND COMPUTING COST

Ciphers	Grain v1	Grain-128	Grain-128a
Online Cost	(#Faults,Cost)	(#Faults,Cost)	(#Faults,Cost)
Location	(0,11)	(0,14)	(0,14)
Pre-computation	$(0, 2^{12})$	$(0, 2^{15})$	$(0, 2^{15})$
NFSR	$(70, 2^{10})$	(56, 128)	(128, 128)
LFSR	$(80, < 2^{21})$	$(256, 2^{21})$	$(256, 2^{21})$
Inversion	(0, r(T))	(0, r(T))	(0, r(T))
Total	$(150, < 2^{21})$	$(312, 2^{21})$	$(384, 2^{21})$

The above equations can be rewritten as follows.

$$s_{i+1}^t = s_i^{t+1}; i = 0, 1, \dots, n-2;$$
 (18)

$$s_0^t = s_n^{t+1} + f^t + s_0^t; (19)$$

$$s_0^t = s_n^{t+1} + f^t + s_0^t;$$

$$b_{i+1}^t = b_i^{t+1}; i = 0, 1, \dots, n-2;$$

$$(10)$$

$$b_0^t = b_{n-1}^{t+1} + g^t + b_0^t;$$
 (21)

These equations hold as both f and g respectively contain s_0^t and b_0^t in all the three versions. Thus we are able to invert the full state up to initialization.

Step 2: When, $t \le 2n$, the output bits, z^t are fed back at $\boldsymbol{s}_{n-1}^{t+1}$ and $\boldsymbol{b}_{n-1}^{t+1}.$ Now, can in all three cases be written in terms of variables,

of variables,
$$s_i^{t+1}, i=0,1,\ldots,n-1 \text{ and } b_{i+1}^t, i=0,1,\ldots,n-1. \text{ For example, } s_{20}^t=s_{21}^{t+1} \text{ and } b_{26}^t=b_{27}^{t+1} \text{ etc.. Hence, once } z^t \text{ is computed,}$$

$$s_0^t = s_{n-1}^{t+1} + f^t + s_0^t + z^t;
 b_0^t = b_{n-1}^{t+1} + g^t + b_0^t + z^t;
 \tag{22}$$

$$b_0^t = b_{n-1}^{t+1} + g^t + b_0^t + z^t; (23)$$

Thus, again being able to invert during initialization also.

This way inverting from iteration 2n to 0, we get back secret $key \text{ in } b_0, b_1, \dots, b_{n-1}.$

V. COMPLEXITY, LIMITATIONS AND ACHIEVEMENTS

In this section, we measure the cost of our proposed attack against three Grain ciphers. This complexity is approximated by the average computing expenses of the process. We also estimate the average number of fault inductions required to completely break the system. Table X summarizes the cost. In this table T refers to the base point of the attack, r(T)represents the cost of reversing Grain for T cycles.

- 1) Fault Location Determination: It is divided into formation of σ table during simulation (algorithm 1) and determination of fault locations in online mode (algorithm 2). The off-line mode requires a space overhead of $128 * 128 = 2^{14}$ for Grain-128 and Grain-128a, and 80 * 80 = 1600 for Grain v1. Following σ formation, location determination is of logarithm complexity giving cost of about 14 units of computations at most. No online faults are required in this phase.
- 2) Pre-computation of Fault Traces: This phase stores precomputed traces for all n fault-locations for the following 2n cycles. Hence, space required in $n*2n*n=2n^3$. Time complexity of this phase is $n * 2n = 2n^2$ Grain

TABLE XI
COMPARISON OF COST OF FAULT ATTACK AGAINST GRAIN

Paper	Cipher	Fault Model	Faults
[4]	Grain-128	Similar	1587
[8]	Grain-128a	Similar	1831
Current	Grain v1	-	150
Current	Grain-128	-	312
Current	Grain-128a	-	384

round operations. This is an off-line operation. Online fault induction is not required in this phase also.

- 3) Determining NFSR Bits: We have faulted a particular location in online mode and determined linear equations by executing algorithm 4 in off-line mode. About 56 faults were required for Grain-128 and 128 units of computation. For Grain-128a each fault yields one equation. Allowing shifts, total number of online faults required is 128 and total computing cost is 128 units. Grain v1 follows same pattern as Grain-128a, requiring 70 online faults and 2¹⁰ units of computation for brute-force.
- 4) Determining LFSR Bits: We obtained linear or small nonlinear equations in this phase. Storing the equations require linear space. Grain-128 and Grain-128a due to same set of utilized equations, requires equal complexity, which is 256 average online faults and computing of about 2²¹ units. Grain v1 requires 80 faults and less than that amount of computing.
- 5) Inverting States: Inverting the state is constant computation per round and storage of 2n bits. Hence, r(T) units of computing is required and constant space is required for this phase irrespective of the version.

It follows from table X, that Grain-128a is the strongest of the three ciphers.

Clearly, the attack can be adapted to Grain-like ciphers. Higher minimum degree g, z will increase attack complexity. Multiple consecutive faults are a more realistic scenario, exploiting this situation may lead to better attacks. Allowing small higher degree equations may also decrease the complexity.

Table XI compares costs of fault attack against Grain presented in literatature with that presented here. It can be noted that in comparison requirements of number of faults is comparatively less in this paper.

In [5], the authors have proposed another attack on Grain family with minimal assumptions. They use SAT solver and non-reproduction of faults. The attack targets both LFSR and NFSR of the cipher. It is claimed that the attack takes 10 or less faults to break the system. However, one strong assumption in the paper is rekeying of the cipher. Ideally, an attacker does not have access to the pins for inputting key into the system. Also the overhead of using a SAT solver is not negligible.

VI. CONCLUSION

In this paper, we have described a fault analysis on the eStream cipher Grain family of stream ciphers. The earlier

fault attacks on Grain target LFSR of the cipher. We have presented a fault attack that faults NFSR. The attack is applied successfully against three Grain ciphers, Grain v1, Grain-128a and Grain-128. It has been seen that a few number of faults (at most 125% of state size) are required which is much lesser than earlier works. Space and time requirements are also minimal. We comment that this attack can be easily adapted on Grain-like ciphers.

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