

# Bool Network: An Open, Distributed, Secure Cross-chain Notary Platform

Zeyuan Yin, Bingsheng Zhang, Jingzhong Xu, Kaiyu Lu, Kui Ren,

**Abstract**—With the advancement of blockchain technology, hundreds of cryptocurrencies have been deployed. The bloom of heterogeneous blockchain platforms brings a new emerging problem: typically, various blockchains are isolated systems, how to securely identify and/or transfer digital properties across blockchains? There are three main kinds of cross-chain approaches: sidechains/relays, notaries, and hashed time-lock contracts. Among them, notary-based cross-chain solutions have the best compatibility and user-friendliness, but they are typically centralized. To resolve this issue, we present Bool Network – an open, distributed, secure cross-chain notary platform powered by MPC-based distributed key management over evolving hidden committees. More specifically, to protect the identities of the committee members, we propose a Ring verifiable random function (*Ring VRF*) protocol, where the real public key of a VRF instance can be hidden among a ring, which may be of independent interest to other cryptographic protocols. Furthermore, all the key management procedures are executed in the TEE, such as Intel SGX, to ensure the privacy and integrity of partial key components. A prototype of the proposed Bool Network is implemented in Rust language, using Polkadot Substrate.

**Index Terms**—cross-chain, evolving committee, threshold cryptography, ring VRF, trusted execution environment

## I. INTRODUCTION

Following the success of Bitcoin, many blockchain-based cryptocurrencies have been developed and deployed. To meet different requirements in various scenarios, a great number of heterogeneous blockchains have emerged. However, each blockchain platform is an isolated system; therefore, interoperability between blockchains become one of the key issues that prevent the blockchain technology from wide adoption.

To address this problem, many cross-chain solutions have been proposed. A typical cross-chain token transfer from a source blockchain to a target blockchain involves three steps: a. locking (or destroying) the source blockchain token; b. cross-chain transfer commitment; c. creation of a representation of the corresponding token on the target blockchain [4]. There are three main categories: sidechains/relays, notaries, and hashed time-lock contracts. First, sidechains/relays [19], [32] rely on a so-called two-way peg to communicate between the sidechain and the mainchain. However, a sidechain/relay is typically associated with one concrete blockchain ecosystem, and it is not compatible with other blockchains. Second, notaries, such as Binance [8], Coinbase [15] and Huobi Global [22], are mostly centralized exchanges; therefore, this

type of methods inevitably introduces the risk of single-point-of-failure. For example, on 7 May 2019, Binance has revealed a large scale security breach, and over 7,000 Bitcoins have been stolen [24]. Meanwhile, decentralized exchange based solutions, e.g., 0x [38] and Uniswap [1], have been proposed. However, for example, Uniswap only supports Ethereum Request for Comment 20 (ERC-20) trades, and they are less convenient to the users than the conventional centralized notaries. Third, hashed time-lock contracts [33] use hashlocks and timelocks to enforce atomicity of operations, but, by its design, the fluctuation of exchange rate may lead to unfair trade. Specifically, the trader who issues the transaction can watch the exchange rate, if the exchange rate is for him/her, he/she will finish the transaction; otherwise, he/she will refuse to publish the secret and the transaction will abort.

On the other hand, the new concept of “blockchain of blockchains” becomes increasingly popular for application-specific homogeneous blockchains. Those frameworks, such as Polkadot [39] and Cosmos [26], naturally support blockchain interoperability within their own community, but not for the other heterogeneous blockchains.

**Our approach.** In this work, we present Bool Network – an open, distributed, secure cross-chain notary platform. It preserves the advantage of centralized notary solutions, e.g., excellent user-convenience and great blockchain compatibility, while making the platform decentralized and open. More specifically, Bool Network is an open blockchain where hidden committees elected by cryptographic sortition are incentivized to handle the cross-chain operations. The committee members can anonymously communicate over blockchain, while their identities are protected from the public. Initially, the committee members collaboratively create an account on each supported blockchain via multi-party computation (MPC). To transfer a token from chain-A to chain-B, the user first transfers the token to Bool Network’s account on chain-A, and sends the transfer request to the Bool Network platform. The committee then jointly sign (by threshold signature scheme) a transaction on chain-B that issues the representative token to the user’s account on chain-B. The system structure of a notary-based cross-chain scheme is shown in Fig. 1.

To achieve evolving and hidden committee, Benhamouda *et al.* [5] proposed a complicated two-committee scheme, where a public nominating committee is first selected by cryptographic sortition, and then each nominating committee members will randomly select a holding committee member. Alternatively, we present a much more efficient solution based on a new notion, called *Ring VRF* (R-VRF). In Ring VRF, the real public key of a verifiable random function (VRF) instance

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TABLE I: Comparison among cross-chain solutions. Here, we denote openness as the property that any party can freely join/leave the system. By decentralization, we mean the transfer cannot be confirmed by a single entity. Fairness refers to the property that no exchange party can benefit from the fluctuation of exchange rate. For compatibility, “low” means only supporting a certain blockchain; “medium” means supporting a certain type of blockchains; “high” means supporting almost all common blockchains. A bullet sign means some solutions in this type satisfy the property.

Solutions	Openness	Decentralization	Hidden Committee	Fairness	Support Heterogeneous Chains	Compatibility
Sidechains/Relays [19]	●	●	–	✓	✓	low
Centralized Exchanges [8], [15], [22]	✗	✗	–	✓	✓	high
Decentralized Exchanges [38], [1]	✓	✓	–	✓	✓	medium
Hashed time-lock contracts [33]	✓	✓	–	✗	✓	medium
Blockchain of Blockchains [39], [26]	✓	✓	–	✓	✗	medium
Fusion [17]	✗	✓	✗	✓	✓	high
Bool Network (this work)	✓	✓	✓	✓	✓	high

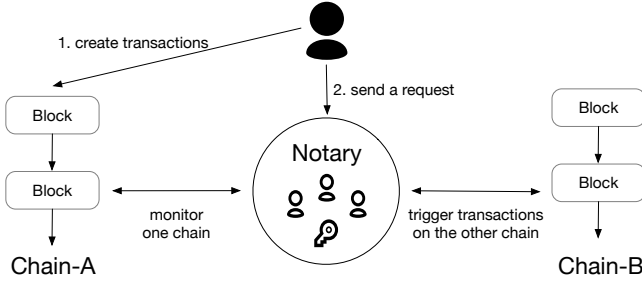


Fig. 1: System structure of a notary-based cross-chain scheme

can be hidden among a ring. More specifically, we design a non-interactive zero knowledge protocol that allow the prover to show that he/she knows the  $sk$  corresponding to one of the public keys among the ring and the output of the pseudorandom function (PRF)  $PRF_{sk}(x)$  is correct. In particular, showing  $PRF_{sk}(x) = v$  w.r.t. the ring  $(pk_0, \dots, pk_{N-1})$ , we have the following relation

$$\mathcal{R}_{VRF} = \left\{ \begin{array}{l} ((pk_0, \dots, pk_{N-1}, x, v), sk, \ell) \mid \\ \ell \in \{0, \dots, N-1\} \wedge (pk_\ell, sk) \in \mathcal{R}_{pk} \wedge \\ v \leftarrow PRF_{sk}(x) \end{array} \right\}$$

**Remark:** Furthermore, to avoid identity leakage from the network-layer, only an anonymous broadcast channel is not enough because the adversary can leverage network delays to identify who is the committee member [25]. A sanitization protocol on the application layer such as SABRE [2], or reliable broadcast mechanisms [9], [10] can be utilized to address this problem, as suggested in [25]. However, this is out of scope of this paper.

In addition, to achieve minimal malware attack interface, we deploy all the key-related programs on a trusted execution environment (TEE), such as Intel SGX, to make sure that even a malicious committee member cannot extract his/her secret shares. Moreover, TEE enforces key share erasure after committee switching. Note that, TEEs can be heterogeneous; namely, the TEEs of the MPC participants can be from different manufactories. Therefore, the threshold cryptosystem used in our scheme ensures that even if side channel attacks against some TEE chips are possible, the adversary cannot

reconstruct the secret as long as the majority of TEEs remain secure.

**Comparison with evolving committee proactive secret sharing.** Evolving committee proactive secret sharing (ECPSS) scheme [5] typically considers proactive/mobile adversary corruption, while the security of the schemes proposed in this paper are proven in the static corruption model. Although our security model is somewhat weaker, it enables much more efficient protocols in practice. In particular, the ECPSS scheme requires two committees, i.e., nominating committee and holding committee, to hide the identities of the holding committee; we only need one committee by utilizing the Ring VRF protocol. Second, in ECPSS the previous-epoch committee member needs to send a proof that the ciphertexts lie on a degree- $(t-1)$  polynomial; In contrast, we let the next-epoch committee member raise a complain if he receives an incorrect share along with a proof of decryption correctness. The NIZK of decryption correctness is much more efficient than the zk proof used in ECPSS.

**Comparison with Fusion.** Fusion [17] is a pioneer that uses distributed control right management (DCRM) to manage the secret keys. In a nutshell, Fusion deploys secret sharing scheme to split the secret keys among multiple nodes, and uses threshold signature scheme to jointly sign the transactions. Compared with our scheme, they have four main drawbacks.

- Fusion’s committee who holds the secret key never changes. The probability that the committee becomes compromised increases along the time.
- The committee members’ identities are publicly known, which makes the system at the risk of coercion or DoS attacks.
- Fusion is not an open platform. Namely, operation nodes cannot freely leave or join Fusion.
- Fusion does not use any hardware guard to protect the secret key. This means that a malicious committee member can easily extract his/her secret shares.

Finally, TABLE I compares popular existing cross-chain schemes with our solution. Here, we denote openness as the property that any party can freely join/leave the system. By decentralization, we mean the transfer cannot be confirmed by a single entity. Moreover, unlike Fusion, the identities of the committee members in Bool Network are hidden from the public; therefore, it offers more security guarantees against

DoS, coercion, and colluding attacks. Fairness refers to the property that no exchange party can benefit from the fluctuation of exchange rate. For compatibility, Sidechains/Relays only support a certain designated mainchain. Decentralized exchanges usually support a particular cryptocurrency token standard, e.g., ERC-20; Hashed time-lock contracts require the blockchain to support smart contracts; Blockchain of blockchains only supports homogeneous blockchains within their community. Therefore, we mark the three aforementioned solutions as medium compatibility. On the contrary, centralized exchanges, Fusion and Bool Network support almost all common blockchains.

## II. PRELIMINARIES

### A. Hybrid Encryption

Our scheme utilizes hybrid encryption over blockchain for peer-to-peer communication. The hybrid encryption scheme HEnc consists of the following 4 PPT algorithms.

- $\sigma \leftarrow \text{Setup}(1^\lambda)$ : take input as security parameter  $\lambda \in \mathbb{N}$ , and output a group parameter  $\sigma$ .
- $(\text{pk}, \text{sk}) \leftarrow \text{KeyGen}(\sigma)$ : pick random  $s \leftarrow \mathbb{Z}_q^*$  and set  $h := g^s$ , and output  $(\text{pk} := (g, h), \text{sk} := s)$ .
- $C \leftarrow \text{Enc}_{\text{pk}}(\sigma; m)$ : pick random  $r \leftarrow \mathbb{Z}_q$ ; compute  $c_1 := g^r$  and  $c_2 := h^r$ ; set  $k \leftarrow \text{hash}(c_2)$ ; compute  $u := \text{AES-GCM}_k(m)$ ; output  $C = (c_1, u)$ .
- $m \leftarrow \text{Dec}_{\text{sk}}(\sigma; C)$ : compute  $c_2 := (c_1)^{\text{sk}}$ ; set  $k \leftarrow \text{hash}(c_2)$ ; output  $m := \text{AES-GCM}_k^{-1}(u)$ .

Clearly, the above hybrid encryption scheme HEnc is IND-CPA secure under the DDH assumption and the semantic security of AES-GCM encryption mode. In the actual implementation, hybrid encryption requires a trusted curve group parameter, and we adopt the well-known curve 25519.

### B. Commitment

A commitment scheme allows a sender to commit to a value, and later he can reveal the value by opening the commitment. A secure commitment scheme shall have two properties: hiding and binding. We denote a commitment as  $c = \text{Com}(m; r)$ , where  $m$  is the committed value(message) and  $r$  is the randomness. The hiding property and binding property are defined as follows:

- Hiding: for any PPT adversary  $\mathcal{A}$ ,  $\Pr[(m_0, m_1) \leftarrow \mathcal{A}; b \leftarrow \{0, 1\}; c \leftarrow \text{Com}(m_b) : \mathcal{A}(c) = b] \approx \frac{1}{2}$
- Binding: for any PPT adversary  $\mathcal{A}$ , the probability  $\Pr[(m_0, r_0, m_1, r_1) \leftarrow \mathcal{A} : m_0 \neq m_1, \text{Com}(m_0; r_0) = \text{Com}(m_1, r_1)] \approx 0$

A Petersen commitment [31] is defined as follows.  $c := \text{Com}_{\text{ck}}(m; r) := g^m h^r$ , where  $\text{ck} = h$  is the commitment key whose discrete logarithm is unknown to the committer. Pedersen commitment is additively homomorphic; namely,  $\text{Com}_{\text{ck}}(m_0; r_0) \cdot \text{Com}_{\text{ck}}(m_1; r_1) = \text{Com}_{\text{ck}}(m_0 + m_1; r_0 + r_1)$ .

In some use case, we don't need homomorphic property. We simple use the salted hash based commitment defined as  $c := \text{Com}(m; r) = \text{hash}(m, r)$ .

### C. Blockchain

Our system is built on top of a blockchain platform. We assume parties can use the underlying blockchain as a non-blocking broadcast channel. Namely, if party  $P_1$  posts a message  $m_1$  over the blockchain at the  $i$ -th round, any other parties will receive it no later than  $(i + \delta)$ -round, where  $\delta$  is a known bound. For recording wealth, the blockchain also serves as an account model ledger so that stake holders can lock some amount of tokens to apply for a committee member. Formally, we abstract the blockchain platform as a UC functionality  $\mathcal{G}_{\text{blockchain}}$  in Fig. 4 in Sec. III, below.

### D. Verifiable Random Functions and Cryptographic Sortition

A verifiable random function (VRF) [29] is a pseudorandom function that enables the key holder to prove the correctness of the output. A VRF scheme VRF consists of the following 3 PPT algorithms:

- $(\text{pk}, \text{sk}) \leftarrow \text{KeyGen}(\lambda)$ : The key generation algorithm produces a public key  $\text{pk}$  and a secret key  $\text{sk}$ .
- $(v, \pi) \leftarrow \text{Eval}_{\text{sk}}(x)$ : The evaluation algorithm takes secret key  $\text{sk}$  and pre-image  $x$ , and image  $v$  and a proof  $\pi$ .
- $0/1 \leftarrow \text{Verify}_{\text{pk}}(x, v, \pi)$ : The verifier algorithm takes input as the public key  $\text{pk}$ , the pre-image  $x$ , the image  $v$ , and the proof  $\pi$ , and it outputs 0 for rejection or 1 for acceptance.

Besides pseudo-randomness, a secure VRF scheme should also have the following properties.

- Completeness: for any  $x$ , given  $(v, \pi) \leftarrow \text{Eval}_{\text{sk}}(x)$ , we have  $\text{Verify}_{\text{pk}}(x, v, \pi) = 1$ .
- Uniqueness: no PPT adversary  $\mathcal{A}$  can output a public key  $\text{pk}$ , a pre-image  $x$ , and  $(v_1, \pi_1, v_2, \pi_2)$  such that  $v_1 \neq v_2$  and  $\text{Verify}_{\text{pk}}(x, v_1, \pi_1) = \text{Verify}_{\text{pk}}(x, v_2, \pi_2) = 1$ .

A formal definition can be found in [29].

It is well-known that cryptographic sortition [13] can be realized by VRF schemes. Over the blockchain, suppose  $(\text{pk}_i, \text{sk}_i)$  are associated with party  $P_i$ . For a public input  $x$ , each party  $P_i$  can compute  $(v_i, \pi_i) \leftarrow \text{VRF.Eval}_{\text{sk}_i}(x)$ . Then,  $v_i$  can be used to select the committee with public verification. In our system, we would like to further hide the committee member's identities by proposing a new notion called Ring VRF (cf. Sec. IV-A, below).

### E. Trusted Execution Environment

Trusted execution environment (TEE) is designed to guarantee confidentiality and integrity of computations. It is an isolated part that can store sensitive data and can issue attestation to prove correctness of computation. In practice, Intel SGX and ARM Trustzone are popular candidates of TEE. Although a few side-channel attacks, e.g. [40], [27], [37], have been explored against those TEE candidates, new designs and fixes are proposed on a monthly basis [7], [34], [28], [30], [36]. Hence, we take TEE as an acceptable hardware guard to ensure privacy of secret key components. In this work, our benchmarks are executed on the Intel SGX platform for its readily deployed remote attestation infrastructure; however, we emphasize that our protocol can also be implemented on any other TEE platforms.

### III. SYSTEM OVERVIEW AND SECURITY MODEL

#### A. System Architecture

In this section, we will give an overview of Bool Network platform with the example of cross-chain transfer between Bitcoin (BTC) and Ethereum (ETH). As depicted in Fig. 2,  $(C_1, C_2)$  denotes the cross-chain channel creation procedures;  $(A_1, A_2, A_3)$  denotes the lock-in procedures;  $(B_1, B_2, B_3)$  denotes the lock-out procedures.

Initially, a user creates an Bool Network account on the Bitcoin blockchain through the Bool Network committee (cf.  $C_1$ ) and a smart contract anchored to the Bool Network account on Ethereum (cf.  $C_2$ ); subsequently, the Bool Network committee members will sign a test message to check the channel's availability.

Each committee member runs light nodes of BTC and ETH to verify transactions. When the user wants to lock-in 1 BTC, he makes a transfer of 1 BTC to the address that was created in step  $C_1$  (cf.  $A_1$ ). After confirming the transaction, the Bool Network committee will use threshold signature to sign a transaction that creates 1 wBTC on Ethereum (cf.  $A_2$ ). Finally, the signed transaction will be submitted to Ethereum (cf.  $A_3$ ). The lock-out process is similar: first, the user destroys 1 wBTC on the Ethereum Anchor contract and specify the beneficiary address (cf.  $B_1$ ). Then, the committee will monitor the event and generate the corresponding Bitcoin transaction (cf.  $B_2$ ). Lastly, the transaction will be broadcast on Bitcoin network (cf.  $B_3$ ).

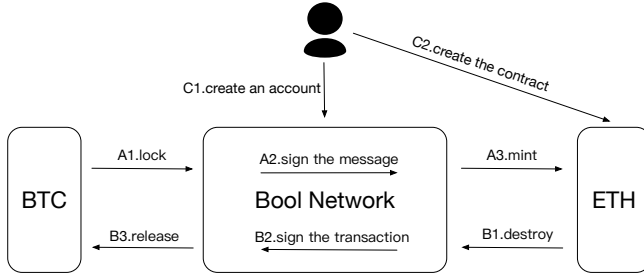


Fig. 2: Bool Network overview

Bool Network supports evolving hidden committee. We assume the underlying blockchain has a weak beacon oracle that can periodically produce an unpredictable but presumably biased random string. For instance, the hash digest of latest blocks. Define an epoch as a pre-defined number of rounds. At the beginning of each epoch, a new committee will be selected by cryptographic sortition using Ring VRF. The old committee will then handover the secret keys to the new committee in a secure and verifiable way. Fig. 3 shows an overview of the evolving hidden committee. More specifically, it consists of the following phases.

1. Registration. Before an epoch starts, all Bool Network stakeholders can register for becoming an committee member. The registration phase works as follows. A stakeholder locks a certain amount of stake together with a public key. Later, the associated public key will be used for cryptographic sortition. For the sake of fairness and animosity, our system utilizes the flat mode, i.e., there is a fixed amount

of stake to be locked per registration. If a stakeholder holds more stake, he/she can register several times to get a larger winning probability.

2. Sortition. For committee sortition, everyone first generates the weak beacon string  $x$  over Bool Network blockchain, which can be the hash of the previous blocks. Then, each stakeholder takes  $x$  as input and computes  $\text{PRF}_{\text{sk}_i}(x)$ . If  $\text{PRF}_{\text{sk}_i}(x) = v < T$ , then  $P_i$  randomly selects a ring  $(\text{pk}_1, \dots, \text{pk}_\ell)$  and posts message  $(\text{pk}_1, \dots, \text{pk}_\ell, v, \pi, \text{epk})$  over the blockchain, where  $\pi$  is the Ring VRF proof,  $v$  is the PRF output,  $\text{epk}$  is a freshly generated ephemeral public key. Winning nodes are elected for next-epoch committee. From then on, they can be communicated with, using the ephemeral public key  $\text{epk}$ . As we can see, the ephemeral public key reveals nothing about the committee member's identity, and the real identity is hidden within the ring  $(\text{pk}_1, \dots, \text{pk}_\ell)$ .
3. Handover. When a new committee is elected, the shared secret keys need to be passed from the previous-epoch committee to the new committee while keeping the public keys unchanged. Our protocol is maliciously secure with identifiable aborts. See sec. IV-E for details.
4. Threshold Signature. All transactions in our Bool Network shall be jointly signed by the committee using a threshold signature scheme. In particular, the threshold signature scheme for ECDSA [23] is adopted from the GG20 scheme by Gennaro and Godfeder [20] where the malicious party can be identified in a protocol abortion.

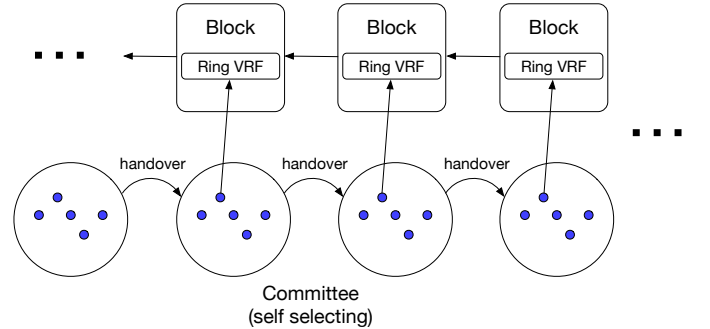


Fig. 3: Evolving committee overview

#### B. Key Management

As we mentioned above, all the key-related programs are deployed on a trusted execution environment (TEE). Specifically, on the initialization stage, a committee member loads the program  $\Pi_{\text{DKG}}$ ,  $\Pi_{\text{handover}}$  and  $\Pi_{\text{sign}}$  into his/her TEE and generates an remote attestation  $\sigma$  which proves that the initialization process is correct. He/she then sends  $\sigma$  to the attestation service and gets the attestation verification report  $\pi$ . Finally, the attestation verification report  $\pi$  is put on the blockchain to be publicly verifiable. Note that merely using cryptographic tools cannot guarantee a malicious committee would faithfully delete his/her secret shares upon request. The deployment of TEE prevents the secret shares from being extracted and enforces key share erasure after the handover protocol, providing better confidentiality of secret keys.

### C. Universal Composability

We will do security analysis under the Universally Composable (UC) [11], [12] model. In the UC framework, a protocol is represented by a set of interactive Turing machines (ITMs), and each ITM represents the program to be run by a participant. In the following, we assume that all ITMs are probabilistic polynomial time (PPT).

The security proof is based on the indistinguishability between the real/hybrid world and the ideal world. Let  $\text{EXEC}_{\Pi, \mathcal{A}, \mathcal{Z}}$  be the execution of protocol  $\Pi$  in the real world with adversary  $\mathcal{A}$  and environment  $\mathcal{Z}$ ; let  $\text{EXEC}_{\mathcal{F}, \mathcal{S}, \mathcal{Z}}$  be the execution in the ideal world interacting with ideal functionality  $\mathcal{F}$ , simulator (ideal adversary)  $\mathcal{S}$  and environment  $\mathcal{Z}$ . We say that  $\Pi$  UC-realizes  $\mathcal{F}$  if for any PPT adversary  $\mathcal{A}$ , there exists a simulator  $\mathcal{S}$  such that no PPT environment  $\mathcal{Z}$  can distinguish between  $\text{EXEC}_{\Pi, \mathcal{A}, \mathcal{Z}}$  and  $\text{EXEC}_{\mathcal{F}, \mathcal{S}, \mathcal{Z}}$ .

### D. UC Ideal Functionalities

Our system utilizes the following UC functionalities as building blocks.

**Blockchain.** As shown in Fig. 4, the functionality  $\mathcal{G}_{\text{blockchain}}$  consists of three interfaces: READ, WRITE and CHECK. Both broadcasting messages and modification of the balance are through the WRITE interface. There is a predicate Validate that ensures the validity of the blockchain content.

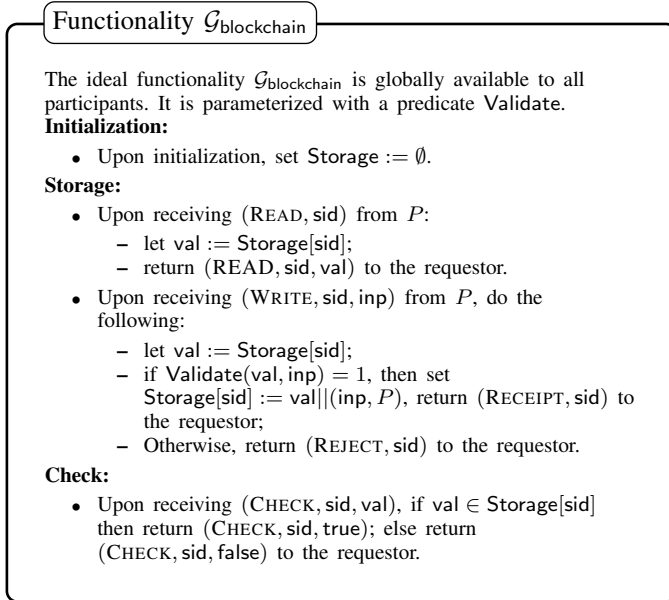


Fig. 4: The ideal blockchain functionality  $\mathcal{G}_{\text{blockchain}}$

**Distributed Key Generation.** We propose a UC secure distributed key generation (DKG) protocol with identifiable aborts in Sec. 10. The DKG functionality is captured by  $\mathcal{F}_{\text{DKG}}[\mathbb{G}]$ . As shown in Fig. 5, the functionality chooses a random global secret  $\text{gsk}$  and shares it to the participants using Shamir's secret sharing. The adversary  $\mathcal{S}$  is able to determine corrupted parties' secret shares, as, in the rushing adversary model, the corrupted parties can send messages after seeing the honest parties' messages in the real protocol.

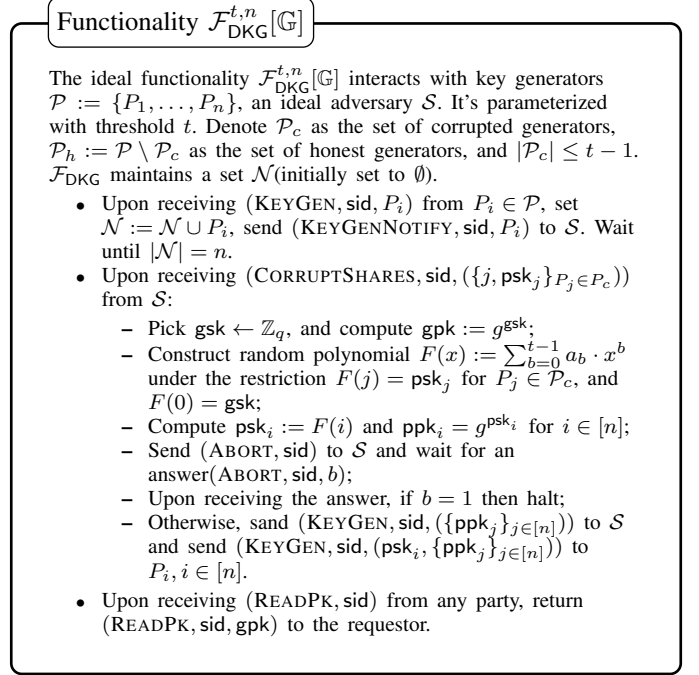


Fig. 5: DKG ideal functionality  $\mathcal{F}_{\text{DKG}}^{t, n}[\mathbb{G}]$

**Anonymous Cryptographic Sortition.** The anonymous sortition functionality  $\mathcal{F}_{\text{Anon-sortition}}^p$  is an abstraction that captures our Ring VRF-based sortition protocol, as depicted in Fig. 6. Unlike the conventional cryptographic sortition, when an elected committee member reveal his/her winning the sortition, he/she does not publish his/her identity. Instead, he/she can publish a message  $m_i$  through the REVEAL interface, which is an ephemeral public key in our scheme.

**Handover.** We design a UC secure handover protocol that allows the old committee to pass the shared secret keys to the new committee. Its functionality is captured by  $\mathcal{F}_{\text{handover}}[\mathbb{G}]$  as depicted in Fig. 7, the adversary can determine corrupted parties' secret shares.

## IV. EVOLVING COMMITTEE

This section will describe the evolving committee secret sharing in detail, as well as Ring VRF  $\Pi_{\text{Ring-VRF}}$  and zk proof for decryption correctness  $\Pi_{\text{Dec}}$  as building blocks.

### A. Ring VRF

In this section, we introduce a new notion, called *Ring Verifiable Random Function* (Ring-VRF). Given a set of public keys  $\{\text{pk}_0, \dots, \text{pk}_{N-1}\}$  and an input  $x \in \{0, 1\}^*$ , R-VRF allows the user to invoke its private key  $\text{sk}$  to generate  $v \leftarrow \text{PRF}_{\text{sk}}(x)$ , and convince the public that

$$\mathcal{R}_{\text{VRF}} = \left\{ \left( (\text{pk}_0, \dots, \text{pk}_{N-1}, x, v), \text{sk}, \ell \right) \mid \begin{array}{l} \ell \in \{0, \dots, N-1\} \wedge (\text{pk}_\ell, \text{sk}) \in \mathcal{R}_{\text{pk}} \wedge \\ v \leftarrow \text{PRF}_{\text{sk}}(x) \end{array} \right\}$$

where  $\mathcal{R}_{\text{pk}} = \{(\text{pk}, \text{sk}) \mid \text{pk} = g^{\text{sk}}\}$ .

We give a formal definition of Ring VRF as follows. It consists of 3 PPT algorithms.

### Functionality $\mathcal{F}_{\text{Anon-sortition}}^p$

The functionality  $\mathcal{F}_{\text{Anon-sortition}}^p$  interacts with a set of parties  $\mathcal{P} := \{P_1, \dots, P_n\}$  and adversary  $\mathcal{S}$ . It is parameterized with a variable  $p \in [0, 1]$ , and tables Pub and  $T$ . Let  $\mathcal{P}_c$  be the set of corrupted parties,  $\mathcal{P}_h := \mathcal{P} \setminus \mathcal{P}_c$  be the honest parties. Initially, set Pub :=  $\emptyset$ ,  $T := \emptyset$ .

#### Initialization:

- Upon receiving (INIT, sid) from the adversary  $\mathcal{S}$ , for each party  $P_i$ , pick a random bit  $b_{P_i}^{\text{sid}}$  with  $\Pr[b_{P_i}^{\text{sid}}] = p$  and set  $T[\text{sid}] := \{\langle P_i, b_{P_i}^{\text{sid}} \rangle\}_{P_i \in \mathcal{P}_c}$ , and then send (INIT, sid,  $T[\text{sid}]$ ) to the adversary  $\mathcal{S}$ .
- Upon receiving (REFRESH, sid) from  $\mathcal{S}$ , last step will be repeated.

#### Query:

- Upon receiving (QUERY, sid) from  $P_i \in \mathcal{P}$ , if  $b_{P_i}^{\text{sid}}$  is initialized, send (QUERY, sid,  $b_{P_i}^{\text{sid}}$ ) to the requestor.

#### Reveal:

- Upon receiving (REVEAL, sid,  $m_i$ ) from  $P_i \in \mathcal{P}$ :
  - If  $b_{P_i}^{\text{sid}} = 1$ , then set Pub[sid] := Pub[sid]  $\cup$   $\{m_i\}$ . Send (REVEAL, sid,  $m_i$ ) to the adversary  $\mathcal{S}$ .
  - Otherwise, ignore the request.
- Upon receiving (STATE, sid) from any party  $P_i \in \mathcal{P}$ , send (STATE, sid, Pub[sid]) to the requestor.

Fig. 6: Anonymous cryptographic sortition ideal functionality

$\mathcal{F}_{\text{Anon-sortition}}^p$

### Functionality $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$

$\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  interacts with previous-epoch committee members  $\mathcal{P} := \{P_1, \dots, P_{c_1}\}$ , next-epoch committee members  $\mathcal{Q} := \{Q_1, \dots, Q_{c_2}\}$  and adversary  $\mathcal{S}$ . It is parameterized with threshold  $t_1, t_2$ , previous-epoch committee size  $c_1$ , next-epoch committee size  $c_2$ . Denote  $\mathcal{P}_c$  as the set of previous-epoch corrupted parties,  $\mathcal{P}_h := \mathcal{P} \setminus \mathcal{P}_c$  as the set of previous-epoch honest parties, and  $|\mathcal{P}_c| \leq t_1 - 1$ . Denote  $\mathcal{Q}_c$  as the set of next-epoch corrupted parties,  $\mathcal{Q}_h := \mathcal{Q} \setminus \mathcal{Q}_c$  as the set of next-epoch honest parties, and  $|\mathcal{Q}_c| \leq t_2 - 1$ . It maintains sets  $\mathcal{M}, \mathcal{N}$  (initially set to  $\emptyset$ ).

- Upon receiving (HANDOVER, sid,  $\overline{\text{psk}}_i, \{\text{ppk}_j^{(i)}\}_{j \in [c_1]})$  from  $P_i$ :
  - Set  $\mathcal{N} := \mathcal{N} \cup i$ , send (HANDOVERNOTIFY, sid,  $P_i$ ) to  $\mathcal{S}$ . Wait until  $|\mathcal{N}| = c_1$ ;
  - Set  $\{\text{ppk}_j\}_{j \in [c_1]}$  as the majority of  $\{\{\text{ppk}_j^{(i)}\}_{j \in [c_1]}\}_{i \in [c_1]}$ ;
  - Set  $\mathcal{M}$  as the first  $t_1$  parties that satisfy  $\text{ppk}_j = g^{\overline{\text{psk}}_j}$ , set  $\text{psk}_j := \overline{\text{psk}}_j$ .
- Upon receiving (CORRUPTSHARES, sid,  $(\{j, \text{npsk}_j\}_{Q_j \in \mathcal{Q}_c})$ ) from  $\mathcal{S}$ :
  - Compute  $\text{gsk} = \prod_{j \in \mathcal{M}} \lambda_j \cdot \text{psk}_j$ , where  $\{\lambda_j\}_{j \in \mathcal{M}}$  are Lagrange coefficients, i.e.,  $\lambda_j := \prod_{\ell \in \mathcal{M} \setminus \{j\}} \frac{\ell - j}{\ell - j}$ ;
  - Construct random polynomial  $F(x) := \sum_{b=0}^{t_2-1} a_b \cdot x^b$  under the restriction  $F(j) = \text{npsk}_j$  for  $Q_j \in \mathcal{Q}_c$ , and  $F(0) = \text{gsk}$ ;
  - Compute  $\text{npsk}_i := F(i)$  and  $\text{nppk}_i = g^{\text{npsk}_i}$  for  $i \in [c_2]$ ;
  - Send (ABORT, sid) to  $\mathcal{S}$  and wait for an answer (ABORT, sid,  $b$ );
  - Upon receiving the answer, if  $b = 1$  then halt;
  - Otherwise, send (HANDOVER, sid,  $(\{\text{nppk}_j\}_{j \in [c_2]})$ ) to  $\mathcal{S}$  and (HANDOVER, sid,  $(\text{npsk}_i, \{\text{nppk}_j\}_{j \in [c_2]})$ ) to  $Q_i, i \in [c_2]$ .

Fig. 7: Handover ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$

- $(\text{pk}, \text{sk}) \leftarrow \text{KeyGen}(\lambda)$ : The key generation algorithm produces a public key  $\text{pk}$  and a secret key  $\text{sk}$ .
- $(v, \pi) \leftarrow \text{Eval}(\text{sk}, x, \{\text{pk}_0, \dots, \text{pk}_{N-1}\})$ : The evaluation algorithm takes as input secret key  $\text{sk}$ ,  $x$  and a set of public keys  $\{\text{pk}_0, \dots, \text{pk}_{N-1}\}$ . It outputs  $v$  and a proof  $\pi$  for  $\mathcal{R}_{\text{VRF}}$ .
- $0/1 \leftarrow \text{Verify}(x, v, \pi)$ : The verifier algorithm takes as input  $(x, v, \pi)$  and outputs 0 or 1.

1) *PRF*: Let  $\mathbb{G}$  be a cyclic group with prime order  $q$ , and denote  $g$  as the generator.  $H : \{0, 1\}^* \mapsto \mathbb{G}$  is a hash function that maps an arbitrary length string to a group element. In this work, we adopt PRF  $\text{PRF} : \{0, 1\}^* \times \mathbb{Z}_q \mapsto \mathbb{G}$  as follows:  $\text{PRF}_{\text{sk}}(x) := H(x)^{\text{sk}}$ . Note that, in the actual implementation,  $H$  can be implemented by the ‘‘Elligator 2’’ mapping proposed in [6].

2) *Our construction*: We first describe the intuition of the protocol. In the first step of our protocol, the prover create a new commitment of  $\text{sk}$ , denoted as  $c = \text{Com}_{\text{ck}}(\text{sk}; t)$ . Then, the protocol can be viewed as two parts run in parallel. One part is inspired by one-out-of-many protocol [21] that ensures  $\text{pk}_\ell$  is in the ring. The other part ensures that  $c$  and  $v$  corresponds to the same  $\text{sk}$ . Combining the two parts together we get the protocol for Ring VRF. In practice, Pedersen commitment can be used as the additive homomorphic commitment scheme, i.e.,  $\text{Com}_{\text{ck}}(a; r) = g^a h^r$ .

In this paragraph we specify the polynomial  $p_i(e)$  used in the protocol. Following the idea of [21], we write  $i = i_1 \dots i_n$  and  $\ell = \ell_1 \dots \ell_n$  in binary, and we let  $\delta_{ij}$  be Kronecker’s delta, i.e.,  $\delta_{\ell\ell} = 1$  and  $\delta_{i\ell} = 0$  for  $i \neq \ell$ . We let  $f_j = \ell_j e + a_j$ , let  $f_{j,1} = f_j = \ell_j e + a_j = \delta_{1\ell_j} e + a_j$  and  $f_{j,0} = e - f_j = (1 - \ell_j)e - a_j = \delta_{0\ell_j} e - a_j$ . Then,  $p_i(e) = \prod_{j=1}^n f_{j, i_j}$  has the form:

$$p_i(e) = \prod_{j=1}^n (\delta_{i_j \ell_j} e) + \prod_{k=0}^{n-1} p_{i,k} e^k = \delta_{ij} e^n + \prod_{k=0}^{n-1} p_{i,k} e^k \quad (1)$$

Finally, Fig. 8 shows the Sigma protocol for the relation  $\mathcal{R}_{\text{VRF}}$ . By Fiat-Shamir heuristic [16] we can transform it into a non-interactive zero knowledge proof. Moreover, we add a tag in the hash input, which is the ephemeral public key in our application scenario.

**Theorem 1.** *Let Com be an additive homomorphic non-interactive commitment scheme that is perfectly hiding and computationally binding with adversarial advantage  $\varepsilon$ . Protocol  $\Pi_{\text{zk-VRF}}$  as described in Fig. 8 for relation  $\mathcal{R}_{\text{VRF}}$  is a three-move public coin zero-knowledge protocol with perfect completeness,  $(n+1)$ -special soundness with adversarial advantage  $\varepsilon$  and perfect special honest verifier zero-knowledge.*

*Proof.* To prove that protocol  $\Pi_{\text{Ring-VRF}}$  is perfect complete observe that  $\prod_{j=1}^n f_{j, i_j}$  is a polynomial in the challenge  $e$  of the form  $p_i(e) = \delta_{ij} e^n + \prod_{k=0}^{n-1} p_{i,k} e^k$ . When  $\text{pk}_\ell = g^{\text{sk}}$ ,  $c_\ell$  is a commitment to 0 we therefore get that  $\prod_{j=1}^n f_{j, \ell_j}$  in the verification equation is a commitment to 0. Since  $c_{d_k} = \prod_i c_i^{p_{i,k}} \text{Com}_{\text{ck}}(0; \rho_k)$ , by the additive homomorphic property of Com, the terms involving  $p_{i,k}$  are all cancelled. Hence, the verification equation  $\prod_i c_i^{\prod_{j=1}^n f_{j, i_j}} \cdot \prod_{k=0}^{n-1} c_{d_k}^{-e^k} =$

Sigma protocol for Ring VRF  $\Pi_{\text{Ring-VRF}}$ 

- CRS:** the commitment key  $\text{ck} = h$ ;  
**Statement:**  $\text{pk}_0, \dots, \text{pk}_{N-1}, x, v$ ;  
**Witness:**  $\text{sk}, \ell$  such that  $\ell \in \{0, \dots, N-1\} \wedge \text{pk}_\ell = g^{\text{sk}} \wedge v = H(x)^{\text{sk}}$ .
- Prover:**
- $t \leftarrow \mathbb{Z}_q, c = g^{\text{sk}} h^t$ ;
  - For  $i = 0, \dots, N-1$ , compute  $c_i = \text{pk}_i / c$ ;
  - For  $j = 1, \dots, n$ 
    - $r_j, a_j, s_j, t_j, \rho_k \leftarrow \mathbb{Z}_q$ ;
    - $c_{\ell_j} = \text{Com}_{\text{ck}}(\ell_j; r_j)$ ;
    - $c_{a_j} = \text{Com}_{\text{ck}}(a_j; s_j)$ ;
    - $c_{b_j} = \text{Com}_{\text{ck}}(\ell_j a_j; t_j)$ ;
    - $c_{d_k} = \prod_i c_i^{p_i, k} \text{Com}_{\text{ck}}(0; \rho_k)$ , using  $k = j-1$  and  $p_{i,k}$  from Eq. 1;
  - $s', t' \leftarrow \mathbb{Z}_q, m_1 = g^{s'} h^{t'}, m_2 = u^{s'}$ , using  $u = H(x)$ ;
  - $P \rightarrow V$ :  
 $(c, c_{\ell_1}, c_{a_1}, c_{b_1}, c_{d_0}, \dots, c_{\ell_n}, c_{a_n}, c_{b_n}, c_{d_{n-1}}, m_1, m_2)$ .
- Verifier:**
- $e \leftarrow \mathbb{Z}_q$ ;
  - In the non-interactive version,  $e = \text{hash}(\text{tag}, \text{ck}, \text{pk}_0, \dots, \text{pk}_{N-1}, r, x, c, c_{\ell_1}, c_{a_1}, c_{b_1}, c_{d_0}, \dots, c_{\ell_n}, c_{a_n}, c_{b_n}, c_{d_{n-1}}, m_1, m_2)$ .
  - $V \rightarrow P$ :  $e$ .
- Prover:**
- For  $j = 1, \dots, n$ 
    - $f_j = \ell_j e + a_j$ ;
    - $z_{a_j} = r_j e + s_j$ ;
    - $z_{b_j} = r_j (e - f_j) + t_j$ ;
    - $z_d = (-t) e^n - \sum_{k=0}^{n-1} \rho_k e^k$ ;
  - $y_1 = s' + \text{sk} \cdot e, y_2 = t' + t e$ ;
  - $P \rightarrow V$ :  $(f_1, z_{a_1}, z_{b_1}, \dots, f_n, z_{a_n}, z_{b_n}, z_d, y_1, y_2)$ .
- Verifier:**
- For  $i = 0, \dots, N-1$ , compute  $c_i = \text{pk}_i / c$ ;
  - For all  $j \in \{1, \dots, n\}$ , check
    - $c_{\ell_j}^e c_{a_j} = \text{Com}_{\text{ck}}(f_j; z_{a_j})$ ;
    - $c_{\ell_j}^{e-f_j} c_{b_j} = \text{Com}_{\text{ck}}(0; z_{b_j})$ ;
  - check  $\prod_i c_i^{\prod_{j=1}^n f_{j,i_j}} \cdot \prod_{k=0}^{n-1} c_{d_k}^{-e^k} = \text{Com}_{\text{ck}}(0; z_d)$ , using  $f_{j,1} = f_j$  and  $f_{j,0} = e - f_j$ ;
  - check  $g^{y_1} h^{y_2} = m_1 c^e, u^{y_1} = m_2 v^e$ , using  $u = H(x)$ ;
  - Output 1 iff all the checks pass.

Fig. 8: Sigma protocol for Ring VRF  $\Pi_{\text{Ring-VRF}}$ 

$\text{Com}_{\text{ck}}(0; z_d)$  always holds. And  $g^{y_1} h^{y_2} = m_1 c^e, u^{y_1} = m_2 v^e$  also hold. Therefore the protocol is complete.

To prove that protocol  $\Pi_{\text{Ring-VRF}}$  is  $(n+1)$ -sound, we show that an adversary with probability  $\varepsilon$  of breaking  $(n+1)$ -soundness can be converted into an adversary that has probability  $\varepsilon$  of breaking the binding property of the commitment scheme.

Suppose the cheating prover creates  $n+1$  accepting responses  $f_1^{(0)}, \dots, y_2^{(0)}, \dots, y_2^{(n)}, \dots, z_d^{(n)}$  to  $n+1$  different challenges  $e^{(0)}, \dots, e^{(n)}$  on the same initial message  $c, c_{\ell_1}, \dots, m_2$ , we first show that  $\ell_j \in \{0, 1\}$ . Pick two responses  $f_j^{(0)}, z_{a_j}^{(0)}, z_{b_j}^{(0)}$  and  $f_j^{(1)}, z_{a_j}^{(1)}, z_{b_j}^{(1)}$  to challenges  $e^{(0)}, e^{(1)}$  on the commitments  $c_{a_j}, c_{b_j}$ . By combining the verification equations we get  $c_{\ell_j}^{e^{(0)}-e^{(1)}} = \text{Com}_{\text{ck}}(f_j^{(0)} - f_j^{(1)}; z_{a_j}^{(0)} - z_{a_j}^{(1)})$  and  $c_{\ell_j}^{e^{(0)}-f_j^{(0)}-e^{(1)}+f_j^{(1)}} = \text{Com}_{\text{ck}}(0; z_{b_j}^{(0)} - z_{b_j}^{(1)})$ .

$z_{b_j}^{(1)})$ . Defining  $\ell_j = \frac{f_j^{(0)}-f_j^{(1)}}{e^{(0)}-e^{(1)}}$  and  $\gamma_j = \frac{z_{a_j}^{(0)}-z_{a_j}^{(1)}}{e^{(0)}-e^{(1)}}$  we extract an opening of  $c_{\ell_j} = \text{Com}_{\text{ck}}(\ell_j; \gamma_j)$ . Furthermore, since  $c_{\ell_j}^{e^{(0)}-f_j^{(0)}-e^{(1)}+f_j^{(1)}} = c_{\ell_j}^{(1-\ell_j)(e^{(0)}-e^{(1)})} = \text{Com}_{\text{ck}}(\ell_j(1-\ell_j)(e^{(0)}-e^{(1)}); \gamma_j(1-\ell_j)(e^{(0)}-e^{(1)})) = \text{Com}_{\text{ck}}(0; z_{b_j}^{(0)} - z_{b_j}^{(1)})$ , we either get a breach of the binding property of the commitment scheme or we have  $\ell_j(1-\ell_j) = 0$ , which implies  $\ell_j \in \{0, 1\}$ .

Let  $a_j$  be the number committed in  $c_{a_j}$ , from the verification equation  $c_{\ell_j}^e c_{a_j} = \text{Com}_{\text{ck}}(f_j; z_{a_j})$  we get that  $f_j^{(0)} = \ell_j e^{(0)} + a_j, \dots, f_j^{(n)} = \ell_j e^{(n)} + a_j$  for all  $j = 1, \dots, n$  unless the adversary breaks the binding property of the commitment scheme, it must hold for all challenges. From  $f_{j,1} = f_j$  and  $f_{j,0} = e - f_j$  we get  $f_{j,1} = \ell_j e + a_j$  and  $f_{j,0} = (1-\ell_j)e - a_j$ . For  $i \neq \ell$  we therefore get that  $\prod_{j=1}^n f_{j,i_j}$  is a degree at most  $n-1$  polynomial  $p_i(e)$ , and for  $i = \ell$  it is a polynomial of the form  $p_\ell(e) = e^n + \dots$ . This means we can rewrite  $\prod_i c_i^{\prod_{j=1}^n f_{j,i_j}} \cdot \prod_{k=0}^{n-1} c_{d_k}^{-e^k} = \text{Com}_{\text{ck}}(0; z_d)$  as

$$c_\ell^{e^n} \cdot \prod_{k=0}^{n-1} c_{d_k}^{e^k} = \text{Com}_{\text{ck}}(0; z_d)$$

for some fixed  $c_{*0}, \dots, c_{*n-1}$ , which can be computed from commitments in the statement and the initial message.

Observe that the vectors  $(1, e^{(\beta)}, \dots, (e^{(\beta)})^n), \beta = 0, \dots, n$  can be viewed as rows in a Vandermonde matrix because  $e^{(0)}, \dots, e^{(n)}$  are all different. The matrix is invertible and we can therefore find a linear combination  $(\alpha_0, \dots, \alpha_n)$  of the rows that gives us the vector  $(0, \dots, 0, 1)$ . Combining the  $n+1$  accepting verification equations we therefore get

$$c_\ell = \prod_{\beta=0}^n (c_\ell^{e^{(\beta)}})^n \cdot \prod_{k=0}^{n-1} (c_{d_k}^{e^{(\beta)k}})^{\alpha_\beta} = \text{Com}_{\text{ck}}(0; \sum_{\beta=0}^n \alpha_\beta z_d^{(\beta)})$$

This gives us an extracted opening of  $c_\ell$  to 0. Since  $c_i = \text{pk}_i / c$ , denote  $\text{pk}_\ell = g^{\text{sk}_\ell}$  we have  $c = g^{\text{sk}_\ell} h^t$ .

As to the PRF part, to show  $c$  and  $v$  corresponds to the same  $\text{sk} = \text{sk}_\ell$ , we pick two responses  $y_1^{(0)}, y_2^{(0)}, y_1^{(1)}, y_2^{(1)}$  to the challenges  $e^{(0)}, e^{(1)}$ . We have  $g^{y_1^{(0)}} h^{y_2^{(0)}} = m_1 c^{e^{(0)}}$ ,  $g^{y_1^{(1)}} h^{y_2^{(1)}} = m_1 c^{e^{(1)}}$ . Divide the two equations we get  $c = g^{(y_1^{(0)}-y_1^{(1)})(e^{(0)}-e^{(1)})^{-1}} h^{(y_2^{(0)}-y_2^{(1)})(e^{(0)}-e^{(1)})^{-1}}$ . Similarly, we have  $u^{y_1^{(0)}} = m_2 v^{e^{(0)}}$ ,  $u^{y_1^{(1)}} = m_2 v^{e^{(1)}}$ . Divide the two equations we get  $v = u^{(y_1^{(0)}-y_1^{(1)})(e^{(0)}-e^{(1)})^{-1}}$ . Thus we extract  $\text{sk} = (y_1^{(0)} - y_1^{(1)})(e^{(0)} - e^{(1)})^{-1}$  and  $t = (y_2^{(0)} - y_2^{(1)})(e^{(0)} - e^{(1)})^{-1}$ .

To prove that protocol  $\Pi_{\text{Ring-VRF}}$  is perfect special honest verifier zero-knowledge, we build a special honest verifier zero-knowledge simulator that is given a challenge  $e \in \{0, 1\}^\lambda$ . First, it picks the elements of the response uniformly at random as  $f_1, \dots, y_2 \leftarrow \mathbb{Z}_q$ . It picks  $c = g^a h^r$  at random and computes  $c_i = \text{pk}_i / c$ . It picks  $c_{\ell_1}, \dots, c_{\ell_n}, c_{d_1}, \dots, c_{d_{n-1}} \leftarrow \text{Com}_{\text{ck}}(0)$  as random commitments to 0. Then, it computes  $c_{a_j} = c_{\ell_j}^{-x} \text{Com}_{\text{ck}}(f_j; z_{a_j})$ ,  $c_{b_j} = c_{\ell_j}^{f_j-x} \text{Com}_{\text{ck}}(0; z_{b_j})$ ,  $c_{d_0} = \prod_i c_i^{\prod_{j=1}^n f_{j,i_j}} \cdot \prod_{k=1}^{n-1} c_{d_k}^{-x^k} \cdot \text{Com}_{\text{ck}}(0; -z_d)$ ,  $m_1 = g^{y_1} h^{y_2} c^{-e}$ ,  $m_2 = u^{y_1} v^{-e}$ . Finally, it returns the simulated initial message and response  $(c, c_{\ell_1}, \dots, m_2, f_1, \dots, y_2)$ .

Now we argue that the simulated proof and the real proof are perfectly indistinguishable. First, in both real proofs and simulated proofs  $f_1, \dots, y_2$  are uniformly random in  $\mathbb{Z}_q$ . Furthermore, by the verification equations,  $c_{a_1}, c_{b_1}, \dots, c_{a_n}, c_{b_n}, c_{d_0}, m_1, m_2$  are determined by on  $f_1, \dots, y_2$  and  $c, c_{\ell_1}, \dots, c_{d_{n-1}}$  both in real and in simulated proofs. Since the commitment scheme Com is perfectly hiding, the adversary cannot distinguish between the real and simulated commitments  $c, c_{\ell_1}, \dots, c_{\ell_n}, c_{d_1}, \dots, c_{d_{n-1}}$ . Hence, the protocol is perfect zero-knowledge.  $\square$

### B. ZK Proof for Hybrid Decryption

In this section, we construct a Sigma protocol to show the correctness of decryption w.r.t. the hybrid encryption scheme, and then use Fiat-Shamir heuristic to transform it into a non-interactive version. The non-interactive zero knowledge protocol will be used later in our DKG protocol and handover protocol. To prove the correctness of decryption, actually, the prover only needs to provide  $c_2 = (c_1)^{\text{sk}}$ . The verifier can then computes  $k \leftarrow \text{hash}(c_2)$  and checks the decryption by  $m = \text{AES-GCM}_k^{-1}(u)$  (cf. II-A). The Sigma protocol is depicted in Fig. 9. Similar to Ring VRF, by Fiat-Shamir heuristic we can transform it into a non-interactive zero knowledge proof. We denote the NIZK protocol as  $\Pi_{\text{NIZKDec}}$ .

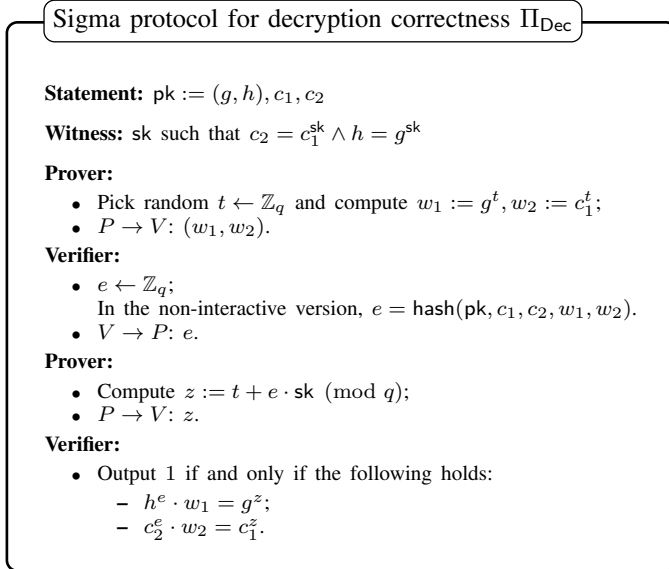


Fig. 9: Sigma protocol for decryption correctness  $\Pi_{\text{Dec}}$

**Theorem 2.** *The Sigma protocol in Fig. 9 for relation  $\mathcal{R}$  is perfectly complete, 2-special sound and perfectly special honest verifier zero-knowledge.*

*Proof.* To see that the Sigma protocol is perfectly complete observe that  $c_2 = c_1^{\text{sk}}$  and  $h = g^{\text{sk}}$ . Thus we have  $g^z = g^t \cdot g^{e \cdot \text{sk}} = g^t \cdot (g^{\text{sk}})^e = h^e \cdot w_1$  and  $c_1^z = c_1^t \cdot c_1^{e \cdot \text{sk}} = c_1^t \cdot (c_1^{\text{sk}})^e = c_2^e \cdot w_2$ .

To see that the Sigma protocol is 2-special sound, we assume the prover respond to two different challenges  $e, e'$  with  $z, z'$ . Divide the verification equation we get  $g^{z-z'} = h^{e-e'}$  and  $c_1^{z-z'} = c_2^{e-e'}$ . From  $g^{z-z'} = h^{e-e'}$  we extract

$\text{sk} = (z - z')(e - e')^{-1}$ . Combining it with  $c_1^{z-z'} = c_2^{e-e'}$  we can prove that  $c_2 = c_1^{\text{sk}}$ .

To see that the Sigma protocol is perfectly special honest verifier zero-knowledge we build a special honest verifier zero-knowledge simulator that is given a challenge  $e$ . It picks  $z \leftarrow \mathbb{Z}_p$ , then computes  $w_1 = g^z/h^e$  and  $w_2 = c_1^z/c_2^e$ . We can see that for a real statement, i.e.,  $c_2 = c_1^{\text{sk}}$ , we have  $w_2 = w_1^{\text{sk}}$  for both real proof and simulated proof, which means real proof and simulated proof have the same distribution.  $\square$

### C. Distributed Key Generation

For distributed key generation, we modify the distributed key generation protocol by Gennaro and Goldfeder [20] to achieve UC secure. We use a NIZK proof and split the broadcast step into two phase. In phase 1 parties broadcast the ciphertexts; In phase 2 parties broadcast the verification components. This allows the ideal world simulator to compute the verification components reversely to achieve indistinguishability. The protocol  $\Pi_{\text{DKG}}^{t,n}$  is depicted in Fig. 10.

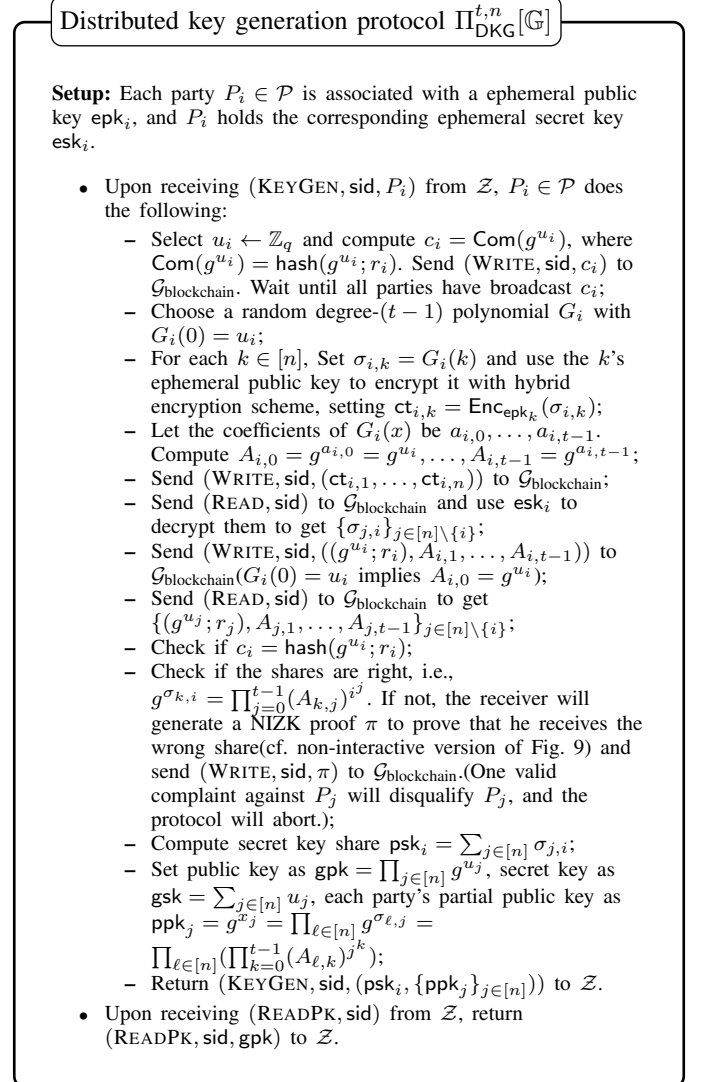


Fig. 10: Distributed key generation protocol  $\Pi_{\text{DKG}}^{t,n}[\mathbb{G}]$



### D. Anonymous Committee Sortition

In the committee sortition process, we hope to hide the committee members so as to avoid coercion or DoS attack. The committee sortition protocol works as follows. For protocol setup, each party owns his/her secret key  $sk$  and there is an agreed-upon random number  $x$  from the blockchain. At the beginning of the protocol, each party computes  $v = \text{PRF}_{sk}(x)$  and check if  $v$  is less than a certain threshold  $T$ , which represents his/her winning the sortition. The sortition winners will broadcast a Ring VRF proof  $\pi$  along with his/her ephemeral public key  $epk$ . Other members watch the blockchain and record the valid ephemeral public keys.

The ideal functionality  $\mathcal{F}_{\text{Anon-sortition}}^p$  can be realized by a Ring VRF-based sortition protocol in the  $\{\mathcal{F}_{\text{RO}}, \mathcal{G}_{\text{blockchain}}\}$ -hybrid model. The simulator  $\mathcal{S}$  performs as follows: Upon receiving  $(\text{INIT}, \text{sid}, T[\text{sid}])$  from  $\mathcal{F}_{\text{Anon-sortition}}^p$ ,  $\mathcal{S}$  simulates  $\mathcal{F}_{\text{RO}}$  so that  $v_i < T$  for  $i : b_{P_i}^{\text{sid}} = 1$ . Upon receiving  $(\text{REVEAL}, \text{sid}, m_i)$  from  $\mathcal{F}_{\text{Anon-sortition}}^p$ ,  $\mathcal{S}$  simulates a Ring VRF proof  $\pi$  and sends  $(\text{WRITE}, \text{sid}, (\pi, m_i))$  to  $\mathcal{G}_{\text{blockchain}}$ . Since  $\mathcal{F}_{\text{RO}}$  is simulated by  $\mathcal{S}$ , the real world and ideal world are indistinguishable.

### E. Handover

The handover protocol utilizes the Shamir secret sharing scheme to re-share the secret keys from the previous committee to the new committee. In particular, it uses Feldman's VSS to achieve verifiability and accountability. Similar to the DKG protocol, a NIZK proof for decryption correctness is used to identify the malicious behaviors and the broadcast step is split into two phase to ensure UC security. The detail of handover protocol is depicted in Fig. 11.

Note that, our handover protocol achieves identifiable abort. Namely, if a malicious committee member does not follow the protocol, the honest party will raise a complain against him/her and he/she will be disqualified. Then, the remaining parties will re-run the protocol so the the protocol will eventually terminate. In Bool Network, these disqualified committee members will be punished when the handover protocol finishes.

### F. Threshold Signature

For threshold signature, we support common signature schemes used in blockchain cryptocurrencies, including BLS [3], Schnorr [35] and ECDSA [23]. To generate a BLS signature, the committee only needs to reconstruct the secret in the exponent. A Schnorr signature can be generated by the additive homomorphism property of Shamir secret sharing. As to the most difficult ECDSA, we adopt the threshold signature protocol proposed by Gennaro and Goldfeder [20], which achieves one round threshold ECDSA with identifiable abort.

## V. SECURITY ANALYSIS

In this section, we will formally prove that our DKG protocol  $\Pi_{\text{DKG}}^{t,n}[\mathbb{G}]$  and handover protocol  $\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  UC-realize DKG ideal functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$  and handover ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$ , respectively.

### The handover protocol $\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$

**Setup:** Each next-epoch committee member  $Q_i \in \mathcal{Q}$  is associated with an ephemeral public key  $\text{epk}_i$ , and  $Q_i$  holds the corresponding ephemeral secret key  $\text{esk}_i$ . The protocol is parameterized with threshold  $t_1, t_2$ , previous-epoch committee size  $c_1$ , next-epoch committee size  $c_2$ . The shares held by  $C_1$  define a degree- $(t_1 - 1)$  polynomial  $F_1$  with  $F_1(0) = \sigma$ , where each seat  $j$  holds a share  $\text{psk}_j = F_1(j)$ . All parties agree on the non-zero evaluations points  $I = \{1, 2, \dots, c_1\}$  used for a  $t_1$ -of- $c_1$  Shamir secret-sharing scheme.

Upon receiving  $(\text{HANDOVER}, \text{sid}, \overline{\text{psk}}_i, \{\text{ppk}_j\}_{j \in [c_1]})$  from  $\mathcal{Z}$ :

- Previous-epoch committee member  $P_i \in \mathcal{P}$  does the following:
  - Send  $(\text{WRITE}, \text{sid}, \{\text{ppk}_j^{(i)}\}_{j \in [c_1]})$  to  $\mathcal{G}_{\text{blockchain}}$ ;
  - Send  $(\text{READ}, \text{sid})$  to  $\mathcal{G}_{\text{blockchain}}$  and set  $\{\text{ppk}_j\}_{j \in [c_1]}$  as the majority of  $\{\{\text{ppk}_j^{(k)}\}_{j \in [c_1]}\}_{k \in [c_1]}$ ;
  - Choose a random degree- $(t_2 - 1)$  polynomial  $G_i$  with  $G_i(0) = \text{psk}_j$ ;
  - For each  $k \in [c_2]$ , Set  $\sigma_{i,k} = G_i(k)$  and use the  $k$ 's ephemeral public key to encrypt it with hybrid encryption scheme, setting  $\text{ct}_{i,k} = \text{Enc}_{\text{epk}_k}(\sigma_{i,k})$ ;
  - Let the coefficients of  $G_i(x)$  be  $a_{i,0}, \dots, a_{i,t_2-1}$ . Compute  $A_{i,1} = g^{a_{i,1}}, \dots, A_{i,t_2-1} = g^{a_{i,t_2-1}}$  ( $A_{i,0} = \text{ppk}_i$ , which is publicly known, so there is no need to broadcast it);
  - Send  $(\text{WRITE}, \text{sid}, (\text{ct}_{i,1}, \dots, \text{ct}_{i,c_2}))$  to  $\mathcal{G}_{\text{blockchain}}$ . Wait until all parties have broadcast  $(\text{ct}_{j,1}, \dots, \text{ct}_{j,c_2})$ ;
  - Send  $(\text{WRITE}, \text{sid}, (A_{i,1}, \dots, A_{i,t_2-1}))$  to  $\mathcal{G}_{\text{blockchain}}$ .
- Next-epoch committee member  $Q_i \in \mathcal{Q}$  does the following:
  - Send  $(\text{READ}, \text{sid})$  to  $\mathcal{G}_{\text{blockchain}}$  and use  $\text{esk}$  to decrypt the ciphertexts to get  $\{\sigma_{j,i}\}_{j \in [c_1]}$ ;
  - Send  $(\text{READ}, \text{sid})$  to  $\mathcal{G}_{\text{blockchain}}$  to get  $\{A_{j,k}\}_{j \in [c_1]}$ ;
  - Check if the shares are right, i.e.,  $g^{\sigma_{j,k}} = \prod_{n=0}^{t_2-1} (A_{j,n})k^n$ , where  $A_{j,0} = \text{ppk}_j$ . If not, the receiver will broadcast a NIZK proof to prove that he receives the wrong share (cf. non-interactive version of Fig. 9); (One valid complain against  $P_j$  will disqualify  $P_j$  and the protocol will abort.)
  - Let the indices of the first  $t_1$  valid messages be set  $\mathcal{I}$ ,  $\mathcal{I} \subseteq [c_1]$  and  $|\mathcal{I}| = t_1$ . Compute the share of the global secret corresponding to seat  $i$  as  $\text{npsk}_i = \sum_{j \in \mathcal{I}} \lambda_j \cdot \sigma_{j,i}$ , where  $\lambda_j := \prod_{\ell \in \mathcal{I} \setminus \{j\}} \frac{\ell}{\ell - j}$ ;
  - Compute each new committee member's partial public key as  $\text{nppk}_k = \prod_{j \in \mathcal{I}} (g^{\sigma_{j,k}})^{\lambda_j} = \prod_{j \in \mathcal{I}} (\prod_{n=0}^{t_2-1} (A_{j,n})k^n)^{\lambda_j}$ ,  $k = 1, \dots, c_2$ , where  $\lambda_j := \prod_{\ell \in \mathcal{I} \setminus \{j\}} \frac{\ell}{\ell - j}$ ;
  - Return  $(\text{HANDOVER}, \text{sid}, (\text{npsk}_i, \{\text{nppk}_j\}_{j \in [c_2]}))$  to  $\mathcal{Z}$ .

Fig. 11: The handover protocol  $\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$

**Theorem 3.** *Let the hybrid encryption scheme  $\text{HEnc}$  be  $\text{IND-CPA}$  secure with adversary advantage  $\text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ . Let the NIZK protocol for decryption correctness  $\Pi_{\text{NIZK}_{\text{Dec}}}$  be statistically sound with soundness error  $\text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$  and perfect zero-knowledge. The protocol  $\Pi_{\text{DKG}}^{t,n}[\mathbb{G}]$  described in fig. 10 UC-realized  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$  against static corruption up to  $t - 1$  parties in the  $\{\mathcal{G}_{\text{blockchain}}, \mathcal{F}_{\text{RO}}\}$ -hybrid model with distinguishing advantage*

$$n^2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda) + n(t-1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda) .$$

*Proof.* We first construct a simulator  $\mathcal{S}$  such that any non-uniform PPT environment  $\mathcal{Z}$  cannot distinguish between the ideal world execution  $\text{EXEC}_{\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}], \mathcal{S}, \mathcal{Z}}$  and the real world execution  $\text{EXEC}_{\Pi_{\text{DKG}}^{t,n}[\mathbb{G}], \mathcal{A}, \mathcal{Z}}^{\mathcal{G}_{\text{blockchain}}, \mathcal{F}_{\text{RO}}}$ . In the ideal world, the parties interact with functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$  and the corrupted parties are controlled by  $\mathcal{S}$ ; in the real world, the parties  $\mathcal{P} = \{P_1, \dots, P_n\}$  run protocol  $\Pi_{\text{DKG}}^{t,n}[\mathbb{G}]$  in the  $\{\mathcal{G}_{\text{blockchain}}, \mathcal{F}_{\text{RO}}\}$ -hybrid world and the corrupted parties are controlled by a dummy adversary  $\mathcal{A}$  who simply forwards messages from/to  $\mathcal{Z}$ .

**Simulator.** The simulator  $\mathcal{S}$  internally runs  $\mathcal{A}$ , forwarding messages to/from the environment  $\mathcal{Z}$ . The simulator simulates the following interactions with  $\mathcal{A}$ :

- Upon receiving (KEYGENNOTIFY, sid,  $P_i$ ) from the ideal functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$  about an honest generator  $P_i$ , the simulator  $\mathcal{S}$  controls  $P_i$  to do the following:
  - Compute  $c_i = \text{Com}(g^0)$ , where  $\text{Com}(g^0) = \text{hash}(g^0; r_i)$ . Send (WRITE, sid,  $c_i$ ) to  $\mathcal{G}_{\text{blockchain}}$ . Wait until all parties have broadcast  $c_i$ .
  - For  $P_k \in \mathcal{P}_h$ , set  $\sigma_{i,k} = 0$ ; for  $P_k \in \mathcal{P}_c$ , select  $r_k \leftarrow \mathbb{Z}_q$  and set  $\sigma_{i,k} = r_k$ . Use the  $k$ 's ephemeral public key to encrypt it with hybrid encryption scheme, setting  $\text{ct}_{i,k} = \text{Enc}_{\text{epk}_k}(\sigma_{i,k})$ ;
  - Send (WRITE, sid,  $(\text{ct}_{i,1}, \dots, \text{ct}_{i,n})$ ) to  $\mathcal{G}_{\text{blockchain}}$ .
- Once the simulated  $\mathcal{G}_{\text{blockchain}}$  receives (WRITE, sid,  $(\text{ct}_{i,1}, \dots, \text{ct}_{i,n})$ ) from all corrupted parties, the simulator  $\mathcal{S}$  does the following:
  - Use honest parties' secret keys to decrypt the corresponding ciphertexts to get  $\sigma_{i,j}$ ,  $P_i \in \mathcal{P}_c$ ,  $P_j \in \mathcal{P}_h$ .
  - For all  $P_j \in \mathcal{P}_c$ , compute  $\text{psk}_j = \sum_{i \in [n]} \sigma_{i,j}$  and send (CORRUPTSHARES, sid,  $(\{j, \text{psk}_j\}_{P_j \in \mathcal{P}_c})$ ) to the ideal functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$ .
- Once the simulated  $\mathcal{G}_{\text{blockchain}}$  receives (WRITE, sid,  $(A_{j,1}, \dots, A_{j,t-1})$ ) from all corrupted parties, the simulator  $\mathcal{S}$  does the following:
  - If  $c_i = \text{hash}(g^{u_i}; r_i)$  and  $g^{\sigma_{i,k}} = \prod_{j=0}^{t-1} (A_{i,j})^{k^j}$  hold for all  $P_i \in \mathcal{P}_c$ , set  $b = 0$ , else set  $b = 1$ . Send (ABORT, sid,  $b$ ) to the ideal functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$ .
- Upon receiving (KEYGEN, sid,  $(\text{psk}_i, \{\text{ppk}_j\}_{j \in [n]})$ ) from the ideal functionality  $\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}]$ , the simulator  $\mathcal{S}$  does the following:
  - Compute  $\{A_{i,j}\}_{P_i \in \mathcal{P}_h, 0 \leq j \leq t-1}$  that satisfies  $g^{\sigma_{i,k}} = \prod_{j=0}^{t-1} (A_{i,j})^{k^j}$ ,  $P_i \in \mathcal{P}_h$ ,  $P_k \in \mathcal{P}_c$  and  $\text{ppk}_i = \prod_{j \in [n]} (\prod_{k=0}^{t-1} (A_{j,k})^{i^k})$ ,  $P_i \in \mathcal{P}_h$ .
  - For all honest parties  $P_i \in \mathcal{P}_h$ , send (WRITE, sid,  $((A_{i,0}, r_i), A_{i,1}, \dots, A_{i,t-1})$ ) to  $\mathcal{G}_{\text{blockchain}}$ , where simulated  $\mathcal{F}_{\text{RO}}$  opens  $c_i$  to  $(A_{i,0}, r_i)$ .
- Once an honest party receives (READPK, sid) from the environment  $\mathcal{Z}$ , compute  $\text{gpk} = \prod_{i \in [n]} \text{ppk}_i$ , return (READPK, sid,  $\text{gpk}$ ).

### Indistinguishability.

The indistinguishability is proven through a series of hybrid worlds  $\mathcal{H}_0, \dots, \mathcal{H}_3$ .

**Hybrid  $\mathcal{H}_0$ :** It is the real world execution  $\text{EXEC}_{\Pi_{\text{DKG}}^{t,n}[\mathbb{G}], \mathcal{A}, \mathcal{Z}}^{\mathcal{G}_{\text{blockchain}}, \mathcal{F}_{\text{RO}}}$ .

**Hybrid  $\mathcal{H}_1$ :**  $\mathcal{H}_1$  is the same as  $\mathcal{H}_0$  except that in  $\mathcal{H}_1$ , the messages  $(\text{ct}_{i,1}, \dots, \text{ct}_{i,n})$  sent by the honest user  $P_i$  are replaced by simulated encryptions of 0.

**Claim:** If the hybrid encryption scheme  $\text{HEnc}$  is IND-CPA secure with adversary advantage  $\text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ ,  $\mathcal{H}_0$  and  $\mathcal{H}_1$  are indistinguishable with distinguishing advantage at most  $n^2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ .

**Proof:** The indistinguishability under chosen plaintext attack means that the adversary cannot distinguish encryption of 0 and encryption of a number. There are at most  $n^2$  simulated ciphertexts, so the advantage in  $\mathcal{H}_1$  is no more than  $n^2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ .

**Hybrid  $\mathcal{H}_2$ :**  $\mathcal{H}_2$  is the same as  $\mathcal{H}_1$  except that in  $\mathcal{H}_2$ , when  $P_j \in \mathcal{P}_c$  produce a valid complaint against an honest party,  $\mathcal{S}$  will abort.

**Claim:** If the NIZK protocol is statistically sound with soundness error  $\text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ ,  $\mathcal{H}_1$  and  $\mathcal{H}_2$  are indistinguishable with distinguishing advantage at most  $n(t-1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .

**Proof:** By the soundness of the NIZK protocol the probability that  $\mathcal{S}$  will give up is negligible. There are at most  $n(t-1)$  statements, so the advantage in  $\mathcal{H}_1$  is no more than  $n(t-1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .

**Hybrid  $\mathcal{H}_3$ :**  $\mathcal{H}_3$  is the same as  $\mathcal{H}_2$  except that in  $\mathcal{H}_3$ ,  $c_i$  is opened with the simulated  $\mathcal{F}_{\text{RO}}$ .

**Claim:**  $\mathcal{H}_2$  and  $\mathcal{H}_3$  are perfectly indistinguishable.

**Proof:** As long as the encoding of  $\mathcal{F}_{\text{RO}}$  has not been exhausted, the simulated  $\mathcal{F}_{\text{RO}}$  is the same as real  $\mathcal{F}_{\text{RO}}$ .

The adversary's view of  $\mathcal{H}_3$  is identical to the simulated view  $\text{EXEC}_{\mathcal{F}_{\text{DKG}}^{t,n}[\mathbb{G}], \mathcal{S}, \mathcal{Z}}$ . Therefore, no PPT  $\mathcal{Z}$  can distinguish the view of the ideal execution from the view of real execution with advantage more than  $n^2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda) + n(t-1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .  $\square$

**Theorem 4.** Let the hybrid encryption scheme  $\text{HEnc}$  be IND-CPA secure with adversary advantage  $\text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ . Let the NIZK protocol for decryption correctness  $\Pi_{\text{NIZK}_{\text{Dec}}}$  be statistically sound with soundness error  $\text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$  and perfect zero-knowledge. The protocol  $\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  described in fig. 11 UC-realized  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  against static corruption up to  $t_1 - 1$  previous-epoch parties and up to  $t_2 - 1$  next-epoch parties in the  $\mathcal{G}_{\text{blockchain}}$ -hybrid model with distinguishing advantage

$$c_1 c_2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda) + c_1 (t_2 - 1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda) .$$

*Proof.* We first construct a simulator  $\mathcal{S}$  such that any non-uniform PPT environment  $\mathcal{Z}$  cannot distinguish between the ideal world execution  $\text{EXEC}_{\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}], \mathcal{S}, \mathcal{Z}}$  and the real world execution  $\text{EXEC}_{\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}], \mathcal{A}, \mathcal{Z}}^{\mathcal{G}_{\text{blockchain}}}$ . In the ideal world, the parties interact with functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  and the corrupted parties are controlled by  $\mathcal{S}$ ; in the real world, the parties  $\mathcal{P} = \{P_1, \dots, P_{c_1}\}$  and  $\mathcal{Q} = \{Q_1, \dots, Q_{c_2}\}$  run protocol  $\Pi_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  in the  $\mathcal{G}_{\text{blockchain}}$ -hybrid world and the corrupted parties are controlled by a dummy adversary  $\mathcal{A}$  who simply forwards messages from/to  $\mathcal{Z}$ .

**Simulator.** The simulator  $\mathcal{S}$  internally runs  $\mathcal{A}$ , forwarding messages to/from the environment  $\mathcal{Z}$ . The simulator simulates the following interactions with  $\mathcal{A}$ :

- Upon receiving (HANDOVERNOTIFY, sid,  $P_i$ ) from the ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$  about an honest generator  $P_i$ , the simulator  $\mathcal{S}$  controls  $P_i$  to do the following:
  - Send (WRITE, sid,  $\{\text{ppk}_j^{(i)}\}_{j \in [c_1]}$ ) to  $\mathcal{G}_{\text{blockchain}}$ ;
  - Send (READ, sid) to  $\mathcal{G}_{\text{blockchain}}$  and set  $\{\text{ppk}_j\}_{j \in [c_1]}$  as the majority of  $\{\{\text{ppk}_j^{(k)}\}_{j \in [c_1]}\}_{k \in [c_1]}$ ;
  - For  $P_k \in \mathcal{P}_h$ , set  $\sigma_{i,k} = 0$ ; for  $P_k \in \mathcal{P}_c$ , select  $r_k \leftarrow \mathbb{Z}_q$  and set  $\sigma_{i,k} = r_k$ . Use the  $k$ 's ephemeral public key to encrypt it with hybrid encryption scheme, setting  $\text{ct}_{i,k} = \text{Enc}_{\text{epk}_k}(\sigma_{i,k})$ ;
  - Send (WRITE, sid,  $(\text{ct}_{i,1}, \dots, \text{ct}_{i,c_2})$ ) to  $\mathcal{G}_{\text{blockchain}}$ .
- Once the simulated  $\mathcal{G}_{\text{blockchain}}$  receives (WRITE, sid,  $(\text{ct}_{i,1}, \dots, \text{ct}_{i,c_2})$ ) from all corrupted parties, the simulator  $\mathcal{S}$  does the following:
  - Use honest parties' secret keys to decrypt the corresponding ciphertexts to get  $\sigma_{i,j}$ ,  $P_i \in \mathcal{P}_c$ ,  $Q_j \in \mathcal{Q}_h$ .
  - Let the indices of the first  $t_1$  valid messages be set  $\mathcal{I}$ ,  $\mathcal{I} \subseteq [c_1]$  and  $|\mathcal{I}| = t_1$ . Compute corrupted parties' new partial secret key as  $\text{npsk}_i = \sum_{j \in \mathcal{I}} \lambda_j \cdot \sigma_{j,i}$ , where  $\lambda_j := \prod_{\ell \in \mathcal{I} \setminus \{j\}} \frac{\ell}{\ell - j}$ ,  $Q_i \in \mathcal{Q}_c$ ;
  - Send (CORRUPTSHARES, sid,  $(\{j, \text{npsk}_j\}_{Q_j \in \mathcal{Q}_c})$ ) to the ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$ .
- Once the simulated  $\mathcal{G}_{\text{blockchain}}$  receives (WRITE, sid,  $((g^{u_i}, r_i), A_{j,1}, \dots, A_{j,t-1})$ ) from all corrupted parties, the simulator  $\mathcal{S}$  does the following:
  - If  $g^{\sigma_{i,k}} = \prod_{n=0}^{t_2-1} (A_{i,n})^{k^n}$  holds for all  $P_i \in \mathcal{P}_c \cap \mathcal{I}$ , set  $b = 0$ , else set  $b = 1$ . Send (ABORT, sid,  $b$ ) to the ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$ .
- Upon receiving (HANDOVER, sid,  $\{\text{nppk}_j\}_{j \in [c_2]}$ ) from the ideal functionality  $\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$ , the simulator  $\mathcal{S}$  does the following:
  - Compute  $\{A_{i,j}\}_{P_i \in \mathcal{P}_h, 1 \leq j \leq t_2-1}$  satisfies  $g^{\sigma_{i,k}} = \prod_{j=0}^{t_2-1} (A_{i,j})^{k^j}$ ,  $P_i \in \mathcal{P}_h$ ,  $Q_k \in \mathcal{Q}_c$  and  $\text{nppk}_k = \prod_{j \in \mathcal{I}} (g^{\sigma_{j,k}})^{\lambda_j} = \prod_{j \in \mathcal{I}} (\prod_{n=0}^{t_2-1} (A_{j,n})^{k^n})^{\lambda_j}$ ,  $Q_k \in \mathcal{Q}_h$ .
  - For all honest parties  $P_i \in \mathcal{P}_h$ , send (WRITE, sid,  $(A_{j,1}, \dots, A_{j,t_2-1})$ ) to  $\mathcal{G}_{\text{blockchain}}$ .

### Indistinguishability.

The indistinguishability is proven through a series of hybrid worlds  $\mathcal{H}_0, \dots, \mathcal{H}_2$ .

**Hybrid  $\mathcal{H}_0$ :** It is the real world protocol execution  $\text{EXEC}_{\text{handover}}^{\mathcal{G}_{\text{blockchain}}, \mathcal{A}, \mathcal{Z}}^{t_1, t_2, c_1, c_2}[\mathbb{G}]$ .

**Hybrid  $\mathcal{H}_1$ :**  $\mathcal{H}_1$  is the same as  $\mathcal{H}_0$  except that in  $\mathcal{H}_1$ , the messages  $(\text{ct}_{j,1}, \dots, \text{ct}_{j,c_2})$  sent by the honest user  $P_i$  are replaced by simulated encryptions of 0.

**Claim:** If the hybrid encryption scheme  $\text{HEnc}$  is IND-CPA secure with adversary advantage  $\text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ ,  $\mathcal{H}_0$  and  $\mathcal{H}_1$  are indistinguishable with distinguishing advantage at most  $c_1 c_2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ .

**Proof:** The indistinguishability under chosen plaintext attack means that the adversary cannot distinguish encryption of 0 and encryption of a number. There are at most  $c_1 c_2$  simulated

ciphertexts, so the advantage in  $\mathcal{H}_1$  is no more than  $c_1 c_2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda)$ .

**Hybrid  $\mathcal{H}_2$ :**  $\mathcal{H}_2$  is the same as  $\mathcal{H}_1$  except that in  $\mathcal{H}_2$ , when  $P_j \in \mathcal{P}_c$  produce a valid complaint against an honest party,  $\mathcal{S}$  will abort.

**Claim:** If the NIZK protocol is statistically sound with soundness error  $\text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ ,  $\mathcal{H}_1$  and  $\mathcal{H}_2$  are indistinguishable with distinguishing advantage at most  $c_1(t_2 - 1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .

**Proof:** By the soundness of the NIZK protocol the probability that  $\mathcal{S}$  will give up is negligible. There are at most  $c_1(t_2 - 1)$  statements, so the advantage in  $\mathcal{H}_1$  is no more than  $c_1(t_2 - 1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .

The adversary's view of  $\mathcal{H}_2$  is identical to the simulated view  $\text{EXEC}_{\text{handover}}^{\mathcal{F}_{\text{handover}}^{t_1, t_2, c_1, c_2}[\mathbb{G}], \mathcal{S}, \mathcal{Z}}$ . Therefore, no PPT  $\mathcal{Z}$  can distinguish the view of the ideal execution from the view of real execution with advantage more than  $c_1 c_2 \cdot \text{Adv}_{\text{HEnc}}^{\text{IND-CPA}}(\mathcal{A}, \lambda) + c_1(t_2 - 1) \cdot \text{Adv}_{\text{NIZK}_{\text{Dec}}}^{\text{sound}}(\mathcal{A}, \lambda)$ .  $\square$

## VI. IMPLEMENTATION AND BENCHMARKS

1) *Test network:* We launched a local network of 30 full nodes with a (10,15)-committee to test Bool Network in the real environment. It worked for several days with a number of successful committee switches.

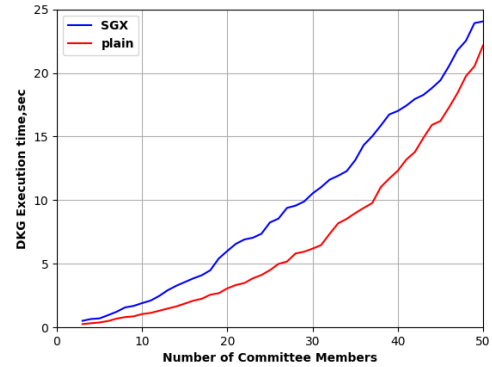


Fig. 12: DKG protocol execution time depending on the number of committee members

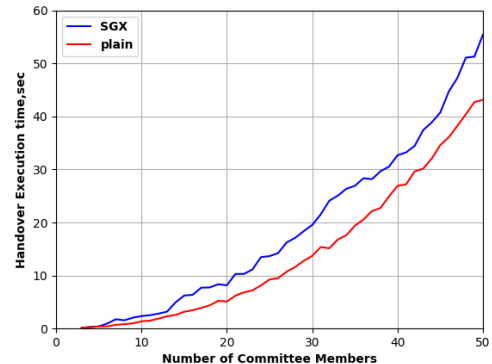


Fig. 13: Handover protocol execution time depending on the number of committee members

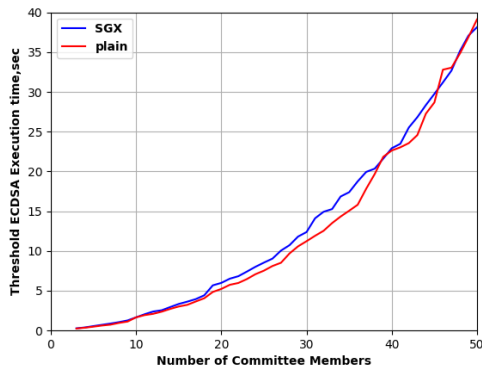


Fig. 14: Threshold signature protocol execution time depending on the number of committee members

2) *Evaluations*: To evaluate the performance of the cryptographic protocols, a special set of tests were conducted on a workstation equipped with Intel Xeon(Ice Lake) Platinum 8369B CPU @2.9 GHz and 64GB RAM running Ubuntu 20.04 LTS. We deploy 3 virtual key servers, which are responsible for executing the protocol, and 1 manage server, which is responsible for giving instructions, on the workstation. Each key server has 3 threads and we divide the task into pieces to simulate the real execution time.

We benchmarked the running time of distributed key generation protocol, handover protocol and threshold ECDSA w/o Intel SGX for different number of committee members: from 3 to 50. Results were given in Fig. 12, Fig. 13, Fig. 14, respectively.

For Ring VRF, we benchmarked the prover running time, verifier running time and proof with respect to different ring size: from 3 to 50. Results were given in Fig. 15.

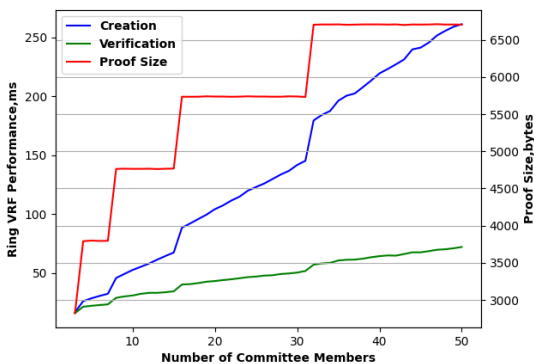


Fig. 15: The prover’s running time, verifier’s running time and the size of the Ring VRF proof

## VII. RELATED WORK

A cross-chain platform is aimed at transferring property or information between different blockchains. Existing cross-chain solutions can be classified into three categories: sidechains/relays, notaries, and hashed time-lock contracts. Sidechains/relays are layer one solutions that allow the main-chain to offload transactions to the sidechain, e.g., [19],

[32]. A notary (e.g., [8], [38], [1]) is responsible to trigger transactions upon monitoring an event on another chain, which usually has better compatibility compared with other solutions. Hashed time-lock contracts [33] ensure the atomicity of transactions by hashlocks and timelocks, but it has the problem of unfair trade due to fluctuation of exchange rate.

For notary schemes, to enhance the security of the secret key, distributed secret key management approach is a potential solution. Generally speaking, in distributed secret key management, there is a committee for managing the secret key and signing transactions, with threshold cryptosystem ensuring fault tolerance.

Fusion [17] is an existing project that uses distributed control right management (DCRM) to manage secret keys. However, in Fusion the committee who holds the secret key does not change, which results in the risk of compromised secret keys after a long period of time.

Benhamouda *et al.* proposed a notion called Evolving-Committee Proactive Secret Sharing [5], where the secret can be passed from the old committee to the new committee, bringing more security to the system. They design a nominating committee, which is self-elected, and a holding committee, which is nominated by the nominating committee and is identity-hidden, but the complicated two-committee scheme makes it theoretic and impractical.

In [18], Ganesh, Orlandi and Tschudi focus on the problem of how to build a private proof-of-stake blockchain. They formally define the ideal functionalities and design protocols to realize these functionalities. However, they use a Merkle tree based zk proof to hide the identities, which is very inefficient.

[14] builds a platform for smart contracts with TEE to address the problem of blockchains’ lacking of confidentiality and poor performance. In this work, we propose a cross-chain platform based on TEE to achieve secret key confidentiality and integrity.

## VIII. CONCLUSION

In this work, we present Bool Network blockchain platform for cross-chain service. In particular, we propose an efficient Ring-VRF scheme that enables evolving committee with hidden identities. Besides, all the key management process are executed in the TEE for confidentiality and integrity. Compared with existing schemes, our system is efficient and can be compatible with all off-the-shelf blockchains.

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