BVOT: Self-Tallying Boardroom Voting with Oblivious Transfer

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***Abstract*—A *boardroom election* is an election with a small number of voters carried out with public communications. We present BVOT, a self-tallying boardroom voting protocol with ballot secrecy, fairness (no tally information is available before the polls close), and dispute-freeness (voters can observe that all voters correctly followed the protocol).**

BVOT works by using a multiparty threshold homomorphic encryption system in which each candidate is associated with a masked unique prime. Each voter engages in an oblivious transfer with an untrusted *distributor*: the voter selects the index of a prime associated with a candidate and receives the selected prime in masked form. The voter then casts their vote by encrypting their masked prime and broadcasting it to everyone. The distributor does not learn the voter's choice, and no one learns the mapping between primes and candidates until the audit phase. By hiding the mapping between primes and candidates, BVOT provides voters with insufficient information to carry out effective cheating. The threshold feature prevents anyone from computing any partial tally—until everyone has voted. Multiplying all votes, their decryption shares, and the unmasking factor yields a product of the primes each raised to the number of votes received.

In contrast to some existing boardroom voting protocols, BVOT does not rely on any zero-knowledge proof; instead, it uses oblivious transfer to assure ballot secrecy and correct vote casting. Also, BVOT can handle multiple candidates in one election. BVOT prevents cheating by hiding crucial information: an attempt to increase the tally of one candidate might increase the tally of another candidate. After all votes are cast, any party can tally the votes.

keywords: Applied cryptography, boardroom voting, election systems, oblivious transfer.

I. Introduction

Many of the proposed high-integrity election systems depend on an election authority to administer elaborate procedures or require voters to carry out complex steps, such as executing and checking *zero-knowledge proofs (ZKPs)*. We propose a new boardroom voting protocol, BVOT, which is self-tallying (any voter can tally the votes) and is based on *oblivious transfer (OT)*.

We focus on electronic boardroom voting, such as that carried out at a meeting of shareholders or a board of directors. A *traditional boardroom election* is an election that takes place in a single room, where the voters can see and hear each other [1]. The election is conducted by an untrusted party, who could be a voter, also present in the room. We address an *electronic version of boardroom elections* in which a small number of voters participate in person or remotely through web browsers or applications, in the absence of a substantial election authority. The small scale of boardroom elections permits the use of protocols and cryptographic primitives that might be impractical at large scale. Examples of boardroom voting include [2], [3], [4], [5]. By contrast, many other voting systems require substantial election authorities (e.g., [6], [7], [8], [9], [10], [11], [12]).

We seek an electronic boardroom voting system that provides each of the following properties [4], [2]:

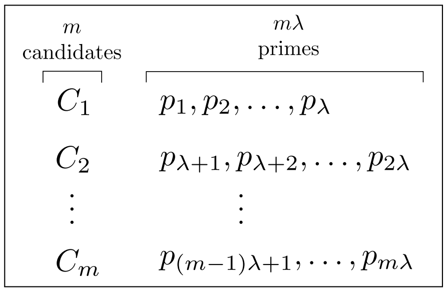
- **Fairness.** None of the voters can learn a complete or partial tally of the votes before casting their vote.
- **Dispute-freeness.** Each voter can observe if the other voters have carried out the the protocol correctly.
- **Perfect ballot secrecy.** How each voter voted remains secret during and after the election, and a partial tally of the votes of any subset of voters is possible only with the collaboration of all of the other voters.
- **Self-tallying.** Any voter can compute the tally.

The main novel feature of BVOT is its use of OT to provide perfect ballot secrecy and ensure correct vote casting. Doing so avoids the need for voters to carry out or verify complex ZKPs. Given that Kilian [13] proved that oblivious transfer is complete for two-party secure computations, it is intriguing to explore applications of OT in voting. Nurmi et al. [14] also used OT in an election system, but only to distribute credentials to voters. BVOT can handle multiple candidates in one election. By contrast, [2], [3], [4], [5] require extensions with additional performance costs to handle multiple candidates.

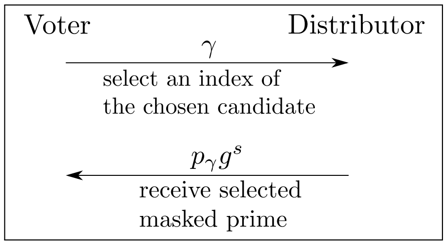
As shown in Figure 1, BVOT works by using a multiparty threshold homomorphic encryption system in which each candidate is associated with a masked unique prime. Each voter engages in an OT with an untrusted *distributor*: the voter selects the index of a prime associated with a candidate and receives the selected prime in masked form. The distributor is untrusted both for privacy and integrity. The voter then casts their vote by encrypting their masked prime and broadcasting it to everyone. The distributor does not learn the voter's choice, and no one learns the mapping between primes and candidates until the audit phase.

By hiding the mapping between primes and candidates, BVOT provides voters with insufficient information to carry out effective cheating. The threshold feature prevents anyone from computing any partial tally—until everyone has voted. Multiplying all votes, their decryption shares, and the unblinding factor yields a product of the primes each raised to the number of votes received. The small size of a boardroom election enables everyone to factor this product. BVOT's novel use of primes to represent ballot choices enables BVOT to handle multiple candidates without extensions.

Our contribution is a remote self-tallying boardroom voting protocol based on OT that is fair and dispute free and enjoys perfect ballot secrecy.


m candidates
$m\lambda$ primes
C_1
$p_1, p_2, \ldots, p_\lambda$
C_2
$p_{\lambda+1}, p_{\lambda+2}, \ldots, p_{2\lambda}$
C_m
$p_{(m-1)\lambda+1}, \ldots, p_{m\lambda}$

(a) Prime-to-candidate mapping. Each prime is associated with a candidate.


Voter
Distributor
γ
select an index of the chosen candidate
$p_\gamma g^s$
receive selected masked prime

(b) Oblivious transfer between voter and distributor.

Fig. 1: In BVOT, there are λ primes associated with each of the m candidates. Each voter selects the index γ of an unknown prime p_γ associated with their chosen candidate and, through an $\text{OT}_1^{\lambda m}$ with an untrusted distributor, receives the selected prime in masked form $p_\gamma g^s$. The distributor does not learn the voter's choice, and no one learns the mapping between primes and candidates until the audit phase.

II. Oblivious Transfer

We briefly review our main building block—*oblivious transer (OT)*—including selected OT protocols and their security and efficiency.

First introduced by Rabin [15], an OT protocol enables a receiver to receive a piece of information from a sequence of pieces of information from a sender, while hiding the selection of information from the sender and hiding the rest of the information from the receiver. Formally, in 1-out-of-2 OT, denoted OT_1^2, the sender has two strings s_0, s_1 and transfers s_b to the receiver, where the receiver selects $b \in \{0, 1\}$ and the following two conditions hold: (1) the sender does not know the value of b, and (2) the receiver does not learn anything about s_{1-b}.

We will use a generalization 1-out-of-n OT, denoted OT_1^n: the sender has n strings and transfers one string to the receiver, without knowing which string it transferred, and the receiver does not learn anything about the other $n-1$ strings.

An *ideal implementation* of OT uses a trusted third party: after obtaining the strings from the sender, and the index choice from the receiver, the trusted party sends the chosen string to the receiver.

OT can be implemented using public-key cryptography without a trusted third party. For example, OT_1^2 can be implemented as follows: the receiver creates two random public keys but knows the private key corresponding to only one of them. The receiver sends the two public keys to the sender. The sender encrypts each string with a different public key and sends the resulting ciphertexts to the receiver. Because the receiver knows the private key corresponding to only one of the public keys, the receiver will be able to decipher only one of the strings, and the receiver will learn nothing about the other string.

Implementing OT with public-key operations, however, is computationally expensive. Seeking faster implementations, researchers have explored the possibility of implementing OT using symmetric-key cryptography, but Impagliazzo and Rudich [16] showed that it is unlikely to find black-box constructions of OT using one-way functions.

Seeking greater efficiency, Bellare and Micali [17] created an OT_1^2 that requires two rounds. Naor and Pinkas [18] reduced the number of exponentiations during run-time in Bellare and Micali from two to one on the sender's side. They also extended OT_1^2 to OT_1^n. In this OT_1^n technique, the sender performs n exponentiations in the initialization step, and uses the resulting values for all subsequent transfers.

Noar and Pinkas [19] showed how to extend an OT_1^2 protocol to an OT_1^n protocol—with $O(n \log n)$ calls to OT_1^2—that provides sender and receiver security computationally, if the underlying OT_1^2 provides sender and receiver security (see Section VI for definitions). Among the most efficient OT protocols that are secure against active adversaries (including possibly sender and/or receiver) are [20], [21], [22]. In BVOT, vote correctness and ballot secrecy rely on the security of OT, so it is crucial that the OT is secure against an active adversary.

Because there are few voters in BVOT, the performance of the underlying OT is tolerable: BVOT uses one OT_1^n for each voter, and these OT_1^ns can be executed in parallel. Chou and Orlandi [21] computed more than $10,000$ OT_1^2s per second using one thread of an Intel Core i7-3537U processor. Even with the overhead of building each OT_1^n from OT_1^2s using Noar and Pinkas's technique, the OT_1^ns in BVOT can be executed sufficiently quickly.

III. Previous and Related Work

We briefly review selected self-tallying boardroom voting protocols that provide ballot secrecy. We also review selected protocols that use OT as a primary building block.

A. Self-Tallying Boardroom Voting Protocols

Kiayias and Yung [2] proposed the first self-tallying boardroom voting protocol with perfect ballot secrecy. Groth [3] simplified the protocol and reduced its computational complexity, preserving the same security properties. Hao et al. [4] proposed a similar self-tallying, dispute-free protocol with perfect ballot secrecy that needs only two rounds of communication.

In the protocols of Hao et al. and Groth, for each $1 \leq i \leq n$, voter V_i chooses a vote $v_i \in \{0, 1\}$ (1 for "yes" and 0 for "no") and computes g^{v_i}, where g is a generator of a group in which the Diffie-Hellman assumption holds [23]. Each voter publishes $g^{v_i} A_i$ as their masked vote, for some secret masking value A_i. Hao generates A_i differently than does Groth. In each protocol, the A_i's are chosen so that $\prod_{i=1}^{n} A_i = 1$. Therefore, $\prod_{i=1}^{n} g^{v_i} A_i = g^{\sum v_i}$. The number of "yes" votes is $\sum v_i$. Because this sum is small, it can be easily calculated given $g^{\sum v_i}$, even assuming the discrete logarithm problem is hard.

In Hao et al. and Groth, using a ZKP, each voter proves that they voted correctly by proving $v_i \in \{0, 1\}$ (and not, for example, $v_i = 2$). The correctness of their protocols depends on this ZKP. By contrast, BVOT does not require such proofs.

The main difference between the protocols of Hao et al. and Groth is how they compute $\prod_{i=1}^{n} g^{v_i} A_i$. In Hao et al., voters publish their masked votes in one round, enabling this product to be computed immediately. In Groth, voters compute the product sequentially. Consequently, Hao et al.'s protocol requires two rounds of communication, whereas Groth requires $n{+}1$ rounds. Our protocol is similar to Hao et al.'s and requires only five rounds of communication.

Szepieniec and Bart [24] proposed a protocol similar to Kiayias and Yung that also provides fairness. Giustolisi et al. [25] showed how to reuse the key-sharing round in Hao et al., reducing the number of rounds to one in all subsequent elections. Adding a commitment round to Hao et al., Khader et al. [5] achieve fairness, and, by introducing a recovery round, they achieve robustness (Section VI-F).

In each of these protocols, because each vote is 0 or 1, the protocol must be executed once for each candidate. Cramer et al. [26] proposed a technique for handling multiple candidates in one protocol execution by using independent generators for the underlying group, one per candidate. Their technique complicates the ZKPs. By contrast, without any ZKPs, BVOT supports multiple candidates in one protocol execution by using a separate small prime integer for each candidate.

B. *Applications of Oblivious Transfer*

OT is a powerful primitive that can be used alone to implement any secure two-party [13] and multiparty [27] secure computation. We briefly point out three examples.

Nurmi et al. [14] used OT_1^n to enable a trusted election authority to distribute a credential to each of the n voters, such that the election authority does not know the credential of any voter. Each voter uses their credential to cast their ballot. OT prevents the election authority from linking voters and their ballots. By contract, BVOT does not depend on a trusted third party, and BVOT uses OT to prevent cheating and to hide primes associated with candidates.

Even et al. used OT to sign contracts [28]. Two parties use OT_1^2 to exchange secrets, where knowledge of the other's secret implies their commitment to the contract. Here, OT guarantees: each party sends its secrets correctly, and both parties simultaneously exchange their secrets.

Fagin et al. [29] used OT to enable two parties to compare their secrets without revealing them (e.g., a user wants to prove their identity using a password but does not trust the medium).

IV. The BVOT Voting System

We explain the BVOT self-tallying boardroom voting system by describing the election organization, our adversarial model, the underlying homomorphic encryption system, the voting protocol, and a small example.

A. *Election Organization*

There is a set of untrusted voters, and one of them is designated as the *distributor*. The distributor generates certain values and uses OT to distribute them to the other voters. After all votes are cast and published, anyone—including voters and non-voters—can tally the votes. Instead of using a bulletin board to post information, for simplicity, BVOT relies on broadcasting messages.

B. *Adversarial Model*

Voters communicate with each other from trusted machines (running a trusted app) over an authenticated channel. A covert polynomial-time adversary (possibly a voter) listens to all communications.

The adversary and voters are cautious: they will follow the protocol and they do not want to be caught misbehaving, but they may try to cheat in favor of some candidate or to learn how others voted. Subject to this constraint, the adversary can behave actively, including as sender or receiver of an OT.

The adversary cannot break standard cryptographic functions. We assume that the adversary does not try to sabotage or delay the election. In particular, in the spirit of boardroom elections, we assume that none of the voters will intentionally lie (e.g., make a false claim of malfeasance) for the purpose of discrediting the election.

C. *The Homomorphic Encryption System*

BVOT uses a multiparty threshold encryption system based on ElGamal, similar to that described by Benaloh [30]. Voters agree on a multiplicative group $G = \mathbb{Z}_q^*$ of order $q - 1$, with generator g, for which group the Diffie-Hellman problem is hard [23]. Here, q is a large prime such that $q - 1$ has at least one large factor.

a) Key Generation.: Each voter V_i chooses a private key $d_i \in_R G$ at random and sends g^{d_i} to all other voters. The product $e = \prod_{i=1}^{n} g^{d_i}$ is the *public key of the set of n voters*. These keys can be used for multiple elections.

b) Encryption.: To encrypt any message M using key e, voter V_j computes $\mathcal{E}_{V_j}(M) = (g^{x_j}, Me^{x_j})$, where $x_j \in_R G$ is a value chosen randomly by V_j.

c) Decryption.: For ciphertext (g^{x_j}, Me^{x_j}), the *decryption share* of voter V_i is $(g^{x_j})^{-d_i}$. To decipher this ciphertext, multiply Me^{x_j} by all decryption shares for this ciphertext:

$$\begin{aligned} Me^{x_j} \prod_{i=1}^{n} (g^{x_j})^{-d_i} &= M \prod_{i=1}^{n} \left(g^{d_i}\right)^{x_j} \prod_{i=1}^{n} (g^{x_j})^{-d_i} \\ &= Mg^0 = M. \end{aligned} \tag{1}$$

D. Boardroom Voting Protocol

We describe the four steps of the voting protocol in detail, in which n voters choose among m candidates. All cryptographic operations take place in a multiplicative group G, as defined in Section IV-C. For each positive integer k, let $\mathbb{Z}_k = \{0, 1, 2, \ldots, k-1\}$.

Step 1. *Election Setup*

a) The voters select a distributor D.

b) The voters agree on a parameter λ, which denotes the number of primes to represent each candidate (having multiple primes per candidate mitigates certain attacks by creating uncertainty which prime a voter selected).

c) D chooses any λm primes $p_1, p_2, \ldots p_{\lambda m}$ such that $b^n < q$, where $b = \max(p_1, p_2, \ldots, p_{\lambda m})$. Hence, the product of any n of the primes is less than q.

d) Using a standard numbering system, D associates the index of each of the primes with some candidate. Let $\mathcal{M} : \mathbb{Z}_{\lambda m} \to \mathbb{Z}_m$ denote this mapping. In particular, primes $p_1, p_2, \ldots, p_\lambda$ are associated with Candidate 1; primes $p_{\lambda+1}, p_{\lambda+2}, \ldots, p_{2\lambda}$ are associated with Candidate 2; and so forth.

e) D selects a mask $s \in_R G$ at random. D commits to $\mathcal{M}$ and s (e.g., by publishing a cryptographic hash of these values).

f) D masks each prime p as pg^s.

Masking the primes mitigates certain attacks involving substituting one prime for another, in hopes of favoring some candidate: the adversary does not know which prime would affect which candidate. Furthermore, the distributor knows the mapping but not which primes are used in the votes, in part because there are multiple primes per candidate.

As an example of an election setup, consider an election with 128 voters held using a group with q having 2048 bits. If $b < 2^{16}$, then there would be 6542 primes available to represent the candidates. In this example, λ can be chosen such that $\lambda m > 128$ (see Section VI-B).

Step 2. *Vote Selection*

a) To select their candidate, each voter engages in an $\text{OT}_1^{\lambda m}$ with D as the sender using the λm masked primes. The candidate selects an index of a prime associated with their candidate, as given by $\mathcal{M}$. The candidate receives the corresponding masked prime.

As a result of the OT, D does not learn the voter's selection.

Step 3. *Vote Casting*

a) Each non-distributor voter V_i publishes the encryption of their chosen masked prime $p_{r_i} g^s$, encrypted with the public key e of the set of voters: $\mathcal{E}_{V_i}(p_{r_i} g^s) = (g^{x_i}, (p_{r_i} g^s) e^{x_i})$, where $x_i \in_R G$ is chosen at random.

b) The distributor $D = V_\delta$ publishes the encryption of their unmasked prime: $\mathcal{E}_{V_\delta}(p_{r_\delta}) = (g^{x_\delta}, p_{r_\delta} e^{x_\delta})$.

Encryption prevents anyone from learning the chosen primes, even after s is eventually revealed.

Step 4. *Vote Tallying*

a) After all votes are cast, D broadcasts $\left(\prod_{k=1}^n g^{x_k}\right)^{-d_\delta}$, which is its decryption share of the product of the g^k values from the encrypted votes from all other voters. Then, each other voter V_i broadcasts $\left(\prod_{k=1}^n g^{x_k}\right)^{-d_i}$, which is its decryption share of this product.

b) D publishes $\mathcal{M}$ (defined in terms of masked primes). From this value, each voter verifies that they received a masked prime that corresponds to their chosen candidate. This step gives each voter an opportunity to file an allegation that they received the wrong masked prime—before the voter learns the tally.

c) D publishes $g^{-(n-1)s}$ and s.

d) Anyone can tally the votes by calculating and factoring the product P of all encrypted votes, the encryption shares, and $g^{-(n-1)s}$. This product is the product of the selected candidate primes, each raised to the number of votes for that prime. That is,

$$\begin{aligned} P = &\left((p_{r_\delta} e^{x_\delta}) \prod_{\substack{i=1 \\ i \neq \delta}}^{n} (p_{r_i} g^s) e^{x_i} \right) \\ &g^{-(n-1)s} \left(\prod_{k=1}^{n} g^{x_k} \right)^{-d_\delta} \prod_{\substack{i=1 \\ i \neq \delta}}^{n} \left(\prod_{k=1}^{n} g^{x_k} \right)^{-d_i} \\ =& p_{r_1} p_{r_2} \cdots p_{r_n} \\ =& p_1^{a_1} p_2^{a_2} \cdots p_{\lambda m}^{a_{\lambda m}}. \end{aligned} \quad (2)$$

The sum of the votes is $a_1 + a_2 + \cdots + a_{\lambda m} = n$, and no candidate prime can be selected more than n times: $0 \leq a_1, a_2, \ldots, a_{\lambda m} \leq n$. Because boardroom elections involve a small number of voters, the primes $p_1, p_2, \ldots, p_{\lambda m}$ and their powers are small. Therefore, P can be easily factored.

E. An Example with Three Candidates and Four Voters

We illustrate BVOT in an election in which four voters (V_1, V_2, V_3, V_4) select among three candidates 1,2,3. Assume V_4 is the distributor D.

For this example, we will represent each candidate by $\lambda = 3$ distinct primes (see Section VI-B). Let these distinct primes be $p_1, p_2, \ldots, p_9$. Distributor D randomly partitions the list of primes to associate three primes with each candidate, cryptographically commits to this association, but does not immediately reveal the association. D selects a mask $s \in_R G$ at random and masks each prime p by computing pg^s.

Using the following standard numbering system, Candidate 1 is associated with indices 1,2,3; Candidate 2 is associated with indices 4,5,6; and Candidate 3 is associated with indices 7,8,9.

Using the standard numbering system described above, each voter selects a candidate by engaging in an OT_1^9 with D to receive one of the nine masked primes. For example, to select Candidate 2, the voter would select index 4, 5, or 6 in the OT. As a result of the OT, the voter receives the masked prime corresponding to their chosen index. D does not learn the selected index, and the voter learns only the selected masked prime.

For each $1 \leq i \leq 3$, let $p_{r_i} g^s$ denote the masked prime received by V_i. Each voter encrypts their masked prime with the public key e of the set of voters. Each voter publishes the resulting ciphertext by sending it to the other voters. For example, V_1 publishes $\mathcal{E}_{V_1}(p_{r_1} g^s) = (g^{x_1}, (p_{r_1} g^s) e^{x_1})$. Similarly, D publishes the encryption of their selected unmasked prime p_{r_4}.

After all encrypted votes are published, D publishes $(\prod_{k=1}^{4} g^{x_k})^{-d_4}$, which is its decryption share of the product of the g^k values from the encrypted votes from the other voters. Next, each of the other voters publishes their decryption share of this product. For example, V_1 publishes $(\prod_{k=1}^{4} g^{x_k})^{-d_1}$.

Finally, D publishes the prime-to-candidate mapping, g^{-3s}, and s. From the published values, anyone can compute the tally by decrypting and unmasking the selected masked primes and computing their product. Specifically, the product, P, of all published encrypted masked primes, decryption shares, and g^{-3s} yields the tally encoded as a product of the nine candidate primes, each raised to the number of votes for that prime. To wit,

$$\begin{aligned} P =&((p_{r_1} g^s) e^{x_1})((p_{r_2} g^s) e^{x_2})((p_{r_3} g^s) e^{x_3})(p_{r_4} e^{x_4}) \\ &(\prod_{k=1}^{4} g^{x_k})^{-d_1}(\prod_{k=1}^{4} g^{x_k})^{-d_2}(\prod_{k=1}^{4} g^{x_k})^{-d_3} \\ &(\prod_{k=1}^{4} g^{x_k})^{-d_4}(g^{-3s}) \\ =&p_{r_1} p_{r_2} \dots p_{r_4} = p_1^{a_1} p_2^{a_2} \dots p_9^{a_9}, \end{aligned} \tag{3}$$

where, for each $1 \leq i \leq 9$, a_i is the number of votes for prime p_i. Thus, $a_1 + a_2 + \cdots + a_9 = 4$ is the sum of the votes. In particular, if the primes for Candidate 1 were p_3, p_4, p_9, then Candidate 1 would have received $a_3 + a_4 + a_9$ votes.

V. Communication and Performance Analysis

We analyze and compare the computational and communication complexity of BVOT with that of selected other self-tallying boardroom voting protocols. Our analysis considers one election with n voters and m candidates, using security parameter λ.

The key-generation step requires one exponentiation per voter. Each voter performs one homomorphic encryption and computes the product of n decryption shares; each of these steps requires one exponentiation. BVOT performs $n-1$ parallel sessions of $\mathrm{OT}_1^{\lambda m}$.

The distributor makes one commitment to the mapping $\mathcal{M}$. In addition to their role as a voter, the distributor performs one more exponentiation to compute $g^{-(n-1)s}$.

BVOT requires five broadcast rounds, three of which are performed solely by the distributor: (1) Each voter broadcasts their public keys. (2) Each voter broadcasts their encrypted vote. (3a) The vote distributor broadcasts its decryption share. (3b) Each voter broadcasts their decryption share. (4a) The vote distributor broadcasts $\mathcal{M}$. (4b) The vote distributor broadcasts $g^{-(n-1)s}$ and s.

Table I compares the computational and communication complexity of BVOT to that of four other selected self-tallying boardroom voting protocols.

TABLE I: Communication and performance of selected self-tallying boardroom voting protocols, for one election with n voters.

	Protocol				
Metric	KY [2]	G [3]	HRZ [4]	KSRH [5] *	BVOT
OT sessions	0	0	0	0	$n-1$
Exponentiations	$2n^2+2n$	$4n$	$2n$	$2n$	$3n$
ZKPs	$2n^2+2n$	$4n$	$2n$	$2n$	0
Broadcast rounds	3	$n+1$	2	3	5

* without robustness

BVOT has significantly better performance than [2] and [3] in that [2] requires a quadratic number of exponentiations and ZKPs, and [3] requires a linear number of broadcast rounds, as a function of the number of voters. BVOT is roughly similar in performance to [4] and [5], with ZKPs replaced with OTs.

BVOT supports multiple candidates, whereas [2], [3], [4], [5] require extensions with additional costs to support multiple candidates. BVOT and [5] are fair, but [2], [3], [4] are not. In Table I, we used a non-robust version of [5] because none of the other protocols are robust; the robust version of [5] requires additional costs.

VI. Security Notes

We discuss the security properties of BVOT, including dispute freeness, perfect ballot secrecy, fairness, robustness, and coercion resistance. First, we give definitions of receiver and sender security for OT, and we explain implications of choosing the parameter λ. Table II summarizes our security comparison of BVOT with four selected other self-tallying boardroom voting systems.

A. Security of Oblivious Transfer

Adapting definitions from Naor and Pinkas [19], we state definitions of receiver and sender security for OT. Receiver security means that the sender does not learn the receiver's choice of strings.

Definition 1: Receiver security in OT. An OT_1^m provides receiver security if and only if, for any probabilistic polynomial-time sender $\mathcal{A}$ with m strings $s_1, s_2, \dots s_m$, given any $1 \leq i < j \leq m$ where the receiver chose either s_i or s_j, $\mathcal{A}$ cannot distinguish whether the receiver chose s_i or s_j.

Sender security means that the receiver does not learn anything other than the string they chose. Sender security is defined in terms of a comparison between the information the receiver learns in the ideal implementation of oblivious transfer and the information the receiver learns in the real implementation.

Definition 2: Sender security in OT. An $\mathrm{OT}_1^{\lambda m}$ provides sender security if and only if, for every probabilistic polynomial-time receiver $\mathcal{B}$, substituting $\mathcal{B}$ in the real implementation of the protocol, there exists a probabilistic polynomial-time machine $\mathcal{B}'$ for the receiver's role in the ideal implementation such that, for every sequence of strings $s_1, s_2, \dots s_{\lambda m}$ of the sender, the outputs of $\mathcal{B}$ and $\mathcal{B}'$ are computationally indistinguishable.

Because secure OT protocols (Definitions 1 and 2) are secure against malicious senders and receivers [20], [21], [22], BVOT sufficiently handles malicious senders and receivers in the OT protocol.

B. Choosing the Number of Primes for a Candidate

The parameter λ specifies the number of primes associated with each candidate. Its purpose is to mitigate the threat that the distributor might try to reduce some candidate's votes and increase some other candidate's votes (without changing the total number of votes) by exploiting its knowledge of the mapping $\mathcal{M}$. Choosing λ such that $\lambda m/n > 1$ is sufficiently large protects against this threat. With this choice of λ, the distributor does not know the voter's selected prime. If the distributor reduces the votes for an unchosen prime, the distributor will be caught.

C. Dispute Freeness

BVOT is dispute free for the following reasons. All communications are public and authenticated, and the threshold feature of the encryption scheme ensures that all votes are included in the tally.

Manipulating the vote counts without detection requires knowledge of at least two distinct primes chosen by voters (e.g., voting with $p_1^2 p_2^{-1}$ rather than with p_1). This knowledge is hidden from the voters through the OT and masking.

If two voters collude by sharing their masked primes, they could unmask their primes (see Section VII-C). Assuming that λ is chosen appropriately, the colluding voters would risk detection if they tried to cheat by adding more than one vote to one of their candidates and subtracting one vote from the other—a result they could have achieved directly without cheating.

The following elements prevent the distributor from engaging in malfeasance without detection: there are λ primes per candidate, so the distributor does not know which primes were chosen. Furthermore, the distributor commits to his vote, mask, and the mapping $\mathcal{M}$.

D. Perfect Ballot Secrecy

BVOT enjoys perfect ballot secrecy for the following reasons. First, the OT hides each voter's candidate selection. Second, each voter encrypts their ballot. The following theorem can be proven:

Theorem 1: Assuming that $\mathrm{OT}_1^{\lambda m}$ provides receiver security and the Diffie-Hellman assumption holds in the underlying group G, for each $1 \leq i \leq n$, voter V_i's ballot $p_{r_i} g^s e^{x_i}$ is indistinguishable from a randomly chosen element in G.

E. Fairness

The threshold property of the encryption scheme ensures that none of the voters can learn anything about the tally until all of the voters publish their decryption share and the distributor publishes the unmasking value. Each voter casts their vote before any voter publishes their decryption share. Thus, BVOT is fair.

F. Robustness

As is true for protocols [2], [3], [4], BVOT is not robust as defined below.

- **Robustness** ensures that the protocol can complete, even if one or more voters attempt to prevent the protocol from completing.

Due to the threshold feature of the encryption scheme, any one voter can prevent anyone from computing the tally by withholding their vote. More generally, Kiayias et al. [2] claimed that no self-tallying voting system that provides ballot secrecy is robust.

G. Coercion Resistance

As is true for protocols [2], [3], [4], [5], BVOT is not coercion resistant [31] as defined below.

- **Coercion resistance** guarantees that a voter cannot prove to the adversary that they followed the adversary's demands.

BVOT is not coercion resistant because a voter can prove how they voted by releasing their masked vote and the random value used in the encryption scheme.

TABLE II: Security properties of selected self-tallying boardroom voting protocols.

	Protocol				
Property	KY [2]	G [3]	HRZ [4]	KSRH [5]	BVOT
Fair	×	×	×	✓	✓
Perfect ballot privacy	✓	✓	✓	✓	✓
Self-tallying	✓	✓	✓	✓	✓
Dispute-free	✓	✓	✓	✓	✓
Robust	×	×	×	✓	×
Coercion resistant	×	×	×	×	×

BVOT has strictly better security properties than [2], [3], [4] in that [2], [3], [4] are not fair. BVOT and [5] have similar security properties, except that [5] is robust. None of the protocols in Table II achieve the challenging property of coercion resistance.

H. Disruptive or Dishonest Voters

It is possible that one or more of the voters (including the distributor) do not faithfully follow the protocol, perhaps in an attempt to cheat or disrupt the election. We discuss three examples of such behaviors. In each case, the disruptive or dishonest behavior can be detected, though the source of the behavior cannot necessarily be determined.

First, the distributor could misbehave, for example, by giving all voters the same masked prime or a different prime from the one specified by the mapping. Before tallying the votes, the other voters could detect such behaviors by examining the revealed mapping and its commitment. It might, however, be impossible to determine whether the distributor misbehaved or the voter lied in reporting malfeasance.

Second, one or more voters might generate e, encrypt, or decrypt incorrectly. Anyone could detect such behaviors by attempting to compute the tally.

Third, instead of voting, for example, with a prime p_1, a voter could attempt to cheat by voting with $p_1^2 p_2^{-1}$, where the voter is uncertain whether anyone else voted with $p_2 \neq p_1$. If no one voted for p_2, then anyone could detect this behavior by noticing that the tally includes a vote of -1 for p_2. It might, however, be impossible to determine which voter caused this anomaly.

As explained in Section IV-B, in the spirit of boardroom elections, we assume that voters will not intentionally engage in such behaviors for the purpose of disrupting the election.

VII. Discussion

We discuss selected major design decisions, explain a variation of BVOT that uses an EA but is still self-tallying, point out an observation about masked primes, and list some open problems.

A. Major Design Decisions

Two of our major design decisions were the following. (1) Instead of relying on each voter to engage in a complex ZKP of vote correctness, we use OT to hide each voter's ballot choice and to provide the voter with the minimum information needed to cast their ballot. (2) We use multiparty threshold homomorphic encryption so that computing the vote tally is possible only when all votes are included.

B. BVOT with Election Authority

BVOT can also be used with a non-voting EA, in which the EA performs the administrative steps of the distributor. If the EA is untrusted (and might exfiltrate secrets), then λ must be chosen as before. If the EA is trusted not to exfiltrate secrets, then $\lambda = 1$ may be chosen. With $\lambda = 1$, BVOT runs faster, performing n sessions of OT_1^m instead of n sessions of $\text{OT}_1^{\lambda m}$.

C. An Observation about Masked Primes

It is important that each voter does not learn more than one masked prime. Otherwise, they could determine the mask value g^s by computing $\gcd(p_i g^s, p_j g^s)$, where $p_i g^s$ and $p_j g^s$ are two distinct masked primes (Section IV-D).

Therefore, one cannot directly substitute a *private information retrieval (PIR)* [32] protocol for OT in BVOT. A straightforward such substitution would not prevent the voter from possibly learning more than one masked prime.

D. Open Problems

Open problems include: (1) Investigate the usability of BVOT. (2) Explore BVOT's possible use (with or without an EA) in particular applications, including as a consensus protocol for a distributed ledger system—for example, Hyperledger Fabric [33]. (3) Design remote boardroom voting protocols that are accountable (no one can make a false claim of malfeasance without detection) and coercion resistant.

VIII. Conclusion

We introduced BVOT, a self-tallying boardroom voting protocol with fairness, perfect ballot secrecy, and dispute-freeness. BVOT is the first boardroom voting protocol to use oblivious transfer to provide these properties. Unlike some existing protocols, BVOT avoids the complex steps of requiring voters to carry out and check ZKPs. BVOT illustrates the power and flexibility of oblivious transfer as a building block in protocol design.

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