Authenticated Garbling and Efficient Maliciously Secure Two-Party Computation

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Abstract

We propose a simple and efficient framework for obtaining efficient constant-round protocols for maliciously secure two-party computation. Our framework uses a function-independent preprocessing phase to generate authenticated information for the two parties; this information is then used to construct a *single* "authenticated" garbled circuit which is transmitted and evaluated.

We also show how to efficiently instantiate the preprocessing phase by designing a highly optimized version of the TinyOT protocol by Nielsen et al. Our overall protocol outperforms existing work in both the single-execution and amortized settings, with or without preprocessing:

- In the single-execution setting, our protocol evaluates an AES circuit with malicious security in 37 ms with an online time of just 1 ms. Previous work with the best online time (also 1 ms) requires 124 ms in total; previous work with the best total time requires 62 ms (with 14 ms online time).
- If we amortize the computation over 1024 executions, each AES computation requires just 6.7 ms with roughly the same online time as above. The best previous work in the amortized setting has roughly the same total time but does not support function-independent preprocessing.

Our work shows that the performance penalty for maliciously secure two-party computation (as compared to semi-honest security) is much smaller than previously believed.

1 Introduction

Protocols for secure two-party computation (2PC) allow two parties to compute an agreed-upon function of their inputs without revealing anything additional to each other. Although originally viewed as impractical, protocols for generic 2PC in the semi-honest setting based on Yao's garbled-circuit protocol [Yao86] have seen tremendous efficiency improvements over the past several years [MNPS04, HEKM11, ZRE15, KS08, KMR14, ALSZ13, BHKR13, PSSW09].

While these results are impressive, semi-honest security—which assumes that both parties follow the protocol honestly yet may try to learn additional information from the execution—is clearly not sufficient for all applications. This has motivated researchers to construct protocols achieving the stronger notion of *malicious* security. One popular approach for designing constant-round maliciously secure protocols is to apply the "cut-and-choose" technique [LP07, sS11, sS13, KSS12, LP11, HKE13, Lin13, Bra13, FJN14, AMPR14] to Yao's garbled-circuit protocol. For statistical security $2^{-\rho}$, the best approaches using this paradigm require ρ garbled circuits (which is optimal); the most efficient instantiation of this approach, by Wang et al. [WMK17], securely evaluates an AES circuit in 62 ms.

The cut-and-choose approach incurs significant overhead when large circuits are evaluated precisely because ρ garbled circuits need to be transmitted (typically, $\rho \geq 40$). In order to mitigate this, recent works have explored secure computation in an *amortized* setting where the same function is evaluated multiple times

AES Evaluation (2.1 ms in the semi-honest setting)							
	Single-Execution Setting			Amortized Setting (1024 executions)			
	[NST17]	[WMK17]	**This paper**	[LR15]	[RR16]	[NST17]	**This paper**
Function-ind. phase	89.6 ms	-	10.9 ms	-	-	13.84 ms	4.9 ms
Function-dep. phase	13.2 ms	28 ms	4.78 ms	74 ms	5.1 ms	0.74 ms	0.53 ms
Online	1.46 ms	14 ms	0.93 ms	7 ms	1.3 ms	1.13 ms	1.23 ms
Total	104.26 ms	42 ms	16.61 ms	81 ms	6.4 ms	15.71 ms	6.66 ms

SHA-256 Evaluation (9.6 ms in the semi-honest setting)							
	Single-Execution Setting			Amortized Setting (1024 executions)			
	[NST17]	[WMK17]	**This paper**	[LR15]	[RR16]	[NST17]	**This paper**
Function-ind. phase	478.5 ms	-	96 ms	-	-	183.5 ms	64.8 ms
Function-dep. phase	164.4 ms	350 ms	51.7 ms	206 ms	48 ms	11.7 ms	8.7 ms
Online	11.2 ms	84 ms	9.3 ms	33 ms	8.4 ms	9.6 ms	11.3 ms
Total	654.1 ms	434 ms	157 ms	239 ms	56.4 ms	204.8 ms	84.8 ms

Table 1: Constant-round 2PC protocols with malicious security. All timings are based on an Amazon EC2 `c4.8xlarge` instance over a LAN, and are averaged over 10 executions. Single-execution times do not include the base-OTs, which require the same time ($\sim$20 ms) for all protocols. Timings for the semi-honest protocol are based on the same garbling code used in our protocol, and also do not include the base-OTs. See Section 7 for more details.

(on different inputs) [HKK$^+$14, LR14, LR15, RR16]. When amortizing over τ executions, only $O(\frac{\rho}{\log \tau})$ garbled circuits are needed per execution. Rindal and Rosulek [RR16] report a time of 6.4 ms to evaluate an AES circuit, amortized over 1024 executions.

More recently, Nielsen and Orlandi [NO16] proposed a protocol with *constant* amortized overhead, but only when $\tau = \Omega(|C|)$. Also, their protocol allows for amortization only over *parallel* executions done at the same time, whereas the works cited above allow amortization even over *sequential* executions, where inputs to the different executions need not be known all at once.

Other techniques for constant-round, maliciously secure two-party computation, with asymptotically better performance than cut-and-choose (without amortization), have also been explored. The LEGO protocol and subsequent optimizations [NO09, FJN$^+$13, FJNT15, HZ15, NST17] are based on a gate-level cut-and-choose approach that can be done during a preprocessing phase before the circuit to be evaluated is known. This class of protocols has good asymptotic performance and small online time; however, the best reported LEGO implementation [NST17] still has a higher end-to-end running time than the best protocol based on the cut-and-choose approach applied at the garbled-circuit level.

The Beaver-Micali-Rogaway compiler [BMR90] provides yet another way to construct constant-round protocols with malicious security [DI05, CKMZ14]. This compiler uses an "outer" secure-computation protocol to generate a garbled circuit that is then evaluated. Lindell et al. [LPSY15, LSS16] suggested applying this idea using SPDZ [DPSZ12] (or based on somewhat homomorphic encryption) as the outer protocol, but did not provide an implementation of the resulting scheme.

There are also protocols whose round complexity is linear in the depth of the circuit being evaluated. The TinyOT protocol [NNOB12] extends the classical GMW protocol [GMW87] by adding information-theoretic MACs to shares held by the parties; The IPS protocol [IPS08] has excellent asymptotic complexity, but its concrete complexity is unclear since it has never been implemented (and appears quite difficult to implement). We remark that the end-to-end time of these protocols suffers significantly from their large round complexity: Even over a LAN, each communication round requires at least 0.5 ms; for evaluating an AES circuit (with a depth of about 50), this means that the time for any linear-round protocol will be at

Protocol	Function-ind. (Comm./Comp.)	Function-dep. (Comm./Comp.)	Online (Comm.)	Online (Comp.) / Storage
Cut-and-choose [Lin13, AMPR14, WMK17]	—	$O(\lvert\mathcal{C}\rvert\rho)$	$O(\lvert\mathcal{I}\rvert\rho)$	$O(\lvert\mathcal{C}\rvert\rho)$
Amortized [HKK+14, LR14]	—	$O\left(\frac{\lvert\mathcal{C}\rvert\rho}{\log\tau}\right)$	$O\left(\frac{\lvert\mathcal{I}\rvert\rho}{\log\tau}\right)$	$O\left(\frac{\lvert\mathcal{C}\rvert\rho}{\log\tau}\right)$
LEGO [NO09, FJN+13]	$O\left(\frac{\lvert\mathcal{C}\rvert\rho}{\log\tau+\log\lvert\mathcal{C}\rvert}\right)$	$O(\lvert\mathcal{C}\rvert)$	$O(\lvert\mathcal{I}\rvert+\lvert\mathcal{O}\rvert)$	$O\left(\frac{\lvert\mathcal{C}\rvert\rho}{\log\tau+\log\lvert\mathcal{C}\rvert}\right)$
SPDZ-BMR [LPSY15, KOS16]*	$O(\lvert\mathcal{C}\rvert\kappa)$	$O(\lvert\mathcal{C}\rvert)$	$O(\lvert\mathcal{I}\rvert+\lvert\mathcal{O}\rvert)$	$O(\lvert\mathcal{C}\rvert)$
This paper (with Section 4.2)	$O\left(\frac{\lvert\mathcal{C}\rvert\rho}{\log\tau+\log\lvert\mathcal{C}\rvert}\right)$	$O(\lvert\mathcal{C}\rvert)$	$\lvert\mathcal{I}\rvert+\lvert\mathcal{O}\rvert$	$O(\lvert\mathcal{C}\rvert)$
This paper (with [IPS08])	$O(\lvert\mathcal{C}\rvert)$			

Table 2: Asymptotic complexity of constant-round 2PC protocols with malicious security. $|\mathcal{C}|, |\mathcal{I}|, |\mathcal{O}|$ are the circuit size, input size, and output size respectively; low-order terms independent of these parameters are ignored. The statistical security parameter is ρ, the computational security parameter is κ, and τ is the number of protocol executions in the amortized setting. Communication (Comm.) is measured as the number of symmetric-key ciphertexts, and computation (Comp.) is measured as the number of symmetric-key operations. "Storage" is the number of symmetric-key ciphertexts generated by the offline stage.
*Although the complexity of function-independent preprocessing can be reduced to $O(|\mathcal{C}|)$ using somewhat homomorphic encryption [DPSZ12], doing so requires a number of *public-key* operations proportional to $|\mathcal{C}|$.

Functionality $\mathcal{F}_{\mathsf{Pre}}$

- Upon receiving Δ_{A} from P_{A} and init from P_{B}, and assuming no values $\Delta_{\mathsf{A}}, \Delta_{\mathsf{B}}$ are currently stored, choose uniform $\Delta_{\mathsf{B}} \in \{0,1\}^{\rho}$ and store $\Delta_{\mathsf{A}}, \Delta_{\mathsf{B}}$. Send Δ_{B} to P_{B}.
- Upon receiving $(\mathsf{random}, r, \mathsf{M}[r], \mathsf{K}[s])$ from P_{A} and random from P_{B}, sample uniform $s \in \{0,1\}$ and set $\mathsf{K}[r] := \mathsf{M}[r] \oplus r\Delta_{\mathsf{B}}$ and $\mathsf{M}[s] := \mathsf{K}[s] \oplus s\Delta_{\mathsf{A}}$. Send $(s, \mathsf{M}[s], \mathsf{K}[r])$ to P_{B}.
- Upon receiving $(\mathtt{AND}, (r_1, \mathsf{M}[r_1], \mathsf{K}[s_1]), (r_2, \mathsf{M}[r_2], \mathsf{K}[s_2]), r_3, \mathsf{M}[r_3], \mathsf{K}[s_3])$ from P_{A} and $(\mathtt{AND}, (s_1, \mathsf{M}[s_1], \mathsf{K}[r_1]), (s_2, \mathsf{M}[s_2], \mathsf{K}[r_2]))$ from P_{B}, verify that $\mathsf{M}[r_i] = \mathsf{K}[r_i] \oplus r_i\Delta_{\mathsf{B}}$ and that $\mathsf{M}[s_i] = \mathsf{K}[s_i] \oplus s_i\Delta_{\mathsf{A}}$ for $i \in \{1,2\}$ and send cheat to P_{B} if not. Otherwise, set $s_3 := r_3 \oplus ((r_1 \oplus s_1) \wedge (r_2 \oplus s_2))$, set $\mathsf{K}[r_3] := \mathsf{M}[r_3] \oplus r_3\Delta_{\mathsf{B}}$, and set $\mathsf{M}[s_3] := \mathsf{K}[s_3] \oplus s_3\Delta_{\mathsf{A}}$. Send $(s_3, \mathsf{M}[s_3], \mathsf{K}[r_3])$ to P_{B}.

Figure 1: The preprocessing functionality, assuming P_{A} is corrupted. (It is defined symmetrically if P_{B} is corrupted. If neither party is corrupted, the functionality is adapted in the obvious way.)

least 25 ms. The situation will be even worse over a WAN.

In Tables 1 and 2, we summarize the efficiency of various constant-round 2PC protocols with malicious security. Table 1 gives the performance of state-of-the-art implementations under fixed hardware and network conditions, while Table 2 reports the asymptotic complexity of various approaches. Following [NST17], we consider executions in three phases:

- **Function-independent preprocessing.** In this phase, the parties need not know their inputs or the function to be computed (beyond an upper bound on the number of gates).
- **Function-dependent preprocessing.** In this phase, the parties know what function they will compute, but do not need to know their inputs.

  Often, the first two phases are combined and referred to simply as the *offline* or *preprocessing* phase.
- **Online phase.** In this phase, the parties evaluate the agreed-upon function on their respective inputs.

1.1 Our Contributions

We propose a new approach for constructing constant-round, maliciously secure 2PC protocols with extremely high efficiency. At a high level (further details are in Section 3), and following ideas of [NNOB12], our protocol uses a function-independent preprocessing phase to realize an ideal functionality that we call $\mathcal{F}_{\mathsf{Pre}}$ (cf. Figure 1). This preprocessing phase is used to set up correlated randomness between the two parties that they can then use during the online phase for information-theoretic authentication of different values. In contrast to [NNOB12], however, the parties in our protocol use this information in the online phase to generate a *single* "authenticated" garbled circuit. As in the semi-honest case, this garbled circuit can then be transmitted and evaluated in just one additional round.

Regardless of how we realize $\mathcal{F}_{\mathsf{Pre}}$, our protocol is extremely efficient in the function-dependent preprocessing phase and the online phase. Specifically, compared to Yao's *semi-honest* garbled-circuit protocol, the cost of the function-dependent preprocessing phase of our protocol is only about $2\times$ higher (assuming 128-bit computational security and 40-bit statistical security), and the cost of the online phase is essentially unchanged.

We show how to instantiate $\mathcal{F}_{\mathsf{Pre}}$ efficiently by developing a highly optimized version of the TinyOT protocol (adapting [NNOB12]), described in Section 4.2. Instantiating our framework in this way, we obtain a protocol with the same asymptotic communication complexity as recent protocols based on LEGO, but with two advantages. First, our protocol has much better *concrete* efficiency (see Table 1 and Section 7). For example, it requires only 16.6 ms total to evaluate AES, a $6\times$ improvement compared to a recent implementation of a LEGO-style approach [NST17]. Furthermore, the storage needed by our protocol is asymptotically smaller (see Table 2), something that is especially important when very large circuits are evaluated.

Instantiating our framework with the realization of $\mathcal{F}_{\mathsf{Pre}}$ described in Section 4.2 yields a protocol with the best concrete efficiency, and is the main focus of this paper. However, we note that our framework can also be instantiated in other ways:

- When $\mathcal{F}_{\mathsf{Pre}}$ is instantiated using the IPS compiler [IPS08] and the bit-OT protocol by Ishai et al. [IKOS09], we obtain a maliciously secure constant-round 2PC protocol with communication complexity $O(|\mathcal{C}|\kappa)$. Up to constant factors, this matches the complexity of *semi-honest* 2PC based on garbled circuits.

  The only previous work [JS07] that achieves similar communication complexity requires a constant number of public-key operations *per gate* of the circuit, and would have concrete performance orders of magnitude worse than our protocol.

- We can also realize $\mathcal{F}_{\mathsf{Pre}}$ using an offline, (semi-)trusted server. In that case we obtain a constant-round protocol for server-aided 2PC with complexity $O(|\mathcal{C}|\kappa)$. Previous work in the same model [MOR16] achieves the same complexity but with number of rounds proportional to the circuit depth.

The results described in this paper—both the idea of constructing an "authenticated" garbled circuit as well as the efficient TinyOT protocol we developed—have already found application in subsequent work [WRK17, HSSV17] on constant-round *multiparty* computation with malicious security.

2 Notation and Preliminaries

We use κ to denote the computational security parameter (i.e., security should hold against attackers running in time $\approx 2^{\kappa}$), and ρ for the statistical security parameter (i.e., an adversary should succeed in cheating with probability at most $2^{-\rho}$). We use $=$ to denote equality and $:=$ to denote assignment. We denote the parties running the 2PC protocol by P_{A} and P_{B}.

A circuit is represented as a list of gates having the format $(\alpha, \beta, \gamma, T)$, where α and β denote the indices of the input wires of the gate, γ is the index of the output wire of the gate, and $T \in \{\oplus, \wedge\}$ is the type of the gate. We use $\mathcal{I}_1$ to denote the set of indices of P_{A}'s input wires, $\mathcal{I}_2$ to denote the set of indices of P_{B}'s input wires, $\mathcal{W}$ to denote the set of indices of the output wires of all AND gates, and $\mathcal{O}$ to denote the set of indices of the output wires of the circuit.

2.1 Information-theoretic MACs

We use the information-theoretic message authentication codes (IT-MACs) of [NNOB12], which we briefly recall. $\mathsf{P_A}$ holds a uniform *global key* $\Delta_\mathsf{A} \in \{0,1\}^\kappa$. A bit b known by $\mathsf{P_B}$ is authenticated by having $\mathsf{P_A}$ hold a uniform key $\mathsf{K}[b]$ and having $\mathsf{P_B}$ hold the corresponding tag $\mathsf{M}[b] := \mathsf{K}[b] \oplus b\Delta_\mathsf{A}$. Symmetrically, $\mathsf{P_B}$ holds an independent global key Δ_B; a bit b known by $\mathsf{P_A}$ is authenticated by having $\mathsf{P_B}$ hold a uniform key $\mathsf{K}[b]$ and having $\mathsf{P_A}$ hold the tag $\mathsf{M}[b] := \mathsf{K}[b] \oplus b\Delta_\mathsf{B}$. We use $[b]_\mathsf{A}$ to denote an authenticated bit known to $\mathsf{P_A}$ (i.e., $[b]_\mathsf{A}$ means that $\mathsf{P_A}$ holds $(b, \mathsf{M}[b])$ and $\mathsf{P_B}$ holds $\mathsf{K}[b]$), and define $[b]_\mathsf{B}$ symmetrically.

Observe that this MAC is XOR-homomorphic: given $[b]_\mathsf{A}$ and $[c]_\mathsf{A}$, the parties can (locally) compute $[b \oplus c]_\mathsf{A}$ by having $\mathsf{P_A}$ compute $\mathsf{M}[b \oplus c] := \mathsf{M}[b] \oplus \mathsf{M}[c]$ and $\mathsf{P_B}$ compute $\mathsf{K}[b \oplus c] := \mathsf{K}[b] \oplus \mathsf{K}[c]$.

It is possible to extend the above idea to authenticate secret values by using XOR-based secret sharing and authenticating each party's share. That is, we can authenticate a bit λ, known to neither party, by letting r, s be uniform subject to $\lambda = r \oplus s$, and then having $\mathsf{P_A}$ hold $(r, \mathsf{M}[r], \mathsf{K}[s])$ and $\mathsf{P_B}$ hold $(s, \mathsf{M}[s], \mathsf{K}[r])$. It can be observed that this scheme is also XOR-homomorphic.

As described in the Introduction, we use a preprocessing phase that realizes a stateful ideal functionality $\mathcal{F}_\mathsf{Pre}$ defined in Figure 1. This functionality is used to set up correlated values between the parties along with their corresponding IT-MACs. The functionality chooses uniform global keys for each party, with the malicious party being allowed to choose its global key. Then, when the parties request a random authenticated bit, the functionality generates an authenticated secret sharing of the random bit $\lambda = r \oplus s$. (The adversary may choose the "random values" it receives, but this does not reveal anything about $r \oplus s$ or the other party's global key to the adversary.) Finally, the parties may also submit authenticated shares for two bits; the functionality then computes a (fresh) authenticated share of the AND of those bits. In the next section we describe our protocol assuming some way of realizing $\mathcal{F}_\mathsf{Pre}$; we defer until Section 4 a discussion of how $\mathcal{F}_\mathsf{Pre}$ can be realized.

3 Protocol Intuition

We give a high-level overview of the core of our protocol in the $\mathcal{F}_\mathsf{Pre}$-hybrid model. Our protocol has the parties compute a garbled circuit in a distributed fashion, where the garbled circuit is "authenticated" in the sense that the circuit generator ($\mathsf{P_A}$ in our case) cannot change the logic of the circuit. We describe the intuition behind our construction in several steps.

We begin by reviewing standard garbled circuits. Each wire α of a circuit is associated with a random "mask" $\lambda_\alpha \in \{0,1\}$ known to $\mathsf{P_A}$. If the actual value of that wire (i.e., the value when the circuit is evaluated on the parties' inputs) is x, then the masked value observed by the circuit evaluator (namely, $\mathsf{P_B}$) on that wire will be $\hat{x} = x \oplus \lambda_\alpha$. Each wire α is also associated with two labels $\mathsf{L}_{\alpha,0}$ and $\mathsf{L}_{\alpha,1} := \mathsf{L}_{\alpha,0} \oplus \Delta$ (using the free-XOR technique [KS08]) known to $\mathsf{P_A}$. If the masked bit on that wire is $\hat{x}$, then $\mathsf{P_B}$ learns $\mathsf{L}_{\alpha,\hat{x}}$.

Let H be a hash function modeled as a random oracle. The garbled table for, e.g., an AND gate $(\alpha, \beta, \gamma, \wedge)$ is given by:

$\hat{x}$ $\hat{y}$	truth table	garbled table
0 0	$\hat{z}_{00} = (\lambda_\alpha \wedge \lambda_\beta) \oplus \lambda_\gamma$	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 00) \oplus (\hat{z}_{00}, \mathsf{L}_{\gamma,\hat{z}_{00}})$
0 1	$\hat{z}_{01} = (\lambda_\alpha \wedge \overline{\lambda_\beta}) \oplus \lambda_\gamma$	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,1}, \gamma, 01) \oplus (\hat{z}_{01}, \mathsf{L}_{\gamma,\hat{z}_{01}})$
1 0	$\hat{z}_{10} = (\overline{\lambda_\alpha} \wedge \lambda_\beta) \oplus \lambda_\gamma$	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,0}, \gamma, 10) \oplus (\hat{z}_{10}, \mathsf{L}_{\gamma,\hat{z}_{10}})$
1 1	$\hat{z}_{11} = (\overline{\lambda_\alpha} \wedge \overline{\lambda_\beta}) \oplus \lambda_\gamma$	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,1}, \gamma, 11) \oplus (\hat{z}_{11}, \mathsf{L}_{\gamma,\hat{z}_{11}})$

$\mathsf{P_B}$, holding $(\hat{x}, \mathsf{L}_{\alpha,\hat{x}})$ and $(\hat{y}, \mathsf{L}_{\beta,\hat{y}})$, evaluates this garbled gate by picking the $(\hat{x}, \hat{y})$-th row and decrypting using the garbled labels it holds, thus obtaining $(\hat{z}, \mathsf{L}_{\gamma,\hat{z}})$.

The standard garbled circuit just described ensures security against a malicious $\mathsf{P_B}$, since (in an intuitive sense) $\mathsf{P_B}$ learns no information about the true values on any of the wires. Unfortunately, it provides no security against a malicious $\mathsf{P_A}$ who can potentially cheat by corrupting rows in the various garbled tables. One particular attack $\mathsf{P_A}$ can carry out is a *selective-failure* attack. Say, for example, that a malicious $\mathsf{P_A}$ corrupts only the $(0,0)$-row of the garbled table for the gate above, and assume $\mathsf{P_B}$ aborts if it detects an

$x \oplus \lambda_\alpha$	$y \oplus \lambda_\beta$	$\mathsf{P_A}$'s share of garbled table	$\mathsf{P_B}$'s share of garbled table
0	0	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 00) \oplus (r_{00}, \mathsf{M}[r_{00}], \mathsf{L}_{\gamma,0} \oplus r_{00}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{00}])$	$(s_{00} = \hat{z}_{00} \oplus r_{00}, \mathsf{K}[r_{00}], \mathsf{M}[s_{00}])$
0	1	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,1}, \gamma, 01) \oplus (r_{01}, \mathsf{M}[r_{01}], \mathsf{L}_{\gamma,0} \oplus r_{01}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{01}])$	$(s_{01} = \hat{z}_{01} \oplus r_{01}, \mathsf{K}[r_{01}], \mathsf{M}[s_{01}])$
1	0	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,0}, \gamma, 10) \oplus (r_{10}, \mathsf{M}[r_{10}], \mathsf{L}_{\gamma,0} \oplus r_{10}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{10}])$	$(s_{10} = \hat{z}_{10} \oplus r_{10}, \mathsf{K}[r_{10}], \mathsf{M}[s_{10}])$
1	1	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,1}, \gamma, 11) \oplus (r_{11}, \mathsf{M}[r_{11}], \mathsf{L}_{\gamma,0} \oplus r_{11}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{11}])$	$(s_{11} = \hat{z}_{11} \oplus r_{11}, \mathsf{K}[r_{11}], \mathsf{M}[s_{11}])$

Table 3: Our final construction of an authenticated garbled table for an AND gate.

error during evaluation. If $\mathsf{P_B}$ aborts, then $\mathsf{P_A}$ learns that the masked values on the input wires of the gate above were $\hat{x} = \hat{y} = 0$, from which it learns that the true values on those wires were λ_α and λ_β.

The selective-failure attack just mentioned can be prevented if the masks are hidden from $\mathsf{P_A}$. (In that case, even if $\mathsf{P_B}$ aborts and $\mathsf{P_A}$ knows the masked wire values, $\mathsf{P_A}$ learns nothing about the true wire values.) Since knowledge of the garbled table would leak information about the masks to $\mathsf{P_A}$, the garbled table must be hidden from $\mathsf{P_A}$ as well. That is, we now want to set up a situation in which $\mathsf{P_A}$ and $\mathsf{P_B}$ hold *secret shares* of the garbled table, as follows:

$\hat{x}$ $\hat{y}$	$\mathsf{P_A}$'s share of garbled table	$\mathsf{P_B}$'s share
0 0	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 00) \oplus (r_{00}, \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{00}})$	$(s_{00} = \hat{z}_{00} \oplus r_{00}, \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{00}})$
0 1	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,1}, \gamma, 01) \oplus (r_{01}, \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{01}})$	$(s_{01} = \hat{z}_{01} \oplus r_{01}, \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{01}})$
1 0	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,0}, \gamma, 10) \oplus (r_{10}, \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{10}})$	$(s_{10} = \hat{z}_{10} \oplus r_{10}, \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{10}})$
1 1	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,1}, \gamma, 11) \oplus (r_{11}, \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{11}})$	$(s_{11} = \hat{z}_{11} \oplus r_{11}, \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{11}})$

(Here, $\mathsf{L}^\mathsf{A}_{\gamma,z}, \mathsf{L}^\mathsf{B}_{\gamma,z}$ represent abstract XOR-shares of $\mathsf{L}_{\gamma,z}$, i.e., $\mathsf{L}_{\gamma,z} = \mathsf{L}^\mathsf{A}_{\gamma,z} \oplus \mathsf{L}^\mathsf{B}_{\gamma,z}$.) Once $\mathsf{P_A}$ sends its shares of all the garbled gates, $\mathsf{P_B}$ can XOR those shares with its own and then evaluate the garbled circuit as before.

Informally, the above ensures *privacy* against a malicious $\mathsf{P_A}$ since (intuitively) the results of any changes $\mathsf{P_A}$ makes to the garbled circuit are *independent* of $\mathsf{P_B}$'s inputs. However, $\mathsf{P_A}$ can still affect *correctness* by, e.g., flipping the masked value in a row. This can be addressed by adding an information-theoretic MAC on $\mathsf{P_A}$'s share of the masked bit. The shares of the garbled table now take the form as shown in the table below.

$\hat{x}$ $\hat{y}$	$\mathsf{P_A}$'s share of garbled table	$\mathsf{P_B}$'s share of garbled table
0 0	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 00) \oplus (r_{00}, \mathsf{M}[r_{00}], \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{00}})$	$(s_{00} = \hat{z}_{00} \oplus r_{00}, \mathsf{K}[r_{00}], \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{00}})$
0 1	$H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,1}, \gamma, 01) \oplus (r_{01}, \mathsf{M}[r_{01}], \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{01}})$	$(s_{01} = \hat{z}_{01} \oplus r_{01}, \mathsf{K}[r_{01}], \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{01}})$
1 0	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,0}, \gamma, 10) \oplus (r_{10}, \mathsf{M}[r_{10}], \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{10}})$	$(s_{10} = \hat{z}_{10} \oplus r_{10}, \mathsf{K}[r_{10}], \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{10}})$
1 1	$H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,1}, \gamma, 11) \oplus (r_{11}, \mathsf{M}[r_{11}], \mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{11}})$	$(s_{11} = \hat{z}_{11} \oplus r_{11}, \mathsf{K}[r_{11}], \mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{11}})$

Once $\mathsf{P_A}$ sends its shares of the garbled circuit to $\mathsf{P_B}$, the garbled circuit can be evaluated as before. Now, however, $\mathsf{P_B}$ will verify the MAC on $\mathsf{P_A}$'s share of each masked bit that it learns. This limits $\mathsf{P_A}$ to only being able to cause $\mathsf{P_B}$ to abort; as before, though, any such abort will occur independently of $\mathsf{P_B}$'s actual input.

Note that $\mathsf{P_A}$'s shares of the wire labels need not be authenticated, since a corrupted wire label will only cause input-independent abort. On the other hand, if $\mathsf{P_B}$ does not abort, the MACs added on r values ensure that $\mathsf{P_B}$ learns correct masked wire value, i.e., $\hat{z}$.

Efficient realization. Although the above idea is powerful, it still remains to design an efficient protocol that allows the parties to distributively compute shares of a garbled table of the above form even when one of the parties is malicious.

One important observation is that if we set $\Delta = \Delta_\mathsf{A}$ then we can secret share $\mathsf{L}_{\gamma,\hat{z}_{00}}$ as

$$\begin{aligned} \mathsf{L}_{\gamma,\hat{z}_{00}} &= \mathsf{L}_{\gamma,0} \oplus \hat{z}_{00}\Delta_\mathsf{A} \\ &= \mathsf{L}_{\gamma,0} \oplus (r_{00} \oplus s_{00})\Delta_\mathsf{A} \end{aligned}$$

$$\begin{aligned}
&= \mathsf{L}_{\gamma,0} \oplus r_{00}\Delta_\mathsf{A} \oplus s_{00}\Delta_\mathsf{A} \\
&= \underbrace{(\mathsf{L}_{\gamma,0} \oplus r_{00}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{00}])}_{\mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{00}}} \oplus \underbrace{(\mathsf{K}[s_{00}] \oplus s_{00}\Delta_\mathsf{A})}_{\mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{00}}}.
\end{aligned}$$

In our construction thus far, $\mathsf{P_A}$ knows $L_{\gamma,0}$ and r_{00} (in addition to knowing Δ_A). Our key insight is that if s_{00} is an authenticated bit known to $\mathsf{P_B}$, then $\mathsf{P_A}$ can locally compute the share $\mathsf{L}^\mathsf{A}_{\gamma,\hat{z}_{00}} := \mathsf{L}_{\gamma,0} \oplus r_{00}\Delta_\mathsf{A} \oplus \mathsf{K}[s_{00}]$ from the information it has, and then the other share $\mathsf{L}^\mathsf{B}_{\gamma,\hat{z}_{00}} := \mathsf{K}[s_{00}] \oplus s_{00}\Delta_\mathsf{A}$ is just the MAC on s_{00} that $\mathsf{P_B}$ already holds! So if we rewrite the garbled table as in Table 3, shares of the table become easy to compute in a distributed fashion.

One final optimization is based on the observation that the masked output values take the following form:

$$\begin{aligned}
\hat{z}_{00} &= (\lambda_\alpha \wedge \lambda_\beta) \oplus \lambda_\gamma \\
\hat{z}_{01} &= (\lambda_\alpha \wedge \overline{\lambda_\beta}) \oplus \lambda_\gamma = \hat{z}_{00} \oplus \lambda_\alpha \\
\hat{z}_{10} &= (\overline{\lambda_\alpha} \wedge \lambda_\beta) \oplus \lambda_\gamma = \hat{z}_{00} \oplus \lambda_\beta \\
\hat{z}_{11} &= (\overline{\lambda_\alpha} \wedge \overline{\lambda_\beta}) \oplus \lambda_\gamma = \hat{z}_{01} \oplus \lambda_\beta \oplus 1.
\end{aligned}$$

Thus, the parties can locally compute authenticated shares $\{r_{ij}, s_{ij}\}$ of the $\{\hat{z}_{i,j}\}$ from authenticated shares of $\lambda_\alpha, \lambda_\beta, \lambda_\gamma$, and $\lambda_\alpha \wedge \lambda_\beta$.

4 Our Framework and Its Instantiations

4.1 Protocol in the $\mathcal{F}_\mathsf{Pre}$-Hybrid Model

In Figure 2, we give the complete description of our main protocol in the $\mathcal{F}_\mathsf{Pre}$-hybrid model. For clarity, we set $\rho = \kappa$, but in Section 6 we describe how arbitrary values of ρ can be supported. Note that the calls to $\mathcal{F}_\mathsf{Pre}$ can be performed in parallel, so the protocol runs in constant rounds. Moreover, $\mathcal{F}_\mathsf{Pre}$ can be instantiated efficiently in constant rounds. We prove security of this protocol in the $\mathcal{F}_\mathsf{Pre}$-hybrid model in the Section 5.

Although our protocol calls $\mathcal{F}_\mathsf{Pre}$ in the function-dependent preprocessing phase, it is easy to push this to the function-independent phase using standard techniques similar to those used with multiplication triples [Bea92].

4.2 Efficiently Realizing $\mathcal{F}_\mathsf{Pre}$

We realize $\mathcal{F}_\mathsf{Pre}$ efficiently using an optimized version of the TinyOT protocol, achieving a 2.7× improvement. Recall that $\mathcal{F}_\mathsf{Pre}$ can be used to output authenticated values $[x_1]_\mathsf{A}$, $[y_1]_\mathsf{A}$, $[z_1]_\mathsf{A}$, $[x_2]_\mathsf{B}$, $[y_2]_\mathsf{B}$, and $[z_2]_\mathsf{B}$ to the parties such that $z_1 \oplus z_2 = (x_1 \oplus x_2) \wedge (y_1 \oplus y_2)$; we refer to these as an *AND triple.* In the original TinyOT protocol, the four terms that result from expanding the right-hand side (namely, x_1y_1, x_1y_2, x_2y_1, and x_2y_2) are computed individually, and then combined. This approach is conceptually simple, but increases the complexity of the protocol. In our new approach, we instead compute AND triples directly. In what follows, we discuss the key ideas of our new protocol; more details can be found in Section 8.

At a high level, we use three steps to compute an AND triple.

1. The parties jointly compute $[x_1]_\mathsf{A}$, $[y_1]_\mathsf{A}$, $[z_1]_\mathsf{A}$, $[x_2]_\mathsf{B}$, $[y_2]_\mathsf{B}$, $[z_2]_\mathsf{B}$, such that if both parties are honest, these form a correct AND triple; if some party cheats, that party can change the value of z_2 but cannot learn the other party's bits.

2. The parties perform a checking protocol that ensures the correctness of every AND triple, while letting the malicious party try to guess the value of x_1 (resp., x_2). Each guess is correct with probability 1/2; any incorrect guess is detected and will cause the other party to abort.

   As a consequence, we can argue that (conditioned on no abort) the malicious party obtains information on at most ρ AND triples except with probability a most $2^{-\rho}$.

Protocol Π_{2pc}

Inputs: In the function-dependent phase, the parties agree on a circuit for a function $f : \{0,1\}^{|\mathcal{I}_1|} \times \{0,1\}^{|\mathcal{I}_2|} \to \{0,1\}^{|\mathcal{O}|}$. In the input-processing phase, $\mathsf{P_A}$ holds $x \in \{0,1\}^{|\mathcal{I}_1|}$ and $\mathsf{P_A}$ holds $y \in \{0,1\}^{|\mathcal{I}_2|}$.

Function-independent preprocessing:

1. $\mathsf{P_A}$ and $\mathsf{P_B}$ send init to $\mathcal{F}_{\mathsf{Pre}}$, which sends Δ_{A} to $\mathsf{P_A}$ and Δ_{B} to $\mathsf{P_B}$.

2. For each wire $w \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{W}$, parties $\mathsf{P_A}$ and $\mathsf{P_B}$ send random to $\mathcal{F}_{\mathsf{Pre}}$. In return, $\mathcal{F}_{\mathsf{Pre}}$ sends $(r_w, \mathsf{M}[r_w], \mathsf{K}[s_w])$ to $\mathsf{P_A}$ and $(s_w, \mathsf{M}[s_w], \mathsf{K}[r_w])$ to $\mathsf{P_B}$, where $\lambda_w = s_w \oplus r_w$. $\mathsf{P_A}$ also picks a uniform κ-bit string $\mathsf{L}_{w,0}$.

Function-dependent preprocessing:

3. For each gate $\mathcal{G} = (\alpha, \beta, \gamma, \oplus)$, $\mathsf{P_A}$ computes $(r_\gamma, \mathsf{M}[r_\gamma], \mathsf{K}[s_\gamma]) := (r_\alpha \oplus r_\beta, \mathsf{M}[r_\alpha] \oplus \mathsf{M}[r_\beta], \mathsf{K}[s_\alpha] \oplus \mathsf{K}[s_\beta])$ and $\mathsf{L}_{\gamma,0} := \mathsf{L}_{\alpha,0} \oplus \mathsf{L}_{\beta,0}$. $\mathsf{P_B}$ computes $(s_\gamma, \mathsf{M}[s_\gamma], \mathsf{K}[r_\gamma]) := (s_\alpha \oplus s_\beta, \mathsf{M}[r_\beta] \oplus \mathsf{M}[r_\beta], \mathsf{K}[r_\alpha] \oplus \mathsf{K}[r_\beta])$.

4. Then, for each gate $\mathcal{G} = (\alpha, \beta, \gamma, \wedge)$:

   (a) $\mathsf{P_A}$ (resp., $\mathsf{P_B}$) sends $(\mathsf{and}, (r_\alpha, \mathsf{M}[r_\alpha], \mathsf{K}[s_\alpha]), (r_\beta, \mathsf{M}[r_\beta], \mathsf{K}[s_\beta]))$ (resp., $(\mathsf{and}, (s_\alpha, \mathsf{M}[s_\alpha], \mathsf{K}[r_\alpha])$, $(s_\beta$, $\mathsf{M}[s_\beta]$, $\mathsf{K}[r_\beta])))$ to $\mathcal{F}_{\mathsf{Pre}}$. In return, $\mathcal{F}_{\mathsf{Pre}}$ sends $(r_\sigma, \mathsf{M}[r_\sigma], \mathsf{K}[s_\sigma])$ to $\mathsf{P_A}$ and $(s_\sigma, \mathsf{M}[s_\sigma], \mathsf{K}[r_\sigma])$ to $\mathsf{P_B}$, where $s_\sigma \oplus r_\sigma = \lambda_\alpha \wedge \lambda_\beta$.

   (b) $\mathsf{P_A}$ computes the following locally:

$$\begin{array}{llll}
(r_{\gamma,0}, \mathsf{M}[r_{\gamma,0}], \mathsf{K}[s_{\gamma,0}]) := (r_\sigma \oplus r_\gamma, & \mathsf{M}[r_\sigma] \oplus \mathsf{M}[r_\gamma], & \mathsf{K}[s_\sigma] \oplus \mathsf{K}[s_\gamma] &) \\
(r_{\gamma,1}, \mathsf{M}[r_{\gamma,1}], \mathsf{K}[s_{\gamma,1}]) := (r_\sigma \oplus r_\gamma \oplus r_\alpha, & \mathsf{M}[r_\sigma] \oplus \mathsf{M}[r_\gamma] \oplus \mathsf{M}[r_\alpha], & \mathsf{K}[s_\sigma] \oplus \mathsf{K}[s_\gamma] \oplus \mathsf{K}[s_\alpha] &) \\
(r_{\gamma,2}, \mathsf{M}[r_{\gamma,2}], \mathsf{K}[s_{\gamma,2}]) := (r_\sigma \oplus r_\gamma \oplus r_\beta, & \mathsf{M}[r_\sigma] \oplus \mathsf{M}[r_\gamma] \oplus \mathsf{M}[r_\beta], & \mathsf{K}[s_\sigma] \oplus \mathsf{K}[s_\gamma] \oplus \mathsf{K}[s_\beta] &) \\
(r_{\gamma,3}, \mathsf{M}[r_{\gamma,3}], \mathsf{K}[s_{\gamma,3}]) := (r_\sigma \oplus r_\gamma \oplus r_\alpha \oplus r_\beta, & \mathsf{M}[r_\sigma] \oplus \mathsf{M}[r_\gamma] \oplus \mathsf{M}[r_\alpha] \oplus \mathsf{M}[r_\beta], & \mathsf{K}[s_\sigma] \oplus \mathsf{K}[s_\gamma] \oplus \mathsf{K}[s_\alpha] \oplus \mathsf{K}[s_\beta] \oplus \Delta_{\mathsf{A}} &)
\end{array}$$

   (c) $\mathsf{P_B}$ computes the following locally:

$$\begin{array}{llll}
(s_{\gamma,0}, \mathsf{M}[s_{\gamma,0}], \mathsf{K}[r_{\gamma,0}]) := (s_\sigma \oplus s_\gamma, & \mathsf{M}[s_\sigma] \oplus \mathsf{M}[s_\gamma], & \mathsf{K}[r_\sigma] \oplus \mathsf{K}[r_\gamma] &) \\
(s_{\gamma,1}, \mathsf{M}[s_{\gamma,1}], \mathsf{K}[r_{\gamma,1}]) := (s_\sigma \oplus s_\gamma \oplus s_\alpha, & \mathsf{M}[s_\sigma] \oplus \mathsf{M}[s_\gamma] \oplus \mathsf{M}[s_\alpha], & \mathsf{K}[r_\sigma] \oplus \mathsf{K}[r_\gamma] \oplus \mathsf{K}[r_\alpha] &) \\
(s_{\gamma,2}, \mathsf{M}[s_{\gamma,2}], \mathsf{K}[r_{\gamma,2}]) := (s_\sigma \oplus s_\gamma \oplus s_\beta, & \mathsf{M}[s_\sigma] \oplus \mathsf{M}[s_\gamma] \oplus \mathsf{M}[s_\beta], & \mathsf{K}[r_\sigma] \oplus \mathsf{K}[r_\gamma] \oplus \mathsf{K}[r_\beta] &) \\
(s_{\gamma,3}, \mathsf{M}[s_{\gamma,3}], \mathsf{K}[r_{\gamma,3}]) := (s_\sigma \oplus s_\gamma \oplus s_\alpha \oplus s_\beta \oplus 1, & \mathsf{M}[s_\sigma] \oplus \mathsf{M}[s_\gamma] \oplus \mathsf{M}[s_\alpha] \oplus \mathsf{M}[s_\beta], & \mathsf{K}[r_\sigma] \oplus \mathsf{K}[r_\gamma] \oplus \mathsf{K}[r_\alpha] \oplus \mathsf{K}[r_\beta] &)
\end{array}$$

   (d) $\mathsf{P_A}$ computes $\mathsf{L}_{\alpha,1} := \mathsf{L}_{\alpha,0} \oplus \Delta_{\mathsf{A}}$ and $\mathsf{L}_{\beta,1} := \mathsf{L}_{\beta,0} \oplus \Delta_{\mathsf{A}}$, and then sends the following to $\mathsf{P_B}$.

$$\begin{array}{lll}
G_{\gamma,0} := H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 0) \oplus (r_{\gamma,0}, & \mathsf{M}[r_{\gamma,0}], & \mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,0}] \oplus r_{\gamma,0}\Delta_{\mathsf{A}}) \\
G_{\gamma,1} := H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,1}, \gamma, 1) \oplus (r_{\gamma,1}, & \mathsf{M}[r_{\gamma,1}], & \mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,1}] \oplus r_{\gamma,1}\Delta_{\mathsf{A}}) \\
G_{\gamma,2} := H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,0}, \gamma, 2) \oplus (r_{\gamma,2}, & \mathsf{M}[r_{\gamma,2}], & \mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,2}] \oplus r_{\gamma,2}\Delta_{\mathsf{A}}) \\
G_{\gamma,3} := H(\mathsf{L}_{\alpha,1}, \mathsf{L}_{\beta,1}, \gamma, 3) \oplus (r_{\gamma,3}, & \mathsf{M}[r_{\gamma,3}], & \mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,3}] \oplus r_{\gamma,3}\Delta_{\mathsf{A}})
\end{array}$$

Input processing:

5. For each $w \in \mathcal{I}_1$, $\mathsf{P_A}$ sends $(r_w, \mathsf{M}[r_w])$ to $\mathsf{P_B}$, who checks that $(r_w, \mathsf{K}[r_w], \mathsf{M}[r_w])$ is valid. $\mathsf{P_B}$ then sends $y_w \oplus \lambda_w := s_w \oplus y_w \oplus r_w$ to $\mathsf{P_A}$. Finally, $\mathsf{P_A}$ sends $\mathsf{L}_{w, y_w \oplus \lambda_w}$ to $\mathsf{P_B}$.

6. For each $w \in \mathcal{I}_2$, $\mathsf{P_B}$ sends $(s_w, \mathsf{M}[s_w])$ to $\mathsf{P_A}$, who checks that $(s_w, \mathsf{K}[s_w], \mathsf{M}[s_w])$ is valid. $\mathsf{P_A}$ then sends $x_w \oplus \lambda_w := s_w \oplus x_w \oplus r_w$ and $\mathsf{L}_{w, x_w \oplus \lambda_w}$ to $\mathsf{P_B}$.

Circuit evaluation:

7. $\mathsf{P_B}$ evaluates the circuit in topological order. For each gate $\mathcal{G} = (\alpha, \beta, \gamma, T)$, $\mathsf{P_B}$ initially holds $(z_\alpha \oplus \lambda_\alpha, \mathsf{L}_{\alpha, z_\alpha \oplus \lambda_\alpha})$ and $(z_\beta \oplus \lambda_\beta, \mathsf{L}_{\beta, z_\beta \oplus \lambda_\beta})$, where z_α, z_β are the underlying values of the wires.

   (a) If $T = \oplus$, $\mathsf{P_B}$ computes $z_\gamma \oplus \lambda_\gamma := (z_\alpha \oplus \lambda_\alpha) \oplus (z_\beta \oplus \lambda_\beta)$ and $\mathsf{L}_{\gamma, z_\gamma \oplus \lambda_\gamma} := \mathsf{L}_{\alpha, z_\alpha \oplus \lambda_\alpha} \oplus \mathsf{L}_{\beta, z_\beta \oplus \lambda_\beta}$.

   (b) If $T = \wedge$, $\mathsf{P_B}$ computes $i := 2(z_\alpha \oplus \lambda_\alpha) + (z_\beta \oplus \lambda_\beta)$ followed by $(r_{\gamma,i}, \mathsf{M}[r_{\gamma,i}], \mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,i}] \oplus r_{\gamma,i}\Delta_{\mathsf{A}}) := G_{\gamma,i} \oplus H(\mathsf{L}_{\alpha, z_\alpha \oplus \lambda_\alpha}, \mathsf{L}_{\beta, z_\beta \oplus \lambda_\beta}, \gamma, i)$. Then $\mathsf{P_B}$ checks that $(r_{\gamma,i}, \mathsf{K}[r_{\gamma,i}], \mathsf{M}[r_{\gamma,i}])$ is valid and, if so, computes $z_\gamma \oplus \lambda_\gamma := (s_{\gamma,i} \oplus r_{\gamma,i})$ and $\mathsf{L}_{\gamma, z_\gamma \oplus \lambda_\gamma} := (\mathsf{L}_{\gamma,0} \oplus \mathsf{K}[s_{\gamma,i}] \oplus r_{\gamma,i}\Delta_{\mathsf{A}}) \oplus \mathsf{M}[s_{\gamma,i}]$.

Output determination:

8. For each $w \in \mathcal{O}$, $\mathsf{P_A}$ sends $(r_w, \mathsf{M}[r_w])$ to $\mathsf{P_B}$, who checks $(r_w, \mathsf{K}[r_w], \mathsf{M}[r_w])$ is valid. If so, $\mathsf{P_B}$ computes $z_w := (\lambda_w \oplus z_w) \oplus r_w \oplus s_w$.

Figure 2: Our protocol in the $\mathcal{F}_{\mathsf{Pre}}$-hybrid model. Here $\rho = \kappa$ for clarity, but this is not needed (cf. Section 6).

3. So far we have described a way for the parties to generate many "leaky" AND triples such that the attacker may have disallowed information on at most ρ of them. We then show how to distill these

into a smaller number of "secure" AND triples, about which the attacker is guaranteed to have no additional information beyond what it is allowed.

In the following, we provide more intuition on each of these steps.

Computing the initial AND triples. We begin by having the parties generate authenticated random bits y_1, z_1, y_2 known to the appropriate party. All that remains is to generate x_1, x_2, and for $\mathsf{P_B}$ to learn $z_2 = (x_1 \oplus x_2) \wedge (y_1 \oplus y_2) \oplus z_1$. To compute this, we use a functionality $\mathcal{F}_{\mathsf{HaAND}}$ that takes as input unauthenticated values of y_1, y_2 and outputs $[x_1]_{\mathsf{A}}, [x_2]_{\mathsf{B}}$ as well as XOR-shares of $x_1 y_2 \oplus x_2 y_1$ to both parties. The parties can then compute an authenticated version of z_2 easily. Details of $\mathcal{F}_{\mathsf{HaAND}}$, and a protocol realizing it, appear in Section 8.1. (The above steps corresponds to steps (1)–(3) in Figure 8.)

Note that a malicious party can easily misbehave by, for example, sending an incorrect value for y_1 (or y_2) to $\mathcal{F}_{\mathsf{HaAND}}$. We deal with this in the next step. Looking ahead, however, we note that this also introduces a selective-failure attack that can leak information to the attacker: if the attacker flips a y-value but the checking step described next does not abort, then it must be the case that $x_1 \oplus x_2 = 0$.

Verifying correctness. Both parties next check correctness of AND triples generated in the previous step. If $x_2 \oplus x_1 = 0$, then we want to check that $z_2 = z_1$; if $x_2 \oplus x_1 = 1$, then we want to to check that $y_1 \oplus z_1 = y_2 \oplus z_2$. However, an obvious problem is that no party knows the value of $x_1 \oplus x_2$, therefore there is no way to know which relationship should be checked. We thus need to construct a checking procedure such that the computation of $\mathsf{P_A}$ is oblivious to x_2, while the computation of $\mathsf{P_B}$ is oblivious to x_1. In Section 8.2, we provide complete description of the protocol as well as a detailed proof. (This corresponds to steps (4)–(5) in Figure 8.)

Combining leaky ANDs. The above check is vulnerable to a selective-failure attack, from which a malicious party can learn the value of x_1/x_2 with one-half probability of being caught. In order to get rid of the leakage, bucketing is performed. Note that in the previous work [NNOB12], bucketing were done on different objects, therefore the key here is to devise a way to combine our new leaky objects. Assuming that two triples are $([x'_1]_{\mathsf{A}}, [y'_1]_{\mathsf{A}}, [z'_1]_{\mathsf{A}}, [x'_2]_{\mathsf{B}}, [y'_2]_{\mathsf{B}}, [z'_2]_{\mathsf{B}})$ and $([x''_1]_{\mathsf{A}}, [y''_1]_{\mathsf{A}}, [z''_1]_{\mathsf{A}}, [x''_2]_{\mathsf{B}}, [y''_2]_{\mathsf{B}}, [z''_2]_{\mathsf{B}})$, we set $[x_1]_{\mathsf{A}} := [x'_1]_{\mathsf{A}} \oplus [x''_1]_{\mathsf{A}}, [x_2]_{\mathsf{B}} := [x'_2]_{\mathsf{B}} \oplus [x''_2]_{\mathsf{B}}$. By doing this, $[x_1]_{\mathsf{A}}, [x_2]_{\mathsf{B}}$ are non-leaky as long as one triple is non-leaky. We can also set $[y_1]_{\mathsf{A}} := [y'_1]_{\mathsf{A}}, [y_2]_{\mathsf{B}} := [y'_2]_{\mathsf{B}}$ and reveal the bit $d := y'_1 \oplus y'_2 \oplus y''_1 \oplus y''_2$, since y's bits are all private. Now observe that

$$\begin{aligned}
(x_1 \oplus x_2)(y_1 \oplus y_2) &= (x'_1 \oplus x'_2 \oplus x''_1 \oplus x''_2)(y'_1 \oplus y'_2) \\
&= (x'_1 \oplus x'_2)(y'_1 \oplus y'_2) \oplus (x''_1 \oplus x''_2)(y'_1 \oplus y'_2) \\
&= (x'_1 \oplus x'_2)(y'_1 \oplus y'_2) \oplus (x''_1 \oplus x''_2)(y''_1 \oplus y''_2) \\
&\quad \oplus (x''_1 \oplus x''_2)(y'_1 \oplus y'_2 \oplus y''_1 \oplus y''_2) \\
&= (z'_1 \oplus z'_2) \oplus (z''_1 \oplus z''_2) \oplus d(x''_1 \oplus x''_2) \\
&= (z'_1 \oplus z''_1 \oplus dx''_1) \oplus (z'_2 \oplus z''_2 \oplus dx''_2)
\end{aligned}$$

Therefore, we could just set $[z_1]_{\mathsf{A}} := [z'_1]_{\mathsf{A}} \oplus [z''_1]_{\mathsf{A}} \oplus d[x''_1]_{\mathsf{A}}$, $[z_2]_{\mathsf{B}} := [z'_2]_{\mathsf{B}} \oplus [z''_2]_{\mathsf{B}} \oplus d[x''_2]_{\mathsf{B}}$. (This corresponds to the protocol in Figure 9.)

4.3 Other Ways to Instantiate $\mathcal{F}_{\mathsf{Pre}}$

We briefly note other ways $\mathcal{F}_{\mathsf{Pre}}$ can be instantiated.

IPS-based instantiation. We obtain better asymptotic performance by instantiating the protocol with the work by Ishai, Prabhakaran and Sahai [IPS08] to realize $\mathcal{F}_{\mathsf{Pre}}$. In the function-dependent preprocessing phase, we need to produce a sharing of λ_i for each wire i, and a sharing of $\lambda_\sigma = (\lambda_\alpha \wedge \lambda_\beta) \oplus \lambda_\gamma$ for each AND gate $(\alpha, \beta, \gamma, \wedge)$. These can be computed by a constant-depth circuit with $O(|C|\kappa)$ gates. For securely evaluating a circuit of depth d and size ℓ, the IPS protocol uses communication complexity (in terms of number of bits) $O(\ell) + poly(\kappa, d, \log \ell)$ and $O(d)$ rounds of communication. When applied to our setting, this translates to a communication complexity of $O(|C|\kappa) + poly(\kappa, \log|C|)$ bits; for sufficiently large circuits, it can be simplified to $O(|C|\kappa)$ bits.

Using a (semi-)trusted server. It is straightforward to instantiate $\mathcal{F}_{\mathsf{Pre}}$ using a (semi-)trusted server. By applying the techniques of Mohassel et al. [MOR16], the offline phase can also be decoupled from the identity of other party; we refer to their paper for further details.

5 Proof of Security

Theorem 5.1. *If H is modeled as a random oracle, the protocol in Figure 2 securely computes f against malicious adversaries with statistical security $2^{-\rho}$ in the $\mathcal{F}_{\mathsf{Pre}}$-hybrid model.*

Proof. We consider separately the case where $\mathsf{P_A}$ or $\mathsf{P_B}$ is malicious.

Malicious $\mathsf{P_A}$. Let $\mathcal{A}$ be an adversary corrupting $\mathsf{P_A}$. We construct a simulator $\mathcal{S}$ that runs $\mathcal{A}$ as a subroutine and plays the role of $\mathsf{P_A}$ in the ideal world involving an ideal functionality $\mathcal{F}$ evaluating f. $\mathcal{S}$ is defined as follows.

1-4 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_B}$, where $\mathcal{S}$ also plays the role of $\mathcal{F}_{\mathsf{Pre}}$, recording all values that are sent to $\mathcal{A}$.

5 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_B}$ using input $y = 0$.

6 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_B}$. For each wire $w \in \mathcal{I}_1$, $\mathcal{S}$ receives $\hat{x}_w$ and computes $x_w = \hat{x}_w \oplus r_w \oplus s_w$, where r_w, s_w are values $\mathcal{S}$ used to play the role of $\mathcal{F}_{\mathsf{Pre}}$ in previous steps. $\mathcal{S}$ sends $x = \{x_w\}_{w\in\mathcal{I}_1}$ to $\mathcal{F}$.

7-8 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_B}$. If $\mathsf{P_B}$ would abort, $\mathcal{S}$ outputs whatever $\mathcal{A}$ outputs and aborts; otherwise $\mathcal{S}$ sends $\mathsf{continue}$ to $\mathcal{F}$.

We now show that the joint distribution over the outputs of $\mathcal{A}$ and the honest $\mathsf{P_B}$ in the real world is indistinguishable from the joint distribution over the outputs of $\mathcal{S}$ and $\mathsf{P_B}$ in the ideal world. We prove this by considering a sequence of experiments, the first of which corresponds to the execution of our protocol and the last of which corresponds to execution in the ideal world, and showing that successive experiments are computationally indistinguishable.

Hybrid$_1$. This is the hybrid-world protocol, where $\mathcal{S}$ plays the role of an honest $\mathsf{P_B}$ using $\mathsf{P_B}$'s actual input y. $\mathcal{S}$ also plays the role of $\mathcal{F}_{\mathsf{Pre}}$.

Hybrid$_2$. Same as **Hybrid$_1$**, except that in step 6, for each wire $w \in \mathcal{I}_1$ the simulator $\mathcal{S}$ receives $\hat{x}_w$ and computes $x_w = \hat{x}_w \oplus r_w \oplus s_w$, where s_w, r_w are values $\mathcal{S}$ used when playing the role of $\mathcal{F}_{\mathsf{Pre}}$. $\mathcal{S}$ sends $x = \{x_w\}_{w\in\mathcal{I}_1}$ to $\mathcal{F}$. If an honest $\mathsf{P_B}$ would abort, $\mathcal{S}$ outputs whatever $\mathcal{A}$ outputs and aborts; otherwise $\mathcal{S}$ sends $\mathsf{continue}$ to $\mathcal{F}$.

The distributions on the view of the adversary in the two experiments above are exactly identical. Lemma 5.1 shows that $\mathsf{P_B}$ generates the same output in both experiments with probability $1 - 2^{-\rho}$.

Hybrid$_3$. Same as **Hybrid$_2$**, except that $\mathcal{S}$ computes $\{s_w\}_{w\in\mathcal{I}_2}$ as follows: $\mathcal{S}$ first randomly pick $\{u_w\}_{w\in\mathcal{I}_2}$, and then computes $s_w := u_w \oplus y_w$.

The above two experiments are identically distributed.

Hybrid$_4$. Same as **Hybrid$_3$**, except that $\mathcal{S}$ uses $y = 0$ as inputs throughout the protocol.

Note that although the value of y in **Hybrid$_3$** and **Hybrid$_4$** are different, the distributions of $s_w \oplus y_w$ are exactly the same. The view of the adversary in the two experiments are therefore the same. We next show that $\mathsf{P_B}$ aborts with the same probability in two experiments.

Observe that the only place where $\mathsf{P_B}$'s abort can possibly depends on y is in step 7(b). However, this abort depends on which row is selected to decrypt, that is the value of $\lambda_\alpha \oplus z_\alpha$ and $\lambda_\beta \oplus z_\beta$, which are chosen uniformly and independently in both experiments. Therefore, the two experiments are identically distributed.

Note that $\mathbf{Hybrid_4}$ corresponds to the ideal-world execution, so this completes the proof for a malicious $\mathsf{P_A}$.

Malicious $\mathsf{P_B}$. Let $\mathcal{A}$ be an adversary corrupting $\mathsf{P_B}$. We construct a simulator $\mathcal{S}$ that runs $\mathcal{A}$ as a subroutine and plays the role of $\mathsf{P_B}$ in the ideal world involving an ideal functionality $\mathcal{F}$ evaluating f. $\mathcal{S}$ is defined as follows.

1-4 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_A}$ and plays the functionality of $\mathcal{F}_{\mathsf{Pre}}$. If an honest $\mathsf{P_A}$ would abort, $\mathcal{S}$ output whatever $\mathcal{A}$ outputs and aborts.

5 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_A}$, receives $\hat{y}_w$from $\mathcal{A}$, and computes $y_w := \hat{y}_w \oplus s_w \oplus r_w$, where s_w, r_w are values $\mathcal{S}$ used when playing the role of $\mathcal{F}_{\mathsf{Pre}}$. $\mathcal{S}$ sends $y = \{y_w\}_{w \in \mathcal{I}_2}$ to $\mathcal{F}$, which sends $z = f(x, y)$ to $\mathcal{S}$.

6 $\mathcal{S}$ interacts with $\mathcal{A}$ acting as an honest $\mathsf{P_A}$ using input $x = 0$. If an honest $\mathsf{P_A}$ would abort, $\mathcal{S}$ output whatever $\mathcal{A}$ outputs and aborts.

8 $\mathcal{S}$ computes $z' = f(0, y)$. For each $w \in \mathcal{O}$, if $z'_w = z_w$, $\mathcal{S}$ sends $(r_w, \mathsf{M}[r_w])$; otherwise, $\mathcal{S}$ sends $(r_w \oplus 1, \mathsf{M}[r_w] \oplus \Delta_\mathsf{B})$, where Δ_B is the value $\mathcal{S}$ used when playing the role of $\mathcal{F}_{\mathsf{Pre}}$.

We now show that the joint distribution over the outputs of $\mathcal{A}$ and the honest $\mathsf{P_A}$ in the real world is indistinguishable from the joint distribution over the outputs of $\mathcal{S}$ and $\mathsf{P_A}$ in the ideal world.

$\mathbf{Hybrid_1}$. Same as the hybrid-world protocol, where $\mathcal{S}$ plays the role of an honest $\mathsf{P_A}$ using the actual input x.

$\mathbf{Hybrid_2}$. Same as $\mathbf{Hybrid_1}$, except that, in step 5, $\mathcal{S}$ receives $y_w \oplus \lambda_w$ from $\mathcal{A}$, and computes $y_w := \hat{y}_w \oplus s_w \oplus r_w$, where s_w, r_w are values $\mathcal{S}$ used when playing the role of $\mathcal{F}_{\mathsf{Pre}}$. $\mathcal{S}$ then sends $y = \{y_w\}_{w \in \mathcal{I}_2}$ to $\mathcal{F}$, and receives $z = f(x, y)$. In Step 8, for each $w \in \mathcal{O}$, $\mathcal{S}$ computes $r'_w := z_w \oplus s_w$, and sends $(r'_w, \mathsf{K}[r'_w] \oplus r'_w \Delta_\mathsf{B})$, where Δ_B is the value $\mathcal{S}$ used to play the role of $\mathcal{F}_{\mathsf{Pre}}$.

$\mathsf{P_A}$ does not have output; furthermore the view of $\mathcal{A}$ does not change between the two **Hybrid**s since the value z that $\mathcal{S}$ obtains from $\mathcal{F}$ is the same as the one $\mathcal{A}$ obtains in $\mathbf{Hybrid_1}$.

$\mathbf{Hybrid_3}$. Same as $\mathbf{Hybrid_2}$, except that in step 6, $\mathcal{S}$ uses $x = 0$ as input.

Note that since $\mathcal{S}$ uses different values for x between two **Hybrid**s, we also need to show that the garbled rows $\mathsf{P_B}$ opened are indistinguishable between two **Hybrid**s. According to Lemma 5.2, $\mathsf{P_B}$ is able to open only one garble rows in each garbled table $G_{\gamma,i}$. Therefore, given that $\{\lambda_w\}_{w \in \mathcal{I}_1 \cup \mathcal{W}}$ values are not known to $\mathsf{P_B}$, masked values and garbled keys are indistinguishable between the two **Hybrid**s.

As $\mathbf{Hybrid_3}$ is the ideal-world execution, the proof is complete. □

Lemma 5.1. *Consider an $\mathcal{A}$ corrupting $\mathsf{P_A}$ and denote $x_w := \hat{x}_w \oplus s_w \oplus r_w$, where $\hat{x}_w$ is the value $\mathcal{A}$ sent to $\mathsf{P_B}$, s_w, r_w are the values from $\mathcal{F}_{\mathsf{Pre}}$. With probability $1 - 2^{-\rho}$, $\mathsf{P_B}$ either aborts or only learns $z = f(x, y)$.*

Proof. Define z^*_w as the correct wire values computed using x defined above and y, z_w as the actual wire values $\mathsf{P_B}$ holds in the evaluation.

We will first show that $\mathsf{P_B}$ learns $\{z^w \oplus \lambda_w = z^*_w \oplus \lambda_w\}_{w \in \mathcal{O}}$ by induction on topology of the circuit.

Base step: It is obvious that $\{z^*_w \oplus \lambda_w = z_w \oplus \lambda_w\}_{w \in \mathcal{I}_1 \cup \mathcal{I}_2}$, unless $\mathcal{A}$ is able to forge an IT-MAC.

Induction step: Now we show that for a gate $(\alpha, \beta, \gamma, T)$, if $\mathsf{P_B}$ has $\{z^*_w \oplus \lambda_w = z_w \oplus \lambda_w\}_{w \in \{\alpha, \beta\}}$, then $\mathsf{P_B}$ also obtains $z^*_\gamma \oplus \lambda_\gamma = z_\gamma \oplus \lambda_\gamma$.

- $T = \oplus$: It is true according to the following: $z^*_\gamma \oplus \lambda_\gamma = (z^*_\alpha \oplus \lambda_\alpha) \oplus (z^*_\beta \oplus \lambda_\beta) = (z_\alpha \oplus \lambda_\alpha) \oplus (z_\beta \oplus \lambda_\beta) z_\gamma \oplus \lambda_\gamma$
- $T = \wedge$: According to the protocol, $\mathsf{P_B}$ will open the garbled row defined by $i := 2(z_\alpha \oplus \lambda_\alpha) + (z_\beta \oplus \lambda_\beta)$. If $\mathsf{P_B}$ learns $z_\gamma \oplus \lambda_\gamma \neq z^*_\gamma \oplus \lambda_\gamma$, then it means that $\mathsf{P_B}$ learns $r^*_{\gamma,i} \neq r_{\gamma,i}$. However, this would mean that $\mathcal{A}$ forges a valid IT-MAC, which only happens with negligible probability.

Now we know that $\mathsf{P_B}$ learns correct masked output. $\mathsf{P_B}$ can therefore learn correct output $f(x, y)$ unless $\mathcal{A}$ is able to flip $\{r_w\}_{w\in\mathcal{O}}$, which, again, happens with negligible probability. □

Lemma 5.2. *Consider an $\mathcal{A}$ corrupting $\mathsf{P_B}$, with negligible, probability, $\mathsf{P_B}$ learns both garbled keys for some wire.*

Proof. The proof is very similar to the proof of security for garbled circuits in the semi-honest setting.

Base step: $\mathsf{P_B}$ can only learn one garbled keys for each input wire, since $\mathsf{P_A}$ only sends one garbled wire, and $\mathsf{P_B}$ cannot learn Δ_A in the protocol.

Induction step: It is obvious that $\mathsf{P_B}$ cannot learn the other label for an XOR gate and so we focus on AND gates. Note that $\mathsf{P_B}$ only learns one garbled key each for input wires α and β. However, each row is encrypted using different combinations of $\{L_{\alpha,b}\}_{b\in\{0,1\}}$ and $\{\mathsf{L}_{\beta,b}\}_{b\in\{0,1\}}$. In order for $\mathsf{P_B}$ to open two rows in the garbled table, $\mathsf{P_B}$ needs to learn both garbled keys for some input wire, which contradict with assumptions in the induction step. □

Global key queries. When instantiated based on the TinyOT protocol, the adversary is also allowed to perform a global key queries: the adversary can, at any time, send (p, Δ') to $\mathcal{F}_{\mathsf{Pre}}$ and be told if $\Delta' = \Delta_p$. It is easy to incorporate this to the simulation above, by simply letting the simulator act as $\mathcal{F}_{\mathsf{Pre}}$ and answer all global key queries honestly. This is feasible since the simulator plays the role of $\mathcal{F}_{\mathsf{Pre}}$ and knows all global keys. In both real and idea world, the probability that an adversary can guess the global key correctly after q queries is always $q/2^\kappa$ (since global keys are random κ-bit strings). Therefore, the global key query only gives the adversary negligible advantage and the rest of the proof follows.

6 Extensions and Optimizations

Reducing the size of the authenticated garbled table. In the original protocol, all MACs and keys are κ-bit values, which may not always be necessary. For ρ-bit statistical security, $\mathsf{M}[r_{00}]$ encrypted in step 4(d) only needs to be of length ρ. Further, the bits $r_{\gamma,i}$ need not be put in the garbled table, since the MAC $\mathsf{M}[r_{\gamma,i}]$ is already enough for $\mathsf{P_B}$ to learn and validate the bit. This reduces the size of a garbled table from $8\kappa + 4$ bits to $4(\kappa + \rho)$ bits.

Partial garbled row reduction. Even with the above optimization, the value $\mathsf{L}_{\gamma,0}$ is still uniform, which means we can further reduce the size of garbled tables using ideas similar to garbled row reduction [PSSW09]. In detail, instead of picking $\mathsf{L}_{\gamma,0}$ randomly, it will be set such that $\mathsf{L}_{\gamma,0} = H(\mathsf{L}_{\alpha,0}, \mathsf{L}_{\beta,0}, \gamma, 0)[0 : \kappa]$, where $X[0 : \kappa]$ refers to the κ least-significant bits of a string X.

Pushing computation to earlier phases. For clarity of presentation, in our description of the protocol we send $\{r_w, \mathsf{M}[r_w]\}_{w\in\mathcal{I}_1}$ and $\{s_w, \mathsf{M}[s_w]\}_{w\mathcal{I}_2}$ in steps 5 and 6. However, they can be sent in step 4 before knowing the input, which reduces the online communication from $|\mathcal{I}|(\kappa + \rho) + |\mathcal{O}|\rho$ to $|\mathcal{I}|\kappa + |\mathcal{O}|\rho$.

7 Evaluation

7.1 Implementation and Evaluation Setup

We implemented our protocol using the EMP-toolkit [WMK16] to verify its efficiency. We will make our implementation publicly available. In the evaluation below, the computational security parameter is set

Bucket size	3	4	5
$\rho = 40$	280K	3.1K	320
$\rho = 64$	1.2B	780K	21K
$\rho = 80$	300B	32M	330K

Table 4: Least number of AND gates needed in the bucketing, for different bucket sizes and statistical security parameters.

Circuit	n_1	n_2	n_3	$\lvert\mathcal{C}\rvert$
AES	128	128	128	6800
SHA-128	256	256	160	37300
SHA-256	256	256	256	90825
Hamming Dist.	1048K	1048K	22	2097K
Integer Mult.	2048	2048	2048	4192K
Sorting	131072	131072	131072	10223K

Table 5: **Circuits used in our evaluation.**

to $\kappa = 128$, and the statistical security parameter is set to $\rho = 40$. Garbling and related operations are implemented using fixed-key AES-NI operations as in Bellare et al. [BHKR13]. Note that our protocol is proven secure in the random oracle model. However, it is merely used to encrypt garbled rows. We could describe everything in terms of abstract encryption. One way to realize the encryption is using H as we describe it (for simplicity), and another way is the JustGarble approach that we use in our implementation. Multithreading, Streaming SIMD Extensions (SSE), and Advanced Vector Extensions (AVX) are also used to improve performance whenever possible. To minimize the bucket size, we calculate the least number of AND gate required for each bucket size and statistical security based on the exact formula, shown in Table 4. We plan to open source the implementation in the near future.

Our implementation consists mainly of three parts:

1. **Authenticated bits.** The protocol to compute authenticated bits is very similar to random OT extension [NNOB12]. Therefore, we adopt the most recent OT extension protocol by Keller et al. [KOS15] along with the optimization of Nielsen et al. [NST17]. The resulting protocol requires $\kappa + \rho$ bits of communication per authenticated bit.

2. $\mathcal{F}_{\mathsf{Pre}}$ **functionality.** In order to improve the running time, we spawn multiple threads that each generate a set of leaky AND gates. After all leaky AND gates are generated, bucketing and combining are done in a single thread.

3. **Our protocol.** The function-independent phase invokes the above two parts to generate random AND triples with IT-MACs. In the function-dependent phase, these random AND triples are used to construct a single garbled table. Note that in the single-execution setting, we use only one thread to construct the garbled circuit; in the amortized setting, we use multiple threads, each constructing a different garbled circuit for the same function but different executions. The online phase is always done using a single thread.

Evaluation setup. Our evaluation focuses on two settings:

- LAN: Amazon EC2 with instance `c4.8xlarge` machines both in the North Virginia region connected with 10 Gbps bandwidth and less than 1ms roundtrip time.

- WAN: One machine in North Virginia and one in Ireland, both of which are of the type `c4.8xlarge`. Single thread communication bandwidth is about 224 Mbps; the maximum total bandwidth is about 3 Gbps with multiple threads.

	LAN				WAN			
	Ind. Phase	Dep. Phase	Online	Total	Ind. Phase	Dep. Phase	Online	Total
AES [WMK17]	-	28 ms	14 ms	42 ms	-	425 ms	416 ms	841 ms
AES [NST17]	89.6 ms	13.2 ms	1.46 ms	104.3 ms	1882 ms	96.7 ms	83.2 ms	2061.9 ms
Here	10.9 ms	4.78 ms	0.93 ms	16.6 ms	821 ms	461 ms	77.2 ms	1359.2 ms
SHA1 [WMK17]	-	139 ms	41 ms	180 ms	-	1414 ms	472 ms	1886 ms
Here	41.4 ms	21.3 ms	3.6 ms	66.3 ms	1288 ms	603 ms	78.4 ms	1969.4 ms
SHA256 [WMK17]	-	350 ms	84 ms	434 ms	-	2997 ms	514 ms	3511 ms
SHA256 [NST17]	478.5 ms	164.4 ms	11.2 ms	654.1 ms	2738 ms	350 ms	93.9 ms	3182 ms
Here	96 ms	51.7 ms	9.3 ms	157 ms	1516 ms	772 ms	88 ms	2376 ms

Table 6: Comparison in the single-execution setting

		LAN				WAN			
	τ	Ind. Phase	Dep. Phase	Online	Total	Ind. Phase	Dep. Phase	Online	Total
[RR16]	32	-	45 ms	1.7ms	46.7 ms	-	282 ms	190 ms	472 ms
	128	-	16 ms	1.5 ms	17.5 ms	-	71 ms	191 ms	262 ms
	1024	-	5.1 ms	1.3 ms	6.4 ms	-	34 ms	189 ms	223 ms
[NST17]	32	54.5 ms	0.85 ms	1.23 ms	56.6 ms	235.8 ms	5.2 ms	83.2 ms	324.2 ms
	128	21.5 ms	0.7 ms	1.2 ms	23.4 ms	95.8 ms	3.9 ms	83.7 ms	183.4 ms
	1024	14.7 ms	0.74 ms	1.13 ms	16.6 ms	42.1 ms	2.1 ms	83.2 ms	127.4 ms
Here	32	8.9 ms	0.6 ms	0.97 ms	10.47 ms	75.2 ms	8.7 ms	76 ms	160 ms
	128	5.4 ms	0.54 ms	0.99 ms	6.93 ms	36.6 ms	8.4 ms	75 ms	120 ms
	1024	4.9 ms	0.53 ms	1.23 ms	6.66 ms	30.0 ms	7.5 ms	76 ms	113.5 ms

Table 7: Evaluation of AES in the amortized setting. τ is the number of executions.

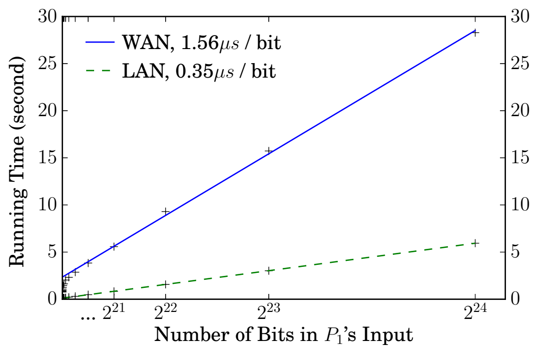
	LAN				WAN			
	Ind. Phase	Dep. Phase	Online	Total	Ind. Phase	Dep. Phase	Online	Total
Hamming Dist.	1867 ms	1226 ms	74 ms	3167 ms	11531 ms	6592 ms	133 ms	18256 ms
Integer Mult.	2860 ms	1921 ms	301 ms	5081 ms	20218 ms	9843 ms	376 ms	30437 ms
Sorting	7096 ms	5508 ms	1021 ms	13625 ms	45155 ms	25582 ms	1918 ms	72655 ms

Table 8: **More examples with a much larger range of input/circuit size.**

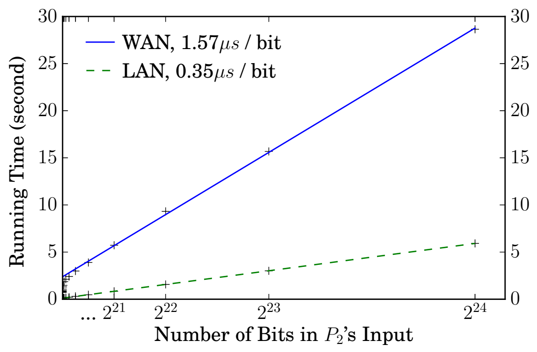
In Section 7.2, we first compare the performance of our protocol with previous protocols in similar settings; here we focus on three circuits commonly used by other works, including AES, SHA-1, and SHA-256 (details in Table 5). Our results show that these circuits may no longer be large enough to serve as the benchmark circuits for malicious 2PC. Therefore, in Section 7.3, we also show the performance of our protocol on some larger circuits (see Table 5). We will make these circuit files publicly available upon publication of our work. In Section 7.4 and Section 7.5, we study the scalability of the protocol and compare the concrete communication complexity of our protocol with prior work.

7.2 Comparison with Previous Work

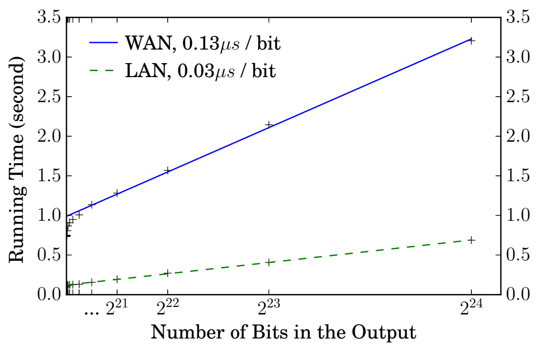
Single-execution setting. First we compare the performance of our protocol to state-of-the-art 2PC protocols in the single-execution setting. In particular, we compare with the protocol of Wang et al. [WMK17], which is based on circuit-level cut-and-choose and is tailored for the single-execution setting, as well as the protocol of Nielsen et al. [NST17], which is based on gate-level cut-and-choose and is able to perform function-independent preprocessing. To make a fair comparison, we ran the implementation by Wang et al. using the same hardware; the results by Nielsen et al. are obtained from their paper, since the hardware configuration is the same. Our reported timings do not include the time for the base-OTs for the same reason as in [NST17]: the performance of base-OTs depends on the details of how the base-OTs are instantiated and is not the focus of our work. For completeness, though, we note that our base-OT implementation (based on the protocol by Chou and Orlandi [CO15]) takes about 20 ms in the LAN setting and 240 ms in the WAN



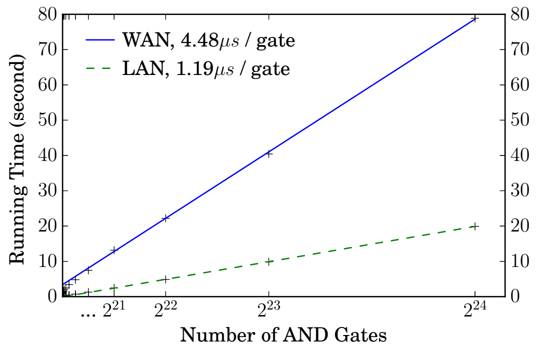
(a) Increasing $\mathsf{P_A}$'s input size (n_1).



(b) Increasing $\mathsf{P_B}$'s input size (n_2).



(c) Increasing output size (n_3).



(d) Increasing circuit size ($|\mathcal{C}|$).

Figure 3: Scalability of our protocol. Initially input sizes and output size are all set to 128 bit with a circuit of size 1024 gate. For each figure, one of the following values increases monotonically: $\mathsf{P_A}$'s input size, $\mathsf{P_B}$'s input size, output size, circuit size.

setting.

As shown in Table 6, our protocol performs better than previous protocols in terms of both overall cost and online time. Compared with the protocol by Wang et al., we achieve a speed up of 2.7× overall and an improvement of about 10× for online time. Compared with the protocol by Nielsen et al., the online cost is roughly the same but our offline time is significantly better: we are 4–7× better in the LAN setting, and 1.3-1.5× better in the WAN setting.

Amortized Setting. We observed that in the amortized setting, our protocol is also better than previous protocols. In particular, we achieve an improvement about 4.5× to 5.5× if only amortized over 32 executions. When the number of executions grows to 1024, [NST17] is no longer better than [RR16] in terms of total time but our protocol still outperforms both protocols: in the LAN setting, the total cost is about the same as [RR16], but most of the computation are done in function-independent phase; in the WAN setting, we are 2× better than [RR16] in terms of total cost and 3× better in terms of online cost.

Comparison with Lindell et al. [LPSY15]. Compared to the recent work of Lindell et al. [LPSY15], our protocol is asymptotically more efficient in the function-independent preprocessing phase; more importantly, the concrete efficiency of our protocol is much better for several reasons: (1) our work is compatible with free-XOR and we do not suffer from any blowup in the size of the circuit being evaluated; (2) Lindell et al. require *five SPDZ-style multiplications* per AND gate of the underlying circuit, while we only need *one TinyOT-style AND* computation per AND gate.

Since the protocol by Lindell et al. is not implemented, we perform a back-of-the-envelope calculation to argue that our protocol is faster. For a circuit of size $|\mathcal{C}|$, their protocol requires $5|\mathcal{C}|$ SPDZ multiplications.

Over a 10 Gbps network, the recent work of Keller et al. [KOS16] can generate in principle 55,000 triples per second using an ideal implementation that fully saturates the network. Therefore, the best end-to-end speed their protocol can achieve in the two-party setting is 11,000 AND gates per second. On the other hand, our actual implementation computes 833,333 AND gates per second as shown by the scalability evaluation in Section 7.4. Therefore, our protocol is at least 75$\times$ better than the best possible implementation of their protocol.

Comparison with linear-round protocols. The AES circuit has depth 50 [LR15]. Therefore, even in the LAN setting with 0.5 ms roundtrip time, and ignoring all computation and communication, any linear-round protocol for securely computing AES would require at least 25 ms, which is already 1.5$\times$ slower than our protocol.

Damgård et al. [DLT14] has the best end-to-end running time among all linear-round protocols. Their protocol only supports amortization by *parallel execution* (where inputs to all executions need to be known at the same time). They report an amortized time for evaluating AES of 14.65 ms per execution, amortized over 680 execution. This is roughly in par with our *single-execution* performance without any preprocessing. When comparing their results to our amortized performance, we are more than 2$\times$ faster, and we are not limited to parallel execution.

A more recent work by Damgård et al. [DNNR16] proposed a protocol with a very efficient online phase. In the LAN setting with similar hardware, it takes 1.09 ms online time to evaluate an AES, which is similar to ours (0.93 ms). They also report 0.47μs online time in the parallel execution setting, which is different from our amortized setting as we discussed above. We cannot compare end-to-end running time since their preprocessing time is not reported. However, we notice that their preprocessing is based on TinyOT, and our optimized TinyOT protocol is much more efficient, which, on the other hand, can also be used to improve their preprocessing phase too.

7.3 Larger Circuits

As we can see from the previous section, evaluating an AES circuit takes less time than generating the base-OT. This means that due to recent advances in 2PC, existing benchmark circuits are no longer large enough for a meaningful evaluation. We propose three new examples and evaluate their performance. The configuration of the circuits are shown in Table 5; we will briefly discuss the functionality of them:

- **Hamming Dist.** Each party inputs a bit string of length 1048576 bits; the output of the circuit is a 22-bit number containing the hamming distance of the two bit string from each party. The circuit complexity is $O(n)$ for n-bit strings.
- **Integer Mult.** Each party inputs a 2048-bit number; the circuit compute the multiplication of them, ignoring the high 2048 bits of the result. The circuit complexity is $O(n^2)$ for n bit numbers.
- **Sorting.** Each party inputs XOR-share of 4096 32-bit numbers; the circuit first XOR them to recover the underlying numbers and then sort the these numbers. The circuit complexity is $O(nl \log^2 n)$ to sort n numbers each with l bits.

Table 8 shows the performance of new examples described above. We can see that the difference of online time between LAN and WAN is about 75 ms, which is roughly the roundtrip time of the WAN network we used. This is also consistent with the fact that our protocol requires only one round of online communication (one message from each party). According to the Table, our protocol is able to sort 4096 32-bit numbers in less than 14 seconds with an online time only 1 second. We would like to note that, the state-of-the-art garbling scheme can evaluate about 20 million AND gate per second. Therefore the same circuit would take about 0.5 second online time even in the semi-honest setting. Other timings can be interpreted similarly.

7.4 Scalability

To explore the concrete performance of our protocol for circuits with different input, output and circuit sizes, we conduct a scalability evaluation: we start with a circuit with input and output sizes of 128 bits and

Circuit	n_1	n_2	n_3	$\lvert\mathcal{C}\rvert$
LAN	0.35	0.35	0.03	1.19
WAN	1.56	1.57	0.13	4.48

Table 9: Scalability of the protocol. All numbers in microseconds.

Protocol	τ	Ind. Phase	Dep. Phase	Online
[RR16]	32	-	3.8 MB	25.8 KB
	128	-	2.5 MB	21.3 KB
	1024	-	1.6 MB	17.0 KB
[NST17]	1	14.9 MB	0.22 MB	16.1 KB
	32	8.7 MB	0.22 MB	16.1 KB
	128	7.2 MB	0.22 MB	16.1 KB
	1024	6.4 MB	0.22 MB	16.1 KB
This Paper	1	2.86 MB	0.57 MB	4.86 KB
	32	2.64 MB	0.57 MB	4.86 KB
	128	2.0 MB	0.57 MB	4.86 KB
	1024	2.0 MB	0.57 MB	4.86 KB

Table 10: Comparison of communication per execution for evaluating an AES circuit. Numbers presented are for the amount of data sent from garbler to evaluator; this reflects the speed in a duplex network. In the setting with a simplex network, the total communication of this work and [RR16] should be doubled for a fair comparison.

1024 AND gates and, at each time, increase one size monotonically up to 2^{24} bits/gates. The result of the evaluation is shown in Figure 3. Trend lines are also included to show the asymptotical performance. Since the bucket size of our protocol reduces as the circuit size increases, these lines are regression of the points when the bucket size is 3.

According to the figures, our implementation scales linearly in the input, output and circuit sizes as expected. We observe that, in the LAN setting, our protocol requires only 0.35 μs to process each input bit and 0.03 μs to process each output bit. Note that this is much better than circuit-level cut-and-choose protocols, mainly for two reasons: 1) Since only one garbled circuit is constructed, only one set of garbled labels need to be transferred; this is an improvement of ρ times. 2) We do not need XOR-Tree or ρ-probe matrix to prevent selective failure, which can incur a huge cost when the input is large [WMK17].

The figures also show that, in the WAN setting, the ratios are about 3–4$\times$ lower than the ratios in the LAN setting. This roughly matches the ratio of network bandwidth between LAN and WAN settings.

7.5 Communication Complexity

We also record the amount of communication used in the protocol based on our implementation. In Table 10 we compare the amount of data sent from garbler to the evaluator with other related works. In detail, we focused on the AES circuit with different number of executions. Our total communication is 3$\times$ to 5$\times$ less than Nielsen et al.'s protocol. Furthermore, our cost in the *single-execution* setting is even half the cost of Nielsen et al.'s protocol when amortized with 1024 executions. Note that for protocols based on cut-and-choose, the total communication to send 40 AES garbled circuit is 8.7 MB, which is already higher than the total communication of our protocol in the single execution setting.

We also observe that our function dependent preprocessing is higher than Nielsen et al.; this is due to the fact that we need to send $3\kappa + 4\rho$ bits per gate while they only need to send 2κ bits. On the other hand, our online communication is extremely small: it is about 3× smaller than in the protocol of Nielsen et al. and 3.5–5.3× smaller than the protocol of Rindal and Rosulek.

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A Improved TinyOT protocol

In this section, we describe an improvement to the TinyOT protocol. For a bucket size of $B = \frac{\rho}{\log |\mathcal{C}|} + 1$, the original protocol requires $14B + 2$ authenticated bits for each AND gate. In the following, we will introduce an improved version where only $6B$ authenticated bits are needed for each AND gate. For a circuit of size 2^{20}, with $\rho = 40$, this is an improvement of $2.7\times$.

A.1 Half Authenticated AND

Before describing the main protocol, we will first show how to compute an AND triple with only x's being authenticated ($\mathcal{F}_{\mathsf{HaAND}}$). This will serve as a building block for the following sections. The functionality $\mathcal{F}_{\mathsf{HaAND}}$ is described in Figure 4. It outputs authenticated bits $[x_1]_{\mathsf{A}}$ and $[x_2]_{\mathsf{B}}$ to the two parties, it also gets y_1 from $\mathsf{P_A}$ and y_2 from $\mathsf{P_B}$ without authentication. The functionality then outputs random shares of $x_1y_2 \oplus x_2y_1$. Looking ahead to the next subsection, this prevents parties from flipping x's, which would

Functionality $\mathcal{F}_{\mathsf{HaAND}}$

1. The box picks random $[x_1]_{\mathsf{A}}$ and $[x_2]_{\mathsf{B}}$ and sends them to the two parties.

2. Upon receiving y_1 from $\mathsf{P_A}$ and y_2 from $\mathsf{P_B}$, the box samples two random bits v_1, v_2 such that $v_1 \oplus v_2 = x_1 y_2 \oplus x_2 y_1$. The box sends v_1 to $\mathsf{P_A}$, v_2 to $\mathsf{P_B}$.

Global Key Queries: The adversary at any point can send some (p, Δ') and will be told if $\Delta' = \Delta_p$.

Figure 4: Functionality $\mathcal{F}_{\mathsf{HaAND}}$ that computes a half authenticated AND triple.

Protocol Π_{HaAND}

Protocol:

1. $\mathsf{P_A}$ and $\mathsf{P_B}$ call $\mathcal{F}_{\mathsf{abit}}$ to obtain $[x_1]_{\mathsf{A}}$ and $[x_2]_{\mathsf{B}}$.

2. $\mathsf{P_A}$ picks random bit s_1 and computes $H_0 := \mathsf{Lsb}(H(\mathsf{K}[x_2])) \oplus s_1$, $H_1 := \mathsf{Lsb}(H(\mathsf{K}[x_2] \oplus \Delta_{\mathsf{A}})) \oplus s_1 \oplus y_1$. $\mathsf{P_A}$ sends (H_0, H_1) to $\mathsf{P_B}$, who computes $s_2 := H_{x_2} \oplus \mathsf{Lsb}(H(\mathsf{M}[x_2]))$.

3. $\mathsf{P_B}$ picks random bit t_1 and computes $H_0 := \mathsf{Lsb}(H(\mathsf{K}[x_1])) \oplus t_1$, $H_1 := \mathsf{Lsb}(H(\mathsf{K}[x_1] \oplus \Delta_{\mathsf{B}})) \oplus t_1 \oplus y_2$. $\mathsf{P_B}$ sends (H_0, H_1) to $\mathsf{P_A}$, who computes $t_2 := H_{x_1} \oplus \mathsf{Lsb}(H(\mathsf{M}[x_1]))$.

4. $\mathsf{P_A}$ computes $v_1 := s_1 \oplus t_2$, $\mathsf{P_B}$ computes $v_2 := s_2 \oplus t_1$.

Figure 5: Protocol Π_{HaAND} instantiating $\mathcal{F}_{\mathsf{HaAND}}$.

cause a selective failure attack on y, but would still allows parties to flip y's, which would cause a selective failure attack on x. The protocol that instantiates this functionality is simple due to the fact that not all bits are authenticated. In the proof, we will essentially show that if an adversary "corrupts" any message, it is equivalent to using some other input.

Lemma A.1. *Assuming H is a random oracle, the protocol in Figure 5 securely implements the functionality in Figure 4 in the $\mathcal{F}_{\mathsf{abit}}$-hybrid model.*

Proof. First we will show the correctness of the protocol. We will show that $s_1 \oplus s_2 = x_2 y_1$ and that $t_1 \oplus t_2 = x_1 y_2$. Without loss of generality, we will show the first equation. There are two cases:

- $x_2 = 0$. In this case, $\mathsf{P_B}$ obtains $s_2 = s_1$.
- $x_2 = 1$. In this case, $\mathsf{P_B}$ obtains $s_2 = s_1 \oplus y_1$.

In both cases, the equation we want to show holds. The other equation can be proven in exactly the same way. The correctness of the protocol follows immediately from these two equations.

In a part below, we will continue to the simulation proof. The proof is straightforward, mainly due to the fact that each party's input is not authenticated and therefore $\mathcal{S}$ can extract the values easily.
Malicious $\mathsf{P_A}$. The simulator works as follows:

1. $\mathcal{S}$ plays the role of $\mathcal{F}_{\mathsf{abit}}$, and stores $[x_1]_{\mathsf{A}}$, $[x_2]_{\mathsf{B}}$.

2. $\mathcal{S}$ receives (H_0, H_1) from $\mathcal{A}$, and computes $s_1 := H_0 \oplus \mathsf{Lsb}(H(\mathsf{K}[x_2]))$, $y_1 := H_1 \oplus s_1 \oplus \mathsf{Lsb}(H(\mathsf{K}[x_2] \oplus \Delta_{\mathsf{A}}))$. $\mathcal{S}$ sends y_1 to $\mathcal{F}_{\mathsf{HaAND}}$ on behalf of $\mathsf{P_A}$ and receives v_1.

3. $\mathcal{S}$ computes $H_{x_1} := \mathsf{Lsb}(H(\mathsf{K}[x_1] \oplus x_1 \Delta_{\mathsf{B}})) \oplus v_1 \oplus s_1$ and picks $H_{1 \oplus x_1}$ randomly, and sends (H_0, H_1) to $\mathsf{P_A}$.

Honest $\mathsf{P_B}$ has the same output according to the correctness proof. It is easy to see that the first two steps are perfect simulation. The last step is also a perfect simulation: the joint distribution of (H_0, H_1) and $\mathsf{P_B}$'s output is perfectly indistinguishable. 1) $\mathsf{P_A}$ only knows either $\mathsf{K}[x_1]$ or $\mathsf{K}[x_1] \oplus \Delta_B$, which means $H_{x_1 \oplus 1}$

Functionality $\mathcal{F}_{\mathsf{LaAND}}$

Honest parties: The box picks random $[x_1]_\mathsf{A}, [y_1]_\mathsf{A}, [z_1]_\mathsf{A}$, and $[x_2]_\mathsf{B}, [y_2]_\mathsf{B}, [z_2]_\mathsf{B}$, such that $(x_1 \oplus x_2) \wedge (y_1 \oplus y_2) = z_1 \oplus z_2$.

Corrupted parties:

1. A corrupted $\mathsf{P_A}$ gets to choose all its randomness. Furthermore, it can send g to the box trying to guess x_2. If $g \neq x_2$ the box output fail and terminates, otherwise the box proceeds as normal.

2. A corrupted $\mathsf{P_B}$ gets to choose all its randomness. Furthermore, it can send g to the box trying to guess x_1. If $g \neq x_1$ the box output fail and terminates, otherwise the box proceeds as normal.

Global Key Queries: The adversary at any point can send some (p, Δ') and will be told if $\Delta' = \Delta_p$.

Figure 6: Functionality $\mathcal{F}_{\mathsf{LaAND}}$ for leaky AND triple generation.

Functionality $\mathcal{F}_{\mathsf{aAND}}$

Honest parties: The box picks random $[x_1]_\mathsf{A}, [y_1]_\mathsf{A}, [z_1]_\mathsf{A}$, and $[x_2]_\mathsf{B}, [y_2]_\mathsf{B}, [z_2]_\mathsf{B}$, such that $(x_1 \oplus x_2) \wedge (y_1 \oplus y_2) = z_1 \oplus z_2$.

Corrupted parties: A corrupted $\mathsf{P_A}$ gets to choose all its randomness.

Global Key Queries: The adversary at any point can send some (p, Δ') and will be told if $\Delta' = \Delta_p$.

Figure 7: Functionality $\mathcal{F}_{\mathsf{aAND}}$ for generating AND triples

remains random as long as H is a random oracle. 2) $\mathsf{P_A}$ obtains from H_{x_1} $v_1 \oplus s_1$, which is the same for both hybrids.

Malicious $\mathsf{P_B}$. The simulation is essentially the same as the case when $\mathsf{P_A}$ is malicious (observing that step 2 and step 3 can be done in any order). □

A.2 New TinyOT Protocol

Assuming that two parties hold $[x_1]_\mathsf{A}$, $[y_1]_\mathsf{A}$, $[x_2]_\mathsf{B}$, $[y_2]_\mathsf{B}$. In the original TinyOT protocol, to compute $(x_1 \oplus x_2)(y_1 \oplus y_2)$, $\mathsf{P_A}$ and $\mathsf{P_B}$ compute $[x_1y_1]_\mathsf{A}$, $[x_2y_2]_\mathsf{B}$, $[x_1y_2+r]_\mathsf{A}$ and $[x_2y_1+r]_\mathsf{B}$ separately, with some random $r \in \{0,1\}$, using various authenticated constructions proposed in their paper. Computing each entry separately incurs a lot of unnecessary cost. We observe that it is possible to compute a whole AND gate directly. Similar to the original TinyOT protocol, we propose a "leaky AND" protocol (Π_{LaAND}), where the adversary is allowed to perform selective-failure attack on one input, and later use bucketing to eliminate such leakage (Π_{aAND}). In the following, we will first discuss the intuition of the leaky AND protocol. The full protocol description is in Figure 8 and Figure 9.

Intuition of our checking protocol. We describe things from the point of view of an honest $\mathsf{P_B}$ holding $x_2 = 0$. Abstractly, the first step is for $\mathsf{P_B}$ to compute values T_0 and U_0 and to send U_0 to $\mathsf{P_A}$; $\mathsf{P_A}$ will then compute V_0 such that if $x_1 = 0$ then $V_0 = T_0$, but if $x_1 = 1$ then $V_0 \oplus U_0 = T_0$. We set things up such that if the AND triple is incorrect, then $\mathsf{P_A}$ cannot compute V_0 correctly. Similar constructs (namely V_1, U_1, and T_1) are also computed for the case when $x_2 = 1$. Now depending on the value of x_1 and x_2, parties need to perform equality comparison between different values, as summarized below.

	$x_1 = 0$	$x_1 = 1$
$x_2 = 0$	$V_0 = T_0$	$V_0 \oplus U_0 = T_0$
$x_2 = 1$	$V_1 = T_1$	$V_1 \oplus U_1 = T_1$

Unfortunately, a direct comparison is not possible since $\mathsf{P_A}$ does not know the value of x_2 and therefore does not know which comparison to perform. Our idea is to transform $\mathsf{P_A}$'s computation such that it is oblivious to x_2. In detail, if $x_1 = 0$, $\mathsf{P_A}$ will compute V_0 as if $x_2 = 0$ and computes V_1 as if $x_2 = 1$. $\mathsf{P_A}$ then encrypts V_0 and V_1 such that $\mathsf{P_B}$ with authenticated x_2 can only decrypt V_{x_2}. $\mathsf{P_B}$ can then compare locally if $V_{x_2} = T_{x_2}$. In the case when $\mathsf{P_A}$ has $x_1 = 1$, $\mathsf{P_A}$ computes and encrypts $V_0 \oplus U_0$ and $V_1 \oplus U_1$ in a similar manner.

Protocol Π_{LaAND}

Protocol:

1. $\mathsf{P_A}$ and $\mathsf{P_B}$ obtain random authenticated bits $[y_1]_\mathsf{A}, [z_1]_\mathsf{A}, [y_2]_\mathsf{B}, [r]_\mathsf{B}$. $\mathsf{P_A}$ and $\mathsf{P_B}$ also calls $\mathcal{F}_{\mathsf{HaAND}}$, receiving $[x_1]_\mathsf{A}$ and $[x_2]_\mathsf{B}$.
2. $\mathsf{P_A}$ sends y_1 to $\mathcal{F}_{\mathsf{HaAND}}$, $\mathsf{P_B}$ sends y_2 to $\mathcal{F}_{\mathsf{HaAND}}$, which sends v_1 to $\mathsf{P_A}$ and v_2 to $\mathsf{P_B}$.
3. $\mathsf{P_A}$ computes $u = v_1 \oplus x_1 y_1$ and sends to $\mathsf{P_B}$. $\mathsf{P_B}$ computes $z_2 := u \oplus x_2 y_2 \oplus v_2$ and sends $d := r \oplus z_2$ to $\mathsf{P_A}$. Two parties compute $[z_2]_\mathsf{B} = [r]_\mathsf{B} \oplus d$.
4. $\mathsf{P_B}$ checks the correctness as follows:
   (a) $\mathsf{P_B}$ computes:
   $T_0 := H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B})$
   $U_0 := T_0 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B})$
   $T_1 := H(\mathsf{K}[x_1], \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B})$
   $U_1 := T_1 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B})$
   (b) $\mathsf{P_B}$ sends U_{x_2} to $\mathsf{P_A}$.
   (c) $\mathsf{P_A}$ randomly picks a κ-bit string R and computes
   $V_0 := H(\mathsf{M}[x_1], \mathsf{M}[z_1])$ $\quad V_1 := H(\mathsf{M}[x_1], \mathsf{M}[z_1] \oplus \mathsf{M}[y_1])$
   $W_{0,0} := H(\mathsf{K}[x_2]) \oplus V_0 \oplus R$ $\quad W_{0,1} := H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_1 \oplus R$
   $W_{1,0} := H(\mathsf{K}[x_2]) \oplus V_1 \oplus U \oplus R$ $\quad W_{1,1} := H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_0 \oplus U \oplus R$
   (d) $\mathsf{P_A}$ sends $W_{x_1,0}, W_{x_1,1}$ to $\mathsf{P_B}$ and sends R to $\mathcal{F}_{\mathsf{EQ}}$.
   (e) $\mathsf{P_B}$ computes $R' := W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_{x_2}$ and sends R' to $\mathcal{F}_{\mathsf{EQ}}$.
5. $\mathsf{P_A}$ checks the correctness as follows:
   (a) $\mathsf{P_A}$ computes:
   $T_0 := H(\mathsf{K}[x_2], \mathsf{K}[z_2] \oplus z_1\Delta_\mathsf{A})$
   $U_0 := T_0 \oplus H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}, \mathsf{K}[y_2] \oplus \mathsf{K}[z_2] \oplus (y_1 \oplus z_1)\Delta_\mathsf{A})$
   $T_1 := H(\mathsf{K}[x_2], \mathsf{K}[y_2] \oplus \mathsf{K}[z_2] \oplus (y_1 \oplus z_1)\Delta_\mathsf{A})$
   $U_1 := T_1 \oplus H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}, \mathsf{K}[z_2] \oplus z_1\Delta_\mathsf{A})$
   (b) $\mathsf{P_A}$ sends U_{x_1} to $\mathsf{P_B}$.
   (c) $\mathsf{P_B}$ randomly picks a κ-bit string R and computes
   $V_0 := H(\mathsf{M}[x_2], \mathsf{M}[z_2])$ $\quad V_1 := H(\mathsf{M}[x_2], \mathsf{M}[z_2] \oplus \mathsf{M}[y_2])$
   $W_{0,0} := H(\mathsf{K}[x_1]) \oplus V_0 \oplus R$ $\quad W_{0,1} := H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}) \oplus V_1 \oplus R$
   $W_{1,0} := H(\mathsf{K}[x_1]) \oplus V_1 \oplus U \oplus R$ $\quad W_{1,1} := H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}) \oplus V_0 \oplus U \oplus R$
   (d) $\mathsf{P_B}$ sends $W_{x_2,0}, W_{x_2,1}$ to $\mathsf{P_A}$ and sends R to $\mathcal{F}_{\mathsf{EQ}}$,
   (e) $\mathsf{P_A}$ computes $R' := W_{x_2,x_1} \oplus H(\mathsf{M}[x_1]) \oplus T_{x_1}$ and sends R' to $\mathcal{F}_{\mathsf{EQ}}$.

Figure 8

Now the only problem is that although a malicious $\mathsf{P_A}$ cannot cheat, a malicious $\mathsf{P_B}$ will not be caught for an incorrect AND relationship because $\mathsf{P_B}$ compare the results locally and $\mathsf{P_A}$ does not know the outcome of the comparison! To solve this, we let $\mathsf{P_A}$ instead send encrypted $V_0 \oplus R$ and $V_1 \oplus R$ for some random R such that $\mathsf{P_B}$ can obtain $V_{x_2} \oplus R$, and learns R from it. Now $\mathsf{P_A}$ and $\mathsf{P_B}$ can check the equality on R using the $\mathcal{F}_{\mathsf{EQ}}$ functionality that allows both parties get the outcome. Finally, the same check will be done in the opposite direction to convince both party the correctness of the triples.

Note that this allows an adversary to perform selective-failure attacks, by sending some corrupted encrypted values. This does not introduce additional leakage, since x's are allowed to be learned by $\mathcal{A}$ anyway . The fact that $\mathcal{A}$ can guess x multiple times also does not help $\mathcal{A}$ in obtaining more information.

A.3 Proof

In the following, we will discuss from a high-level view how the proof works for the new TinyOT protocol. We will focus on the security of Π_{LaAND} protocol, since the security of Π_{aAND} is fairly straightforward given the proof in the original paper [NNOB12].

Protocol Π_{aAND}

Protocol:

1. $\mathsf{P_A}$ and $\mathsf{P_B}$ call $\mathcal{F}_{\mathsf{LaAND}}$ $\ell' = \ell B$ times and obtains $\{[x_1^i]_\mathsf{A}, [y_1^i]_\mathsf{A}, [z_1^i]_\mathsf{A}, [x_2^i]_\mathsf{B}, [y_2^i]_\mathsf{B}, [z_2^i]_\mathsf{B}\}_{i=1}^{\ell'}$.
2. $\mathsf{P_A}$ and $\mathsf{P_B}$ randomly partition all objects into ℓ buckets, each with B objects.
3. For each bucket, two parties combine B Leaky ANDs into one non-leaky AND. To combine two leaky ANDs, namely $([x_1']_\mathsf{A}, [y_1']_\mathsf{A}, [z_1']_\mathsf{A}, [x_2']_\mathsf{B}, [y_2']_\mathsf{B}, [z_2']_\mathsf{B})$ and $[x_1'']_\mathsf{A}, [y_1'']_\mathsf{A}, [z_1'']_\mathsf{A}, [x_2'']_\mathsf{B}, [y_2'']_\mathsf{B}, [z_2'']_\mathsf{B}$
   (a) Two parties reveal $d' := y_1' \oplus y_1'', d'' = y_2' \oplus y_2''$ with their MAC checked, and compute $d := d' \oplus d''$.
   (b) Set $[x_1]_\mathsf{A} := [x_1']_\mathsf{A} \oplus [x_1'']_\mathsf{A}$, $[x_2]_\mathsf{B} := [x_2']_\mathsf{B} \oplus [x_2'']_\mathsf{B}$, $[y_1]_\mathsf{A} := [y_1']_\mathsf{A}$, $[y_2]_\mathsf{A} := [y_2']_\mathsf{A}$, $[z_1]_\mathsf{A} := [z_1']_\mathsf{A} \oplus [z_1'']_\mathsf{A} \oplus d[x_1'']_\mathsf{A}$, $[z_2]_\mathsf{B} := [z_2']_\mathsf{B} \oplus [z_2'']_\mathsf{B} \oplus d[x_2'']_\mathsf{B}$.

   Two parties iterate all B leaky objects, by taking the resulted object and combine with the next element.

Figure 9: Protocol Π_{aAND} instantiating $\mathcal{F}_{\mathsf{aAND}}$.

Correctness

Without loss of generality, we want to show that if both players followed the protocol then in step 4.e that $W_{x_1,x_2} \oplus \mathsf{M}[x_2] \oplus T_{x_2} = R$. Checks in step 5 are perfectly symmetric to ones in step 4. We will proceed on a case per case basis.

Case 1: $x_1 = 0, x_2 = 0$
The value of x_1, x_2 means that $\mathsf{M}[x_1] = \mathsf{K}[x_1]$ and that $\mathsf{M}[x_2] = \mathsf{K}[x_2]$. Since $x_1 \oplus x_2 = 0$, we know that $z_1 = z_2$, which further implies that

$$\mathsf{M}[z_1] = \mathsf{K}[z_1] \oplus z_1\Delta_\mathsf{B} = \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}$$

The equation holds based on the following:

$$\begin{aligned}
&W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_{x_2} \\
&\quad = H(\mathsf{K}[x_2]) \oplus V_0 \oplus R \oplus H(\mathsf{M}[x_2]) \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}) \\
&\quad = V_0 \oplus T_0 \oplus R \\
&\quad = H(\mathsf{M}[x_1], \mathsf{M}[z_1]) \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}) \oplus R \\
&\quad = R
\end{aligned}$$

Case 2: $x_1 = 0, x_2 = 1$
Similar to the previous case, we know that $\mathsf{M}[x_1] = \mathsf{K}[x_1]$ and that $\mathsf{M}[x_2] = \mathsf{K}[x_2] \oplus \Delta_\mathsf{B}$. $x_1 \oplus x_2 = 1$ also implies that

$$\begin{aligned}
&\mathsf{M}[z_1] \oplus \mathsf{M}[y_1] \\
&\quad = \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_1 \oplus z_1)\Delta_\mathsf{B} \\
&\quad = \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B}
\end{aligned}$$

The equation holds based on the following:

$$\begin{aligned}
&W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_{x_2} \\
&\quad = W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_1 \\
&\quad = H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_1 \oplus R \oplus H(\mathsf{M}[x_2]) \oplus T_1 \\
&\quad = V_1 \oplus T_1 \oplus R \\
&\quad = H(\mathsf{M}[x_1], \mathsf{M}[z_1] \oplus \mathsf{M}[y_1]) \\
&\qquad \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B} \oplus \mathsf{K}[y_1] \oplus y_2\Delta_\mathsf{B}) \oplus R \\
&\quad = R
\end{aligned}$$

Case 3: $x_1 = 1, x_2 = 0$
Similar to the previous cases, we know that $\mathsf{M}[x_1] = \mathsf{K}[x_1] \oplus \Delta_\mathsf{B}$, $\mathsf{M}[x_2] = \mathsf{K}[x_2]$ and that $\mathsf{M}[z_1] \oplus \mathsf{M}[y_1] = \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B}$, which will be used to prove the following:

$$\begin{aligned}
&W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_{x_2} \\
&\quad = W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_0 \\
&\quad = H(\mathsf{K}[x_2]) \oplus V_1 \oplus U \oplus R \oplus H(\mathsf{M}[x_2]) \oplus T_0 \\
&\quad = V_1 \oplus U \oplus R \oplus T_0 \\
&\quad = H(\mathsf{M}[x_1], \mathsf{M}[z_1] \oplus \mathsf{M}[y_1]) \oplus R \oplus T_0 \\
&\qquad \oplus T_0 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B}) \\
&\quad = R
\end{aligned}$$

Case 4: $x_1 = 1, x_2 = 1$
Similar to the previous cases, we know that $\mathsf{M}[x_1] = \mathsf{K}[x_1] \oplus \Delta_\mathsf{B}$, $\mathsf{M}[x_2] = \mathsf{K}[x_2] \oplus \Delta_\mathsf{B}$ and that $\mathsf{M}[z_1] = \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}$, which will be used to prove the following:

$$\begin{aligned}
&W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_{x_2} \\
&\quad = W_{x_1,x_2} \oplus H(\mathsf{M}[x_2]) \oplus T_1 \\
&\quad = H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_0 \oplus U \oplus R \oplus H(\mathsf{M}[x_2]) \oplus T_1 \\
&\quad = V_0 \oplus U \oplus R \oplus T_1 \\
&\quad = H(\mathsf{M}[x_1], \mathsf{M}[z_1]) \oplus R \oplus T_1 \\
&\qquad \oplus T_1 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}) \\
&\quad = R
\end{aligned}$$

Unforgeability

Lemma A.2. *If $(x_1 \oplus x_2) \wedge (y_1 \oplus y_2) \neq (z_1 \oplus z_2)$ then the protocol will result in an abort except with negligible probability.*

We will proceed on a case per case basis. We assume that P_B is honest and that the adversary corrupts P_A. By symmetry, this would also show that the protocol would abort when P_B is corrupt and P_A is honest.

Case 1: $x_1 = 0, x_2 = 0$
The adversary to pass the test would have to produce a pair R and $W_{0,0}$ such that:

$$\begin{aligned}
W_{0,0} &= H(\mathsf{M}[x_2]) \oplus T_{x_2} \oplus R \\
W_{0,0} &= H(\mathsf{M}[x_2]) \oplus R \\
&\quad \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B})
\end{aligned}$$

Since $z_1 \oplus z_2 = 1$, the last line requires the adversary to compute $\mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B} = \mathsf{M}[z_1] \oplus \Delta_\mathsf{B}$. This is equivalent to forging a mac and is thus infeasible. Alternatively, the adversary could try to compute T_0 from $U_0 = T_0 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B})$. Fortunately, since $\mathsf{K}[x_1] \oplus \Delta_\mathsf{B} = \mathsf{M}[x_1] \oplus \Delta_\mathsf{B}$. This is also infeasible. This implies that an adversary cannot pass the test.

Case 2: $x_1 = 0, x_2 = 1$
The adversary to pass the test would have to produce a pair R and $W_{0,1}$ such that:

$$\begin{aligned}
W_{0,1} &= H(\mathsf{M}[x_2]) \oplus T_{x_2} \oplus R \\
W_{0,1} &= H(\mathsf{M}[x_2]) \oplus R \\
&\quad \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B} \oplus \mathsf{K}[y_1] \oplus y_2\Delta_\mathsf{B})
\end{aligned}$$

However, since $z_1 \oplus z_2 \oplus y_1 \oplus y_2 = 1$, the last line requires the adversary to compute $\mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (z_2 \oplus y_2)\Delta_\mathsf{B} = \mathsf{M}[y_1] \oplus \mathsf{M}[z_1] \oplus \Delta_\mathsf{B}$. This is equivalent to forging a mac tag which is infeasible. Alternatively, the adversary could try to compute T_1 from $U_1 = T_1 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B})$. Fortunately, since $\mathsf{K}[x_1] \oplus \Delta_\mathsf{B} = \mathsf{M}[x_1] \oplus \Delta_\mathsf{B}$. This is also infeasible. This implies that an adversary cannot pass the test.

Case 3: $x_1 = 1, x_2 = 0$

The adversary to pass the test would have to produce R, $W_{1,0}$ such that:

$$\begin{aligned} W_{1,0} &= H(\mathsf{M}[x_2]) \oplus T_{x_2} \oplus R \\ W_{1,0} &= H(\mathsf{M}[x_2]) \oplus R \\ &\quad \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B}) \end{aligned}$$

Since $x_1 = 1$, the last line requires the adversary to compute $\mathsf{K}[x_1] = \mathsf{M}[x_1] \oplus \Delta_\mathsf{B}$. This is equivalent to forging a mac tag which is infeasible. Alternatively, the adversary could try to compute T_0 from $U_0 = T_0 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B})$. Fortunately, since $y_1 \oplus y_2 \oplus z_1 \oplus z_2 = 1$ then $\mathsf{K}[y_1] \oplus \mathsf{K}[z_1] \oplus (y_2 \oplus z_2)\Delta_\mathsf{B} = \mathsf{M}[y_1] \oplus \mathsf{M}[z_1] \oplus \Delta_\mathsf{B}$ This is also infeasible. This implies that an adversary cannot pass the test.

Case 4: $x_1 = 1, x_2 = 1$

The adversary to pass the test would have to produce R and $W_{1,1}$ such that:

$$\begin{aligned} W_{1,1} &= H(\mathsf{M}[x_2]) \oplus T_{x_2} \oplus R \\ W_{1,1} &= H(\mathsf{M}[x_2]) \oplus R \\ &\quad \oplus H(\mathsf{K}[x_1], \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B} \oplus \mathsf{K}[y_1] \oplus y_2\Delta_\mathsf{B}) \end{aligned}$$

Since $x_1 = 1$, the last line requires the adversary to compute $\mathsf{K}[x_1] = \mathsf{M}[x_1] \oplus \Delta_\mathsf{B}$. This is equivalent to forging a mac tag which is infeasible. Alternatively, the adversary could try to compute T_1 from $U_1 = T_1 \oplus H(\mathsf{K}[x_1] \oplus \Delta_\mathsf{B}, \mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B})$. Fortunately, since $z_1 \oplus z_2 = 1$ then $\mathsf{K}[z_1] \oplus z_2\Delta_\mathsf{B} = \mathsf{M}[z_1] \oplus \Delta_\mathsf{B}$. Thus, this is also infeasible.

Completed proof

Now we will proceed with the complete proof.

Lemma A.3. *The protocol in Figure 8 securely implements the functionality in Figure 6 against corrupted* $\mathsf{P_A}$ *in the* $(\mathcal{F}_\mathsf{abit}, \mathcal{F}_\mathsf{HaAND}, \mathcal{F}_\mathsf{EQ})$*-Hybrid model.*

Proof. We will construct a simulator as follows:

1 $\mathcal{S}$ interacts with $\mathcal{A}$ and receives $(x_1, \mathsf{M}[x_1])$, $(y_1, \mathsf{M}[y_1])$, $(z_1, \mathsf{M}[z_1])$, $\mathsf{K}[x_2]$, $\mathsf{K}[y_2]$, $\mathsf{K}[r]$, and Δ_A that $\mathcal{A}$ sent to $\mathcal{F}_\mathsf{abit}$. $\mathcal{S}$ picks a random bit s, sets $\mathsf{K}[z_2] := \mathsf{K}[r] \oplus s\Delta_\mathsf{A}$, and sends $(x_1, \mathsf{M}[x_1])$, $(y_1, \mathsf{M}[y_1])$, $(z_1, \mathsf{M}[z_1])$, $\mathsf{K}[x_2], \mathsf{K}[y_2], \mathsf{K}[z_2], \Delta_\mathsf{A})$ to $\mathcal{F}_\mathsf{LaAND}$, which sends $(x_2, \mathsf{M}[x_2])$,$(y_2, \mathsf{M}[y_2])$, $(z_2, \mathsf{M}[z_2])$, $\mathsf{K}[x_1]$, $\mathsf{K}[y_1]$, $\mathsf{K}[z_1]$, Δ_B to $\mathsf{P_B}$.

2-3 $\mathcal{S}$ plays the role of $\mathcal{F}_\mathsf{HaAND}$ obtaining the inputs from $\mathcal{A}$, namely y_1' and the value $\mathcal{A}$ sent, namely u'. $\mathcal{S}$ uses y_1 and u to denote the value that an honest $\mathsf{P_B}$ would use. If $y_1' \neq y_1, u' \neq u$, $\mathcal{S}$ sets $g_0 = 1 \oplus x_1$, if $y_1' \neq y_1, u' = u$, $\mathcal{S}$ sets $g_0 = x_1$.

4 $\mathcal{S}$ sends a random U^* to $\mathcal{A}$, and receives some W_0, W_1 and computes some R_0, R_1, such that, if $x_1 = 0$, $W_0 := H(\mathsf{K}[x_2]) \oplus V_0 \oplus R_0, W_1 := H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_1 \oplus R_1$; otherwise, $W_0 := H(\mathsf{K}[x_2]) \oplus V_1 \oplus U^* \oplus R_0$ and $W_1 := H(\mathsf{K}[x_2] \oplus \Delta_\mathsf{A}) \oplus V_0 \oplus U^* \oplus R_1$.

$\mathcal{S}$ also obtains R that $\mathcal{A}$ sent to $\mathcal{F}_\mathsf{EQ}$. If R does not equal to either R_0 or R_1, $\mathcal{S}$ aborts; otherwise $\mathcal{S}$ computes g_1 such that $R \neq R_{g_1}$ for some $g_1 \in \{0, 1\}$.

5 $\mathcal{S}$ receives U, picks random W_0^*, W_1^* and sends them to $\mathcal{A}$. $\mathcal{S}$ obtains R' that $\mathcal{A}$ sent to $\mathcal{F}_{\mathsf{EQ}}$.

- If both U, R' are honestly computed, $\mathcal{S}$ proceeds as normal.
- If U is not honestly computed and that $R' = W_{x_1}^* \oplus H(\mathsf{M}[x_1]) \oplus T_{x_1}$ is honestly computed, $\mathcal{S}$ set $g_2 = 0$
- If either of the following is true: 1) $x_1 = 0$ and $R' = W_{x_1}^* \oplus H(\mathsf{M}[x_1]) \oplus U \oplus H(\mathsf{K}[x_1] \oplus \Delta_{\mathsf{B}}, \mathsf{K}[y_1] \oplus (y_2 \oplus z_2)\Delta_{\mathsf{B}})$; 2) $x_1 = 1$ and $R' = W_{x_1}^* \oplus H(\mathsf{M}[x_1]) \oplus U \oplus H(\mathsf{K}[x_1] \oplus \Delta_{\mathsf{B}}, \mathsf{K}[z_1] \oplus z_2\Delta_{\mathsf{B}})$, $\mathcal{S}$ sets $g_2 = 1$.
- Otherwise $\mathcal{S}$ aborts.

6 For each value $g \in \{g_0, g_1, g_2\}$, if $g \neq \perp$, $\mathcal{S}$ sends g to $\mathcal{F}_{\mathsf{LaAND}}$. If $\mathcal{F}_{\mathsf{LaAND}}$ abort after any guess, $\mathcal{S}$ aborts.

Note that the first 3 steps are perfect simulations. However, an malicious $\mathsf{P_A}$ can flip the value of y_1 and/or u used. According to the unforgeability proof, the protocol will abort if the relationship $(x_1 \oplus x_2) \wedge (y_1 \oplus y_2) \oplus (z_1 \oplus z_2) = 0$ does not hold. Therefore, if $\mathcal{A}$ flip y_1, it is essentially guessing that $x_1 \oplus x_2 = 0$; if $\mathcal{A}$ flip both y_1 and u, it is guessing that $x_1 \oplus x_2 = 1$. Such selective failure attack is extracted by $\mathcal{S}$ and answered accordingly.

In step 4, U^* is sent in the simulation, while U_{x_2} is sent. This is a perfect simulation unless both of the input to random oracle in U_{x_2} get queried. This does not happen during the protocol, since Δ_{B} in not known to $\mathcal{A}$. In step 5, W_0^*, W_1^* are sent in the simulation, while $W_{x_2,0}, W_{x_2,0}$ are sent in the real protocol. This is also a perfect simulation unless $\mathsf{P_A}$ gets Δ_{B}: both R and one of $H(\mathsf{K}[x_1])$ and $H(\mathsf{K}[x_1] \oplus \Delta_{\mathsf{B}})$ are random.

Another difference is that $\mathsf{P_B}$ always aborts in the simulation if G_{x_2,y_2} is not honestly computed. This is also the case in the real protocol unless $\mathcal{A}$ learns Δ_{B}. □

Lemma A.4. *The protocol in Figure 8 securely implements the functionality in Figure 6 against corrupted* $\mathsf{P_B}$ *in the* $(\mathcal{F}_{\mathsf{abit}}, \mathcal{F}_{\mathsf{HaAND}}, \mathcal{F}_{\mathsf{EQ}})$*-Hybrid model.*

Proof. We will construct a simulator as follows:

1. $\mathcal{S}$ interacts with $\mathcal{A}$ and receive $(x_2, \mathsf{M}[x_2])$, $(y_2, \mathsf{M}[y_2])$, $(r, \mathsf{M}[r])$, $\mathsf{K}[x_1]$, $\mathsf{K}[y_1]$, $\mathsf{K}[z_1]$, Δ_{B} that $\mathcal{A}$ sent to $\mathcal{F}_{\mathsf{abit}}$. $\mathcal{S}$ picks a random bit s, sets $(z_2, \mathsf{M}[z_2]) := (r \oplus s, \mathsf{M}[z_2] \oplus s\Delta_{\mathsf{B}})$, and sends $(x_2, \mathsf{M}[x_2])$, $(y_2, \mathsf{M}[y_2])$, $(z_2, \mathsf{M}[z_2])$, $\mathsf{K}[x_1]$, $\mathsf{K}[y_1]$, $\mathsf{K}[z_1]$) to $\mathcal{F}_{\mathsf{LaAND}}$, which sends $(x_1, \mathsf{M}[x_1])$, $(y_1, \mathsf{M}[y_1])$, $(z_1, \mathsf{M}[z_1])$, $\mathsf{K}[x_2]$, $\mathsf{K}[y_2]$, $\mathsf{K}[z_2]$) to $\mathsf{P_B}$.

2-3 $\mathcal{S}$ plays the role of $\mathcal{F}_{\mathsf{HaAND}}$ and obtains y_2' $\mathcal{A}$ sent. $\mathcal{S}$ also obtains d' sent by $\mathsf{P_B}$. Denoting y_2', d as values an honest $\mathsf{P_B}$ would use, if $y_2' \neq y_2, d' \neq d$, $\mathcal{S}$ sets $g_0 = 1 \oplus x_2$, if $y_2' \neq y_2, d' = d$, $\mathcal{S}$ sets $g_0 = x_2$.

4-6 Note that step 4 and step 5 of the protocol are the same with the exception that the roles of $\mathsf{P_A}$ and $\mathsf{P_B}$ are switched. We denote S' the simulator that was defined for the case where $\mathsf{P_A}$ is corrupted. $\mathcal{S}$ will employ in step 4 the same strategy that was employed by S' in step 5. $\mathcal{S}$ will employ in step 5, the same strategy that was employed by S' in step 4.

The first three steps are perfect simulation, with a malicious $\mathsf{P_B}$ having a chance to perform a selective failure attack similar to when $\mathsf{P_A}$ is malicious. If $\mathsf{P_B}$ flip y_2, it is guessing that $x_1 \oplus x_2 = 0$; if $\mathsf{P_B}$ flip y_2 and d, $\mathsf{P_B}$ is guessing $x_1 \oplus x_2 = 1$. The proof for step 4 and 5 are the same as the proof for malicious $\mathsf{P_A}$ (with order of steps switched). □

A.4 More optimizations.

Note that the protocol description in Figure 8 does not include all possible optimizations for ease of understanding. In the following we will briefly discuss additional optimizations.

1. For clarity, R was chosen randomly in Π_{LaAND}. It is possible to perform garbled row reduction so that $W_{0,0}, W_{1,0}$ are zero. This saves two ciphertexts per leaky AND.
2. Only ρ bits of the R and U values need to be sent.