Experimental Study of DIGIPASS GO3 and the Security of Authentication

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Abstract. Based on the analysis of 6-digit one-time passwords(OTP) generated by DIGI-PASS GO3 we were able to reconstruct the synchronisation system of the token, the OTP generating algorithm and the verification protocol in details essential for an attack. The OTPs are more predictable than expected. A forgery attack is described. We argue the attack success probability is 8^{-5} . That is much higher than 10^{-6} which may be expected if all the digits are independent and uniformly distributed. Under natural assumptions even in a relatively small bank or company with 10^4 customers the number of compromised accounts during a year may be more than 100.

1 Introduction

The remote authentication is commonly multi-way. It is based on a combination of the customer's identifier, his static password and a dynamic one-time password(OTP) generated by a security token, or read from a one-time password card, or communicated by a mobile device. Static passwords are at least theoretically may be tapped by a malicious software like trojans with keystroke logging for example, and used for an attack at some later point. Therefore the authentication security is largely based on the security of the one-time password. Theoretically, the latter is insecure if the next one-time passwords. For instance, 6-digit password may be predicted with probability given a number of previous one-time passwords. For instance, 6-digit password may be predicted with probability 10^{-6} anyway. If it is possible to predict with a significantly larger probability, then there is an internal weakness in the generator or in the verification protocol. The number of attempted wrong log-ins is usually bounded. In practical terms that makes the implementation of the attack more difficult, but does not make that impossible.

DIGIPASS is a one-time password generator manufactured by VASCO [1]. In what follows we study 6-digit combinations produced by DIGIPASS GO3. According to [1], the token is used by lots of customers all over the world, for instance, in Norway, Belgium, Ireland, United Kingdom, Netherlands, Denmark, Saudi Arabia, South Africa etc. It is used in Norwegian banks as Sparebanken Vest, as described in Sect. 4, and DNB NOR. The latter is one of the largest banks in Norway with 2.3 millions of retail customers and 198000 corporate customers. The device uses strong crypto algorithms as DES, TDES and AES and supports event and time based authentication. The detailed description of the algorithm was not published.

Based on the experiments with the token, we show 6-digit combinations from DIGIPASS GO3 are dependent and more predictable than expected. The left most digit is used for synchronisation: it indicates a time interval where the token was pressed to generate the OTP. The left most digit of the current OTP depends on the left most digit of the previous OTP and the time t elapsed since the previous OTP was generated. The left most digit is predictable with probability almost 1 if t

is close to a multiple of 64 seconds. Also we found the token may generate the same combination if $50 \le t < 64$ seconds and the probability of that depends on t. The last 5 digits of the 6-digit combination are not uniformly distributed: the digits 0, 1, 2, 3, 4, 5 appear with probability 1/8, while 6, 7, 8, 9 appear with probability 1/16.

We were able to reconstruct the synchronisation system of the token, the OTP generating algorithm and the verification protocol, see Sect. 2.5 and Sect. 2.6, in details essential for an attack. Our reconstruction fits well with the experimental data. In particular, it explains the OTP repetition and the token synchronisation. Though the algorithm implemented in DIGIPASS GO3 is somewhat similar to that published in [3] and described in Sect. 5, there is a difference. Due to the synchronisation provided by the left most digit, the verifier needs only one OTP comparison to authenticate the token.

A forgery attack is described in Sect. 3. The left most digit does not affect the security of the authentication. Therefore, the probability of one forged authentication is 8^{-5} instead of 10^{-6} as one may expect.

The attack may be automated and applied against any number of customers which use the token. We study the effect of the attack under natural assumptions that customer identifiers and static passwords are known to the adversary who may use malicious software as trojans etc to procure them, see for instance [4]. Also we assume the adversary is able to attack after each time the customer uses the token for authentication. Under those assumptions even in a relatively small bank or company with 10^4 customers the number of compromised accounts during a year may be more than 100.

2 Experimental Study of OTPs by DIGIPASS GO3

The experiments were produced by pressing the token and the stopwatch simultaneously at some time steps. A very little fluctuation between the real pressing time and that indicated in the tables below is possible. The data in the tables was produced by the same DIGIPASS GO3 token. Some of the experiments were repeated for other tokens with similar results.

The customer presses the token to generate an OTP. During the next 40 seconds this OTP is kept on the screen and then disappears. If one then presses the token in time between 40 and 50 seconds the same OTP reappears. If one presses in ≥ 50 seconds a new OTP is generated. The OTP may repeat if the token is pressed in between 50 and 64 seconds again as described in the next Sect. 2.1.

2.1 OTPs at time steps within [0:50+,1:03+)

Tables 5-18 present 6-digit combinations produced at time step t (format min:sec), where $0: 50 \le t < 0: 51$ (denoted t = 0: 50+, Table 5), $0: 51 \le t < 0: 52$ (denoted t = 0: 51+, Table 6) and so on.

For instance, 29 combinations in Table 5 were produced at time step t = 0 : 50+. The first OTP 240445 indicates starting time 0 : 00.00, the next 302168 was generated after 50.35 sec, the next 498773 was generated after another 50.41 sec and so on. Surprisingly, the DIGIPASS happens to repeat the combination. For instance, 642055 repeats after 0 : 50+ seconds again etc. That does not seem affect the security of the authentication as the server(verifier) does not accept repeated combination as correct. The sequence of 6-digit combinations in the tables may be split into intervals, each interval ends with a repeated 6-digit combination. For t = 0 : 50+ the length

of the intervals is 4 or 5. The right most column in the tables contains a sequence of differences modulo 10 between the left most digits in subsequent 6-digit combinations. That is called a pattern of the sequence. The pattern may be split into intervals accordingly.

For t = 0:51+ the length of the intervals is 5 or 6. For t = 0:52+ the length of the intervals is 5 or 6 again but the pattern is different. For t = 0:53+ the length of the intervals is 6 or 7 and so on. Finally, for t = 1:03+ the length of an interval may be larger than 67.

So when pressing at time step t within [0: 50+, 1: 03+) the first digit of the combination increases by 1 or is the same and the whole combination repeats. The pattern changes when t changes every second. In particular, the probability that 0 appears in the pattern is steadily decreasing while the time step is increasing by a second. For t around 1: 04 = 64 seconds the left most digit increases by 1 with probability very close to 1.

2.2 OTP at time intervals t within [1:04+,2:07+)

When the pressing at time steps within [1:04+, 2:07+) the first digit of the combination increases by 1 or 2 and the pattern changes every second as above. Tables 19-23 represent 6-digit combinations after pressing at time steps t, where t = 1:04+, 1:21+, 1:22+, 1:54+, 2:07+. The combinations may be split into the intervals. In Tables 19,20,21 the interval ends each time when the left most digits increases by 2 modulo 10. In Tables 22,23 the interval ends each time when the left most digits increases by 1 modulo 10. For t = 1:21+ the pattern includes intervals of length 3 and 4. For t = 1:22+ there are intervals of length 3 and 4 again, but the pattern is different. In particular, the intervals of length 3 appear more often than the intervals of length 4. The pattern for t = 1:54+is very similar to the pattern for t = 0:50+ with changing 0, 1 by 1, 2.

2.3 Pattern in General

Based on the experiments with the time steps t within [0:50+, 1:03+) and within [1:04+, 2:07+), and by extrapolating to any time interval we conclude the following.

- 1. The smallest time interval used in the measurements and the computations by DIGIPASS and the server is a second.
- 2. For $(1:04) \times A \leq t < (1:04) \times (A+1)$ the first digit of the combination increases by A or A+1 modulo 10. Each such interval incorporates 64 seconds and is equivalently represented as

$$t \in [64 \times A, 64 \times (A+1)). \tag{1}$$

Within (1) the pattern only includes digits A, A + 1 modulo 10 and is changing when t is changing every second.

- 3. The patterns are similar for time shifts equal to a multiple of 64 seconds. In other words, the patterns are similar for time steps t and t+64 seconds after substituting A, A+1 by A+1, A+2 respectively for any t.
- 4. Assume an OTP was generated and the next OTP was generated in t seconds. The first digit of the combination may increase by A modulo 10 only if

$$t \in [64 \times (A-1), 64 \times (A+1)).$$
(2)

5. The first digit likely increases by A - 1 modulo 10 in the beginning of the time interval (2), by A + 1 modulo 10 in the end. In the middle it mostly increases by A modulo 10. So if t is about 64 A seconds, the left most digit increases by A modulo 10 with probability close to 1. By symmetry, the probability of getting an increase by A modulo 10 within the interval (2) is 1/2.

Those observations are supported by experiments with random time steps in Table 1 and the time step 10 : 46 in Table 24.

2.4 Synchronising Function

Let (t_0, a_0) be some initial values. Let $(t_i, a_i), i = 1, 2, ...$ be the sequence of tuples, where t_i is the time when an OTP was generated and a_i is its left most digit. We claim there is a function $f(t_l, t_k)$ such that for any $l > k \ge 0$

1.
$$f(t_l, t_k) \in \{h_{lk} - 1, h_{lk}\},$$
 where $h_{lk} - 1 \le \frac{t_l - t_k}{64} < h_{lk},$
2. $a_l - a_k \equiv f(t_l, t_k) \mod 10,$
3. and $f(t_l, t_k) = \sum_{l=1}^{l-1} f(t_{i+1}, t_i).$ (3)

 $\overline{i=k}$

Though the function f may also depend on some other parameters, that does not affect our conclusions. The value of $f(t_l, t_k)$ represents the number of "time steps" between t_l and t_k and is similar to the time step function used in the well-known time-based OTP algorithm described in Sect. 5. However in contrast, $f(t_l, t_k)$ may have two values according to property 1 of f.

For example, the following Table 1 data was produced by pressing the token for 20 random time intervals $t_i - t_{i-1}$ within 1 and 10 minutes or so. The 4-th column of the table presents integers h - 1, h such that $h - 1 \leq \frac{t_i - t_{i-1}}{64} < h$, where $t_i - t_{i-1}$ was transformed into seconds. The 5-th column presents the pattern of the sequence of 6-digit combinations. The 6-th column gives the value of $f(t_i, t_{i-1}) \in \{h - 1, h\}$ according to the definition above.

Let us compute $f(t_{11}, t_5)$. As $t_{11} - t_5 = 30 : 27 = 1827$ seconds, we have $28 \le \frac{t_{11} - t_5}{64} < 29$. By (3), we compute $f(t_{11}, t_5) = 5 + 1 + 3 + 9 + 7 + 4 = 29$ and see that $a_{11} - a_5 \equiv f(t_{11}, t_5) \mod 10$. Therefore all the properties in the definition of f are satisfied for $f(t_{11}, t_5)$. Similarly, one computes $f(t_l, t_k)$ for any other t_l, t_k in Table 1 and in any other table of this paper and checks it always satisfies the definition.

2.5 How does the token generate an OTP

In this section we reconstruct the way how an OTP is generated by the token. Let t_0 be some initial moment of time and A_0 an initial value. Let t_{i-1} be the time of generating the (i-1)-th OTP and let A_{i-1} be an auxiliary integer number. Both t_{i-1} and A_{i-1} are kept by the token before the next pair is computed. The next OTP a_i, X_i is generated at time $t_i, i \ge 1$ by

$$A_i = A_{i-1} + f(t_i, t_{i-1}),$$

$$a_i \equiv A_i \mod 10,$$

$$X_i = E_K(A_i),$$

i	comb.	$t_i - t_{i-1}$	$\frac{t_i - t_{i-1}}{64}$	$a_i - a_{i-1} \mod 10$	$f(t_i, t_{i-1})$
0	565201				
1	057045	6:00	5,6	5	5
2	031320	10:01	9,10	0	10
3	317587	3:11	2,3	3	3
4	291277	10:05	9,10	9	9
5	433132	2:15	2,3	2	2
6	911213	5:08	4,5	5	5
7	041125	1:17	1,2	1	1
8	319430	2:26	2,3	3	3
9	253057	10:16	9,10	9	9
10	987234	7:31	7,8	7	7
11	398564	3:49	3,4	4	4
12	070423	7:32	7,8	7	7
13	216702	1:43	1,2	2	2
14	542368	3:29	3,4	3	3
15	914109	4:35	4,5	4	4
16	293821	3:28	3,4	3	3
17	348346	1:05	1,2	1	1
18	913123	5:39	5,6	6	6
19	611331	8:10	7,8	7	7
20	501416	9:37	9,10	9	9

Table 1. 6-digit combinations at random time intervals

where a_i is the OTP left most digit and X_i is a 5-digit combination, 6 digits overall. The function E_K is based on an encryption algorithm and depends on a secrete key. By induction and the property of f in Sect. 2.4,

$$A_{i} = A_{i-1} + f(t_{i}, t_{i-1}) = A_{0} + f(t_{i-1}, t_{0}) + f(t_{i}, t_{i-1}) = A_{0} + f(t_{i}, t_{0}).$$

$$\tag{4}$$

The algorithm above is similar to one described in Sect. 5, where the number of time steps is a steadily increasing function: it increases by 1 after each fixed time interval. In contrast, $f(t_i, t_0)$ may have two values for t_i within the same time interval of 64 seconds, see Sect. 2.4.

The algorithm fits well with the properties of DIGIPASS GO3 found in Sect. 2. In particular, if $f(t_i, t_{i-1}) = 0$, then $A_i = A_{i-1}$ and the whole OTP repeats. The latter is only possible for $t_i - t_{i-1} < 64$ seconds by the definition of f. Also the algorithm explains why the token may be pressed any number of times without authentication and the authentication still works after, see the next Sect. 2.6 for details.

2.6 How does the verifier check the OTP

In what follows we reconstruct the server(verifier) action in authentication. When a customer logs in he introduces his identifier and static passwords into a pop up window on the monitor of his computer. He then presses the DIGIPASS, reads an OTP, introduces that into another pop up window and hits the return key. When verifying the server is to solve the following three problems.

1. Handle the delay between generating the OTP and when the server gets it for authentication.

- 2. Check if the OTP was produced by the token assigned to that customer.
- 3. Assume a customer generates several OTPs without log in. The server should be able to authenticate that customer with the same token later.

Handling a delay. Let t be the time of generating an OTP and t' the time it comes to the server for authentication. There should be an acceptable delay time interval t' - t < T. We found T is 480 seconds by the following experiment. An OTP was generated and then introduced to the system with a delay of t'' and the server reaction was observed. The results are in Table 2. In this experiment T is 8 minutes, that is 480 seconds. Interestingly, after the OTP was introduced with a delay of 7:59+ and accepted, the synchronisation between the DIGIPASS and the server got lost. A new token had to be used.

Table 2. Delay handling at the verifier

t''	12:40+	10:00+	9:11+	8:00+	7:59+	7:04+	6:02+	5:06+
status	reject	reject	reject	reject	accept	accept	accept	accept

Defining the value of f within the delay. Let t_0 be an initial time and A_0 an initial integer number, both are available for the prover and the verifier. Let t be the time when the OTP a, Xwas generated. Let t' be the time when it comes to the server for verification. We show the server is able to recover $f(t, t_0)$ if t' - t < 480 seconds. Really, let t' belong to the interval

$$B - 1 \le \frac{t' - t_0}{64} < B \tag{5}$$

for some B. Therefore if t' - t < 480 seconds, then

$$B - 9 \le \frac{t - t_0}{64} < B.$$
(6)

 So

$$f(t, t_0) \in \{B - 9, \dots, B - 1, B\}$$

by the definition of f. As $f(t, t_0) \equiv a - A_0 \mod 10$, the server finds

$$f(t, t_0) = A - A_0. (7)$$

Simultaneously, the verifier learns an interval of at most 128 seconds for t:

$$(f(t, t_0) - 1) \, 64 \le t - t_0 < (f(t, t_0) + 1) \, 64.$$

Moreover it is likely that

 $t \approx f(t, t_0) \, 64 + t_0$

seconds by property 5 in Sect. 2.3.

Verification protocol In this description we skip the verification of the customer's static passwords. Let t be the moment of time when the token generates a new OTP a, X and t' be the moment of time when it comes for verification. The following protocol authenticates the token.

- 1. Compute B by (5).
- 2. Find $f(t, t_0) = A A_0$ from (7) and therefore A.
- 3. Compute

 $X' = E_K(A)$

and check if X = X'. If equality, then the OTP is accepted, otherwise rejected.

By the properties of f and by (4), A only depends on A_0, t_0 and t. So the customer may press the token any number of times without log in and that won't affect the next authentication.

2.7 Last 5 digits distribution

Let

abcdef

be a 6-digit combinations produced by DIGIPASS GO3. According to the analysis in previous sections, the first digit a is used for synchronisation. We study the distribution of the other digits b, c, d, e, f taken separately. There are 814 OTPs without repetitions in all the tables of this article. We count the number of times a decimal digit appears in each of the last five positions in those OTPs. The data is collected in Table 3, where the positions are denoted by b, c, d, e, f. The distributions

Table 3. The experimental digit distribution in the last 5 positions

	0	1	2	~	-	5	~		~	~
						105				
						80				
						103				
						97				
f	97	110	110	108	100	83	51	57	54	44

are not uniform. The digits 0, 1, 2, 3, 4, 5 appear twice more often on the average than 6, 7, 8, 9. Our explanation is the following. The last 5 digits are produced from a 20-bit string of pseudorandom data. One decimal digit per each subsequent 4-bit string. The latter represents a number from $\{0, 1, \ldots, 15\}$ and therefore a decimal digit after reduction modulo 10. As 4-bit strings are distributed uniformly, the distribution of decimal digits is as 0, 1, 2, 3, 4, 5 have probability 1/8 and 6, 7, 8, 9 have probability 1/16. That fits well with the experimental Table 3 data.

3 Basic Attack

We may assume customer's static passwords are known to the adversary. In what follows a basic algorithm to forge the dynamic password generated by a DIGIPASS is presented and the probability of success is calculated. Let t' be the time the latest OTP generated by the customer came for authentication, it may be known to the adversary or guessed as well.

- 1. Take random $a \mod 10$.
- 2. Generate 5 random digits

$$b, c, d, e, f \in \{0, 1, 2, 3, 4, 5\}$$

Put X = b c d e f.

3. Introduce the forged OTP a, X.

To avoid possible security checks at the verifier, one may supply the forged OTP at time > t' + 64 seconds or so for instance. In order to authenticate the server constructs a value A such that $A \equiv a \mod 10$, computes $X' = E_K(A)$ and matches X and X'.

According to Sect. 2.7, the probability X = X' is 8^{-5} . The probability p(x) of the attack success in case of x independent attempts to log in is

$$p(x) = 1 - \left(1 - \frac{1}{8^5}\right)^x.$$

3.1 One customer is targeted

When the customer finishes his session of authentications, the adversary communicates with the server by introducing the customer's identifier and his static password and tries to start a new session of authentications with forged OTP according to the algorithm in Section 3. Also there might be a possibility for the adversary to forge the OTP during the session started by the customer. If not there should be a possibility for him to interrupt the customer's session and then try to start a new session with a forged OTP. Therefore we assume the adversary is able to start forging the customer's authentication after each finished authentication by the customer.

For instance, in Norwegian Sparebanken Vest, see Sect. 4, each transaction requires an authentication. Let a customer use the remote authentication once per month to accomplish 10 authentications, e.g., to start a new session and to pay 9 bills. Therefore the number of forged attempts to authenticate is about 120 r during a year per customer, where r is the number of allowed attempts of authentication. In Sparebanken Vest r = 3. So the attack success probability applied during a year is then p(120r). If the attacker wants the attack to go unveiled he should wait for a new authentication by the customer to start a series of r-1 attempt forged authentication. The success probability is then p(120(r-1)) for one year.

Table 4. The success probability during a year	Table 4.	The	success	probab	oility	during	a year
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r	1	2	3	4	5	6
p(120 r)	0.0036	0.0072	0.0109	0.0145	0.0181	0.0217

3.2 Many customers are targeted

Let N customers use the remote authentication to accomplish 10 authentications per month on the average, that is 120 authentications per year. The average number of successful forged authentications is then about Np(120r) during one year. The attack may be automated. If it works against

one customer that should work against $N = 10^6$ or so of them. For r = 3 we have

$$Np(120 \times 3) \approx N \times 0.010926$$

of customer's accounts will get compromised on the average. If $N = 10^6$ then the number of compromised accounts is around 10926 during one year. For $N = 10^4$ the number of compromised accounts during one year is around 109.

4 Authentication in a Norwegian Bank

There are 3 ways for a customer in Norwegian Sparebanken Vest to authenticate himself and get access to his account for internet banking. They are

- 1. "BankID-innlogging" requires the social security number, BankID password of the customer's choice and a one-time password from DIGIPASS GO3.
- 2. "BankID på mobil" requires the social security number, PIN-code and a mobile phone with a BankID certificate on that.
- 3. "Alternativ innlogging" requires the social security number, internet banking password of the customer's choice and a one-time password generated by DIGIPASS GO3 or read from a one-time password card.

Two of the three authentication protocols require a one-time password generator.

5 Published One-Time Password Algorithms

Two One-Time Password(OTP) algorithms are published in [2,3]. The first HMAC-based One-Time Password (HOTP) algorithm specifies event-based OTP algorithm. The other(TOTP) is en extension of the HOTP algorithm to support the time-based moving factor. We briefly describe the latter. The prover(token) and the verifier(authentication server) use the same time-step value X. There must be a unique secret key K for each prover. Therefore,

$$TOTP = HOTP(K, T) = Truncate(HMAC("crypto", K, T)),$$
(8)

where T is an integer number which represents the number of time steps between the initial counter time T_0 and the current Unix time. More specifically,

$$T = \lfloor (\text{Current Unix Time} - T_0) / X \rfloor.$$

The server should compare OTP not only with the receiving time-stamps but also the past timestamps that are within a transition delay window, specified by the protocol. The function "Truncate" is specified in [2] for HMAC-SHA-1 as "crypto" in (8).

References

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- 3. www.ietf.org/rfc/rfc6238.txt
- M. Adham, A. Azodi, Y. Desmedt and I. Karaolis, How to Attack Two-Factor Authentication Internet Banking, in FC 2013, LNCS 7859, pp. 322–328, 2013

6 Appendices

Table 5. 6-digit combinations at 0:50+

i	comb.		7	818140	1	15	475446	1			
0	240445		8	943630	1	16	534182	1	23	141253	1
1	302168	1	9	943630	0	17	621339	1	24	141253	0
2	498773	1	10	091211	1	18	747108	1	25	251821	1
3	591327	1	11	147884	1	19	747108	0	26	370755	1
4	642055	1	12	220372	1	20	829372	1	27	455412	1
5	642055	0	13	388126	1	21	904089	1	28	455412	0
6	773743	1	14	388126	0	22	012530	1			

Table 6. 6-digit combinations at 0:51+

		_								
i	comb.	_ 21	319740	1	42	062056	1	63	756821	1
0	613525	$\frac{21}{22}$		1		130310	1		813067	
1	770281	1		-	-		-	-		
2	813348	1 1	544489	1		-0.00-	1		915018	
3	900173	$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \end{bmatrix}$	684950	1	-	011101	1		0000-0	
4	061082	⁻ 25	751093	1	46	377484	0	67	000513	0
5	001082 061082	1 26	751093	0	47	405446	1	68	123010	1
Ŭ		. 27	843031	1	48	510028	1	69	230168	1
6		$\frac{1}{28}$	953144	1	49	655673	1	70	336542	1
7	214421	1 29 29	091433	1	50	722642	1	71	444814	1
8	330592	1 30	102531	1	51	869254	1	72	444814	
9	446002	1 31 31	102531 102531	$\hat{0}$	-	869254	-	· ·	524691	1
10	446002	$0 \begin{vmatrix} 31 \\ 32 \end{vmatrix}$		Ĭ I	-		-		672561	1
11	577502	1 1		1	~~	0000	1			-
12	660042	1 1	344032	1	-	000-00	1		774225	-
13	711359	$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 34 \\ -1 \end{bmatrix}$		1			1		804713	
14	873243	- 135	520503	$1 \mid$	56	292465	1	77	804713	0
$14 \\ 15$		_ 36	520503	0	57	292465	0	78	906800	1
		$\begin{bmatrix} 0 \\ 1 \end{bmatrix} 37$	659320	1	58	325694	1	79	099042	1
16	000-00	$\frac{1}{38}$	755504	1	59	444324	1	80	158124	1
17	081487	1 39		1	60	546511	1	81	211271	1
18	127404	1	956211	1		627135	1		343838	-
19	252525	1 40 41 41		-	-	627135	0	-	343838	
20	252525	0	950211	U	02	027155	0	00	545050	U
L	ı – 1									

Table 7. 6-digit combinations at 0:52+

i	comb.		8	407177 1	17 142656 1	26 951011 1
0	736910		9	529021 1	18 205424 1	20 951011 1 27 951011 0
1	826434	1	10	690643 1	19 344929 1	27 951011 0 28 092344 1
2	938934	1	11	690643 0	20 449877 1	28 092344 1 29 103750 1
3	092613	1	12	725702 1	21 588742 1	$30\ 273100\ 1$
4	114863	1	13	835136 1	22 588742 0	31 325527 1
5	114863	0	14	911041 1	23 654000 1	31 323327 1 32 446575 1
6	242575	1	15	065804 1	24 731026 1	33 446575 0
$\overline{7}$	310480	1	16	065804 0	25 852133 1	001100100

Table 8. 6-digit combinations at 0:53+

$\begin{array}{c c} i & \text{comb.} \\ \hline 0 & 543141 \\ 1 & 632547 & 1 \\ 2 & 750122 & 1 \\ 3 & 820133 & 1 \\ 4 & 971984 & 1 \\ 5 & 023743 & 1 \\ 6 & 023743 & 0 \\ 7 & 100270 & 1 \\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} 6 & 023743 & 0 \\ 7 & 123658 & 1 \\ 8 & 213661 & 1 \end{array}$	$\begin{array}{c} 16 & 909506 & 1 \\ 17 & 034130 & 1 \\ 18 & 034130 & 0 \end{array}$		$\begin{array}{c} 35 534404 \\ 36 683124 \\ 37 683124 \\ 0 \end{array}$

Table 9. 6-digit combinations at 0:54+

i comb.	8 128426 1	17 930741 1	26 714284 1
0 460553	9 273175 1	18 015819 1	27 714284 0
1 522574 1	10 325114 1	19 145630 1	28 817215 1
2 605910 1	11 428472 1	20 145630 0	29 950261 1
3 708033 1	12 548721 1	21 210016 1	30 070143 1
4 891210 1	13 685321 1	22 305020 1	31 156623 1
5 982543 1	14 685321 0	23 422742 1	32 209504 1
6 019508 1	15 744745 1	24 533817 1	33 311863 1
7 019508 0	16 893417 1	25 643375 1	34 311863 0

Table 10. 6-digit combinations at 0:55+

i	comb.		7	615196 0	15 320845 0	23 063637 1
0	041004		8	756362	16 424153 1	24 172283 1
1	129955	1	9	890229 1	17 589022 1	25 215258 1
2	236319	1	10	932420 1	18 640131 1	26 324114 1
3	373335	1	11	051043 1	19 784457 1	27 440322 1
4	483103	1	12	130620 1	20 820301 1	28 511341 1
5	591790	1	13	251594 1	21 900508 1	29 649452 1
6	615196	1	14	320845 1	22 900508 0	30 649452 0

Table 11. 6-digit combinations at 0:56+

$\begin{array}{c c} i \text{ comb.} \\ \hline 0 232615 \\ 1 345751 1 \\ 2 476543 1 \\ 3 581103 1 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 13 & 443090 & 1 \\ 14 & 591452 & 1 \\ 15 & 643205 & 1 \\ 16 & 748041 & 1 \\ 17 & 748041 & 0 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} 3 & 581103 & 1 \\ 4 & 694218 & 1 \\ 5 & 700474 & 1 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 17 & 748041 & 0 \\ 18 & 810307 & 1 \\ 19 & 901680 & 1 \end{array}$	$\begin{array}{c} 23 & 350105 \\ 24 & 451132 \\ 25 & 451132 \\ \end{array} \\ 0$

Table 12. 6-digit combinations at 0:57+

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{bmatrix} 0 & 742005 \\ 1 & 801269 \\ 2 & 906986 \\ 1 & 17 & 305590 \\ 3 & 026145 \\ 1 & 18 \\ 452553 \\ 1 & 33 \\ 752402 \\ 1 & 33 \\ 752402 \\ 1 & 33 \\ 752402 \\ 1 & 45 \\ 45 \\ 859096 \\ 46 \\ 915222 \\ 47 \\ 044338 \\ 1 & 33 \\ 752402 \\ 1 & 47 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 45 \\ 1 & 18 \\ 1 &$	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\frac{1}{3}$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	31
	- 1 -
	. I ~
52732231 20 546102 1 35 914573 1 49 165276	- 1 -
63693511 216179491 360692411 50262014	
74547261 227945721 371331691 51304038	1
8 572390 1 23 806460 1 38 133169 0 52 409315	1
95723900 249042051 392876041 53504916	1
	1
	1
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	s 0
$13 956443 1 \\ 28 234253 0 \\ 43 673364 1 \\ 01 001200$	1°

Table 13. 6-digit combinations at 0:58+

i comb.	6 331435 1	13 941303 1	
0 710424	7 430511 1	14 044900 1	20 611601 1
1 832534 1	8 574450 1	15 194233 1	20 011001 1 21 714464 1
2 954513 1	9 646207 1	16 215335 1	217144041 228761421
3 051144 1	10 762324 1	17 381041 1	23 876142 0 23 876142 0
4 144831 1	11 762324 0	18 450269 1	23 870142 0
5 225274 1	12 811080 1	19 524832 1	

Table 14. 6-digit combinations at 0:59+

i	comb.		10	171118	1	21	109515	1	32	108404	1
0	132935		11	231314	1	22	266521	1	33	275774	1
1	279542	1	12	343154	1	23	344521	1	34	353233	1
2	344200	1	13	421393	1	24	456243	1	35	404253	1
3	413388	1	14	421393	0	25	557915	1	36	578471	1
4	578023	1	15	553101	1	26	612537	1	37	646855	1
5	085158	1	16	635170	1	27	786251	1	38	712803	1
6	731544	1	17	759238	1	28	786251	0	39	849321	1
7	815031	1	18	815107	1	29	836131	1	40	921401	1
8	945111	1	19	965207	1		957731		41	061879	1
9	064830	1	20	000298	1	31	051304	1	42	061879	0

Table 15. 6-digit combinations at 1:00+

i comb.	6 323641 1	13 940556 1	20 693002 1
0 724454	7 454143 1	14 335577 0	21 711011 1
1 861432 1	8 454143 1	15 162684 1	22 843759 1
2 907425 1	9 580265 1	16 285611 1	23 959331 1
3 033217 1	10 648310 1	17 391576 1	24 019107 1
4 126058 1	11 752930 1	18 422613 1	25 192847 1
5 235589 1	12 864301 1	19 558204 1	26 192847 1

Table 16. 6-digit combinations at 1:01+

i comb.	13 133123 1	27 440132 0	41 841782 1
0 803133	14 243252 1	28 509292 1	42 954933 1
1 922672 1	15 327112 1	29 680841 1	
2 062016 1	16 464543 1	30 750414 1	43 028616 1
3 141753 1	17 573035 1	31 860648 1	44 157851 1
42765541	18 665709 1	32 986760 1	45 253403 1
$5\ 315994\ 1$	19 731074 1	33 034146 1	46 316240 1
6 401737 1	20 895420 1	34 184532 1	47 453212 1
7 590233 1	21 973368 1	35 221552 1	48 533493 1
8 634440 1	22 065620 1	36 348100 1	49 610200 1
97464171	23 181246 1	$37\ 405404\ 1$	50 754655 1
10 844471 1	23 131240 1 24 232721 1	37 403404 1 38 527308 1	51 840223 1
			52 944388 1
11 927342 1	25 304903 1	39 641451 1	53 944388 0
12 023403 1	26 440132 1	40 707232 1	

Table 17. 6-digit combinations at 1:02+

i comb. 16 634277 1 33 268	8803 1 50 936521 1
0 049789 17 752091 1 34 374	$49901 \begin{bmatrix} 50 & 930521 \\ 51 & 017643 \end{bmatrix}$
1254541 188102671 35414	4499 1 5 5 5 5 5 5 5 5
2 249724 1 19 918023 1 36 54	52 107207 1 52 107207 1 52 107207 1 50 50 50 50 50 50 50 50 50 50 50 50 50
3 336253 1 20 045143 1 37 683	532538501
4 434262 1 21 110766 1 38 705	54221 54 308923 1
55340111122250180139879	55 442758 1
6 634963 1 23 390264 1 40 988	56153531111
	$\begin{bmatrix} 5030 \\ 8402 \\ 1 \end{bmatrix} \begin{bmatrix} 57 \\ 694302 \\ 1 \end{bmatrix}$
	58 704092 1
8 864346 1 25 407684 0 42 184	59 808352 1
9 904542 1 26 517745 1 43 203	60 956510 1
10 055374 1 $ 27 621362 1 $ $ 44 345$	
11 109826 1 $ 28 741133 1 $ $ 45 463$	52691 621548621
12 250174 1 $ 29 825148 1 $ $ 46 502$	$2087 1 \\ 63 223165 1 \\ 63 22$
13 393648 1 30 921056 1 47 603	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
14 472334 1 31 021283 1 48 724	403411 1 1 1 1 1
15 522314 1 32 132300 1 49 82	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Table 18. 6-digit combinations at 1:03+

					_			_			
i	comb.		17	736565	1	35	562882	1	53	354324	1
0	197308	ור	18	877010	1	36	696411	1	54	450806	1
1	229653	1	19	932921	1	37	793512	1	55	551977	1
2	314183	1	20	001830	1	38	856523	1	56	682610	1
3	314183	0	21	117801	1	39	954494	1	57	710400	1
4	472242	1	22	233531	1	40	001432	1	58	830638	1
5	548311	1	23	390642	1	41	159710	1	59	963248	1
6	653232	1	24	414560	1	42	253131	1	60	059241	1
7	751238	1	25	519185	1	43	343330	1	61	110925	1
8	853136	1	26	623444	1	44	453233	1	62	217217	1
9	994925	1	27	726137	1	45	564870	1	63	344198	1
10	003939	1	28	815144	1	46	604627	1	64	407524	1
11	136055	1	29	913133	1	47	775411	1	65	552941	1
12	242150	1	30	050445	1	48	842291	1	66	631306	1
13	377020	1	31	154051	1	49	967451	1	67	764802	1
14	433284	1	32	205540	1	50	005706	1	68	880970	1
15	589263	1	33	371497	1	51	102313	1	69	921905	1
16	697642	1	34	428420	1	52	240554	1	70	063039	1

Table 19. 6-digit combinations at 1:04+

i co	mb.	16 803105	1	33 834055 1	
					50 525517 1
	9612	17 964970	1	34 952044 1	51 632156 1
1 352	2122 1	18 085128	1	35 006042 1	51 002100 1 52 701365 1
2 430)378 1	19 149115	1	36 114194 1	
3 501	426 1	20 202403	1	37 200429 1	53 823442 1
0000	47261	21 393555		38 332250 1	54 967919 1
-					55 057862 1
5 770	0000 1	22 434318		39 434921 1	56 117521 1
6 858	8070 1	23 544247	1	40 541209 1	57 234149 1
7 954	4795 1	24 900020	1	41 628292 1	
8 054	4725 1	25 075758	1	42 721117 1	58 315074 1
		26 131647	-	438450511	59 452005 1
	1035				60 530501 1
10 215	5597 1	27 236059	1	44 922050 1	61 633500 1
11 324	4103 1	28 361743	1	45 024254 1	
12 431	417 1	29 487607	1	46 189460 1	62 720237 1
-	3145 1	30 550510		47 203522 1	63 898292 1
		00000000	-		64 906445 1
-	4507 1	31 640136		48 333231 1	65 155602 2
15 744	4115 1	32 788015	1	49 432931 1	

Table 20. 6-digit combinations at 1:21+

i	comb.		10 988222	1	21	334242	1			
0	729021		11 124510	2	22	530457	2	32	759044	1
1	862954	1	12 201595	1	23	672523	1	33	993272	2
2	984329	1	13 320271	1	24	739122	1	34	041015	1
3	027332	1	14 444368	1	25	904289	2	35	188531	1
4	283083	2	15 648544	2	26	066584	1	36	338403	2
5	354734	1	16 730542	1	27	126007	1	37	419244	1
6	430327	1	17 888677	1	28	252851	1	38	520143	1
$\overline{7}$	620593	2	18 011941	2	29	426375	2	39	623183	1
8	732308	1	19 135073	1	30	522133	1	40	834127	2
9	830515	1	20 272046	1	31	602702	1			

Table 21. 6-digit combinations at 1:22+

i	comb.		13	965143	1	ſ	27	841140	2				
0	373763		14	112015	2		28	992450	1	41	675	541	2
1	406375	1	15	237007	1		29	081502	1	42	710	104	1
2	533966	1	16	331278	1		30	102493	1	43	863	144	1
3	704802	2	17	515093	2		31	312450	2	44	953	202	1
4	875121	1	18	640216	1		32	439123	1	45	155	572	2
5	927450	1	19	751552	1		33	516114	1	46	243	962	1
6	032654	1	20	858301	1		34	772509	2	47	391	080	1
7	299538	2	21	015311	2		35	864425	1	48	534	333	2
8	325157	1	22	126288	1		36	963344	1	49	642	176	1
9	441409	1	23	200451	1		37	073140	1	50	772	069	1
10	650483	2	24	421201	2		38	209594	2	51	857	507	1
11	717743	1	25	593095	1		39	336225	1	52	008	457	2
12	883525	1	26	635180	1		40	443141	1				

Table 22. 6-digit combinations at 1:54+

i comb.	10 655822 2	21 674041 2	32 504147 1
0 834101	11 810212 2	22 802045 2	33 739322 2
1 090148 2	12 050308 2	23 985324 1	34 940505 2
2 292408 2	13 215335 2	24 122040 2	35 126602 2
3 428030 2	14 310162 1	25 355406 2	36 324350 2
$4\ 531451\ 1$	15 595105 2	26 523265 2	37 487094 1
5 766274 2	16 749532 2	27 751112 2	38 639081 2
6 941733 2	17 935564 2	28 860390 1	39 851054 2
7 135251 2	18 028050 1	29 009811 2	40 053215 2
8 375411 2	19 203259 2	30 203422 2	41 250881 2
9 430301 1	20 488972 2	31 430329 2	42 362597 1

Table 23. 6-digit combinations at 2:07+

$i \mod$		4 000508 2	9 000822 2	14 900609 2
0 27333	5	5 296752 2	10 114160 1	15 154812 2
1 48315	8 2	6 431516 2	11 379050 2	16 303585 2
2 64105	1 2	7 643494 2	12 571410 2	$10\ 505500\ 2$ $17\ 529381\ 2$
3 80383	5 2	8 833238 2	13 781454 2	17 0200012

Table 24. 6-digit combinations at 10:46+

i comb.	4 461315 0	9 576113 1	14 572212 0
0 452170 0	$5\ 432355\ 0$	10 559829 0	14 572212 0 15 598353 0
$1\ 421660\ 0$	6 420151 0	11 513224 0	165963420
2 462345 0	7 431628 0	12 540014 0	$10 \ 530542 \ 0 \ 17 \ 531571 \ 0$
3 431757 0	8 422360 0	13 523659 0	17 331371 0